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ENFORCEMENT OF TAX LAWS: AN AUCTION THEORY APPROACH

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Enforcement of tax laws: an auction theory approach

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Abstract

Tax liabilities of taxpayers are their private information. From this evidence arises an adverse selection problem which tax agencies ought to deal with in the most efficient way. In this article it is considered a tax agency directing its audits to the lowest declared incomes. This strategy implicitly awards audit-free prizes to the higher declarers; a mechanism that can be analyzed with an auction theory approach. The model leads to comparative statics results close to the ones emerging with a cut-off rule but allows the tax administration to set with certainty the audit budget and permits to investigate the relation between audit budget and compliance.

Keywords: Audit; Tax Evasion; Incentives; Enforcement

JEL *classification*: **H2**; **H26**; **C72; K34**

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1 Introduction

Tax collection entails asymmetric information between tax administration and taxpayers. In fact, taxpayer's liability is private information of taxpayer. Developed countries tax-collection schemes are based on the self-declaration of liabilities made by taxpayers and, in order to induce truthful declarations, audits are performed and fines can be levied upon uncompliant taxpayers. The scientific literature has pointed out that a rise in the probability of audit and/or in the entity of fine deters tax evasion. Since the entity of the fine cannot assume infinite value for practical (liquidity constraint), efficiency (presence of errors in audits and in declaration) and equity issues (ex-post inequality), the audit probability, along with audit strategy, has a prominent role in ensuring compliance. If the tax administration can commit to a certain audit rule, the interaction between taxpayer and tax administration can be studied thanks to game theory. One of the basic model utilized to study enforcement considers a cut-off rule, its assumptions consists in: uniformly distributed income in the interval $[0, b]$, proportional tax rate on income, proportional fine on the amount under declared and a tax administration exclusively concerned with revenues maximization. Tax administration sets a threshold and commits itself to auditing every taxpayer submitting declaration lower than it. As a consequence, every taxpayer with an income under the threshold declares truthfully, because audited with certainty and every taxpayer with an income above the threshold declare exactly the threshold to prevent audits. Optimal threshold ought to be lower than t/s , a level that ensures full compliance, and has to equate the marginal revenue with marginal cost of raising the threshold. In the model considered the marginal revenues from shifting upwards the threshold is equal to t multiplied for the number of taxpayers whom income is above the threshold y^* while the marginal cost is represented by the cost of the audit c multiplied for the probability of audit t/s . Hence, the optimal threshold y^* increases in the maximal income, decreases in the cost of the audit and can be expressed as $y^* = b - c/s$. Above this point revenue from prosecution is lower than costs of auditing; thus prosecution should not be done. In this model the setting of the threshold is considered without accounting for the means necessary to ensure it. This assumption identifies a state-dictator capable of realizing its will whatever it is and imposes to assume the absence of a budget constraint for auditing policies. Hence tax administration is considered capable of auditing as many taxpayers it is necessary to insure his will. Clearly, this is not the case because tax administration has only limited resources to be devoted to enforcement. Since to produce credible threat material resource have to be utilized an analysis of the effects on compliance due to a rising in the number of audits is a compelling question. Furthermore, an inquiry on the relations between resources utilized and compliance induced allows to study the level of compliance that can be achieved given the tax administration budget and given the strategy of tax auditing. Another consequence of this approach is to allow the tax administration to define with certainty its audit budget while this can only be done in expected value when utilizing a cut off rule. Moreover, the tax agency overcomes also the problem of having reliable information about the taxpayers controlled. Thanks to statistical studies it is possible to know the distribution of incomes for different classes of taxpayer and thus estimate the number of audit that are going to be undertaken after the declaration phase under a cut off rule. Nonetheless, this process is costly and not necessarily accurate due to the elusive nature of the information under analysis. Conversely, a tax administration committing to audit a certain number of the lowest declaration, not only knows with certainty its costs, but also induces taxpayers to utilize their belief about the relative position occupied by their income while declaring. As a result, the declaration reflects the information on the distribution of income of the subjects directly involved, presumably more reliable and less costly than ones acquired with surveys. The article is organized as follows: In the second paragraph the model is presented along with the declaration function, in the third paragraph some comparative statics results are discussed while in the fourth paragraph the results are summarized and some consideration about the hypothesis introduced in the model are discussed.

2 The model

It is considered a model where tax agency performs a fixed number of controls k on the lowest declarations submitted by taxpayers. If a taxpayer is found to have under-declared his liabilities, he is forced to compliance at a proportional tax rate t on his real income and is levied a fine, defined as s, proportional to the amount under declared. Conversely, if taxpayer is not audited, he enjoys a reduction of his tax liability equal to the evaded tax debt. Assume a population T composed by n taxpayers $T = [1,2 \dots n]$ where every taxpayer is characterized by an identical and independent random variable $Y_{i \in T}$ defined on $[0, b]$ representing his pre-tax income. The realization of $Y_{i \in T}$ defines the type y_i of the i-th taxpayer determining his behaviour. The analysis is carried out referring to a generic type $w \in [0, b]$. As the distribution function of income is common knowledge between taxpayers it can be written:

$$F(w) = \Pr\,(Y_i < w)$$

And, assumed that the declaration $D(w)$ made by a type w taxpayer increases in taxpayer's type w , it is assumed that:

$$\Pr[D(w) \geq D(Y_i)] = \Pr(Y_i < w)\ , \qquad i \in C$$

Since tax agency audits the k lowest declaration ($k \in T$), audited declarations (the lowest ones) can be identified with $\{m_1, m_2, \dots m_k\}$

$$m_0 \equiv \emptyset$$
$$m_1 \equiv \big(D(y_h) \in NA_0 | D(y_h) < D(y_j)\ \forall\ D(y_j) \in NA_0\ ;\ \ h \neq j\big)$$
$$m_2 \equiv \big(D(y_h) \in NA_1 | D(y_h) < D(y_j)\ \forall\ D(y_j) \in NA_1\ ;\ h \neq j\big)$$
$$\dots$$
$$m_k \equiv \big(D(y_h) \in NA_{k-1} | D(y_h) < D(y_j)\ \forall\ D(y_j) \in NA_{k-1}\ ;\ h \neq j\big)$$

where NA_k is the set of non audited declarations if k audits are performed:

$$NA_0 = \{D(y_1), D(y_2) \dots D(y_n)\}$$
$$NA_1 = NA_0 \backslash \{m_1\}$$
$$NA_2 = NA_0 \backslash \{m_1, m_2\}$$
$$\dots$$
$$NA_k = NA_0 \backslash \{m_1, m_2, \dots, m_k\}$$

It is possible to define the probability of not being audited $P(w, k, n)$ as:

$$P(w, k, n) = 1 - \Pr\,(D(w) \leq m_k)$$

Hence, audit probability when tax agency performs only one audit is equal to the probability of submitting the lowest declaration. Recalling that the probability for a declaration to be higher than the income $Y_{i \in T}$ is identified with $F(w) = \Pr\ (D(w) > Y_{i \in T})$ and holding the independence of the realization of the income of taxpayers, taxpayer's probability to be audited when $k = 1$ is:

$$Pr(D(w) < m_1) = [1 - F(w)]^{n-1}[F(w)]^0$$

while if $k = 2$ the probability of being audited is equal to the probability of submitting the lowest declaration plus the probability of the $n - 1$ events where the second lowest declaration is submitted.

$$Pr(D(w) < m_2) = Pr(D(w) < m_1) + (n - 1)[1 - F(w)]^{n-2}[F(w)]^1]$$

Thus, the distribution function of the audit probability can be identified with:

$$Pr(D(w) < m_k) = \sum_{x=0}^{k-1}\binom{n}{x}[F(w)]^x[1 - F(w)]^{n-x-1} \quad (1)$$

It is now possible to define the expected after-tax income of generic taxpayer i :

$$E[Y_i{}^{at}] = [P(w,k,n)]\{y_i - tD(w)\} - [1 - P(w,k,n)]\{y_i - tD(w) - s(y_i - D(w))\} \quad (2)$$

Where

y_i , $i \in T$ is the income of taxpayer

$0.1 > t > 0.7$ is the tax rate

$0.75 > s > 1,5$ is the fine (surcharge) specified on the amount under declared.

The choice of the range of values for s and t ensures a non-trivial analysis.

The first part of (2) multiplies the probability of not being audited for payoff achievable through evasion in the state of the world where taxpayer is not subject to audit. Conversely, the second term multiplies the probability of audit for an after-tax disposable income where true liabilities are paid along with a fine on the amount under-declared. Similarly to the Allingham and Sandmo model (1972), evasion is considered as a gamble and its profitability and its riskiness increase in proportion with the under-declaration of tax liability. However, in the present model the amount under declared influences not only the fine but also the audit probability which depends on the realization of taxpayer's income and on the relative position of the income in the distribution of realized incomes.

Thus, taxpayer faces a trade-off between evasion and compliance that depends on the number of audit performed relative to the size of the population but also on the beliefs regarding the distribution of incomes and the behaviour of other taxpayers.
Maximizing (2) respective to w while considering only strategies where taxpayers truthfully declare the type them are behaving as $D(w) = w$ leads to:

$$\max_{w} E[Y^{at}]$$

$$\frac{\partial E[Y^{at}]}{\partial w} = \alpha D'(w) - P(y_i, k, n)D'(w) - P'_1(y_i, k, n)D(w) + P'_1(y_i, k, n)y_i = 0 \quad (3)$$

where:

$\alpha \equiv \frac{s-t}{s}$

$P'_1(y_i, k, n) \equiv \frac{dP(y_i,k,n)}{dy_i}$

$D'(w) \equiv \frac{dD(w)}{dw}$

The solution of this maximization problem leads to the optimal declaration function :

$$D^*(y_i) = \begin{cases} y_i & if\ y_i \leq y^\circ \\ \frac{\int_{y^\circ}^{y_i} P'_1(z,k,n)z\, dz}{P(y_i,k,n)-\alpha} & if\ y_i > y^\circ \end{cases} \quad (4)$$

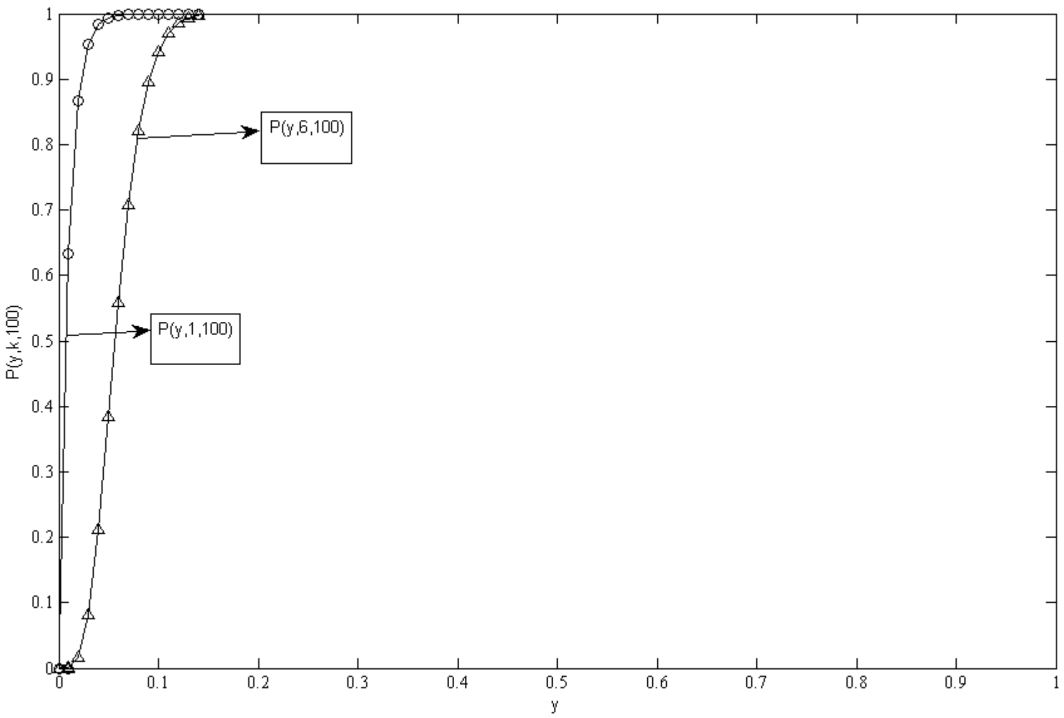
where

$y^\circ \equiv \{y \in [0, b] | P(y^\circ, k, n) = \alpha, \}$

Proof, see appendix [a].
Low income taxpayers: those characterized by $y \leq y^\circ$, truthfully declare their liabilities because are subject to a (relatively) high audit probability. In fact, the expected payoff from evasion is negative under y° due to the high value of the expected fine. Conversely, for incomes higher than y° the expected payoff from evasion is positive. As a consequence, taxpayers with a relatively high income under-declare a fraction of their real income.
To investigate the behaviour of $D^*(y_i)$ respective to y_i, consider the differential (Proof, see appendix [b]):

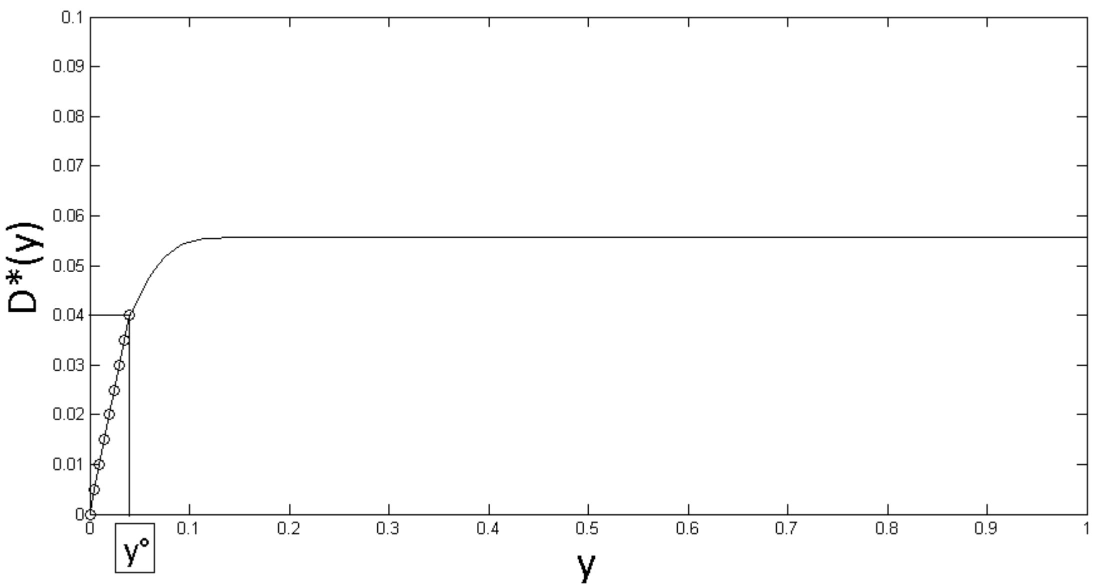
$$\frac{\partial D^*(y_i)}{\partial y_i}|y_i > y° = \frac{P'_1(y_i,k,n)\ \left[\alpha(y° - y_i) + \int_{y°}^{y_i} P(z,k,n)\,\mathrm{dz}\right]}{(P(y_i,k,n)-\alpha)^2} \geq 0$$

Since $P'_1(y_i,k,n)$ is defined positive and $(P(y_i,k,n)-\alpha)^2$ is necessarily positive only the terms in brackets deserve attention. While $\int_{y°}^{y_i} P(z,k,n)\mathrm{dz}$ is positive since $P(y_i,k,n)$ is defined positive, $\alpha(y° - y_i) < 0$ because it is considered the case $y_i > y°$. Nevertheless, it is possible to infer that for every $y \in (y°, 1]$ the term in bracket is positive. Approximating the integral with a sum leads to an increment of $P(y_i,k,n) > \alpha$ and a decrement of $\alpha(y° - y_i) = \alpha$ for any unit of income considered. As a consequence, it is possible to state that the derivative is non negative. Nonetheless, it has to be noted that the value of the derivative is close to zero for the great majority of the incomes because of the value assumed by $P'_1(y_i,k,n)$. In fact, if the number of audits is similar to the one adopted in developed countries (ranging to 0.5% to a maximum of 3%, see Skinner and Slemrod (1985)) the probability of not being audited covers its range of variation in a relatively small interval of the distribution of incomes.



Graph 1: behaviour of $P(y_i,k,n)$for different values of k

Even considering an unreasonably high probability of being audited to give robustness to this consideration, the incomes characterized by a $P'_1(y_i,k,n)$ higher than zero are slightly more than ten percentiles, slightly more than five percentiles if we focus on the ones interested by this changing (the ones above $y°$). Thus, the increase in the declaration following by a rise of income is limited to few percentiles above $y°$ while the rest of the incomes are unaffected. It can be concluded that the declaration function has a 45° slope until $y°$, a slope <45° for few percentiles above $y°$ and is a flat line for higher values. To give an insight of the shape of $D^*(y_i)$ it is presented in Graph2 the result of a simulation made assuming an uniform distribution of incomes (the values of the parameters can be found in [2] while a discussion of the chosen values is provided in the concluding remarks; the qualitative results presented in the graph holds if parameters pertain to their respective domains). For incomes under $y°$ the function overlaps with the 45° line while above $y°$ it continues increasing but with a progressively reduced intensity leading to a flat line. Increasing declaration above $y°$ is performed by taxpayers in consideration of the rise in the probability of not being audited it causes. Thus, as $P(y_i,k,n)$ reaches the unity, the optimal declaration becomes a constant. As a consequence, the ratio under-declaration to income increases with income.



Graph 2: Optimal declaration function

[2] $s = 1{,}1\,; t = 0{,}5\,;\ \alpha = 0{,}46\,; n = 1800\ k = 40\,;\ \frac{k}{n} = 2{,}2\%\,, Y_i \sim U[0{,}1]$

It can be seen that the optimal declaration function is a continuous (proof, see appendix [c]) and increasingly monotone. As a consequence it is invertible leading to a time-consistency problem respective to the audit strategy of tax agency: for a tax agency concerned with revenue maximization it is ex-post efficient to audit only the highest declarations.

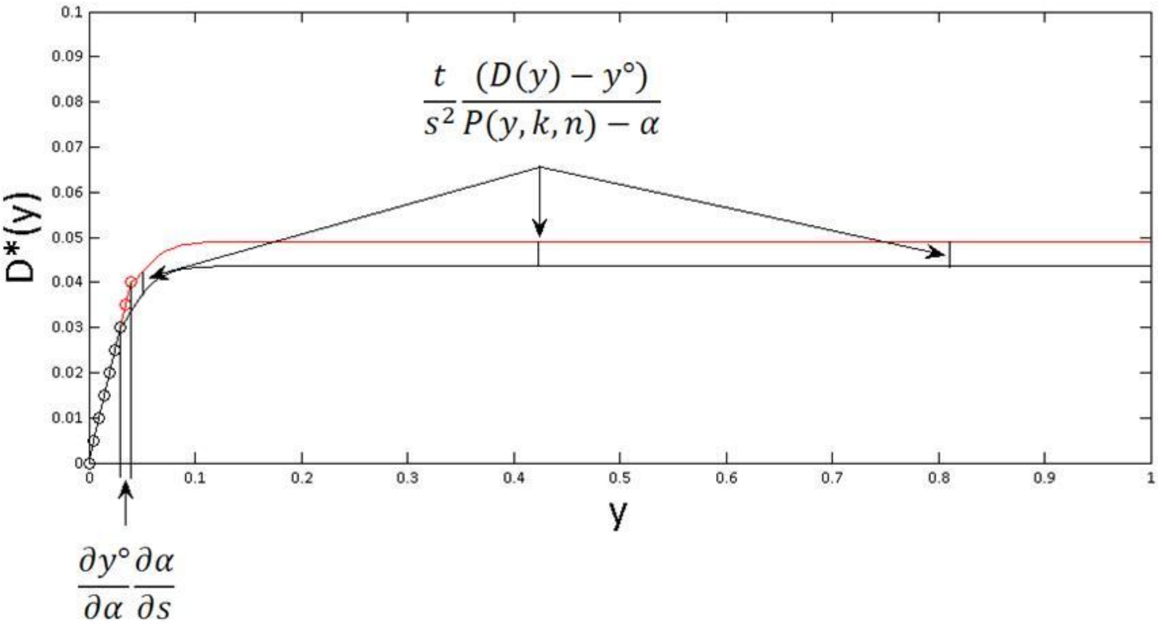
3 Some comparative statics

Here are analyzed the effects that a change in some of the enforcement/tax-setting variables cause on taxpayer's declaration.

An increase of s, causing a rise in α, has two effects on the optimal declaration function. Firstly, it leads to a growth of the area of truthful declaration by its rising effect on $y°$. This happens because the tougher fine determines a negative expected payoff from evasion on a wider range of incomes. Secondly, it leads to an higher compliance for taxpayer that still evade because to achieve the same expected fine taxpayers have to rise their declarations (proof, see appendix [d]). Graph3 shows two optimal declaration function where the one in red is obtained utilizing the same parameters of the black one but with a rise of 20% in s (the values utilized for the black line in the graph of this paragraph can be found in [3]). The first effect noted above, the growth of the area of truthful declaration, is emphasized in the graph by the bottom arrow that points at the additional percentiles on which the red line overlaps with the 45° lines respective to the black one. The second effect is highlighted by the three top arrows

[3] $s = 0{,}45$; $t = 0{,}45$; $\alpha = 0{,}5263$; $n = 1800$; $k = 50$; $\frac{k}{n} = 3{,}6\%$; $Y_i \sim U[0{,}1]$

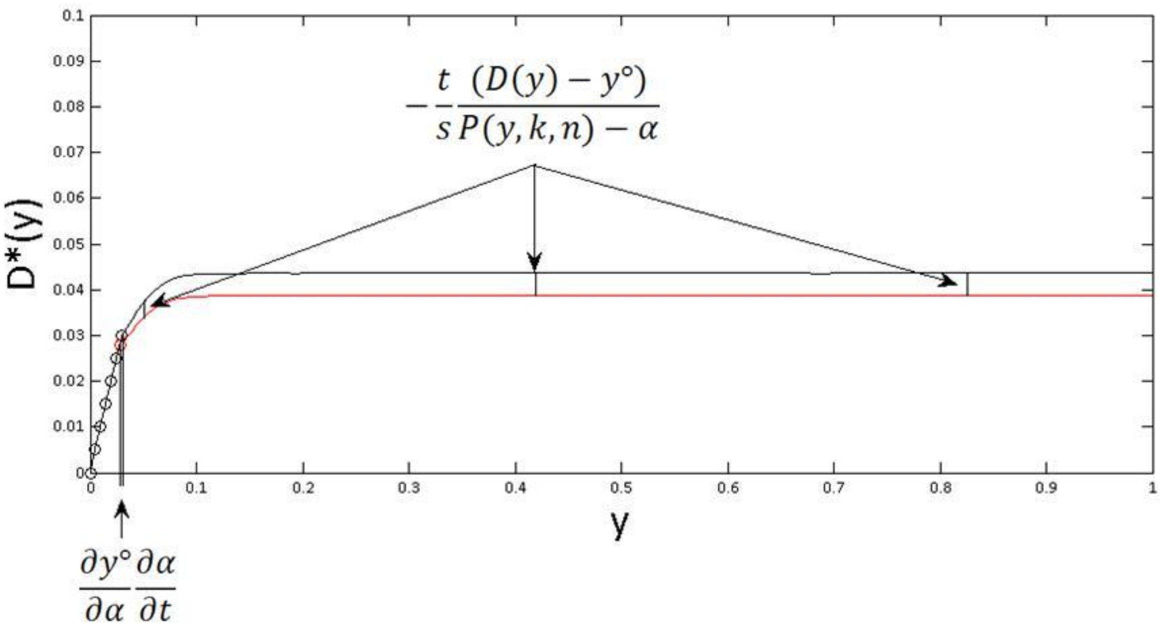
pointing at the increase of the declaration that is determined on the incomes above $y°$.



Graph 3: Effect on $D^*(y_i)$ of an increment of s

A modification in t produces opposite results: as t increases, the expected fine per dollar evaded decreases along with the payoff of compliance and taxpayer re-allocates its bets substituting compliance with evasion. Hence, a reduction in the area of truthful declaration and an increase of the amount under-declared by evaders follows.

Proof, see appendix [e].



Graph 4: Effect on $D^{*}(y_i)$ of an increment of t

In Graph4 it can be seen both effects following from a 20% increase of t.

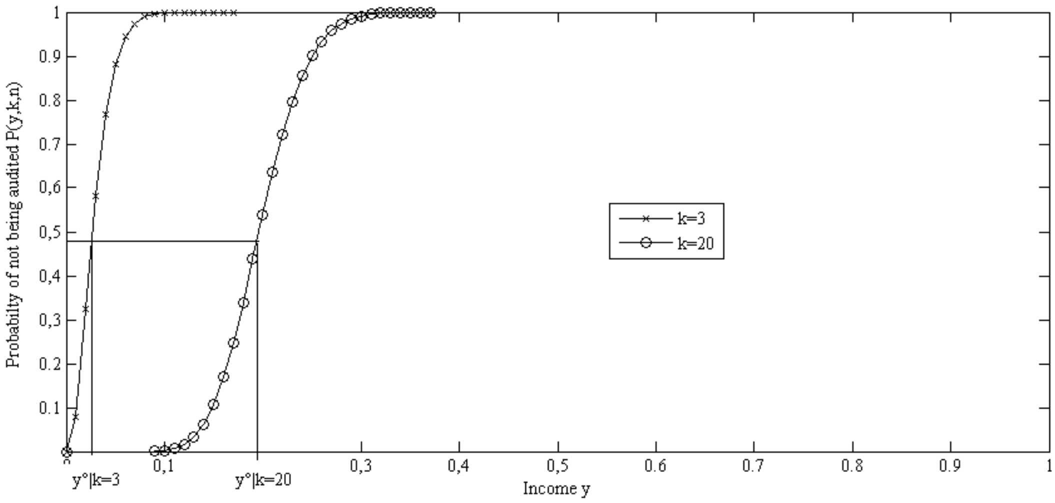
Notably, the magnitude of the effects determined by t and s on $D^{*}(y_i)$ differs only in their impact on α. Since $\frac{\partial \alpha}{\partial s} = t/s^2$, $\frac{\partial \alpha}{\partial t} = -t/s$ and given the ranges of variation of s and t , nothing can be said on which of the two parameters produces the most effective change on the optimal declaration function.

Finally, the effect of a changing in k is considered. Increasing the number of audited declarationst modifies $P(y_i, k, n)$: passing from $\tilde{k}$ audits to $\tilde{k}$+1 leads to a decrement in the probability of not being audited :

$$\frac{\Delta P(y_i,\tilde{k},n)}{\Delta k} = P\big(y_i, \tilde{k}+1\,, n\big) - P\big(y_i, \tilde{k}, n\big) = -\binom{n}{k}[[1-F(y_i)]^{n-\tilde{k}-1}[F(y_i)]^{\tilde{k}}] \qquad (5)$$

Since the cumulative probability distribution of being audited is monotonically increasing, an increment of k causes a rise in the audit probability (given y_i and n) which is reflected in a reduction of $P(y_i, k, n)$ for every value of y_i. As a consequence, it is necessary a higher y_i to achieve the same probability of not being audited. Hence, the rage of incomes that lead to a correct declaration grows:

$$P\left(y°_{|k=\tilde{k}}, \tilde{k}, n\right) = \alpha \wedge P\left(y°_{|k=\tilde{k}+1}, \tilde{k}+1, n\right) = \alpha \Rightarrow y°_{|k=\tilde{k}+1} > y°_{|k=\tilde{k}} \quad (6)$$

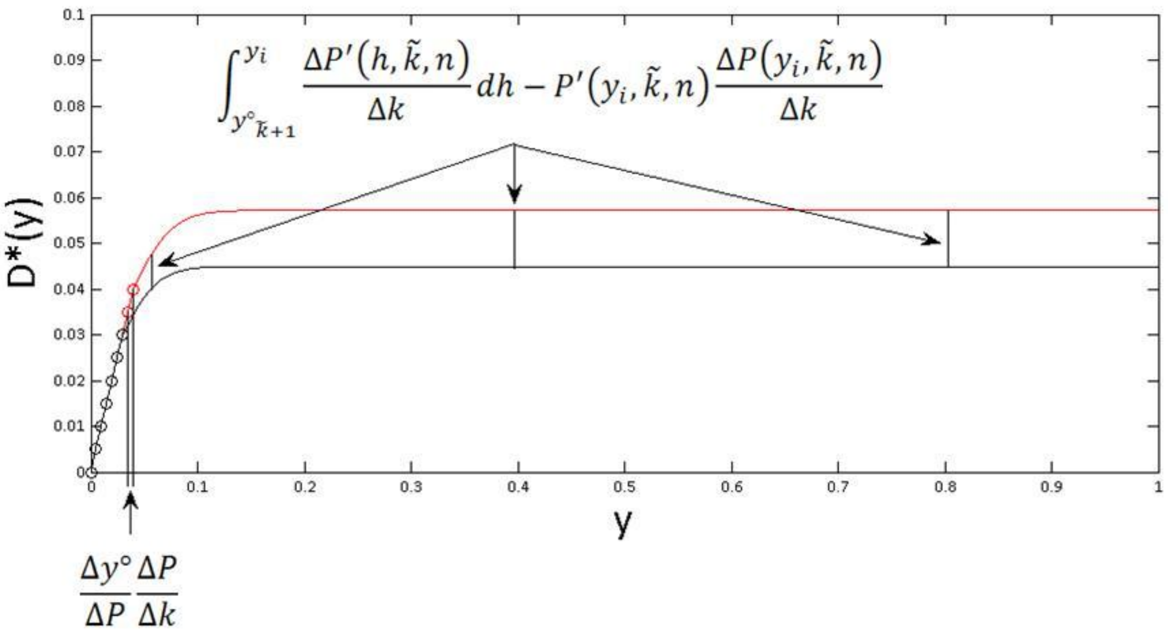


Graph 5: graphical illustration of (6)

Graph 2 shows that $y°_{k=3} < y°_{k=20}$ (the marked difference in the number of audit is provided only to magnify the effects on $y°$).

The effects of a rise in k on the optimal declaration function are qualitatively the same of the ones caused by a rise in s: an increase in the area of truthful declaration and an enhancement of the compliance of the evaders.

Proof, see appendix [f].



Graph 6: Effect on $D^{*}(y_i)$ of an increment of k

Where the second term is positive for (5), while the first one :

$\frac{\Delta P'(y_i,\tilde{k},n)}{\Delta k} = P'\left(y_i, \tilde{k}+1\,, n\right) - P'\left(y_i, \tilde{k}, n\right)$

is non-negative if :

$$F(y_i) \geq \frac{k}{n-1}$$

And hence it can be inferred that the differential is positive for the major part of the income distribution in tax-setting characterized by a number of audits ranging from 0.5% to 3% of the number of declarations.

4 Concluding remarks

The model considered accounts for the strategic interaction arising between tax administration and taxpayer due to enforcement policies. As a consequence of the rule of enforcement adopted: auditing the lowest declaration, the evasion decision can be considered as an auction where the tax agency offers $n-k$ prizes that consists in not being audited.

From one hand, choice of under-declaring affects positively the expected gain from evasion in the state of the world where taxpayer is not audited. From the other hand, it increases the expected loss in the state of the world where taxpayer is audited in two different ways: rising the amount of the fine levied and by increasing the probability of being audited (the lower the declaration, the higher the probability of submitting one of the lowest k declarations). Higher income taxpayer, facing a low probability of reporting one of the k lowest declaration, are subject to a low expected fine and hence are incentivised to under-declare their liabilities. Conversely, lower income taxpayers are more probably subject to audit and their expected fine is severely boosted by an increase in under-declaration. Hence, those taxpayer declare truthfully.

The simple model presented shows neat comparative static results. A growth of k or s lead to an enhanced compliance. As the expected fine from under-declaration rises, in the first case for the increase of the fine itself the second one for the higher probability for the fine to be levied, the area of truthful declaration is expanded and the ratio under-declared amount to real income is reduced for taxpayers that are not deterred from evasion.

Conversely, an increase in t leads to a reduced expected fine for every euro evaded and, rising the payoff from evasion, induces a substitution from declaration to concealment.

The result of a declaration increasing in taxpayer income results from the specification of the audit strategy and can be considered as a positive result of the enforcement policy. Nevertheless, for incomes above $y°$ the optimal declaration function entails a strictly increasing under-declaration to income ratio.

Some equity issues concerning the audit strategy adopted by tax agency has to be discussed. While auditing the lowest declarations preserves the horizontal equity because taxpayers that declare the same liabilities face the same audit probability, vertical equity issues arise for the regressive bias introduced by the auditing policy. In fact, since taxpayer's liabilities can be considered private information in an adverse selection problem, optimal taxpayer's behaviour is to pretend to have low liabilities. Thus, to deter under-declaration it is necessary to set an audit probability that decreases with declaration and the regressive bias cannot be eliminated.

Substituting the value observed in real tax setting into the model permits to conclude that evasion should be widespread. This result, in sharp contrast with reality, can be referred to the limits of the Taxpayer-As-a-Gambler (TAG) models in the explanation of the evasion phenomenon. The assumption that the evasion decision is driven only by monetary consideration occurring in the single spot between taxpayer and government represented by tax declaration is blatantly inadequate. Nevertheless, the implementation of ethical, moral and social motivation in economic models is threatened by the risk to provide ad-hoc solutions lacking the generality expected from a positive theory [an extended discussion of this topic can be found in Cowell (2004)]. The model also predicts that in real tax settings audits are performed only below the $y°$ threshold and thus do not produce any revenue. This conclusion neglects the fact that in reality taxpayers to be audited are chosen not only on the basis of the declaration but in the light of additional information. Still, the present model considers a strategy of auditing that exploits the information known by the taxpayer themselves and is of particular interest in the cases where information about taxpayers, and the economic variables which their income depends on, are difficultly obtainable or highly costly (like when the taxpayers considered are firms). Furthermore, the specification of an audit strategy permits to deal with the reasonable assumption of a fixed audit budget allowing investigating the level of compliance achievable.

The parameters utilized for the graphs are referred to the ones reported by Skinner and Slemrod (2004) while for n it is been adopted a value close to the one reported by the Italian Agenzia delle Entrate in its studi di settore (2010)[4] relative to households with declared incomes up to 30.000 euro pertaining to Val d'Aosta.

The next step in future research should evaluate the impact on the revenues caused by a variation of the relevant parameters of the model. Another interesting issue is represented by the analysis of the design of audit classes. Since this topic is out of the means of the present work it is only noted that $P(y_i, 2{,}100) \ll P(y_i, 20{,}1000)$ with severe implication on the implementability of the presented audit strategy in classes characterized by an high number of taxpayers. This consideration also arises a problem of optimal design of audit classes that is of particular interest for its implication on the Studi di Settore performed by the Italian tax agency.

Appendix

[a]

To prove (4) two cases are analyzed separately.

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[4]http://www.finanze.it/export/finanze/Per_conoscere_il_fisco/studi_statistiche/studi_settore/index.htm

Case I, $y \leq y°$

Utilizing the revelation principle we assume that taxpayers truthfully declare the type they are behaving as :

$D^*(w) = w$

Simplifying (2) leads to :

$EY^{at} = (s-t)D(w) + s[(y_i - D(w)]P(w,k,n)$

That, for the revelation principal can be re-stated as:

$EY^{at} = (s-t)w + s[(y_i - w)]P(w,k,n)$

Maximizing respective to w:

$$\frac{\partial EY^{at}}{\partial w} = (s-t) + s[(y_i - w)P'_1(w,k,n) - P(w,k,n)] = 0$$

$$s\left[(y_i - w)P'_1(w,k,n) + \frac{(s-t)}{s} - P(w,k,n)\right] = 0$$

Where $\alpha = \frac{(s-t)}{s}$

The first term in the square brackets is necessarily greater than zero because the probability distribution function is defined positive while over declaring is never performed since entails losses only.

The second term in the square brackets is positive by definition since $if\ y \leq y° \rightarrow P(w,k,n) \leq \alpha$

Hence, the derivative respective to w is positive and taxpayer behaves as the highest type excluding over-declaration: its true one ■

Case II, $y > y°$

Recalling (3)

$$\frac{\partial E[Y^{at}]}{\partial w} = \alpha D'(w) - P(w,k,n)D'(w) - P'_1(w,k,n)D(w) + P'_1(w,k,n)w = 0$$

Integrating between $y°$ and y

$$\int_{y°}^{y_i} \alpha D'(z) - P(z,k,n)D'(z) - P'_1(z,k,n)D(z) + P'_1(z,k,n)z \text{ dz} = 0$$

$$\alpha[D(y_i) - D(y°)] - [P(y_i,k,n)D(y_i) - P(y°,k,n)D(y°)] + \int_{y°}^{y_i} P'_1(z,k,n)z\,\mathrm{dz} = 0$$

$$D^*(y_i)|y_i > y° = \frac{\int_{y°}^{y_i} P'_1(z,k,n)z\,\mathrm{dz}}{P(y_i,k,n) - \alpha} \blacksquare$$

[b]

Differentiating the declaration respective to income :

$$\frac{D^*(y_i)|y_i > y}{\partial y_i} = \frac{P'_1(y_i,k,n)y_i(P(y_i,k,n) - \alpha) - (P'_1(y_i,k,n)\int_{y°}^{y_i} P'_1(z,k,n)z\,\mathrm{dz}}{(P(y_i,k,n) - \alpha)^2}$$

$$\frac{P'_1(y_i,k,n)\left[y_iP(y_i,k,n) - y_i\alpha - \int_{y°}^{y_i} P'_1(z,k,n)z\,\mathrm{dz}\right]}{(P(y_i,k,n) - \alpha)^2}$$

$$\frac{P'_1(y_i,k,n)\left[y_iP(y_i,k,n) - y_i\alpha - \ y_iP(y_i,k,n) + y°\alpha + \int_{y°}^{y_i} P(z,k,n)\,\mathrm{dz}\right]}{(P(y_i,k,n) - \alpha)^2}$$

$$\frac{P'_1(y_i,k,n)\ \left[\alpha(y° - y_i) + \int_{y°}^{y_i} P(z,k,n)\,\mathrm{dz}\right]}{(P(y_i,k,n) - \alpha)^2} \blacksquare$$

[c]

$$\lim_{y_i \to y°} \frac{D^*(y_i)}{\partial y_i} = \frac{P'_1(y°,k,n)y°}{P'_1(y°,k,n)} = y° \blacksquare$$

[d]

Recalling that $\alpha = \frac{s-t}{s}$ it is possible to rewrite (4) as:

$$D^*(y_i)|y_i > y° = \frac{\int_{y°}^{y_i} P'_1(z,k,n)z\,\mathrm{dz}}{P(y_i,k,n) - \frac{s-t}{s}} \qquad (7)$$

Differentiating (7) respective to s leads to:

$$\frac{\partial D^*(y_i)|y_i > y^\circ}{\partial s} = \frac{\left\{\frac{t}{s^2}\left[\left(-\alpha\frac{\partial y^\circ}{\partial \alpha} - y^\circ\right) + \alpha\frac{\partial y^\circ}{\partial \alpha}\right]\right\}(P(y_i,k,n) - \alpha) + \frac{t}{s^2}\int_{y^\circ}^{y_i} P'_1(z,k,n)z\,\mathrm{d}z}{(P(y_i,k,n) - \alpha)^2} =$$

$$= \frac{-\frac{t}{s^2}y^\circ(P(y_i,k,n) - \alpha) + \frac{t}{s^2}\int_{y^\circ}^{y_i} P'_1(z,k,n)z\,\mathrm{d}z}{(P(y_i,k,n) - \alpha)^2} =$$

$$= \frac{\frac{t}{s^2}(D^*(y_i) - y^\circ)}{P(y_i,k,n) - \alpha} \blacksquare$$

Since $\frac{t}{s^2}$ and $P(y_i,k,n) - \alpha$ are greater than zero, $D^*(y^\circ) = y^\circ$ (see appendix [c]) and $\frac{\partial D^*(y_i)}{\partial y_i} \geq 0 \; \forall \; y_i \in (y^\circ, b]$ the differential of the optimal declaration respective to the fine is positive.

[e]

Differentiating (7) respective to t leads to:

$$\frac{\partial D^*(y_i)|y_i > y^\circ}{\partial t} = \frac{\left\{-\frac{1}{s}\left[-y^\circ + \alpha\frac{\partial y^\circ}{\partial \alpha}\right]\right\}(P(y_i,k,n) - \alpha) - \frac{1}{s}\int_{y^\circ}^{y_i} P'_1(z,k,n)z\,\mathrm{d}z}{(P(y_i,k,n) - \alpha)^2} =$$

$$= \frac{-\frac{1}{s}(D^*(y_i) - y^\circ)}{P(y_i,k,n) - \alpha} \blacksquare$$

Since $-\frac{1}{s}$ is smaller than zero and in the light of what has been said in appendix [d] the derivative of the optimal declaration respective to tax rate is negative.

[f]

Identify optimal declaration function when the number of audits is equal to $\tilde{k}$ with:

$$D^*_{\tilde{k}}(y_i)|y_i > y^\circ = \frac{\int_{y^\circ_{\tilde{k}}}^{y_i} P'_1(z,\tilde{k},n)z\,\mathrm{d}z}{P(y_i,\tilde{k},n) - \alpha}$$

And the optimal declaration function when $\tilde{k}+1$ audits are performed with:

$$D^*_{\tilde{k}+1}(y_i)|y_i > y° = \frac{\int_{y°_{\tilde{k}+1}}^{y_i} P'_1(z,\tilde{k}+1,n)z\,\mathrm{dz}}{P(y_i,\tilde{k}+1,n)-\alpha}$$

Since $y°_{\tilde{k}} < y°_{\tilde{k}+1}$ the optimal declaration function when $\tilde{k}+1$ audits are undertaken is characterized by an extension of the truthful declaration area respective to the case where $\tilde{k}$ audits are performed.

Hence, in order to study the effect on the declaration of an unitary increase of the audit it is considered the difference of the two declaration for incomes belonging to the interval $(y°_{\tilde{k}+1}, b]$

$$D^*_{\tilde{k}+1}(y_i) - D^*_{\tilde{k}+1}(y_i) \mid y_i \in (y°_{\tilde{k}+1}, b] =$$

$$= \frac{[P(y_i,\tilde{k},n)-\alpha]\int_{y°_{\tilde{k}+1}}^{y_i} P'_1(z,\tilde{k}+1,n)z\,dz - [P(y_i,\tilde{k}+1,n)-\alpha]\int_{y°_{\tilde{k}+1}}^{y_i} P'_1(z,\tilde{k},n)z\,dz}{[P(y_i,\tilde{k},n)-\alpha][P(y_i,\tilde{k}+1,n)-\alpha]}$$

(8)

Defining:

$$\frac{\Delta P'(y_i,\tilde{k},n)}{\Delta k} = P'(y_i,\tilde{k}+1\,,n) - P'(y_i,\tilde{k},n) =$$

$$= \binom{n}{k} f(y_i)\{[1-F(y_i)]^{n-\tilde{k}-2}[F(y_i)]^{\tilde{k}-1}[(n-\tilde{k}-1)F(y_i) - k[1-F(y_i)]]\} \quad (9)$$

Where $\frac{\partial F(y_i)}{\partial y_i} = f(y_i)$

And given (5), (8) can be re stated as:

$$\int_{y°_{\tilde{k}+1}}^{y_i} \frac{\Delta P'(h,\tilde{k},n)}{\Delta k} dh - P'(y_i,\tilde{k},n)\frac{\Delta P(y_i,\tilde{k},n)}{\Delta k} \blacksquare$$

Bibliography

Allingham, M.G. e Sandmo, A. [1972]:"Income tax evasion: a theoretical analysis", Journal of public economics, 1, 323-38.

Andreoni, James, Brian Erard, and Jonathan Feinstein. [1998]: "Tax Compliance." The Journal of Economic v Literature 36, no. 2: 818-60.

Cowell, Frank [2004]: "Carrots and sticks in enforcement," The crisis in tax administration. The Brookings Institution, Washington, DC, pp. 230-275. ISBN 0-815701233

Marchese, C.[2004]:"Taxation, Black Markets and Other Unintended Consequences," Handbook of Public Finance,237-277. Kluwer Academic Publishers.

Skinner, J, e J. Slemrod, [1985]. "An Economic Perspective on Tax Evasion," National Tax Journal, 38.

Snow, A. and Warren, R. S. [2005]:"Ambiguity about audit probability, Tax Compliance and Taxpayer Welfare" Economic Inquiry, 43: 865–871. doi: 10.1093/ei/cbi066