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## Political Activism, Party Coalitions and Electoral Competition

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#### Abstract

The paper contributes to the small but growing literature on electoral competition when political parties are regarded as coalitions. We present a model of coalition formation that involves reasons for policy-motivated activists to provide resources to, and be an integral part of, a particular coalition as an alternative to lobbying from outside. We link two types of literature: one focusing on the consequences of coalition formation for the nature of equilibrium in the absence of campaign contributions, and the other where competing parties that are strictly electorally oriented are provided with resources by contributors who are always external to them. In this framework, we consider how changes in party valence and in the regulation of campaign finance affect the nature of equilibrium platforms and the degree of party activism.

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## 1 Introduction

This paper presents a model of coalition formation and political competition which includes reasons for policy-motivated activists to provide resources to particular party coalitions from inside the coalition rather than from outside. It addresses questions of the role of activists in coalition formation and of the effects on political equilibrium of the resources such party activists, who could also contribute from outside the coalition, bring with them. Specific comparative static experiments with the framework we develop concern the effects of changes in party valence and in the regulation of campaign finance on the nature of equilibrium platforms and the degree of party activism.

The model incorporates ideas about coalition formation due to Roemer (2001) and Aldrich (1983 a,b), and on the role of policy-motivated activists offering contributions from outside vote-maximizing parties by Schofield (2003), Schofield, Miller and Martin (2003), Austen-Smith (1987) and others.<sup>1</sup> In Roemer's framework, each party coalition is composed of two essential fractions that we shall call opportunists and activists. Opportunists desire only to maximize the probability of the coalition's victory. They have no interest in policy per se. Activists, or militants as Roemer calls them, only care about policy and are not concerned about winning elections. They propose a policy as close as possible to their ideal point, and use electoral competition as a forum to advertise and agitate for their preferred choice  $^2$ . In the spatial voting models of Schofield et al, Austen-Smith and others, candidates for office balance the need for political resources provided by contributors from outside the party with the desire to move towards the center of the mass of voters. In these models, while centrist policies win votes, they do little to earn the support of external contributors who are ideologically extreme and who supply vital electoral resources. Thus, as Schofield, Miller and Martin (2003, 219) put it: "rational political candidates attempt to balance the need for resources with the need to take winning

<sup>&</sup>lt;sup>1</sup>On coalition formation, see also Levy (2004). Many authors have considered the impact on equilibrium of activists who contribute campaign resources, including Moon (2004), Grossman and Helpman (2001) and Mitchell and Munger (1991). We note that we do not consider the role of inherent asymmetries in access to political resources as does Moon, nor do we consider in as much detail as do Grossman and Helpman (2001) the exchange between lobbyists outside party coalitions and the government, in order to focus on the role of activists in coalition formation which is not part of their models.

 $<sup>^{2}</sup>$ Roemer also considers a third coalition member - the reformists - who maximize expected utility. However, reformists turn out to be a combination of opportunists and activists and for this reason become superfluous in the analysis.

#### policy positions<sup>3</sup>."

A major difference between the coalition model presented by Roemer and the spatial voting models of Schofield et al and Austen-Smith and others, is that in the former, activists are inside the coalition and do not explicitly provide political resources while in the latter, they operate from outside and do contribute money or time. This difference is the starting point for our analysis, which combines ideas from both types of approaches in a framework where activists, who always provide crucial political resources, may act both inside and outside party coalitions.<sup>4</sup>

It is not difficult to see why a policy-motivated activist may want to be, at different times, both inside and outside. Activists who join a coalition want to acquire some agenda control in the policy formation process concerning those issues where they have a major stake. They may also expect to profit from inside access to areas of policy somewhat related to their special interests on a day by day basis. An example is the Greens in some European Union countries who were outside parties until the middle eighties, when they started to formally enter party coalitions. The Greens contributed by involving large groups of people and collecting resources from supporters outside. Another example is the rich Italian entrepreneur and politician Silvio Berlusconi who supported parties from outside in the eighties and joined a party coalition in the nineties, becoming the Prime Minister. Berlusconi contributed to the party through his influence over the media and with money.

There is a direct price to be paid for gaining inside power of course, and there is also an indirect cost stemming from the fact that activists, when inside, can only give resources to their own coalition. Activists will stay outside - and when they do we shall call them *lobbyists* for convenience - if they care mainly about one or very few issues and do not require constant access to deal with them. Lobbyists consider gaining

<sup>&</sup>lt;sup>3</sup>They continue: "Voters chose among candidates for both policy and non policy reasons. Policy motivations of voters pull candidates towards the center. However, centrist policies do little to earn the support of party activists, who are more ideologically extreme than the median voter, and who supply vital electoral resources. Candidates realize that the resources obtained from party activists make them more attractive, independent of policy positions. This implies that candidates must balance the attractiveness of activists' resources against the centrist tug of voters."

<sup>&</sup>lt;sup>4</sup>In a recent interesting paper, Roemer (2005) also considers the role of campaign finance using a model of explicit party formation. His model differs from that developed here in the way in which the electoral productivity of political resources is modelled. In Roemer's model, there are uninformed voters that both parties must address, while we model uncertainty as part of a non-policy related 'valence' that differs across parties. Roemer considers in some detail how political contributions inside a party coalition are co-ordinated, while our model emphasizes an insider-outsider choice by party contributors, as discussed below.

agenda control power too costly, and prefer to influence one or possibly both coalitions on a key issue by offering campaign contributions or other political resources. While insiders must pay for continued access to power, for example by getting a certain number of people elected as their representatives or agents in the legislature, or by providing time and money from an enlarged set of party workers, when they are outside as lobbyists they need only to contribute directly to one or both parties in order to influence a single issue.

As for the opportunists, they want resources from activists, and are prepared to some extent to adapt their policy platform to the preferences of activists for this reason. Moreover, opportunists know that when activists are inside the party coalition, they do not contribute to the opposition.

Capturing the full richness of the coalition formation process in this situation is a complex matter. At the outset, it is necessary to model the decision of contributors to participate inside as activists or stay outside as lobbyists, and this we do using a participation constraint. We assume that activists bargain directly over policy and campaign and other resource contributions with one group of opportunists in a Nash bargaining game. When inside, activists are assumed to maximize their utility defined over their own party's possible platforms. As lobbyists outside, they are modelled as expected utility maximizers who may contribute to one or both parties and who take both party positions directly into account. Moreover, while lobbyists have an influence on equilibrium platforms, we assume (as in Austen-Smith 1987) that they do not bargain with the parties, offering most of their contribution to the party that promises the platform - taken as given at the time of the contribution that is closest to their ideal point.

The reasons for introducing the asymmetry in behavioral assumptions about activists and lobbyists are both substantive and practical. Substantively, when activists are inside the coalition, dedication to the coalition as reflected here by simple utility optimization gives them more bargaining power in the decision-making process. From a modelling point of view, were activists to maximize expected utility defined over the actions of competing coalitions, the policy-oriented aspect of their utility (which leads them to desire somewhat extreme policy positions), would conflict with their desire to benefit to some extent from the expected actions of the other party, and for this reason make the model much more complex.

Equilibrium in the competition between two party coalitions of opportunists and activists is modelled using a probabilistic spatial voting framework where, as is now common in this literature, the probability of winning elections is the sum of a policy-related component, and a non-policy related component referred to as the party 'valance', and where political resources provided by activists or lobbyists can increase a party's valence.<sup>5</sup> A consequence for coalition formation of adopting this structure is that activists will gain greater control over the platform when they contribute more, or when the non-policy related valance is particularly important because of uncertainty about its platform or how it will conduct itself once in office, so that the coalition more urgently needs money to face the competition.

In this Nash bargaining, Nash non-cooperative competition framework pioneered by Roemer (2001) - which combines elements of the party coalition approach to political competition with probabilistic spatial voting in the presence of lobbying - we consider the importance of activists and of the nature of electoral uncertainty for equilibrium policy outcomes. We also investigate the consequences for the degree of activism and equilibrium platforms of public financing and regulation of elections, a matter of substantial concern in discussions of electoral financing law. On the regulation of party finance, it is particularly interesting to note that the German law, as discussed by von Arnim and Schurig (2004, 40-41), is explicitly formulated to limit public funding in order to strengthen the need for parties to maintain their links with citizens<sup>6</sup>. We shall consider whether or not public financing of campaigns and regulation of lobbying are conducive to the presence of activism, or whether it drives 'idealism' out of politics. In all of this, our emphasis is on the role and consequences of activism inside coalitions since much of the existing literature focusses on how contributions from lobbyists outside party coalitions influence equilibrium outcomes.<sup>7</sup>

The paper proceeds in two main steps. First we develop a basic model of coalition formation in which parties negotiate inside a coalition over policy and contributions. Here contributions to parties from outside lobbyists are assumed to be forbidden by regulation. In a second step, we extend the model by introducing an outside option for activists, who

<sup>&</sup>lt;sup>5</sup>Adams, Merrill and Grofman (2005) provide a thorough recent review of the spatial voting literature. On party valence see also Ansolabehere and Snyder (2000), Groseclose (2001) and Stokes (1993)

<sup>&</sup>lt;sup>6</sup>For a recent experimental study on campaign finance see Houser and Stratmann (2006).

<sup>&</sup>lt;sup>7</sup>In so doing, we place less emphasis than does Roemer on the question of the existence of equilibrium, and less on the details of lobbying from outside as in Grossman and Helpman(2000). We may also note that in contrast to Felli and Merlo's (2004) paper on endogenous lobbying, coalitions here are formed before the election in the context of a spatial voting approach. Extensions in all of these directions are left for further research.

may choose to contribute to either party without being a member of any coalition. The basic model, including analysis of coalition formation and the electoral equilibrium when outside contributions to parties are prohibited, is presented in sections two and three. In section four and five we analyze this model, considering how asymmetric changes in the degree of uncertainty that the competing parties must cope with and in public financing of electoral campaigns affect the electoral equilibrium and the degree of activism as measured by total contributions by party activists. Outside options for activists are introduced formally into the basic model in section six, and the implications of this for electoral equilibrium and the degree of activism are reconsidered in this and section seven. Section eight summarizes the policy implications of several types of campaign finance regimes for policy convergence and the degree of activism, and section nine concludes.

## 2 The Model without outside contributions

Consider an economy with N voters and two parties, i and j. Inside each party two factions, named opportunists and activists, form a coalition and choose policy platform and contributions needed for campaigning. Activists play the role of party financiers, if an agreement is not reached, opportunists will run for the election alone without the activists. In this case, opportunists can move policy position without commitments and use only public financing for campaigning if it is available<sup>8</sup>. On the contrary, activists will never run alone for the election since they are not interested in winning office. Furthermore, activists do not receive public financing<sup>9</sup>.

As noted above, we distinguish two cases. In the first case, we assume that parties do not receive external contributions apart from any public financing. In this case, opportunists must form a coalition with a group of activists if they want resources for campaigning in addition to public financing. Likewise, activists can affect policy only by becoming part of a party-coalition. In this context, we study how policy platforms are affected by valence and the role of public and private financing of political parties.

In the second case, we extend the model by allowing outside contributions from lobbyists. In this case, if the coalition doesn't reach an

<sup>&</sup>lt;sup>8</sup>Note that this is a first important difference with respect to Roemer (2001). In particular, in Roemer, if a coalition doesn't reach an agreement over policy platform, the opponents win the election by default.

<sup>&</sup>lt;sup>9</sup>A further development of this model could allow activists to form their own party and gain public contribution if they do not reach an agreement with the opportunists.

agreement, activists may still influence policy by contributing from outside. Activists (as lobbyists) now face a trade-off between contributing from inside or from outside. When outside, as lobbyists, they loose the possibility of negotiating over policy, but can give money to support either party i or j. Outside, they are price-takers in the sense that consider policy platforms as given and maximize expected utility by supporting the party that is closer to their ideal point. Activists gain the power to negotiate over policy and contributions only when they are inside one coalition.

Similarly, in the second and full model, opportunists can attract external private contributions if they don't form a coalition with their activists. The opportunists' dilemma is whether they should accept contributions from outside or from inside the coalition. For them, an agreement that brings activists inside has the advantage of reducing contributions in favour of the opposing coalition because activists contribute only to one coalition when they are part of it.

In the electoral equilibrium, whether with or without lobbying, the two coalitions choose policy positions and contributions simultaneously. The electoral equilibrium is represented as the simultaneous combination of Nash bargaining inside the two coalitions and a Nash equilibrium between coalitions. When the two factions of the two symmetric coalitions are bargaining inside, we assume they consider as given the policy platform and contributions chosen by the opposing coalition.

We now consider the nature of the participants in details.

## 2.1 Voters

As is now common in the spatial voting literature, we assume voting behavior depends on policy and non-policy related variables. The indirect utility of voter h,

$$U_{h_k} = U_{h_k} \left( u_h \left( s_h^*, s_k \right); \gamma_k, c, c_k; \xi_k \right),$$

is a function of the policy  $s_k$  announced by party k = i, j, voters h's bliss point,  $s_h^*$ , and of two non-policy components<sup>10</sup>  $\gamma_k$  and  $\xi_k$ . The parameter  $\gamma_k$ , which has been called valence (Ansolabehere and Snyder 2000; Groseclose 2001; Schofield 2003), can be affected by the money  $C_k(c, c_k)$  that party k spends for the electoral campaign, where c and  $c_k$ represent respectively public and private financing of political parties.

<sup>&</sup>lt;sup>10</sup>Examples of non-policy components are the personal qualities of the candidates like personality, honesty, reputation, charisma, appeal, prestige, celebrity or the uncertainty about policy platforms.

For each party k there is an "innate" valence, which is best represented by a stochastic error term,  $\xi_k$ , (Schofield 2003) and that is not affected by any type of contribution.

Accordingly, the first part,  $u_h$ , of voter h's utility depends directly on the policy position of coalition k, such that<sup>11</sup>

$$u_h(s_k) = -(s_h^* - s_k)^2,$$
 (1)

where  $s_h^*$  is the ideal point of voter h.

The non-policy related part - or party valence - is assumed to be probabilistic from the perspective of parties, and can be thought of as a random variable that has two components. One reflects voter uncertainty about what each party proposes, or will do once elected. As do Schofield (2003) and Austen-Smith (1987), we model this first component as a function of political resources or contributions, on the assumption that more money can be used to reduce the perceived variance of the announced policy platform and so enhance the electoral attractiveness of the party platform. The second random component is randomly distributed over the population and is essentially outside each coalition's influence.

Accordingly, and making use of Austen-Smith's formulation, the utility  $U_{h_k}$  of voter h is:

$$U_{h_{k}} = u_{h}(s_{k}) - \tau_{h} \frac{\gamma_{k}}{c + c_{k}} + \xi_{k}; \quad \text{with } k = i, j$$

The second term  $\tau_h \frac{\gamma_k}{c+c_k}$  can be affected by the money and/or time,  $c_k$ , which activists make available to coalition k. Here, for later consideration, we also introduce public financing c which may also be used to increase party valence; that is, to reduce the impact of uncertainty on a party's expected vote. The parameter  $\tau_h$  measures the impact on the individual voter of the valence component<sup>12</sup> of coalition k,  $\gamma_k$ , and of political contributions  $c_k$  and c. The third term  $\xi_k$  is not affected by the money and time spent during the electoral campaign. It represents the "innate" valence or the evaluation of each political coalition by the voter on non-policy matters such as a candidate's personality. The difference in the non-policy evaluation of the two coalitions,

$$b_h = \xi_j - \xi_i,\tag{2}$$

 $<sup>^{11}\</sup>mathrm{We}$  use the quadratic form as is common in the literature, Schofield and Miller (2003), etc..

 $<sup>^{12}\</sup>tau_h$  says everyone is affected differently by a given amount of advertising  $(c+c_k)$ .

represents a utility bias in favour of coalition j that is independent of policy positions.

The probability  $q_{hi}$  that citizen h votes for coalition i, given party platforms  $(s_i, s_j)$  and resources  $(c_i, c_j)$ , is

$$q_{hi}\left(\cdot \mid s_{i}, s_{j}; c_{i}, c_{j}\right) = \begin{cases} 1 \text{ if } \left(u_{h}\left(s_{i}\right) - u_{h}\left(s_{j}\right)\right) - \tau_{h}\left(\frac{\gamma_{i}}{c+c_{i}} - \frac{\gamma_{j}}{c+c_{j}}\right) > b_{h}, \\ 0 \text{ otherwise.} \end{cases}$$

$$(3)$$

where, the valence differential  $\left(\frac{\gamma_i}{c+c_i} - \frac{\gamma_j}{c+c_j}\right)$  can be affected by contributions  $c_k$  and c.

## 2.2 Opportunists

We consider a spatial voting model in which party i chooses policy position on the left-hand side of the mass of voters and party j on the right-hand side. Opportunists care only about winning the election, so that they maximize the probability of winning represented by the party's expected votes,

$$EV_{k} = EV_{k}\left(s_{i}, s_{j}; \gamma_{i}; \gamma_{j}; c, c_{i}, c_{j}; \xi_{i}, \xi_{j}\right),$$

where k = i, j and  $EV_i = N - EV_j$ . Each party, the opportunists, receives a public financing for their activities under the form of a lump-sum transfer c from the state. In addition to public financing, opportunists receive private contributions,  $c_k$ , from the activists that join the party. Accordingly, opportunists' expected votes is a function of public financing, the policy platforms chosen by the two coalitions, parties' valences and activists' contributions.

Party opportunists choose policies so as to continuously maximize their expected votes given the opposing coalition's position. As shown in the Appendix, the expected vote of party *i* from *N* voters for the case in which  $b_h$  is uniformly distributed<sup>13</sup> can be expressed as:

$$EV_i(s, c, c_i, c_j) = \sum_{h=1}^N \theta_h \left\{ \left[ \left( u_h(s_i) - u_h(s_j) \right) - \tau_h \left( \frac{\gamma_i}{c + c_i} - \frac{\gamma_j}{c + c_j} \right) \right] - b_h^{\min} \right\};$$
(4)

where,  $(u_h(s_i) - u_h(s_j)) - \tau_h\left(\frac{\gamma_i}{c+c_i} - \frac{\gamma_j}{c+c_j}\right)$  represents the individual net welfare or utility differential when voter h votes for party i;  $\theta_h$  is the individual sensitivity of the probability of voting to a change in individual

 $<sup>^{13}</sup>$  "The voting is called probabilistic because parties only know the probability distribution of voter's biases." (Roemer 2006).

welfare;  $b_h^{\min}$  is the minimum value of the bias term  $b_h$ , which doesn't play any role in the following analysis. For the "standard" derivation of equation 4 see the Appendix.

Let  $\rho_i = \rho_i(S, C) \in [0, 1]$  be the probability that candidate *i* wins the election, given policy announcements  $S = (s_i, s_j)$  and total political resources  $C = (c_i, c_j, c)$ . Following Calvert (1986), Hinich (1977) and others, for large *N* and no abstention, maximizing coalition *i*'s expected votes is equivalent to maximizing *i*'s probability of electoral success. Therefore, assuming *N* large, we can write  $\rho_i(S, C) \equiv EV_i(s_i, s_j)/N$ , where  $\frac{\partial EV_i}{\partial \gamma_i} < 0$ ,  $\frac{\partial EV_i}{\partial \gamma_i} > 0$  and  $EV_j = N - EV_i$ .

## 2.3 Activists

The activists (or lobbyists) care only about policy and are unconcerned about winning the election. Their purpose is to influence policy platforms. They are ready to contribute with money or time to a party in exchange for policy and can be either ideology or profit motivated.

In contrast to Aldrich's (1983a, 1983b) theory<sup>14</sup>, we assume that when activists are inside the coalition they are not price-takers. They are partners in the party coalition with the opportunists and give money, time, or other resources in exchange for policy. The activists' utility from being part of coalition k, (k = i or j) is given by:

$$u_k = v_k \left( s_k \right) - c_k \tag{5}$$

where, s is the policy and  $c_k$  is the activists' contribution to their own coalition. In order to distinguish activists in different parties, we assume for convenience that  $\frac{\partial v_i(s)}{\partial s} < 0$  and  $\frac{\partial v_j(s)}{\partial s} > 0$ , i.e., activists in party *i* want a platform s that is small or to the 'left', while those in party *j* want a platform that is further to the 'right'. We can use contributions  $c_k$  as an index of the degree of activism or idealism within each coalition.

#### 2.4 Policy convergence in the absence of activists

At this point, it is helpful to quickly analyze the political equilibrium when there are no activists and opportunists compete only with policy

<sup>&</sup>lt;sup>14</sup>Aldrich (1983a, 1983b) provides one of the first models that incorporates partisan activists into the spatial theory of electoral competition (see Hinich, 1983). In Aldrich's model, policies are not chosen by opportunists who maximize the probability of winning election, but are determined exclusively by citizens-activists. Citizens choose whether to become activists in one of two political parties or abstain. Activists are price takers, in the sense that they may influence the average position of the party only infinitesimally by deciding to join it. A party is seen as a distribution of its activists, who join the party not to change its goals but to support and help in their realization. Results predict a cleavage between the two parties' activists who are "relatively cohesive internally and relatively distinctive externally".

positioning. In this situation platforms converge. This is the standard Downsian (1957) result in a spatial voting model.

In this case, party i maximizes its expected vote:

$$\max_{s_i} EV_i(s_i, s_j, c) = \sum_{h=1}^{N} \theta_h \left\{ \left[ \left( -(s_h^* - s_i)^2 + (s_h^* - s_j)^2 \right) - \frac{\tau_h}{c} \left( \gamma_i - \gamma_j \right) \right] - b_h^{\min} \right\} \right\}$$
(6)

while party j maximizes  $(N - EV_i(S))$ 

Since the first order conditions for the choice of platform are the same for both parties, that is

$$2\sum_{h=1}^{N} \theta_h \left( s_h^* - s_k \right) = 0, \tag{7}$$

they propose identical platforms in the Nash equilibrium. Note also that here, public financing of parties, if it exists and is outside of the parties' control, does *not* influence equilibrium platforms.

When activists or lobbyists are introduced in this model, they will pull the two policy positions apart. Recall that we assume that activists *i* want to push the policy platform as much as possible to the left (since  $\frac{\partial v_i(s)}{\partial s} < 0$ ), while activists *j* try to push to the right  $(\frac{\partial v_j(s)}{\partial s} > 0)$ .

## 3 Bargaining in the coalition without external financing

Here, besides public financing when available, the only way for the opportunists to generate resources is to form a coalition with a group of activists. We assume that in absence of an agreement, a party can still use public financing if it is available.

## 3.1 Opportunists' disagreement utility and net gain

Opportunists will gain from reaching a political agreement with activists since the money they provide can be used to reduce uncertainty and thus increase the party's valence. Accordingly, the opportunists' disagreement utility (or threat point), which determines their bargaining  $strength^{15}$ , is:

$$EV_i^d(s,c) = \sum_{h=1}^N \theta_h \left\{ \left[ \left( u_h\left(\widetilde{s}_i\right) - u_h\left(\widetilde{s}_j\right) \right) - \tau_h \left( \frac{\gamma_i}{c} - \frac{\gamma_j}{c+c_j} \right) \right] - b_h^{\min} \right\}.$$
(8)

Without an agreement opportunists i use public financing to reduce the negative effect of valence or uncertainty. Note that  $s_i$  and  $s_j$  are chosen by bargaining inside the two parties and are different from  $\tilde{s}_i$  and  $\tilde{s}_j$ , which represent policy chosen by party i and j when a coalition fails to come to an agreement and doesn't form. We model policy choices only for the case in which the two coalitions reach an internal agreement and consider, therefore,  $\tilde{s}_i$  and  $\tilde{s}_j$  as given.

Party *i* always assumes that party *j* reaches an agreement and viceversa. So that, from the point of view of party *i*, party *j* can count on  $c + c_j$ , while party *j* assumes that party *i* can count on  $c + c_i$ . As a result, opportunists *j* disagreement utility is

$$EV_{j}^{d}(s,c) = \sum_{h=1}^{N} \theta_{h} \left\{ \left[ \left( u_{h}\left(\widetilde{s_{j}}\right) - u_{h}\left(\widetilde{s_{i}}\right) \right) - \tau_{h} \left( \frac{\gamma_{j}}{c} - \frac{\gamma_{i}}{c+c_{i}} \right) \right] - b_{h}^{\min} \right\}.$$
(9)

For a successful coalition to form, opportunists' net gain,  $\psi_k = EV_k(S,C) - EV_k^d(S,C)$ , must be positive: i.e., considering only opportunists i,

$$\psi_{i} = \sum_{h=1}^{N} \theta_{h} \left\{ \left( u_{h}\left(s_{i}\right) - u_{h}\left(s_{j}\right) \right) - \left( u_{h}\left(\widetilde{s}_{i}\right) - u_{h}\left(\widetilde{s}_{j}\right) \right) - \tau_{h}\gamma_{i} \left( \frac{1}{c+c_{i}} - \frac{1}{c} \right) \right\}$$
(10)

must be > 0. The net policy differential  $(u_h(s_i) - u_h(s_j)) - (u_h(\tilde{s}_i) - u_h(\tilde{s}_j))$ represents the part of the opportunists' net gain from cooperating that depends exclusively on the differences in policy positions offered to activists and to lobbyists. The term  $-\sum \tau_h \gamma_i \left(\frac{1}{c+c_i} - \frac{1}{c}\right)$  in equation 10 measures opportunists *i*'s net gain of party valence of opportunists *i* running for the election as part of a coalition, with respect to the situation in which they compete without forming a coalition<sup>16</sup>.

<sup>&</sup>lt;sup>15</sup>The greater the opportunists' disagreement utility, the more substantial their bargaining power because they then have less to gain in having the activists inside a coalition.

<sup>&</sup>lt;sup>16</sup>Note that the net net gain for opportunisits in party *i* increases with  $c_i$  and declines with  $\gamma_i$ , *c* and  $\tau_h$ .

#### 3.2 Activists' net gain

Activists' inside options represent what they would get from reaching a political agreement with party opportunists. Thus, activists' utility from being part of party *i*'s coalition is given by (5). Now, what is the utility activists *i* get if they do not reach an agreement with the opportunists i?<sup>17</sup> In the case of disagreement, activists cannot influence policy. However, they can still have a positive expected utility,  $\Omega_i$ , which we define as follows:

$$\Omega_{i} = \pi_{i} v_{i} \left( \widetilde{s}_{i} \right) + \left( 1 - \pi_{i} \right) v_{i} \left( \widetilde{s}_{i} \right)$$

where  $\pi_i$  is the probability that party *i* wins the election when activists *i* are not part of the coalition. Since we focus on modelling the case in which both parties form a coalition, we consider  $\Omega_i$ ,  $\tilde{s}_i$  and  $\tilde{s}_j$  as given.<sup>18</sup>

The net gain,  $\phi_i$ , is then

$$\phi_i = v_i \left( s_i \right) - c_i - \Omega_i \tag{11}$$

We shall assume that  $\phi_i$  is decreasing and concave in policy  $s_i$  Similarly,  $\phi_i$  is increasing an concave in policy  $s_j$ .<sup>19</sup>

Activists will participate in negotiation only when  $c_i < v_i(s_i) - \Omega_i$ .

## 3.3 Coalition formation and electoral equilibrium

Suppose now that coalition j proposes policy  $s_j$ , which generate an internal contribution level of  $c_j$ . The two factions of coalition i now bargain to an equilibrium: we take Nash bargaining as the procedure.<sup>20</sup> The Nash bargaining solution between these two factions of party i is the pair of policy and contribution  $(s_i, c_i)$  that maximizes the following Nash product:

$$\max_{s_i,c_i} \ln \psi_i \left( s_i, \overline{s_j}, c_i, \overline{c_j} \right) + \ln \phi_i \left( s_i, \overline{s_j}, c_i \right)$$

<sup>19</sup>Formally, our assumptions are:  $\frac{\partial \phi_i}{\partial s_i} < 0$  and  $\frac{\partial \phi_i}{\partial s_i^2} < 0$ ;  $\frac{\partial \phi_i}{\partial s_j} < 0$  and  $\frac{\partial^2 \phi_i}{\partial s_j^2} < 0$ ;  $\frac{\partial \phi_j}{\partial s_j^2} < 0$ ;  $\frac{\partial \phi_j}{\partial s_i^2} < 0$ . <sup>20</sup>Since both coalitions are bargaining simultaneously, it simplifies the analysis

<sup>&</sup>lt;sup>17</sup>In this model, it is natural that activists i never form a coalition with opportunists j.

j. <sup>18</sup>Note that, when activists are part of a coalition they are not expected utility maximizers. A main reason is that within the coalition, the opportunists are those who care only about the probability of winning; therefore, if activists include in their valuation such a probability they will give a bargaining advantage to the opportunists, which would make themselves worst-off in the bargaining outcome.

<sup>&</sup>lt;sup>20</sup>Since both coalitions are bargaining simultaneously, it simplifies the analysis further to assume, as we do, that potential members of each coalition assume that the other coalition will reach an agreement.

where,  $\psi_i(s_i, \overline{s_j}, c_i, \overline{c_j})$  and  $\phi_i(s_i, \overline{s_j}, c_i)$  are defined as in 10 and 11.

Similarly, facing a policy  $s_i$  and contribution  $c_i$  from party i, party j's opportunists and activists Nash-bargain to a  $(s_j, c_j)$  that solves

$$\max_{s_j,c_j} \ln \psi_i\left(\overline{s_i}, s_j, \overline{c_i}, c_j\right) + \ln \phi_i\left(\overline{s_i}, s_j, c_j\right)$$

We define a Nash-bargaining equilibrium in the two party game as a pair of policies  $(s_i, s_j)$  and contributions  $(c_i, c_j)$  such that, facing  $s_j$ and  $c_j$ , party *i*'s factions Nash bargain to  $s_i$  and  $c_i$ , and facing  $s_i$  and  $c_i$ , party *j*'s factions Nash-bargain to  $s_j$  and  $c_j$ .

For coalition *i*, the required first order condition with respect to  $s_i$  is

$$\frac{\sum \theta_h u_h^{'}(s_i)}{\psi_i} + \frac{v_i^{'}(s_i)}{\phi_i} = 0; \qquad (12)$$

the first order condition with respect to  $c_i$  is

$$\frac{\frac{\gamma_i}{(c+c_i)^2} \sum \theta_h \tau_h}{\psi_i} - \frac{1}{\phi_i} = 0.$$
(13)

It is interesting to note that  $\frac{\partial \phi_i}{\partial s_i} = v'_i(s_i) < 0$  because activists are better off the smaller  $s_i$  by assumption. By subtraction this implies  $\frac{\partial \psi_i}{\partial s_i} > 0$ . The negotiation here implies that inside coalition *i*, both parties compromise with respect to policy and contributions. Opportunists move away from the centre of the mass of voters and activists get less policy then they would like if they could choose policy without having to bargain (in which case they would set policy so that  $v'_i(s_i) = 0$ ).

Taking the ratios of (12) and (13), the Nash bargaining equilibrium inside coalition *i* can be described by (14),

$$\frac{\sum_{i=1}^{n} \theta_h u_h^{\dagger}(s_i)}{\frac{\gamma_i}{(c+c_i)^2} \sum_{i=1}^{n} \theta_h \tau_h} + v_i^{\dagger}(s_i) = 0.$$
(14)

Similarly, for coalition j, the Nash bargaining equilibrium is

$$\frac{\sum_{j=1}^{n} \theta_h u_h'(s_j)}{\left(\frac{\gamma_j}{(c+c_j)^2} \sum_{j=1}^{n} \theta_h \tau_h} + v_j'(s_j) = 0, \qquad (15)$$

which differs from equation (14) because the signs on the partial derivatives with respect to s are reversed. It is useful to note that in (14) and (15)  $\sum \theta_h u'_h(s_k)$  is the marginal change in expected votes with respect to  $s_k$  and  $\frac{\gamma_i}{(c+c_k)^2} \sum \theta_h \tau_h$ is the marginal change in expected votes with respect to  $c_k$ . Thus, the ratios  $MRS^{\psi_k}_{s_k,c_k} \equiv \frac{\sum \theta_h u'_h(s_k)}{\frac{\gamma_k}{(c+c_k)^2} \sum \theta_h \tau_h} \equiv \frac{\frac{\partial EV_k}{\partial s_k}}{\frac{\partial EV_k}{\partial c_k}}$  and  $MRS^{\phi_k}_{s_k,c_k} \equiv \frac{v'_k(s_k)}{1} \equiv \frac{\frac{\partial u_k}{\partial s_k}}{\frac{\partial u_k}{\partial c_k}}$ represent the marginal rate of substitution between policy and contributions of the two negotiating factions. For later use, it is helpful to write the Nash bargaining equilibrium in the following way:  $MRS^{\psi_k}_{S_k,C_k} \equiv MRS^{\phi_k}_{S_k,C_k}$  the marginal rate of substitutions in each coalition, (14) and (15), of opportunists and activists must be equal in equilibrium.

The *electoral equilibrium* is a configuration of policy positions and activists contributions in both coalitions such that no member of either coalition can improve their own situation either with respect to their coalition partner or with respect to the opposing party.

The electoral equilibrium occurs when (14) and (15) hold simultaneously, so that the following clearly holds in equilibrium:

$$\frac{\sum_{i}^{\gamma_{i}} \theta_{h} u_{h}^{'}(s_{i})}{\frac{\gamma_{i}}{(c+c_{i})^{2}} \sum_{i}^{\gamma_{i}} \theta_{h} \tau_{h}} + v_{i}^{'}(s_{i}) - \frac{\sum_{i}^{\gamma_{j}} \theta_{h} u_{h}^{'}(s_{j})}{\frac{\gamma_{j}}{(c+c_{j})^{2}} \sum_{i}^{\gamma_{j}} \theta_{h} \tau_{h}} - v_{j}^{'}(s_{j}) = 0 \qquad (16)$$

The electoral equilibrium is a configuration of policy positions and activists contributions in both coalitions such that no member of either coalition can improve their own situation either with respect to their coalition partner or with respect to the opposing party.

Condition (16) is used in the Appendix to derive comparative statics results.

## 4 The consequences of asymmetric shocks in party valence

The parameter  $\gamma$  represents electoral uncertainty facing a political party. One also may think of  $\gamma$  as the electoral "wind" that blows with more or less strength against (or in favour) of a coalition. A larger  $\gamma_k$  represents a worsening in the political situation for coalition k. In this section, we study the consequences for activism and policy platforms of changes in the degree of uncertainty  $\gamma_k$ .

The following Proposition shows that an asymmetric shock to uncertainty causes party platforms divergence. The second Proposition deals with the consequences for the degree of party activism. **Proposition 1** An increase in the uncertainty  $\gamma_k$  facing one coalition causes greater divergence in equilibrium policy platforms.

$$\begin{cases} \frac{ds_i}{d\gamma_k \text{ other variables constant}} < 0\\ \frac{ds_j}{d\gamma_k \text{ other variables constant}} > 0 \end{cases}$$
(17)

with k = i, j.

#### **Proof.** See Appendix. ■

An increase in  $\gamma_i$  means that party *i* faces a worsening environment, where more money is needed to deal with greater uncertainty about its platform. As a consequence, i's expected vote declines and this affects its inter-coalition bargaining. Opportunists i react to the increase in uncertainty by giving up policy position to get more resources from their activists, and thus move further to the left of the mass of the voters. Interestingly, coalition j also moves, and to the right, so that the distance  $s_i - s_j$  increases. Since coalition *i* is more extreme, opportunists in party *j* can afford to trade policy position for money from its activists while still improving their total expected vote. Activists in j want this move to the right because their original negotiation led them to compromise with their opportunists. Opportunists and Activists in coalition j have now the possibility for mutual gains due to the worsening in the situation of coalition i by increasing  $s_i$ . Nash bargaining requires this mutual beneficial exchange of policy for resources occurs if the coalition is to survive.

The impact of asymmetric shocks in the non-policy variables on activists' contributions is reported in the following Proposition, which confirms the increase in activism or idealism expected in both coalitions; where, activism or idealism are measured by the amount of resources (or effort) contributed by activists.

**Proposition 2** If outside contributions are not allowed, the degree of party activism increases when there is an asymmetric increment in the uncertainty facing any coalition.

$$\begin{cases} \frac{dc_i}{d\gamma_k \text{ other variables constant}} > 0\\ \frac{dc_j}{d\gamma_k \text{ other variables constant}} > 0 \end{cases}$$
(18)

with k = i, j.

**Proof.** See appendix.

Thus, greater asymmetric uncertainty leads to more idealism in parties and to more money in political campaigns.

#### 5 Public financing of electoral campaigns

We have allowed for the possibility of public financing as an alternative to financing by activists. The following Proposition shows what happens when public financing increases, and confirms the fears of the German Electoral Commission that public financing tends to drive idealism (or activism) out of political parties.

**Proposition 3** An equilibrium in which external private contributions are not allowed, is one in which an increase in public financing leads to both policy platform convergence and less activism/contributions,

$$\begin{cases} \frac{ds_i}{dc \text{ other variables constant}} > 0\\ \frac{ds_j}{dc \text{ other variables constant}} < 0\\ \frac{dc_k}{dc \text{ other variables constant}} < 0 \end{cases}$$
(19)

with k = 1, j.

#### **Proof.** See Appendix.

Party opportunists become more independent when they receive greater public funding. They do not need activists' contribution as before and for this reason they tend to converge more. Consequently, when outside contributions are prevented, activism will increase.

Similar effects, but with opposite signs, of changes in public financing are obtained when symmetric shocks in the gammas occur, as stated in the following Corollary.

**Corollary 1** An equilibrium in which external private contributions are not allowed, is one in which a symmetric increase of uncertainty leads to policy divergence and more activists' contributions,

$$\begin{cases}
\frac{ds_i}{d(\Delta\gamma_i = \Delta\gamma_j)} & < 0 \\
\frac{ds_j}{d(\Delta\gamma_i = \Delta\gamma_j)} & > 0 \\
\frac{dc_k}{d(\Delta\gamma_i = \Delta\gamma_j)} & \\
\frac{dc_k}{d(\Delta\gamma_i = \Delta\gamma_j)} & \\
\frac{dc_k}{other \ variables \ constant} & > 0
\end{cases}$$
(20)

with k = 1, j.

#### **Proof.** See Appendix. ■

Symmetric shocks in the non policy components lead to policy divergence because both parties need more money and/or effort from their activists. The difference with Proposition 1 is in the reasons why policy platforms diverge. An asymmetric increase in the uncertainty leads the damaged party to diverge in order to gain more contributions, while the opposing party diverges because of the Nash bargaining equilibrium, which imposes a mutual beneficial move between the two factions.

It can be easily shown that equivalent to the symmetric shock in the gammas is the case of changes in voters sensitivity to the non-policy components,  $\tau_h$ .

## 6 Bargaining inside a coalition with outside options

We extend the model by allowing external private contributions by lobby ists who can also choose to be activists inside a coalition. In a bargaining model, this means that opportunists either get  $c_i$  if they reach an agreement with the activists, or  $\tilde{c_i}^i$  if activists *i* contribute as lobby ists from outside. Similarly, activists can now affect their outside option by supporting either party when they are outside a coalition, so that activists *i* will contribute either  $c_i$  to coalition *i* if they reach an agreement with the opportunists *i* or  $\tilde{c_i}^i$  to party *i* and  $\tilde{c_i}^j$  to party *j* otherwise.

We do not explicitly solve for the parties' outside options<sup>21</sup>; we are interested rather in the inside versus outside productivity of money and its impact on the bargaining equilibrium inside each coalition. In order to proceed, we introduce a necessary participation condition for activists which requires that activists are indifferent between being inside or outside a coalition when for any given amount of money they spend inside, they gain the same utility if they spend the same amount outside.

The opportunists' disagreement utility defined in equation (8) then becomes:

$$\widehat{EV}_{i}^{d}(s,c) = \sum_{h=1}^{N} \theta_{h} \left\{ \left[ \left( u_{h}\left(\widetilde{s}_{i}\right) - u_{h}\left(\widetilde{s}_{j}\right) \right) - \tau_{h} \left( \frac{\gamma_{i}}{c + \widetilde{c}_{i}^{i}} - \frac{\gamma_{j}}{c + c_{j} + \widetilde{c}_{i}^{j}} \right) \right] - b_{h}^{\min} \right\};$$

<sup>&</sup>lt;sup>21</sup>A more explicit modelling of both opportunists' and activists' outside options is left for future research. One extension of the model, for example, could explore the strategic behavuoir of opportunists who could tactically affect the formation of the opposing coalition by moving policy  $\tilde{s}_k$  advantageously. For instance, they can alter the bargaining power of the two partners forming the competing coalition by moving policy position in favour of the opposing activists. Activists in the opposing coalition will then have a better incentive to move outside or might use their increased strength to claim more policy in their favour.

Similarly, a future development may consider a situation in which a group of activists engage in a double bargaining with the two parties.

where,  $(\tilde{c}_i^{\ i} + \tilde{c}_i^{\ j})$  are activists' total contributions when they are outside as lobbyists. The greater the opportunists' disagreement utility or outside option, the more substantial their bargaining power because they then have less to gain in having the activists inside a coalition.<sup>22</sup>

Opportunists' net gain becomes:

$$\widehat{\psi}_{i} = \sum_{h=1}^{N} \theta_{h} \left\{ \left( u_{h}\left( s_{i} \right) - u_{h}\left( s_{j} \right) \right) - \left( u_{h}\left( \widetilde{s}_{i} \right) - u_{h}\left( \widetilde{s}_{j} \right) \right) + \tau_{h} \left[ \gamma_{j} \left( \frac{1}{c + c_{j}} - \frac{1}{c + c_{j} + \widetilde{c}_{i}^{j}} \right) - \gamma_{i} \left( \frac{1}{c + c_{i}} - \frac{1}{c + \widetilde{c}_{i}^{i}} \right) \right]$$
(21)

where a tilde over a platform s denotes a policy position taken as given by lobbyists, rather than a policy over which opportunists and activists are bargaining. The term  $(u_h(s_i) - u_h(s_j)) - (u_h(\tilde{s}_i) - u_h(\tilde{s}_j))$ represents the part of the opportunists' net gain from cooperating that depends exclusively on the differences in policy positions offered to activists and to lobbyists. The term  $\gamma_j \left(\frac{1}{c+c_j} - \frac{1}{c+c_j+\tilde{c}_i^j}\right) - \gamma_i \left(\frac{1}{c+c_i} - \frac{1}{c+\tilde{c}_i^i}\right)$ in equation (21) measures the impact on the net gain of party valences, respectively of coalition j and i, of opportunists i running for the election as part of a coalition, with respect to the situation in which they compete without forming a coalition. Note that:  $\gamma_j \left(\frac{1}{c+c_j} - \frac{1}{c+c_j+\tilde{c}_i^j}\right) \ge 0$  and  $\gamma_i \left(\frac{1}{c+c_i} - \frac{1}{c+\tilde{c}_i^i}\right) \leqslant 0$ . In particular,  $-\gamma_i \left(\frac{1}{c+c_i} - \frac{1}{c+\tilde{c}_i^i}\right) \ge 0$  if  $c_i \ge \tilde{c}_i^{j}$ and  $-\gamma_i \left(\frac{1}{c+c_i} - \frac{1}{c+\tilde{c}_i^i}\right) < 0$  if  $c_i < \tilde{c}_i^{j}$ . This last inequality suggests that it may be advantageous for party i to form a coalition with activists only in order to deny funds to the opposition provided by these activists if they were left outside as lobbyists.

The necessary participation condition for activists to join a party is that for any given amount they contribute to the coalition, they must gain inside at least as much as they would if they spent the same amount lobbying outside. If the amount of money spent inside is  $c_i$  and that spent outside on i and j is  $\tilde{c}_i^i + \tilde{c}_i^j$ , activists compare the inside and outside options for  $c_i = \tilde{c}_i^i + \tilde{c}_i^j$ . We can now write activists' outside option or gain as a lobbyist from spending  $c_i$  as:

$$\widehat{\Omega}_{i} = \pi_{i} \left( c_{i} - \widetilde{c}_{i}^{j} \right) v_{i} \left( \widetilde{s}_{i} \right) + \left( 1 - \pi_{i} \left( c_{i} - \widetilde{c}_{i}^{j} \right) \right) v_{i} \left( \widetilde{s}_{j} \right) - c_{i}, \qquad (22)$$

<sup>&</sup>lt;sup>22</sup>We assume that the only information opportunists know about the outside lobbying behavior of the activists is that they will distribute a certain amount of resources  $\widetilde{c_i}^i + \widetilde{c_i}^j$  to the parties. Opportunists by assumption do not strategize over the amount of money activists may give outside as lobbyist. Relaxing this assumption involves substantial complexity and is left for future research.

where  $\pi_i(\tilde{c}_i^i) = \pi_i(c_i - \tilde{c}_i^j)$  is the probability that party *i* gets elected when lobbyist *i* contributes  $\tilde{c}_i^i$ , and  $\tilde{s}_i$  is the policy offered to lobbyists by opportunists *i*. We note that to simplify the model, in the Nash bargaining in coalition *i*, the policy offered to activists if they move outside, as well as that of the opposing coalition, is taken as given in the bargaining process.

The activists' outside option can also be written in the following form<sup>23</sup>:  $\widehat{\Omega}_i = v_i(\widetilde{s}_j) + \pi_i(c_i - \widetilde{c}_i^j)(v_i(\widetilde{s}_i) - v_i(\widetilde{s}_j)) - c_i$ . As lobbyists, activists will get at least  $v_i(\widetilde{s}_j)$  for sure, plus the expected difference between the policy implemented by coalition *i* and the policy implemented by coalition *j*, minus  $c_i$ . Clearly lobbyists will not contribute from outside if platforms converge because they would not get more than the common policy  $\widetilde{s}$  while still paying for it. Activist's net gain  $\widehat{\phi}_i$  is then  $v_i(s_i) - c_i - \widehat{\Omega}_i$  or

$$\widehat{\phi}_{i} = v_{i}\left(s_{i}\right) - v_{i}\left(\widetilde{s}_{j}\right) - \pi_{i}\left(c_{i} - \widetilde{c}_{i}^{j}\right)\left(v_{i}\left(\widetilde{s}_{i}\right) - v_{i}\left(\widetilde{s}_{j}\right)\right)$$
(23)

The activists' participation condition for coalition *i* can be obtained by setting the net gain equal to zero; i.e.,  $v_i(s_i) = \pi_i (c_i - \tilde{c}_i^j) v_i(\tilde{s}_i) + (1 - \pi_i (c_i - \tilde{c}_i^j)) v_i(\tilde{s}_j)$ .

For coalition k = i, j, the bargaining solution now solves the following maximization problem

$$\max_{s_k,c_k} \widehat{\psi}_k \widehat{\phi}_k \tag{24}$$

After repeating the same steps in the Section 3.3, the new electoral equilibrium can be conveniently stated using equation (25), which is just a restatement of equation (16) using MRSs:

$$MRS_{s_{i},c_{i}}^{\psi_{i}} + MRS_{s_{i},c_{i}}^{\phi_{i}} = MRS_{s_{j},c_{j}}^{\psi_{j}} + MRS_{s_{j},c_{j}}^{\phi_{j}}.$$
 (25)

Policies platforms  $(s_i^*, s_j^*)$  and contributions  $(c_i^*, c_j^*)$  constituting a Nash electoral equilibrium satisfy equation  $(25)^{24}$ .

<sup>23</sup>Given  $c_i$ , the outside option is maximized when  $\frac{\partial \pi_i(c_i^i)}{\partial c_i^j} v_i(\widetilde{s}_i) = 1$  and

 $\frac{\partial \pi_{i}\left(\widetilde{c_{i}^{j}}\right)}{\partial \widetilde{c_{i}^{j}}}v_{i}\left(\widetilde{s_{j}}\right) = 1, \text{ which implies } \frac{\partial \pi_{i}\left(\widetilde{c_{i}^{j}}\right)}{\partial \widetilde{c_{i}^{j}}}v_{i}\left(\widetilde{s_{i}}\right) = \frac{\partial \pi_{j}\left(\widetilde{c_{i}^{j}}\right)}{\partial \widetilde{c_{i}^{j}}}v_{i}\left(\widetilde{s_{j}}\right).$  From outside, activists equalize the marginal expected utilities from contributing to either coalition.

<sup>&</sup>lt;sup>24</sup>For the proof see Appendix.

## 7 Activism, party financing and outside options

It can be verified that the comparative statics with respect to policy platforms when any exogenous parameter changes is not affected by the introduction of an outside option<sup>25</sup>. However, the impact of asymmetric shocks in the non-policy variables on activists' contributions is not straightforward. The main reason is that the more activists' contribute inside, the higher is the value of their outside option too<sup>26</sup>. The following Proposition captures this solution:

**Proposition 4** In equilibrium, if outside private financing is allowed, activists' contributions may either increase or decrease when there is a shock in the degree of uncertainty facing any coalition. In particular,

$$\begin{cases} \frac{dc_i}{d\gamma_k \text{ other variables constant}} \begin{pmatrix} > 0 \text{ if } \frac{\partial MRS^{\psi_i}}{\partial c_i} > \frac{\partial MRS^{\phi_i}}{\partial c_i} \\ < 0 \text{ if } \frac{\partial MRS^{\psi_i}}{\partial c_i} < \frac{\partial MRS^{\phi_i}}{\partial c_i} \\ \end{pmatrix} \\ \frac{dc_j}{d\gamma_k \text{ other variables constant}} \begin{pmatrix} > 0 \text{ if } \frac{\partial MRS^{\psi_j}}{\partial c_j} > \frac{\partial MRS^{\phi_j}}{\partial c_j} \\ < 0 \text{ if } \frac{\partial MRS^{\psi_j}}{\partial c_j} < \frac{\partial MRS^{\phi_j}}{\partial c_j} \end{cases}, \qquad (26)$$

with k = i, j.

#### **Proof.** See Appendix.

In order to get the intuition behind the above proposition, we compare it with Proposition 2. If outside contributions are forbidden or limited by campaign regulation, then results are simple and straightforward: without activists' outside contributions the more the non-policy component matters, the more money and activism will be involved in coalitions. Similarly, uncertainty increases the amount of money spent in the electoral campaign no matter who is benefitting from the non-policy related factors.

The difference between the two results is in the impact of the outside option that creates the ambiguity. An increase in the inside contributions by activists produces an increase in the value of the outside option because the probability,  $\pi_i$ , that party *i* would win the election if activists were outside increases in  $c_i$  at a decreasing rate. As a result, activists' net gain from contributing inside the coalition decreases in  $c_i$ at an increasing rate. This means that  $c_i$  increases in  $\gamma_k$  if the marginal rate of substitutions are such that activists are also compensated for the

<sup>&</sup>lt;sup>25</sup>See Appendix.

<sup>&</sup>lt;sup>26</sup>A consequence of the introduction of an outside option is that activists' net gain is decreasing and convex in contribution; i.e.  $\partial \phi_i / \partial c_i < 0$  and  $\partial \phi_i / \partial c_i^2 > 0$ .

increasing utility they would get from outside, and this happens when  $\frac{\partial MRS^{\psi_i}}{\partial c_i} > \frac{\partial MRS^{\phi_j}}{\partial c_i}$ . The same logic applies to  $c_j$ .

Similarly, positive and symmetric shocks in the gammas lead to the following changes in the sign of activists' contributions:

$$\frac{dc_k}{d\left(\Delta\gamma_i = \Delta\gamma_j\right)_{\text{other variables constant}}} \begin{cases} > 0 \text{ if } \frac{\partial MRS^{\psi_k}}{\partial c_k} > \frac{\partial MRS^{\psi_k}}{\partial c_k} \\ < 0 \text{ if } \frac{\partial MRS^{\psi_k}}{\partial c_k} < \frac{\partial MRS^{\psi_k}}{\partial c_k} \end{cases}$$
(27)

with  $k = i, j.^{27}$  Instead, we obtain a different result when we consider shocks in the level of public financing<sup>28</sup>:

$$\frac{dc_k}{dc}_{\text{other variables constant}} \begin{cases} < 0 \text{ if } \frac{\partial MRS^{\psi_k}}{\partial c_k} > \frac{\partial MRS^{\phi_k}}{\partial c_k} \\ > 0 \text{ if } \frac{\partial MRS^{\psi_k}}{\partial c_k} < \frac{\partial MRS^{\phi_k}}{\partial c_k} \end{cases} .$$
(28)

In contrast to result (19), result (28) suggests that inside contributions are not necessarily reduced when an outside option is available. In particular, activists have a higher marginal benefit from giving contributions when parties converge to the centre. For this reason, they may be willing to contribute more in order to reduce convergence.

## 8 Consequences of the regulation of campaign finance

It is useful to draw together the results derived above concerning the consequences of public financing and regulation of party financing. The following table summarizes the impact on electoral equilibrium and the degree of activism of public financing and of regulation of outside contributions by lobbyists.

	Outside financing prohibited	Outside financing allowed
Public financing	convergence; $c_k \downarrow$	convergence; $c_k \Uparrow \Downarrow$
No public financing	divergence; $c_k \uparrow$	divergence; $c_k \uparrow \downarrow$

 $<sup>^{27}</sup>$ For the proof, see Appendix.

<sup>&</sup>lt;sup>28</sup>For the proof, see Appendix.

The table distinguishes between the four cases that have been covered, depending on whether public financing is provided and whether or not contributions from lobbyists are allowed. As the first row in the table illustrates, a regime with public financing is characterized by more centrist policies regardless of the regulation of lobbying, because parties do not need to cater to policy activists to the same extent to generate resources. When lobbying is explicitly prohibited, activism (or idealism) is definitely driven out of political parties. When it is allowed, parties still tend to converge but activism, as measured by the size of contributions, may or may not decline depending, as we have seen, on the marginal rates of substitution between policy position and resources inside and outside established in the equilibrium. The case in which public financing is absent, shown in the second row of the table, is opposite to the first row except with respect to the degree of activism when outside lobbying is allowed.

The results summarized in the table indicate that the question of whether or not policy should be designed to encourage activism in party politics, an issue raised by the German legislation pointed to in the Introduction, does not have a straightforward answer. Our results show that the most activism within parties will occur if outside financing is severely restricted and public financing is curtailed. But this encourages polarization of party platforms, and thus wider swings in government policies, which may be undesirable. Recent Canadian campaign finance legislation, to take another example, provides generous public financing while severely restricting outside contributions. It can be expected to lead to greater policy convergence and stability but also to less idealism in politics, a trade-off that may not be desirable.

## 9 Concluding remarks

We have presented a model of party coalition formation and political competition which includes reasons for policy-motivated activists to provide resources to particular party coalitions from inside the coalition rather than from outside. The model combines Nash-bargaining as the basis of party coalition formation and Nash non-cooperative competition between parties, in a manner pioneered by Roemer (2001). The model addresses questions of the role of activists in coalition formation and of the effects on political equilibrium of the resources party activists bring with them, allowing for the possibility that activists may choose instead to contribute to one or both parties from outside as 'lobbyists'. Specific comparative static experiments with the framework we have developed concern the effects on equilibrium platforms and the degree of party activism of asymmetric changes in party valence and in the regulation and public financing of campaign finance. In constructing this model, we have linked two types of literature: one focusing on the consequences of coalition formation for the nature of equilibrium in the absence of campaign contributions, and the other where competing parties that are strictly electorally oriented are provided with resources by contributors who are always external to them.

We have shown that in this Nash bargaining, Nash competition framework, an asymmetric shock to one party's electoral valence leads to divergence of party platforms in equilibrium. The opportunists in the party suffering the shock require more resources from their activists, and these can be had only by delivering to them a more extreme policy platform. The fact that the opposing coalition that indirectly benefits from this asymmetric shock also moves its platform in the opposite direction is less straightforward. This result depends crucially on the assumption that parties are coalitions formed by Nash bargaining, a structure which requires both opportunists and activists in that party to share the gains of the now more favorable political environment.

More uncertainty, whether as a result of asymmetric or symmetric shocks to valence, always increases activists' participation when outside contributions to parties are forbidden by regulation. However, when the option of contributing to one or both parties from outside is available, we cannot conclude that increased uncertainty (which increases activists bargaining power in the coalition) drives activism, as measured by activists' resource contributions, out of the party coalitions. The reason is that now, as activists contribute more to any coalition, the value of their outside option also changes, and the degree of activism in equilibrium then depends on marginal rates of substitution between policy and political resources, inside and outside party coalitions, that are established in equilibrium and which could in principle be consistent with a wide variety of outcomes. We have drawn out the implications of these results for the regulation of campaign finance, suggesting that there is an important trade-off between policy polarization and activism in politics that needs to be addressed by policy makers.

The paper contributes to the small but growing literature on electoral competition when parties are explicitly regarded as coalitions. Many interesting avenues of research remain, including the incorporation of a fuller treatment of the lobbying alternative.

## 10 Appendix

## 10.1 Derivation of opportunists' expected vote function 4.

For the derivation of equation 4, we have assumed the utility bias of the "innate" valence, represented in 2, is perceived as a random variable from the perspective of parties, and have assumed that it has a uniform distribution<sup>29</sup> over the real interval  $[b_h^{\min}, b_h^{\max}]$  such that<sup>30</sup>

$$b_{h}^{\min} < \left(u_{h}\left(s_{i}\right) - u_{h}\left(s_{j}\right)\right) - \tau_{h}\left(\frac{\gamma_{i}}{c + c_{i}} - \frac{\gamma_{j}}{c + c_{j}}\right) < b_{h}^{\max}.$$

The assumption that the utility differential is always contained in the interval on which  $b_h$  is defined ensures that every voter always has a positive probability of voting for it.

If the cumulative distribution function for  $b_h$  is  $D(U_{h_i} - U_{h_j})$ , then the probability as seen by a party coalition that h votes for coalition iis given by the probability that  $b_h$  is less than the utility differential in favour of coalition i,  $U_{h_i} - U_{h_j}$ :

$$D(\cdot) = \theta_h \left\{ \left[ \left( u_h(s_i) - u_h(s_j) \right) - \tau_h \left( \frac{\gamma_i}{c + c_i} - \frac{\gamma_j}{c + c_j} \right) \right] - b_h^{\min} \right\}$$
(29)

where  $\theta_h = 1/(b_h^{\min} - b_h^{\max})$  is the sensitivity of the probability of voting to a change in individual welfare. Voters "whose behavior is perceived to be more sensitive to changes in welfare will be given more attention by party strategists" (Coughlin (2002), Hettich and Winer, 1999). The probability that h votes for coalition j is 1 - D. Using 29, the expected vote of coalition i from N voters can be expressed as in formula 4.

## 10.2 Proves of the comparative statics without external contributions

Denote with G the electoral equilibrium represented in equation 16, such that:

$$G = \frac{\sum_{i=1}^{n} \theta_{h} u_{h}'(s_{i})}{\frac{\gamma_{i}}{(c+c_{i})^{2}} \sum_{i=1}^{n} \theta_{h} \tau_{h}} + v_{i}'(s_{i}) - \frac{\sum_{i=1}^{n} \theta_{h} u_{h}'(s_{j})}{\frac{\gamma_{j}}{(c+c_{j})^{2}} \sum_{i=1}^{n} \theta_{h} \tau_{h}} - v_{j}'(s_{j}) = 0.$$
(30)

 $^{29}\mathrm{Note}$  that the area of the uniform distribution is one.

<sup>&</sup>lt;sup>30</sup>For further discussion of the admissible types of probability functions, see for example Enelow and Hinich (1989) and others.

Then,

$$G_{s_i} = \frac{\sum_{i=1}^{n} \theta_h u_h''(s_i)}{\frac{\gamma_i}{(c+c_i)^2} \sum_{i=1}^{n} \theta_h \tau_h} + v_i''(s_i) < 0$$

and similarly,  $G_{s_j} > 0$ ;

$$G_{\gamma_i} = -\frac{\frac{1}{(c+c_i)^2}\sum \theta_h \tau_h \sum \theta_h u'_h \overset{+}{\left(s_i\right)}}{\left[\frac{\gamma_i}{(c+c_i)^2}\sum \theta_h \tau_h\right]^2} < 0$$

and similarly,  $G_{\gamma_j} < 0$ ;

$$G_{d\gamma_{i}=\gamma_{j}=\gamma} = -\frac{\frac{1}{(c+c_{i})^{2}}\sum\theta_{h}\tau_{h}\sum\theta_{h}u_{h}^{+}\stackrel{+}{(s_{i})}}{\left[\frac{\gamma}{(c+c_{i})^{2}}\sum\theta_{h}\tau_{h}\right]^{2}} + \frac{\frac{1}{(c+c_{j})^{2}}\sum\theta_{h}\tau_{h}\sum\theta_{h}u_{h}^{'}\stackrel{-}{(s_{j})}}{\left[\frac{\gamma}{(c+c_{j})^{2}}\sum\theta_{h}\tau_{h}\right]^{2}} < 0$$

$$G_{c} = \frac{\frac{2\gamma_{i}}{(c+c_{i})^{3}}\sum\theta_{h}\tau_{h}\sum\theta_{h}u_{h}^{+}\stackrel{+}{(s_{i})}}{\left[\frac{\gamma_{i}}{(c+c_{i})^{2}}\sum\theta_{h}\tau_{h}\right]^{2}} - \frac{\frac{2\gamma_{j}}{(c+c_{j})^{3}}\sum\theta_{h}u_{h}^{'}\stackrel{-}{(s_{j})}}{\frac{\gamma_{j}}{(c+c_{j})^{2}}\sum\theta_{h}\tau_{h}} > 0$$

$$G_{c_{i}} = \frac{\frac{2\gamma_{i}}{(c+c_{i})^{3}}\sum\theta_{h}\tau_{h}\sum\theta_{h}u_{h}^{'}\stackrel{+}{(s_{i})}}{\left[\frac{\gamma_{i}}{(c+c_{i})^{2}}\sum\theta_{h}\tau_{h}\right]^{2}} > 0$$

and similarly,  $G_{c_j} > 0$ .

Combinations of the above differentiations of the G function leads to the comparative statics reported below.

**Proof of Proposition 1.** It is now straightforward to verify that

$$\begin{cases}
\frac{ds_i}{d\gamma_k \text{ other variables constant}} = -\frac{G_{\gamma_k}}{G_{s_i}} < 0 \\
\frac{ds_j}{d\gamma_k \text{ other variables constant}} = -\frac{G_{\gamma_k}}{G_{s_j}} > 0
\end{cases},$$
(31)

with k = i, j. **Proof of Proposition 2.** 

with k = i, j.

#### **Proof of Proposition 3.**

$$\begin{cases} \frac{ds_i}{dc \text{ other variables constant}} = -\frac{G_c}{G_{s_i}} > 0\\ \frac{ds_j}{dc \text{ other variables constant}} = -\frac{G_c}{G_{s_j}} < 0\\ \frac{dc_k}{dc \text{ other variables constant}} = -\frac{\frac{H}{G_c}}{G_{c_k}} < 0\\ + \end{cases},$$

with k = i, j. **Proof of Corollary 1.** 

$$\begin{cases} \frac{ds_i}{d(\Delta\gamma_i = \Delta\gamma_j)}_{\text{other variables constant}} = -\frac{G_{\Delta\gamma_i = \Delta\gamma_j}}{G_{s_i}} < 0\\ \frac{ds_j}{d(\Delta\gamma_i = \Delta\gamma_j)}_{\text{other variables constant}} = -\frac{G_{\Delta\gamma_i = \Delta\gamma_j}}{G_{s_j}} > 0\\ \frac{dc_k}{d(\Delta\gamma_i = \Delta\gamma_j)}_{\text{other variables constant}} = -\frac{G_{\Delta\gamma_i = \Delta\gamma_j}}{G_{s_j}} > 0 \end{cases}$$

with k = i, j.

## 10.3 Proof of the Nash non-cooperative electoral equilibrium represented by equation (25)

Let  $MRS_{s_k,c_k}^{\widehat{\psi}_k} \equiv \frac{\partial \widehat{\psi}_k}{\partial s_k} / \frac{\partial \widehat{\psi}_k}{\partial c_k} = \frac{\sum_{i=1}^{n} \theta_i u_h'(s_k)}{\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \theta_i u_h'(s_k)} \equiv \frac{\partial EV_k}{\partial c_k} / \frac{\partial EV_k}{\partial c_k}$  be the marginal rate of substitution of policy position and contributions for opportunists k and let  $MRS_{s_k,c_k}^{\widehat{\phi}_k} \equiv \frac{\partial \widehat{\phi}_k}{\partial s_k} / \frac{\partial \widehat{\phi}_k}{\partial c_k} = \frac{v_k'(s_k)}{\partial \widehat{\phi}_k / \partial c_k} \neq \frac{\partial u_k}{\partial s_k} / \frac{\partial u_k}{\partial c_k}$  be the marginal rate of substitution of policy position and contributions for activists k, with k = i, j. All the MRS are based on the participants' net gains. However, if the MRS of the opportunists is also equivalent to that computed on the opportunists' expected vote, the MRS of the activists, in this case, is not equivalent to that calculated on the activists' preferences.

The Nash bargaining equilibrium inside coalition k can then be described by the equation

$$MRS_{s_k,c_k}^{\widehat{\psi}_k} = MRS_{s_k,c_k}^{\widehat{\phi}_k},\tag{32}$$

which states that, inside each coalition, the bargaining equilibrium is one in which the marginal rates of substitution between the party platform and activists' contributions are equal for each coalition partner.

The electoral equilibrium in (25) is a straightforward combination of condition (32) for coalition i and j.

# 10.4 Proves of the comparative statics with outside options

We denote equilibrium (25) with  $F(\cdot)$ , such that

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$$F = \frac{\sum_{i=1}^{n} \theta_{h} u_{h}^{i}(s_{i})}{\sum_{i=1}^{n} (c_{i}+c_{i})^{2}} \sum_{i=1}^{n} \theta_{h} \tau_{h}^{i} - \frac{v_{i}^{'}(s_{i})}{-\pi_{i}^{'}(c_{i}-\widetilde{c}_{i}^{j})(v_{i}(\widetilde{s}_{i})-v_{i}(\widetilde{s}_{j}))} - \frac{\sum_{i=1}^{n} \theta_{h} u_{h}^{i}(s_{j})}{\sum_{i=1}^{n} \theta_{h} \tau_{h}^{i}} + \frac{v_{j}^{'}(s_{j})}{-\pi_{j}^{'}(c_{j}-\widetilde{c}_{j}^{i})(v_{j}(\widetilde{s}_{j})-v_{j}(\widetilde{s}_{i}))} = 0.$$
(33)

Then,

$$F_{s_{i}} = \frac{\sum_{i} \theta_{h} u_{h}^{\prime\prime}\left(s_{i}\right)}{\frac{\gamma_{i}}{\left(c+c_{i}\right)^{2}} \sum_{+} \theta_{h} \tau_{h}} - \frac{v_{i}^{\prime\prime}\left(s_{i}\right)}{-\pi_{i}^{\prime}\left(c_{i}-\widetilde{c}_{i}^{j}\right)\left(v_{i}\left(\widetilde{s}_{i}\right)-v_{i}\left(\widetilde{s}_{j}\right)\right)} < 0$$

and similarly,

$$F_{s_j} = -\frac{\sum_{j=1}^{n} \theta_h u_h''(s_j)}{\frac{\gamma_j}{(c+c_j)^2} \sum_{j=1}^{n} \theta_h \tau_h} + \frac{v_j''(s_j)}{-\pi_j'(c_j - \widetilde{c}_j^i)(v_j(\widetilde{s}_j) - v_j(\widetilde{s}_i))} > 0;$$

$$\begin{split} F_{\gamma_i} &= -\frac{\sum_{i=1}^{n} \theta_h u_h'\left(s_i\right)}{\frac{\gamma_i^2}{c+c_i} \sum_{i=1}^{n} \theta_h \tau_h} < 0, \\ F_{\gamma_j} &= \frac{\sum_{i=1}^{n} \theta_h u_h'\left(s_j\right)}{\frac{\gamma_j^2}{c+c_j} \sum_{i=1}^{n} \theta_h \tau_h} < 0 \end{split}$$

and similarly,

$$F_{\Delta\gamma_i=\Delta\gamma_j} = -\frac{\sum_{\substack{\gamma_i^2\\c+c_i}} \overset{+}{\sum_{\substack{\gamma_i^2\\c+c_i}}} \frac{\theta_h u_h'(s_i)}{\frac{\gamma_j^2}{c+c_j}} + \frac{\sum_{\substack{\gamma_j^2\\c+c_j}} \overset{-}{\sum_{\substack{\gamma_i^2\\c+c_j}}} \frac{\theta_h u_h'(s_j)}{\frac{\gamma_i^2}{c+c_j}} < 0;$$

$$F_{c} = + \frac{2\sum_{\substack{\gamma_{i} \\ \overline{c+c_{i}}}} \frac{\theta_{h}u_{h}'(s_{i})}{\Phi_{h}\tau_{h}}}{\frac{\gamma_{j}}{c+c_{j}}\sum_{\substack{\gamma_{i} \\ \overline{c+c_{j}}}} \frac{\varphi_{h}u_{h}'(s_{j})}{\Phi_{h}u_{h}'(s_{i})} > 0;$$

$$F_{c_{i}} = \frac{2\sum_{\substack{\gamma_{i} \\ \overline{c+c_{i}}}} \frac{\theta_{h}u_{h}'(s_{i})}{\Phi_{h}\tau_{h}}}{\frac{\pi_{i}''\left(c_{i} - \widetilde{c}_{i}^{j}\right)v_{i}'(s_{i})}{\left(-\pi_{i}'\left(c_{i} - \widetilde{c}_{i}^{j}\right)\right)^{2}\left(v_{i}\left(\widetilde{s_{i}}\right) - v_{i}\left(\widetilde{s_{j}}\right)\right)}} \begin{pmatrix} > 0 \text{ if } \frac{\partial MRS^{\psi_{i}}}{\partial c_{i}} > \frac{\partial MRS^{\phi_{i}}}{\partial c_{i}}}{\langle -\pi_{i}'\left(c_{i} - \widetilde{c}_{i}^{j}\right)\right)^{2}\left(v_{i}\left(\widetilde{s_{i}}\right) - v_{i}\left(\widetilde{s_{j}}\right)\right)}} \begin{pmatrix} > 0 \text{ if } \frac{\partial MRS^{\psi_{i}}}{\partial c_{i}} > \frac{\partial MRS^{\phi_{i}}}{\partial c_{i}}}{\langle -\pi_{i}'\left(c_{i} - \widetilde{c}_{i}^{j}\right)\right)^{2}\left(v_{i}\left(\widetilde{s_{i}}\right) - v_{i}\left(\widetilde{s_{j}}\right)\right)}} \begin{pmatrix} > 0 \text{ if } \frac{\partial MRS^{\psi_{i}}}{\partial c_{i}} > \frac{\partial MRS^{\phi_{i}}}{\partial c_{i}}}{\langle -\pi_{i}'\left(c_{i} - \widetilde{c}_{i}^{j}\right)\right)^{2}\left(v_{i}\left(\widetilde{s_{i}}\right) - v_{i}\left(\widetilde{s_{j}}\right)\right)} \begin{pmatrix} > 0 \text{ if } \frac{\partial MRS^{\psi_{i}}}{\partial c_{i}} > \frac{\partial MRS^{\phi_{i}}}{\partial c_{i}}}{\langle -\pi_{i}'\left(c_{i} - \widetilde{c}_{i}^{j}\right)\right)^{2}\left(v_{i}\left(\widetilde{s_{i}}\right) - v_{i}\left(\widetilde{s_{j}}\right)\right)} \begin{pmatrix} > 0 \text{ if } \frac{\partial MRS^{\psi_{i}}}{\partial c_{i}} > \frac{\partial MRS^{\phi_{i}}}{\partial c_{i}}} \rangle \\ < 0 \text{ if } \frac{\partial MRS^{\psi_{i}}}{\partial c_{i}} < \frac{\partial MRS^{\phi_{i}}}{\partial c_{i}}} \end{pmatrix}$$

and similarly,

$$F_{c_j} = -\frac{2\sum \theta_h u_h'\left(s_j\right)}{\frac{\gamma_j}{c+c_j} \left(\sum_{+} \theta_h \tau_h\right)^2} + \frac{\pi_j''\left(c_j - \widetilde{c}_j^i\right) v_j'\left(s_j\right)}{\left(-\pi_j'\left(c_j - \widetilde{c}_j^i\right)\right)^2 \left(v_j\left(\widetilde{s}_j\right) - v_j\left(\widetilde{s}_i\right)\right)} \begin{pmatrix} > 0 \text{ if } \frac{\partial MRS^{\psi_j}}{\partial c_j} > \frac{\partial MRS^{\phi_j}}{\partial c_j} \\ < 0 \text{ if } \frac{\partial MRS^{\psi_j}}{\partial c_j} < \frac{\partial MRS^{\phi_j}}{\partial c_j} \end{pmatrix}$$

Combinations of the above differentiations of the F function leads to the comparative statics studied below<sup>31</sup>.

Proves of policy divergence with outside option, when there are either symmetric or asymmetric shocks in the gammas. With symmetric shocks, the changes in policy platforms are given by

$$\begin{cases} \frac{ds_i}{d\gamma_k \text{ other variables constant}} &= -\frac{F\gamma_k}{F_{s_i}} < 0\\ \frac{ds_j}{d\gamma_k \text{ other variables constant}} &= -\frac{F\gamma_k}{F_{s_j}} > 0 \end{cases};$$

similarly, with asymmetric shocks we get

$$\begin{cases} \frac{ds_i}{d(\Delta\gamma_i = \Delta\gamma_j)}_{\text{other variables constant}} = -\frac{F_{\Delta\gamma_i = \Delta\gamma_j}}{F_{s_i}} < 0\\ \frac{ds_j}{d(\Delta\gamma_i = \Delta\gamma_j)}_{\text{other variables constant}} = -\frac{F_{\Delta\gamma_i = \Delta\gamma_j}}{F_{s_j}} > 0 \end{cases}$$

with k = i, j.

**Proof of convergence with public financing and outside option.** The proof is given by

$$\begin{cases} \frac{ds_i}{dc \text{ other variables constant}} = -\frac{F_c}{F_{s_i}} > 0\\ \frac{ds_j}{dc \text{ other variables constant}} = -\frac{F_c}{F_{s_j}} < 0 \end{cases}.$$

 $<sup>\</sup>frac{\cdot s_j}{^{31}\text{We now simplify the notation as follows: } MRS^{\widehat{\psi}_k} = MRS^{\widehat{\psi}_k}_{s_k,c_k} \text{ and } MRS^{\widehat{\phi}_k} = MRS^{\widehat{\phi}_k}_{s_k,c_k}.$ 

**Proof of Proposition 4.** It is now straightforward to verify that

$$\frac{dc_i}{d\gamma_k_{\text{ other variables constant}}} = -\frac{\bar{F_{\gamma_k}}}{\bar{F_{c_i}}} \begin{cases} > 0 \text{ if } \frac{\partial MRS^{\psi_i}}{\partial c_i} > \frac{\partial MRS^{\phi_i}}{\partial c_i} \\ < 0 \text{ if } \frac{\partial MRS^{\psi_i}}{\partial c_i} < \frac{\partial MRS^{\phi_i}}{\partial c_i} \end{cases}$$

and

$$\frac{dc_j}{d\gamma_k_{\text{other variables constant}}} = -\frac{\bar{F_{\gamma_k}}}{F_{c_j}} \begin{cases} > 0 \text{ if } \frac{\partial MRS^{\psi_j}}{\partial c_j} > \frac{\partial MRS^{\phi_j}}{\partial c_j} \\ < 0 \text{ if } \frac{\partial MRS^{\psi_j}}{\partial c_j} < \frac{\partial MRS^{\phi_j}}{\partial c_j} \end{cases},$$

with k = i, j. **Proof of result 27.** It can be easily checked that

$$\frac{dc_k}{d\left(\Delta\gamma_i = \Delta\gamma_j\right)}_{\text{other variables constant}} = -\frac{F_{\Delta\gamma_i = \Delta\gamma_j}}{F_{c_k}} \begin{cases} > 0 \text{ if } \frac{\partial MRS^{\psi_k}}{\partial c_k} > \frac{\partial MRS^{\phi_k}}{\partial c_k} \\ < 0 \text{ if } \frac{\partial MRS^{\psi_k}}{\partial c_k} < \frac{\partial MRS^{\phi_k}}{\partial c_k} \end{cases},$$

with k = i, j. **Proof of result 28.** It is straightforward to verify that

$$\frac{dc_k}{dc_{\text{ other variables constant}}} = -\frac{\overset{+}{F_c}}{\underset{\pm}{F_c}} \begin{cases} < 0 \text{ if } \frac{\partial MRS^{\psi_k}}{\partial c_k} > \frac{\partial MRS^{\phi_k}}{\partial c_k} \\ > 0 \text{ if } \frac{\partial MRS^{\psi_k}}{\partial c_k} < \frac{\partial MRS^{\phi_k}}{\partial c_k} \end{cases},$$

with k = i, j.

## References

- Adams, J. F., Merrill III, S., Grofman, B., (2005). "A Unified Theory of Party Competition. A Cross-National Analysis Integrating Spatial and Behavioral Factors". Cambridge University Press.
- [2] Aldrich, John, (1995). *Why parties?* Chicago: University of Chicago.
- [3] Aldrich, John, (1983a). "A Downsian Spatial Model with Party Activism." *Public Choice*, 41: 63-100.
- [4] Aldrich, John, (1983b). "A Spatial Model with Party Activists: Implication for Electoral Dynamics." *American Political Science Re*view, pp. 974-990.
- [5] Ansolabehere, S., and Snyder, J., M., (2000). "Valence Politics and Equilibrium in Spatial Election Models." *Public Choice*, 103: 327-336.
- [6] Austen Smith, David, (1987). "Interest groups, campaign contributions, and probabilistic voting". *Public choice*, No. 54.
- [7] Austen-Smith, David and Banks, Jeffrey S. (2005). "Positive Political Theory II: Strategy and Structure". Michigan Studies in Political Analysis. Ann Arbor: University of Michigan Press.
- [8] Banks, J. and Duggan and M. Le Breton (2002). "Bounds for Mixed Strategy Equilibria and the Spatial Model of Elections." *Journal of Economic Theory*, 103: 88-105.
- [9] Baron, D., (1994). "Electoral competition with informed and uninformed voters" *American Political Science Review* 88, 33-47.
- [10] Calvert, R. (1985). "Robustness of the Multidimensional Voting Model: Candidates, Motivations, Uncertainty and Convergence." *American Journal of Political Science*, 29: 69-85.
- [11] Coate, S., 2004. "Pareto-Improving Campaign Finance Policy," *American Economic Review*, American Economic Association, vol. 94(3), pages 628-655.
- [12] Coughlin, Peter, (1992). Probabilistic Voting Theory. Cambridge University Press.
- [13] Coughlin, Peter, and Nitzan, Shmuel, (1981). "Electoral Outcomes with Probabilistic Voting and Nash Social Welfare Maxima". Journal of Public Economics 15, 133-121.
- [14] Downs, Anthony, (1957). "An Economic Theory of Political Action in a Democracy." Journal of Political Economy, 135-150.
- [15] Enelow, James M., and Hinich, Melvin J., (1989) "A General Probabilistic Spatial Theory of Elections"; *Public Choice*; Vol. 61, No. 2; 101-113; #2250.
- [16] Herbert von Arnim, H., Schurig, M., (2004). "The European Party Financing Regulation". Recht: Forschung una Wissenschaft, LIT

VERLAG Munster.

- [17] Hettich, Walter, and Winer, Stanley, L., (1999). Democratic Choice and Taxation. A Theoretical and Empirical Analysis. Cambridge University Press.
- [18] Hinich, M., (1977). "Equilibrium in spatial voting: The Median Voter Theorem is an artefact." *Journal of Economic Theory*, 16: 208-219.
- [19] Hinich, M., (1983). "Comment on the Aldrich Paper". Public Choice, 41: 101-102.
- [20] Houser, D., and Stratmann, T., (2006). "Selling Favours in the Lab: Experiments on Campaign Finance Reform" CESifo Working Paper No. 1727.
- [21] Groseclose, T., (2001). "A Model of Candidate Location When One Candidate Has a Valence Advantage." *American Journal of Political Science*, Vol. 45, No. 4, 862-886.
- [22] Felli, Leonardo, and Merlo, Antonio, (2006): "Endogenous Lobbying," Journal of the European Economic Association, 4(1):180-215.
- [23] Levy, Gilat, (2004). "A Model of Political Parties", Journal of Economic Theory, Vol. 115(2), 2004, pp. 250-277.
- [24] Lindbeck, A., and Weibull, J. W., (1987). "Balanced-Budget Redistribution as the outcome of Political Competition". *Public Choice* 52:273-97.
- [25] Miller, Gary, and Schofield, Norman (2003). "Activists and Partisan Realignment in the United States," *American Political Science Review*, vol. 97, No. 2.
- [26] Mitchell, William, C. and Munger, Michael, C. (1991). "Economic Models of Interests Groups: An Introductory Survey". American Journal of Political Science, Vol. 35, No. 2, pp. 512-546.
- [27] Moon, W., (200). "Party Activists, Campaign Resources and Candidate Position Taking: Theory, Tests and Applications." B.J.Pol. 34, 611-633.
- [28] Roemer, E., John (2001). Political Competition. Theory and Applications. Cambridge, Massachusetts; London, England: Harvard University Press.
- [29] Roemer, E., John (2005). "Political equilibrium with private and/or public finance: A comparison of institutions". Mimeo, Department of Political Science and Economics, Yale University.
- [30] Schofield, Norman, (2005). "The Mean Voter Theorem under Proportional and Plurality Rule". Washington University.
- [31] Schofield, Norman, (2003). "Valence Competition in the Spatial Stochastic Model." *Journal of Theoretical politics*, 15(4): 371-383.
- [32] Schofield, Norman, and Sened, Itai, (2003). "Modelling the interac-

tion of Parties, Activist and Voters: Why is the Political Centre so Empty?" Center in Political Economy, Washington University.

- [33] Schofield, Norman, and Miller, Gary, and Martin, Andrew (2003).
   "Critical Elections and Political Realignments in the USA: 1860-2000," *Political Studies*, vol. 51, 217-240.
- [34] Stokes, D. (1963). "Spatial Models and Party Competition." American Political Science Review. 57: 368-377.
- [35] Stokes, D. (1993). "Valence politics," in Electoral Politics. (D. Kavanagh, ed.) Oxford: Clarendon Press.