XVIII CONFERENZA

SERVIZI PUBBLICI.
Nuove tendenze nella regolamentazione, nella produzione e nel finanziamento

Pavia, Università, 14 - 15 settembre 2006

# DEREGULATED WHOLESALE ELECTRICITY PRICES IN ITALY AN EMPIRICAL ANALYSIS

BRUNO BOSCO, LUCIA PARISIO and MATTEO PELAGATTI

pubblicazione internet realizzata con contributo della COMPAGNIA di San Paolo

# Deregulated Wholesale Electricity Prices in Italy
## *An Empirical Analysis*

Bruno Bosco, Lucia Parisio, Matteo Pelagatti
Università degli Studi di Milano-Bicocca

## Abstract

*In this paper we analyze a time series of daily average prices in the Italian electricity market, which started to operate as a Pool in April 2004. Our objective is to model the high degree of autocorrelation and the multiple seasonalities in electricity prices. We use periodic time series models with GARCH disturbances and leptokurtic distributions and compare their performance with more classical ARMA-GARCH processes. The within-year seasonal variation is modelled using the low-frequency components of physical quantities, which are very regular throughout the sample. Our results reveal that much of the variability in the price series is explained by the interactions between deterministic multiple seasonalities. Periodic AR-GARCH models seem to perform quite well in mimicking the features of the stochastic part of the price process.*

**Key words**: *Electricity auctions, Periodic Time Series, Conditional Heteroskedasticity, Multiple Seasonalities.*

**JEL codes**: C22, D44, L94, Q40.

## Introduction

Electricity prices as they are now determined in regulated (generally, Pool) markets, where private operators have replaced previously well established public enterprises, present specific behavioural characteristics. Prices in these Pool markets differ from the previous prices which were fixed by governments or public agencies. Despite the very limited storability and transportability of electricity, government-determined prices incorporated little uncertainty in their dynamics as they were generally capped by the imposition of price ceilings, resulting from the implementation of welfare-improving tariff policies. Market determined electricity prices, on the other hand, are strongly affected by the impossibility to arbitrage between time and space, and consequently are very volatile.

Time series of current electricity prices differ quite substantially from prices determined in the markets for financial assets or other type of commodities, since electricity cannot be treated like a stock. Hence, electricity prices have specific and somewhat unique characteristics (e.g. strong seasonalities and mean reversion), which in the recent past was the motivation for using time series modelling to study the specific features of their time patterns and to evaluate how prices are affected by temporal demand-supply imbalances, seasonality, transmission congestion and, to a lesser extent, by the features of the mechanism that generates the data (type of auction employed, price rule, degree of market concentration, etc.).

In Italy the privatization of the former public (quasi-) monopolist eventually led to the creation of an electricity Pool with some specific legal characteristics such as the presence on the demand side of a single buyer and which began operations in April 2004.

In this paper we try to describe the price dynamics of the Italian Pool and compare our findings with those obtained from analysis of other European markets. We also suggest a new methodology for dealing with multiple seasonalities that may prove useful for future econometric analysis of electricity prices. The stochastic part of the price evolution is modelled through periodic AR-GARCH processes, which are able to capture the different memories of each day of the week of the past.

Our results can be exploited directly by companies and traders for forecasting and for improving their bidding strategies, by regulators for identifying possible anomalous pricing behaviours, and, indirectly, by hedgers for the pricing of energy derivatives.

The paper is organized as follows. We first discuss the main characteristics of some European electricity markets, including the Italian Pool, for which data availability permits time series analysis. We emphasize differences in the organization and regulation of the markets as well as in their production structure (specifically, electricity generation), which might be important for explaining differences in the econometric results. We then review the existing econometric literature. The empirical analysis starts with a description of the general characteristics of the Italian data and concentrates on modelling the deterministic component of the data. We go on to describe and justify the choice of stochastic models and methods employed to account for the dynamics in the Italian prices. The paper concludes by presenting the results accompanied by some detailed comments.

## The electricity markets in Italy and other European countries

The Italian electricity market (IPEX) is organized as a Pool system, managed by a market operator (GME), that collects the bids, determines the merit order for the dispatch of electricity and is responsible for all auxiliary services. The Pool, initially planned to come into force in January 2001, actually began operations at the end of March 2004, as a one-side market. A single buyer, constituted in 1999, was responsible for guaranteeing the supply of electricity to a set of captive customers. Demand-side bidding was introduced in January 2005.

IPEX was the most recent Pool market to be created in Europe. Following the establishment in 1991 of the England and Wales (E&W) Electricity Pool, as a result of the liberalization of the British electricity market, competitive electricity markets have been organized in many European markets: [Nord Pool], Austria, France, Germany, Netherlands, Spain. The key features of most of these changes, including the Italian Pool market, were the privatization and restructuring of the existing vertically integrated monopolistic supplier. The second step was to organize the exchange of physical electricity in competitive wholesale spot markets through auctions. Competition was introduced at the retail level while transmission/distribution, which were still considered natural monopolies, remained under government regulation. This reorganization of the industry led to a separation of potentially competitive elements from natural monopolies.

The wholesale exchange of electricity poses some problems for regulators in relation to market architecture and design. First a decision must be made about whether to opt for a centralized Pool or for a decentralized market. In the former arrangement, all the electricity is allocated through the Pool; thus bilateral contracts are not allowed. All operators, both on the demand and the supply sides, submit hourly or half-hourly bids which are matched by a procedure that minimizes the cost of despatch. A decentralized electricity market such as NETA (England) or the Californian market, on the other hand, is organized as a series of voluntary forward and spot markets and bilateral contracting is allowed. The advantages of a Pool over a decentralized market are that demand and supply are continuously matched so that all coordination problems disappear. Advocates of the decentralized market structure, however, emphasize that the Pool may be affected by strategic bidding on the part of those operators with market power and, as a consequence, Pool prices do not generally reveal costs. Whilst theoretically this issue is still being debated, at an empirical level we find many examples, especially in Europe, of non-mandatory electricity Pools, where bilateral contracts are allowed. Such arrangements are probably motivated by the desire to capture the advantages of both types of schemes.

Electricity Pools work as multi-unit uniform price auctions: operators submit price/quantity offers, which are aggregated by the market operator in order to form a demand (where demand side bidding is allowed) and a supply curve. The equilibrium price and quantity are then determined by the usual crossing condition and all despatched producers receive the same System Marginal Price (SMP), equal to the bid made by the marginal unit in operation. This means that a) 24 or 48 auctions are held the day before delivery, one for each hour or half hour of the following day; b) all units that have been selected by the auction receive the SMP for the whole quantity they sell.

A characteristic that is common to the European electricity exchanges is the existence of demand side bidding. However, the opening of the bidding process to demand has not proceeded at the same pace in all countries. As already mentioned, in the Italian market, for instance, demand side bidding for a small portion of eligible consumers was only introduced from 1st January 2005.However, in line with the European Directive 2003/54/EC, all customers must be considered to be eligible by 1st July 2007. This means that on that date all consumers ought to be able to buy electricity directly in the day-ahead market.

In all European countries the electricity Pool market is managed by a market operator (MO, auctioneer), who collects bids and organizes the dispatch of units in a cost minimizing way. This auction-based dispatching does not take transmission conditions into account and so congestion may occur. The main features of the mechanisms implemented to manage congestion have changed in favour of a system compatible with bid-price-based optimization. When congestion occurs in the transmission line the market operator and the Transmission System Operators (TSO) try to relieve it at the minimum possible cost, in a market based way.

The electricity markets in Europe differ significantly in terms of their underlying production structure. This is significant since issues related to the market design become less severe when the industry is per se more competitive. It is well known that electricity can be generated in a variety of ways and using different types of inputs, which may be either renewable or non-renewable. The cost of the unit of energy supplied depends upon the technology and this influences the shape of the marginal cost function of the system and, hence, the system marginal price.

The productive mix of the electricity generating industry is thought to influence the market power of firms, their strategic behaviour and, finally, the price of energy. Nord Pool comprises countries in which a high percentage of production comes from hydro resources (56.7%) and in Austria this proportion is 69% of total production. Spain and France have similar proportions (11.8% and 11% respectively) of hydroelectric production, but France has a very high percentage of nuclear production (78%). The Netherlands and Germany account for only a small quota of hydroelectric production (0.1% and 4.2% respectively). France and Germany have recently installed wind plants. In Italy 13% of total gross production of electricity is obtained from hydro sources and around 80% from oil, gas and solid sources.

The Nordic area appears to have a more competitive power market and also the highest percentage of hydro plants installed. It is not surprising then, that prices in Finland, Sweden and Norway are well below the EU average. The French market is characterized by a high level of concentration (EDF has 90% share of the market) and by strong consumer protection, which results in low regulated tariffs. Power Next accounts for a small proportion of the total energy consumed (low level of liquidity, about 3%). This situation is similar in Germany, where only 11% of energy is traded on EEX. All the other European markets, including Italy, appear to be fairly concentrated and have a low liquidity share.

We can conclude that, across Europe, the level of concentration in generation is still high and this creates scope for market power and the ability to influence prices. The strong position of the incumbent operators has not been eroded in any significant way by investments in generation of new entrants. New generation assets normally entail significant investment costs, which are seen as a major barrier to entry. Uncertainties (high price volatility) associated with the power exchanges have also been seen as barriers to entry. Generation is a key issue for competition in the European electricity markets. The generators, due to the characteristics of the electricity market (the non-storability of electricity, the high inelasticity of demand, the very wide spectrum of costs of production and prices equal to the highest offer made on the power exchanges), are able to influence prices through the use of the generation capacity available to them either because they are indispensable or by forcing recourse to more expensive sources of supply by a withdrawal of capacity. Withdrawal of capacity is profitable if the cost of not producing is more than compensated for by the increase in SMP. A large number of low-cost plants adopt this strategy. In the case that they are indispensable for meeting demand, it is possible to raise SMP even with a relatively small portfolio of plants, depending on other offer constraints (e.g. the location of units). Therefore, the behaviour of generators can impact significantly on the level of prices, even at low levels of concentration.

## The existing literature

The modifications to the organisation of electricity markets have stimulated empirical studies of electricity prices both inside and outside of Europe.

Bhanot (2000) analyzes electric power prices in 12 Californian regional markets. The objective is to characterize and explain the high degree of autocorrelation and seasonality in power prices and address issues that are pertinent to the valuation and hedging of power-based financial contracts. He shows that price behaviour changes with each regional market, so that a firm that seeks to price or hedge power-based contracts must use instruments from the region in which it operates.

Escribano et al. (2002) use average daily prices for several markets (Nord Pool, Argentina, Victoria, New Zealand, Spain) and propose a general and flexible model that allows for deterministic seasonality, mean reversion, jumps and conditional heteroscedasticity. They use six nested versions of their model to analyze price behaviour in the different markets. Their results indicate that an AR(1)-GARCH(1,1) with jumps performs better than other versions.

Lucia and Schwartz (2002) present a model, which permits the definition of analytical formulae for derivative (futures) pricing. They employ dummy variables and sinusoids to deal with the seasonalities and an AR(1) for the autocorrelation structure.

Wilkinson and Winsen (2002) use Australian data to conduct a non-parametric test for the equality of peak and off-peak prices and for log-normality. They obtain mixed evidence: the null hypothesis of equal day effects and log-normality is rejected for some sub-sample periods and not for others.

Koopman at al. (2005) using European data, argue that there is no need to model conditional variance, when the conditional expectation of the price time series is properly modelled by means of periodic autoregressive (PAR) processes. They model seasonalities using sinusoids and weekday dummies. A PAR(1) model seems to be sufficient to fit the stochastic part of their data. They find evidence of mean reversion in the stochastic part of the model and periodic long memory in the North Pool prices.

Knittel and Roberts (2005) study the distributional and temporal properties of California prices, using several common asset price specifications (as well as other less conventional models). Results reveal several specific characteristics unique to electricity prices. They use zonal hourly electricity prices (Euro/MWh), when there is a separate market price in each zone. However, with no congestion arbitrage across zones this drives the price to a converging level. The degree of divergence then is an indicator of no arbitrage opportunities and when a high degree of correlation across "zonal" prices exists, then just one zone or the national time series of prices can be used.

Fabra and Toro (2005) use a Markov-Switching model on Spanish data to investigate collusive vs. cooperative behaviour of bidders.

The above literature generally adopts  a sort of two-step procedure. A preliminary data analysis is conducted; inspection of the data reveal the main characteristics of the dynamics of the electricity prices. On the basis of this examination it is invariably clear that the model applied in the second step of the analysis should integrate multiple seasonalities and reflect phenomena such as short (sometimes long) memory, mean reversion, high price-dependent volatility and leptokurtosis. As discussed above, methods to deal with seasonalities range from the use of dummy variables to the application of sinusoids at low seasonal frequencies (usually just the dominant $2\pi/365$ and the first harmonic $4\pi/365$).

In what follows we discuss the results from some previous studies by clustering them into sets of specific issues.

**Seasonality**

Real-time balancing and dependency on cyclical demand impose several different seasonal patterns on electricity prices (within day, week, year) almost everywhere. Deidersen and Trück (2002) study price series for Germany, New Zealand and Spain and report a strong intra-day pattern with a peak around midday. Moreover, they find that monthly mean prices are higher during daytime, and weekly seasonal patterns underline the presence of weekend effects. Annual seasonality is also present, with winter prices always higher than prices recorded in other seasons. Also Knittel and Roberts (2005) find that Californian electricity prices show intra-day seasonality and a "summer" (rather than a winter) effect, while Bhanot (2000), using US wholesale transaction prices recorded from 1$^{st}$ January 1995 to 1$^{st}$ June 1998 show that the seasonal means for peak and off-peak prices exhibit significant variations across the 12 months and across delivery points.

**Volatility**

Storage and transmission problems and the need for markets to be balanced in real time are responsible for an unusually high volatility. All the above reported empirical studies emphasize that there is a positive correlation between the standard deviation and the mean of the price process, making volatility dependent on the price level. Furthermore, many time series exhibit some *volatility clustering* making models for conditional heteroscedasticity opportune. When demand approaches and exceeds the limits of the system's generation capacity, prices are high and more volatile. Many authors model this using log-prices.

**Mean reversion**

By mean reversion we mean the absence of stochastic trends or martingale-like behaviour. This is a distinctive feature of electricity prices with respect to other commodity prices. Electricity prices do not behave like martingales, and the non-deterministic part of the data generating process does not seem to contain unit roots (e.g. no random walk like behaviour). When *hourly* prices rise they have to move downwards again in a relatively short time. It is thought that they oscillate around some "equilibrium" mean (possibly deterministically time varying). This makes a crucial difference with respect to other financial markets. The speed of reversion is also quite informative in regulatory terms, since it reveals the time needed by the supply side of the market to react to unanticipated events, or the time needed for the event to be "absorbed" by the system. The mean reverting nature of electricity prices is generally explained by market fundamentals. Mean reversion, a deterministic trend, and multiple seasonalities are integrated in virtually all the model proposed by the cited authors, where the price (or log-price) process is additively decomposed into a deterministic and a mean-reverting stochastic component.

**Spikes and jumps**

These are attributed to sudden and strong increases in demand, when supply is at the limit of generation capacity or there is an unexpected break down in large assets. Depending on demand and supply conditions spikes can also be negative. According to Deidersen and Trück (2002) they are less frequent in market with high levels of hydropower generation. Nevertheless, spikes are quite pervasive and it is their presence that impairs the forecasting properties of the models described in the literature. These extreme values can be modelled in discrete time by using a stochastic process with leptokurtic marginal distributions, or in continuous time by introducing jumps in the process. A somewhat different approach is that of Byström (2005), who models extreme price changes in the Nord Pool and estimates tail quantiles by filtering the return series and then applying an extreme value theory model to the residuals. As in other studies, the performance of the estimates improves when the model explicitly relates the volatility of the data to the within-year seasonal trends.

## Preliminary analysis of the Italian data

In this section[1] we study electricity prices recorded in Italy from 1st April 2004 to 15th January 2006. The data are sampled hourly, but in this study we use daily means[2].
The daily prices are depicted in Figure 1 together with the total demand for electricity (daily means as well).

Figure 1 here

**Figure 1. Daily means of hourly prices (line, unit = Euros per MWh) and of hourly demand (dots, unit = MWh $\times$ 1000) for electricity**

The strong within-week seasonality of the prices is clearly due to the seasonality present in electricity consumption. Indeed, the unitary price of electricity changes according to the volume to be produced, in the pattern depicted roughly in Figure 2.

Figure 2 here

**Figure 2. Scatter plot of electricity prices and demand with non parametric loess fit.**

Figures 1 and 3 clear show that in 2004 prices were significantly more volatile than in subsequent years. This may be ascribed to a learning phase that traders underwent and to the regulatory changes that took place in January 2005.In addition, in the first 10-15 days of 2005 there was an abrupt increase in prices not supported by a corresponding rise in demand. This episode provoked an inquiry by the antitrust authority. The remainder of the time series shows greater regularity.

[1]The computations in this section were carried out using EViews 4 (regressions and low-pass filtering through the *state-space* object), GiveWin 2 (kernel density estimates and sample autocorrelations), PcGive 10 (descriptive statistics and normality test).
[2]In most of the existing literature only daily means are considered.

Figure 3 here

**Figure 3. Weekly time series of the seven days.**

Table 1 reports some descriptive statistics, normality tests and graphs for the weekly time series for each day. It is interesting that Tuesday-Friday exhibit very similar behaviour (see Figure 3). From the normality tests (a modified version of the Jarque-Bera statistic as in Doornick and Hansen (1994)), it could be assumed that the Monday-Saturday data are normal, but the kernel estimates suggest the presence of multi-modality for all densities. This is due to the presence of seasonality within the year, and a possible trend, which make the data generating process non-stationary and the marginal densities not well-defined. A further problem might be the presence of weekday holidays, when the price of electricity is generally very low, thus producing negative skewness.

From the sample autocorrelation function (ACF) reported in Table 1, there is evidence of a high persistence of linear memory at weekly lags. This suggests three things: i) the presence of a deterministic within-week seasonality; ii) the presence of seven seasonal unit roots; iii) the presence of multiple periodic unit roots (Franses and Paap , 2004, ch.4). In the literature only the first hypothesis has been considered.

Table 1 here

**Table 1. Descriptive statistics, normality tests for each day**

In order to deal with within-year seasonality the cited authors used monthly dummies or sinusoids with frequencies $2\pi/365$ and $4\pi/365$. Since this seasonality is due to the low-frequency components in electricity demand, and these components tend to be very regular across years, it is sensible to apply them directly rather than approximating for them.

By observing the electricity demand series in Figure 1 it can be seen that there is a higher-than-average consumption in winter and summer with sudden decreases in the two main vacation periods: Christmas holidays (in Italy typically 24$^{th}$ December -6$^{th}$ January) and August. In order to successfully extract the features described, we designed a low-pass filter with two different cut-off frequencies: a lower one for “normal” periods and higher one for vacation times. As a result, the extracted time series is rather smooth most of the time, but it does not average out the negative peaks in the two vacation times. The slight trend in the extracted component of consumption was eliminated by imposing the same value for 31$^{st}$ December 2004 and 31$^{st}$ December 2005 and adjusting all other daily values by linear discounting. Technical details related to the filtering are reported in the Appendix. The low-pass filtered series is depicted in Figure 4.

Figure 4 here

**Figure 4. Band-pass filtered electricity demand of years 2005 (line) and 2004 (dots).**

If we assume, at least for the moment, that the price data are generated by the sum of a deterministic component (seasonalities and trend) and a (well behaved) stationary process, the least square estimates of the regression of the prices on the deterministic components are consistent and asymptotically normal (CAN) and the asymptotic covariance matrix of the estimators may be consistently estimated (see for example Hamilton, 1994, pp. 282-283).
We estimated the following nested regressions:

$$y_t = \tau \cdot t + \sum_{i=1}^{7} \delta_{0,i} \cdot D_{i,t} + \delta_s \cdot S_t + \eta_t \tag{1}$$

$$y_t = \tau \cdot t + \sum_{i=1}^{7} \left( \delta_{0,i} \cdot D_{i,t} + \delta_{1,i} \cdot D_{i,t} S_t \right) + \eta_t \tag{2}$$

$$y_t = \tau \cdot t + \sum_{i=1}^{7} \left( \delta_{0,i} \cdot D_{i,t} + \delta_{1,i} \cdot D_{i,t} S_t + \delta_{2,i} \cdot D_{i,t} S_t^2 \right) + \eta_t \tag{3}$$

where $D_{i,t}$ is the daily dummy of day $i = 1,...,7$ ($1 = Monday$, $7 = Sunday$), $S_t$ is the seasonal variable of Figure 4 and $\eta_t$ is a stationary process with absolutely summable covariances. In equation (1) the within-year seasonality ($S_t$) enters linearly and cannot influence the within-week seasonality, in equation (2) $S_t$ enters linearly and influences the within-week seasonality, while in equation (3) $S_t$ enters quadratically and influences the within-week seasonality. Table 2 reports summary statistics for the three regression models and tests for the validity of the constraints imposing the equality of all the parameters relative to the days Tuesday-Friday.

| Whole sample | eq. (1) | eq. (2) | constr | eq. (3) | constr |
|---|---|---|---|---|---|
| $R^2$ | 0.67 | 0.68 | 0.68 | 0.70 | 0.69 |
| S.E. of Regression | 7.93 | 7.85 | 7.83 | 7.68 | 7.65 |
| LogLik | -2281 | -2272 | -2273 | -2254 | -2255 |
| BIC | 7.06 | 7.09 | 7.03 | 7.10 | 7.02 |
| Wald Test Sig * | | 0.52 | | 0.55 | |
| Feb2005-Jan2006 | eq. (1) | eq. (2) | constr | eq. (3) | constr |
| $R^2$ | 0.84 | 0.87 | 0.87 | 0.92 | 0.92 |
| S.E. of Regression | 4.97 | 4.52 | 4.51 | 3.49 | 3.48 |
| LogLik | -1050 | -1014 | -1016 | -920 | -924 |
| BIC | 6.17 | 6.06 | 5.98 | 5.64 | 5.51 |
| Wald Test Sig * | | 0.32 | | 0.23 | |

* Wald test for the equality of all the parameters relative to Tuesday-Friday.

**Table 2. Diagnostics for the regression models of equations (1)-(3).**

The models have been fitted to the whole sample and to the sub-sample 1st February 2005 through 15th January 2006. In both samples the constrained model (3) outperforms the others according to the Schwartz' Bayesian Information Criterion. It is striking that the performance of all the models drastically worsens when the whole sample is considered: for the best fitting model, the standard error of regression more than doubles, while the $R^2$ is 20% smaller. These and other considerations lead us to conclude that omitting the first ten months of observations should produce more accurate models and predictions.

# Stochastic models for daily electricity prices

In this section[3] we formulate and test a set of models for the Italian price time series. These models encompass the deterministic and the stochastic components, in order to incorporate the memory present in the regression residuals, which plays an important role in short term forecasting and in derivative pricing.

Figure 5 here

**Figure 5. ACF and PACF of regression errors of model (3) with constrains.**

The sample's ACF and PACF functions show the presence of linear memory both in the

[3]The computations in this section were carried out using EViews 4 (conditional ML estimates of the Reg-PAR-GARCH models through the *logl* object) and R 2.1.0 with the *pear* package (sample periodic autocorrelations and partial autocorrelations).

errors and the squared errors, suggesting the opportunity of ARMA-GARCH models. Maybe curiously, the model in this family that seems to fit the data best is the constrained regression (3) with AR(1,6)-GARCH(1,1) errors:

$$\eta_t = \phi_1 \eta_{t-1} + \phi_6 \eta_{t-6} + \sigma_t z_t \tag{4}$$

$$\sigma_t^2 = \omega + \alpha \sigma_{t-1}^2 z_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{5}$$

with $z_t$ i.i.d. standard normal process.

Another interesting class of models, that could fit the different characteristics of the daily prices better than simple ARMA is that of periodic ARMA-GARCH[4]. A periodic ARMA is an ARMA model with periodicity (with seasons). For example a PARMA(1,1) with period 7 is

$$y_t = \mu_t + \phi_{1,t}(y_{t-1} - \mu_{t-1}) + \sigma_t z_t + \theta_{1,t}\sigma_{t-1} z_{t-1} \tag{6}$$

with $z_t$ i.i.d. and

$$\mu_{t+7} = \mu_t, \quad \phi_{t+7} = \phi_t, \quad \sigma_{t+7} = \sigma_t, \quad \theta_{t+7} = \theta_t \tag{7}$$

A periodic ARMA is a non-stationary model since mean, variance and ARMA filter depend on time. Nevertheless a PARMA model for daily data has a VARMA representation for the vector of the seven weekly time series. If the VARMA representation has a causal stationary solution, then the process is said to be *periodically stationary*. For details on periodic time series models refer to the monograph of Franses and Paap (2004) and to the articles of Jones and Brelsford (1967), Pagano (1978), Tiao and Grupe (1980) and Vecchia and Ballerini (1991).

A PARMA model may be enriched with a periodic GARCH-type structure Bollerslev and Ghysels (1994) by opportunely redefining $\sigma_t$ in equation (6). From our analyses we concluded that a non-periodic GARCH-type process should suffice. By looking at the vast GARCH library, we picked the EGARCH of Nelson (1991) since it is easier to adapt to a periodically unconditional variance, it allows for asymmetry (which implies skewness in the unconditional distribution) and does not impose constraints on the parameters:

$$\log \sigma_t^2 = \log \bar{\sigma}_t^2 + \alpha(|z_{t-1}| - E|z_{t-1}|) + \lambda z_{t-1} + \beta(\log \sigma_{t-1}^2 - \log \bar{\sigma}_{t-1}^2), \tag{8}$$

[4]The first (and so far only) application of a PAR model to daily electricity prices was carried out by [12].

with $\bar{\sigma}_{t+7}^2 = \bar{\sigma}_t^2$.

The identification of the order of a PARMA model may be based on the periodic ACF and PACF functions. The seven periodic autocorrelation functions of the periodically stationary process of period 7, $x_t$, are defined by

$$\gamma_t(k) = E\left( \frac{x_t - \mu_t}{\sigma_t} \cdot \frac{x_{t-k} - \mu_{t-k}}{\sigma_{t-k}} \right)$$

with $\gamma_{t+7}(k) = \gamma_t(k)$. The periodic ACF and PACF are straightforward generalizations of non-periodic ones (for definitions and algorithms refer to Sakai, 1982). Figure 6 depicts the sample ACF and PACF of the estimated regression (3) errors.

Figure 6 here

**Figure 6. Sample periodic ACF (bar) and PACF (line) of the estimated regression (3) errors.**

It is interesting to observe how the linear memory changes according to the day of the week., Particularly interesting is the high lag 3 partial autocorrelation of Monday (for which the preceding Friday carries most of the information), and the scant influence of previous days on Saturday. The considerable variation in the behaviour of the periodic autocorrelation functions is the main justification for the use of PARMA models.

# Estimation results

The process of finding a good model for the Italian data was incremental: we began with simple models and added complexity gradually, in order to match features of the data that appeared during the modelling process and had not been included in previous versions.

- M1 Reg-AR(1,6)-GARCH(1,1), $z_t \sim N(0,1)$;
- M2 Reg-PAR(1), $z_t \sim N(0,1)$;
- M3 Reg-PAR(1), $z_t \sim GED$[5];

[5]The Generalized Error Distribution (GED) (sometimes referred to as Exponential Power Distribution) has a shape parameter, $r$, that for $r = 2$ makes it a Normal distribution, for $r \in (0,2)$ makes it leptokurtic and for

- M4 Reg-PAR(5), $z_t \sim GED$ with

$$\begin{aligned}
Mon: &\quad \eta_t = \phi_{1,1}\eta_{t-1} + \phi_{3,1}\eta_{t-3} + \sigma_1 z_t \\
Tue: &\quad \eta_t = \phi_{1,2}\eta_{t-1} + \sigma_2 z_t \\
Wed: &\quad \eta_t = \phi_{1,3}\eta_{t-1} + \sigma_3 z_t \\
Thu: &\quad \eta_t = \phi_{1,4}\eta_{t-1} + \sigma_4 z_t \\
Fri: &\quad \eta_t = \phi_{1,5}\eta_{t-1} + \phi_{2,5}\eta_{t-5} + \sigma_5 z_t \\
Sat: &\quad \eta_t = \phi_{1,6}\eta_{t-1} + \sigma_6 z_t \\
Sun: &\quad \eta_t = \phi_{1,7}\eta_{t-1} + \sigma_7 z_t;
\end{aligned}$$

- M5 like Mod4. but with EGARCH(1,1) of equation (8).

Table 3 reports some goodness-of-fit statistics and diagnostic tests for the five models plus a constrained version of M5 (the insignificant parameters and the insignificantly different parameters are constrained).

| | M1. | M2. | M3. | M4. | M5. | M5.c |
|---|---|---|---|---|---|---|
| LogLik | -834 | -840 | -813 | -804 | -792 | -798 |
| N. of Coefs. | 15 | 27 | 28 | 30 | 33 | 23 |
| AIC | 4.88 | 4.97 | 4.82 | 4.78 | 4.73 | 4.71 |
| BIC | 5.08 | 5.27 | 5.13 | 5.11 | 5.09 | 4.96 |
| $Q(10)$ Sig. | 0.229 | 0.007 | 0.000 | 0.000 | 0.005 | 0.005 |
| $Q(10)^2$ Sig. | 0.855 | 0.012 | 0.221 | 0.724 | 0.989 | 0.989 |

$Q(10)$ is the lag 10 Box-Lijung statistics on the standardized residulas.
$Q(10)^2$ is the lag 10 Box-Lijung statistics on the squared standardized residuals.

**Table 3. Goodness-of-fit statistics of the various models.**

Model M5., particularly in its constrained version, seems to outperform the others, although the simple AR-GARCH works reasonably well, if one is led by Schwartz' BIC. Table 4 reports the constrained estimates. The asymmetry parameter of the EGARCH was eliminated since it was not significant.

$r \in (,\infty)$ turns it into a platokurtic distribution (cfr. [14]). The GED we used has zero mean and unit variance.

| | Coefficient | Std.Error | t-Ratio | Prob. |
|---|---|---|---|---|
| $\phi_{1,1}$ | 0.251 | 0.098 | 2.555 | 0.011 |
| $\phi_{1,2} = \phi_{1,4} = \phi_{1,5}$ | 0.776 | 0.047 | 16.504 | 0.000 |
| $\phi_{1,3}$ | 0.985 | 0.073 | 13.475 | 0.000 |
| $\phi_{1,7}$ | 0.827 | 0.103 | 8.034 | 0.000 |
| $\phi_{2,5}$ | 0.211 | 0.075 | 2.823 | 0.005 |
| $\phi_{3,1}$ | 0.388 | 0.073 | 5.288 | 0.000 |
| $\alpha$ | 0.361 | 0.101 | 3.561 | 0.000 |
| $\beta$ | 0.695 | 0.150 | 4.621 | 0.000 |
| $\sigma_1 = \ldots = \sigma_7$ | 1.349 | 0.368 | 3.662 | 0.000 |
| GED's $r$ | 0.925 | 0.082 | 11.296 | 0.000 |

**Table 4. Estimates of model M5. constrained (only the parameters of the stochastic part are reported).**

The results from this application can be contrasted with those reported by the many authors who studied other European Pools. Daily equilibrium prices in Italy were found by us to be high, non normally distributed and leptokurtic. This shape is similar to that found for the Nord Pool by both Byström (2005) and Lucia and Schwartz (2002), and by Escribano et al. (2002) for the Spanish Pool. Mean reversion[6] is another characteristic of the Italian Pool and its speed is more similar to that found in Spain by Escribano et al. (2002) than that revealed by several authors for the Nord Pool. Differences with respect to Spain and Nord Pool lie in the seasonal volatility of the Italian data as well as in the jump process, possibly due to the differences in the structure of production capacity. In terms of seasonality, Italian data display some peculiar characteristics mainly due to the high concentration of vacations in August and over Christmas, which accounted for in an original way.

As in Koopman et al. (2005) we found evidence of periodicity in the autoregressive coefficients: the prices of the weekdays correlate very weakly with previous weekend prices and the prices over weekends are not correlated with previous weekday prices. Koopman et al. (2005) found long memory in Nord Pool prices, but this feature cannot be inferred from the sample periodic ACF of Italian prices (cfr. Figure 6).

Thus, some features that determine Italian price behaviour are market specific, while some are

[6]It is hard to believe that the price of oil, which follows an integrated process, does not affect the price of electricity. If this is true, mean reversion is not a sensible hypothesis. What we may affirm is that for the limited length of our time series, the (possible) underlying unit root process takes account of a negligible degree of variance with respect to other components: seasonalities and short memory noise, in particular. A better description of this might be *local mean reversion*.

common to other markets. This implies that perspective comparative works should incorporate into the analysis specific aspects of the electricity generation/demand structure, such as differences in technological and climatic conditions, as well as some common elements such as oil price, that affect local markets in different ways.

# Conclusion

The analysis of the Italian electricity prices carried out in this study has enabled a good understanding of the most relevant features of the data. The first finding is the significant changes to the data generating process from mid January 2005. This may be due to the learning time needed by companies and traders involved and by the change in the regulation that took place at that time.

Another peculiarity of Italian prices are the drops that occur over the Christmas and summer vacations, which render the use of few sinusoids or monthly dummies unfit for modelling within-year seasonality. An original methodology to deal with this issue was developed. We also modelled the interaction between within-year seasonality within-week seasonality.

A slow but significant (increasing) linear trend in prices was noted and fitted. The reasons for this increasing price trend may lie in the accompanying growth in the prices of hydrocarbon-based energy sources.

Leptokurtic PAR-GARCH models seem most appropriate to show the different amount of memory of past observations that each weekday carries, as well as the presence of spikes and some form of volatility clustering.

Although the limited length of the price time series raises some questions, the models developed in this paper seem to perform quite well.

# Appendix

In order to filter the low frequencies of the daily time series of electricity demand, we designed a partially model-based low-pass filter with time varying cut-off frequency. We used the model

$$y_t = \mu_t + \gamma_t^{(1)} + \gamma_t^{(2)} + \gamma_t^{(3)} + \varepsilon_t \tag{9}$$

$$\mu_t = \mu_{t-1} + \beta_{t-1} \qquad (10)$$

$$\beta_t = \beta_{t-1} + \zeta_t \qquad (11)$$

$$\begin{bmatrix} \gamma_t^{(i)} \\ \tilde{\gamma}_t^{(i)} \end{bmatrix} = \begin{bmatrix} \cos\omega_j & \sin\omega_j \\ -\sin\omega_j & \cos\omega_j \end{bmatrix} \begin{bmatrix} \gamma_{t-1}^{(j)} \\ \tilde{\gamma}_{t-1}^{(j)} \end{bmatrix} + \begin{bmatrix} \kappa_t^{(j)} \\ \tilde{\kappa}_t^{(j)} \end{bmatrix} \qquad (12)$$

with $\omega_j = j \cdot 2\pi/7$, $j = 1,2,3$, $\varepsilon_t \sim (0, \sigma_\varepsilon^2)$, $\zeta_t \sim (0, \sigma_\zeta^2)$, $\kappa_t^{(j)}, \tilde{\kappa}_t^{(j)} \sim (0, \sigma_\kappa^2)$. $\mu_t$ captures the low frequency movements in which we are interested, the $\gamma_t$'s take care of the within-week seasonality and $\varepsilon_t$ is white noise. Since the cut-off frequency of the low-pass filter for extracting $\mu_t$ is determined by the signal-to-noise ratio $\rho = \sigma_\zeta^2 / \sigma_\varepsilon^2$, we fixed it to 1600 for "normal" days and to 100 for Christmas and summer vacation times (24 December-6 January and July-September). The other unknown variances have been estimated by ML. The filtered series was produced by the Kalman smoother.

The gain of the filter is given by

$$G(\lambda) = \frac{\left[\rho(2 - 2\cos\lambda)^2\right]^{-1}}{1 + \left[\rho(2 - 2\cos\lambda)^2\right]^{-1} + S(\lambda)}$$

where

$$S(\lambda) = \sum_{j=1}^{3} \left[ r \left( \frac{4(\cos\lambda - \cos\omega_j)^2}{1 - 2\cos\omega_j \cos\lambda + \cos^2\omega_j} \right)^2 \right]^{-1},$$

$\rho$ is defined as above, $r$ is the signal-to-noise ratio relative to the seasonal component (the estimated value is 48.660.207, meaning that the weekly seasonality is practically time-invariant) and $\omega_j = 2\pi j/7$.

The resulting cutoff frequency for normal times is $0.05\pi$ corresponding to a period of circa 40 days. The cutoff frequency for vacation days is $0.10\pi$ (ca. 20 days).

# Acknowledgements

This paper was written within the research programme on the regulation of the electricity sector in Italy. We would like to thank the University of Milan-Bicocca for financing the research from the FAR 2003 (ex 60%) grant and MURST for a PRIN 2004 grant.
We thank REF (Researches in Economics and Finance, Milan, www.ref-online.it/eng/) for providing us with the Italian data, and Pia Saraceno, Claudia Checchi, Edoardo Settimio and Mara Zanini in particular for comments and suggestions. All other data discussed in the paper were obtained from publications downloaded from the web site of the International Energy Agency (www.iea.org/Textbase/stats/rd.asp).
An earlier version of this paper was presented at the Sixty-First International Atlantic Economic Conference. We thank Peter van der Hoek, Kinga Mazur, Martin McGuire and Michael Ye for useful comments.

# References

[1] Bhanot, K. "Behavior of power prices: implications for the valuation and hedging of financial contracts," *Journal of Risk*, 2 (3), Spring 2000, 43–62.
[2] Bollerslev, T.; Ghysels E. "On periodic autoregressive conditional heteroskedasticity." September 1994, CIRANO Scientific Series No. 94s-3, Montréal.
[3] Byström, H. "Extreme value theory and extremely large electricity price changes," *International Review of Economics and Finance*, 14 (1), 2005, 41–55.
[4] Deidersen, J.; Trück, S. "Energy price dynamics. Quantitative studies and stochastic processes." 2002, Technical Report TR-ISWM-12/2002, University of Karlsruhe.
[5] Doornik, J. A.; Hansen H. "A practical test for univariate and multivariate normality." November 1994, Working Paper, Nuffield College, Oxford.
[6] Escribano, A.; Peña J. I.; Villaplana P. "Modelling electricity prices: international evidence." June 2002, Economic Series 08, Working Paper 02-27, Universidad Carlos III de Madrid.
[7] Fabra, N.; Toro J. "Price wars and collusion in the Spanish electricity market," April 2005, *International Journal of Industrial Organization*, 23 (3-4), 155–181.
[8] Franses, P. H.; Paap R. *Periodic Time Series Models*, Oxford UK: Oxford University Press, 2004.
[9] Hamilton J. D. *Time Series Analysis*, Princeton NJ: Princeton University Press, 1994.
[10] Jones, R. H.; Brelsford W. M. "Time series with periodic structure," December 1967, *Biometrika*, 54 (3-4), 403–408.
[11] Knittel, C.; Roberts M. R. "An empirical examination of restructured electricity prices," September 2005, *Energy Economics*, 27 (5), 791–817.
[12] Koopman, S. J.; Ooms M.; Carnero M. A. "Periodic Seasonal Reg-ARFIMA-GARCH Models for Daily Electricity Spot Prices." September 2005, Discussion Paper 2005-091/4, Tinbergen Institute.
[13] Lucia, J.; Schwartz E. "Electricity prices and power derivatives: evidence from the Nordic Power Exchange," January 2002, *Review of Derivatives Research*, 5 (1), 5–50.
[14] Nelson, D. B. "Conditional Heteroskedasticity in Asset Returns: a New Approach," March 1991, *Econometrica*, 59 (2), 347–370.
[15] Pagano, M. "On Periodic and Multiple Autoregressions," November 1978, *Annals of*

*Statistics*, 6 (6), 1310–1317.
[16] Sakai, H. "Circular Lattice Filtering Using Pagano's Method," April 1982, *IEEE Transactions on Acoustic, Speech, and Signal Processing*, 30 (2), 279–287.
[17] Tiao, G. C.; Grupe, M. R. "Hidden periodic autoregressive-moving average models in time series data," August 1980, *Biometrika*, 67 (2), 365–373.
[18] Vecchia, A. V.; Ballerini R. "Testing for periodic autocorrelations in seasonal time series data," March 1991, *Biometrika*, 78 (1), 53–63.
[19] Wilkinson, L.; Winsen J. "What can we learn from a statistical analysis of electricity prices in New South Wales," April 2002, *The Electricity Journal*, 15 (3), 60–6.

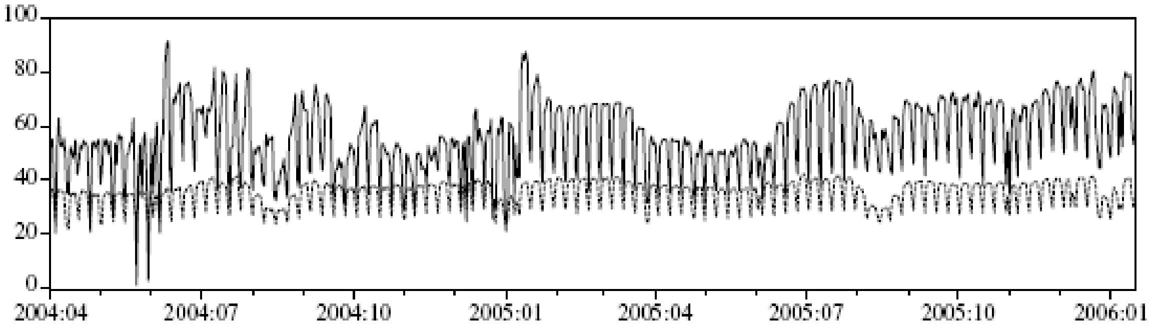



*Figure 1.* Daily means of hourly prices (line, unit = Euros per MWh) and of hourly demand (dots, unit = MWh×1000) for electricity

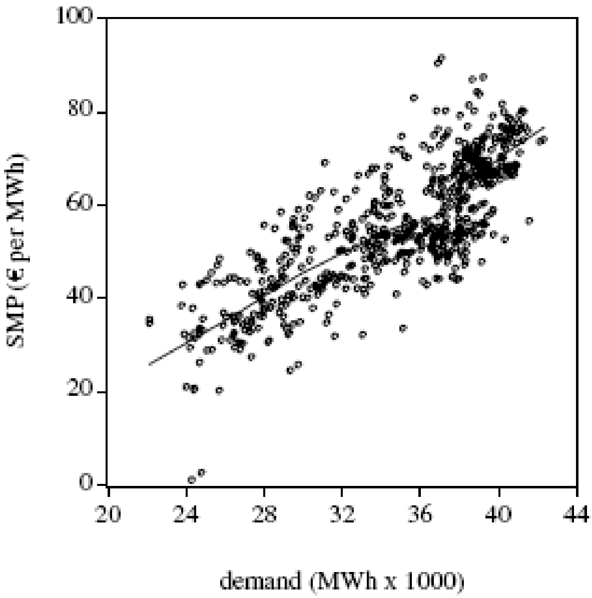



*Figure 2.* Scatter plot of electricity prices and demand with non parametric loess fit.

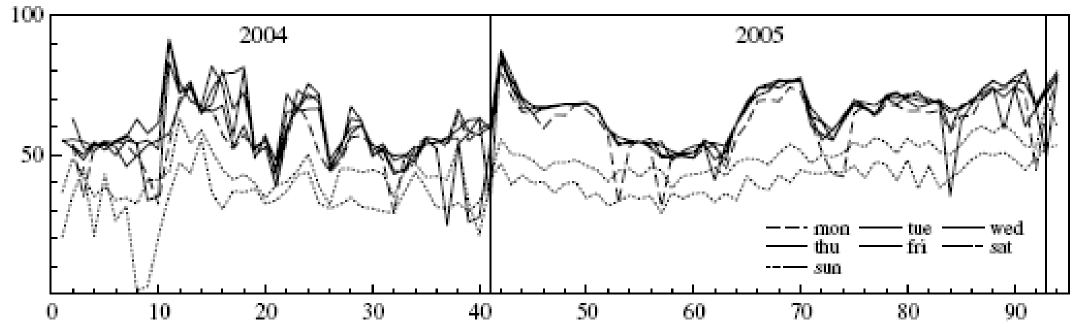
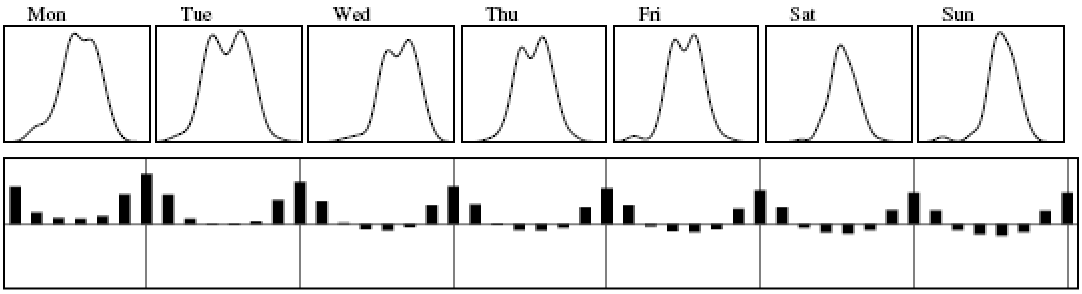



*Figure 3.* Weekly time series of the seven days.

Table I. Descriptive statistics, normality tests and kernel density estimates (top graph) for each day and sample ACF of the whole time series (bottom graph).

| | mon | tue | wed | thu | fri | sat | sun |
|---|---|---|---|---|---|---|---|
| Means | 58.04 | 61.96 | 62.24 | 62.84 | 61.94 | 46.66 | 37.93 |
| St.Dev. | 10.92 | 10.06 | 11.42 | 11.08 | 11.17 | 7.83 | 8.75 |
| Skew | -0.487 | -0.113 | -0.488 | -0.054 | -0.363 | -0.073 | -1.334 |
| Kurt | 2.977 | 2.449 | 3.220 | 2.595 | 3.804 | 3.643 | 7.453 |
| Min | 29.06 | 35.16 | 24.68 | 32.89 | 25.87 | 21.14 | 1.21 |
| Max | 80.26 | 86.91 | 84.25 | 90.38 | 91.49 | 69.09 | 55.75 |
| Norm (sig) | 0.103 | 0.590 | 0.141 | 0.881 | 0.099 | 0.166 | 0.000 |



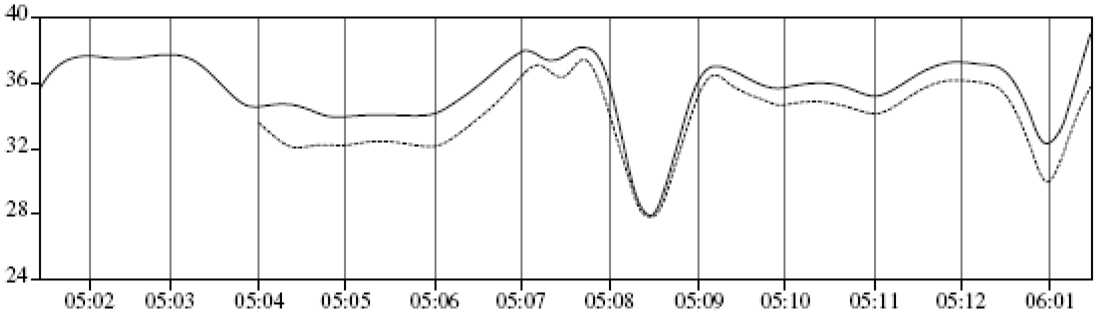




*Figure 4.* Band-pass filtered electricity demand of years 2005 (line) and 2004 (dots).

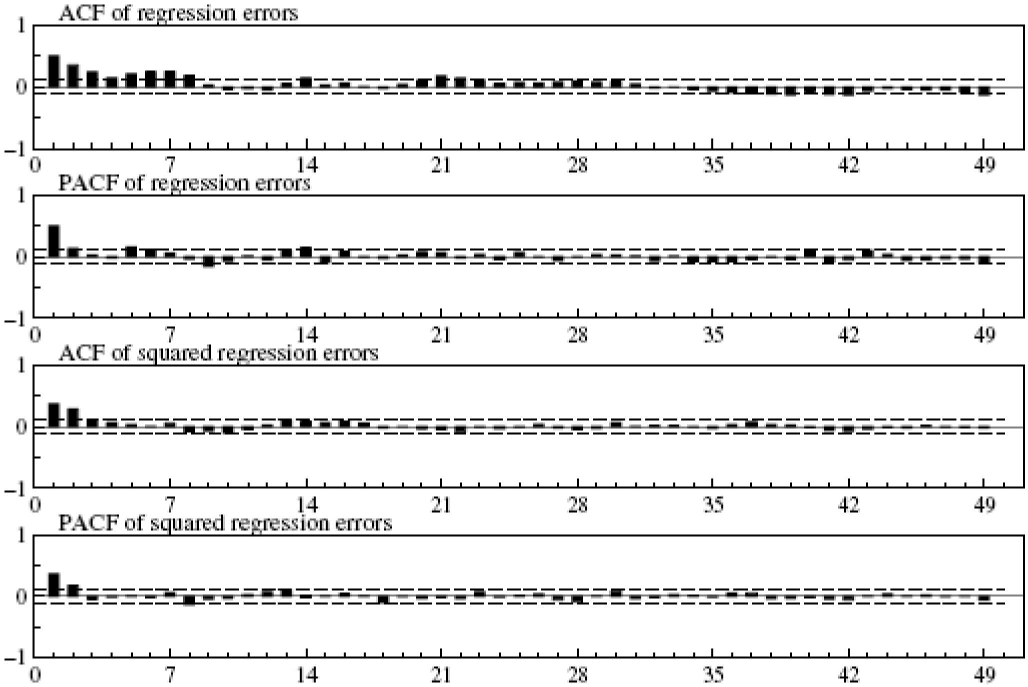



*Figure 5.* ACF and PACF of regression errors of model (3) with constrains.

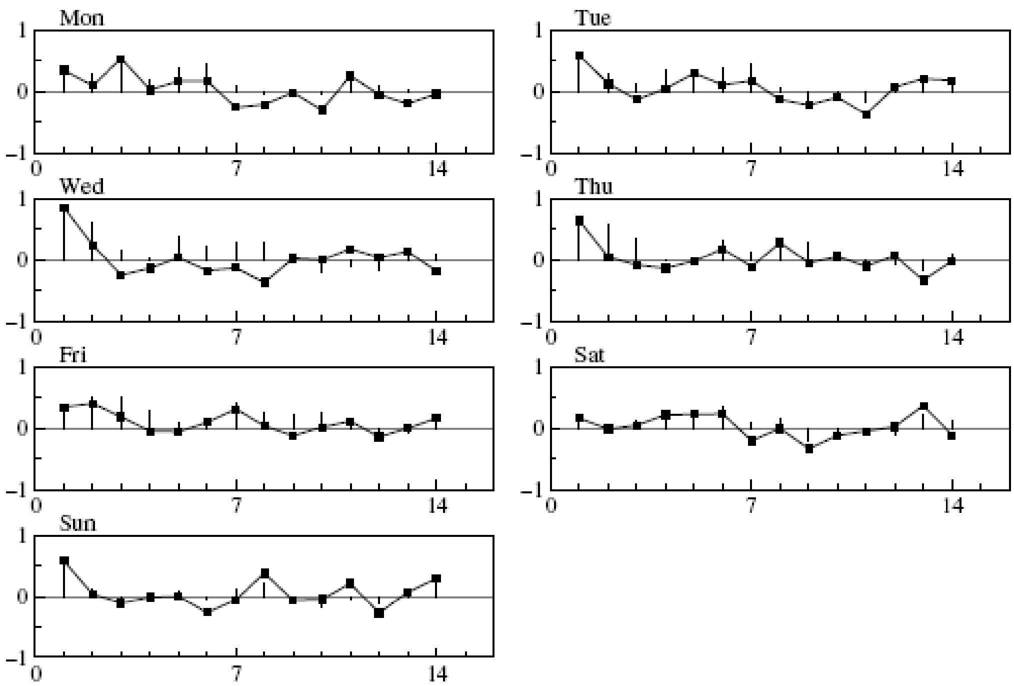



*Figure 6.* Sample periodic ACF (bar) and PACF (line) of the estimated regression (3) errors.