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PARTIAL REGULATION IN VERTICALLY DIFFERENTIATED INDUSTRIES

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Partial Regulation in Vertically Differentiated Industries

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Very Preliminary and Incomplete

Abstract

In this paper we provide a theoretical foundation for the price-and-quality cap regulation of the recently liberalized utilities. We model a *partially regulated oligopoly* where vertically differentiated services, such as high-speed and low-speed train services, are provided by a regulated incumbent and a strategic unregulated entrant competing in price and quality. This stylization is rather flexible in that it may equally well represent competition between regulated and unregulated industries, such as regulated train operators and deregulated air carriers. We establish that the price and quality weights in the cap need to depend also on the market served by the entrant, despite the latter is not directly concerned by the partial regulation. The conclusion is drawn that the regulators of the sectors under scrutiny should be allowed to use information about the overall industries, rather than the sole incumbents.

Keywords: Price-and-Quality Cap; Partial Regulation; Vertical Differentiation
J.E.L. Classification Numbers: L11, L13, L51, L9

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"*Price cap regulation rates as one of the success stories of applied economic theory. (...) it strikes a very good compromise between the theoretically rigorous foundation of the theory of optimal regulation for multiproduct firms (...) and the practitioner's requirement of the simple, easy-to-understand, easy-to-apply rule.*" (De Fraja and Iozzi [16], p.1)

1 Introduction

In a seminal piece of work, Laffont and Tirole [21] show that capping the prices of a multiproduct monopolist by means of a constraint in which any such price is attributed a weight equal (or proportional) to the correctly forecasted optimal quantities induces the regulated firm to charge the Ramsey prices, so that the second-best optimum for a monopolistic market is entailed. This result "restates" the one tracing back to Brennan [10] that the price cap emerges as the solution to a problem of welfare-constrained profit maximization, output quantities being the appropriate weights to be attributed to the allowable deviations from the socially desirable prices.

The ones previously mentioned are definitely not the sole research studies about the so-called *ideal price cap*. Indeed, numerous other authors have concentrated on the subject. Some such papers generically look at industries providing services of general interest. Among others, one may recall De Fraja and Iozzi [16] and [15] as well as Iozzi, Poritz and Valentini [20]. As for the works which, instead, focus on particular sectors, one may remember Billette de Villemeur, Cremer, Roy and Toledano [8], who investigate the price cap regulation in the postal sector.

Firms that are compelled to meet regulatory obligations often provide services of socially suboptimal quality. Such a phenomenon has been demonstrated on a theoretical ground as well as observed in several real-world situations. In consideration of this, regulators tend to exert special effort for controlling the quality supplied by the regulated operators. The concern for quality is particularly strong when utilities are subject to price cap regulation, due to the incentive the latter provides to cost cutting, which may translate into reductions in the level of offered quality. In one of the contributions previously listed, De Fraja and Iozzi [15] recall that, as from Rovizzi and Thompson (1992), a noticeable reduction in quality was registered in British Telecom's services immediately after privatization, as soon as the company was subject to price cap regulation without specific quality provisions. Then the Authors address the issue by laying down a theoretical foundation of the price-and-quality regulation, which emerges by integrating the price cap with the quality dimension. As the latter is introduced to obtain a global price-and-quality cap, it is found that appropriate price weights are still the optimal quantities, whereas opportune quality weights are given by the consumer marginal surplus evaluated at the optimal prices and qualities[1].

[1] The list of papers about price cap regulation we mention is far from exhaustive. There also exists a

Observe that the overall economic foundation of the ideal price(-and-quality) cap, to which all the contributions previously mentioned belong, refers to monopolistic industries. This circumstance is easily explained. After the second World War, almost all governments chose to structure the provision of services of general interest as State controlled-and-owned vertically integrated monopolies. Over time, this organizational model has lost momentum, due to the dissatisfactory performance experienced in terms of investment, efficiency, incentives and the growing budgetary problems. In the recent decades, network industries, such as telecommunications, water, energy and transportation, have either undergone a major process of privatization[2] or become commercial (hence, profit-oriented rather then welfare-concerned) players to be regulated, ownership being (at least formally) separated from regulatory tasks. Having in mind these real-world scenarios, economists have devoted close attention to monopoly regulation in general, and to price cap monopoly regulation in particular, with and without quality adjustments.

However, in a plurality of cases, the segments of the network industries that are not concerned by subadditive technologies have also been opened up to competition. As a consequence of the liberalization process, *partially regulated oligopolies* arise, where the incumbent firms (namely, the former monopolists) are subject to regulatory obligations, whereas the entrants are allowed to operate uncontrolled, though strategic. Such policies are meant, on one side, to promote access and (some) competition and, on the other side, to guarantee the collectivity with a reliable supply of the concerned services at affordable prices, by making the incumbents less market powerful but still financially viable. Biglaiser and Ma [6] provide the example of AT&T, which operates as a regulated dominant firm in the long-distance telecommunications market, where it competes with the unregulated MCI and Spring[3].

A crucial question arises. Can we expect the by now established monopoly regulation to be valid also for imperfectly competitive partially regulated sectors? Clearly, there is no economic reason for this to be *a priori* the case. This is confirmed by the conclusions achieved by Biglaiser and Ma [6] who, investigating the optimal regulation of a dominant firm playing the market game against an unregulated competitor, show that the regulatory programme is sensitive to the presence of a second operator endowed with market power. New constraints and trade-offs add up in the regulatory process, beyond those faced in

full class of works which focus on the practical application of the price cap regulation, either in a static or in a dynamic version. Some such papers are Vogelsang and Finsinger [28], Littlechild [19], Foreman [17] and Billette de Villemeur [7]. However, we do not strictly hinge on the literature background about implementation, to which those studies belong. For the same reason, Brennan [10] and De Fraja and Iozzi [15] are solely referred to as for the theoretical foundation of the price cap and of the price-and-quality cap respectively, not as for the implementation schemes they propose. More generally, see Vogelsang [27] for a survey of the literature about incentive regulation in general and its implementation over the last two decades. See Armstrong and Sappington [2] as well.

[2] See Martimort [23] for a discussion about the costs and benefits associated with privatization in an incentive theory perspective.

[3] See also Helm and Jenkinson [18], who report that regimes of partial regulation apply to railways and truck freight transport in Argentina and in the USA, as well as to the natural gas and oil sectors in Germany, Finland and Hong Kong. Other forms of partial regulation are those concerning former monopolies currently engaged both in regulated and unregulated activities in different market segments.

the monopoly regulation, and the result is ultimately a *third-best* one. This finding, which emerges from a Bayesian context of optimal regulation under asymmetric information, conveys a lesson that usefully applies to price cap regulation as well. Provided that the latter lacks a normative basis with reference to the new relevant situations, the necessity arises to revise the underlying theory, so that the implementation practice can then be updated accordingly.

Notably, this need is envisaged already in Brennan [10]. Indeed, on one side, his results are specified to easily extend to scenarios where a competitive fringe is present, the relevant output quantities being just those of the regulated firm, rather than total market quantities. On the other side, conclusions are recognized to be hardly applicable to situations where competing firms are *not* price takers[4].

The ultimate goal of the present paper is to provide a theoretical foundation for the price-and-quality cap regulation in oligopolistic environments, where a regulated dominant firm (a Stackelberg leader) competes in price and quality with one (or more) unregulated follower(s) not behaving as price taker(s). In particular, we focus on a market where vertically differentiated services are supplied to a population of consumers exhibiting heterogeneous valuations for quality. This is meant to account for the importance quality takes in the provision of basically all services of general interest.

The interpretation of our model is twofold. First of all, it represents competition between asymmetrically regulated operators *within* some given industry. To fix ideas, one may think about a regulated dominant train operator competing with one or more unregulated rivals, whose services display different qualities, such as high-speed and low-speed train services. Secondly, our model stylizes competition between regulated and unregulated industries. Insisting on transportation, one may consider the inter-modal competition between regulated train operators and deregulated air carriers, whose services differ as for speed, frequency, scheduling reliability, comfort.

Albeit transportation is particularly well placed to illustrate the flexibility of our model, it is noteworthy that the latter does apply to the majority of the utilities. Indeed, as further examples of relevant quality dimensions, one may recall the continuity and constancy of the supply of Internet services as well as the reaction lags in electricity generation and provision[5].

Within the context so far described, we first characterize the *(constrained) optimal partial regulatory policy*. This preliminary step is necessary because, as we mentioned, due to the presence of the rivals, such a policy is no longer the well-known second-best monopoly one. In particular, Ramsey pricing no longer yields (constrained) efficiency. Therefore, the second-best optimum for a monopoly does not constitute the objective of the regulatory mechanism. Relevant target is, instead, the policy which arises whenever a welfare-maximizing firm is in the position of a Stackelberg leader with respect to one (or more) profit-maximizing rival(s), facing the requirement that its profits be non-negative.

[4] Check footnote 6 at page 144 and footnote 10 at page 145 in Brennan [10].

[5] Crampes and Moreaux [11] stress that this quality aspect of the energy provision introduces a heterogeneity dimension in the generated electricity, which is otherwise an homogeneous product.

In this perspective, our work is close to the mixed oligopoly models, namely Rees [25] and Bos [9], in which the public firm is taken to be a first mover *vis-à-vis* the private operator(s)[6].

At later stage, we illustrate how the established optimal policy should be decentralized to the incumbent by means of an ideal price-and-quality cap, the truly practical implementation issue being, instead, left for further research. Our major finding is that, despite partial regulation does not directly concern the follower(s), decentralization of the optimal policy to the dominant operator requires that weights be attributed to price and quality, which depend not only on the portion of market that is served by the incumbent, but also on the one that is covered by the unregulated competitor(s) as well as on the pertaining ranges of quality valuations. A non-negligible implication is drawn from this conclusion. The regulatory bodies of the incumbents in liberalized industries should be allowed to use the available knowledge (if any) and/or to extract information (otherwise) about the overall industry, rather than being restricted to solely access and rely upon the (available) information about the targeted operators.

The paper is organized as follows. Section 2 presents the general theoretical framework. In particular, it details over consumer preferences and behaviour and producer technologies and profit functions. Section 3 illustrates the impact of the incumbent's actions on the entrant's decision and characterizes the optimal partial regulatory policy in a Stackelberg oligopoly. Section 4 focuses on the decentralization of the target policy by means of an appropriate price-and-quality cap. The latter is further investigated as for the special case of unit demand, which returns especially useful and intuitive insights. Section 5 concludes.

2 The Model

We consider an industry where two firms provide vertically differentiated products. One firm, denoted I, is the incumbent of the sector, the former monopolist which has started facing competition after the liberalization of the industry. The second firm, denoted F, is a newly entered operator. The two providers play a Stackelberg game, the incumbent in the role of the leader and the entrant in that of the follower. Strategic variables are prices (p_I and p_F) and qualities (q_I and q_F). Given the latter, the goods that are provided by the two firms turn out to be perfect substitutes.

The population of consumers is heterogeneous in that each individual can be identified by means of a parameter θ. The latter expresses the personal valuation for the quality of the product she purchases. The characteristic θ is distributed on the compact interval $\left[\underline{\theta}, \overline{\theta}\right]$, with $\underline{\theta} > 0$, density $f(\theta)$ and cumulative distribution function $F(\theta)$. Given her quality

[6] In particular, addressing the issue of which pricing policy a public firm should pursue if it faces constraints (namely, the requirement of operating at zero profits), Bos [9] stresses that the firm should opt for a *modified Ramsey-pricing rule.* In their turn, Beato and Mas Colell [4] show that the solution to a homogeneous-product Stackelberg game, where the public firm moves first, corresponds to average-cost pricing for the public firm. Indeed, since the latter selects its output before the private competitor, it chooses so that, at equilibrium, it obtains zero profits. See Nett [24] for a survey about the homogeneous-good mixed oligopoly literature.

valuation, the $\theta-$consumer patronizing firm $i \in \{I, F\}$ faces the so-called generalized price $\widetilde{p}_i \equiv p_i - \theta q_i$, which is equal to the unit monetary price (p_i) net of the individual benefit associated with the product quality (θq_i). The $\theta-$consumer prefers purchasing the good from firm i, rather than from firm j, whenever she bears a smaller generalized price $(\widetilde{p}_i < \widetilde{p}_j)$ by doing so.

The marginal consumer, who is indifferent between the two operators, is characterized by the parameter value

$$\theta_m \equiv \frac{p_i - p_j}{q_i - q_j}, \tag{1}$$

where $q_i > q_j$ and $p_i \geq p_j$ without loss of generality. Individuals whose θ exceeds θ_m patronize firm i, whereas individuals whose θ is smaller than θ_m select operator j.

2.1 Consumer Surplus and Demands

The net surplus the $\theta-$client obtains from the consumption of x_i units of the good purchased from firm i is given by

$$S_\theta (x_i) = U (x_i) - \widetilde{p}_i x_i, \ \forall i \in \{I, F\}, \tag{2}$$

where $U (x_i)$ measures the gross utility and $\widetilde{p}_i x_i$ the overall generalized cost. The optimal quantity of good i for the client under scrutiny is pinned down by maximizing (2) with respect to x_i, which yields

$$\frac{\partial U}{\partial x_i} = \widetilde{p}_i, \ \forall i \in \{I, F\}, \tag{3}$$

showing that the optimal quantity $x_i (p_i, q_i; \theta)$ is, indeed, a function of the generalized price.

Relying upon (3), it is possible to establish the relationship existing between price and quality impact on consumed quantity. To see this, let us first notice that (3) holds for any p_i and any q_i. We can then differentiate both sides with respect to p_i and to q_i, which yields

$$\frac{\partial^2 U}{\partial x_i^2} \frac{\partial x_i}{\partial p_i} = 1$$

and

$$\frac{\partial^2 U}{\partial x_i^2} \frac{\partial x_i}{\partial q_i} = -\theta$$

respectively. The expressions above reveal that a unitary increase in price p_i (resp., quality q_i) induces a unitary increment (resp., a decrement equal to θ) in the marginal utility of the service, through the variation intervened in the consumer demand. We next combine those equalities and obtain

$$\frac{\partial x_i / \partial q_i}{-\partial x_i / \partial p_i} = \theta, \ \forall i \in \{I, F\}, \tag{4}$$

which suggests that for the consumer demand to remain unchanged, as the price p_i is increased by one unit, the quality q_i should be augmented by an amount equal to her valuation for the quality itself (namely θ).

Aggregate demands are then immediately obtained by summing over the relevant ranges of θ's. They are given by

$$X_j(p_i, q_i, p_j, q_j) = \int_{\underline{\theta}}^{\theta_m} x_j(p_j, q_j; \theta) f(\theta) \, d\theta \tag{5a}$$

and

$$X_i(p_i, q_i, p_j, q_j) = \int_{\theta_m}^{\overline{\theta}} x_i(p_i, q_i; \theta) f(\theta) \, d\theta \tag{5b}$$

for firm j and i respectively. The properties they display are rather standard. We hereafter briefly recall them for any $i, j \in \{I, F\}$:

1. firm i's demand decreases in its own price p_i $\left(\frac{\partial X_i}{\partial p_i} < 0\right)$;
2. firm i's demand increases in its own quality q_i $\left(\frac{\partial X_i}{\partial q_i} > 0\right)$;
3. firm i's demand increases in the rival price p_j $\left(\frac{\partial X_i}{\partial p_j} > 0\right)$;
4. firm i's demand decreases in the rival quality q_j $\left(\frac{\partial X_i}{\partial q_j} < 0\right)$.

Moreover, it is straightforward to obtain the aggregate consumer surplus as a function of prices and qualities. At this aim, it is enough to plug the individual demand pinned down by (3) into the surplus function (2) and then sum over the relevant ranges of θ, which ultimately returns

$$\begin{aligned} V(p_i, q_i, p_j, q_j) &= \int_{\underline{\theta}}^{\theta_m} \left[U(x_j(p_j, q_j; \theta)) - (p_j - \theta q_j) x_j(p_j, q_j; \theta)\right] f(\theta) \, d\theta \\ &+ \int_{\theta_m}^{\overline{\theta}} \left[U(x_i(p_i, q_i; \theta)) - (p_i - \theta q_i) x_i(p_i, q_i; \theta)\right] f(\theta) \, d\theta. \end{aligned} \tag{6}$$

2.2 Technologies and Profits

Firm i's cost function is given by $C_i(X_i, q_i)$, $i \in \{I, F\}$, which is increasing both in the production level $\left(\frac{\partial C_i}{\partial X_i} > 0\right)$ and in the offered quality $\left(\frac{\partial C_i}{\partial q_i} > 0\right)$.

We assume that $C_i(\cdot, +\infty) = +\infty$. This equality means that providing very high quality is infinitely costly for the operator, hence such a quality is never actually provided on the market. We further suppose that firms never offer zero quality. Taken together, these hypotheses ensure that $q_i \in (0, \overline{q})$, $\forall i \in \{I, F\}$, $\overline{q}$ identifying a finite quality level.

Finally, defining firm i's revenues as $R_i \equiv p_i X_i$, the profit function is written

$$\pi_i(p_i, q_i, p_j, q_j) = R_i - C_i(X_i, q_i), \ \forall i \in \{I, F\}. \tag{7}$$

3 The (Constrained) Optimal Partial Regulatory Policy

The present section is devoted to the characterization of the *(constrained) optimal partial regulatory policy*. By the latter we mean the policy which would materialize in a mixed duopoly where a welfare-maximizing (public) firm were to play the market game as a Stackelberg leader *vis-à-vis* a profit-maximizing (private) competitor, under a non-negative profit constraint.

As the follower's reaction to the incumbent's policy is taken into account, the analysis is to be performed backward. Therefore, we start by investigating the entrant's behaviour and we subsequently look for the leader's price-and-quality bundle.

3.1 The Price-and-Quality Policy of Firm F

Firm F behaves as a follower *vis-à-vis* the incumbent. Therefore, it takes firm I's price and quality as given and optimizes its own accordingly.

The first-order condition for a maximum of (7) with respect to price p_F is given by

$$p_F - \frac{\partial C_F}{\partial X_F} = -\frac{X_F}{\partial X_F / \partial p_F},$$

or, equivalently, by

$$\frac{p_F - \partial C_F / \partial X_F}{p_F} = \frac{1}{\varepsilon_F^{Xp}}, \tag{8}$$

where we have defined $\varepsilon_F^{Xp} \equiv \frac{p_F}{X_F}\left(-\frac{\partial X_F}{\partial p_F}\right)$ the (absolute value of the) demand elasticity to price. It is straightforward to recognize in (8) the monopoly inverse elasticity rule. This suggests that firm F acts as a monopolist *vis-à-vis* the portion of market demand it faces.

Furthermore, the first-order condition with respect to quality q_F is given by

$$p_F \frac{\partial X_F}{\partial q_F} = \frac{\partial C_F}{\partial X_F}\frac{\partial X_F}{\partial q_F} + \frac{\partial C_F}{\partial q_F}. \tag{9}$$

As (9) reveals, q_F is chosen so that the marginal revenue of quality (the left-hand side) equals the marginal cost of quality (the right-hand side). (9) can be rearranged and combined with the first-order condition with respect to p_F in order to obtain

$$X_F \left(\frac{\partial X_F / \partial q_F}{-\partial X_F / \partial p_F}\right) = \frac{\partial C_F}{\partial q_F}. \tag{10}$$

We denote $\left(p_F^{PR}, q_F^{PR}\right)$ the price-and-quality pair firm F selects to satisfy the above conditions. This choice is anticipated while determining the leader's policy.

3.2 The Partial Regulatory Policy

We next characterize the incumbent's policy, taking the follower's behaviour (as illustrated in the previous Section) into account. For sake of shortness, let us denote **p** and **q** the vectors of overall market prices and qualities respectively. The (constrained) optimal partial regulatory policy is pinned down by maximizing the unweighed social welfare function

$$W(\mathbf{p},\mathbf{q}) = V(\mathbf{p},\mathbf{q}) + \pi_I(\mathbf{p},\mathbf{q}) + \pi_F(\mathbf{p},\mathbf{q}) \tag{11}$$

with respect to p_I and q_I, under the constraint that the incumbent makes non-negative profits $(\pi_I \geq 0)$.

Let λ the Lagrange multiplier associated with firm I's budget constraint. The first-order condition for a constrained maximum of (11) with respect to price p_I is given by

$$\frac{d\pi_I}{dp_I} = \left(\frac{1}{1+\lambda}\right)\left(-\frac{\partial V}{\partial p_I} - \frac{\partial \pi_F}{\partial p_I}\right). \tag{12}$$

The left-hand side of (12) is the *total* derivative of firm I's profits with respect to p_I, which also internalizes the indirect impact a variation in the own price induces on profits π_I through the rival's price p_F and quality q_F; it writes as

$$\frac{d\pi_I}{dp_I} = \frac{\partial \pi_I}{\partial p_I} + \frac{\partial \pi_I}{\partial p_F}\frac{dp_F}{dp_I} + \frac{\partial \pi_I}{\partial q_F}\frac{dq_F}{dp_I}.$$

Moreover, the derivative of firm F's profits with respect to p_I appearing in the right-hand side of (12) specifies as

$$\begin{aligned} \frac{\partial \pi_F}{\partial p_I} &= \left(p_F - \frac{\partial C_F}{\partial X_F}\right)\frac{\partial X_F}{\partial p_I} \\ &= X_F\left(\frac{\partial X_F/\partial p_I}{-\partial X_F/\partial p_F}\right), \end{aligned} \tag{13}$$

where we have used the first-order condition of firm F's profits with respect to p_F (see previous Section). Finally, supposing that firm I serves the consumers whose θ belongs to the interval $\left[\theta_m, \overline{\theta}\right]$ and firm F those with $\theta \in [\underline{\theta}, \theta_m)$, we can specify the derivative of the aggregate consumer surplus as (the negative of) the incumbent's aggregate demand. Indeed, from Roy's identity it follows that

$$\frac{\partial V}{\partial p_I} = \int_{\theta_m}^{\overline{\theta}} -x_I f(\theta)\, d\theta = -X_I. \tag{14}$$

The assumption that a regulated incumbent provides higher quality, hence it sells its products to the customers exhibiting a relatively larger valuation for quality, and receives a price that is sufficiently large to remunerate such a quality provision, seems to be rather reasonable and will be maintained all along the analysis. Nevertheless, it is noteworthy that, *mutatis mutandis*, the investigation would similarly be performed in the event that the unregulated follower served the market segment $\left[\theta_m, \overline{\theta}\right]$.

Plugging (13) and (14) into (12) ultimately yields

$$\frac{d\pi_I}{dp_I} = \left(\frac{1}{1+\lambda}\right)\left[X_I - X_F\left(\frac{\partial X_F/\partial p_I}{-\partial X_F/\partial p_F}\right)\right]. \tag{15}$$

Turning next to the second relevant dimension, the first-order condition for a constrained maximum of (11) with respect to quality q_I is given by

$$\frac{d\pi_I}{dq_I} = -\left(\frac{1}{1+\lambda}\right)\left(\frac{\partial V}{\partial q_I} + \frac{\partial \pi_F}{\partial q_I}\right). \tag{16}$$

The left-hand side of (16) is the total derivative of profits π_I with respect to quality q_I. It is written

$$\frac{d\pi_I}{dq_I} = \frac{\partial \pi_I}{\partial q_I} + \frac{\partial \pi_I}{\partial q_E}\frac{dq_E}{dq_I} + \frac{\partial \pi_I}{\partial p_E}\frac{dp_E}{dq_I},$$

that is as the sum of the direct and the indirect effect of a quality variation on profits. Furthermore, in the right-hand side of (16), we find both the derivative of aggregate consumer surplus and that of firm F's profits with respect to the incumbent's quality. As for the former, we have

$$\frac{\partial V}{\partial q_I} = \int_{\theta_m}^{\bar{\theta}} \theta x_I f(\theta)\, d\theta \equiv \widetilde{\theta}_I X_I, \tag{17}$$

meaning that the variation a quality increase induces in consumer surplus equals the weighed sum of the quantities consumed by firm I's infra-marginal clients, weights being their personal appreciations for quality. The $\widetilde{\theta}_I$ in (17) is defined as

$$\widetilde{\theta}_I \equiv \frac{\int_{\theta_m}^{\bar{\theta}} \theta x_I f(\theta)\, d\theta}{\int_{\theta_m}^{\bar{\theta}} x_I f(\theta)\, d\theta},$$

hence it expresses the average valuation of the quality q_I in the population of consumers served by the dominant firm. It follows that the marginal consumer surplus $\partial V/\partial q_I$ can ultimately be expressed as the product between the average quality appreciation of those same consumers and the total number of units they purchase from firm I $\left(\widetilde{\theta}_I X_I\right)$.

In turn, as for the derivative of the rival profits with respect to q_I, relying upon the first-order condition for a maximum of π_F with respect to p_F, we can write

$$\frac{\partial \pi_F}{\partial q_I} = X_F\left(\frac{\partial X_F/\partial q_I}{-\partial X_F/\partial p_F}\right), \tag{18}$$

Replacing (17) and (18) into (16), we ultimately obtain

$$\frac{d\pi_I}{dq_I} = -\left(\frac{1}{1+\lambda}\right)\left[\widetilde{\theta}_I X_I + X_F\left(\frac{\partial X_F/\partial q_I}{-\partial X_F/\partial p_F}\right)\right]. \tag{19}$$

In definitive, the optimal partial regulatory policy under the incumbent's budget constraint is given by the price-and-quality combination $\left(p_I^{PR}, q_I^{PR}\right)$ which simultaneously satisfies the pair of conditions (15) and (19).

4 Decentralization through a Price-and-Quality Cap

In this section, we propose a mechanism that allows to decentralize the allocation identified above. The incumbent is left free to choose both prices and qualities provided that a "price-and-quality" cap constraint is satisfied. Formally, this means that the incumbent solves the programme

$$\begin{cases} \underset{\{p_I,q_I\}}{Max}\ \pi_I \\ s.t. \\ \alpha p_I - \beta q_I \leq P \end{cases} . \tag{20}$$

Let μ the multiplier associated with the price-and-quality constraint in (20). The first-order condition for a constrained maximum of π_I with respect to p_I is given by

$$\frac{d\pi_I}{dp_I} = \mu\alpha, \tag{21}$$

suggesting that firm I equals its marginal profits $\left(\frac{d\pi_I}{dp_I}\right)$ to the shadow cost of the regulatory constraint (μ) multiplied by the weight that is attributed to price in the latter constraint (α). Therefore, given $\alpha > 0$, the regulatory constraint is tightened as the incumbent's profits increase following to a price increment.

On the other hand, the first-order condition with respect to q_I writes as

$$\frac{d\pi_I}{dq_I} = -\mu\beta, \tag{22}$$

revealing that the incumbent's marginal profits $\left(\frac{d\pi_I}{dq_I}\right)$ should be equal to the (negative of the) shadow cost of the regulatory constraint (μ) multiplied by the weight that is attributed to quality (β). According to (22), given $\beta > 0$, the regulatory constraint is relaxed as firm I's profits decrease following to a quality increase.

For the imposition of the price-and-quality cap to ultimately implement the partial regulatory policy $\left(p_I^{PR}, q_I^{PR}\right)$ as previously characterized, hence for the choice of the rival pair $\left(p_F^{PR}, q_F^{PR}\right)$ to be induced, it must be the case that

$$\mu = \frac{1}{1+\lambda^{PR}} \tag{23a}$$

together with

$$\alpha = X_I^{PR} - X_F^{PR}\left(\frac{\partial X_F^{PR}/\partial p_I}{-\partial X_F^{PR}/\partial p_F}\right) \tag{23b}$$

and with

$$\beta = \widetilde{\theta}_I^{PR} X_I^{PR} + X_F^{PR}\left(\frac{\partial X_F^{PR}/\partial q_I}{-\partial X_F^{PR}/\partial p_F}\right), \tag{23c}$$

where the presence of the superscript PR indicates that terms are evaluated at the partial regulation prices and qualities. Notice that the derivatives $\partial X_F^{PR}/\partial p_I$ and $\partial X_F^{PR}/\partial q_I$ in the right-hand sides of the previous expressions both express marginal variations, as firm

F's infra-marginal consumption units are not concerned by changes in the incumbent's price and quality. More precisely, we have

$$\frac{\partial X_F^{PR}}{\partial p_I} = x_m^{PR} f\left(\theta_m^{PR}\right) \frac{d\theta_m^{PR}}{dp_I} \tag{24a}$$

as well as

$$\frac{\partial X_F^{PR}}{\partial q_I} = x_m^{PR} f\left(\theta_m^{PR}\right) \frac{d\theta_m^{PR}}{dq_I}, \tag{24b}$$

where x_m^{PR} measures the consumption of the marginal client, the one characterized by

$$\theta_m^{PR} \equiv \frac{p_I^{PR} - p_F^{PR}}{q_I^{PR} - q_F^{PR}},$$

at those prices and qualities, the relevant density being $f\left(\theta_m^{PR}\right)$. Plugging (24a) and (24b) into (23b) and (23c) ultimately returns more interpretable expressions for the regulatory price and quality weights, that is

$$\alpha = X_I^{PR} - X_F^{PR} \frac{x_m^{PR} f\left(\theta_m^{PR}\right) \frac{d\theta_m^{PR}}{dp_I}}{\left[\int_{\underline{\theta}}^{\theta_m} -\frac{\partial x_F^{PR}}{\partial p_F} f(\theta)\, d\theta + x_m^{PR} f\left(\theta_m^{PR}\right) \frac{\partial \theta_m^{PR}}{\partial p_F}\right]} \tag{25a}$$

and

$$\beta = \widetilde{\theta}_I^{PR} X_I^{PR} - X_F^{PR} \frac{x_m^{PR} f\left(\theta_m^{PR}\right) \frac{d\theta_m^{PR}}{dq_I}}{\left[\int_{\underline{\theta}}^{\theta_m} -\frac{\partial x_F^{PR}}{\partial p_F} f(\theta)\, d\theta + x_m^{PR} f\left(\theta_m^{PR}\right) \frac{\partial \theta_m^{PR}}{\partial p_F}\right]} \tag{25b}$$

respectively. Let us closely investigate each of the weights we have found.

According to (25a), the appropriate price weight α is given by the difference between two terms. The first term is the incumbent's demand evaluated at $\left(\mathbf{p}^{PR}, \mathbf{q}^{PR}\right)$, namely X_I^{PR}. The second term consists in the demand faced by firm E evaluated at $\left(\mathbf{p}^{PR}, \mathbf{q}^{PR}\right)$, namely X_F^{PR}, as multiplied by a ratio which displays the marginal variation that is induced in demand X_F^{PR} by an increase in the incumbent's price at the numerator and the (absolute value of the) overall (marginal and infra-marginal) variation that is caused in the same demand by an increase in the competitor's price at the denominator. The (cross) effect of price p_I on the follower's demand is less important than the (own) effect of price p_F. Therefore, the ratio under scrutiny is smaller than unity. This involves that the incumbent's demand is attributed a larger relevance than the rival's in the composition of the price weight. In definitive, α is obtained by subtracting a properly calibrated portion of the entrant's demand, at the desirable prices and qualities, from the dominant operator's.

Furthermore, (25b) suggests that also the quality weight β equals the difference between two terms. As already mentioned, the first term, namely $\widetilde{\theta}_I^{PR} X_I^{PR}$, is an aggregate measure of the quality appreciation of firm I's consumers. The second term is given by the competitor's demand X_F^{PR} as multiplied by a ratio between the marginal variation that is induced in demand X_F^{PR} by an increase in the incumbent's quality and, again, the (absolute value of the) overall (marginal and infra-marginal) variation that is caused in the same demand by an increase in the competitor's price.

Observe that it is harder to draw further considerations about the ultimate composition of β, than it is about α, because a direct comparison between numerator and denominator of the ratio just described cannot be performed in the general scenario under scrutiny. Nevertheless, a clear message emerges from (25a) and (25b), namely that, for either weight to be properly determined, the relevant market conditions about the incumbent *and* the follower are to be taken into account. Therefore, while under regulated monopoly relevant such conditions are the overall market conditions, under partially regulated oligopoly relevant conditions are the ones pertaining to each active player and to the market as a whole.

In the next Section, we investigate a specific case that returns particularly intuitive insights for some interesting real-world environments.

4.1 The Unit Demand Case

We hereafter focus on the case where each customer has to allocate only one unit of consumption to her preferred operator[7]. This kind of consumption decision fits the transportation sector especially well. To see this, consider that, in some given day, an individual who needs to make a travel and faces two operators, chooses the one from which she will purchase her ticket. As an alternative way to view the situation, consider the case of an individual to whom two transportation modes are available, who selects one such mode to perform a single travel.

In the unit demand scenario, we have

$$\frac{\partial X_F^{PR}}{\partial p_I} = f\left(\theta_m^{PR}\right)\frac{d\theta_m^{PR}}{dp_I}$$

as well as

$$\frac{\partial X_F^{PR}}{\partial q_I} = f\left(\theta_m^{PR}\right)\frac{d\theta_m^{PR}}{dq_I}.$$

It is possible to show that, under sufficiently mild conditions (see Appendix), the equality

$$\frac{d\theta_m^{PR}}{dq_I} = -\theta_m^{PR}\frac{d\theta_m^{PR}}{dp_I}$$

is satisfied, involving that

$$\begin{aligned}\frac{\partial X_F^{PR}}{\partial q_I} &= -\theta_m^{PR} f\left(\theta_m^{PR}\right)\frac{d\theta_m^{PR}}{dp_I} \\ &= -\theta_m^{PR}\frac{\partial X_F^{PR}}{\partial p_I}.\end{aligned}$$

[7] As the reader should recall, for each individual, the preferred operator is the one which ensures the lower generalized cost, given her personal valuation for quality.

Moreover, whenever $\theta_m^{PR} = \partial p_F / \partial q_F$ (as it is, indeed, along the follower's isoprofit curve), it turns out that

$$\frac{\partial X_F^{PR}}{\partial p_I} = -\frac{\partial X_F^{PR}}{\partial p_F}.$$[8]

It follows that we have

$$\frac{\partial X_F^{PR} / \partial p_I}{-\partial X_F^{PR} / \partial p_F} = 1,$$

which leads to the result

$$\frac{\partial \pi_F^{PR}}{\partial p_I} = X_F^{PR}, \tag{26}$$

i.e. the follower's marginal profit at $\left(\mathbf{p}^{PR}, \mathbf{q}^{PR}\right)$, as p_I is increased, just equals its demand at $\left(\mathbf{p}^{PR}, \mathbf{q}^{PR}\right)$. On the other hand, with $\theta_m^{PR} = \partial p_F / \partial q_F$, we can as well compute

$$\frac{\partial X_F^{PR} / \partial q_I}{-\partial X_F^{PR} / \partial p_F} = -\theta_m^{PR}.$$[9]

Therefore, under the same circumstance as above, we have

$$\frac{\partial \pi_F^{PR}}{\partial q_I} = -\theta_m^{PR} X_F^{PR}. \tag{27}$$

Replacing (26) into (25a) and (27) into (25b) respectively yields

$$\alpha^U = X_I^{PR} - X_F^{PR} \tag{28a}$$

and

$$\beta^U = \widetilde{\theta}_I^{PR} X_I^{PR} - \theta_m^{PR} X_F^{PR} \tag{28b}$$

where the superscript U stays for *unit demand* and

$$\widetilde{\theta}_I^{PR} \equiv \frac{\int_{\theta_m^{PR}}^{\bar{\theta}} \theta f(\theta)\, d\theta}{\int_{\theta_m^{PR}}^{\bar{\theta}} f(\theta)\, d\theta}$$

expresses the average valuation of quality q_I^{PR} in the population of incumbent's customers. Let us hereafter comment on (28a) and (28b).

[8] To see this point, it is necessary to recall that we have $-\frac{\partial X_F^{PR}}{\partial p_F} = \frac{f\left(\theta_m^{PR}\right)}{q_I^{PR} - q_F^{PR}}$ together with $\frac{\partial X_F^{PR}}{\partial p_I} = \frac{f\left(\theta_m^{PR}\right)}{q_I^{PR} - q_F^{PR}} \left(1 - \frac{\partial p_F}{\partial p_I} + \theta_m^{PR} \frac{\partial q_F}{\partial p_I}\right)$. Hence, for the equality in the text to hold, it must be the case that $1 - \frac{\partial p_F}{\partial p_I} + \theta_m^{PR} \frac{\partial q_F}{\partial p_I} = 1$,which does occur as $\theta_m^{PR} = \frac{\partial p_F}{\partial q_F}$.

[9] To see this point as well, the reader should remember that it is $\frac{\partial X_F^{PR}}{\partial q_I} = \frac{f\left(\theta_m^{PR}\right)}{q_I^{PR} - q_F^{PR}} \left(-\theta_m^{PR} - \frac{\partial p_F}{\partial q_I} + \theta_m^{PR} \frac{\partial q_F}{\partial q_I}\right)$. Hence, for the equality in the text to be satisfied, it must be the case that $\left(-\theta_m^{PR} - \frac{\partial p_F}{\partial q_I} + \theta_m^{PR} \frac{\partial q_F}{\partial q_I}\right) = -\theta_m^{PR}$, which does occur with $\theta_m^{PR} = \frac{\partial p_F}{\partial q_F}$.

A very intuitive result is embodied in (28a). The appropriate weight to be attributed to the incumbent's price is simply the difference between the incumbent's demand $\left(X_I^{PR}\right)$ and the rival's demand $\left(X_F^{PR}\right)$, which are evaluated at the optimal partial regulation prices and qualities $\left(\mathbf{p}^{PR}, \mathbf{q}^{PR}\right)$. Interestingly enough, in the current scenario, equal relevance is recognized to the operators' demands. Hence, the follower's demand needs be entirely subtracted from the dominant firm's.

In turn, (28b) suggests that the proper weight to be assigned to the incumbent's quality equals the difference between the aggregate valuation of such a quality $\left(\widetilde{\theta}_I^{PR} X_I^{PR}\right)$ and a measure of the aggregate valuation of firm F's quality $\left(\theta_m^{PR} X_F^{PR}\right)$. In particular, in this aggregate measure, the marginal valuation $\left(\theta_m^{PR}\right)$ is used to weight all the consumption units $\left(X_F^{PR}\right)$. Recall that θ_m^{PR} is the upper bound of the range of θ's served by the follower. Therefore, while the volume of the incumbent's services is weighed with the overall interval of relevant θ's, the one of the rival's services is wholly weighed with the highest possible value of θ. This reveals that the valuation of quality q_F^{PR}, which is relevant to the composition of β^U, "overstates" the one in the population of firm F's clients, whereas not so is as for the valuation of the regulated quality.

5 Conclusions

In this paper, we have provided a theoretical foundation for the price-and-quality cap regulation of the recently liberalized utilities. For this purpose, we have stylized a *partially regulated oligopoly* where vertically differentiated services, such as high-speed and low-speed train services, are provided by a regulated incumbent (the Stackelberg leader) and a strategic unregulated provider (the follower) competing in price and quality. We have as well proposed an alternative interpretation of this model in terms of competition between regulated and unregulated industries, namely regulated train operators and deregulated air carriers.

Within the context described above, we have first characterized the *(constrained) optimal partial regulatory policy* as the target policy for the industry regulation. In the environment under scrutiny, this is the policy which arises whenever a welfare-maximizing firm is in the position of a Stackelberg leader with respect to one (or more) profit-maximizing rival(s) and is compelled to the requirement that its profits be non-negative.

We have subsequently addressed the issue of properly decentralizing the target policy. We have established that the appropriate weights to be inserted in the incumbent's price-and-quality cap depend both on the "optimal" demand of the regulated incumbent and on that faced by the follower, despite the latter is not directly concerned by the partial regulation. In particular, in the special case of unit demand, under mild conditions, the price weight is simply the difference between the leader's and the follower's demand at the optimal prices and qualities. On the other hand, the quality weight is given by the difference between the aggregate valuation of the regulated quality and a measure of the aggregate valuation of the rival quality that overstates the real valuation in the population of the follower's clients.

In order to better highlight the core contribution of our investigation, we find it useful to compare our results about the appropriate price weight with some points raised in Brennan [10]. The latter [10] explains that, when the regulated firm faces a price-taking competitive fringe, as long as fringe profits are included in the social welfare, the relevant output quantity in the price cap is just that of the regulated firm. If, instead, fringe profits are not considered to be a social welfare component, then the relevant output quantity is the total market supply, precisely as in a monopoly. Our analysis shows that things are different in the presence of an unregulated strategic follower. In particular, for the price cap to be desirably set, the incumbent's optimal demand has to be partially or totally diminished by the competitor's optimal demand. The intuition behind this finding is that, since rival profits contribute to social welfare just as the surplus accruing to any other economic agent and since the rival operator is not a price-taker, the weight to be attributed to the incumbent's price in the cap needs to ensure that the follower be not crowded out and that, at the same time, the distortion induced by the market power it exerts be minimized. Similar intuition underlies the proper composition of the quality weight, but this part of the cap is not comparable to any outcome in Brennan [10], where attention is devoted to a pure price cap and the quality issue remains unaddressed.

Finally, hinging on the results of our analysis, an important conclusion can be drawn as for the regulatory process of the sectors under scrutiny. Regulatory bodies implementing the "partial version" of the price-and-quality cap should be allowed to use information about both the concerned industry as a whole and each of its active players. Indeed, contrary to what loose intuition may suggest, relevant information for the regulatory process is not just the one about the dominant firm, even in a world where this firm is the sole regulated agent.

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A Appendix

Let us consider the unit demand case and suppose that $\theta_m = \frac{p_I - p_F}{q_I - q_F}$. Firm I serves the θ's belonging to the interval $\left[\theta_m, \overline{\theta}\right]$, firm F the ones within the interval $\left[\underline{\theta}, \theta_m\right)$. In this scenario, the equalities

$$\frac{\partial X_F}{\partial p_F} = f\left(\theta_m\right) \frac{\partial \theta_m}{\partial p_F} = -\frac{f\left(\theta_m\right)}{q_I - q_F}$$

and

$$\frac{\partial X_F}{\partial q_F} = f\left(\theta_m\right) \frac{\partial \theta_m}{\partial q_F} = \theta_m \frac{f\left(\theta_m\right)}{q_I - q_F}$$

are true. Therefore, we can write

$$\frac{\partial X_F}{\partial q_F} = f\left(\theta_m\right) \frac{\partial \theta_m}{\partial q_F} = \theta_m \frac{f\left(\theta_m\right)}{q_I - q_F} = -\theta_m \frac{\partial X_F}{\partial p_F} = -\theta_m f\left(\theta_m\right) \frac{\partial \theta_m}{\partial p_F}$$

or, equivalently,

$$\frac{\partial \theta_m}{\partial q_F} = -\theta_m \frac{\partial \theta_m}{\partial p_F}. \tag{29}$$

We hereafter check whether and, if so, under which conditions, one as well has

$$\frac{d\theta_m}{dq_I} = -\theta_m \frac{d\theta_m}{dp_I}. \tag{30}$$

For this purpose, we first compute the total derivative of θ_m with respect to p_I and obtain

$$\begin{aligned}
\frac{d\theta_m}{dp_I} &= \frac{\partial \theta_m}{\partial p_I} + \frac{\partial \theta_m}{\partial p_F}\frac{\partial p_F}{\partial p_I} + \frac{\partial \theta_m}{\partial q_F}\frac{\partial q_F}{\partial p_I} \\
&= \frac{1}{q_I - q_F} - \frac{1}{q_I - q_F}\frac{\partial p_F}{\partial p_I} + \frac{\theta_m}{q_I - q_F}\frac{\partial q_F}{\partial p_I} \\
&= \frac{1}{q_I - q_F}\left(1 - \frac{\partial p_F}{\partial p_I} + \theta_m \frac{\partial q_F}{\partial p_I}\right).
\end{aligned} \tag{31a}$$

We next calculate the total derivative of θ_m with respect to q_I, which is given by

$$\begin{aligned}
\frac{d\theta_m}{dq_I} &= \frac{\partial \theta_m}{\partial q_I} + \frac{\partial \theta_m}{\partial p_F}\frac{\partial p_F}{\partial q_I} + \frac{\partial \theta_m}{\partial q_F}\frac{\partial q_F}{\partial q_I} \\
&= -\frac{\theta_m}{q_I - q_F} - \frac{1}{q_I - q_F}\frac{\partial p_F}{\partial q_I} + \frac{\theta_m}{q_I - q_F}\frac{\partial q_F}{\partial q_I} \\
&= \frac{1}{q_I - q_F}\left(-\theta_m - \frac{\partial p_F}{\partial q_I} + \theta_m \frac{\partial q_F}{\partial q_I}\right).
\end{aligned} \tag{31b}$$

The first-order condition for a maximum of π_F with respect to p_F is given by

$$\left(p_F - \frac{\partial C_F}{\partial X_F}\right)\frac{f\left(\theta_m\right)}{q_I - q_F} = F\left(\theta_m\right).$$

Moreover, the first-order condition for a maximum of π_F with respect to q_F is written

$$\left(p_F - \frac{\partial C_F}{\partial X_F}\right)\frac{f\left(\theta_m\right)}{q_I - q_F} = \frac{\partial C_F}{\partial q_F}\frac{1}{\theta_m}.$$

Combining the two conditions above, it follows that

$$\theta_m F(\theta_m) = \frac{\partial C_F}{\partial q_F}. \tag{32}$$

Differentiating both sides of (32) with respect to p_I yields

$$\frac{d\theta_m}{dp_I}F(\theta_m) + \theta_m f(\theta_m)\frac{d\theta_m}{dp_I} = \frac{\partial^2 C_F}{\partial q_F^2}\frac{dq_F}{dp_I} + \frac{\partial^2 C_F}{\partial q_F \partial X_F}\left(\frac{\partial X_F}{\partial p_F}\frac{dp_F}{dp_I} + \frac{\partial X_F}{\partial q_F}\frac{dq_F}{dp_I}\right).$$

Recalling that we have

$$\frac{\partial X_F}{\partial p_F} = -\frac{f(\theta_m)}{q_I - q_F}$$

together with

$$\frac{\partial X_F}{\partial q_F} = \theta_m \frac{f(\theta_m)}{q_I - q_F},$$

we can ultimately write

$$\frac{d\theta_m}{dp_I} = \frac{\frac{\partial^2 C_F}{\partial q_F^2}\frac{dq_F}{dp_I} + \frac{\partial^2 C_F}{\partial q_F \partial X_F}\frac{f(\theta_m)}{q_I - q_F}\left(\theta_m \frac{dq_F}{dp_I} - \frac{dp_F}{dp_I}\right)}{F(\theta_m) + \theta_m f(\theta_m)}. \tag{33}$$

Let us next differentiate both sides of (32) with respect to q_I, which returns

$$\frac{d\theta_m}{dq_I}F(\theta_m) + \theta_m f(\theta_m)\frac{d\theta_m}{dq_I} = \frac{\partial^2 C_F}{\partial q_F^2}\frac{dq_F}{dq_I} + \frac{\partial^2 C_F}{\partial q_F \partial X_F}\left(\frac{\partial X_F}{\partial p_F}\frac{dp_F}{dq_I} + \frac{\partial X_F}{\partial q_F}\frac{dq_F}{dq_I}\right).$$

This is equivalent to

$$\frac{d\theta_m}{dq_I}\left[F(\theta_m) + \theta_m f(\theta_m)\right] = \frac{\partial^2 C_F}{\partial q_F^2}\frac{dq_F}{dq_I} + \frac{f(\theta_m)}{q_I - q_F}\frac{\partial^2 C_F}{\partial q_F \partial X_F}\left(\theta_m \frac{dq_F}{dq_I} - \frac{dp_F}{dq_I}\right),$$

which finally yields

$$\frac{d\theta_m}{dq_I} = \frac{\frac{\partial^2 C_F}{\partial q_F^2}\frac{dq_F}{dq_I} + \frac{f(\theta_m)}{q_I - q_F}\frac{\partial^2 C_F}{\partial q_F \partial X_F}\left(\theta_m \frac{dq_F}{dq_I} - \frac{dp_F}{dq_I}\right)}{F(\theta_m) + \theta_m f(\theta_m)}. \tag{34}$$

Relying upon (31a) and (31b), we can write

$$\begin{aligned}&\left[F(\theta_m) + \theta_m f(\theta_m)\right]\left(1 - \frac{dp_F}{dp_I} + \theta_m \frac{dq_F}{dp_I}\right)\\ = &\quad (q_I - q_F)\left[\frac{\partial^2 C_F}{\partial q_F^2}\frac{dq_F}{dp_I} + \frac{\partial^2 C_F}{\partial q_F \partial X_F}\frac{f(\theta_m)}{q_I - q_F}\left(\theta_m \frac{dq_F}{dp_I} - \frac{dp_F}{dp_I}\right)\right]\end{aligned}$$

together with

$$\begin{aligned}&\left[F(\theta_m) + \theta_m f(\theta_m)\right]\left(-\theta_m - \frac{dp_F}{dq_I} + \theta_m \frac{dq_F}{dq_I}\right)\\ = &\quad (q_I - q_F)\left[\frac{\partial^2 C_F}{\partial q_F^2}\frac{dq_F}{dq_I} + \frac{f(\theta_m)}{q_I - q_F}\frac{\partial^2 C_F}{\partial q_F \partial X_F}\left(\theta_m \frac{dq_F}{dq_I} - \frac{dp_F}{dq_I}\right)\right].\end{aligned}$$

Hence, we obtain

$$
\begin{aligned}
& \left[F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)\right] \\
= & \left[F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)-f\left(\theta_m\right) \frac{\partial^2 C_F}{\partial q_F \partial X_F}\right]\left(\frac{d p_F}{d p_I}-\theta_m \frac{d q_F}{d p_I}\right)+\left(q_I-q_F\right) \frac{\partial^2 C_F}{\partial q_F^2} \frac{d q_F}{d p_I}
\end{aligned}
$$

as well as

$$
\begin{aligned}
& -\theta_m\left[F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)\right] \\
= & \left[F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)-f\left(\theta_m\right) \frac{\partial^2 C_F}{\partial q_F \partial X_F}\right]\left(\frac{d p_F}{d q_I}-\theta_m \frac{d q_F}{d q_I}\right)+\left(q_I-q_F\right) \frac{\partial^2 C_F}{\partial q_F^2} \frac{d q_F}{d q_I}.
\end{aligned}
$$

It follows that

$$
\frac{d p_F}{d p_I}-\theta_m \frac{d q_F}{d p_I}=\frac{F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)-\left(q_I-q_F\right) \frac{\partial^2 C_F}{\partial q_F^2} \frac{d q_F}{d p_I}}{F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)-f\left(\theta_m\right) \frac{\partial^2 C_F}{\partial q_F \partial X_F}}
$$

and

$$
\frac{d p_F}{d q_I}-\theta_m \frac{d q_F}{d q_I}=\frac{-\theta_m\left[F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)\right]-\left(q_I-q_F\right) \frac{\partial^2 C_F}{\partial q_F^2} \frac{d q_F}{d q_I}}{F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)-f\left(\theta_m\right) \frac{\partial^2 C_F}{\partial q_F \partial X_F}}.
$$

If $C_F(\cdot)$ is linear in q_F (*i.e.*, if it is $\left(\partial^2 C_F / \partial q_F^2\right) \equiv 0$), then one has

$$
\begin{aligned}
\left(q_I-q_F\right) \frac{d \theta_m}{d p_I} & = 1-\frac{d p_F}{d p_I}+\theta_m \frac{d q_F}{d p_I} \\
& = 1-\frac{F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)}{F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)-f\left(\theta_m\right) \frac{\partial^2 C_F}{\partial q_F \partial X_F}} \\
& = \frac{F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)-f\left(\theta_m\right) \frac{\partial^2 C_F}{\partial q_F \partial X_F}-F\left(\theta_m\right)-\theta_m f\left(\theta_m\right)}{F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)-f\left(\theta_m\right) \frac{\partial^2 C_F}{\partial q_F \partial X_F}} \\
& = \frac{-f\left(\theta_m\right) \frac{\partial^2 C_F}{\partial q_F \partial X_F}}{F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)-f\left(\theta_m\right) \frac{\partial^2 C_F}{\partial q_F \partial X_F}}
\end{aligned}
$$

and

$$
\begin{aligned}
\left(q_I-q_F\right) \frac{d \theta_m}{d q_I} & = -\theta_m-\frac{d p_F}{d q_I}+\theta_m \frac{d q_F}{d q_I} \\
& = -\theta_m\left[1-\frac{F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)}{F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)-f\left(\theta_m\right) \frac{\partial^2 C_F}{\partial q_F \partial X_F}}\right] \\
& = -\theta_m\left[\frac{F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)-f\left(\theta_m\right) \frac{\partial^2 C_F}{\partial q_F \partial X_F}-F\left(\theta_m\right)-\theta_m f\left(\theta_m\right)}{F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)-f\left(\theta_m\right) \frac{\partial^2 C_F}{\partial q_F \partial X_F}}\right] \\
& = \frac{\theta_m f\left(\theta_m\right) \frac{\partial^2 C_F}{\partial q_F \partial X_F}}{F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)-f\left(\theta_m\right) \frac{\partial^2 C_F}{\partial q_F \partial X_F}}.
\end{aligned}
$$

It follows that

$$\frac{d\theta_m}{dq_I} = -\theta_m \frac{d\theta_m}{dp_I}.$$

Let us next look at the case where $\left(\partial^2 C_F / \partial q_F^2\right) \neq 0$. In particular, from (32) we have

$$\begin{aligned}
\frac{\partial^2 C_F}{\partial q_F^2} &= \frac{\partial}{\partial q_F}\left[\theta_m F\left(\theta_m\right)\right] \\
&= \frac{\partial \theta_m}{\partial q_F} F\left(\theta_m\right) + \theta_m \frac{\partial F\left(\theta_m\right)}{\partial \theta_m} \frac{\partial \theta_m}{\partial q_F} \\
&= \frac{\partial \theta_m}{\partial q_F}\left[F\left(\theta_m\right) + \theta_m f\left(\theta_m\right)\right] \\
&= \frac{\theta_m}{q_I - q_F}\left[F\left(\theta_m\right) + \theta_m f\left(\theta_m\right)\right].
\end{aligned}$$

Recalling that $X_F = F\left(\theta_m\right)$, from (32) we also deduce that

$$\begin{aligned}
\frac{\partial^2 C_F}{\partial q_F \partial X_F} &= \frac{\partial}{\partial X_F}\left[\theta_m F\left(\theta_m\right)\right] \\
&= \frac{\partial}{\partial F\left(\theta_m\right)}\left[\theta_m F\left(\theta_m\right)\right] \\
&= \theta_m.
\end{aligned}$$

Using these results, we can write

$$\begin{aligned}
\frac{dp_F}{dp_I} - \theta_m \frac{dq_F}{dp_I} &= \frac{F\left(\theta_m\right) + \theta_m f\left(\theta_m\right) - \left(q_I - q_F\right) \frac{\theta_m}{q_I - q_F}\left[F\left(\theta_m\right) + \theta_m f\left(\theta_m\right)\right] \frac{dq_F}{dp_I}}{F\left(\theta_m\right) + \theta_m f\left(\theta_m\right) - \theta_m f\left(\theta_m\right)} \\
&= \frac{F\left(\theta_m\right) + \theta_m f\left(\theta_m\right) - \theta_m\left[F\left(\theta_m\right) + \theta_m f\left(\theta_m\right)\right] \frac{dq_F}{dp_I}}{F\left(\theta_m\right)} \\
&= \frac{F\left(\theta_m\right) + \theta_m f\left(\theta_m\right)}{F\left(\theta_m\right)}\left(1 - \theta_m \frac{dq_F}{dp_I}\right)
\end{aligned}$$

as well as

$$\begin{aligned}
\frac{dp_F}{dq_I} - \theta_m \frac{dq_F}{dq_I} &= \frac{-\theta_m\left[F\left(\theta_m\right) + \theta_m f\left(\theta_m\right)\right] - \theta_m\left[F\left(\theta_m\right) + \theta_m f\left(\theta_m\right)\right] \frac{dq_F}{dq_I}}{F\left(\theta_m\right) + \theta_m f\left(\theta_m\right) - \theta_m f\left(\theta_m\right)} \\
&= -\theta_m \frac{F\left(\theta_m\right) + \theta_m f\left(\theta_m\right)}{F\left(\theta_m\right)}\left(1 + \frac{dq_F}{dq_I}\right).
\end{aligned}$$

It follows that

$$
\begin{aligned}
(q_I - q_F)\frac{d\theta_m}{dp_I} &= 1 - \frac{dp_F}{dp_I} + \theta_m \frac{dq_F}{dp_I} \\
&= 1 - \frac{F(\theta_m) + \theta_m f(\theta_m)}{F(\theta_m)} \left(1 - \theta_m \frac{dq_F}{dp_I}\right) \\
&= \frac{F(\theta_m) - [F(\theta_m) + \theta_m f(\theta_m)]\left(1 - \theta_m \frac{dq_F}{dp_I}\right)}{F(\theta_m)} \\
&= \frac{F(\theta_m) - F(\theta_m) - \theta_m f(\theta_m) + \theta_m [F(\theta_m) + \theta_m f(\theta_m)] \frac{dq_F}{dp_I}}{F(\theta_m)} \\
&= \theta_m \frac{-f(\theta_m) + [F(\theta_m) + \theta_m f(\theta_m)] \frac{dq_F}{dp_I}}{F(\theta_m)} \\
&= \frac{\theta_m}{F(\theta_m)} \left\{ [F(\theta_m) + \theta_m f(\theta_m)] \frac{dq_F}{dp_I} - f(\theta_m) \right\}
\end{aligned}
$$

and so that

$$
\frac{d\theta_m}{dp_I} = \frac{\theta_m}{(q_I - q_F) F(\theta_m)} \left\{ [F(\theta_m) + \theta_m f(\theta_m)] \frac{dq_F}{dp_I} - f(\theta_m) \right\}.
$$

It also follows that

$$
\begin{aligned}
(q_I - q_F)\frac{d\theta_m}{dq_I} &= -\theta_m - \frac{dp_F}{dq_I} + \theta_m \frac{dq_F}{dq_I} \\
&= -\theta_m + \theta_m \frac{F(\theta_m) + \theta_m f(\theta_m)}{F(\theta_m)} \left(1 + \frac{dq_F}{dq_I}\right) \\
&= \frac{-\theta_m F(\theta_m) + \left[\theta_m F(\theta_m) + \theta_m^2 f(\theta_m)\right]\left(1 + \frac{dq_F}{dq_I}\right)}{F(\theta_m)} \\
&= \frac{-\theta_m F(\theta_m) + \theta_m F(\theta_m) + \theta_m^2 f(\theta_m) + \left[\theta_m F(\theta_m) + \theta_m^2 f(\theta_m)\right] \frac{dq_F}{dq_I}}{F(\theta_m)} \\
&= \frac{\theta_m^2 f(\theta_m) + \left[\theta_m F(\theta_m) + \theta_m^2 f(\theta_m)\right] \frac{dq_F}{dq_I}}{F(\theta_m)} \\
&= \frac{\theta_m}{F(\theta_m)} \left\{ [F(\theta_m) + \theta_m f(\theta_m)] \frac{dq_F}{dq_I} + \theta_m f(\theta_m) \right\}
\end{aligned}
$$

and so that

$$
\frac{d\theta_m}{dq_I} = \frac{\theta_m}{(q_I - q_F) F(\theta_m)} \left\{ [F(\theta_m) + \theta_m f(\theta_m)] \frac{dq_F}{dq_I} + \theta_m f(\theta_m) \right\}.
$$

Therefore, in the general case, for the condition

$$
\frac{d\theta_m}{dq_I} = -\theta_m \frac{d\theta_m}{dp_I}
$$

to hold, it must be the case that

$$\begin{aligned}
\left[F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)\right] \frac{dq_F}{dq_I}+\theta_m f\left(\theta_m\right) &= -\theta_m\left\{\left[F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)\right] \frac{dq_F}{dp_I}-f\left(\theta_m\right)\right\} \\
&= -\theta_m\left[F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)\right] \frac{dq_F}{dp_I}+\theta_m f\left(\theta_m\right),
\end{aligned}$$

that is, we need to have

$$\left[F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)\right] \frac{dq_F}{dq_I}=-\theta_m\left[F\left(\theta_m\right)+\theta_m f\left(\theta_m\right)\right] \frac{dq_F}{dp_I}$$

or, equivalently,

$$-\frac{\frac{dq_F}{dq_I}}{\frac{dq_F}{dp_I}}=-\frac{dp_I}{dq_I}=\theta_m.$$