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S-BASED TAXATION UNDER DEFAULT RISK

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S-Based Taxation under Default Risk

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Abstract

This article studies the characteristics of a S-based tax system under default risk. In particular we show that its neutrality properties depend on whether debt is protected or unprotected. In the former case, this system is neutral. In the latter case, where default timing is optimally chosen by shareholders, the S-based system is neutral with respect to real decisions only if the firm's and the lender's tax rate are equal. However, the shareholders' decision to default is always distorted.

JEL classification: H25 and H32.

Keywords: capital structure, corporate taxation, neutrality, option pricing.

1 Introduction

One of the basic neutral tax designs is the 'imputed income method' (see Boadway and Bruce, 1984). Over the last decade this method has become popular in Europe. At the beginning of the '90s, the Nordic countries introduced Dual Income Tax (DIT) which allows a lower tax rate on capital income. In the same period, the IFS Capital Taxes Group (1991) proposed the Allowance for Corporate Equity (ACE) and subsequently other countries, such as Austria, Croatia and Italy, applied dual tax systems.[1] More recently, the Skauge Committee has proposed in Norway a tax system which shares some important features with the ACE tax system.[2]

The imputed income system allows two alternative treatments of the cost of debt. On the one hand, debt is deductible at the risk-free rate: this system will be named R-based. On the other hand, debt is deductible at the interest rate actually paid: we will call it the S-based system[3]. This option, which is quite common in existing tax systems, is supported by Keen and King (2002). They stress the fact that the computation of the tax base is easy as it is based on book values. Moreover they argue that the deduction of actual interest expenses does not distort firms' choices as long as debt is competitively supplied. In this case all rents accrue to shareholders, and taxation is neutral. This point is proven by Bond and Devereux (2003) (hereafter BD), who show that an S-based system is neutral under default risk even if the firm's and the lender's tax rate are different[4]. However, BD point out that their result holds on condition that capital markets are perfectly efficient and information is symmetric. If, instead, capital markets are imperfect and in particular information is asymmetric, borrowers and lenders might collude in order to avoid taxation. In this case, the government

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[1] For further details see e.g. Bordignon et al. (2001), Cnossen (2005), Fehr and Wiegard (2001), Keen and King (2002), Keen (2003) and Zodrow (2003).

[2] The Norwegian proposal entails the introduction of a Rate-of-Return Allowance (RRA) at the level of domestic personal shareholders, rather than at the corporate level. The RRA is calculated as the risk-free interest rate multiplied by the basis of the share (see Sφrensen, 2003).

[3] For a comparison between the S-based and the R-based system see Becker and Fuest (2005).

[4] This property makes dual systems attractive at least in a closed economy (see Bond, 2000). As recently proven by Panteghini (2004), however, the imputed method can be attractive even in an open economy if multinational firms can decide not only whether but also when to invest.

would require a greater amount of information to fight tax avoidance.

In this article we aim to show that the neutrality properties of the S-based system are less general than thought, and that distortions may arise even under perfect credit market efficiency and symmetric information. To show this we will depart from BD, who treat default as an exogenous event, and introduce two well-known default conditions. The first one refers to *protected debt.* In this case default may be triggered when the firms' asset value falls to the debt's value. This means that debt is approximated with a positive net-covenant.[5] The second default condition regards *unprotected debt*, and implies that default timing is optimally chosen by shareholders. When the firm's net cash flow is negative, shareholders may have the opportunity to inject further equity capital in order to meet the firm's debt obligations. As long as they issue new capital and pay the interest rate they can exploit future recoveries in the firm's profitability. In this case, shareholders behave as if they owned a put option, whose exercise leads to default.[6] It will be shown that neutrality holds only under fully protected debt financing. If, instead, debt is unprotected, a twofold distortion may arise. Firstly, investment may be distorted unless a uniform tax rate is levied on both shareholders and the lender. Secondly, taxation always affects shareholders' decision to default.

The article is structured as follows. Section 2 introduces a continuous-time model and analyzes both protected and unprotected debt. Section 3 describes the effects of S-based taxation on the firm's value. Section 4 derives the neutrality results under protected and unprotected debt financing. Section 5 summarizes the results and discusses their policy implications.

2   The model

In line with Leland (1994) we apply a continuous-time model describing the investment decision made by a representative firm. At time 0, the firm decides whether to undertake an infinitely-lived project, whose cost is I: investment is thus undertaken if the project Net Present Value is positive.[7]

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[5] For further details see Brennan and Schwartz (1978), and Smith and Wardner (1977).

[6] As pointed out by Leland (1994) both protected and unprotected debt are widely used. In particular, minimum net-worth requirements, implied by protected debt, are common in short-term debt financing, whereas long-term debt instruments are usually unprotected or only partially protected.

[7] For simplicity, we assume that the firm cannot postpone its investment decision.

The firm's EBIT at time t, defined as Π_t, is subject to a shock and, in particular, follows a geometric Brownian motion

$$\frac{d\Pi_t}{\Pi_t} = \sigma dz \text{ with } \Pi_0 > 0, \tag{1}$$

where σ is the instantaneous standard deviation of $\frac{d\Pi_t}{\Pi_t}$.[8]

The assumptions on the firm's capital structure are the following:

1. the risk-free interest rate r is fixed;

2. credit markets are perfectly competitive;

3. information is symmetric;

4. at time 0, the firm borrows some resources and pays a constant coupon $C \leq \Pi_0$, which cannot be renegotiated.

The first three assumptions describe a perfectly efficient setting. Assumption 4 deserves some comment. It is worth noting that setting C and then computing the market value of debt is equivalent to set first, the value of debt and then, compute the equilibrium interest rate.[9] Assumption 4 also means that debt maturity is in line with the investment lifetime.[10]

Default occurs when the firm does not meet its debt obligation. In this case it is expropriated by the lender. Let us next introduce the two default conditions.

Definition 1 *Under protected debt financing, default takes place when Π_t falls to an exogenously given threshold point $\widetilde{\Pi}^p$.*

As will be shown, full debt protection is ensured if the threshold point $\widetilde{\Pi}^p$ is equal to C.

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[8]With no loss of generality, we assume that the drift of the Brownian motion is nil.

[9]Notice that the introduction of any trade-off is beyond the scope of this article. In the absence of any distress cost, therefore, C is not the result of an optimal choice but rather is exogenously given.

[10]This assumption is realistic. As shown for instance by Graham and Harvey (2001) more than 63% of the US firms surveyed state that debt maturity is aimed at matching with assets' lifetime. On the other hand, BD assume a one-period debt financing strategy for a two-period investment project. However, they argue that the quality of results does not change if a new debt issue in the following period is assumed (see Bond and Devereux, 1999).

Definition 2 *Under unprotected debt financing, the threshold point $\widetilde{\Pi}^u$ is chosen optimally by shareholders at time 0.*

In both cases, the existence of symmetric information means that, at time 0, the creditor knows $\widetilde{\Pi}^i$ with $i = p, u$ before lending.

3   The tax system

Let us next introduce the S-based system, which allows the deduction of both effective interest payments and the opportunity cost of equity finance. Under full loss-offset[11], the tax base will be

$$\left\{ \Pi_t - C - r\left[I - D^i\left(\widetilde{\Pi}^i; \Pi_0, C\right)\right]\right\} \text{ for } i = p, u \text{ and } t \geq 0.$$

As can be seen the relief on equity finance is equal to the default-free interest rate times the book value of equity[12], $\left[I - D^i\left(\widetilde{\Pi}^i; \Pi_0, C\right)\right]$, that is the difference between the historical cost of the investment project and the initial value of debt.[13]

Contrary to cash-flow systems, tax benefits are distributed along the investment's lifetime. This implies that, in the event of default, shareholders would fail to obtain a full tax benefit, unless an *ad hoc* rebate were granted. For this reason, BD propose a rebate equal to

$$R^i\left(\widetilde{\Pi}^i\right) = \tau\left(I - \frac{\widetilde{\Pi}^i}{r}\right) \text{ for } i = p, u, \tag{2}$$

which is computed when default takes place. As shown in (2) the rebate is equal to the tax rate τ multiplied by the difference between the book value of the asset, i.e. I, and the net present value of the subsequent pre-tax cash flow in the event of default, i.e. $\frac{\widetilde{\Pi}^i}{r}$. This means that, under symmetric information, the government observes $\widetilde{\Pi}^i$ and then sets the rebate, thereby ensuring full compensation.

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[11] For a discussion on the effects of no loss-offset, see e.g. Panteghini (2001, 2005).

[12] This rule, which only requires the knowledge of book values, is in line with the ACE tax proposal. See e.g. Devereux and Freeman (1991).

[13] According to assumption 4, $D^i\left(\widetilde{\Pi}^i; \Pi_0, C\right)$ represents both the market and book value of debt at time 0.

3.1 The value of debt

Like the firm, the lender is subject to S-based tax, though its tax rate may differ from the firm's one.

Before default, the lender' base is given by the difference between the interest payment (i.e. the coupon) and the opportunity cost of debt, i.e. $rD^i\left(\widetilde{\Pi}^i; \Pi_0, C\right)$. In line with BD, we assume that when default occurs, the tax relief is proportional to the net present value of the subsequent pre-tax cash flows of the project. In this case, the opportunity cost of debt will be r times the market value of debt when $\Pi = \widetilde{\Pi}^i$, i.e. $D^i\left(\widetilde{\Pi}^i; \widetilde{\Pi}^i, C\right)$. Given the tax rate h the lender's after-tax cash flow will then be

$$
\begin{cases}
C - h\left[C - rD^i\left(\widetilde{\Pi}^i; \Pi_0, C\right)\right] & \text{before default,} \\
\Pi_t - h\left[\Pi_t - rD^i\left(\widetilde{\Pi}^i; \widetilde{\Pi}^i, C\right)\right] & \text{after default.}
\end{cases}
$$

Under market efficiency, the value of debt at time 0 is such that the following non-arbitrage condition (see Appendix A)

$$
rD^i\left(\widetilde{\Pi}^i; \Pi_0, C\right) = C + \frac{\frac{\sigma^2}{2}\Pi^2 D_{\Pi\Pi}^i\left(\widetilde{\Pi}^i; \Pi_0, C\right)}{1 - h} \tag{3}
$$

holds. Equation (3) entails the equality between a risk-free asset whose return is r and a risky asset (the lender's credit) whose return is the effective interest rate, net of the default risk premium. As can be seen, taxation may be distortive since any change in the rate h must be offset by a change in the value of debt in order for condition (3) to hold. Solving (3) we can write the value of debt at time 0 (see Appendix A) as a weighted average between the cash flow received before default and that received after:

$$
D^i\left(\widetilde{\Pi}^i; \Pi_0, C\right) = \left[1 - c\left(\widetilde{\Pi}^i; \Pi_0\right)\right]\frac{C}{r} + c\left(\widetilde{\Pi}^i; \Pi_0\right)\frac{\widetilde{\Pi}^i}{r}. \tag{4}
$$

The weight is $c\left(\widetilde{\Pi}^i; \Pi_0\right) \equiv \frac{b\left(\widetilde{\Pi}^i; \Pi_0\right)}{1 - h + hb\left(\widetilde{\Pi}^i; \Pi_0\right)}$, where $b\left(\widetilde{\Pi}^i; \Pi_0\right) \equiv \left(\frac{\Pi_0}{\widetilde{\Pi}^i}\right)^{\beta_2}$, and $\beta_2 < 0$ is the negative root of the characteristic equation

$$
\phi\left(\beta\right) \equiv \frac{\sigma^2}{2}\beta(\beta - 1) - r = 0. \tag{5}
$$

The term $c\left(\widetilde{\Pi}^i; \Pi_0\right)$ measures the present value of 1 Euro contingent on the event default. As can be seen, it differs from the discount factor $b\left(\widetilde{\Pi}^i; \Pi_0\right)$. This is due to the fact that an increase in the rate h raises the after-tax risk premium and, consequently, $c\left(\widetilde{\Pi}^i; \Pi_0\right)$. Therefore the inequality $b\left(\widetilde{\Pi}^i; \Pi_0\right) < c\left(\widetilde{\Pi}^i; \Pi_0\right)$ holds.

Let us next analyze the impact of the default conditions on the value of debt. By definition, full protection implies that the value of debt is equal to that of a default risk-free asset. Using (4) we can thus obtain

$$\frac{C}{D^p\left(\widetilde{\Pi}^p; \Pi_0, C\right)} = r. \tag{6}$$

Equality (6) implies that full protection holds if $\widetilde{\Pi}^p = C$. It is thus clear that taxation does not affect the value of debt. This is the case implicitly analyzed by BD.

When debt is unprotected, the trigger point $\widetilde{\Pi}^u$ is not necessarily equal to C. In this case, default entails that the lender's expected cash flow changes from C to $\widetilde{\Pi}^u$. Using (4) we obtain the effective interest rate as the sum of the risk-free interest rate and the default risk premium:

$$\frac{C}{D^u\left(\widetilde{\Pi}^u; \Pi_0, C\right)} = r + \underbrace{c\left(\widetilde{\Pi}^u; \Pi_0\right) \frac{C - \widetilde{\Pi}^u}{r}}_{\text{default risk premium}}. \tag{7}$$

3.2 The value of equity

To compute the value of equity we must introduce a boundary condition regarding default. We know that at point $\Pi = \widetilde{\Pi}^i$ shareholders are expropriated, and, therefore, their claim is just equal to the tax rebate. Defining $E^i\left(\widetilde{\Pi}^i; \Pi_t, C\right)$ as the value of equity therefore, we can write the following Value Matching Condition[14]

$$E^i\left(\widetilde{\Pi}^i; \widetilde{\Pi}^i, C\right) = R^i\left(\widetilde{\Pi}^i\right) \text{ for } i = p, u. \tag{8}$$

---

[14]For details on this condition see Leland (1994) and Dixit and Pindyck (1994).

Applying condition (8) (see Appendix B) we can compute the value of equity

$$E^i\left(\widetilde{\Pi}^i;\Pi_t,C\right) = \frac{Y^i\left(\widetilde{\Pi}^i;\Pi_t,C\right)}{r} + b\left(\widetilde{\Pi}^i;\Pi_t\right)\left[R^i\left(\widetilde{\Pi}^i\right) - \frac{Y^i\left(\widetilde{\Pi}^i;\widetilde{\Pi}^i,C\right)}{r}\right] \quad \text{for } t \geq 0,$$

(9)

where

$$Y^i\left(\widetilde{\Pi}^i;\Pi_t,C\right) \equiv (1-\tau)(\Pi_t - C) + \tau r\left[I - D^i\left(\widetilde{\Pi}^i;\Pi_0,C\right)\right]$$

is the net-of-tax cash flow. As shown in (9), the value of equity consists of two terms. The first term, $\frac{Y^i\left(\widetilde{\Pi}^i;\Pi_0,C\right)}{r}$, is a perpetual rent proportional to after-tax cash flow. The second term measures the overall effect of default. This component is equal to the product between the discount factor $b\left(\widetilde{\Pi}^i;\Pi_t\right)$ and the shareholders' net tax rebate $\left[R^i\left(\widetilde{\Pi}^i\right) - \frac{Y^i\left(\widetilde{\Pi}^i;\widetilde{\Pi}^i,C\right)}{r}\right]$, i.e. the tax rebate received in the event of default minus future cash flow lost by expropriation. As can be seen, the shareholders' discount factor is $b\left(\widetilde{\Pi}^i;\Pi_t\right)$, and thus differs from the lender's one.[15]

4   Neutrality results

Let us next analyze the neutrality properties of the S-based tax system. Here we deal with both real and financial neutrality, when the firm's decisions are made at time $t = 0$. To define real neutrality let us write the firm's Net Present Value

$$NPV^i\left(\widetilde{\Pi}^i;\Pi_0,C\right) = E^i\left(\widetilde{\Pi}^i;\Pi_0,C\right) + D^i\left(\widetilde{\Pi}^i;\Pi_0,C\right) - I.$$

(10)

Following Brown (1948) we can thus state that

Condition 1 *Real neutrality holds if*

$$NPV^i\left(\widetilde{\Pi}^i;\Pi_0,C\right) = (1-\tau)NPV^*\left(\Pi_0\right),$$

(11)

where $NPV^*\left(\Pi_0\right) \equiv \frac{\Pi_0}{r} - I$ is the Net Present Value in the absence of taxation.

---

[15]Remember that $c\left(\widetilde{\Pi}^i;\Pi_t\right) > b\left(\widetilde{\Pi}^i;\Pi_t\right)$.

As pointed out by Johansson (1969, p.104), equation (11) entails that an 'identical ranking of alternative investments is obtained in a pre-tax and post-tax profitability analysis'. Moreover, condition (11) implies that the Modigliani-Miller Theorem holds.

Substituting (4) and (9) into (10) yields

$$NPV^i\left(\widetilde{\Pi}^i; \Pi_0, C\right) = (1 - \tau)\, NPV^*\left(\Pi_0\right) + X\left(\widetilde{\Pi}^i; \Pi_0, C\right) \qquad (12)$$

where

$$X\left(\widetilde{\Pi}^i; \Pi_0, C\right) \equiv \left\{(1 - \tau)\left[1 - b\left(\widetilde{\Pi}^i; \Pi_0, C\right)\right]\left[D^i\left(\widetilde{\Pi}^i; \Pi_0, C\right) - \frac{C}{r}\right]\right\} +$$
$$+ \left\{b\left(\widetilde{\Pi}^i; \Pi_0, C\right)\left[D^i\left(\widetilde{\Pi}^i; \Pi_0, C\right) - D^i\left(\widetilde{\Pi}^i; \widetilde{\Pi}^i, C\right)\right]\right\}$$

measures the overall tax distortion. This distortion consists of two terms. The first one measures the present value of the after-tax risk premium before default. The second term is given by the expected devaluation of debt conditional on the event of debt. If therefore $X\left(\widetilde{\Pi}^i; \Pi_0, C\right) = 0$, real neutrality is ensured.

The second neutrality condition regards unprotected debt. In this case, indeed, shareholders can decide when to default. Defining $\widetilde{\Pi}^u_{LF}$ as the default trigger point in the absence of taxation, we can thus state that

Condition 2 *Financial neutrality holds if default timing is not affected by taxation, i.e. $\widetilde{\Pi}^u = \widetilde{\Pi}^u_{LF}$.*

4.1 Protected debt

Let us start with protected debt financing. We can write the following

Proposition 1 *If debt is fully protected the S-based system is neutral.*

Proof. See Appendix C. ∎

As pointed out in Section 3.1 full protection is ensured if $\widetilde{\Pi}^p = C$. This implies that $D^p\left(\widetilde{\Pi}^p; \Pi_0, C\right) = D^p\left(C; \Pi_0, C\right)$. Moreover, the lender's post-default claim is equal to $D^i\left(\widetilde{\Pi}^i; \widetilde{\Pi}^i, C\right) = \frac{\widetilde{\Pi}^i}{r}$, $i = u, p$ (see Appendix C). Therefore the shareholders' rebate is equal to $R^p(C) = \tau\left(I - \frac{C}{r}\right)$: this makes $X\left(\widetilde{\Pi}^p; \Pi_0, C\right)$ fall to zero. The intuition behind this result is straightforward:

the cash flow tax and equivalently, any imputed income tax systems can be thought of as ensuring relief for both the risk-free rate and the risk premium (see Devereux, 2003). As shown in (6), however, the default risk premium falls to zero and any distortive effect of h on the value of debt vanishes.

4.2   Unprotected debt

Proposition 1 is in line with BD's findings. When debt is unprotected, however, results are quite different. In this case, the default trigger point $\widetilde{\Pi}^u$ is optimally chosen by maximizing the value of equity, i.e.

$$\max_{\widetilde{\Pi}^u} E^u\left(\widetilde{\Pi}^u; \Pi_0, C\right). \tag{13}$$

Solving (13) we obtain

Proposition 2 *Under unprotected debt financing the inequalities* $\widetilde{\Pi}^u < \widetilde{\Pi}^u_{LF} < C$ *hold* $\forall \tau$.

   Proof. See Appendix D. ∎

   Proposition 2 shows that shareholders postpone their default decision. To get the intuition behind the result it is worth noting that $\widetilde{\Pi}^u_{LF} < C$, namely that, without taxation, the default option is exercised when the net cash flow is negative. This is due to the fact that default is an irreversible choice[16]: shareholders are aware that the exercise of the put option entails the *irreversible* loss of any opportunity to exploit future profit recoveries. Under the S-based system, the lower the point $\widetilde{\Pi}^u$ is, the greater is the value of the rebate received by shareholders. Not surprisingly therefore we have $\widetilde{\Pi}^u < \widetilde{\Pi}^u_{LF}$, i.e. default timing is delayed.[17]

   Let us next focus on real effects. We can prove the following:

Proposition 3 *Under unprotected debt financing a real distortion arises if* $h \neq \tau$, *i.e.*

$$NPV^u\left(\widetilde{\Pi}^u; \Pi_0, C\right) - (1 - \tau) NPV^*\left(\Pi_0\right) \propto (\tau - h). \tag{14}$$

---

[16]Remember that the default trigger point $\widetilde{\Pi}^u$ is set at time 0 and is known by the lender.

[17]Notice that this behaviour is stimulated by full loss-offset provisions. In this case, loss-making firms enjoy a tax benefit and may be induced to further postpone default.

Proof. See Appendix E. ∎

The intuition behind Proposition 3 is as follows. Assume that initially the equality $\tau = h$ holds. Then suppose that h is cut. This tax rate change has a twofold effect. On the one hand, it reduces the value of debt $D^u\left(\widetilde{\Pi}^u; \Pi_0, C\right)$, and, on the other hand, raises the shareholders tax benefit $\tau r\left[I - D^u\left(\widetilde{\Pi}^u; \Pi_0, C\right)\right]$, thereby increasing the value of equity. Proposition 3 thus shows that the decrease in $D^u\left(\widetilde{\Pi}^u; \Pi_0, C\right)$ is overcompensated by the increase in $E^u\left(\widetilde{\Pi}^u; \Pi_0, C\right)$, so inequality $NPV^u\left(\widetilde{\Pi}^u; \Pi_0, C\right) > (1-\tau) NPV^*\left(\Pi_0\right)$ holds and we have overinvestment.[18] The converse is true if h is raised.

The above result is not surprising if we disregard the firm's ownership and rather, focus on the project value. We can in fact state that inequality $\tau > h$ is equivalent to an expected tax rate cut occurring whenever Π falls to $\widetilde{\Pi}^u$. This expected tax cut thus stimulates investment.[19] The converse is true if $\tau < h$.

5   Concluding remarks and policy implications

In this article we have shown that the neutrality properties of an S-based system depend on the default condition assumed. In particular, we have shown that both real and financial neutrality hold when debt is protected.

When debt is unprotected, results are different. Real neutrality is ensured only under uniform taxation. Moreover, financial neutrality never holds, as shareholders are induced to delay their default decision.

In a companion article (Panteghini, 2004) we showed that an R-based system, allowing for deduction of debt at the risk-free rate, ensures both real and financial neutrality irrespective of whether debt is protected or unprotected. This allows us to conclude that in terms of neutrality, a R-based system is preferable to a S-based one even in perfectly efficient capital markets.

---

[18]Notice that under full debt protection, the default premium is nil and, therefore, this distortive effect disappears.

[19]A similar point is made by Bond and Devereux (1995) where they show that any expected future tax rate change is distortive.

A The computation of (3) and (4)

Using dynamic programming, the value of debt can be written as follows

$$
D^i\left(\widetilde{\Pi}^i;\Pi_t,C\right) = \begin{cases} \left\{C - h\left[C - rD^i\left(\Pi_0,C\right)\right]\right\}dt + e^{-rdt}\xi\left[D^i(\Pi_t + d\Pi_t,C)\right] & \text{before default,} \\ \\ \left[\Pi_t - h\left(\Pi - rD^i\left(\widetilde{\Pi}^i,C\right)\right)\right]dt + e^{-rdt}\xi\left[D^i(\Pi_t + d\Pi_t,C)\right] & \text{otherwise.} \end{cases}
$$

$$(15)$$

where $\xi\left[.\right]$ is the expectation operator.

Let us first focus on the pre-default value of debt. Expanding the RHS of (15) and applying Itô's Lemma, one obtains

$$
\begin{aligned}
D^i\left(\widetilde{\Pi}^i;\Pi_t,C\right) &= \left[(1-h)\,C + hrD^i\left(\widetilde{\Pi}^i;\Pi_0,C\right)\right]dt+ \\
&+ (1-rdt)\left[D^i\left(\widetilde{\Pi}^i;\Pi_t,C\right) + \tfrac{\sigma^2}{2}D^i_{\Pi\Pi}\left(\widetilde{\Pi}^i;\Pi_t,C\right)\right].
\end{aligned}
$$

$$(16)$$

Recalling that $(dt)^2 \to 0$ one easily obtains the following non-arbitrage condition

$$
rD^i\left(\widetilde{\Pi}^i;\Pi_t,C\right) = \left[(1-h)\,C + hrD^i\left(\widetilde{\Pi}^i;\Pi_0,C\right)\right] + \frac{\sigma^2}{2}\Pi^2 D^i_{\Pi\Pi}\left(\widetilde{\Pi}^i;\Pi_t,C\right).
$$

$$(17)$$

Manipulating (17) one easily obtains (3).

Let us next solve (17). Thus before default we obtain

$$
D^i\left(\widetilde{\Pi}^i;\Pi_t,C\right) = \frac{(1-h)\,C + hrD^i\left(\widetilde{\Pi}^i;\Pi_0,C\right)}{r} + \sum_{j=1}^{2}G^i_j\Pi_t^{\beta_j},
$$

$$(18)$$

where β_1 and β_2 are the positive and negative root of the characteristic equation (5), respectively. After default the value of debt is

$$
D^i\left(\widetilde{\Pi}^i;\Pi_t,C\right) = \frac{(1-h)\,\Pi_t + hrD^i\left(\widetilde{\Pi}^i;\widetilde{\Pi}^i,C\right)}{r} + \sum_{j=1}^{2}L^i_j\Pi_t^{\beta_j}.
$$

$$(19)$$

We can now compute the values of G^i_j and L^i_j for $j = 1, 2$. To do so we must introduce two boundary conditions, which are in line with the assumption

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of perfectly efficient capital markets.[20] Firstly, we assume that no financial bubbles exist. This means that $G_1^i = L_1^i = 0$. Secondly, we assume that when $\Pi_t = 0$, the lender's claim is worth the tax benefit, namely

$$D^i\left(\widetilde{\Pi}^i; 0, C\right) = hD^i\left(\widetilde{\Pi}^i; \widetilde{\Pi}^i, C\right).$$

This implies that $L_2^i = 0$. Using (19) we thus have

$$D^i\left(\widetilde{\Pi}^i; \widetilde{\Pi}^i, C\right) = \frac{\widetilde{\Pi}^i}{r}. \tag{20}$$

To compute G_2^i, let the two branches (18) and (19) meet at point $\Pi_t = \widetilde{\Pi}^i$. Using (20) we thus have

$$D^i\left(\widetilde{\Pi}^i; \widetilde{\Pi}^i, C\right) = \frac{(1-h)\,C + hr D^i\left(\widetilde{\Pi}^i; \Pi_0, C\right)}{r} + G_2^i\widetilde{\Pi}^{i^{\beta_2}} = \frac{\widetilde{\Pi}^i}{r}. \tag{21}$$

Using (18) and setting $\Pi = \Pi_0$ yields the value of debt at time 0

$$D^i\left(\widetilde{\Pi}^i; \Pi_0, C\right) = \frac{C}{r} + \frac{G_2^i\Pi_0^{\beta_2}}{1-h}. \tag{22}$$

Substituting (22) into (21), and solving for G_2^i we have

$$G_2^i = \frac{\widetilde{\Pi}^i - C}{r}\left[\frac{h}{1-h}\Pi_0^{\beta_2} + \widetilde{\Pi}^{i^{\beta_2}}\right]^{-1}. \tag{23}$$

Rearranging yields (4).

B   The computation of (9)

The firm's value of equity can be written as

$$E^i\left(\widetilde{\Pi}^i; \Pi_t, C\right) = \begin{cases} Y^i\left(\widetilde{\Pi}^i; \Pi_t, C\right)dt + e^{-rdt}\xi\left[E(\widetilde{\Pi}^i; \Pi_t + d\Pi_t, C)\right] & \text{before default,} \\[2em] R^i\left(\widetilde{\Pi}^i\right) & \text{otherwise.} \end{cases} \tag{24}$$

---

[20] For further details on this boundary conditions see e.g. Dixit and Pindyck (1994).

As shown in (24), in the event of default, shareholders are expropriated and the value of their claim is simply equal to $R^i\left(\widetilde{\Pi}^i\right)$.

Let us next focus on the pre-default case. Expanding the RHS of (24), one obtains the following Bellman equation

$$rE^i\left(\widetilde{\Pi}^i;\Pi_t,C\right) = Y^i\left(\widetilde{\Pi}^i;\Pi_t,C\right) + \frac{\sigma^2}{2}\Pi^2 E^i_{\Pi\Pi}\left(\widetilde{\Pi}^i;\Pi_t,C\right). \qquad (25)$$

where

$$Y^i\left(\widetilde{\Pi}^i;\Pi_t,C\right) \equiv (1-\tau)(\Pi_t - C) + \tau r\left[I - D^i\left(\widetilde{\Pi}^i;\Pi_0,C\right)\right].$$

Solving (25) yields

$$E^i\left(\widetilde{\Pi}^i;\Pi_t,C\right) = \frac{Y^i\left(\widetilde{\Pi}^i;\Pi_t,C\right)}{r} + \sum_{j=1}^{2}F^i_j\Pi_t^{\beta_2} \qquad (26)$$

Let us next compute F^i_j for $j = 1,2$. Notice that, in the absence of any bubble, we have $F^i_1 = 0$. To compute F^i_2 we use (26) and apply the Value Matching Condition (8). We thus obtain

$$\frac{Y^i\left(\widetilde{\Pi}^i;\widetilde{\Pi}^i,C\right)}{r} + F^i_2\widetilde{\Pi}^{i\beta_2} = R^i\left(\widetilde{\Pi}^i\right) = \tau\left(I - \frac{\widetilde{\Pi}^i}{r}\right),$$

which yields

$$F^i_2 = \left[R^i\left(\widetilde{\Pi}^i\right) - \frac{Y^i\left(\widetilde{\Pi}^i;\widetilde{\Pi}^i,C\right)}{r}\right]\left(\widetilde{\Pi}^i\right)^{-\beta_2}. \qquad (27)$$

Substituting (27) into (26), at point Π_0 yields (9).

C Proof of Proposition 1

Substituting (6) into (4), we easily obtain the equality $\widetilde{\Pi}^p = C$. This means that

$$D^p\left(\widetilde{\Pi}^p;\Pi_0,C\right) = D^p\left(C;\Pi_0,C\right) = \frac{C}{r}. \qquad (28)$$

Moreover using (20) we obtains the following inequalities

$$D^p\left(\widetilde{\Pi}^p;\widetilde{\Pi}^p,C\right) = D^p\left(C;C,C\right) = \frac{C}{r}. \qquad (29)$$

Substituting (28) and (29) into (10) yields (11). This proves Proposition 1.∎

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D   Proof of Proposition 2

Recall (9) and (27), and rewrite the value of equity as

$$E^u\left(\widetilde{\Pi}^u; \Pi_0, C\right) = \left[\frac{(1-\tau)\Pi_0 - C}{r} + \tau I\right] + H\left(\widetilde{\Pi}^u; \Pi_0, C\right)$$

with

$$H\left(\widetilde{\Pi}^u; \Pi_0, C\right) \equiv -\tau \frac{G_2^u \Pi_0^{\beta_2}}{1-h}\left[1 - \left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right] - \frac{\left(\widetilde{\Pi}^u - C\right)}{r}\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}$$

and

$$\frac{G_2^u \Pi_0^{\beta_2}}{1-h} = \frac{\widetilde{\Pi}^u - C}{r} \frac{\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}}{\left[(1-h) + h\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right]}.$$

Compute now the first order condition of problem (13). It is clear that

$$\frac{\partial E^u\left(\widetilde{\Pi}^u; \Pi_0, C\right)}{\partial \widetilde{\Pi}^u} = \frac{\partial H\left(\widetilde{\Pi}^u; \Pi_0, C\right)}{\partial \widetilde{\Pi}^u} = 0.$$

where

$$\frac{\partial H\left(\widetilde{\Pi}^u; \Pi_0, C\right)}{\partial \widetilde{\Pi}^u} = \frac{\partial}{\partial \widetilde{\Pi}^u}\left[-\frac{\left(\widetilde{\Pi}^u - C\right)}{r}\left(\frac{\Pi}{\widetilde{\Pi}^u}\right)^{\beta_2}\right]\left\{\frac{1 + (\tau - h)\left[1 - \left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right]}{\left[(1-h) + h\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right]}\right\} +$$

$$- \left[\frac{\left(\widetilde{\Pi}^u - C\right)}{r}\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right] \cdot \frac{\partial}{\partial \widetilde{\Pi}^u}\left\{\frac{1 + (\tau - h)\left[1 - \left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right]}{\left[(1-h) + h\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right]}\right\}$$

with

$$\frac{\partial}{\partial \widetilde{\Pi}^u}\left[-\frac{\left(\widetilde{\Pi}^u - C\right)}{r}\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right] = -\frac{(1-\beta_2)}{r}\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\left(\widetilde{\Pi}^u\right)^{-1}\left(\widetilde{\Pi}^u - \frac{\beta_2}{\beta_2 - 1}C\right),$$

and

$$\frac{\partial}{\partial \widetilde{\Pi}^u}\left\{1 + \tau\frac{\left[1 - \left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right]}{\left[(1-h) + h\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right]}\right\} = \left(\frac{\tau}{h}\right)\frac{\beta_2\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\left(\widetilde{\Pi}^u\right)^{-1}}{\left\{1 - h\left[1 - \left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right]\right\}^2} < 0.$$

We thus have

$$
\frac{\partial H\left(\widetilde{\Pi}^u;\Pi_0,C\right)}{\partial \widetilde{\Pi}^u} = -\frac{(1-\beta_2)}{r}\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\left(\widetilde{\Pi}^u\right)^{-1}\left\{\widetilde{\Pi}^u - \frac{\beta_2}{\beta_2-1}C\right\}\left\{1+\tau\frac{\left[1-\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right]}{\left[(1-h)+h\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right]}\right\} -
$$
$$
- \left[\frac{\left(\widetilde{\Pi}^u-C\right)}{r}\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right]\left(\frac{\tau}{h}\right)\frac{\beta_2\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\left(\widetilde{\Pi}^u\right)^{-1}}{\left\{1-h\left[1-\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right]\right\}^2} = 0
$$

(30)

Using (30) we obtain the following equation

$$
\widetilde{\Pi}^u = \frac{\beta_2}{\beta_2-1}C + \left(\frac{\tau}{h}\right)\frac{\frac{\beta_2}{\beta_2-1}\left(\widetilde{\Pi}^u-C\right)\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}}{\left\{1-h\left[1-\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right]\right\}\left\{1+(\tau-h)\left[1-\left(\frac{\Pi_0}{\widetilde{\Pi}^u}\right)^{\beta_2}\right]\right\}}.
$$

(31)

To analyze the distortive effect of taxation, recall (30), and set $\tau = h = 0$. We thus obtain the *laissez-faire* threshold point:

$$
\widetilde{\Pi}^u_{LF} = \frac{\beta_2}{\beta_2-1}C
$$

(32)

By contradiction we can now prove that $\widetilde{\Pi}^u < \widetilde{\Pi}^u_{LF}$. Assume *ab absurdo* that $\widetilde{\Pi}^u > C$. In this case equation (31) holds, but it is straightforward to see that $H\left(\widetilde{\Pi}^u;\Pi_0,C\right) < 0$. This entails that no solution $\widetilde{\Pi}^u > C$ ensures a maximum. Moreover, assume *ab absurdo* that $\frac{\beta_2}{\beta_2-1}C < \widetilde{\Pi}^u < C$. In this case (31) does not hold: thus no solution can be found. Therefore, the solution must be such that inequality $\widetilde{\Pi}^u < \frac{\beta_2}{\beta_2-1}C$ holds. In this case, indeed, equation (31) holds, and $H\left(\widetilde{\Pi}^u;\Pi_0,C\right) > 0$. This proves Proposition 2.∎

E   Proof of Proposition 3

Using (4) we can rewrite (12) as

$$
NPV^u\left(\widetilde{\Pi}^u;\Pi_0,C\right) = (1-\tau)NPV^*\left(\Pi_0\right) + X\left(\widetilde{\Pi}^u;\Pi_0,C\right)
$$

(33)

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where

$$X\left(\widetilde{\Pi}^u; \Pi_0, C\right) \equiv \left\{(1-\tau)\left[1 - b\left(\widetilde{\Pi}^u; \Pi_0, C\right)\right]\left[D^u\left(\widetilde{\Pi}^u; \Pi_0, C\right) - \frac{C}{r}\right]\right\} +$$
$$+ \left\{b\left(\widetilde{\Pi}^u; \Pi_0, C\right)\left[D^u\left(\widetilde{\Pi}^u; \Pi_0, C\right) - \frac{\widetilde{\Pi}^u}{r}\right]\right\}$$

Let us substitute (4) into (33) and rearrange, so as to obtain

$$NPV^u\left(\widetilde{\Pi}^u; \Pi_0, C\right) = (1-\tau)\, NPV^*\left(\Pi_0\right) - \frac{\left(\widetilde{\Pi}^u - C\right)}{r}\, b\left(\widetilde{\Pi}^u; \Pi_0\right) \cdot \frac{(\tau - h)\left[1 - b\left(\widetilde{\Pi}^u; \Pi_0\right)\right]}{1 - h\left[1 - b\left(\widetilde{\Pi}^u; \Pi_0\right)\right]}.$$

Since $\widetilde{\Pi}^u < C$, we can write (14), thereby proving Proposition 3.∎

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