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POVERTY ALLEVIATION IN A WELFARIST FRAMEWORK OF OPTIMAL LINEAR INCOME TAXATION

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Abstract

This paper compares the optimal linear income tax structure in a welfarist setting with the one derived in a non welfarist approach. In order to have a flexible framework able to encompass both the poverty minimisation and the social welfare maximisation objectives, a censored social welfare function, in which only variations in incomes below the poverty line affect social welfare, is used. Adopting the main assumptions of the model developed by Atkinson (1973), conditions under which the poverty-minimising marginal tax rate is higher than the welfare-maximising one are shown. However, we find that the opposite case may occur under specific conditions on income distribution. In particular, the redistribution resulting from the maximisation of an utilitarian welfare function is higher than the one obtained pursuing a poverty alleviation goal when the poor are close enough to the poverty line.

1 Introduction

The literature on optimal income taxation, which began with the seminal paper of Mirrlees (1971), characterises the optimal tax schedule that satisfies equity goals, taking into account the disincentive effects of taxation on labour effort. Following the tradition in welfare economics, society's values about equity and fairness are embodied in a social welfare function, which depends solely on the individuals' utility. Social preferences are expressed through the relative weights assigned to different agents' well-being and range from giving equal weight to all individuals to only being concerned with the welfare of the most disadvantaged person. According to this perspective, poverty matters only to the extent that redistributive goals imply that a higher weight is given to additional income going to those on lower incomes than on higher incomes. There is no concern for poverty reduction as a distinct social objective. Since it evaluates the impact of different fiscal policies only on social welfare, the literature on optimal income taxation is commonly defined as "welfarist".

This approach has been criticized on several grounds.

A first line of criticism is developed within the policy debate around fiscal reforms. While the optimal tax literature takes into account the values of both net income and leisure in individual utility functions in determining the optimal tax schedule, the policy debate gives little weight to the disutility the poor may experience from work. Policy discussion generally focuses only on incomes. A view not assigning a positive value to leisure of the poor may lead to optimal marginal tax rates that are lower than the welfarist ones, so to incentive labour effort[1].

From a theoretical point of view, the welfarist formulation gives a too narrow perspective by assuming that the objectives of policy can be fully represented by a function based solely on individual well-being. Other ethical principles may be deemed as valuable in judging different policy options. As Sen (1982) argues, welfarism is "informationally restrictive" since it rejects any information unrelated to individual utility. Non-utility information, such as the possibility to exercise one's rights, the freedom to express one's view or the right to be non-exploited or discriminated, on the contrary, is necessary for social welfare judgements and objectives.

"Indeed, some moral principles are formulated without making any use of utility information at all, e. g., "equal pay for equal work", "non-exploitation", etc.,

[1] "*While the conventional optimal tax literature takes into account the values of both net income and leisure in the individual welfare functions, the policy discussion focusses almost exclusively on incomes... Even when work incentives are discussed explicitly it is the implications for government revenue and individual incomes that are paramount; little weight is typically given to such disutility as the poor experience from working.*" (Kanbur, Keen e Tuomala [1994], p.1615-1616)

and it is easy to demonstrate that these principles would conflict with welfarism, which makes the utility information decisive... In its uncompromising rejection of the relevance of non-utility information welfarism is indeed a very limiting approach.[2]" [Sen (1982), p.340]

Alternative approaches using non-utility information have been developed following Sen's suggestions. These approaches are usually referred to as "nonwelfarist". Among others, in a series of papers, Kanbur, Keen and Tuomala (Kanbur (1987), Kanbur e Keen (1989), Kanbur, Keen and Tuomala (1994, 1995)) have examined the implication of replacing the social welfare maximisation objective with the poverty minimisation one. The optimal structure of marginal tax rates is then defined with respect to this objective function.

This change of social goal has two major implications.

Firstly, poverty is measured as a function of income rather than individual utility: a person is poor if his disposable income falls short of the minimum that would allow him to attain a given "decent" standard of living. This is coherent with the view that considers the minimum income not only in terms of consumption goods, but as the necessary condition to satisfy certain fundamental needs, such as the rights to meet one's nutritional requirements, to be clothed and sheltered, or the power to participate in the social life of the community. According to this perspective, in line with the conventional policy debate, poverty indexes assign no value to the consumption of leisure by the poor.

Secondly, the non welfarist approach focuses tightly on the incomes at the bottom of the distribution, while it ignores incomes above poverty threshold. The welfarist approach, on the contrary, gives some weight to the welfare levels of all individuals, even if weights may be differentiated in favour of low incomes. At the opposite extreme, the Rawlsian social welfare function only considers the welfare of the worst-off individual.

The adoption of a poverty minimising objective leads to a pattern of marginal tax rates potentially different from that resulting in the welfarist tradition. Kanbur et al. (1994), examining the shape of the non linear tax schedule that most reduces a given poverty index, conclude that, under certain conditions, over some interval at the bottom of the income distribution the marginal tax rate is lower under the non welfarist perspective than under the welfarist one. More specifically, the optimal marginal rate at the lower end of the scale should be negative, even though this result is not confirmed by their simulations. They

[2]Sen (1982) goes on arguing that *"often utility information is very difficult to obtain both because of problems of measurability and comparability, as well as because of the well-known difficulties in inducing honest revelation of preferences... In contrast some of the non-utility information, e. g., "equal pay for equal work" is being observed, or what primary goods people have, may be a lot easier to obtain. Thus the restriction imposed by welfarism is not only ethically limiting, it can be deeply problematic also from the point of view of data availability, making this restriction "doubly" regrettable"* [Sen (1982), p. 340-41]

note, however, that their framework is not suited for a comparison between the two approaches. While the non welfarist minimand is independent of the cardinalisation of utility, the welfarist maximand is not. Consequently, in the latter case, no simple results on the optimal tax structure may be provided.

In the economics literature welfarist and non welfarist goals have generally been adopted each to the exclusion of the other. On the contrary, people may apply several different criteria when judging policy options. As Atkinson (1995) argues, it is important to extend the range of social objectives to include non-welfarist goals, namely those not based only on considerations of individual welfare. These objectives, such as securing individual freedom and independence, or the right to receive reward for effort, do not replace concern for social welfare, that remains a relevant principle to be considered in social policy evaluations. In order to combine multiple principles, Atkinson suggests, among other solutions, the use of a higher-level social maximand that trades off different criteria, social welfare included. The crucial issue is which form this social objective function should have[3].

Following Atkinson's suggestion, this paper provides a method to include the non welfarist goal of poverty alleviation in a welfarist analytical setting. Such a framework enables us to directly compare the optimal linear income tax structures resulting under the welfarist and non welfarist social objectives.

In the poverty literature, deprivation is commonly defined as shortage of income with respect to a prespecified level, the poverty line, conventionally agreed. Pursuing poverty alleviation goals implies to minimise an aggregate poverty index which is the sum of all individual poverty measures. Consequently, the impact of anti-poverty policies on social welfare cannot be evaluated. Likewise, as just noted, poverty reduction cannot be considered as a social goal in the welfarist context.

For a welfarist analytical framework to accommodate such objective, the latter must be "welfarised"[4]. To this purpose, poverty has been defined in terms of utility rather than income, by specifying the "poverty line utility" as the utility enjoyed by the individual whose income is equal to the poverty line. In this way, a person is poor if the difference between the poverty threshold utility and individual utility is positive. The poverty minimisation implies to minimise this difference.

Further, we show that the minimisation of the second poverty measure of Clark *et al.* (1981) is equal to the maximisation of a censored function of individ-

[3] Creedy (1996) compares the effects on poverty and welfare of the introduction of a linear income tax and a minimum income guaranteed scheme. His analysis is carried out by using some well-known inequality and progressivity measures; in this way, Creedy (1996) departs from the literature, in which target efficiency measures of Beckerman (1979) were traditionally used.

[4] This term has been borrowed from Wane (2001). In his paper, poverty is "welfarised" since it is considered as a negative externality, which reduces individual utilities.

ual incomes. By substituting incomes with a specific utility function we obtain a censored social welfare function. This specification is flexible enough to encompass both the non welfarist and the welfarist goals, since it retains the standard social welfare framework while considering the poverty minimisation objective. It comes from a crucial feature of the censored social welfare function, in which only utility variations below the poverty threshold (i.e. redistribution in favour of the poor) can affect collective welfare. Different social objectives may, consequently, be analysed simply shifting the threshold along the income distribution.

Finally, conditions under which the poverty-minimising marginal tax rate is higher than the welfare-maximising one are shown. The comparison between different optimal linear income tax structures is carried out adopting the main assumption of the model developed by Atkinson (1973), whose relatively simple formulation allows to determine the fiscal parameters' value under different assumptions on income distribution and on the degree of inequality social aversion.

The higher redistribution resulting from the adoption of the poverty alleviation goal may appear obvious. In this perspective, the effects of marginal tax rate's increases on higher incomes do not affect collective welfare, while a higher lump sum subsidy which increases low incomes positively influences social wellbeing.

However, we find that the opposite case may occur under specific conditions on income distribution. In particular, we show that the redistribution resulting from the maximisation of an utilitarian welfare function is higher than the one obtained pursuing a poverty alleviation goal when the poor are close enough to the poverty line.

The remainder of the paper is organised as follows. The next section presents the general structure of the economy and introduces the censored social welfare function. Section 3 provides conditions under which the optimal marginal tax rate is higher when a poverty alleviation goal is pursued; in this section, numerical simulations for specific poverty lines and parameters' values are conducted. Section 4 investigates particular cases in which the previous result may be overturned and the welfare maximising marginal tax rate can be higher than the poverty minimising one. Section 5 concludes.

2 The basic model

We restrict our analysis to consider a government that levies a linear income tax, such that the provision of a lump sum benefit is combined with a proportional tax on all income.

Following Atkinson (1973), it is assumed that individual's earnings depend only on earning ability (n) and on the number of years of education received (S); hours of work are supposed to be fixed. In particular, the individual has zero earnings when undergoing education while earns a constant amount $z(n,S) = nS$ when he is at work. He retires (and does not receive any pension benefit) after R years.

The linear income tax function $T(z)$ is:

$$T(z) = (1-\beta)\, z - \alpha$$

where $(1-\beta) = t$ is the proportional (constant) marginal rate and α is the guaranteed minimum income, equal for all individuals.

Individual income generating possibilities (abilities) are distributed according to the density function $f(n)$, assumed to be Pareto in form:

$$f(n) = \mu \underline{n}^{\mu} n^{-\mu-1}$$

where $\underline{n}$ is the lowest ability and μ (with $\mu \geq 2$) reflects the degree of abilities' dispersion around the mean value.

The individual problem is then to maximise the present value of lifetime income (I), discounted at the interest rate i:

$$\max_S I(n) = \int_S^{R+S} [z - T(z)]\, e^{-it} dt \tag{1}$$

or, solving the integral calculus:

$$\max_S I(n) = A\,(\alpha + \beta n S)\, e^{-iS} \tag{2}$$

where

$$A = \frac{(1-e^{-iR})}{i}$$

The first order condition gives:

$$\begin{cases} S^* = \dfrac{1}{i} - \dfrac{\alpha}{\beta n} & \text{for } n \geq \dfrac{\alpha i}{\beta} = n_0 \\ S^* = 0 & \text{for } n \leq n_0 \end{cases} \tag{3}$$

It follows that the optimal level of S is negatively correlated to the subsidy α and to the marginal tax rate $(1-\beta)$, while it is positively correlated to the individual ability.

The resulting level of lifetime income is then:

$$\begin{cases} I^* = A\left(\dfrac{\beta n}{i}\right) e^{\left(\frac{\alpha i}{\beta n}-1\right)} & \text{for } n \geq n_0 \\ I^* = A\alpha & \text{for } n \leq n_0 \end{cases} \tag{4}$$

Assuming that $n_0 \leq \underline{n}$, such as $S^* > 0$ for all individuals, the government revenue constraint is:

$$\int_0^\infty T(z)\, f(n)\, dn = \int_0^\infty [(1-\beta)\, z - \alpha]\, f(n)\, dn = 0$$

or equivalently:

$$\beta = 1 - \frac{n_0}{n_m} \tag{5}$$

where n_m denotes the mean value. This formulation implies that the population is constant in size and the government is only concerned by redistribution.

Pursuing a welfarist objective, Atkinson (1973) assumes that the government maximises social welfare, given by the sum of individual utilities, supposed to be iso-elastic in form:

$$U(I) = \frac{I^{1-\rho}}{1-\rho}$$

and ρ is the income marginal utility elasticity.

Consequently, the social welfare function is:

$$\max_{\alpha,\beta} W = \int_{\underline{n}}^{\infty} U[I(n)] f(n) dn \tag{6}$$

Under the hypothesis of $\rho = 1$, the government maximand becomes a logarithmic function:

$$\max_{\alpha,\beta} W = \log\beta + \int_{\underline{n}}^{\infty} \log\left(\frac{An}{i}\right) f(n)dn + \int_{\underline{n}}^{\infty} \left(\frac{n_0}{n} - 1\right) f(n)dn \tag{7}$$

The goal of the government is therefore to choose the tax schedule that maximises this function under the revenue constraint (eq. 5).

The first order condition is:

$$\beta^* = \left(\int_{\underline{n}}^{\infty} \frac{n_m}{n} f(n)dn \right)^{-1} \tag{8}$$

By substituting the Pareto density function for $f(n)$, the resulting marginal tax rate is:

$$(1-\beta) = \frac{1}{\mu^2} \tag{9}$$

This implies that the optimal marginal tax rate increases as μ decreases. As the intuition confirms, a more unequal ability distribution, highlighted by a lower value of μ, may justify a more redistributive tax function, through a higher marginal rate and subsidy.

As ρ increases, the optimal tax rate rises. For a Rawlsian welfare function, the optimal rate is:

$$(1-\beta) = \frac{1}{\mu}$$

2.1 The censored social welfare function

In order to accomodate a poverty alleviation objective in this welfarist setting, we firstly need to introduce a poverty measure to be minimised. Several measures of deprivation have been developed in the poverty literature[5]. For our purposes, we focus on the second index proposed by Clark, Hemming and Ulph (1981), since it presents some useful properties. Specifically, in its derivation, Clark *et al.* (1981) assume a social evaluation function defined on a censored income distribution (Takayama (1979)), where for all incomes below the poverty line the effective value is considered, while all incomes above the threshold are set equal to the poverty line:

$$x_i = \min\{x_i, \pi\}$$

where x_i is the individual income and π is the poverty line.

Consequently, the social evaluation function for individual incomes is:

$$u(x_i) = \frac{1}{\varepsilon}\left[\min(x_i, \pi)\right]^{\varepsilon}, \qquad \varepsilon \leq 1$$

where the parameter ε measures the aversion to inequality among incomes of the poor (Foster and Sen (1997)).

On the basis of the equally distributed equivalent income for the censored income distribution (x^{eq}), the CHU measure is obtained:

$$P_{\varepsilon} = 1 - \left(\frac{x^{eq}}{\pi}\right)$$

This may also be expressed as:

$$P_{\varepsilon} = 1 - \frac{1}{\pi}\left[\frac{1}{N}\sum_{i=1}^{n}\left(\min(x_i, \pi)^{\varepsilon}\right)\right]^{\frac{1}{\varepsilon}}$$

[5] A survey of the poverty literature is beyond the scope of this paper. The interested reader is referred to Ravallion (1994), Seidl (1988) and Zheng (1997), among others.

This index, therefore, attaches an increasing weight to lower incomes when $\varepsilon < 0$; for $\varepsilon \to -\infty$ only the highest poverty gap matters.

Let us consider a monotonic transformation of the CHU index, defined on individual abilities (n):

$$P_\varepsilon = \int_0^\infty 1 - \left(\frac{x^{eq}}{\pi}\right)^\varepsilon f(n)dn$$

It can be shown that minimising the CHU poverty index is equivalent to maximise a censored income function. That is:

$$\min_x P_\varepsilon = 1 - \frac{\varepsilon}{\pi^\varepsilon}\int_0^\infty \left(\frac{(x^{eq})^\varepsilon}{\varepsilon}\right) f(n)dn$$

is equal to:

$$\max_x (1 - P_\varepsilon) = +\frac{\varepsilon}{\pi^\varepsilon}\int_0^\infty \left(\frac{(x^{eq})^\varepsilon}{\varepsilon}\right) f(n)dn$$

where:

$$x^{eq} = \begin{cases} x & \text{for } x < \pi \\ \pi & \text{for } x \geq \pi \end{cases}$$

The censored function can alternatively be written as:

$$(1 - P_\varepsilon) = \int_0^{\tilde{n}} \frac{1}{\varepsilon}\left(x^\varepsilon\right) f(n)d(n) + \int_{\tilde{n}}^\infty \frac{1}{\varepsilon}\left(\pi^\varepsilon\right) f(n)dn \tag{10}$$

where $\tilde{n}$ is the ability of the person that has an income equal to the poverty line. This formulation implies that only income variations below the poverty threshold can affect the function.

In order to "welfarise" the poverty alleviation goal, we need to define deprivation in terms of utility and not of income. In our setting, an individual is poor if his utility is below a prespecified level, defined as the poverty line utility, $U(\pi)$. In this way, we depart from the conventional view of the poverty literature, in which deprivation is defined as income shortfall from a predetermined threshold.

Assuming that the individual utility function is expressed as in Atkinson (1973), the lifetime net income $I(n)$ can be substituted for x in eq. (10). In this way, each of the two integrals represents individual utilities, respectively of those below and above the poverty line. π may be interpreted as the lifetime net income below which individuals are considered poor.

Consequently, eq. (10) can be defined as a censored social welfare function:

$$W = \int_{\underline{n}}^{\tilde{n}} \frac{1}{\varepsilon}\left[I(n)\right]^\varepsilon f(n)dn + \int_{\tilde{n}}^\infty \frac{1}{\varepsilon}\left(\pi^\varepsilon\right) f(n)dn \tag{11}$$

where $\tilde{n}$, in this case, is the ability of the individual whose lifetime income is equal to π.

Since lifetime incomes of those above the poverty line are set equal to the threshold, only redistributions that affect incomes below π may modify social welfare. This feature distinguishes the censored social welfare function from the others: in standard social welfare functions, indeed, collective welfare is affected by transfers on all income distribution. Meanwhile, this formulation is less extreme than the Rawlsian one, in which only the welfare of the worst-off individual matters.

As in Atkinson (1973), ε (the elasticity of the marginal utility of income) measures the degree of income inequality aversion, even if, in eq. (11), it applies only on incomes of the poor. Accordingly, in the censored welfare function, social preferences about redistribution are expressed by both the parameter ε and the poverty line. When π shifts along the income distribution, the range of incomes over which preferences apply changes and different redistributive goals can be examined. Specifically, when $\pi \rightarrow \infty$, the basic utilitarian social welfare function (eq. 6) is obtained[6].

3 A welfarist model with poverty

This section considers the case in which poverty alleviation is the main goal of the government redistributive intervention. In the following, when poverty reduction is the social objective, a label PR is used, to distinguish it from the standard welfare maximisation problem.

Let us denote by π the lifetime post-tax income defining the poverty line:

$$\pi = I(\tilde{n}) = A\left[\alpha + \beta\left(\tilde{n}S^{*}\right)\right]e^{-iS^{*}} \tag{12}$$

From eq.(12), it results that:

$$\tilde{n} = \frac{\pi/A \cdot e^{iS^{*}} - \alpha}{\beta S^{*}}$$

i.e. the threshold ability level, for a given poverty line, depends on the fiscal parameters (β and α).

The maximisation program of the government can be restated:

$$W = \int_{\underline{n}}^{\tilde{n}} \frac{1}{\varepsilon}\left[I(n)\right]^{\varepsilon} f(n)dn + \int_{\tilde{n}}^{\infty} \frac{1}{\varepsilon}\left(\pi^{\varepsilon}\right) f(n)dn$$

Assigning different values to ε, the shape of the social welfare function changes. However, in order to have comparable results, we consider only the Atkinson's

[6] On the contrary, when $\pi \rightarrow 0$, the social welfare function tends to the maxi-min form.

central case[7]. Assuming that $\varepsilon = (1-\rho) = 0$, the censored social welfare function is logarithmic in form.

The objective of a poverty minimising government is therefore:

$$\max_{\alpha,\beta} \log W = \int_{\underline{n}}^{\tilde{n}} \log\left[I\left(n\right)\right] f\left(n\right) dn + \int_{\tilde{n}}^{\infty} \log \pi f\left(n\right) dn$$

$$s.t. \qquad \beta = 1 - \frac{n_0}{n_m}$$

Substituting the expression for $I\left(n\right)$ and differentiating, the first order condition defines the optimal value for the fiscal parameter β:

$$\beta^*_{PR} = F\left(\tilde{n}\right)\left(\int_{\underline{n}}^{\tilde{n}} \frac{n_m}{n} f(n) dn\right)^{-1}$$

A comparison with the equivalent expression in the standard welfarist model of the previous section (eq. 8) highlights that, in the latter case, welfare gains and losses induced by changes in fiscal parameters affect all the population, while in the poverty reduction case only effects on those below the poverty line matter.

After some manipulations, β^*_{PR} can be expressed in terms of β^*:

$$\beta^*_{PR} = \beta^* \cdot F\left(\tilde{n}\right)\left[1 + \Upsilon\left(\tilde{n}\right)\right] \tag{13}$$

where:

$$\Upsilon\left(\tilde{n}\right) \equiv \frac{\displaystyle\int_{\tilde{n}}^{\infty} \frac{n_m}{n} f(n) dn}{\displaystyle\int_{\underline{n}}^{\tilde{n}} \frac{n_m}{n} f(n) dn}$$

Eq. (13) implies that marginal tax rates resulting in the poverty reduction and the standard welfarist approaches tends to be equal as $\tilde{n}$ increases. This outcome ensues directly from the nature of the censored function. When the threshold ability level moves to the upper tail of the distribution, the censored social welfare function approximates the utilitarian one. In the limit case, when $\tilde{n} \rightarrow \infty$, the gap between the two functions collapses. It must be noticed that, in this case, when society is indifferent to poverty or the government does not care about deprivation, one deals with the standard welfarist optimal linear tax problem. The censored social welfare function allows thus to develop a more general setting, within which the utilitarian formulation represents a special case.

[7] It is easy to demonstrate that subsequent results hold even when the more general case of $\varepsilon \neq 0$ is considered.

In the poverty reduction perspective, the problem of social welfare maximisation may have either an internal solution ($\underline{n} \leq \tilde{n} \leq \infty$) or a corner solution ($\tilde{n} = \underline{n}$). The latter case, however, is trivial. If the individual with the lowest ability has an income equal to the conventionally agreed poverty line, the government does not need to redistribute in order to alleviate poverty, since no-one is poor in that society.

In the following, it will be shown that conditions under which the internal solution prevails depend on the level of the poverty line[8].

Substituting for the Pareto income distribution, the optimal value for β^*_{PR} is derived:

$$\beta^*_{PR} = \left(1 - \frac{1}{\mu^2}\right)\gamma \tag{14}$$

where

$$\gamma \equiv \frac{\left[1 - \left(\frac{\underline{n}}{\tilde{n}}\right)^{\mu}\right]}{\left[1 - \left(\frac{\underline{n}}{\tilde{n}}\right)^{\mu+1}\right]}$$

and $\gamma < 1$ for every $\tilde{n} > \underline{n}$.

Considering that the optimal value for β^* in the standard welfarist model was $(1 - \frac{1}{\mu^2})$, it follows that:

$$\beta^*_{PR} < \beta^*$$

or, equivalently, that the poverty minimising marginal tax rate is higher than the (standard) welfarist one.

3.1 Numerical simulations

The model discussed in the preceding section shows that maximising a censored social welfare function gives a higher marginal tax rate than maximising an usual utilitarian function. However, this result is still stated in a general (implicit) form, since the optimal value of β depends on the value of $\tilde{n}$, which in turn depends again on β.

[8] Specifically, it will be argue that $\tilde{n}$ of equilibrium is an increasing function of π and a critical value, $\hat{\pi}$, exists such that:

$$\begin{array}{lccl} \text{if} & \pi & \leq & \hat{\pi} \Longrightarrow \text{ the problem has a corner solution } \tilde{n} = \underline{n} \\ \text{if} & \pi & > & \hat{\pi} \Longrightarrow \text{ the problem has an internal solution } \tilde{n} > \underline{n} \end{array}$$

The explicit solution for the fiscal parameter results from the following system of equations[9]:

$$\beta^*_{PR} = \left(1 - \frac{1}{\mu^2}\right)\gamma \tag{15}$$

$$\tilde{n}^* = \frac{\pi/A \cdot e^{iS^*} - \alpha^*}{\beta^*_{PR} S^*} \tag{16}$$

$$\alpha^* = \frac{\beta^*_{PR}\left(1 - \beta^*_{PR}\right) n_m}{i} \tag{17}$$

where

$$S^* = \frac{1}{i} - \frac{\alpha^*}{\beta^*_{PR}\tilde{n}^*}$$

and α^* is the optimal value for the lump sum subsidy, given β^*_{PR}[10].

Substituting α^* and S^* in eq. (16), and given the eq. (15), the system may be simplified[11]. The value of $\widetilde{n}$ which balances the system, therefore, results from the following expression:

$$\frac{h\gamma\widetilde{n}}{i} = \frac{\pi}{A} e^{\left\{1 - \frac{(1 - h\gamma)\, n_m}{\tilde{n}}\right\}} \tag{18}$$

in which $h \equiv \left(1 - \frac{1}{\mu^2}\right)$.

In eq. (18), $\widetilde{n}$ is an implicit function of known parameters. If we denote with $\Phi(\tilde{n})$ the left hand side and with $\Lambda(\tilde{n})$ the right hand side of eq. (18), the ensuing properties come:

Proposition 1:
i) $\Phi(\tilde{n})$ *is such that* $\Phi'(\tilde{n}) > 0$ *and*

$$\lim_{\tilde{n} \to \underline{n}} \Phi = \frac{h\left(\frac{\mu}{\mu+1}\right)\underline{n}}{i} = \underline{\Phi}$$

$$\lim_{\tilde{n} \to \infty} \Phi = \infty$$

ii) $\Lambda(\tilde{n})$ *is such that* $\Lambda'(\tilde{n}) > 0$ *and*

[9] It is easily verified that the expression for β satisfies the constraint under which all individuals have a positive number of hours of education.($\underline{n} \geq n_0$).

[10] α^* in eq. (17) derives from the revenue constraint, by substituting the expression for n_0.

[11] In the following, to simplify the exposition, we miss out to denote with star the optimal value of the variables.

$$\underline{\Lambda} = \frac{\pi}{A} e^{\left\{ 1 - \frac{\left[1 - h\left(\frac{\mu}{\mu+1}\right)\right] n_m}{\underline{n}} \right\}}$$

$$\bar{\Lambda} = \frac{\pi}{A} e$$

Proof: (see Appendix 1)

As we argue in the previous section, the social welfare maximisation problem in the poverty reduction case may have either an interior or a corner solution. Which solution prevails depends on the prespecified poverty threshold with respect to a critical level, resulting from the limit value, for $\tilde{n} \to \underline{n}$, of the difference between Φ and Λ. Specifically, it may be shown that the poverty line critical level $\hat{\pi}$ is given by (see Appendix 2):

$$\hat{\pi} = \frac{A}{i} \frac{(\mu - 1)\, \underline{n}}{\mu} e^{-\left(\frac{\mu - 2}{\mu - 1}\right)}$$

For an internal solution, the optimal value of $\widetilde{n}$ may be identified at the intersection point between Φ and Λ.

Specifying some value for the parameters (μ, $\underline{n}$, i, R), numerical simulations allows us to verify the following:

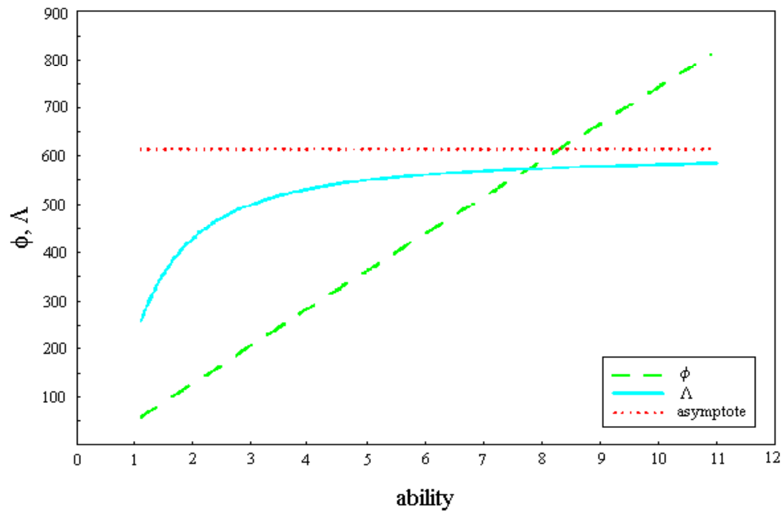
Proposition 2: *given the critical value of the poverty line*
i) if $\pi = \hat{\pi}$, the $\Lambda(\tilde{n})$ crosses the $\Phi(\tilde{n})$ at $\tilde{n} = \underline{n}$;
ii) if $\pi < \hat{\pi} \Longrightarrow \Lambda(\tilde{n}) < \Phi(\tilde{n})$ for every n, the $\Lambda(\tilde{n})$ does not meet the $\Phi(\tilde{n})$ at all. In this case, again, the corner solution holds, $\tilde{n} = \underline{n}$;
iii) if $\pi > \hat{\pi}$, the intersection between the curves Φ and Λ identifies the $\tilde{n}$ of equilibrium. In this case, the internal solution holds, $\tilde{n} > \underline{n}$;
iv) the intersection, when it exists, is unique.

Proof: See Appendix 3

In the following graphics, the function $\Lambda(\tilde{n})$ is in bold, while the $\Phi(\tilde{n})$ is in dashed form (the dotted line represents the asymptote of $\Lambda(\tilde{n})$). Fig. 1 shows the case of an internal solution, identified by the point in which Φ meets Λ[12].

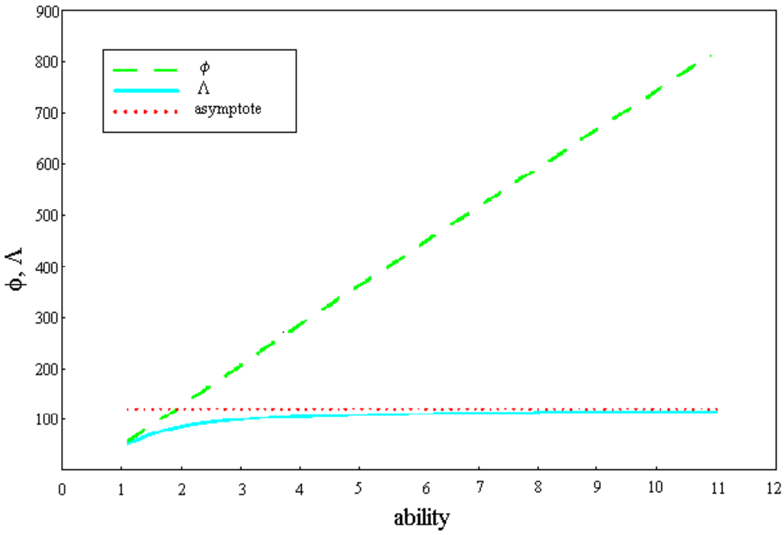
[12]This simulation assumes the following parameters values: $\underline{n} = 1$, $i = 0.01$, $\mu = 2$, $A = 25$, $\pi = 5000$.

Fig.1 Optimal threshold ability



As illustrated by simulations, when the level of π rises, the function Λ (and its asymptote) shifts upwards. Correspondingly, the intersection point moves rightwards and $\tilde{n}$ of equilibrium rises. On the contrary, when the poverty line is reduced, the function Λ gradually shifts downwards and the value of the threshold ability decreases. The corner solution is displayed in Fig. 2[13].

Fig. 2 Optimal threshold ability (corner solution)



Intuitively, the lower is the poverty line, the less are the poor, given the income distribution. In the limit case, if the poverty line is low enough, no one

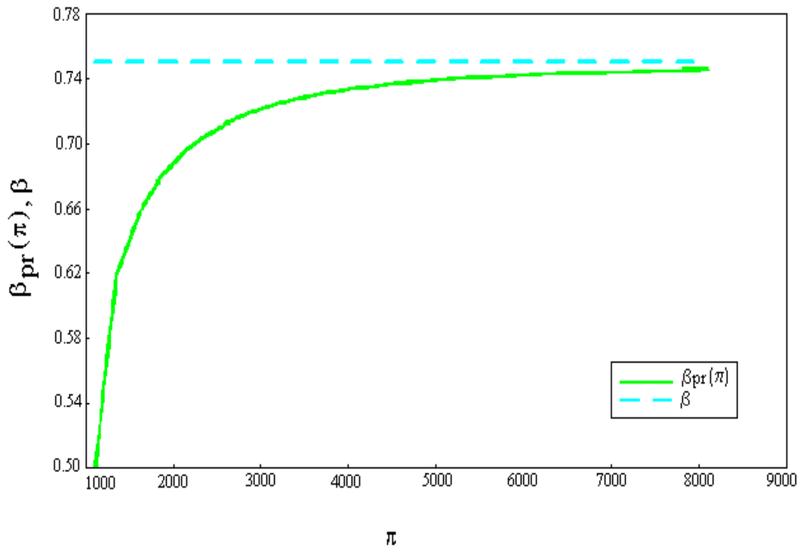
[13] This simulation has been carried out by assuming $\pi = 1000$, given other parameters values.

could be considered poor. In the censored social welfare function, when $\tilde{n} = \underline{n}$, all individuals have the same income, equal to π. This implies that income variations, induced by fiscal policies, do not affect collective well-being at all.

Since γ is an increasing function of $\tilde{n}$ and $\beta_{PR} = h\gamma$, it follows that β_{PR} is also an increasing function of $\tilde{n}$. Consequently, the marginal tax rate decreases as $\tilde{n}$ rises.

The bold line in Fig. 3 represents the optimal values of the fiscal parameter as π changes, while the Atkinson's solution is in dashed form. As shown graphically, β_{PR} is an increasing function of π and, for $\pi \to \infty$, β_{PR} tends to the standard welfarist result.

Fig.3 Optimal fiscal parameters



It is possible to provide a clear economic interpretation to this pattern. If the poverty line is relatively low (as long as $\pi > \hat{\pi}$), the target group of the poverty reduction intervention is given by the lowest income individuals. In this case, $\tilde{n}$ (and β_{PR}) are low, while the marginal tax rate is high. Respecting the revenue constraint, the high fiscal pressure allows to provide a significant (considerable) subsidy. The intuition behind comes straightforward from the properties of the censored welfare function: increases of the marginal rate affecting incomes above the threshold do not have any effect on social welfare, since individual utilities above the poverty line are all set equal to π itself[14]. On the contrary, changes of fiscal parameters modify utilities below the threshold. At the lowest incomes, the positive effect on net income of a higher subsidy exceeds the negative effect of a higher marginal tax rate. From the social welfare maximisation point of

[14]Paradoxically, it could be possible to reduce individual incomes/utilities up to the poverty line level. As long as none of the "richest" crosses down the threshold, an equalizing taxation would not affect social welfare.

view, when the objective is poverty alleviation, a highly redistributive linear income taxation is then justified. When the poverty line is set at higher levels of income, the range of incomes upon which the negative effects of high tax rates matter grows. The beneficial effects of redistribution have to be balanced with its increasing costs, which affect a broader share of the population. $\tilde{n}$ (and β_{PR}) are higher, while the optimal marginal tax rate (and the optimal subsidy provided) is lower. In the limit case, when $\pi \to \infty$ ($\tilde{n} \to \infty$), all individual utilities are influenced by fiscal changes and the linear structure tends to be equal to the Atkinson's one.

Summing up, when the target group is restricted to the poorest, the optimal tax-transfer structure is highly redistributive, taxing heavily the richest and subsidising the poorest. The redistributive power of the tax schedule decreases as the poverty line rises.

Finally, Fig. 4.a, b, c display how the intersection between $\Phi(\tilde{n})$ and $\Lambda(\tilde{n})$ changes for different values of the poverty line. They confirm that the internal solution, when it exists, is unique.

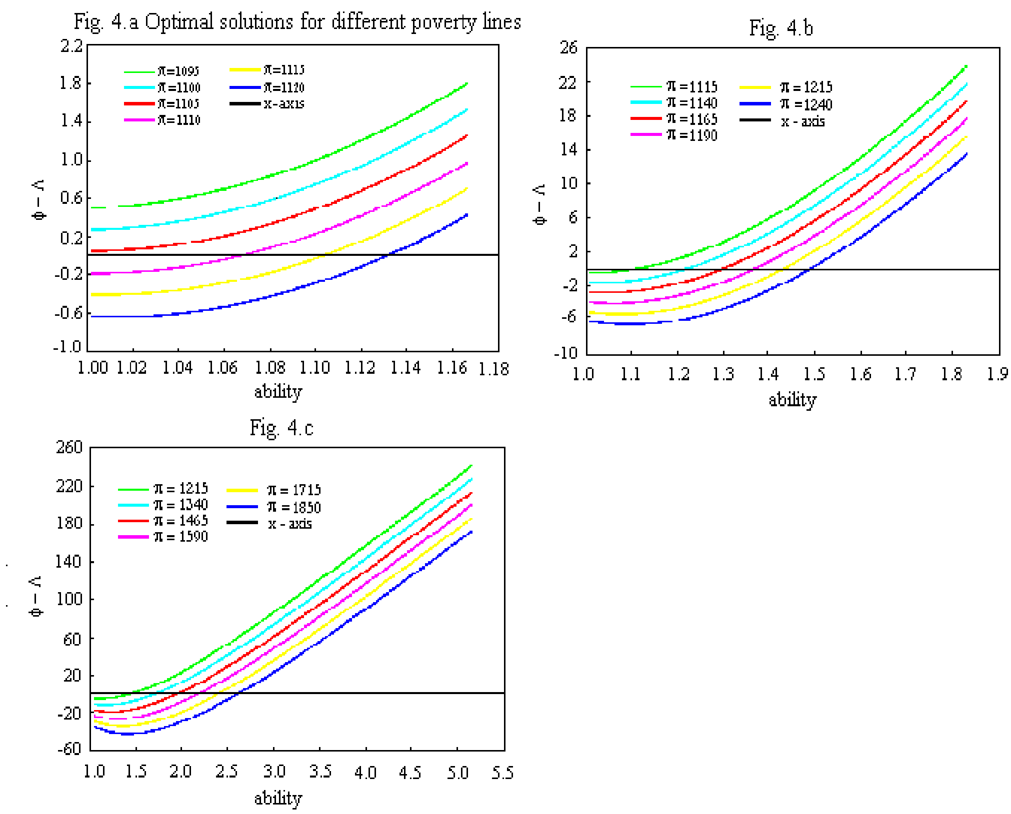


Fig. 4.a Optimal solutions for different poverty lines

Fig. 4.b

Fig. 4.c

Results discussed in this section are summarised in the following:

Proposition 3:

i) the optimal linear marginal tax rate is higher when social preferences are aimed at minimising poverty rather than at maximising a standard social welfare function;

ii) the optimal value of the poverty reduction marginal tax rate is a decreasing function of the poverty line. This means that redistribution is higher when the poverty line is set at a low income level and decreses as the threshold shifts upwards;

iii) when $\pi \to \infty$, the optimal linear tax structure tends to approximate the Atkinson's one.

4 A special case

Previous findings may appear intuitive: a higher redistribution seems to be a reasonable outcome when social preferences are such to privilege the poverty alleviation goal.

However, it is possible to identify conditions under which the result can be overturned and the poverty minimising marginal tax rate may be lower than the standard welfare maximising one. To do so, we assume that the economy consists of only two individuals: the "poor" has an income a little below the poverty line, while the "rich" is relatively far from the threshold. This implies that only a little income transfer from the rich to the poor is needed to raise the latter up to the poverty line, reducing the poverty gap to zero. In other words, we are supposing that a "small size redistribution" is sufficient in order to minimise deprivation. This assumption about the gap between the poor and the poverty line is crucial for results derived in this section.

We denote by n_1 the ability of the poor, while n_2 (where $n_2 = kn_1$, with $k > 1$) is the ability of the rich. As in the previous section, we assume that both individuals have a positive number of years of education received, i.e. $n_1 \geqslant n_0 = \dfrac{\alpha i}{\beta}$.

Let us define $I(n_j)$, with $j = 1, 2$, the optimal lifetime post-tax income of each individual, given the value of β satisfying the revenue constraint[15]:

$$I(n_j) = A\left(\frac{\beta n_j}{i}\right) e^{\left(\frac{(1-\beta)(n_1+n_2)}{2n_j} - 1\right)}$$

[15] Since we are supposing that both individuals choose a positive number of years of education, the revenue constraint is given by:

$$\beta = 1 - \frac{n_0}{n_m} = 1 - \frac{\alpha}{\beta} \cdot \frac{2i}{n_1 + n_2}$$

It is easy to verify the following proposition:

Proposition 4:
i) for $\beta \in (0,1)$, the optimal income of the poor, $I(n_1)$, increases up to reach its maximum value at $\beta = \hat{\beta}_1 = \frac{2}{1+k}$, and then decreases;
ii) for $\beta \in (0,1)$, the optimal income of the rich, $I(n_2)$, is always increasing[16]*;*
iii) for $k < \overline{\overline{k}}$, $I(n_2)$ meets $I(n_1)$ when the latter increases[17] (See Fig. 5.a).

Proof: See Appendix 4

The constraint under which both individuals choose a positive number of years of education, given the revenue constraint, implies that the solution of the censored welfare function maximisation must satisfy the condition that $\beta_{PR} > \beta_{istr} = \frac{k-1}{k+1}$ (the proof is given in Appendix 5). It is easily verified that $\beta_{istr} \gtreqless \hat{\beta}_1$ if and only if $k \gtreqless 3$.

In the following, we assume, for analytical simplicity, that $k < \overline{\overline{k}}$; specifically, we initially suppose that $k = 3$.

The case of "small size redistribution" we are considering takes place when the poverty line is set within the interval $[\pi_{\min}, \pi_{\max}]$, where $\pi_{\min} = A\left(\frac{n_1}{i}\right)e^{-1}$ is the income of the poor in the absence of any tax-transfer policy, while $\pi_{\max} = A\left(\frac{k-1}{k+1}\right)\left(\frac{n_1}{i}\right)$ is the income level resulting from the intersection between $I(n_1)$ and the line corresponding to the education constraint (β_{istr}). The case is shown in Fig. 5.a.

For each π within the interval, the problem of the censored social welfare function maximisation becomes:

$$\max_{\alpha,\beta} W = \log I(n_1) + \log(\pi) \tag{19}$$

subject to the public revenue constraint:

$$\beta_{PR} = 1 - \frac{\alpha}{\beta_{PR}} \cdot \frac{2i}{n_1 + n_2}$$

[16] The function $I(n_2)$ reaches its maximum value when $\hat{\beta}_2 = \frac{2k}{(1+k)} > 1$.

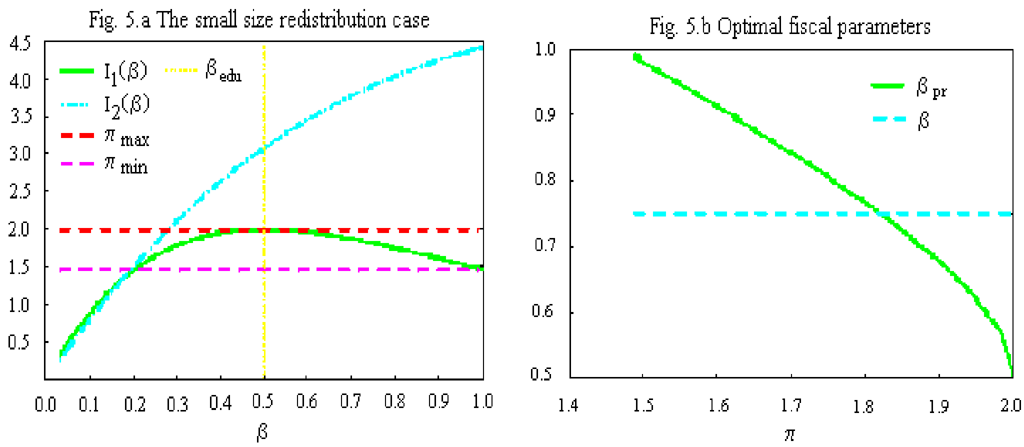
[17] The critical value $\overline{\overline{k}}$ is given by:

$$k = \exp\left[\frac{(k-1)^2}{2k}\right]$$

Besides the solution $k = 1$ (rejected by our assumption), this equation has a unique solution in the range of acceptable values for k. Specifically, $\overline{\overline{k}} = 5,033$.

Solving the problem, the function is maximised for the value of $\beta_{PR}(\pi) \geqslant \beta_{istr}$ such that the poor is lifted exactly to the poverty line, while the rich continues to stay above. Note that, once reached the poverty alleviation goal, the government is indifferent to other policies implying more redistribution, i.e. marginal tax rates higher than the ones reducing the poverty gap to zero.

The optimal tax rate is an increasing function of the poverty line, as shown in Fig. 5.b. As it is intuitive, the lower the income level at which the threshold is set, the lower the tax rate needed to the poor to escape from poverty.



In order to compare our function with the outcome resulting under a standard welfarist approach, the following maximisation has to be solved:

$$\begin{aligned} \max W &= \log I(n_1) + \log I(n_2) \\ s.t. \qquad \beta &= 1 - \frac{\alpha}{\beta} \cdot \frac{2i}{n_1 + n_2} \end{aligned}$$

In this case, the optimal value for the fiscal parameter is easily obtained:

$$\beta = \frac{4n_1 n_2}{(n_1 + n_2)^2} = \frac{4k}{(1+k)^2}$$

Fig. 5.b shows the range of poverty lines values for which the poverty minimising marginal tax rate is lower than the one resulting from the usual welfarist approach. Specifically, this outcome is found when π is low, and a little transfer is needed to eradicate poverty. When the poverty line is higher, by contrast, a

higher marginal rate is required, even under our hypothesis of "small size redistribution".

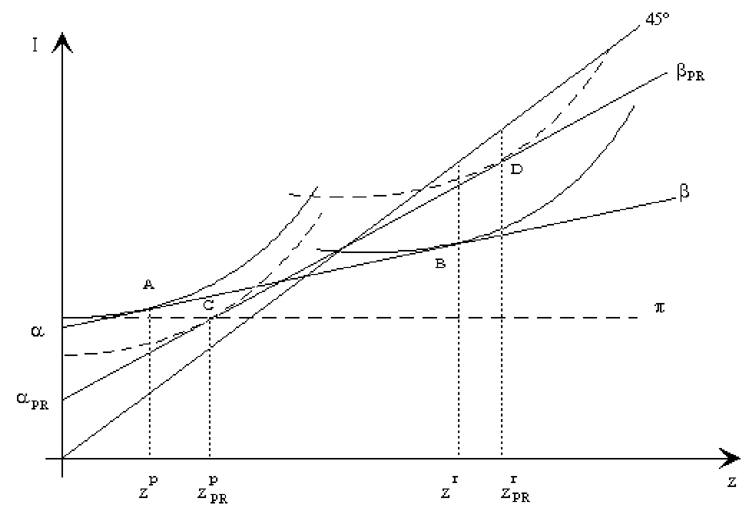
It may be of interest to better explain this outcome, which seems to go against our previous remarks. The case for a less redistributive linear tax structure emerging in a poverty alleviation framework is again a consequence of the kind of function we are considering. While the standard social welfare function, such as the utilitarian Atkinson's one, is adverse to income inequality over all the distribution (i.e., the parameter reflecting the degree of aversion to inequality applies on all income levels along the whole distribution), the censored welfare function attaches a positive weight only to inequalities between incomes of the poor and the poverty line. In our two individuals' example, it means that the utilitarian function considers the whole income gap between the poor and the rich, while the censored function considers the gap between the income of the poor and the threshold only. Given the value of the inequality aversion parameter, if we suppose a gap between individual incomes large enough and a small gap between the lower income and the poverty line (the so-called "small size redistribution"), a higher redistribution may be required following the goal of inequality reduction, rather than poverty reduction.

For values of $k \neq 3$, the range of π for which the case of "small size redistribution" takes place changes, since $\pi_{\max}$ is identified by the highest between $\hat{\beta}_1$ and β_{istr}[18]. The set of optimal values for β_{PR} also changes: the lowest value is equal to the highest between $\hat{\beta}_1$ and β_{istr}. However, even when $k \neq 3$, some solutions for the censored welfare function maximisation are less redistributive than the standard welfarist one.

Fig. 6 provides an example of linear tax structures obtained under the two perspectives. The line α-β identifies the tax-transfer scheme resulting from the maximisation of a standard social welfare function, while the α_{PR}-β_{PR} is the poverty minimising linear tax schedule. Consistently with the previous analysis, the poverty reduction function has an higher slope (given by β_{PR}) and a lower vertical intercept (equal to the lump sum subsidy) than the line α-β.

[18]Specifically, for $k > 3$, $\beta_{istr} > \hat{\beta}_1$ and then $\pi_{\max=}\pi\left(\beta_{istr}\right)$. By contrast, when $k < 3$, $\beta_{istr} < \hat{\beta}_1$ and $\pi_{\max=}\pi\left(\hat{\beta}_1\right)$.

Fig. 6 Optimal linear income tax structures



In the conventional utilitarian framework, the optimal individual allocations are given by A and B, which identify z^p and z^r, the before-tax income levels, respectively of the poor and the rich. Likewise, C and D (to which correspond z^p_{PR} and z^r_{PR}) represent the optimal solutions for the censored welfare function maximisation. In both cases, the difference between after-tax and before-tax incomes for the poor is equal to the same difference for the rich.

It is interesting to note that, given the functional form for z, the pre-tax income is an increasing function of β. As a consequence, a lower marginal tax rate gives place a higher before-tax income. Furthermore, since the income elasticity to changes of β is increasing as the ability decreases, the effect of a marginal rate lowering on z is higher for the lowest ability. It explains why $(z^p_{PR} - z^p) > (z^r_{PR} - z^r)$.

5 Conclusion

This paper aims at developing a unified framework to accomodate the typically non welfarist goal of poverty reduction in a welfarist analytical setting. This framework has then been used to define the optimal linear income tax function, when social preferences are oriented to the poverty alleviation objective.

The innovative feature of our approach mainly lies in the adoption of a censored social welfare function, that allows to treat the standard utilitarian income tax problem as a special case. It relies on the nature of the censored function: since social welfare is affected only by changes in well-being of those below the poverty line, different distributive preferences may be assessed simply by shifting the threshold along the income distribution. Consequently, a direct comparison between optimal tax structures resulting under the welfarist and non welfarist perspectives has been carried out.

Analytical investigations, corroborated by numerical simulations, highlight conditions under which the poverty minimising income tax structure is more redistributive than the standard welfare maximising (utilitarian) one. More precisely, the optimal value of the poverty reducing marginal tax rate increases when the poverty line shifts downwards. This is coherent with the properties of the censored social welfare function. A more redistributive linear income tax schedule is justified when the target group is made up of the poorest.

Under more restrictive assumptions on the income distribution, however, special cases may emerge and previous findings may be overturned. Specifically, if we suppose that the lowest income is close to the poverty line, a little income transfer (a "small size redistribution") from the richest to the poorest can be sufficient to minimise deprivation. Besides, if the income gap between the rich and the poor is broad enough, an utilitarian welfare maximisation goal may require a higher redistribution. In this context, therefore, the optimal linear income tax scheme resulting from the maximisation of an utilitarian welfare function can imply a higher marginal tax rate (and subsidy) than the one derived pursuing a poverty alleviation objective.

Appendix 1:

Proof of Proposition 1.

To analyse the behaviour of $\Phi(\tilde{n})$ we initially consider the function $\gamma(\tilde{n}) = \left[\dfrac{1-\left(\frac{n}{\tilde{n}}\right)^{\mu}}{1-\left(\frac{n}{\tilde{n}}\right)^{\mu+1}}\right]$.

The first derivative of γ is given by:

$$\gamma'(\tilde{n}) = \frac{\frac{1}{\tilde{n}}\left(\frac{n}{\tilde{n}}\right)^{\mu}\left\{\mu-\left(\frac{n}{\tilde{n}}\right)\left[(\mu+1)-\left(\frac{n}{\tilde{n}}\right)^{\mu}\right]\right\}}{\left[1-\left(\frac{n}{\tilde{n}}\right)^{\mu+1}\right]^{2}}$$

Since the denominator, $\dfrac{1}{\tilde{n}}$ and $\left(\dfrac{n}{\tilde{n}}\right)^{\mu}$ are all strictly positive, the following holds

$$\gamma' \gtreqless 0 \Longleftrightarrow \left\{\mu-\left(\frac{n}{\tilde{n}}\right)\left[(\mu+1)-\left(\frac{n}{\tilde{n}}\right)^{\mu}\right]\right\} \gtreqless 0 \tag{20}$$

It is then possible to show that, when $\mu \geq 2$:

$$\mu \geq \left(\frac{n}{\tilde{n}}\right)\left[(\mu+1)-\left(\frac{n}{\tilde{n}}\right)^{\mu}\right]$$

so that eq. 20 is satisfied for ≥ 0.

Moreover, it is easily verified that, when $\tilde{n} \rightarrow \underline{n}$, $\gamma \rightarrow \left(\frac{\mu}{\mu+1}\right) < 1$, while for $\tilde{n} \rightarrow \infty$, $\gamma \rightarrow 1$.

Since it is defined as a product of increasing functions, $\Phi(\tilde{n})$ is strictly increasing in $\tilde{n}$ and it lies between an inferior value $\dfrac{h\left(\frac{\mu}{\mu+1}\right)\underline{n}}{i} = \underline{\Phi}$ and ∞.

Secondly, it can be shown that Λ is a strictly increasing function and it goes from an inferior value, given by:

$$\lim_{\tilde{n}\rightarrow\underline{n}} \Lambda(\tilde{n}) = \frac{\pi}{A} e^{\left\{1-\frac{\left[1-h\left(\frac{\mu}{\mu+1}\right)\right]n_m}{\underline{n}}\right\}} = \underline{\Lambda}$$

and an upper value, to which it tends asymptotically:

$$\lim_{\tilde{n}\rightarrow\infty} \Lambda(\tilde{n}) = \frac{\pi}{A} e = \bar{\Lambda}$$

The first derivative of Λ is given by:

$$\Lambda'(\tilde{n}) = \frac{\pi}{A} e^{\left\{1-\frac{(1-h\gamma)n_m}{\tilde{n}}\right\}} \cdot \frac{n_m\left(h\gamma'\tilde{n}+1-h\gamma\right)}{\tilde{n}^2} \tag{21}$$

Since the first term is always positive, the sign of eq. (21) depends on $(h\gamma'\tilde{n}+1-h\gamma)$, in which $(1-h\gamma) > 0$ and $\gamma' > 0$. It follows that:

$$h\gamma'\tilde{n}+1-h\gamma > 0 \qquad \Rightarrow \qquad \Lambda'(\tilde{n}) > 0$$

Appendix 2:

The critical value of the poverty line, $\hat{\pi}$, is obtained by calculating the limit of the difference $(\Phi - \Lambda)$, as $\tilde{n} \rightarrow \underline{n}$:

$$\lim_{\tilde{n}\rightarrow\underline{n}} (\Phi - \Lambda) = \frac{h\gamma\tilde{n}}{i} - \frac{\pi}{A} e^{\left\{1-\frac{(1-h\gamma)n_m}{\tilde{n}}\right\}}$$

Substituting expressions for h and n_m and considering that $\underset{\tilde{n}\rightarrow\underline{n}}{lim}\ \gamma = \frac{\mu}{\mu+1}$ previous equation becomes:

$$\lim_{\tilde{n}\rightarrow\underline{n}} (\Phi - \Lambda) = \frac{1}{i}\left(\frac{\mu-1}{\mu}\right) \cdot \underline{n} - \frac{\pi}{A} e^{\left\{\frac{\mu-2}{\mu-1}\right\}} \tag{22}$$

$\hat{\pi}$ is the poverty line level for which eq.22 is equal to zero:

$$\hat{\pi} = \frac{A}{i}\frac{(\mu - 1)\,\underline{n}}{\mu} e^{-\left(\frac{\mu - 2}{\mu - 1}\right)}$$

Appendix 3:

Numerical simulations allow to verify the uniqueness of the intersection between the curves $\Phi(\tilde{n})$ and $\Lambda(\tilde{n})$, when it exists. Specifically, simulations have been carried out for alternative values of the parameter μ of the Pareto income distribution ($\mu = 2$, $\mu = 3$, $\mu = 4$), and for a wide range of poverty lines levels. Other parameters (A, i e $\underline{n}$) have been held constant, since their variation does not influence our testing.

In all cases, the intersection between $\Phi(\tilde{n})$ and $\Lambda(\tilde{n})$, corresponding to the points where the curve $(\Phi(\tilde{n}) - \Lambda(\tilde{n}))$ meets the x-axis, has been proved to be unique.

Fig. 4. a, b, c show the case in which we consider $\mu = 2$: under this assumption, the critical value of the poverty line below which $\Phi(\tilde{n})$ and $\Lambda(\tilde{n})$ do not meet is around 1106. In particular, the first simulation supposes that the poverty line may change from 1095 and 1120. The pattern of the difference $(\Phi(\tilde{n}) - \Lambda(\tilde{n}))$ is increasing for all these values and $\tilde{n}$, when it exists, vary in a range of values between 1,07 e 1,14. It is worth to note that, for $\pi < 1106$, the two functions do not meet. In the Fig. 4.b, the intersection always exists, since the threshold assumes values between 1115 e 1240, and is unique. In this case, the pattern of the difference $(\Phi(\tilde{n}) - \Lambda(\tilde{n}))$ highlights a U-form for high levels of the poverty line, i.e. the function is decreasing for low values of $\tilde{n}$ and then always increasing. In all cases, however, the curve meets the x-axis when it increases. The Fig. 4.c, where higher values of π are considered, confirms the U-form of the curve. The possibility of a non-uniqueness of the intersection, however, is ruled out: for high levels of π, the intersection between the curve $(\Phi(\tilde{n}) - \Lambda(\tilde{n}))$ and the horizontal axis takes place for high values of $\tilde{n}$, for which the curve is always increasing. On the other hand, if we lower π, so the intersection corresponds to lower values of $\tilde{n}$, the U-pattern tends to vanish (as shown by Fig. 4.a).

Even if results are not provided here, simulations carried out for $\mu > 3$ support previous findings, highlighting patterns similar to the previous ones.

Appendix 4:

Proof of Proposition 4:

By derivating the function $I(n_1)$ with respect to β we obtain:

$$\frac{\partial I(n_1)}{\partial \beta} = A\left(\frac{n_1}{i}\right) e^{\left\{\frac{(1-\beta)(n_1+n_2)}{2n_1} - 1\right\}} \left[1 - \frac{\beta(n_1+n_2)}{2n_1}\right]$$

It follows that:

$$\frac{\partial I(n_1)}{\partial \beta} \gtreqless 0 \Longleftrightarrow \beta \lesseqgtr \frac{2n_1}{n_1+n_2}$$

Recalling that $n_2 = kn_1$, where $k > 1$, then

$$\frac{\partial I(n_1)}{\partial \beta} \gtreqless 0 \Longleftrightarrow \beta \lesseqgtr \frac{2}{1+k} \equiv \hat{\beta}_1 \tag{23}$$

with $\hat{\beta}_1 < 1$, since $k > 1$.

Eq. 23 implies that the function $I(n_1)$ is increasing up to reach its maximum value at $\hat{\beta}_1$, and then decreases. Specifically, in $\hat{\beta}_1$ the function is equal to:

$$\hat{I}\left(n_1 \mid \beta = \hat{\beta}_1\right) = A\left(\frac{2}{1+k}\right)\frac{n_1}{i}e^{\left(\frac{k-3}{2}\right)}$$

Since $\hat{\beta}_1 < 1$, $\hat{I}(n_1)$ lies within the interval of acceptable values for β (note that $0 < \beta < 1$).

When $\beta = 1$, $I(n_1)$ is given by:

$$I(n_1 \mid \beta = 1) = A\left(\frac{n_1}{i}\right)e^{-1}$$

As far as the derivative with respect to β of the lifetime income function for the second individual is concerned, it follows that:

$$\frac{\partial I(n_2)}{\partial \beta} \gtreqless 0 \Longleftrightarrow \beta \lesseqgtr \frac{2k}{(1+k)} \equiv \hat{\beta}_2 \tag{24}$$

where $\hat{\beta}_2 > 1$, since $k > 1$.

According to Eq. 24, the function $I(n_2)$ increases up to reach $\hat{\beta}_2$, at which the income of the richest individual is maximum. However, since $\hat{\beta}_2 > 1$, it lies outside of the range of possible values for β. In particular, when $\beta = 1$, $I(n_2)$ is equal to:

$$I(n_2 \mid \beta = 1) = A\left(\frac{n_2}{i}\right)e^{-1} = A\left(\frac{kn_1}{i}\right)e^{-1}$$

The range of values for which $I(n_1) \geq I(n_2)$ is given by:

$$n_1 e^{\left\{\frac{(1-\beta)(n_1+n_2)}{2n_1}-1\right\}} \geq n_2 e^{\left\{\frac{(1-\beta)(n_1+n_2)}{2n_1}-1\right\}}$$

or equivalently

$$\beta \leq 1 - \frac{2k}{k^2 - 1} \log k$$

Consequently, $I(n_1)$ meets $I(n_2)$ for values of $\beta < \hat{\beta}_1$ when

$$k > e^{\frac{(k-1)^2}{2k}}$$

It is then easy to verify that previous inequality holds for $1 \leq k \leq 5,033$.

Appendix 5:

In order to have both individuals choosing a positive number of years of education, the condition the solution of the censored welfare function maximisation must satisfy is derived from the revenue constraint, by substituting the value of α in $n_1 \geq n_0$. Since we assumed $n_2 = kn_1$ it follows that the education constraint implies:

$$\beta \geq \frac{k-1}{k+1} \equiv \beta_{istr}$$

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