THE TRADE-OFF BETWEEN PUBLIC FINANCING AND ACTIVISM IN PARTY COALITIONS

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May 26, 2011

Abstract

This paper presents a model of party coalition formation between policy-motivated activists and office-seeking opportunists. In this framework, I consider how changes in party valence and public financing of political parties shape the equilibrium inside coalitions. Results show that, in equilibrium, opportunists and activists have the same marginal rate of substitution between policy position and activists' contribution. An asymmetric worsening of one party's valence leads to divergence of its policy platform and a higher degree of activism. Furthermore, public financing of political parties drives activism or idealism out of politics. As a consequence, public financing is an important policy instrument to regulate the trade-off between the degree of activism in politics and the independence of political parties from lobbying. Besides, electoral platforms bend towards the policy position of the opposition when public financing is asymmetric.

Key words: activists, idealism, lobbyists, coalition formation, Nash bargaining, party valence, polarization.

JEL Classifications: D70, D72, D78.

"*Historically, however, parties are associated with particular ideologies – presumably the views, or preferences, of the coalition of citizens whom they, in some way, represent. So the Downsian model is missing something important – perhaps the essence – of democratic competition*". (Roemer, 2004)

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1 Introduction

This paper presents a model of coalition formation and political competition which includes reasons for policy-motivated activists to provide resources to a party coalition from inside the coalition. The model incorporates ideas about coalition formation by Aldrich (1983a) and, especially, by Roemer (2001), and about the role of policy-motivated activists who offer contributions from outside vote-maximizing parties by Austen-Smith (1987), Schofield, Miller and Martin (2003), Schofield (2007) and others. In Roemer's framework, each party is made up of two essential groups that I shall call opportunists and activists. Opportunists sole desire is to maximize expected votes and thereby the chance of electoral success. They have no interest in policy per se. Activists, or militants as Roemer calls them, are only concerned with policy and not with winning elections. They propose a policy as close as possible to their ideal, and use electoral competition as a forum to advertise and agitate for their preferred aim[1].

It is not difficult to see why an activist may want to be inside. Joining a coalition allows activists to acquire agenda control in the policy formation process regarding those issues where they have a major stake. They may also hope to profit from inside access to areas of policy somehow related to their special interests on a day by day basis.

As for the opportunists with whom activists must coalesce, they seek resources for electoral purposes and are prepared to adapt their policy platform to some extent to the preferences of activists in exchange for their support.

Capturing the full richness of the coalition formation process in this context is a complex matter. We assume that activists bargain directly with one group of opportunists in a Nash bargaining game, exchanging money and time for a preferred policy positioning by the party.

Equilibrium in the electoral competition between the two coalitions of opportunists and activists is modelled using a probabilistic spatial voting framework where, as is now common in this literature, the probability of winning elections is the sum of a policy and a non-policy related component referred to as the party "valence". In this model, political resources provided by activists or lobbyists and public financing can be used to improve a party's valence, for example through advertising.[2] As consequence of coalition formation adopting this structure, activists will gain greater control over their party's platform when they contribute more, or when the non-policy related valence is particularly important because of "uncertainty" about such things as the degree to which its campaign promises are credible, so that the coalition has a greater need for resources in order to face the competition.

[1] Roemer also considers a third coalition partner called reformists who maximize their expected utility, and who can be thought of as a combination of opportunists and activists. I do not include this type of actor here.

[2] See Adams, Merrill and Grofman (2005) and Schofiled and Sened (2006) for recent overviews of the spatial voting literature.

In this Nash bargaining, Nash non-cooperative competition framework, which combines elements of the coalition approach to political competition with spatial voting in the presence of lobbying, the role of activists and of electoral "uncertainty" for equilibrium policy outcomes is considered. Particular attention is drawn to how the assumption of Nash bargaining as the basis for coalition formation shapes the degree of activism as measured by resource contributions, the policy choices of the parties and the nature of equilibrium platforms.

In addition to activists' contribution to political parties, the framework herein developed is well suited to consideration of the consequences of public financing. In this respect it is interesting to note that the German law, as discussed by von Arnim and Schurig (2004, 40-41), is explicitly formulated to limit public funding in order to enhance the incentive parties have to maintain their links with citizens and, I add, especially with activists, in order to raise the necessary resources. The purpose here is to consider whether or not public financing of campaigns is conducive to the presence of activism, or whether they drive "idealism" out of politics.

Aldrich (1983a, 1983b) provides one of the first models that incorporates partisan activists into the spatial theory of electoral competition. In Aldrich's model, policies are not chosen by opportunists who maximize the probability of winning election, but are determined exclusively by citizens-activists. Citizens choose whether to become activists in one of two political parties or abstain. Activists are price takers, in the sense that they may only influence the average position of the party infinitesimally by deciding to join it. A party is seen as a distribution of its activists, who join the party not to change its goals but to support and help it in their realization. Results predict a cleavage between the two parties' activists who are "relatively cohesive internally and relatively distinctive externally". In contrast to Aldrich (1983a, 1983b) here it is assumed that when activists are inside the coalition they are not price-takers.

Roemer (2006) proposed a theory of party competition where citizens join a party and influence its policy by contributing money. With respect to both Roemer (2006) and Aldrich (1983a, 1983b), this paper develops a theory of party coalition competition in which policy platform is a compromise between activists' and opportunists' most preferred policy positions.

The paper proceeds as follows. Section two develops a basic spatial voting model in which two parties composed only of opportunists or expected vote maximizers compete, and derive as a benchmark the resulting equilibrium in which policy platforms converge. Section three presents a model of coalition formation by Nash bargaining and electoral competition in which opportunists and activists negotiate, assuming that resource contributions by lobbyists from outside the parties are prohibited by regulation. Using this second model, section four and five investigate the consequences for the degree of activism (measured by the contributions of activists) and the nature of equilibrium platforms of asymmetric changes in party valence and of public financing of electoral campaigns. Section six concludes. The appendix contains derivations and proofs.

2 A Basic Model: Opportunists Compete for Office Alone

I begin with a basic framework in which there are a finite number of citizens, all of whom vote, and two competing parties - labelled 1 and 2 - consisting only of opportunists who maximize the share of the total vote that they expect to receive.[3]

The timing of events in this basic model is as follows[4]: at stage one the two parties simultaneously announce their electoral platforms s_1 and s_2; at stage two, each citizen evaluates these platforms and casts his or her vote sincerely for the party that promises to deliver the highest economic well-being; and finally, at a third stage, the party that wins the election implements their announced policy. In what follows we consider the nature of the participants in detail and then, as usual, solve the model backwards.

2.1 Voters

As is now common in the spatial voting literature, I assume that voting behavior *as seen by the parties* depends on the utility for voters that is generated by policy choices as well as on non-policy related factors. The policy-related part of the utility of a voter h, u_h, depends directly on the multi-dimensional policy platform of party k, s_k, such that

$$u_h(s_k) = -\left(s_h^* - s_k\right)^2, \qquad k = 1, 2 \tag{1}$$

where s_h^* is the ideal point of the voter.[5]

The non-policy related component, also referred to as the party valence, consists of two parts. One reflects voter "credibility or trustworthiness", for example about what each party proposes to do once elected, and is denoted by $\gamma_k > 0$. A higher γ_k means more scepticism facing party k. The key aspect of this part of the valence term is that it can be affected by the expenditure of resources - money or the time provided by activists or through public financing - during an election campaign, as in Austen-Smith (1987) or Schofield (2003).[6] The second component, ξ_k^h, represents an innate valence or evaluation of each party by the voter on all other non-policy matters, and cannot be affected by the use of resources.[7] This second part is assumed by the parties to be randomly distributed over the population in a manner described below.

Following Austen-Smith's formulation, the total utility U_h of a voter when the

[3] This model replicates the standard probabilistic spatial voting approach analyzed by Coughlin and Nitzan (1981), Enelow and Hinich (1989), Hettich and Winer (1999), Adams, Merrill and Grofman (2005), Austen-Smith and Banks (2005), Schofield and Sened (2006) and others, and can be seen as a formalization of the approach to political competition initiated by Downs (1957).

[4] For a further discussion about the timing of the game see also Person and Tabellini (2000).

[5] The quadratic form of u_h is mathematically convenient and is also commonly used in the literature.

[6] On the nature and role of party 'valance', see for example Stokes (1963, 1992), Ansolabehere and Snyder (2000), Groseclose (2001), Schofield (2003) and Schofield and Sened (2006).

[7] Some authors refer to γ as a party's *valence* and to ξ as an error term.

platform of party k is implemented can then be written as

$$U_h = u_h(s_k) - \frac{\gamma_k}{c} + \xi_k^h, \qquad \text{with } k = 1, 2. \tag{2}$$

where $c > 0$ represents publicly provided political resources that can be used to reduce the adverse electoral consequences of uncertainty about a party's platform or candidates.[8]

For convenience in what follows, I note also that

$$b_h = \xi_2^h - \xi_1^h \tag{3}$$

represents a stochastic utility bias of the voter in favour of party 2, which is independent of policy positions. Parties are assumed to know the distribution of this bias but do not, of course, know for sure how any particular voter will behave at the polls.

Each voter casts his or her ballot for the party whose platform, coupled with its associated non-policy characteristics gives him the highest utility and, if indifferent, tosses a coin. Thus the probability q_h that citizen h votes for party 1, given party platforms (s_1, s_2) and resources used in political campaigning c is:

$$q_h = \begin{cases} 1 \text{ if } \left(u_h(s_1) - u_h(s_2)\right) - \frac{\gamma_1 - \gamma_2}{c} > b_h \\ \frac{1}{2} \text{ if } \left(u_h(s_1) - u_h(s_2)\right) - \frac{\gamma_1 - \gamma_2}{c} = b_h \\ 0 \text{ otherwise} \end{cases}. \tag{4}$$

2.2 The vote share objective of opportunists

To derive the objective of the parties, I simplify further by assuming that the population consists of $J = 1, 2, ..., I$ groups or types, with everyone in group J having the same preferences.[9] The population share of group J is α^J, with $\sum_J \alpha^J = 1$. I must also be more specific about the distribution of the bias term for each voter defined in equation (3) as seen by the parties; I assume it has a group specific, uniform and independent distribution over the interval[10] $\left[-\frac{1}{2\theta^J}, \frac{1}{2\theta^J}\right]$. Here the parameter θ^J, which is the height of the uniform distribution, represents the density of voters in group J. This parameter multiplied by the corresponding population share will serve as an index of the effective influence of voters in this group on the equilibrium policy outcome.

Define the "swing voter" in group J as the one whose bias term b^J is such that he or she is just indifferent between the promises of either party: $b^J = \left(u^J(s_1) - u^J(s_2)\right) -$

[8] Note that I have assumed for simplicity that $\frac{\gamma_k}{c}$ has the same impact on all voters. Furthermore, I allow for funding by activists in the subsequent model.

[9] This simplification too is often used in the spatial voting literature. See, for example, Persson and Tabellini (2000, chapter 3).

[10] The force of this assumption is to insure that every voter always has a positive probability of voting for both parties, even if it is small.

$\frac{\gamma_1-\gamma_2}{c}$. Since all voters in group J with a bias $b^{hJ} \leq b^J$ vote for party 1, the vote share of party 1 is then given by the sum over the groups of the probabilities that $b^{hJ} \leq b^J$. Using the cumulative distribution of the independently distributed bias terms, party 1's expected vote share thus is

$$\pi_1 = \sum_J \alpha^J \theta^J \left\{ \left[\left(u^J (s_1) - u^J (s_2) \right) - \frac{\gamma_1 - \gamma_2}{c} \right] + \frac{1}{2\theta^J} \right\}. \tag{5}$$

Party 2's expected vote share is $\pi_2 = 1 - \pi_1$.

2.3 A Downsian electoral equilibrium

As a reference for what is to follow, it is helpful at this point to characterize the political equilibrium when only opportunists compete. The result is stated in the following Lemma.

Lemma 1 *Two strictly office-seeking parties offer the same policy platform in an electoral equilibrium:*

$$s^O = \frac{\sum_J \alpha^J \theta^J s^{*J}}{\sum_J \alpha^J \theta^J}. \tag{6}$$

Proof. Please see the Appendix. ■

In this case the two parties converge on a platform that maximizes a weighted average of the ideal points of the voters in each group, with political weights $\alpha^J \theta^J$ determined by the density of voters in each group, a result of the type first derived by Coughlin and Nitzan (1981) and recently developed by Schofield (2007).

Furthermore, in the present context, public financing of parties c, if it exists and is outside of party control, does *not* influence equilibrium platforms. We shall see that when parties are regarded as coalitions, public financing will matter for platform positioning because it affects the bargain structure between coalition partners.

3 Coalition Formation

Now I allow for coalition formation assuming that lobbying from outside is prohibited; that is, I assume the only way for activists to participate to the policy formation process is to be part of a party coalition.[11] Similarly, besides public financing, the only way for opportunists to generate political resources is to form a coalition with a group of activists. I recall that in contrast to the opportunists, activists care only about policy, and they are ready to contribute money or time to a party in exchange for a preferred policy position. Basically, activists represent the ideological part of the coalition. They are not driven about winning the election. They rather care about promoting their ideology. Activists belong to a particular party and obtain a utility from affecting the policy of their own party. Given a policy platform s_k,

[11] An interesting generalization of this model would be to allow activists and lobbyists to be either inside or outside parties.

activists k's utility is $v_k(s_k)$. In order to distinguish activists in the different parties, I assume for convenience that $\frac{\partial v_1(s)}{\partial s} < 0$ and $\frac{\partial v_2(s)}{\partial s} > 0$; i.e., that activists in party 1 want a platform that is further to the "left", while those in party 2 want a platform that is further to the "right". I shall use the size of contributions by activists c_k as an index of the degree of activism or idealism within each coalition.[12]

The timing of events in this model of coalition formation is different from the Downsian model presented earlier only in stage one, where opportunists and activists in each party now simultaneously form a coalition and choose a policy platform s_k and contributions by activists c_k. Since the two coalitions move simultaneously, I assume for tractability that everyone believes that the opposing coalition will reach an agreement.

3.1 Bargaining in a coalition

I proceed by first analyzing separately the net gains for opportunists and activists from being part of a party coalition. The Nash bargaining - Nash competition electoral equilibrium can then be characterized.

3.1.1 Opportunists' net gain from coalition formation

Opportunists will gain from reaching a political agreement with activists since the time, effort and money the latter provide can be used to reduce uncertainty and thus increase the party's valence and chances of electoral success. Two strategic scenarios are possible. In the first, opportunists do not reach an agreement with the activists and run for election alone. As a consequence, they can count only on public financing if it is available, and can choose a policy platform without compromising with a group of activists. In this case, the opportunists of party 1 choose a platform s_1^d to solve

$$\max_{s_1^d} \sum_J \alpha^J \theta^J \left\{ \left[\left(u^J\left(s_1^d\right) - u^J\left(s_2\right) \right) - \left(\frac{\gamma_1}{c} - \frac{\gamma_2}{c + c_2} \right) \right] + \frac{1}{2\theta^J} \right\}. \tag{7}$$

It is straightforward to verify that $s_1^d = s^O$; where s^O is defined in Lemma 1. Thus, the opportunists' payoff for implementing this strategy, which represents their threat point and determines their bargaining strength, is

$$\pi_1^d = \sum_J \alpha^J \theta^J \left\{ \left[\left(u^J\left(\frac{\sum_J \alpha^J \theta^J s^{*J}}{\sum_J \alpha^J \theta^J} \right) - u^J\left(s_2\right) \right) - \left(\frac{\gamma_1}{c} - \frac{\gamma_2}{c + c_2} \right) \right] + \frac{1}{2\theta^J} \right\}. \tag{8}$$

Note that without an agreement, opportunists in party 1 use only public financing (if available) to reduce the negative effect of uncertainty on their valence, while by assumption (that the other coalition always forms) opportunists in the other party

[12] Note that, as for all voters, activists' utility could be represented by $u_k(s_k) = v_k(s_k) - c_k = -\left(s_{Ak}^* - s_k\right)^2 - c_k$, where $v_k(s_k) = -\left(s_{Ak}^* - s_k\right)^2$, while $s_{A_1}^* < s_1$and $s_{A_2}^* > s_2$ represent activists' ideal points. The simpler formulation in the text is sufficient for our purposes.

also count on contributions from their own activists so that their resources are equal to $c+c_2$. Note that the threat point I use is different than the one used by Roemer (2001). In Roemer, opportunists loose the election by assumption if they do not form a coalition, so that their disagreement utility is zero. Instead, here I assume that without an agreement opportunists can still run for the electoral competition and have some positive probability of either winning the election or gaining sits in the legislature since they can count on public financing an choose polity platform freely.

The second of the two scenarios is that the opportunists form a coalition with the activists. In that case, they can count on both public contributions as well as those from activists, $c+c_1$. The opportunists payoff from reaching an agreement with the activists is then

$$\pi_1 = \sum_J \alpha^J \theta^J \left\{ \left[\left(u^J(s_1) - u^J(s_2)\right) - \left(\frac{\gamma_1}{c+c_1} - \frac{\gamma_2}{c+c_2}\right)\right] + \frac{1}{2\theta^J} \right\}. \qquad (9)$$

For a successful coalition to form, the net gain of opportunists in any party, $\psi_k = \pi_k - \pi_k^d$, must be positive. For party 1 this means that

$$\psi_1 = \sum_J \alpha^J \theta^J \left[\left(u^J(s_1) - u^J\left(s^O\right)\right) - \gamma_1 \left(\frac{1}{c+c_1} - \frac{1}{c}\right)\right] > 0. \qquad (10)$$

Here, the policy differential $\left(u^J(s_1) - u^J\left(s_1^d\right)\right)$ is policy that opportunists give up to bring activists into the coalition. It represents the part of the opportunists' net gain from cooperating that depends exclusively on the difference in policy positions with and without the cooperation of activists. The term $-\gamma_1\left(\frac{1}{c+c_1} - \frac{1}{c}\right)$ in equation (10) measures the valence gain of opportunists 1 for being in the coalition, relative to the situation in which they compete alone. The overall net gain for opportunists increases with both c_1 and γ_1 and declines with public financing c. The net gain for opportunists of party 2 is analogous to that in party 1. Note that opportunists' net gain is independent from policy position and activists' contribution of the opposing coalition. Basically, in the bargaining situation, opportunists look at how much they move away from the first best policy in (6) and how much they can improve their valence exposure when they reach an agreement.[13]

3.1.2 Activists' net gain from coalition formation

Activists also face two possible scenarios: either they are successful in forming a coalition, or they do not join any party. In the latter case they are prohibited from making contributions from outside, and I simplify this situation by assuming that activists never run for election alone. Thus if activists do not join a coalition, they

[13] Note that in this model, policy position chosen by a coalition does not affect the bargaining net gain of the opposing coalition due to the dominant strategy described in Lemma 1. This allows us to separate the internal logic of coalition formation from the external reaction to a policy change in the opposing coalition. I leave the formulation of a more general analysis out for future research.

take no further action and save their contributions. Furthermore, in the present model, it is natural that activists predisposed to join party 1 never form a coalition with opportunists of party 2. I assume that activists get utility from influencing the policy of the belonging coalition.

What is the utility that activists get if they do not reach an agreement with the opportunists? In the case of disagreement, activists cannot influence policy and they get $u_k^d = v_k\left(s^O\right)$.[14]

On the other hand, if the activists join a coalition, their inside option is what they would get from reaching an agreement with the party's opportunists. Thus, utility for activists from being part of a coalition is $u_k = v_k\left(s_k\right) - c_k$, where c_k is the activists' contribution to their own party. For activists k, their net gain, ϕ_k, is[15]

$$\phi_k = v_k\left(s_k\right) - v_k\left(s^O\right) - c_k; \quad \text{with } k = 1, 2, \tag{11}$$

and a group of activists will participate in negotiations only when ϕ_k is positive. Note that ϕ_1 is decreasing and concave in s_1 and that ϕ_2 is increasing and concave in policy s_2.[16]

3.2 The electoral equilibrium

Suppose now that coalition 2 proposes policy s_2, which generates an internal contribution level of c_2. The two factions of party coalition 1 then Nash-bargain to an equilibrium taking what happens in coalition 2 as given.[17] The bargaining outcome for party 1 is the policy and contribution pair (s_1, c_1) that maximizes the Nash product:

$$\max_{s_1, c_1} \left(\psi_1 \phi_1\right),$$

where, $\psi_1\left(s_1, \overline{s_2}, c_1, \overline{c_2}\right)$ and $\phi_1\left(s_1, \overline{s_2}, c_1\right)$ are defined in (10) and (11). Similarly, party 2's opportunists and activists Nash-bargain to a solution (s_2, c_2) that solves

$$\max_{s_2, c_2} \left(\psi_2 \phi_2\right).$$

I define an equilibrium in the electoral contest as a pair of policies (s_1^*, s_2^*) and contributions (c_1^*, c_2^*) such that, facing s_2^* and c_2^*, party 1's factions Nash-bargain to

[14] In Roemer (2001) activists' disagreement utility is $u_k^d = v_k\left(s_{-k}\right)$ because in his model the opposing coalition wins the election by assumption.

[15] In the bargaining situation, one might argue that activists care about the policy platform they get from their own party and the difference between what they get if they win the election and what they get if the other coalition wins the election, i.e. their utility is $v_k\left(s_k, s_k - s_{-k}\right)$, with $k = 1, 2$. In this paper we put for tractability all the weight on policy platform s_k and leave the alternative analysis for future research.

[16] Formally: $\frac{\partial \phi_1}{\partial s_1} = \frac{\partial v_1(s_1)}{\partial s_1} < 0$ and $\frac{\partial \phi_1}{\partial s_1^2} = \frac{\partial v_1(s_1)}{\partial s_1^2} < 0$; $\frac{\partial \phi_2}{\partial s_2} = \frac{\partial v_2(s_2)}{\partial s_2} > 0$ and $\frac{\partial \phi_2}{\partial s_2^2} = \frac{\partial v_2(s_2)}{\partial s_2^2} < 0$.

[17] We recall here that it simplifies the analysis to assume, as we do, that potential members of each coalition always think that the other coalition will reach an agreement.

s_1^* and c_1^*, and facing s_1^* and c_1^*, party 2's factions Nash-bargain to s_2^* and c_2^*. For coalition 1, the required first order condition with respect to s_1 is

$$\frac{\sum \alpha^J \theta^J \overset{+}{u^{J\prime}}(s_1)}{\psi_1} + \frac{\overset{-}{v_1'}(s_1)}{\phi_1} = 0, \tag{12}$$

where for convenience of the reader the signs of partial derivatives are indicated. And the first order condition with respect to c_1 is

$$\frac{\frac{\gamma_1}{(c+c_1)^2} \sum \alpha^J \theta^J}{\psi_1} - \frac{1}{\phi_1} = 0. \tag{13}$$

It is important to note that $\frac{\partial \phi_1}{\partial s_1} = v_1'(s_1) < 0$ because activists are better off the smaller s_1 by assumption. By subtraction this implies $\frac{\partial \psi_1}{\partial s_1} > 0$. Thus, as usual, negotiation here entails that inside a coalition, both partners compromise with respect to policy and contributions. Opportunists move away from the centre of the mass of voters and activists get less policy then they would like if they could choose policy without having to bargain (in which case they would set policy so that $v_1'(s_1) = 0$).

The electoral equilibrium described in the previous paragraph can be represented in several forms. The form I use to compute the comparative statics is the following system of four non-linear equations in four unknowns:

$$\begin{cases} \frac{\sum \alpha^J \theta^J \overset{+}{u^{J\prime}}(s_1)}{\psi_1} + \frac{\overset{-}{v_1'}(s_1)}{\phi_1} = 0 \\ \frac{\frac{\gamma_1}{(c+c_1)^2} \sum \alpha^J \theta^J}{\psi_1} - \frac{1}{\phi_1} = 0 \\ \frac{\sum \alpha^J \theta^J \overset{-}{u^{J\prime}}(s_2)}{\psi_2} + \frac{\overset{+}{v_2'}(s_2)}{\phi_2} = 0 \\ \frac{\frac{\gamma_2}{(c+c_2)^2} \sum \alpha^J \theta^J}{\psi_2} - \frac{1}{\phi_2} = 0 \end{cases}, \tag{14}$$

where the first two equations are the first order conditions (12) and (13), which define the Nash bargaining equilibrium inside coalition 1. Similarly, the last two equations define the Nash bargaining equilibrium inside coalition 2.

The above system leads to the following Proposition.

Proposition 1 *In equilibrium, the condition for coalition formation is that the marginal rate of substitution between policy position and activists' contribution must be the same for both opportunists and activists in each party; i.e.:*

$$MRS^{\psi_k} = MRS^{\phi_k}, \qquad \textit{with } k = 1, 2. \tag{15}$$

In order to prove Proposition 1, divide the left-hand and right-hand side of the first two and the last two equations in system (14), which gives the following generalized equation:

$$\frac{\sum \alpha^J \theta^J u^{J\prime}(s_k)}{\frac{\gamma_k}{(c+c_k)^2} \sum \alpha^J \theta^J} = \frac{v_k'(s_k)}{-1}, \qquad \text{with } k = 1, 2. \tag{16}$$

The term $\sum \alpha^J \theta^J u^{J\prime}(s_k)$ represents the marginal change in the expected vote share with respect to s_k, and $\frac{\gamma_k}{(c+c_k)^2} \sum \alpha^J \theta^J$ is the marginal change in the expected vote share with respect to c_k. Thus for opportunists, the ratio $MRS^{\psi_k} = \frac{\sum \alpha^J \theta^J u^{J\prime}(s_k)}{\frac{\gamma_k}{(c+c_k)^2} \sum \alpha^J \theta^J}$ represents the marginal rate of substitution in terms of expected vote shares between policy and contributions. Similarly for activists, $MRS^{\phi_k} = -\frac{v_k'(s_k)}{1}$.

Note that the following inequalities and partial derivatives of the marginal rates of substitution hold: $MRS^{\psi_1} > 0; MRS^{\phi_1} > 0; MRS^{\psi_2} < 0; MRS^{\phi_2} < 0; \frac{\partial MRS^{\psi_k}}{\partial s_k} < 0; \frac{\partial MRS^{\phi_k}}{\partial s_k} > 0; \frac{\partial MRS^{\psi_1}}{\partial c_1} > 0; \frac{\partial MRS^{\psi_2}}{\partial c_2} < 0; \frac{\partial MRS^{\phi_k}}{\partial c_k} = 0; \frac{\partial MRS^{\psi_1}}{\partial \gamma_1} < 0; \frac{\partial MRS^{\psi_2}}{\partial \gamma_2} > 0; \frac{\partial MRS^{\phi_k}}{\partial \gamma_k} = 0; \frac{\partial MRS^{\psi_1}}{\partial c} > 0; \frac{\partial MRS^{\psi_2}}{\partial c} < 0; \frac{\partial MRS^{\phi_k}}{\partial c} = 0$.

The equilibrium conditions can be used to study the effects on equilibrium policy platforms and the degree of activism due to changes in the electoral scepticism or valence, γ_k, and public financing, c, to which we shall turn.

4 The Consequences of Asymmetric Shocks in Party Valence

The following proposition shows the impact of an increase in the degree of "trustworthiness" for party 1, represented by an increase in γ_1, on both party platforms and the degree of party activism.

Proposition 2 *An adverse (asymmetric) shock in voters' scepticism γ_k facing one coalition causes a divergence in the equilibrium policy platforms of coalition k and an increase in the degree of activism in coalition k, such that,*

$$\begin{cases} \frac{ds_1}{d\gamma_1} < 0 \\ \frac{ds_2}{d\gamma_2} > 0 \\ \frac{dc_k}{d\gamma_k} > 0 \end{cases}, \tag{17}$$

with $k =$ 1,2.

Proof. In the Appendix. ■

An increase in γ_1, say, means that party 1 faces an environment where more money is needed to deal with greater scepticism in the minds of voters regarding its platform. Opportunists in the party react to this increase in scepticism by giving up policy position in order to generate more resources from their activists, and thus move further to the left of the mass of voters. It is easy to verify that, in this context, the coalition that is not directly affected by the asymmetric shock will not move its policy position. A reason is that opportunists' net gain is not affected by the other party's valence. In Ansolabehere and Snyder (2000), Aragones and Palfrey (2002, 2003), and Groseclose (1999), the candidate with a valence disadvantage chooses a more extreme position, as the disadvantaged party does here. In the absence of coalition politics, they show that the advantaged candidate locates more centrally, which is similar to what happens in the present model.[18]

Furthermore, when opportunists of coalition k face a worsening in the non policy component, they diverge from the mass of voters by moving policy position in favour of their activists. In this way, they get more contributions from the activists, which compensate the worsening in the valence of the party.

Following Proposition 1, a worsening in γ_1 declines the MRS between policy position and activists' contribution of opportunists 1, while the MRS of the activists is unaffected. Therefore, the agreement inside coalition 1 must be renegotiated. Assume the two factions re-establish equilibrium by adjusting policy position, that is, they change s_1 and keep all the other variables constant. In order to reach a new agreement, the opportunists of coalition 1, who are now more willing to negotiate than before, move policy toward the left to please the coalition's activists. This movement to the left establishes a new equilibrium as the MRS of the opportunists increases and that of the activists declines. On the contrary, a movement to the right would take away the marginal rates of substitution of the two factions and, therefore, the possibility to come to a new agreement.

Similarly, assume that the two factions re-establish the equilibrium by adjusting activists' contribution, that is, they change c_1 and keep all the other variables constant. In order to compensate an increase in γ_1, c_1 must increase. In this way, the MRS of the opportunists increases and equates to that of the activists, which does not change. Conversely, the gap between the two marginal rates of substitution increases when c_1 decreases. The same logic applies, mutatis mutandi, to coalition 2 for changes in γ_2.

5 Public Financing and Party Coalitions

Allowing for possible public financing as an alternative means of subsidizing by activists, the following result shows what happens when public financing becomes more generous in the coalition model.

[18] It could be interesting to generalize this model in order to incorporate the reaction of a coalition to changes in the policy position of the opposing coalition. However, this would add complexity to the model and I have left it out for future research.

Proposition 3 *An increase in public financing leads to convergence in policy platforms and a decrease in the degree of activism:*

$$\begin{cases} \frac{ds_1}{dc} > 0 \\ \frac{ds_2}{dc} < 0 \\ \frac{dc_k}{dc} < 0 \end{cases}, \tag{18}$$

with $k = 1, 2$.

Proof. In the Appendix. ■

Party opportunists become more independent when they receive greater public funding. They do not need contributions from activists as they did before, and for this reason they tend to adopt policies that are more central in a bid to increase their vote share. In other words, the Nash bargaining outcome shifts in favor of the opportunists. The coalition adopts a more centre of the road policy, replacing contributions from activists to some extent with public funds. Thus public financing, which is irrelevant when parties are strictly vote-oriented, now interacts with coalition formation to influence policy choices.

Propositions 1 and 3 can also be viewed together to better understand the logic of party coalition. An increase in public financing increases the MRS of opportunists 1, who are more willing to move policy position towards the centre of the mass of voters. Similarly, an increases in c declines the MRS of opportunists 2, who are less willing to move away from the centre. In addition, a change in c has no direct effect on the MRS of activists in both coalitions. Assume that the coalitions re-establish their internal equilibrium represented by equation (15) by adjusting policy position and keeping, simultaneously, all other variables constant. An increase in s_1 establishes a new equilibrium inside coalition 1 as the MRS of opportunists 1 declines and the MRS of activists 1 increases. Similarly, a decrease in s_2 establishes a new equilibrium inside coalition 2 because the MRS of the opportunists increases and the MRS of the activists decreases.

In the same way, assume that the coalitions re-establish their internal equilibrium represented by equation (15) by adjusting the amount of activists' contribution and keeping other variables constant. Less contribution establishes a new equilibrium in coalition 1 because the MRS of the opportunists declines and that of the activists does not change. Likewise, less contribution establishes a new equilibrium in coalition 2 because the MRS of the opportunists increases and that of the activists remains constant.

It is worthy of noting that in this analysis symmetric shocks in the gammas are just a symmetric combination of asymmetric shocks.

5.1 Asymmetric financing of political parties

This section shows the change in policy platforms when public financing of political parties is asymmetric. Public financing can be linked to a number of parameters such as, for example, the number of sits in Parliament.

Here, I assume that an exogenous rule, or law, assigns a share δ_k of the total amount of public financing c to party k, where $\delta_k \in [0,1]$, $k = 1,2$ and $\delta_1 = 1 - \delta_2$. The share of public financing received by party 1 is $\delta_1 c = (1-\delta_2)\,c$ and the share received by party 2 is $\delta_2 c = (1-\delta_1)\,c$. The parameter δ_k is exogenously determined and can be interpreted as the share of Parliamentary seats holded by party k.[19] In this case, δ_k can also be written under the following forms: $\delta_1 = \frac{n_1}{n_1+n_2}$ and $\delta_2 = \frac{n_2}{n_1+n_2}$, where $n_1$and n_2 are the number of seats in Parliament. If $n_1 > n_2$, then coalition 1 is the incumbent party, which implies $\delta_2 < 0.5 < \delta_1$.

Proposition 4 *If public financing is proportional to parties' dimension, the bigger a party is, the more its policy platform converges towards that of the opposing party, while its activism declines. Conversely, the smaller the party is, the more extreme its policy platform is, while its activism increases. In formulas,*

$$\begin{cases} \frac{ds_1}{d\delta_1} > 0 \\ \frac{ds_2}{d\delta_1} > 0 \\ \frac{dc_1}{d\delta_1} < 0 \\ \frac{dc_2}{d\delta_1} > 0 \end{cases}, \tag{19}$$

and

$$\begin{cases} \frac{ds_1}{d\delta_2} < 0 \\ \frac{ds_2}{d\delta_2} < 0 \\ \frac{dc_1}{d\delta_2} > 0 \\ \frac{dc_2}{d\delta_2} < 0 \end{cases}. \tag{20}$$

When public financing is asymmetric, the party that receives less public funding becomes more extreme to stimulate support from its activists. While, the coalition with larger public funding converges towards the policy platform of the opposing party. This, in turn, implies that party coaltions present policy platforms that are more preferred by the activists of the coaltion with less public financing.

Therefore, the consequence of linking public financing, for instance, to the number of parliamentary sits is the shift of the electoral equilibrium towards the policy position of the opposition. Furthermore, the more competitive is the electoral equilibrium, which means that winners and looser obtain a close number of sits, the more policy platform converge to the centre of the mass of voters.

PROOF. In order to introduce asymmetric publich financing into the model we can rewrite equation (16) under the following form

$$MRS^{\psi_k} = \frac{\sum \alpha^J \theta^J u^{J\prime}(s_k)}{(\delta_k c + c_k)^{-2}\gamma_k \sum \alpha^J \theta^J} = \frac{v_k'(s_k)}{-1} = MRS^{\phi_k}, \qquad \text{with } k = 1,2, \tag{21}$$

[19] Alternatively, δ_k can be interpreted as any dimension index for party k such as, for instance, the number of parties' members.

where, $\frac{\partial MRS^{\psi_1}}{\partial \delta_1} > 0$, $\frac{\partial MRS^{\psi_2}}{\partial \delta_2} < 0$, $\frac{\partial MRS^{\psi_1}}{\partial \delta_2} < 0$, $\frac{\partial MRS^{\psi_2}}{\partial \delta_1} > 0$.

Consider the case in which the share of public financing in favour of party 1, δ_1, increases. The MRS of opportunists 1 increases, while the MRS of activists 1 does not change. Now, assume that the two factions of coalition 1 decide to restablish a new equilibrium by adjusting policy position s_1 and keeping all other variables constant. If, for example, they increase s_1, the MRS declines for the opportunists and increases for the activists. Therefore, an increase in s_1 can restore the equilibrium. On the contrary, a decline in s_1 will not restore the equilibrium becasue it increases the MRS for the oppurtinists and decreases the MRS of the activists. Now, assume that coalition 1 decides to restore the equilibrium by renegotiating the level of activism c_1 and keeping all the other variable constant. If, for example, c_1 increases, a new equilibrium cannot be reached because the MRS increases for the opportunists but it does not change for the activists. Instead, a decrease in c_1 is the only way to restore the equilibrium because the MRS declines for the opportunists until it reaches the previous equilibrium value.

Similarly, following the assumption that δ_2 declines as δ_1 increases, it is easy to analyse the impact of an increase in δ_1 on the equilibrium insied coalitition 2. The MRS increases for opportunists 2, while it does not change for activists 2. If coalition 2 restores the equilibrium by adjusting policy positon and keeping all other variables constant, they will increase s_2 because, in this way, the MRS declines for the opportunists and increases for the activists. Similarly, if they decide to restore the equilibrium by changing activits' contribution and keeping all the other variables constant, they wil increase c_2 because, in this way, the MRS declines for the opporutnists until it equates that of the activists, which does not change.

The same logic can be applied to prove the results in (20).

6 Coalition formartion with outside option

7 Concluding Remarks

This paper contributes to the small but growing literature on electoral competition when parties are explicitly regarded as coalitions in which policy-motivated activists provide resources to parties from inside a coalition. The model combines Nash-bargaining as the basis of party formation with Nash non-cooperative competition between parties in a spatial voting framework, in a manner suggested by Roemer (2001). In this Nash bargaining-Nash competition framework, I have explored the consequences for activism of changes in party valences and in the amount of public financing of political parties. The model covers a gap between two types of literature: one that focuses on the consequences of coalition formation for the nature of equilibrium platforms in the absence of campaign contributions, and the other where competing parties that are strictly electorally oriented are provided with resources by contributors who are always external to them.

The model shows that, in equilibrium, the two factions forming the two competing coalitions, the office-motivated opportunists and the policy-motivated activists, have the same marginal rate of substitution between policy position and activists' contribution. In this framework, an asymmetric shock to one party's valence leads to divergence of its policy platform. The opportunists in the party suffering the shock require more resources from their activists, and these can be had only by offering them a more extreme policy platform.

The analysis predicts that public financing of political parties drives activism or idealism out of politics. An increase in public financing makes political parties more autonomous from their activists. As a consequence, policy platforms converge to the centre of the mass of voters and the degree of activism declines. Therefore, public financing of political parties can be used as a policy instrument to regulate the trade-off between the degree of activism in politics and the independence of parties from lobbying.

Many interesting avenues of research remain, including the incorporation of a fuller treatment of lobbying alternatives. The model can be extended, for example, to incorporate two types of literature, one in which parties compete alone as points and lobbyists are always outside and the other in which parties are regarded as coalitions and lobbyists are inside them as in this paper. This can be done by allowing activists to lobby from either inside or outside parties. In this context, one could explore, for instance, the strategic behavior of opportunists who could alter the bargaining power of the two partners forming the competing coalition by moving policy position in favour of the opposing activists or contributors. Activists in the opposing coalition will then have a greater incentive to move outside, or might use their increased strength to claim more policy in their favour.

Furthermore, in this paper, activists care about policy platform provided by their own party only. A more complex behavior can be provided by modelling activists when they are concerned about the difference between the policy platform of their party and that of the opposing one.

Moreover, the question of how party financing should be regulated in view of the trade-off between idealism or activism in party organization and policy platform polarization also deserves further attention.

In order to solve in a relatively simple and tractable manner the problem of equilibrium formation inside coalitions, here a model with an electoral equilibrium with a dominant strategy has been used, in which opportunists always want to converge to the centre of the mass of voters regardless of what the opposing party does. It could be interesting to generalize this analysis by replacing the electoral equilibrium with dominant strategy with one in which parties always react to changes on the side of their competitors. This would add complexity, but also new insights on how the three simultaneous equilibriums, two inside the coalitions and the third between coalitions, interact.

Acknowledgments

I thank with gratitude Stanley Winer with whom I started to work on models of political competition. Without him I would have never written this paper. I also thank for helpful comments, Abhinay Muthoo, Christos Kotsogiannis, Andrea Galeotti, Ken Burdett, Francesco Forte, Guido Ortona and participants at seminars at the Universities of Essex, York and Exeter. Financial support provided by the Department of POLIS of the University of Eastern Piedmont, the Region of Piedmont and the Canada Research Chair in Public Policy while a post-doctoral fellow and the hospitality offered by the School of Public Policy and Administration and the Department of Economics at Carleton University are gratefully acknowledged.

8 Appendix

Proof. [Proof of Lemma 1] Party 1 maximizes its expected vote share:

$$\max_{s_1} \pi_1 = \sum_J \alpha^J \theta^J \left\{ -\left(s^{*J} - s_1\right)^2 + \left(s^{*J} - s_2\right)^2 - \frac{\gamma_1 - \gamma_2}{c} + \frac{1}{2\theta^J} \right\}, \tag{22}$$

while party 2 maximizes $(1 - \pi_1(s))$. Since the first order conditions for the choice of vote share maximizing platforms are the same for both parties, that is

$$2 \sum_J \alpha^J \theta^J \left(s^{*J} - s_k^O\right) = 0, \quad \text{with } k = 1{,}2, \tag{23}$$

the policy platforms of the parties are identical in the Nash electoral equilibrium. ■

8.1 Electoral equilibrium

The introduction of two new definitions, which simplify the comparative statics, and some general computations are now given before proceeding with the proofs.

Definition 1 *Define the "policy elasticity" of opportunists' net gain from reaching an agreement over policy platform* s_k *with* $\epsilon_{s_k}^{\psi_k} = \frac{\partial \psi_k / \partial s_k}{\psi_k / s_k}$*, where* $k = 1, 2$*. Similarly, define the "contribution elasticity" of opportunists' net gain from reaching an agreement over contribution* c_k *with* $\epsilon_{c_k}^{\psi_k} = \frac{\partial \psi_k / \partial c_k}{\psi_k / c_k}$*, where* $k = 1, 2$*.*

The elasticities measure the percent change of opportunists' net gain from reaching an agreement, ψ_k, relative to the percent change of either s_k or c_k.

Definition 2 *Define the "policy elasticity" of activists' net gain from reaching an agreement over policy platform* s_k *with* $\epsilon_{s_k}^{\phi_k} = \frac{\partial \phi_k / \partial s_k}{\phi_k / s_k}$*, where* $k = 1, 2$*. Similarly, define the "contribution elasticity" of activists' net gain from reaching an agreement over contribution* c_k *with* $\epsilon_{c_k}^{\phi_k} = \frac{\partial \phi_k / \partial c_k}{\phi_k / c_k}$*, where* $k = 1, 2$*.*

The elasticities measure the percent change of activists' net gain from reaching an agreement, ϕ_k, relative to the percent change of s_k or c_k.

The electoral equilibrium system (14) can be rewritten in a new form, which will be useful in the comparative statics. In equilibrium, the policy and contribution elasticity of the net gains of opportunists k are respectively equal in absolute value to the policy and contribution elasticity of the net gains of activists k and take opposite signs; that is,

$$\begin{cases} \epsilon_{s_k}^{\psi_k} = -\epsilon_{s_k}^{\phi_k} \\ \epsilon_{c_k}^{\psi_k} = -\epsilon_{c_k}^{\phi_k} \end{cases}, \quad \text{with } k = 1, 2. \tag{24}$$

Basically, perturbations of any exogenous variable like public financing or parties' valences will re-establish equilibrium by restoring the equalities in (24). Note that by dividing the two elasticities on the left-hand side and the two on the right-hand side as follows, $\frac{\epsilon_{s_k}^{\psi_k}}{\epsilon_{c_k}^{\psi_k}} = \frac{\epsilon_{s_k}^{\phi_k}}{\epsilon_{c_k}^{\phi_k}}$, we obtain the same formulation of Proposition 1.

Furthermore, it is useful to denote with F^k, with $k = 1, 2, 3, 4$, the four first order conditions in (14), as follows:

$$F^1 = \frac{\sum \alpha^J \theta^J u^{J\prime}(s_1)}{\psi_1} + \frac{v_1'(s_1)}{\phi_1} = \frac{\frac{\partial \psi_1}{\partial s_1}}{\psi_1} + \frac{\frac{\partial \phi_1}{\partial s_1}}{\phi_1} = \epsilon_{s_1}^{\psi_1} + \epsilon_{s_1}^{\phi_1} = 0, \tag{25}$$

$$F^2 = \frac{\frac{\gamma_1}{(c+c_1)^2} \sum \alpha^J \theta^J}{\psi_1} - \frac{1}{\phi_1} = \frac{\frac{\partial \psi_1}{\partial c_1}}{\psi_1} + \frac{\frac{\partial \phi_1}{\partial c_1}}{\phi_1} = \epsilon_{c_1}^{\psi_1} + \epsilon_{c_1}^{\phi_1} = 0, \tag{26}$$

$$F^3 = \frac{\sum \alpha^J \theta^J u^{J\prime}(s_2)}{\psi_2} + \frac{v_2'(s_2)}{\phi_2} = \frac{\frac{\partial \psi_2}{\partial s_2}}{\psi_2} + \frac{\frac{\partial \phi_2}{\partial s_2}}{\phi_2} = \epsilon_{s_2}^{\psi_2} + \epsilon_{s_2}^{\phi_2} = 0, \tag{27}$$

$$F^4 = \frac{\frac{\gamma_2}{(c+c_2)^2} \sum \alpha^J \theta^J}{\psi_2} - \frac{1}{\phi_2} = \frac{\frac{\partial \psi_2}{\partial c_2}}{\psi_2} + \frac{\frac{\partial \phi_2}{\partial c_2}}{\phi_1} = \epsilon_{c_2}^{\psi 2} + \epsilon_{c_2}^{\phi_2} = 0. \tag{28}$$

8.1.1 Partial derivatives

We report below the differentiation of the F^k functions with respect to the endogenous and exogenous variables.

$$\frac{\partial F^1}{\partial s_1} = \frac{\frac{\partial \psi_1}{\partial s_1^2}}{\psi_1} + \frac{\frac{\partial \phi_1}{\partial s_1^2}}{\phi_1} - 2\left(\frac{\epsilon_{s_1}^{\psi_1}}{s_1}\right)^2 < 0; \tag{29}$$

$$\frac{\partial F^1}{\partial c_1} = \frac{\partial F^2}{\partial s_1} = -\frac{\frac{\partial \psi_1}{\partial s_1}\frac{\partial \psi_1}{\partial c_1}}{\psi_1^2} - \frac{\frac{\partial \phi_1}{\partial s_1}\frac{\partial \phi_1}{\partial c_1}}{\phi_1^2} = -\frac{2\epsilon_{s_1}^{\psi_1}\epsilon_{c_1}^{\psi_1}}{s_1 c_1} < 0; \tag{30}$$

$$\frac{\partial F^1}{\partial \gamma_1} = -\frac{\frac{\partial \psi_1}{\partial \gamma_1}\frac{\partial \psi_1}{\partial s_1}}{\psi_1^2} = -\frac{\frac{\partial \psi_1}{\partial \gamma_1}\epsilon_{s_1}^{\psi_1}}{\psi_1 s_1} < 0; \tag{31}$$

$$\frac{\partial F^1}{\partial c} = -\frac{\frac{\partial \psi_1}{\partial c}\frac{\partial \psi_1}{\partial s_1}}{\psi_1^2} = -\frac{\epsilon_c^{\psi_1}}{c}\frac{\epsilon_{s_1}^{\psi_1}}{s_1} > 0; \tag{32}$$

$$\frac{\partial F^2}{\partial s_1} = \frac{\partial F^1}{\partial c_1} = -\frac{\frac{\partial \psi_1}{\partial c_1}\frac{\partial \psi_1}{\partial s_1}}{\psi_1^2} - \frac{\frac{\partial \phi_1}{\partial c_1}\frac{\partial \phi_1}{\partial s_1}}{\phi_1^2} = -\frac{2\epsilon_{s_1}^{\psi_1}\epsilon_{c_1}^{\psi_1}}{s_1 c_1} < 0; \tag{33}$$

$$\frac{\partial F^2}{\partial c_1} = \frac{\frac{\partial\left(\frac{\partial \psi_1}{\partial c_1}\right)}{\partial c_1}\psi_1 - \left(\frac{\partial \psi_1}{\partial c_1}\right)^2}{\psi_1^2} + \frac{-\left(\frac{\partial \phi_1}{\partial c_1}\right)^2}{\phi_1^2} = \frac{\frac{\partial\left(\frac{\partial \psi_1}{\partial c_1}\right)}{\partial c_1}}{\psi_1} - 2\left(\frac{\epsilon_{c_1}^{\psi_1}}{c_1}\right)^2 < 0; \tag{34}$$

$$\frac{\partial F^2}{\partial \gamma_1} = \frac{\partial \left(\epsilon_{c_1}^{\psi_1}\right)}{\partial \gamma_1} = \frac{\frac{\partial\left(\frac{\partial \psi_1}{\partial c_1}\right)}{\partial \gamma_1}}{\psi_1} - \frac{\frac{\partial \psi_1}{\partial \gamma_1}\epsilon_{c_1}^{\psi_1}}{\psi_1 c_1} = \frac{\left(\frac{\sum \alpha^J \theta^J}{c+c_1}\right)^2 \left(u^J\left(s_1\right) - u^J\left(s^O\right)\right)}{\psi_1^2} < 0;$$

$$\frac{\partial F^2}{\partial c} = \frac{\partial \epsilon_{c_1}^{\psi_1}}{\partial c} = \frac{\psi_1 \frac{\partial\left(\frac{\partial \psi_1}{\partial c_1}\right)}{\partial c} - \frac{\partial \psi_1}{\partial c}\frac{\partial \psi_1}{\partial c_1}}{\psi_1^2} \lesseqqgtr 0; \tag{35}$$

$$\frac{\partial F^3}{\partial s_2} = \frac{\psi_2 \sum \alpha^J \theta^J u^{J\prime\prime}\left(s_2\right) - \left(\frac{\partial \psi_2}{\partial s_2}\right)^2}{\psi_2^2} + \frac{\phi_2 v_2^{\prime\prime}\left(s_2\right) - \left(v_2^{\prime}\left(s_2\right)\right)^2}{\phi_2^2} = \frac{\frac{\partial\left(\frac{\partial \psi_2}{\partial s_2}\right)}{\partial s_2}}{\psi_2} + \frac{\frac{\partial\left(\frac{\partial \phi_2}{\partial s_2}\right)}{\partial s_2}}{\phi_2} - 2\left(\frac{\epsilon_{s_2}^{\psi_2}}{s_2}\right)^2 < 0; \tag{36}$$

$$\frac{\partial F^3}{\partial c_2} = \frac{\partial F^4}{\partial s_2} = -\frac{\frac{\partial \psi_2}{\partial c_2}\frac{\partial \psi_2}{\partial s_2}}{\psi_2^2} - \frac{\frac{\partial \phi_2}{\partial c_2}\frac{\partial \phi_2}{\partial s_2}}{\phi_2^2} = -2\frac{\epsilon_{c_2}^{\psi_2}\epsilon_{s_2}^{\psi_2}}{c_2 s_2} > 0; \tag{37}$$

$$\frac{\partial F^3}{\partial c} = -\frac{\frac{\partial \psi_2}{\partial c}\frac{\partial \psi_2}{\partial s_2}}{\psi_2^2} < 0; \tag{38}$$

$$\frac{\partial F^4}{\partial s_2} = \frac{\partial F^3}{\partial c_2} = -\frac{\frac{\partial \psi_2}{\partial s_2}\frac{\partial \psi_2}{\partial c_2}}{\psi_2^2} - \frac{\frac{\partial \phi_2}{\partial s_2}\frac{\partial \phi_2}{\partial c_2}}{\phi_2^2} = -\frac{2\epsilon_{s_2}^{\psi_2}\epsilon_{c_2}^{\psi_2}}{c_2 s_2} > 0; \tag{39}$$

$$\frac{\partial F^4}{\partial c_2} = \frac{\frac{\partial\left(\frac{\partial \psi_2}{\partial c_2}\right)}{\partial c_2}}{\psi_2} - 2\frac{\left(\epsilon_{c_2}^{\psi_2}\right)^2}{c_2^2} < 0; \tag{40}$$

$$\frac{\partial F^4}{\partial c} = \frac{\partial \epsilon_{c_2}^{\psi_2}}{\partial c} = \frac{\psi_2 \frac{\frac{\partial \psi_2}{\partial c_2}}{\partial c} - \frac{\partial \psi_2}{\partial c}\frac{\partial \psi_2}{\partial c_2}}{\psi_{2^2}} \lesseqqgtr 0; \tag{41}$$

$$\frac{\partial F^3}{\partial \gamma_2} = -\frac{\frac{\partial \psi_2}{\partial \gamma_2}\frac{\partial \psi_2}{\partial s_2}}{\psi_2^2} = -\frac{\frac{\partial \psi_2}{\partial \gamma_2}\epsilon_{s_2}^{\psi_2}}{\psi_2 s_2} > 0; \tag{42}$$

$$\frac{\partial F^4}{\partial \gamma_2} = \frac{\frac{\psi_2 \sum \alpha^J \theta^J}{\left(c+c_2\right)^2} - \frac{\partial \psi_2}{\partial \gamma_2}\frac{\partial \psi_2}{\partial c_2}}{\psi_2^2} = \frac{\left(\frac{\sum_J \alpha^J \theta^J}{c+c_2}\right)^2 \left(u^J\left(s_2\right) - u^J\left(s^O\right)\right)}{\psi_2^2} < 0. \tag{43}$$

Furthermore, the following partial derivatives are equal to zero: $\frac{\partial F^1}{\partial s_2} = 0$; $\frac{\partial F^1}{\partial c_2} = 0$; $\frac{\partial F^1}{\partial \gamma_2} = 0$; $\frac{\partial F^2}{\partial s_2} = 0$; $\frac{\partial F^2}{\partial c_2} = 0$; $\frac{\partial F^2}{\partial \gamma_2} = 0$; $\frac{\partial F^3}{\partial s_1} = 0$; $\frac{\partial F^3}{\partial c_1} = 0$; $\frac{\partial F^3}{\partial \gamma_1} = 0$; $\frac{\partial F^4}{\partial s_1} = 0$; $\frac{\partial F^4}{\partial c_1}$; $\frac{\partial F^4}{\partial \gamma_1} = 0$.

Proof. [Proof of Proposition 2] Following Chiang (1984, pp. 210-227), suppose we now hold the exogenous variables and parameters fixed with the exception of γ_1. Then, we may write the following matrix equation:

$$\begin{bmatrix} \frac{\partial F^1}{\partial s_1} & \frac{\partial F^1}{\partial c_1} & 0 & 0 \\ \frac{\partial F^2}{\partial s_1} & \frac{\partial F^2}{\partial c_1} & 0 & 0 \\ 0 & 0 & \frac{\partial F^3}{\partial s_2} & \frac{\partial F^3}{\partial c_2} \\ 0 & 0 & \frac{\partial F^4}{\partial s_2} & \frac{\partial F^4}{\partial c_2} \end{bmatrix} \begin{bmatrix} \frac{\partial s_1}{\partial \gamma_1} \\ \frac{\partial c_1}{\partial \gamma_1} \\ \frac{\partial s_2}{\partial \gamma_1} \\ \frac{\partial c_2}{\partial \gamma_1} \end{bmatrix} = \begin{bmatrix} -\frac{\partial F^1}{\partial \gamma_1} \\ -\frac{\partial F^2}{\partial \gamma_1} \\ 0 \\ 0 \end{bmatrix}. \tag{44}$$

The determinant of the Jacobian matrix,

$$|J| = \begin{bmatrix} \frac{\partial F^1}{\partial s_1} & \frac{\partial F^1}{\partial c_1} & 0 & 0 \\ \frac{\partial F^2}{\partial s_1} & \frac{\partial F^2}{\partial c_1} & 0 & 0 \\ 0 & 0 & \frac{\partial F^3}{\partial s_2} & \frac{\partial F^3}{\partial c_2} \\ 0 & 0 & \frac{\partial F^4}{\partial s_2} & \frac{\partial F^4}{\partial c_2} \end{bmatrix}, \tag{45}$$

is

$$|J| = \left(\frac{\partial F^1}{\partial s_1} \frac{\partial F^2}{\partial c_1} - \frac{\partial F^1}{\partial c_1} \frac{\partial F^2}{\partial s_1} \right) \left(\frac{\partial F^3}{\partial s_2} \frac{\partial F^4}{\partial c_2} - \frac{\partial F^3}{\partial c_2} \frac{\partial F^4}{\partial s_2} \right) > 0, \tag{46}$$

The Jacobian matrix is non zero for $\frac{\partial F^1}{\partial s_1} \frac{\partial F^2}{\partial c_1} \neq \frac{\partial F^1}{\partial c_1} \frac{\partial F^2}{\partial s_1}$ and $\frac{\partial F^3}{\partial s_2} \frac{\partial F^4}{\partial c_2} \neq \frac{\partial F^3}{\partial c_2} \frac{\partial F^4}{\partial s_2}$. Furthermore, $|J|$ is positive. To see this, we first study the sign of

$$\frac{\partial F^1}{\partial s_1} \frac{\partial F^2}{\partial c_1} - \frac{\partial F^1}{\partial c_1} \frac{\partial F^2}{\partial s_1} =$$

$$= \frac{\frac{\partial \psi_1}{\partial c_1^2}}{\psi_1} \left[\frac{\frac{\partial \psi_1}{\partial s_1^2}}{\psi_1} + \frac{\frac{\partial \phi_1}{\partial s_1^2}}{\phi_1} - 2 \left(\frac{\epsilon_{s_1}^{\psi_1}}{s_1} \right)^2 \right] - 2 \left(\frac{\epsilon_{c_1}^{\psi_1}}{c_1} \right)^2 \left[\frac{\frac{\partial \psi_1}{\partial s_1^2}}{\psi_1} + \frac{\frac{\partial \phi_1}{\partial s_1^2}}{\phi_1} \right] > 0. \tag{47}$$

Similarly,

$$\frac{\partial F^3}{\partial s_2} \frac{\partial F^4}{\partial c_2} - \frac{\partial F^3}{\partial c_2} \frac{\partial F^4}{\partial s_2} =$$

$$= \frac{\frac{\overset{-}{\partial} \psi_2}{\partial c_2^2}}{\psi_2} \left[\frac{\frac{\partial \psi_2}{\partial s_2^2}}{\psi_2} + \frac{\frac{\partial \phi_2}{\partial s_2^2}}{\phi_2} - 2 \left(\frac{\epsilon_{s_2}^{\psi_2}}{s_2} \right)^2 \right] - 2 \left(\frac{\epsilon_{c_2}^{\psi_2}}{c_2} \right)^2 \left[\frac{\frac{\partial \psi_2}{\partial s_2^2}}{\psi_2} + \frac{\frac{\partial \phi_2}{\partial s_2^2}}{\phi_2} \right] > 0. \tag{48}$$

Thus, the sign of the Jacobian matrix is positive.

By Cramer's rule, and using (46), we then find the solution to be

$$\frac{ds_1}{d\gamma_1} = \frac{\begin{bmatrix} -\frac{\partial F^1}{\partial \gamma_1} & \frac{\partial F^1}{\partial c_1} & 0 & 0 \\ -\frac{\partial F^2}{\partial \gamma_1} & \frac{\partial F^2}{\partial c_1} & 0 & 0 \\ 0 & 0 & \frac{\partial F^3}{\partial s_2} & \frac{\partial F^3}{\partial c_2} \\ 0 & 0 & \frac{\partial F^4}{\partial s_2} & \frac{\partial F^4}{\partial c_2} \end{bmatrix}}{\left(\frac{\partial F^1}{\partial s_1}\frac{\partial F^2}{\partial c_1} - \frac{\partial F^1}{\partial c_1}\frac{\partial F^2}{\partial s_1}\right)\left(\frac{\partial F^3}{\partial s_2}\frac{\partial F^4}{\partial c_2} - \frac{\partial F^3}{\partial c_2}\frac{\partial F^4}{\partial s_2}\right)};$$

The determinant of the numerator is

$$\left(\frac{\partial F^2}{\partial c_1}\frac{\partial F^1}{\partial \gamma_1} - \frac{\partial F^1}{\partial c_1}\frac{\partial F^2}{\partial \gamma_1}\right)\left(\frac{\partial F^3}{\partial c_2}\frac{\partial F^4}{\partial s_2} - \frac{\partial F^3}{\partial s_2}\frac{\partial F^4}{\partial c_2}\right).$$

Consequently,

$$\frac{ds_1}{d\gamma_1} = \frac{\frac{\partial F^1}{\partial c_1}\frac{\partial F^2}{\partial \gamma_1} - \frac{\partial F^1}{\partial \gamma_1}\frac{\partial F^2}{\partial c_1}}{\frac{\partial F^1}{\partial s_1}\frac{\partial F^2}{\partial c_1} - \frac{\partial F^1}{\partial c_1}\frac{\partial F^2}{\partial s_1}}. \tag{49}$$

Thus, given the positive sign of (47), the sign of the comparative statics depends on the sign of the numerator, which we rewrite under the following form:

$$\frac{\partial F^1}{\partial c_1}\frac{\partial F^2}{\partial \gamma_1} - \frac{\partial F^1}{\partial \gamma_1}\frac{\partial F^2}{\partial c_1} = -\frac{2\epsilon_{s_1}^{\psi_1}\epsilon_{c_1}^{\psi_1}\frac{\partial\left(\frac{\partial \psi_1}{\partial c_1}\right)}{\partial \gamma_1}}{s_1 c_1 \psi_1} + \frac{\epsilon_{s_1}^{\psi_1}\frac{\partial \psi_1}{\partial \gamma_1}\frac{\partial \psi_1}{\partial c_1^2}}{\psi_1^2 s_1} < 0. \tag{50}$$

As a consequence, $\frac{ds_1}{d\gamma_1} < 0$.

Similarly, after repeating the same steps to study the sign of $\frac{ds_2}{d\gamma_2}$, we get

$$\frac{ds_2}{d\gamma_2} = \frac{\frac{\partial F^3}{\partial c_2}\frac{\partial F^4}{\partial \gamma_2} - \frac{\partial F^3}{\partial \gamma_2}\frac{\partial F^4}{\partial c_2}}{\frac{\partial F^3}{\partial s_2}\frac{\partial F^4}{\partial c_2} - \frac{\partial F^3}{\partial c_2}\frac{\partial F^4}{\partial s_2}}. \tag{51}$$

Therefore, given the positive sign of (48), the sign of the comparative statics depends on the sign of the numerator, which we rewrite under the following form:

$$\frac{\partial F^3}{\partial c_2}\frac{\partial F^4}{\partial \gamma_2} - \frac{\partial F^3}{\partial \gamma_2}\frac{\partial F^4}{\partial c_2} = -2\frac{\epsilon_{c_2}^{\psi_2}\epsilon_{s_2}^{\psi_2}\frac{\partial \frac{\partial \psi_2}{\partial c_2}}{\partial \gamma_2}}{c_2 s_2 \psi_2} + \frac{\epsilon_{s_2}^{\psi_2}\frac{\partial \psi_2}{\partial \gamma_2}\frac{\partial \psi_2}{\partial c_2^2}}{\psi_2^2 s_2} > 0. \tag{52}$$

As a result, $\frac{\partial s_2}{\partial \gamma_2} > 0$.

Furthermore, the study of the sign of $\frac{dc_k}{d\gamma_k}$ leads to the following results:

$$\frac{dc_1}{d\gamma_1} = \frac{\frac{\partial F^1}{\partial \gamma_1}\frac{\partial F^2}{\partial s_1} - \frac{\partial F^1}{\partial s_1}\frac{\partial F^2}{\partial \gamma_1}}{\frac{\partial F^1}{\partial s_1}\frac{\partial F^2}{\partial c_1} - \frac{\partial F^1}{\partial c_1}\frac{\partial F^2}{\partial s_1}} \gtreqless 0 \tag{53}$$

and

$$\frac{dc_2}{d\gamma_2} = \frac{\frac{\partial F^3}{\partial \gamma_2}\frac{\partial F^4}{\partial s_2} - \frac{\partial F^3}{\partial s_2}\frac{\partial F^4}{\partial \gamma_2}}{\frac{\partial F^3}{\partial s_2}\frac{\partial F^4}{\partial c_2} - \frac{\partial F^3}{\partial c_2}\frac{\partial F^4}{\partial s_2}} \lesseqqgtr 0. \tag{54}$$

While, the denominators of (53) and (54) are positive, the algebraic study of the sign of the two numerators is not a simple task. Therefore, we proceed in the proof by contradiction. It is worth noting that the proof by contradiction, which does not require complex calculus, can also be easily extended to any other proof in the paper. Specifically, here, it is necessary to prove that, in equilibrium, $dc_k/d\gamma_k > 0$, which implies $\frac{\partial F^1}{\partial \gamma_1}\frac{\partial F^2}{\partial s_1} - \frac{\partial F^1}{\partial s_1}\frac{\partial F^2}{\partial \gamma_1} > 0$ and $\frac{\partial F^3}{\partial \gamma_2}\frac{\partial F^4}{\partial s_2} - \frac{\partial F^3}{\partial s_2}\frac{\partial F^4}{\partial \gamma_2} > 0$. In order to do so let us consider the case in which $dc_k/d\gamma_k$ is negative and study whether this is feasible in equlibrium. Proposition 1 states that in equilibrium the MRS is the same for activists and opportunists in each coalition. Furthermore, according to equilibrium equation (16), the marginal rates of substitution are positive for both activists and opportunists of coalition 1 and negative for activists and opportunists of coalition 2. Focusing on coalition 1, an increase in γ_1 causes a break in the initial equilibrium because the marginal rate of substitution declines for opportunists 1, while it does not change for activists 1; i.e.: $\frac{\partial MRS^{\psi_1}}{\partial \gamma_1} < 0$ and $\frac{\partial MRS^{\phi_1}}{\partial \gamma_1} = 0$. In order to re-establish a new equilibrium, the coalition needs to renegotiate the amount of activists' contribution c_k. Since $\frac{\partial MRS^{\psi_1}}{\partial c_1} > 0$ and $\frac{\partial MRS^{\phi_1}}{\partial c_1} = 0$, the equilibrium can be re-established only with an increase in c_1. This, in turn, implies that $dc_1/d\gamma_1$ must be positive. Therefore, the case in which $dc_k/d\gamma_k$ is negative contradicts the equilibrium conditions (15) and (16). This proves the Proposition.

The same logic can be replicated, mutatis mutandi, for the case of a decrease in γ_1 and for changes in γ_2.

Furthermore, note that $\frac{ds_1}{d\gamma_2} = 0$, $\frac{ds_2}{d\gamma_1} = 0$, $\frac{dc_1}{d\gamma_2} = 0$ and $\frac{dc_2}{d\gamma_1} = 0$. ■

Proof. [Proof of Proposition 3] The comparative statics with regard to public financing is given by the solutions to the following system in matrix form:

$$\begin{bmatrix} \frac{\partial F^1}{\partial s_1} & \frac{\partial F^1}{\partial c_1} & 0 & 0 \\ \frac{\partial F^2}{\partial s_1} & \frac{\partial F^2}{\partial c_1} & 0 & 0 \\ 0 & 0 & \frac{\partial F^3}{\partial s_2} & \frac{\partial F^3}{\partial c_2} \\ 0 & 0 & \frac{\partial F^4}{\partial s_2} & \frac{\partial F^4}{\partial c_2} \end{bmatrix} \begin{bmatrix} \frac{\partial s_1}{\partial c} \\ \frac{\partial c_1}{\partial c} \\ \frac{\partial s_2}{\partial c} \\ \frac{\partial c_2}{\partial c} \end{bmatrix} = \begin{bmatrix} -\left(\frac{\partial F^1}{\partial c}\right) \\ -\left(\frac{\partial F^2}{\partial c}\right) \\ -\left(\frac{\partial F^3}{\partial c}\right) \\ -\left(\frac{\partial F^4}{\partial c}\right) \end{bmatrix}. \tag{55}$$

For s_1 we get

$$\frac{\partial s_1}{\partial c} = \frac{\begin{bmatrix} -\frac{\partial F^1}{\partial c} & \frac{\partial F^1}{\partial c_1} & 0 & 0 \\ -\frac{\partial F^2}{\partial c} & \frac{\partial F^2}{\partial c_1} & 0 & 0 \\ -\frac{\partial F^3}{\partial c} & 0 & \frac{\partial F^3}{\partial s_2} & \frac{\partial F^3}{\partial c_2} \\ -\frac{\partial F^4}{\partial c} & 0 & \frac{\partial F^4}{\partial s_2} & \frac{\partial F^4}{\partial c_2} \end{bmatrix}}{|J|},$$

which gives

$$\frac{\partial s_1}{\partial c} = -\frac{\frac{\partial F^1}{\partial c}\frac{\partial F^2}{\partial c_1} - \frac{\partial F^1}{\partial c_1}\frac{\partial F^2}{\partial c}}{\frac{\partial F^1}{\partial s_1}\frac{\partial F^2}{\partial c_1} - \frac{\partial F^1}{\partial c_1}\frac{\partial F^2}{\partial s_1}}. \tag{56}$$

We already know from (47) the denominator is positive. The sign of the numerator is studied below:

$$-\frac{\partial F^1}{\partial c}\frac{\partial F^2}{\partial c_1} + \frac{\partial F^1}{\partial c_1}\frac{\partial F^2}{\partial c} = \frac{\frac{\partial \psi_1}{\partial c}\epsilon^{\psi_1}_{s_1}\frac{\partial\left(\frac{\partial \psi_1}{\partial c_1}\right)}{\partial c_1}}{s_1\psi_1^2} - \frac{2\epsilon^{\psi_1}_{s_1}\epsilon^{\psi_1}_{c_1}\frac{\partial\left(\frac{\partial \psi_1}{\partial c_1}\right)}{\partial c}}{s_1 c_1 \psi_1} > 0. \tag{57}$$

This, in turn, implies that $\frac{\partial s_1}{\partial c} > 0$.

Similarly, for s_2 the comparative static is

$$\frac{\partial s_2}{\partial c} = -\frac{\frac{\partial F^3}{\partial c}\frac{\partial F^4}{\partial c_2} - \frac{\partial F^3}{\partial c_2}\frac{\partial F^4}{\partial c}}{\frac{\partial F^3}{\partial s_2}\frac{\partial F^4}{\partial c_2} - \frac{\partial F^3}{\partial c_2}\frac{\partial F^4}{\partial s_2}}, \tag{58}$$

where, the denominator is positive and the numerator is negative, as equation (59) shows.

$$\frac{\partial F^3}{\partial c_2}\frac{\partial F^4}{\partial c} - \frac{\partial F^3}{\partial c}\frac{\partial F^4}{\partial c_2} = +\frac{\frac{\partial \psi_2}{\partial c}\epsilon^{\psi_2}_{s_2}\frac{\partial\left(\frac{\partial \psi_2}{\partial c_2}\right)}{\partial c_2}}{s_2\psi_2^2} - \frac{2\epsilon^{\psi_2}_{s_2}\epsilon^{\psi_2}_{c_2}\frac{\partial\left(\frac{\partial \psi_2}{\partial c_2}\right)}{\partial c}}{s_2 c_2 \psi_2} < 0. \tag{59}$$

As a result,

$$\frac{\partial s_2}{\partial c} < 0.$$

Furthermore, the study of the sign of $\frac{dc_k}{dc}$ leads to the following results:

$$\frac{\partial c_1}{\partial c} = \frac{\frac{\partial F^1}{\partial c}\frac{\partial F^2}{\partial s_1} - \frac{\partial F^1}{\partial s_1}\frac{\partial F^2}{\partial c}}{\frac{\partial F^1}{\partial s_1}\frac{\partial F^2}{\partial c_1} - \frac{\partial F^1}{\partial c_1}\frac{\partial F^2}{\partial s_1}} \gtreqless 0 \tag{60}$$

and

$$\frac{\partial c_2}{\partial c} = \frac{\frac{\partial F^3}{\partial c}\frac{\partial F^4}{\partial s_2} - \frac{\partial F^3}{\partial s_2}\frac{\partial F^4}{\partial c}}{\frac{\partial F^3}{\partial s_2}\frac{\partial F^4}{\partial c_2} - \frac{\partial F^3}{\partial c_2}\frac{\partial F^4}{\partial s_2}} \gtreqless 0. \tag{61}$$

Whil the denominators of (60) and (61) are positive, the algebraic study of the sign of the two numerators is not simple. Therefore, proceeding in the proof by contradiction, we need to prove that, in equilibrium, $dc_k/dc < 0$, which implies $\frac{\partial F^1}{\partial c}\frac{\partial F^2}{\partial s_1} - \frac{\partial F^1}{\partial s_1}\frac{\partial F^2}{\partial c} < 0$ and $\frac{\partial F^3}{\partial c}\frac{\partial F^4}{\partial s_2} - \frac{\partial F^3}{\partial s_2}\frac{\partial F^4}{\partial c} < 0$. In order to do so we consider the case in which dc_k/dc is positive and study whether this is feasible in equilibrium. Recall from Proposition 1 that in equilibrium the marginal rates of substitution are the same for both activists and opportunists in each coalition. Furthermore, according to equilibrium equation (16), the marginal rates of substitution are positive for both activists and opportunists of coalition 1 and negative for activists and opportunists of coalition 2. Turning our attention to coalition 1, we see that an increase in c, for example, causes a break in the initial equilibrium because the MRS increases for opportunists 1, while it does not change for activists 1; i.e.: $\frac{\partial MRS^{\psi_1}}{\partial c} > 0$ and $\frac{\partial MRS^{\phi_1}}{\partial c} = 0$. In order to re-establish a new equilibrium, the coalition needs to renegotiate the amount of activists' contribution c_k. Since $\frac{\partial MRS^{\psi_1}}{\partial c_1} > 0$ and $\frac{\partial MRS^{\phi_1}}{\partial c_1} = 0$, the equilibrium can be re-established only with a decrease in c_1. This, in turn, implies that dc_1/dc must be negative. Therefore, the case in which dc_k/dc is positive contradicts the equilibrium conditions (15) and (16). This proves the proposition.

The same logic can be replicated, mutatis mutandi, for the case of a decrease in c and for studying the impact on c_2 of changes in c. ■

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