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# MARKET STRUCTURE AND VAT EVASION WITH TRADITIONAL AND E-COMMERCE

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# Market structure and VAT evasion with traditional and e-commerce\*

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**Abstract.** This paper considers the interaction between market structure and Value Added Tax (VAT) evasion in a monopolistically competitive economy composed of two industries. The paper interprets an increase in the intra-brand elasticity of substitution in one industry as the diffusion of e-commerce in that industry. The paper shows that e-commerce diffusion, by increasing the cost of not fully recovering the VAT paid on intermediate inputs, lowers the incentives to evade the VAT on final sales for the firms operating in the affected industry. Moreover, e-commerce diffusion changes the equilibrium number of firms and their size in both industries. These effects impact on the level and distribution of VAT revenues between industries. By using European Union macroeconomic panel data for 2000-2006, the paper finds evidence of a negative correlation between internet diffusion, which is taken as a proxy for the diffusion of e-commerce, and the VAT gap share, which is taken as a proxy for VAT evasion.

**Keywords**: Monopolistic competition, traditional and e-commerce, VAT evasion **JEL codes**: H26, L11

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### 1. Introduction

This paper considers the equilibrium market structure changes following from the introduction of ecommerce and their impact on Value Added Tax (VAT) compliance. There is a large literature studying the effects of e-commerce on equilibrium prices, but fewer papers investigate its impact on market structure. A notable exception is Goldmanis et al. (2010), who find that the introduction of e-commerce within an industry, by provoking a downward shift in consumer search costs, leads to lower prices and shifts market shares away from less efficient towards more efficient producers. Testing their model on US industry data, these authors find that e-commerce diffusion generally lowers the total number of active firms, with smaller firms exiting the industry and larger firms becoming more dominant. This paper develops a standard general equilibrium model of a monopolistically competitive economy generating predictions on the diffusion of ecommerce that are consistent with Goldmanis et al's findings. More specifically, in this paper the economy is composed of a competitive sector (for example, agriculture) and two monopolistically competitive industries. One industry produces physical goods that can be sold both by traditional bricks-and-mortar shops and over the internet (for example, cars, computer hardware, electronic equipment). The other industry produces goods that can be sold by traditional shops only (for example, shoes). The paper proxies the diffusion of ecommerce in the former industry by considering an increase in the industrial intra-brand substitution elasticity. This assumption seems consistent with both the hypothesis that internet commerce makes markets more competitive and the empirical finding that internet users tend to be highly price sensitive.<sup>1</sup> The paper also assumes that the elasticity of substitution is higher within than between industries and that there is a unique standard VAT rate on industrial goods.

Solving the model in general equilibrium under free entry and exit for the full VAT compliance case, the paper shows that the diffusion of e-commerce has pro-competitive effects for the entire industrial sector, due to a price index externality that brings about price and wage reductions in both industries. Changes in relative consumption demands imply that the industry with e-commerce undergoes an increase both in the total demand for its composite commodity and in the size of active firms, with the number of active firms

<sup>&</sup>lt;sup>1</sup> Several studies find very high price-sensitivity of demand in US internet markets, see for example Goolsbee (2000) and Ellison and Ellison (2009) for computer parts, Chevalier and Goolsbee (2003) for booksellers, Granados et al. (2009) for air travel tickets. However, these results are not univocal. For example, Alm and Melnik (2005) derive more conservative estimates. Moreover, Ellison and Ellison (2009) point out that "obfuscation" strategies by sellers (e.g. adds-on for updates and delivery fees) can reduce the price elasticity of demand. Alternatively, one can interpret the increase in the intra-brand substitution elasticity as the pro-competitive effect of either trade liberalisation (see for example Danthine and Hunt, 1994) or product market reforms (see for example Blanchard and Giavazzi, 2003).

declining at the same time. In the industry with traditional sales only, the demand for its composite commodity falls and the number of active firms declines. As a result, output per firm remains unaffected. These changes in market structure provoked by e-commerce diffusion have important implications for VAT issues. Firstly, under full VAT compliance the paper shows that increasing the VAT rate lowers the equilibrium number of industrial firms, but not their size, implying a shrinking industrial sector's size. Second, the paper considers the impact of e-commerce on VAT compliance.

The paper shows that, in free-entry equilibrium, under some conditions e-commerce diffusion in one industry lowers the optimal degree of VAT evasion on final sales in that industry. The mechanism is similar to the one pointed out by Keen (2008) for studying governmental incentives to introduce the VAT in developing countries. The idea is that firms evading the VAT on final sales are unable to recover the VAT paid on their purchases of intermediate inputs. In Keen (2008), firms operate within a fully competitive economy and their evasion decision is "all or nothing". In the current paper, firms are differentiated monopolies that may optimally choose to under report a fraction of their revenue. Thus, firms are always able to recover part of the VAT paid on their purchases of intermediate inputs. However, for each unit of under-reported final sales, the corresponding cost of un-recovered VAT is proportional to the real producer price of intermediate inputs. As long as this real price is an increasing function of the industry-specific intrabrand substitution elasticity, e-commerce diffusion raises the cost of VAT evasion and induces more compliance, other things being equal. This feature of the model depends on the invoice-credit mechanism of the VAT.<sup>2</sup> To be sure, the paper also shows that, when the VAT is formally equivalent to a retail sales tax (RST), the evasion decision is unaffected by e-commerce diffusion.<sup>3</sup> Then, the paper derives the general equilibrium implications of e-commerce diffusion on prices and wages, on the number and size of industrial firms, on VAT revenues and on the real revenue maximising VAT rate.

Although the tax evasion literature is mainly concerned with personal income tax evasion (see Andreoni et. al. 1998, for a survey), there are by now several papers focusing on indirect tax evasion by firms. Earlier models include Virmani (1989) and Cremer and Gahavari (1993), assuming perfect

 $<sup>^{2}</sup>$  The invoice-credit mechanism means exactly that the VAT paid on the purchases of intermediate inputs can be credited against the VAT paid on final sales. This mechanism operates in the majority of countries adopting the VAT.

<sup>&</sup>lt;sup>3</sup> When the VAT is equivalent to a RST, it can be shown that e-commerce diffusion lowers VAT evasion if the probability of detection is made conditional on the average firm's size, see Appendix 3 for details.

competition, and Marrelli (1984), focusing on monopoly firms. All of these models take the number of firms as given. More recently, Goerke and Runkel (2011) have considered indirect tax evasion in a Cournot oligopoly market with endogenous number of firms.<sup>4</sup> VAT evasion has been analysed in formal models by, among others, Fedeli and Forte (1999, jointly with income tax evasion), Keen and Smith (2006), Keen (2008), Hashimzade et. al (2010). However, to the best of our knowledge, the existing literature has never considered the general equilibrium implications of VAT evasion in a two-sector monopolistically competitive economy with endogenous number of firms. A notable exception is Davies and Paz (2011) who, in contrast to our paper, allow for differences in firm productivity. They find that VAT evasion is related to firm efficiency, with less efficient firms submerging and more efficient firms being fully compliant. However, in their model the VAT is formally equivalent to a RST,<sup>5</sup> and they do not consider the effects of changes in the intra-brand substitution elasticity on firm evasion incentives. Alm and Sennoga (2010) also study tax evasion in general equilibrium. However, their focus is different. By developing a fully-fledged computable general equilibrium model à la Harberger, with one formal and one informal (untaxed) sector, they consider the distributional consequences of personal income tax evasion. Their main result is that, with heterogeneous households, the individual gains from tax evasion are washed away by the general equilibrium adjustments in relative prices.<sup>6</sup> This paper, by considering a representative household and by using a simplified and symmetric production technology, looks instead at the general equilibrium implications of ecommerce-induced changes in household preferences that may directly impact on firm evasion choices.

This paper takes also a different view than usually done in the literature as regards the relationship between indirect taxation and e-commerce. The existing literature, starting from the assumption that indirect tax evasion is easier when sales are made over the internet, is mainly concerned with the possible revenue losses associated with e-commerce diffusion. Thus, it focuses on the desirability of differential or preferential tax treatment of e-commerce relatively to traditional commerce (see Zodrow, 2006, for a discussion). In this paper, instead, the diffusion of e-commerce is interpreted as a pro-competitive change in market conditions that may lead to lower VAT evasion. If this is indeed the case, one policy implication of

<sup>&</sup>lt;sup>4</sup> Profit tax evasion in oligopolistic markets is analysed, for example, by Marrelli and Martina (1988), Goerke and Runkel (2006) and Bayer and Cowell (2009).

<sup>&</sup>lt;sup>5</sup> On the contrary, De Paula and Scheinkman (2010) model the VAT chain with heterogeneous firms. They also find that incentives to evade are larger for less efficient firms. However, they consider competitive markets.

<sup>&</sup>lt;sup>6</sup> Alm and Turner (2011, unpublished) find similar results when studying firm evasion of the RST.

the paper is that governments willing to tackle VAT evasion should encourage, rather than fear, the diffusion of e-commerce. To be sure, although e-commerce-related tax revenue losses are a serious concern for politicians, their practical relevance might have been exaggerated in recent years, especially as regards non-digital goods (see Alm and Melnik, 2009, for the US; and National Audit Office, 2006: 10 for the UK).

The paper is organised as follows. Section 2 presents the model. Section 3 derives the equilibrium in the industrial sector under free entry and exit, full VAT compliance, and by taking into account of general equilibrium effects. Section 4 considers firm VAT evasion. Section 5 presents evidence of a negative correlation between internet diffusion (a proxy for e-commerce diffusion) and the VAT gap share (a proxy for VAT evasion) based on macroeconomic panel data for EU countries in 2000-2006. Section 6 concludes.

### 2. The model

Consider a closed economy composed of an agricultural sector, producing a homogeneous good sold to consumers, a monopolistically competitive industrial sector, producing differentiated commodities, and a capital good sector, producing a capital good sold to industrial firms. The economy is populated by a continuum of H households indexed by h=1, 2... H sharing the following identical preferences:

$$\begin{split} U_{h} &= \left[\frac{Z_{h}/P}{1-\alpha}\right]^{1-\alpha} \left\{ \left[\frac{X_{h}}{(1-\beta)\alpha}\right]^{1-\beta} \left[\frac{C_{h}}{\alpha\beta}\right]^{\beta} \right\}^{\alpha} - \theta\ell, \ 0 < \alpha < 1, \ 0 < \beta < 1 \\ C_{h} &= \left[\frac{\varepsilon}{\varepsilon-1}\right]_{i=1}^{2} C_{hi} \frac{\varepsilon-1}{\varepsilon}, \ \varepsilon > 1 \\ C_{hi} &= n_{i} \frac{-1}{\sigma_{i}-1} \left[\sum_{j=1}^{n_{i}} c_{hij} \frac{\sigma_{i}-1}{\sigma_{i}}\right]^{\frac{\sigma_{i}}{\sigma_{i}-1}}, \ i = 1, 2, \ j = 1, 2...n_{i}, \ \sigma_{i} > 1, \ \sigma_{i} > \varepsilon > 1/(1-\beta) \\ P_{i} &= \left[n_{i}^{-1} \sum_{j=1}^{n_{i}} p_{ij}^{1-\sigma_{i}}\right]^{\frac{1}{1-\sigma_{i}}}, \ P_{c} &= \left[2^{-1} \left(P_{1}^{1-\varepsilon} + P_{2}^{1-\varepsilon}\right)\right]^{\frac{1}{1-\varepsilon}}, \ P = p_{X}^{1-\beta} P_{c}^{\beta} \end{split}$$

$$(1)$$

 $Z_h$  is a non-produced good providing liquidity services to households and that can be interpreted as being "money".<sup>7</sup>  $X_h$  is consumption of the homogeneous agricultural good.  $C_h$  is consumption of the industrial good. Each household offers one unit of labour,  $\ell=1$ , with constant disutility equal to  $0 < \theta < 1$ . Equation (1) adopts a quasi-linear utility function to avoid any effect of the labour supply decision on the VAT structure.

<sup>&</sup>lt;sup>7</sup> The non-produced good is needed, together with the assumption of an exogenous price in the agricultural sector, for closing the model in general equilibrium, see section 3 below.

Consumption of industrial goods  $C_h$  is a constant elasticity of substitution function of consumption in the two industries i=1, 2. Consumption in each industry is a Dixit-Stiglitz (1977) CES index of the different j=1, 2...n<sub>i</sub> industrial brands. The number of brands n<sub>i</sub> will be determined endogenously below.  $\sigma > 1$  is the industry *i* intra-brand elasticity of substitution.<sup>8</sup> Choosing the price of the non-produced good Z as the numéraire implies the consumption price index P of equation (1). To simplify the analysis, we assume that industry i=1 produces physical goods that can be sold both by traditional bricks-and-mortar shops and over the internet (for example, cars, computer hardware, electronic equipment). Industry i=2 produces instead goods that can be sold by traditional shops only (for example, shoes).<sup>9</sup> This assumption, together with the one made below that the diffusion of e-commerce in a given industry can be proxied by an increase in the industry-specific inter-brand substitution elasticity, allows us to capture the stylized fact that e-commerce may disseminate differently across different industries (see Goldmanis et al, 2010, for US evidence).

Industrial goods face a common VAT rate, whereas a zero VAT rate is applied to the non-industrial sectors. This assumption, as far as the European Union is concerned, seems broadly consistent with the observation that the national standard VAT rate applies rather uniformly to industrial production.<sup>10</sup> We also assume that the intra-brand substitution elasticity is larger within than between industries, implying  $\sigma_i > \epsilon$  Lastly, we impose that the substitution elasticity between composite industrial goods is sufficiently high, namely  $\epsilon > 1/(1-\beta)$ . We shall discuss this condition in section 3 below.

With identical household preferences, under the utility function (1) we can consider a representative household. Dropping the h subscript and interpreting consumption variables in equation (1) as economy-wide variables, the representative household faces the following aggregate budget constraint:

$$Z + p_{x}X + \sum_{j=1}^{n_{1}} p_{1j}c_{1j} + \sum_{j=1}^{n_{2}} p_{2j}c_{2j} \le wL + \Pi + Z_{0} - T \equiv \Omega$$
(2),

where  $\Omega$  is aggregate wealth and L is total employment. We assume a competitive labour market, with perfect labour mobility and a common wage w. Thus, wL is aggregate labour income,  $\Pi$  is aggregate profits,  $Z_0$  is the initial aggregate endowment of the non-produced good. T is a lump-sum tax (transfer, if T is

<sup>&</sup>lt;sup>8</sup> The normalisation of the CES index eliminates the "taste for diversity" effect. As is well known, this makes the VAT-inclusive price indexes  $P_i$  and  $P_c$  independent of the number of brands, see equations (7) and (7bis) below.

<sup>&</sup>lt;sup>9</sup> We abstract from the idea that, depending on the mode of purchase, customers may perceive differently the same kind of physical good, see Zodrow (2006) for such an approach.

<sup>&</sup>lt;sup>10</sup> Reduced and super-reduced VAT rates are applied to a limited range of goods, see European Commission (2010).

negative, in which case it consists of redistributed tax receipts). Utility maximisation for given  $\Omega$  implies the following aggregate demand functions:

$$Z/P = (1 - \alpha)\Omega$$

$$X = \alpha(1 - \beta)\Omega/p_X$$

$$C = \alpha\beta\Omega/P_C$$

$$C_i = \left(\frac{P_i}{P_c}\right)^{-\varepsilon} \frac{C}{2}, \quad i = 1, 2, \quad c_{ij} = \left(\frac{P_{ij}}{P_i}\right)^{-\sigma_i} \frac{C_i}{n_i}, i = 1, 2; j = 1, 2, ..., n_i$$
(3)

From equation (1), the labour supply decision of the household corresponds to the participation decision. If the real consumption wage w/P is greater than or equal to the marginal disutility of labour  $\theta$ , the representative household offers H units of labour services. If the real consumption wage is less than the disutility of labour, the household does not participate in the labour market. In what follows, we assume that producers face a perfectly elastic supply of labour in the economy, implying that w/P= $\theta$  and H<L.

Turning to the producers' side, we first consider the agricultural sector. We assume that a representative competitive firm produces a single homogeneous good with a decreasing returns to labour technology, implying the following profit function  $\Pi_A = p_X X - w X^{\frac{1}{\gamma}}$ , where  $0 < \gamma < 1$  represents the constant elasticity of output with respect to labour input. Profit maximisation yields equilibrium output

$$X_{A} = \left(\frac{p_{x}\gamma}{\theta P}\right)^{\frac{\gamma}{1-\gamma}}$$
(4),

where  $w=\theta P$  by imposing equilibrium in the labour market. To characterise the general equilibrium solution below, we shall assume that the agricultural product price  $p_x$  is exogenously fixed. This would be the case, for example, if the agricultural price is administered by the government in a closed economy (or by the European Commission under the EU common agricultural policy), or if it represents a price that is fixed in international markets for a small-open economy. To simplify notation, we set  $p_x=1$  without loss of generality.

In the industrial sector, each differentiated brand is produced under increasing returns to scale and monopolistic competition by using both a fixed requirement of overhead capital F, which is needed to setting up firms (for example, plants, buildings, managerial labour), and a variable labour input requirement L<sub>ij</sub>, with

i=1, 2 and j=1, 2, ... $n_i$ .<sup>11</sup> Overhead capital is produced in the capital good sector. Following Dixon and Santoni (1995), we assume that a representative firm produces and sells overhead capital to firms in the industrial sector under constant returns to labour, with one unit of labour producing F units of overhead capital. The market demand for capital goods is determined by the number of firms operating in the industrial sector. Thus, aggregate output-employment in the capital good sector, indexed by K, is given by  $L_K = (n_1+n_2)F$ . As long as the representative firm makes no profits, the equilibrium price is  $p_K=w=\theta P$ . We assume, without loss of generality, that the VAT tax rate for capital goods is equal to zero.<sup>12</sup> In the next sections, we shall determine equilibrium in the industrial sector by making the number of firms endogenous. We shall study the effects of internet diffusion, firstly under full VAT compliance, then under VAT evasion.

### 3. Full VAT compliance. General equilibrium under free entry and exit.

In the industrial sector, each monopolistically competitive firm produces one unit of output  $x_{ij}$  by using one unit of labour input  $L_{ij}$ , provided it has F units of overhead capital already installed. Each unit of the final good sold in the product market is subject to an ad valorem tax rate  $\tau \in (0, 1)$  corresponding to the VAT tax rate  $s \equiv \tau/(1-\tau)$ .<sup>13</sup> The typical monopoly firm  $j=1,2...n_i$  in industry i=1, 2 maximises profits subject to the demand constraint given by equation (3) above. Thus, it solves

$$p_{ij} \arg \max \pi_{ij} = \frac{1}{1+s} \Big[ p_{ij} - w(1+s) \Big] x_{ij} - wF \text{ s.t. } x_{ij} = c_{ij} = \left( \frac{p_{ij}}{P_i} \right)^{-\sigma_i} \frac{C_i}{n_i}$$
(5).

If the number of firms operating in the same industry is sufficiently large, the Dixit-Stiglitz assumption of aggregate price-taking behaviour holds true. Thus, the typical firms sets the optimal VAT inclusive price

$$p_{ij} = \left(\frac{\sigma_i}{\sigma_i - 1}\right) \theta P(1+s), \quad i = 1, 2; j = 1, 2, ... n_i$$
 (6).

<sup>&</sup>lt;sup>11</sup> We shall provide two interpretations for the labour input requirement. In section 3, when considering full VAT compliance, it will be interpreted as labour hired at the wage w. In section 4, when dealing with VAT evasion, it will be also interpreted as an intermediate input the firm produces by buying labour services at the VAT inclusive arm's length price  $P_L=w+\tau P_L$ , where  $\tau$  is the single VAT rate. The reason for these different interpretations shall become clear below.

<sup>&</sup>lt;sup>12</sup> As long as, with a single rate and invoice-credit method, the VAT paid on capital goods is always credited to the firm.

<sup>&</sup>lt;sup>13</sup> In this section, modelling the VAT as an ad valorem retail sales tax does not matter. However, if labour is interpreted as an intermediate input bought in the market, with VAT evasion things turn out to be different, see section 4 below.

As expected, the optimal price is a decreasing function of the industrial intra-brand elasticity of substitution and an increasing function of both the VAT tax rate and the marginal production costs. These latter are w= $\theta$ P under the assumption of labour market clearing occurring at less than full employment, or L<H. In a symmetric partial equilibrium within each industry, firms set the same price and choose the same output level c<sub>ij</sub>=C<sub>i</sub>/n<sub>i</sub>. This implies P<sub>i</sub>=p<sub>ij</sub> from equation (1). Substituting equation (6) into (1) and rearranging, yields the equilibrium price index for the industrial commodity, P<sub>c</sub>:

$$P_{c} = \left[2^{-1}\left[\left(\frac{\sigma_{1}}{\sigma_{1}-1}\right)^{1-\varepsilon} + \left(\frac{\sigma_{2}}{\sigma_{2}-1}\right)^{1-\varepsilon}\right]\right]^{\frac{1}{(1-\varepsilon)(1-\beta)}} \left[\theta(1+s)\right]^{\frac{1}{(1-\beta)}}$$
(7).

Given that  $\varepsilon > 1$  and  $0 < \beta < 1$ , this price index, thus the consumer price index  $P = P_c^{\beta}$ , is a decreasing function of the industrial substitution elasticity  $\sigma_i$  and an increasing function of both the constant disutility of labour  $\theta$  and the VAT tax rate *s*, as expected. Substituting the equilibrium equations (3), (6) and (7) into (5), and using the zero-profit condition, yields the equilibrium number of firms in industry i=1, 2 under free entry and exit

$$n_{i} = \frac{C_{i}}{F} \left[\frac{1}{\sigma_{i}-1}\right] = \left[\frac{1}{\sigma_{i}-1}\right] \left\{ \underbrace{\left(\frac{\sigma_{i}}{\sigma_{i}-1}\right)^{-\varepsilon}}_{C_{i}} \left[2^{-1} \left[\left(\frac{\sigma_{i}}{\sigma_{i}-1}\right)^{1-\varepsilon} + \left(\frac{\sigma_{-i}}{\sigma_{-i}-1}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon(1-\beta)-1}{(1-\varepsilon)(1-\beta)}}_{C_{i}}\right] \Phi\left[\theta(1+s)\right] / F\right\} 8$$

 $\varepsilon > 1/(1-\beta)$  from previous restrictions and  $\Phi \equiv [\alpha\beta]\Omega/2$ . Under our assumptions, aggregate wealth  $\Omega$  is constant in general equilibrium, thus  $\Phi$  can be treated as a parameter. More specifically, by using the incomeexpenditure identity wL+ $\Pi$ =C+Z, it turns out that  $\Omega = (Z_0-T)/(1-\alpha)$  in general equilibrium, where  $Z_0$  is the initial aggregate endowment of the non-produced good, T is the lump-sum tax (if any) and  $\alpha$  is the expenditure share on produced goods.<sup>14</sup>

Let us now interpret an increase in  $\sigma_1$  as the diffusion of e-commerce in sector 1. Our hypothesis is that e-commerce, by lowering consumer search costs in industry 1 and by facilitating price comparisons, makes it easier intra-brand substitutability for consumers. By inspecting equations (6), (7) and (8), yields

**Proposition 1. Full VAT compliance with entry and exit. Impact of e-commerce on industrial equilibrium.** Assume that before the introduction of e-commerce,  $\sigma_i = \sigma$  for i=1, 2. Interpret an increase in the elasticity of substitution  $\sigma_1$  as the diffusion of e-commerce in industry 1. It follows that: i) Equilibrium prices

<sup>&</sup>lt;sup>14</sup> Nominal VAT receipts are  $R=\tau\alpha\beta[Z_0-T]/(1-\alpha)$ , irrespective of their redistribution to households (when -T=R). Given that R is independent of the elasticity of substitution parameters, it is unaffected by internet diffusion, see below.

in each industry and the industrial consumption price index both fall. ii) In industry 1, the equilibrium number of firms falls; the equilibrium size of firms rises. iii) In industry 2, the equilibrium number of firms falls; the equilibrium size of firms is unaffected.

Proof. See the Appendix 1.

The intuition for Proposition 1 is as follows. An increase in industry 1's intra-brand substitution elasticity  $\sigma_1$ has pro-competitive effects for the entire industrial sector, but these are larger for industry 1. As long as the price mark-up falls in industry 1, the reduction in industry 1 prices brings about an industrial consumption price-index externality (i.e. Pc falls), which in turn generates a consumption price index externality (i.e. P also falls). As a result, equilibrium marginal labour costs are lowered (i.e. w=0P falls). This general equilibrium effect, operating through the competitive wage, lowers prices in both industries. Moreover, given that the agricultural price is fixed, the relative price of industrial goods  $(P_c/p_x)$  falls, shifting consumer's demand towards the industrial sector. But the increase in the demand for industrial goods is unevenly distributed between industries. As long as the relative price P<sub>1</sub>/ P<sub>2</sub> falls, demand shifts from sector 2 to sector 1. This implies an overall increase in the demand for the composite commodity produced in industry 1. The size of this demand increase and of the corresponding demand reduction for industry 2's composite good,  $dlnC_1/dd\sigma_1 = - dlnC_2/d\sigma_1$ , depends on the value taken by the between-industry substitution elasticity  $\varepsilon$ . If this is sufficiently large, namely if  $\varepsilon > 1/(1-\beta)$ , the between-industry relative price effect dominates the industrial consumption price index externality. Thus, total demand for the composite commodity produced in industry 2 falls. As a result, firms exit sector 2, namely  $dn_2/d\sigma_1 \le 0$ . The equilibrium size of active firms remains unchanged, though, namely  $d(C_2/n_2)/d\sigma_1=0$ . This is because, under symmetry, the reduction in the number of active monopolies is proportional to the reduction in the total demand for the composite good in sector 2. In industry 1, the demand increase causes an increase in the size of the typical active firm,  $d(C_1/n_1)/d\sigma_1 > 0$ , while the reduction in the price mark-up provokes firm's exit, d  $n_1/d\sigma_1 < 0$  (see Blanchard and Giavazzi, 2003, as well).

The results of Proposition 1 are consistent with the main findings of Goldmanis et al. (2010). In their partial-equilibrium search model with heterogeneous households and firms, these authors show that e-commerce diffusion, namely a leftward shift in the consumers' search costs distribution, lowers both average prices and price dispersion and raises industry concentration. Their empirical analysis for three US industries (namely, travel agencies, auto-dealers and bookshops) in 1994-2003 finds that e-commerce diffusion

generally causes a reduction in the number of small establishments, while having no impact or increasing the size of large firms. In this section, we have developed a general equilibrium model of a monopolistically competitive economy showing that the diffusion of e-commerce within an industry can affect market structure in other industries as well through relative price changes and general equilibrium spillover effects. As long as we have a symmetric model, we cannot of course investigate the impact of e-commerce on price dispersion and the distribution of firms' size within each industry. However, Proposition 1 generates predictions that are consistent with the existing empirical evidence of the e-commerce effects on industrial entry-exit patterns.

From equations (6), (7) and (8), the impact of VAT rate changes on market equilibrium is given by

**Proposition 2. Full VAT compliance with entry and exit. VAT tax rates effects on industrial equilibrium**. An increase in the common VAT rate has the following effects: i) Equilibrium prices in each industry and the industrial consumption price index both rise. ii) The equilibrium number of firms falls in each industry; there is no effect on the firms' size. Thus, the absolute size of the industrial sector falls. iii) The real VAT revenue maximising tax rate is  $0 < \tau^* = (1-\beta) < 1$  without tax rebates and independent of internet diffusion; it is equal to  $\tau^{**} < \tau^*$  with tax rebates.

*Proof.* See the Appendix 1.

An increase in the tax rate under full VAT compliance has the effect of lowering the equilibrium number of firms operating in both industries under free entry and exit, while having no effect on the size of active firms. As a result, the tax base, namely the size of the industrial sector, falls. This is because an increase in the tax rate provokes an increase in industrial prices and in the consumer price index. This generates an increase in the marginal costs faced by each active firm, leading eventually to less entry. Moreover, provided that aggregate nominal wealth  $\Omega = (Z_0-T)/(1-\alpha)$  remains unchanged, the demand for the composite industrial good C falls. These results are consistent with those derived by Schröder (2004) in a model with entry and exit under monopolistic competition. Schröder (2004, Lemma 2) shows that ad valorem taxes lower firms' entry, while leaving output per firm unaffected, if compared with equal-yield specific taxes.<sup>15</sup> However, his mechanism differs from ours. As long as his paper considers a single industry and assumes exogenous constant marginal production costs, there are no effects of tax-induced price changes on marginal costs, thus on firms' profitability and entry, which instead occur here.<sup>16</sup> Proposition 1iii) states that the VAT-revenue-

<sup>&</sup>lt;sup>15</sup> See Doi and Futagami (2004: 273). Ad valorem taxes induce entry with Cournot firms, see Anderson et al. (2001).

<sup>&</sup>lt;sup>16</sup> Schröder (2004) takes explicitly into account of redistributed tax revenues in general equilibrium.

maximising tax rate is given by the standard condition equating the elasticity of the tax base with respect to the tax rate to unity. Without tax rebates, this implies that the optimal tax rate is equal to the size of the non-VAT sector,  $\tau^*=1-\beta$ . The intuition is that the general price-index externality of a higher VAT rate, thus the tax elasticity of the real tax base, is lower the higher is the non-VAT sector size. When the tax revenue is lump-sum redistributed to households, the tax base becomes more elastic, thus lowering the optimal tax rate.

Lastly, we consider the effects of e-commerce diffusion in sector 1 on the VAT revenue. It turns out that there are effects on neither the nominal VAT revenue nor the VAT- revenue-maximising tax rate (see Proposition 2iii). However, internet diffusion shifts the tax base from sector 2 to sector 1, as it lowers the relative price of sector 1's composite commodity. Moreover, given that internet diffusion leads to a reduction in consumer prices in general equilibrium, the real VAT revenue increases accordingly. This is shown in

**Proposition 3. Full VAT compliance with entry and exit. Effects of e-commerce on VAT revenue.** Starting from a full symmetric equilibrium with  $\sigma_i = \sigma$  and for i=1, 2, internet diffusion in sector 1 raises real VAT revenues, it has no effects on nominal VAT revenues, and it shifts nominal VAT revenues from sector 2 to sector 1.

Proof. See the Appendix 1.

## 4. VAT evasion and industrial structure with traditional and e-commerce

Consider now VAT evasion in the industrial sector in a standard model à la Allingham and Sandmo (see Andreoni et al., 1998: 823, for a survey, and the references cited in the introduction). Assume that monopolistically competitive firms are asked to report their revenue to the government. If they are honest, they report revenue  $p_{ij}x_{ij}=p_{ij}c_{ij}$  and pay the VAT, but they may choose to under report their revenue instead.<sup>17</sup> Let  $0 \le \delta_{ij} \le 1$  be the fraction of revenue a typical firm j of sector i does not report, such that  $\delta_{ij}=0$  implies honest tax reporting. Firms must engage in some explicit and costly concealment activity if they wish to evade the VAT, otherwise the tax authority is able to detect manifest under reporting by observing at no cost their product and labour markets activities.<sup>18</sup> Following the literature, assume that concealment costs are

<sup>&</sup>lt;sup>17</sup> Under declaring the amount of VAT due constitutes a sizeable part of VAT evasion. For example, in the UK underdeclarations account for about 39% of the total UK VAT losses in 2001-2002, see National Audit Office (2004: 11).

<sup>&</sup>lt;sup>18</sup> For example, the tax authority could observe firms' employment and labour costs directly from payrolls. By knowing firms' technologies, it would be able to infer the true level of sales. Concealment activities by firms avoid this.

convex in the amount of the VAT evaded (see below).<sup>19</sup> Rationales for this assumption include the need of hiring extra and better paid personnel (e.g. better accountants and lawyers, guards) that keep account of, store and move around increasing amounts of physical goods that must be concealed from the tax authorities; the need of paying higher bribes to corrupt audit customs officials; the need of making higher side payments to bank employees when firms are using multiple bank accounts for hiding increasing amounts of unofficial financial transactions. These costs may also include non-pecuniary costs of tax evasion (e.g. jail terms) that are usually in place when evasion is made by altering account books and/or it is above a given threshold set by law. The tax authority can always obtain information on true revenue levels by undertaking costly auditing of firms, which is financed using part of the VAT revenues. Thus, to enforce VAT compliance, the tax authority audits firms with an exogenous probability of detection  $0<\lambda_i<1$ , i=1, 2, corresponding to random auditing at the industry level. Once discovered, tax evaders must pay the evaded VAT plus a penalty. The penalty is common to both industries and proportional to the amount of tax being evaded (see below).

To address VAT *evasion*, we need to modify the interpretation of the production function for industrial firms. Without intermediate variable inputs, the VAT is equivalent to a retail sales tax (RST) in our model. Several authors analysing VAT evasion actually model the VAT as a tax paid by formal sector firms on final sales (see, for example, Emran and Stiglitz, 2005, and Davies and Paz, 2011).<sup>20</sup> However, as noted by Fedeli and Forte (1999) and Keen (2008) among others, the VAT is not equivalent to a RST in the presence of intermediate inputs, unless the chain of crediting and refunding associated with the VAT is unbroken. This point is relevant when analysing VAT evasion under the invoice-credit mechanism. Given that the VAT is charged at each stage of the production process, when a firm evades the VAT on its final sales it is unlikely that it will be able to claim full credit or refund for the VAT paid on its intermediate inputs. For example, in the presence of a simple linear technology, claiming full credit on inputs while under reporting final sales at the same time may be easily perceived as a signal of potential VAT evasion. As Keen (2008: 1893) points out, if a firm cannot claim full credit or refund, the VAT paid on the intermediate inputs purchased from VAT-compliant firms is equivalent to an input tax levied at the VAT rate. Thus, when

<sup>&</sup>lt;sup>19</sup> See for example Virmani (1989), Cremer and Gahavari (1993) and Stöwhase and Traxler (2005). As is well known, if firms maximise expected profits and the punishment function is linear in the amount being evaded, this assumption is needed to find an internal solution to the firm's evasion problem, see Virmani (1989, Proposition 4). Hashimzade et al. (2010) derive endogenously convex concealment costs in a model of VAT frauds.

<sup>&</sup>lt;sup>20</sup> These papers compare tariffs vs. the VAT in developing countries. They model VAT evasion as an "all or nothing" choice. Thus, formal firms pay the VAT (which is modelled as a RST), while informal firms evade it completely.

choosing how much VAT to evade on its final sales, the firm trades off the gain from under reporting its final sales with the loss from not fully recovering the VAT paid on the intermediate inputs bought in the market place, other things being equal.

To incorporate this feature of the VAT system without complicating the model, we assume that, once it has installed F units of overhead capital, a typical monopoly firm produces one unit of output  $x_{ij}$  by using one unit of intermediate input  $L_{ij}$ . Intermediate inputs are purchased in a competitive market at the arm's length price  $P_L=w+\tau P_L$ , where w is the VAT-free price of one input unit and  $\tau$  is the single VAT rate.<sup>21</sup> As a result of the former assumptions, the typical monopoly firm j=1, 2,...n<sub>i</sub> in industry i=1, 2 chooses the output level  $x_{ij}$  and the fraction of unreported revenue,  $\delta_{ij}$ , to solve the problem:

$$x_{ij}, \delta_i \arg \max \pi_{ij}^e s.t.$$
equation (3), where

$$\pi_{ij}^{e} = \underbrace{\left[1 - \lambda_{i}\right]}_{\text{probability of not being detected}} \left\{ \underbrace{\left[\frac{p_{ij}x_{ij} - P_{L}x_{ij} - wF}{\text{gross profits}}\right] - \underbrace{\psi_{ij}x_{ij}(1 - \delta_{ij})}_{\text{VAT paid on VAT credit claimed on reported purchases of intermediate inputs}} \Phi - \underbrace{p_{ij}x_{ij}\delta_{ij}^{2}/2}_{\text{concealment costs}}\right\} + \frac{\lambda_{i}}{\frac{p_{ij}x_{ij}}{probability of being detected}} \left\{ \underbrace{\left[\frac{p_{ij}x_{ij} - P_{L}x_{ij} - wF}{q_{ij}}\right] - \underbrace{\psi_{ij}x_{ij}(1 - \delta_{ij})}_{\text{reported revenue on reported purchases of intermediate inputs}} \Phi - \underbrace{p_{ij}x_{ij}\delta_{ij}^{2}/2}_{\text{concealment costs}} + \underbrace{\lambda_{i}}_{q_{ij}} + \underbrace{\lambda_{i}}_{q_{ij}} \underbrace{\frac{\left[\frac{p_{ij}x_{ij} - P_{L}x_{ij} - wF}{q_{ij}}\right] - \underbrace{\psi_{ij}x_{ij}(1 - \delta_{ij})}_{\text{reported revenue on reported purchases of intermediate inputs}} \Phi - \underbrace{p_{ij}x_{ij}\delta_{ij}^{2}/2}_{q_{ij}} + \underbrace{\psi_{L}x_{ij}\delta_{ij}}_{q_{ij}} \underbrace{\frac{p_{ij}x_{ij}\delta_{ij}}{q_{ij}} + \underbrace{\psi_{L}x_{ij}\delta_{ij}}_{q_{ij}} K\Phi - \underbrace{(\psi - 1)}_{q_{ij}} \underbrace{\frac{p_{ij}x_{ij}\delta_{ij}}{q_{ij}}}_{q_{ij}} \underbrace{\frac{p_{ij}x_{ij}\delta_{ij}}{q_{ij}} + \underbrace{\psi_{L}x_{ij}\delta_{ij}}_{q_{ij}} K\Phi - \underbrace{(\psi - 1)}_{q_{ij}} \underbrace{\frac{p_{ij}x_{ij}\delta_{ij}}{q_{ij}}}_{q_{ij}} \underbrace{\frac{p_{ij}x_{ij}\delta_{ij}}{q_{ij}}}_{q_{ij}} \underbrace{\frac{p_{ij}x_{ij}\delta_{ij}}{q_{ij}} + \underbrace{\psi_{L}x_{ij}\delta_{ij}}_{q_{ij}} K\Phi - \underbrace{(\psi - 1)}_{q_{ij}} \underbrace{\frac{p_{ij}x_{ij}\delta_{ij}}{q_{ij}}}_{q_{ij}} \underbrace{\frac{p_{ij}x_{ij}\delta_{ij}}{q_{ij}}}_{q_{ij}} \underbrace{\frac{p_{ij}x_{ij}\delta_{ij}}{q_{ij}} \underbrace{\frac{p_{ij}x_{ij}\delta_{ij}}{q_{ij}}}_{q_{ij}} \underbrace{\frac{p_{ij}x_{ij}\delta_{ij}}{q_{ij}}}_{q_{ij}$$

Expected nominal profits can be written as

$$\pi_{ij}^{e} = \left[ \left( p_{ij} - w \right) x_{ij} - wF \right] - p_{ij} x_{ij} \left[ \tau \left( 1 - \delta_i + \lambda_i \psi \delta_{ij} \right) + \delta_{ij}^2 / 2 \right] - \tau P_L x_{ij} \delta_{ij} (1 - \lambda K) \Phi, \text{ with } \Phi = \{0, 1\} \text{ and } K = \{0, 1\}$$

$$\tag{9}$$

Equation (9) allows us to consider formally both the VAT-RST-equivalent (namely, the VAT in the absence of intermediate inputs) for  $\Phi$ =0, and the VAT, for  $\Phi$ =1. Three further features of the above problem are worthy noting. Firstly, we assume that concealment costs are quadratic in the proportion of revenue being evaded,  $\delta_{ij}$ , implying that the marginal concealment cost is increasing and convex in  $\delta_{ij}$ . Second, we assume a

<sup>&</sup>lt;sup>21</sup> This amounts at assuming that workers behave like individual competitive firms selling one unit of labour services to industrial firms at a VAT inclusive price  $P_L$ . Industrial firms transform these services, without additional costs, into one unit of output, provided that they have F units of capital, see also footnote 11 above.

linear penalty scheme, with  $\psi$ -1>0 representing the penalty rate, which is the same for the two industries. Lastly, the dummy variable K={0, 1} allows us to consider two different punishment schemes. Under the first scheme, K=1, the "non-adversarial" tax authority fully credits the VAT paid on intermediate inputs before applying the fine. Under the second scheme, K=0, the "adversarial" tax authority does not credit the VAT paid on unreported intermediate inputs, implying that the firm faces the input tax fully.

From equation (9), it is clear that a firm's evasion decision is independent of its decision on the profit-maximising level of sales only for  $\Phi$ =0, when the VAT-RST-equivalent applies. This "separability" result is standard in the literature of firm evasion of indirect taxes (see, for example, Sandmo, 2005) and here stems from the joint assumptions of a fixed probability of auditing and of effective expected VAT payments being proportional to the firm's true revenue level. However, as soon an invoice-credit VAT scheme is considered, the separability result disappears. This is because the fraction of revenue evaded now depends on the producer's real price, thus on the actual optimal level of firm sales.

For the VAT-RST-equivalent case, the solution to the problem is:

$$\delta_{ij}^{*} = \delta_{i}^{*} = \tau [1 - \lambda_{i} \psi] \quad \text{if } \lambda_{i} \psi < 1$$

$$= 0 \quad \text{otherwise}$$

$$p_{ij} = \left(\frac{\sigma_{i}}{\sigma_{i} - 1}\right) \theta \left(1 + s_{i}^{\text{RST}}\right), \quad i = 1, 2; j = 1, 2, \dots n_{i} \qquad (10),$$

$$1 + s_{i}^{\text{RST}} \equiv \frac{1}{1 - t_{i} e^{*}} < \frac{1}{1 - \tau}$$

$$0 \le t_{i}^{e^{*}} \equiv \tau \left[1 - \delta^{*}_{i} + \lambda_{i} \psi \delta^{*}_{i}\right] + (\delta_{i}^{*})^{2} / 2 = \tau \left[1 - \tau (1 - \lambda_{i} \psi)^{2} / 2\right] \le \tau$$

where we impose the labour market clearing condition w=0P. Clearly, because the detection probability is sector-specific, all firms within a given industry j=1, 2 choose the same level of evasion  $\delta_{ij}^* = \delta_i^*$ . As one would expect, equation (10) shows that the optimal fraction of undeclared revenue  $\delta_i^*$  is an increasing function of the statutory tax rate  $\tau$ , and a decreasing function of both the detection probability  $0 < \lambda_i < 1$  and the harshness of the penalty  $\psi > 1$ . As pointed out by Cremer and Gahavari (1993: 264),  $\lambda_i \psi < 1$  is a necessary condition for an interior solution to the tax evasion problem. If this condition does not hold, the firm reports honestly its revenue by choosing  $\delta_i^*=0$ . Equation (10) also shows that  $\delta_i^*<1$ , implying that the firm never submerges. Three further things are worthy noting. Firstly, finding an internal solution to the tax evasion problem implies that the equilibrium effective expected tax rate  $t_i^{e*}$  is less than the statutory rate, namely  $t_i^{e*} < \tau$ . If this were not the case, the firm would find it optimal to report truthfully its revenue by choosing  $\delta_i$ =0 and pay the required tax bill. Second, when the probability of detection  $\lambda_i$  is exogenous, internet diffusion within an industry (i.e. a reduction in  $\sigma_i$ ) has no effect on the firm's decision to evade in both industries. Internet diffusion affects market equilibrium only through its effects on the price mark-up, thus on equilibrium prices, similarly to the full compliance case (see section 3 above). Lastly, the optimal producer's real price under evasion of the VAT-RST-equivalent,  $p_{ij}/P$ , is lower than under full VAT compliance, as long as  $1+s_i^{RST} < (1+s) \equiv 1/(1-\tau)$ . We shall return on this point below. These results are summarised in:

Lemma 1. VAT Evasion with VAT-RST-equivalent. Assume an exogenous detection probability  $\lambda_i$  in industry i=1,2. If the VAT is formally equivalent to a retail sales tax (RST): i) *Honest reporting*, the firm reports honestly its revenue if the probability of detection is  $\lambda_i \ge 1/\psi$ . ii) *Optimal degree of tax evasion*, if  $\lambda_i < 1/\psi$ , the firm chooses to under-report a fraction  $0 < \delta_i^* < 1$  of its revenue. This fraction is increasing in the statutory tax rate  $\tau$ , decreasing in the probability of detection  $\lambda_i$  and in the penalty rate  $\psi > 1$ , and independent of the substitution elasticity  $\sigma_i$ .

Turning to the VAT, the analysis becomes slightly more complicated, as long as, when  $\Phi$ =1, the FOC for tax evasion is quadratic in  $\delta_{ij}$ . However, we can find sufficient conditions leading to an internal solution. As shown in the Appendix 2, this solution is

$$\delta_{ij}^{*} = \delta_{i}^{*} = \frac{-B + \sqrt{B^{2} - 4AC}}{2A} \equiv \delta_{i}^{*} (\sigma_{1}, \lambda_{1}, \psi, \tau) < 1$$

$$A \equiv \tau (1 - \lambda_{i}K)[\sigma + 1]/2 > 0$$

$$B \equiv \sigma (1 - \tau) - \tau^{2} (1 - \lambda_{i}K)(1 - \lambda_{i}\psi) > 0$$

$$C \equiv \tau (1 - \tau)[\sigma_{i}\lambda_{i}(\psi - K) - (1 - \lambda_{i}K)] < 0, \quad K = \{0, 1\}$$

$$\text{if } \lambda_{i} < \frac{1}{\sigma_{i}(\psi - 1) + 1} \quad \text{and } \tau \leq 0.5;$$

$$(11.1)$$

$$\delta_{ii}^{*} = \delta_{i}^{*} = 0$$

otherwise

$$p_{ij} = \left(\frac{\sigma_i}{\sigma_i - 1}\right) \theta \mathbb{P}(1 + s_i^{VAT})$$

$$(1 + s_i^{VAT}) = \frac{1}{(1 - \tau)} \left[\frac{1 - \tau + \tau \delta_i^* (1 - \lambda_i K)}{1 - t_i^e *}\right] > \frac{1}{1 - \tau}, \quad i = 1, 2; j = 1, 2, ... n_i$$

$$0 \le t_i^e^* = \tau \left[1 - \delta^*_i + \lambda_i \psi \delta^*_i\right] + (\delta_i^*)^2 / 2 \le \tau$$
(11.2),

where we have made use of  $P_L=w/(1-\tau)$  and of the market clearing condition w= $\theta P$ . By inspecting the optimal degree of tax evasion, it turns out that, providing the detection probability and the VAT rate are

sufficiently low (see the restrictions reported in equation 11.1),<sup>22</sup> the firm optimally under-reports a fraction of its revenue, without submerging. The restriction imposed on the detection probability,  $\lambda_i < 1/[\sigma_i (\psi-1)+1]$ , is less likely to be satisfied the higher is the industry elasticity of substitution  $\sigma_i$ . This implies that, for example, if  $\sigma_i=4$  and  $\psi=2$  (100% fine), a detection probability equal to  $\lambda_i \ge 0.2$  would induce honest reporting, whereas if  $\sigma_i=5$  and  $\psi=2$ , it is enough that  $\lambda_i \ge 0.167$  for this to be the case. Note that, under a RST, the corresponding cut-off value is less restrictive and equal to  $\lambda_i \ge 0.5$ . This means that, under a VAT scheme internet diffusion in sector 1 makes it more likely that a firm in this sector reports honestly its revenue, other things being equal. Moreover, it can be shown that, for  $0<\delta_1 *<1$ , internet diffusion lowers the optimal degree of tax evasion, namely  $\partial \delta_1 * / \partial \sigma_1 < 0$ , other things being equal (see the Appendix 2). The intuition for this result is as follows. Under a VAT scheme, the marginal cost of evasion is higher than under a RST as long as, by underreporting its final sales, the firm is unable to recover fully the VAT paid on intermediate inputs. This additional cost of VAT evasion is proportional to the producer's real price of intermediate inputs,  $P_1/p_{ij}$ . In turn, from the optimal marginal pricing condition, and other things being equal, this real price is an increasing function of substitution elasticity  $\sigma_1$ . Thus, internet diffusion, by raising the producer's real price of inputs, lowers the firm's incentives to evade the VAT on final sales.

To gain a further understanding of the determinants of firm evasion under the VAT scheme, Tables 2.1 and 2.2 below report numerical examples showing  $\delta_i^*$  as a function of the relevant parameters. For the industry 1 substitution elasticity, we set  $\sigma_i$ ={2.4, 3.5}, representing respectively the lowest median elasticity (for the UK) and the median of the median elasticity of substitution (for Denmark) in import- competing industries as being estimated by Broda et al. (2004, Table 4) for the EU-17. For the statutory tax rate, we set  $\tau$ ={0.2, 0.25), where the first number represents the median standard VAT tax rate for the EU-25 in 2006, whereas the second number is the standard VAT rate for Denmark and Sweden. For the penalty rate, we set  $\psi$ ={1.3, 2}, where the first number represents a 30% fine on VAT evasion, which may signal a "cooperative" agreement between tax evaders and the tax administration,<sup>23</sup> while the second number represents a 100% fine. Finally, for the probability of detection, we set  $\lambda_i$ ={0.05, 0.1, 0.15, 0.20}.

<sup>&</sup>lt;sup>22</sup> They are sufficient conditions for an internal solution to the firm's problem and for the SOCS to be satisfied, respectively, see the Appendix 2 for details.

<sup>&</sup>lt;sup>23</sup> This is indeed the case in the UK, where since 2002 "new civil procedures" have been introduced for evaders who fully cooperate with customs in case of under-reporting. These procedures imply that "any penalty imposed will not normally exceed 20 per cent of the tax evaded", National Audit Office (2004: 17).

	λ <sub>1</sub> =0.05	λ <sub>1</sub> =0.05	λ <sub>1</sub> =0.1	λ <sub>1</sub> =0.1	λ <sub>1</sub> =0.15	λ <sub>1</sub> =0.15	λ <sub>1</sub> =0.2	λ <sub>1</sub> =0.2
	ψ=1.3	ψ=2	ψ=1.3	ψ=2	ψ=1.3	ψ=2	ψ=1.3	ψ=2
σ <sub>1</sub> =2.4	7.66	6.96	6.94	5.55	6.21	4.11	5.49	2.68
σ1=3.5	5.15	4.45	4.5	3.16	3.7	1.75	3.39	0.57
Evasion under VAT-RST- equivalent	18.7	18	17.4	16	16.1	14	14.8	12

Table 2.1 Optimal fraction of un-reported revenue  $\delta_1^*$  when  $\tau=0.2$  and K=1 (per cent values)

*Note*:  $\sigma_1$  = elasticity of substitution in industry 1,  $\lambda_1$  = detection probability in industry 1,  $\psi$  = penalty rate;  $\tau$ =statutory VAT rate; K=1 "non-adversarial" tax authority; Evasion under VAT-RST equivalent= $\tau(1 - \lambda_i \psi)$ .

	λ <sub>1</sub> =0.05	λ <sub>1</sub> =0.05	λ <sub>1</sub> =0.1	λ <sub>1</sub> =0.1	λ <sub>1</sub> =0.15	λ <sub>1</sub> =0.15	λ <sub>1</sub> =0.2	λ <sub>1</sub> =0.2
	ψ=1.3	ψ=2	ψ=1.3	ψ=2	ψ=1.3	ψ=2	ψ=1.3	ψ=2
σ <sub>1</sub> =2.4	9.6	8.73	8.7	6.95	7.79	5.16	6.9	3.37
σ1=3.5	6.46	5.59	5.72	3.96	4.98	2.35	4.24	0.72
Evasion under VAT- RST- equivalent	23.38	22.5	21.75	20	20.13	17.5	18.5	15

Table 2.2 Optimal fraction of un-reported revenue  $\delta_i^*$  when  $\tau=0.25$  and K=1 (per cent values)

*Note*:  $\sigma_1$  = elasticity of substitution in industry 1,  $\lambda_1$  = detection probability in industry 1,  $\psi$  = penalty rate;  $\tau$ =statutory VAT rate; K=1 "non-adversarial" tax authority; Evasion under VAT-RST equivalent=  $\tau(1 - \lambda_i \psi)$ .

The numerical results presented in Table 2 show that the share of un-reported revenue  $\delta_i^*$  is a decreasing function of the elasticity of substitution  $\sigma_i$ , of the penalty rate  $\psi$  and of the probability of detection  $\lambda_i$ , and an increasing function of the statutory tax rate  $\tau$ , other things being equal. Moreover, it turns out that tax evasion will be lower when firms are facing an "adversarial" tax authority, namely when K=0, other things being equal (not shown in Table 2, see the Appendix 2). Of course, the latter effect depends on the fact that the firm perceives a higher expected marginal cost of tax evasion when K=0, since it is unable to fully recover the VAT paid on intermediate inputs once discovered evading. Tables 2.1 and 2.2 also show that, other things being equal, tax evasion will be lower under the VAT than under the VAT-RST-equivalent.

Turning to the typical firm's optimal pricing rule, comparison of equations (11.2) and (6) shows that the producer's real price  $p_{ij}/P$  will be *higher* under VAT evasion than under VAT compliance, other things being equal, as long as  $(1+s_i^{VAT})>(1+s)\equiv 1/(1-\tau)$ . Equation (11) also shows that internet diffusion in sector 1,

namely an increase in the substitution elasticity  $\sigma_1$ , induces sector 1's firms to lower their optimal price, namely  $\partial p_{i1}/\partial \sigma_1 < 0$ , for given consumer price index P. The mechanism is two-fold. Firstly, there is a standard mark-up effect, that is also present under full VAT compliance and under evasion with the VAT-RSTequivalent: a bigger elasticity of substitution lowers the price mark-up. Second, there is a tax evasion effect, which is specific to the VAT evasion case: a bigger elasticity of substitution lowers the fraction of unreported revenue,  $\partial \delta_1/\partial \sigma_1 < 0$ , and a lower fraction of unreported revenue induces firms to set lower prices,  $\partial p_{i1}/\partial \delta_1 > 0$  (see the Appendix 2). But price moderation at the firm level gives rise to a bigger price externality in general equilibrium than in the full compliance case (see below). This analysis is summarised in

**Lemma 2. VAT Evasion**. Assume an exogenous detection probability  $\lambda_i$  in industry i=1,2. If the VAT is not formally equivalent to a RST: i) *Honest reporting*, the firm reports honestly its revenue if the probability of detection is  $\lambda_i \ge 1/[\sigma_i (\psi-1)+1]$ . ii) *Optimal degree of tax evasion*: If  $\lambda_i < 1/[\sigma_i (\psi-1)+1]$  and the statutory VAT rate is  $\tau < 0.5$ , the firm chooses to under-report a fraction  $0 < \delta_i^* < 1$  of its revenue. This fraction is increasing in the statutory tax rate  $\tau$ , and decreasing in the probability of detection  $\lambda_i$ , in the penalty rate  $\psi > 1$ , and in the substitution elasticity  $\sigma_i$ ; it is lower when K=0 implying an "adversarial" tax authority.

Let us now interpret  $\delta^*$  as the optimal VAT gap share, namely as the difference between the VAT revenue under full compliance and the actual VAT receipts under evasion, divided by the VAT revenue under full compliance. Lemma 1 and 2 imply

**Proposition 4. VAT gap share and internet diffusion**. Assume an exogenous detection probability  $\lambda_i$  in industry i=1,2. Internet diffusion in sector 1, namely an increase in the intra-brand substitution elasticity  $\sigma_1$  i) has no effect on the VAT gap share, if the VAT is formally equivalent to a RST; ii) if the VAT is not formally equivalent to a RST, it makes it more likely that sector 1's firms report honestly their revenue. If firms choose under-reporting, internet diffusion lowers the VAT gap share, by reducing it in sector 1, while leaving it unaffected in sector 2.

# 4.1 General equilibrium solution

We now consider the general equilibrium solution of the model by also imposing in each industry a symmetric equilibrium under free entry and exit. Firstly, note that in symmetric equilibrium  $p_{ij}=P_i$ , thus  $x_{ij}=C_i/n_i$ . As a result, the average firm size in both industries does depend neither on tax evasion nor on whether we consider the RST or the VAT. Actually, imposing the zero profit condition in equation (9), using  $x_{ij}=C_i/n_i$  and either equation (10) for the RST or equation (11) for the VAT, equation (8) still holds. As a consequence, the average firm size depends only on overhead capital and the industry-specific substitution elasticity  $C_i/n_i=F(\sigma_i-1)$ , as one would expect in this framework. However, tax evasion will influence the equilibrium number of firms, as long as, by affecting equilibrium prices and quantities, it will impact on the

total demand for the composite industrial commodity  $C_i$ . Because tax evasion is sector-specific, equilibrium equations become slightly more complicated than for the full VAT compliance case. The equilibrium price for the typical firm in sector i=1, 2,  $p_i^{EV}$ , the industrial price index  $P_c^{EV}$ , and the number of firms  $n_i^{EV}$ , where the superscript EV stands for "evasion" and k=VAT, RST, become:

$$p_{ij}^{EV,k} = p_i^{EV,k} = \left(\frac{\sigma_i}{\sigma_i - 1}\right) \theta \left(P_c^{EV,k}\right)^{\beta} (1 + s_i^{e,k}), \quad i = 1, 2; j = 1, 2, ... n_i$$
(6 bis)

$$P_{\mathcal{C}}^{EV,k} = \left[2^{-1}\left[\left(\frac{\sigma_{1}}{\sigma_{1}-1}(1+s_{1}^{e,k})\right)^{1-\varepsilon} + \left(\frac{\sigma_{2}}{\sigma_{2}-1}(1+s_{2}^{e,k})\right)^{1-\varepsilon}\right]\right]^{\frac{1}{(1-\varepsilon)(1-\beta)}} \left[\theta\right] \frac{1}{(1-\beta)}$$
(7bis)

1

$$n_{i}^{EV, k} = \frac{C_{i}^{EV, k}}{F} \left[ \frac{1}{\sigma_{i} - 1} \right]$$
(8bis)
$$= FV k = \left( \left[ \frac{\sigma_{i}}{\sigma_{i}} - \frac{\sigma_{i}}{\sigma_{i}} - \frac{\sigma_{i}}{\sigma_{i}} \right]^{-\varepsilon} \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right] = \left[ \frac{\varepsilon(1 - \beta) - 1}{(1 - \varepsilon)(1 - \beta)} \right]$$

$$C_{i}^{EV, k} = \left(\frac{\sigma_{i}}{\sigma_{i}-1}(1+s_{i}^{e,k})\right)^{-\varepsilon} \left[2^{-1}\left[\left(\frac{\sigma_{i}}{\sigma_{i}-1}(1+s_{i}^{e,k})\right)^{1-\varepsilon} + \left(\frac{\sigma_{j}}{\sigma_{j}-1}(1+s_{j}^{e,k})\right)^{1-\varepsilon}\right]^{(1-\varepsilon)(1-\beta)}\right] \Phi[\theta]\frac{1}{\beta-1}$$

 $s_i^{e, RST}$ ,  $s_i^{e, RST}$ , by comparison of equations (6) and (10), and  $s_i^{e, VAT}$ >s, by comparison of equations (6) and (11), it turns out that  $s_i^{e, VAT}$ >s>  $s_i^{e, RST}$ . From these inequalities, using equations (6) to (8) and (6bis) to (8bis), yields

Lemma 3. VAT evasion. General equilibrium under free entry and exit. Assume an exogenous detection probability  $\lambda_i$  and consider a symmetric equilibrium within each industry i= 1, 2. If comparing full VAT compliance with evasion under the VAT-RST-equivalent and with evasion under the VAT it turns out that: i) *The average firm size* is equal to  $C_i/n_i=F(\sigma_i-1)$ , irrespective of the regime. ii) *Equilibrium prices* are higher under VAT evasion than full VAT compliance than VAT-RST-equivalent evasion, namely  $P^{EV,VAT}>P>P^{EV,RST}$ . iii) *Consumption levels* in industry i are lower under VAT evasion than full VAT compliance than VAT-RST-equivalent evasion than full VAT compliance than VAT-RST-equivalent evasion than full VAT evasion, namely  $C_i^{EV, VAT} < C_i < C_i^{EV, RST}$ . iv) *The equilibrium number of firms* in each industry is lower under VAT evasion than full VAT compliance than VAT-RST-equivalent evasion, namely  $n_i^{EV,VAT} < n_i < n_i^{EV,RST}$ .

The ranking of regimes show that, on the one hand, under VAT evasion equilibrium prices are the largest, while consumption levels and sales and the number of active firms are the lowest in both industries i=1, 2. The intuition is that, under VAT evasion, firms can shift on consumer prices the implicit input tax resulting from their inability to recover fully the VAT paid on their purchases of intermediate inputs. In general equilibrium and other things being equal, this implies the highest competitive wage. Because firms face now higher marginal costs, the number of active firms that are able can operate in a free entry-exit equilibrium is the lowest. In symmetric equilibrium, this means the lowest consumption levels/sales of industrial goods.

However, equilibrium prices are the lowest, consumption levels/sales and the number of active firms are the largest under the VAT-RST-equivalent and evasion. The intuition for this result is as follows. As long as equilibrium prices are proportional to the effective tax rate and this is lower than the standard tax rate, with VAT evasion (and providing  $\lambda_{i}(.)\psi<1$ ) they are also lower, implying higher consumption levels in both sectors if compared with the full VAT compliance case. As a consequence, more firms are active in symmetric equilibrium under free entry and exit and evasion. Note that the larger number of firms observed in equilibrium in this case does not depend on efficiency considerations. Because the equilibrium is symmetric, all firms are the same and it is not the case that evasion allows less efficient firms, by selling goods at lower prices, to compete vis-à-vis more efficient VAT paying firms.

The presence of VAT evasion affects the impact of e-commerce on the size and sectoral distribution of VAT revenue. These effects are summarised in

**Proposition 5. Effects of e-commerce on VAT revenue under free entry and exit and VAT evasion.** Starting from a full symmetric equilibrium with  $\sigma_i = \sigma$  and  $\lambda_i = \lambda$  for i=1, 2, for a given tax rate, e-commerce diffusion in sector 1 raises both real and nominal VAT revenues and shifts nominal VAT revenues from sector 2 to sector 1.

*Proof*: See the Appendix 2.

The impact on the consumer price index of internet diffusion now comprises two effects. The first effect, operating with full VAT compliance as well,<sup>24</sup> is the standard reduction in the price mark-up, provoking reductions in equilibrium prices and wages, thus in the equilibrium price index, due to enhanced competition. The second effect is a compliance effect in sector 1 operating exclusively with VAT evasion (see Proposition 4). The compliance effect reinforces price moderation, as long as  $\partial s_1^* / \partial \sigma_1 < 0$  at a symmetric equilibrium. Thus, internet diffusion provokes a bigger proportionate reduction in the consumer price index with VAT  $\partial \ln P^{EV,VAT} / \partial \sigma_1 = \beta \partial \ln P_c^{EV,VAT} / \partial \sigma_1 < 0$ namely evasion full VAT compliance. with than  $|\partial \ln P^{EV,VAT}/\partial \sigma_1| > |\partial \ln P/\partial \sigma_1|$ . As a result, internet diffusion raises both nominal and real VAT revenues. Nominal revenue rises because the increase in sector 1's tax base outweighs the reduction in sector 2's.

The analysis made so far assumes that the VAT rate is given. To be sure, this would be the case if the tax authorities set it equal to the real revenue maximising tax rate under full VAT compliance (see Proposition 2iii above). However, the real revenue maximising tax rate under evasion is likely to be

<sup>&</sup>lt;sup>24</sup> With an exogenous detection probability, the effects of internet diffusion are identical under full VAT compliance and under VAT-RST-equivalent evasion. Thus, we compare VAT evasion only with the former regime.

different. In order to make a comparison between compliance and evasion, we consider the real VAT revenue under evasion in a full symmetric equilibrium with entry and exit, namely  $R^{EV, k}/P^{EV,k}=2R_i^{EV,k}/P^{EV,k}=2\Phi\tau(1-\delta^*)/\{(\sigma/\sigma-1)(1-s^{e^*,k})^{-1}\theta\}^{\beta/(1-\beta)}$ , with k=RST, VAT. Maximisation with respect to the tax rate  $\tau$  yields:  $\partial \ln (R^{EV}/P^{EV})/\partial \tau = 1/\tau -[(\partial \delta^*/\partial \tau) /(1-\delta^*)+ (\beta/1-\beta)(\partial s^{e^*k}/\partial \tau)/(1-s^{e^*k})]=0$ . The first RHS term represents the marginal benefit of a higher tax rate. This is the same term we would obtain under full VAT compliance. The second RHS term in square brackets is the marginal cost of a higher tax rate, namely the reduction in the tax base, which depends on two further terms. The first term is the reduction in the nominal tax base provoked by higher VAT under-reporting, as long as  $\partial \delta^*/\partial \tau > 0$  from Lemma 1 and 2. The second term is the reduction in the real tax base due the tax-induced increase in the consumer price index. This term is regime-specific and is weighted by  $\beta$ -the relative size of the industrial sector- capturing the extent of the tax externality on the consumer price index. To get sharper results, we evaluate the marginal cost under evasion in the neighbourhood of the full compliance equilibrium with no tax rebates,  $\delta^*=0$  and  $t^{e*}=\tau^*=(1-\beta)$ , yielding

**Proposition 6 VAT evasion. Real revenue maximising tax rate and internet diffusion.** Starting from a full symmetric equilibrium with  $\sigma_1 = \sigma_2 = \sigma$ ,  $\lambda_1 = \lambda_2 = \lambda$ , in the neighbourhood of the full compliance equilibrium with  $\delta^*=0$ ,  $t^{e*}=\tau^*=(1-\beta)$  and  $\beta>1/2$ : i) the real revenue maximising tax rate is:  $\tau^{EV, RST} < \tau^{EV, VAT} < \tau^*=(1-\beta)$ ; ii) under VAT evasion, internet diffusion in sector 1 raises the optimal tax rate towards its full VAT compliance level.

Proof: See the Appendix 2.

Proposition 6 shows that the real revenue maximising tax rate is lower with evasion than full compliance<sup>25</sup> and lower with VAT-RST-equivalent taxation than with the VAT. The reason is that the price externality will be larger in the former than latter case, other things being equal. Note the condition  $\beta$ >1/2: in the neighbourhood of the full compliance equilibrium it is needed to insure that  $\partial \delta^*/\partial \tau$ >0 under VAT evasion. To illustrate Proposition 6, Table 3 below presents numerical examples. For given detection probability  $\lambda$ =0.1, penalty rate,  $\psi$ =2, and assuming a "non-adversarial" tax authority under VAT evasion, K=1, Table 3 sets  $\sigma = \{5.5, 6\}$  and  $(1 - \beta) = \{0.2, 0.25\}$ , such that  $\sigma$ >1/(1- $\beta$ ). Table 3 shows that internet diffusion raises the optimal VAT rate under evasion, as it lowers the marginal impact of the tax rate on the optimal degree of VAT evasion,  $\partial \delta^{*2}/\partial \tau \partial \sigma$ <0, thus lowering the government's marginal cost of increasing the tax rate.

<sup>&</sup>lt;sup>25</sup> This result is consistent with Agha and Haughton (1996: 306, Table 4) findings for 17 OECD countries in 1987 that revenue maximising VAT rates are lower with evasion.

VAT-RST equivalent evasion	$\tau^{*EV,RST}=0.172$	$\tau^{*EV, RST} = 0.208$
$\tau^{*^{EV, RST}} = (1-\beta)/[1+(1-\beta)(1-\lambda\psi)].$		
VAT evasion: σ= 5.5	0.198	0.247
VAT evasion: σ= 6	0.199	0.248
Full VAT compliance:	$\tau^{*}=0.2$	τ* =0.25
<b>τ*=(1-β)</b>		

### Table 3. Real revenue maximising tax rates with evasion

Note: Each cell from the second to the fourth row represents the optimal tax rates under VAT evasion,

 $τ^{*EV, VAT} = (1-β)/{[1+(1-β)(1-2β)[σλ(ψ-1)-(1-λ)]/σβ} > τ^{*EV} = (1-β)/[1+(1-β)(1-λψ)].$ 

Tax rates are computed in the neighbourhood of the full VAT compliance equilibrium with  $\delta^{*=0}$ ,  $t^{e^{*}} = \tau = 1 - \beta < 1$ ;  $\beta > 1/2$ .

### 5. Are internet diffusion and the VAT gap share negatively correlated in the EU- 24?

The analysis made so far suggests that internet diffusion, provided that it raises the intra-brand substitution elasticity, should lower the VAT gap share (see Proposition 4). This section tries to assess whether it exists a broad negative correlation between the variables of interest in the European Union (EU-24). Whether or not internet diffusion is associated in practice with more market competition is a controversial issue. Although the general consensus is that internet markets tend to increase the price sensitivity among consumers, the empirical evidence is not univocal (see for example Ellison and Ellison, 2009). Moreover, the literature is mainly focused on US data. Despite the fact that little is known regarding the price elasticity of demand for internet markets in the European Union, our maintained hypothesis is that increasing internet diffusion should be associated with more e-commerce and with a higher price sensitivity of demand. On this basis, Figure 1 below presents a simple cross-sectional scatter plot of the correlation between internet diffusion and the VAT gap share in the EU-24 (EU-25 with the exclusion of Cyprus). One major drawback of this analysis is that it uses macroeconomic data, as long as we are unable to identify the correlation of interest at the industrial level. More specifically, we use the 2006 country-level VAT gap share as being computed by Reckon LLP (2009: 9, Table 3) in a study written for the European Commission. The VAT gap share is defined as the difference between the theoretical amounts of VAT that should be paid, based on national accounts data, and the actual VAT receipts divided by the theoretical VAT liability.<sup>26</sup> Internet diffusion is proxied by the number of estimated internet users per 100 inhabitants, as being computed by ITU (a United

<sup>&</sup>lt;sup>26</sup> Although this "top-down" approach to measuring VAT compliance suffers from several problems related to the accuracy of national accounts data, see Keen and Smith (2006) for a discussion, it is widely used in empirical works, see for example Agha and Houghton (1996), Nam et al. (2003) and Christie and Holzner (2006).

Nation agency). To weaken potential endogeneity problems, internet diffusion is measured using average yearly data for 2000-2004. It turns out that the correlation between the variables of interest is negative (-0.436), as we would expect. Figure 1 also plots the bivariate OLS regression line, together with 95% confidence intervals. It turns out that the slope coefficient is negative and statistically significant at the 5% level. On average, EU countries with a bigger share of internet users are more likely to have a lower VAT gap share. <sup>27</sup> Of course, one should be cautious in interpreting these results as evidence in favour of a casual relationship running from internet diffusion, thus e-commerce diffusion, to the VAT gap. However, Figure 1 suggests us that this negative correlation should be further investigated.<sup>28</sup>

To make a further step, Table 4 below presents Panel data estimates of the correlation of interest exploiting the country by time variability of the data over the period 2000-2006. On the basis of Hausman



Figure 1. VAT GAP share 2006 and internet users 2000-2004 in the EU-24

**Note:** VAT gap share is the difference between the theoretical amounts of VAT that should be paid, based on national accounts data, and the actual VAT receipts divided by the theoretical VAT liability. Source Reckon LLP (2009). Internet Users: Average number of internet users per 100 inhabitants for 2000-2004. Source: ITU (2011).

<sup>&</sup>lt;sup>27</sup> The bivariate OLS regression is as follows: *VAT GAP<sub>i</sub>* = **20.04** (4.81)-**0.247** (0.10) *Average Internet Users 2000-2004*<sub>i</sub>. Heteroskedasticity-consistent standard errors in brackets, N=24, F (1, 22)= 5.85 (p-value 0.02), R<sup>2</sup> = 0.19.

<sup>&</sup>lt;sup>28</sup> We also experimented with the proportion of respondents answering YES to the Eurobarometer question: "Please tell me if you have purchased any goods or services in the past 12 months by distance in (Your country) or elsewhere in any of the following ways...? Via the Internet (website, email, etc.)". These data are only available for 2006 and 2008-2010. The correlation between the VAT gap share and e-commerce diffusion in 2006 remains negative -0.5072. The results of the bivariate OLS regression confirms our analysis: *VAT GAP 2006*<sub>i</sub> = **18.88** (3.99)-**0.298** (0.114) E-commerce share  $2006_i$ . Heteroskedasticity-consistent standard errors in brackets, N=24, F (1, 22)= 6.87 (p-value 0.016), R<sup>2</sup>=0.2572.

test statistics, showing no correlation between random effects and the explanatory variables,<sup>29</sup> Table 4 shows the results of random effects estimates. The negative correlation between the one-year lagged value of internet diffusion and the VAT gap share remains robust, once introducing control variables widely used in the literature. These variables are the standard VAT rate (a proxy for the VAT burden), the World Bank index of corruption control (a proxy for political-economy determinants of the detection probability), the log of real GDP per capita (a proxy for the level of economic development), and the log population (a proxy for country size). Although our main concern is to show the existence of a negative correlation between internet diffusion and the VAT gap share, we briefly comment on the effects on the latter of the control variables. The standard VAT rate enters with a negative sign and is in general statistically significant, the opposite of what our model predicts. A possible interpretation for this result is that the standard VAT rate is a bad empirical proxy for the VAT burden. An alternative interpretation follows Friedman et al.'s (2000: 475; 483). These authors, using a sample of 69 countries in the 1990s, find that higher tax rates- including a higher VAT rate- are associated with a lower share of the unofficial economy. They claim that higher tax rates have opposite effects on firms' incentives to go underground. On the one hand, they increase the

Independent Variables	(1)	(2)	(3)	(4)
Internet diffusion	-0.184***	-0.193***	-0.1531**	-0.138**
-1	(0.064)	(0.062)	(0.0602)	(0.066)
Standard VAT Rates		-0.388***	-0.361**	-0.420***
		(0.148)	(0.173)	(0.150)
Corruption Control			-3.669**	-1.687
_			(1.457)	(2.231)
Log Population				0.927
				(0.797)
Log Real GDP				-6.301*
per capita				(3.838)
CONSTANT	18.665***	26.483***	29.715***	82.898***
	(2.121)	(3.885)	(4.725)	(39.860)
Time dummies	YES	YES	YES	YES
Observations	144	144	144	144
Number of countries	24	24	24	24
Overall R <sup>2</sup>	0.304	0.270	0.422	0.480
Hausman test statistics $\chi^2$	p-value=0.155	p-value=0.297	p-value=0.05	p-value=0.1391

# Table 4 Random effects estimates Dependent variable: Vat gap share

*Note*: Robust standard errors (clustered by country) in brackets. \* p<0.10, \*\* p<0.05, \*\*\* p<0.01, where p is the marginal probability level. Time dummy omitted 2001. The Hausman tests for fixed effects vs. random effects are run without clustering. For variable definitions and sources, see Appendix 4.

<sup>&</sup>lt;sup>29</sup> But marginally for column 3 in Table 4. Fixed effects estimates, allowing for country specific intercepts, and Pooled OLS estimates, allowing for a common intercept term, are available from the authors on request.

incentives to avoid a higher tax burden. On the other hand, they increase the benefits to remain official, in so far as higher tax rates are associated with higher tax revenues, thus with more provision of productivityenhancing public goods, including a strong legal environment. The World Bank index of corruption control enters with the expected positive sign (as a higher value for this index is associated with higher corruption control thus, other things being equal, with more effective auditing), but it becomes statistically insignificant when controlling for economic conditions. Finally, population and per capita GDP enter with the expected positive and negative signs, respectively, as expected, but only the latter is significant at the 10% level.

## 6. Conclusion

This paper has considered the interaction between market structure and VAT evasion in a monopolistically competitive economy composed of two industries. The paper has interpreted an increase in the intra-brand substitution elasticity in one industry as the diffusion of e-commerce in that industry. The paper has shown that, when the VAT is not formally equivalent to a RST, e-commerce diffusion raises both VAT compliance and the real VAT revenue. The first finding has been tested on macroeconomic data for the EU-24. The empirical analysis has found evidence of a negative correlation between the proportion of inhabitants who are internet users, which we take as a proxy for e-commerce diffusion, and the VAT gap share, which we take as a proxy for VAT evasion, over the years 2000-2006.

There are at least two important limitations to our analysis. As regards the model, we have restricted the analysis to symmetric equilibria. However, one would expect that less efficient (or smaller) firms would have more incentives to evade the VAT, either as a way for competing with more efficient (or larger) firms (see for example De Paula and Scheinkman, 2010) or because they are less closely monitored by the tax authorities (see Keen and Smith, 2006, and European Commission, 2005: 22). Adding this heterogeneity will be our next modelling step. As regards the empirical analysis, one would need industry data to test the hypothesis that internet diffusion has pro-competitive effects leading to higher VAT compliance. Notwithstanding these limitations, a policy message that can be drawn from this paper is that policymakers, rather than focusing exclusively on the potential VAT revenue losses that may be associated with e-commerce, should regard e-commerce diffusion as a potential and additional market instrument for contrasting VAT evasion.

## Appendices

### **Appendix 1 Full VAT compliance**

**Proof of Proposition 1.** From equation (6), the price mark-up over marginal costs is a decreasing function of the elasticity of substitution, but  $dP_c/d\sigma_1 < 0$ , which establishes part i). Differentiating (8) with respect to  $\sigma_1$  and evaluating the result at  $\sigma_1 = \sigma_2 = \sigma$ , implying a full-symmetric equilibrium with  $p_{ij}=P_i=P_c$ , yields:  $d\ln n_1/d\sigma_1 = d\ln C_1/d\sigma_1 - [1/(\sigma_1-1)]=[(\epsilon-\sigma)(1-\beta)+1-\sigma(1-\beta)]/[2\sigma(\sigma-1)(1-\beta)]<0$ , as long as  $\sigma > \epsilon > 1/(1-\beta)$ . The effect on firm size is positive:  $d\ln (C_1/n_1)/d\sigma_1 = F>0$ , establishing part ii). Turning to industry 2,  $d\ln n_2/d\sigma_1 = - [\epsilon(1-\beta)-1]/[2\sigma(\sigma-1)(1-\beta)]<0$  and  $d\ln (C_2/n_2)/d\sigma_1 = 0$ , which establishes part iii).

**Proof of Proposition 2.** From equations (6), (7), (8) yields:  $dlnP_c/ds=1/(1-\beta)(1+s)>0$ ,  $dlnP_i/ds=\beta dlnP_c/ds+1/(1+s)>0$ ;  $dlnn_i/ds=-1/(1-\beta)(1+s)<0$ ;  $dln (C_i/n_i)/ds=0$  establishing i) and iii). Real VAT revenues are R/P= $\tau\alpha\beta[Z_0-T]/P_c^{\beta}$ . Using (1), maximisation for  $\tau$  yields:  $\tau^*=1-\beta$  for T>0,  $\tau^{**}<\tau^*$  for -T=R yielding iii).

**Proof of Proposition 3.** Nominal VAT revenues are  $R = R_1 + R_2$ , with  $R_1 = \tau P_1 C_1$ ,  $R_2 = \tau P_2 C_2$  being revenues in the two sectors; real VAT revenues are R/P. The effects of internet diffusion in sector 1 on VAT revenues is  $d(R/P)/d\sigma_1 = (1/P)dR/d\sigma_1 - (R/P)dlnP/d\sigma_1$ . The first term represents the effect of internet diffusion on nominal revenue, while the second term is the effect on the consumer price index. Using equations (2), (3), (6), (7) and (8), the nominal VAT revenue effect, evaluated at an initial symmetric equilibrium with  $\sigma_1 = \sigma_2 = \sigma$  and  $\lambda_1 = \lambda_2 = \lambda$ , is zero:  $dR/d\sigma_1 = 0$  as long as  $dR_1/d\sigma_1 = -dR_2/d\sigma_1 = \tau \alpha\beta\Omega(\epsilon-1)\sigma(\sigma-1)/4>0$ . The consumer price index effect at a symmetric equilibrium is:  $dlnP/d\sigma_1 = -\{[1-\beta] [2 \sigma(\sigma-1)]\}^{-1} < 0$ . These results imply  $d(R/P)/d\sigma_1 > 0$ .

#### **Appendix 2 VAT evasion**

**Proof of equation (11)** Using equation (10), the FOC and SOC for the optimal degree of VAT evasion are  $\frac{\partial \pi_{ij}}{\partial \delta_{ij}} = -p_{ij}x_{ij}[-\tau + \lambda_i\psi_i\tau + \delta^*_{ij}] - \tau P_L x_{ij}(1 - \lambda K) = 0$   $\frac{\partial^2 \pi_{ij}}{\partial \delta_{ij}^2} = -p_{ij}x_{ij}\left[1 - (\partial \ln p_{ij}/\partial \delta_{ij})\tau [P_L(1 - \lambda K)/p_{ij}]\right] =$   $= -p_{ij}x_{ij}\left\{1 - \left(\frac{\sigma_i - 1}{\sigma_i^2}\right)\left(1 - \tau + \tau \delta^*_{ij}(1 - \lambda_i\psi_i) - \delta^*_{ij}^2/2\right)\left(\frac{\tau(1 - \lambda K)}{1 - \tau + \tau \delta^*_{ij}(1 - \lambda K)}\right)^2\right\}$ 

where  $\sigma_i > 1$ ,  $\lambda_i < 1$ ,  $\psi_i > 1$ ,  $\delta^*_{ij} \le 1$ . A sufficient condition for the SOC<0 is  $\tau \le 0.5$ , in which case all the three round bracketed terms in the second equation are lower than unity, thus the term is curly brackets is positively signed. The FOC can be written as:  $\delta^*_{ij} = \tau(1 - \lambda_i \psi_i) + \tau(P_L / p_{ij})(1 - \lambda K)$ . The last RHS term represents the marginal cost of VAT evasion on final sales that is related to the VAT paid, but not recovered, on intermediate inputs. This term is absent when the VAT is formally equivalent to the RST. This extra cost of VAT evasion is proportional to the real price of intermediate inputs and to the adjusted probability of not being detected. For K=0, we have an "adversarial" tax authority: once evasion is detected, the firm receives no compensation for the unrecovered part of the VAT paid on its intermediates. If K=1, the tax authority, before applying the penalty to the tax evader, fully credits the VAT paid on intermediates. Thus, this component of the marginal cost of VAT evasion is smaller in the latter than in the former case. The real price of intermediate inputs is derived from the FOC for the optimal output level:

$$\partial \pi_{ij} / \partial x_{ij} = p_{ij} \left[ 1 - (\partial \ln p_{ij} / \partial \ln x_{ij}) \left[ \left( 1 - \tau + \tau \delta^*_{ij} (1 - \lambda_i \psi_i) - \delta^*_{ij}^2 / 2 \right) \right] - P_L \left[ 1 - \tau + \tau \delta^*_{ij} (1 - \lambda_i K) \right] = 0,$$
  
yielding 
$$\frac{P_L}{p_{ij}} = \left[ \frac{\sigma_i - 1}{\sigma_i} \right] \left[ \frac{\left( 1 - \tau + \tau \delta^*_{ij} (1 - \lambda_i \psi_i) - \delta^*_{ij}^2 / 2 \right)}{1 - \tau + \tau \delta^*_{ij} (1 - \lambda_i K)} \right] < 1$$

Using this latter expression into the FOC for the optimal degree of VAT evasion and rearranging yields:

$$\begin{aligned} A(\delta_{ij}^{*})^{2} + B\delta_{i}^{*} + C &= 0 \\ A &\equiv \tau (1 - \lambda_{i}K)[\sigma + 1]/2 > 0; \ B &\equiv \sigma (1 - \tau) - \tau^{2} (1 - \lambda_{i}K)(1 - \lambda_{i}\psi) > 0 \quad \text{for } \tau < 0.618 \text{ (sufficient condition);} \\ C &\equiv \tau (1 - \tau)[\sigma_{i}\lambda_{i}(\psi - K) - (1 - \lambda_{i}K)], \text{ with } K = \{0, 1\} \end{aligned}$$

where we impose  $\tau \le 0.5$  such that the SOC holds. Thus, if C $\ge 0$ , the optimal degree of VAT evasion will be  $\delta^*_{ij}=0$ , whereas it will be  $0<\delta^*_{ij}<1$  if C<0, corresponding to the positive root of the quadratic equation. The necessary and sufficient condition for honest reporting is thus C $\ge 0$ , namely  $\lambda_i \ge 1/[\sigma_i(\psi_i-K)+K]$ , K={0, 1}. A sufficient condition for honest reporting is  $\lambda_i \ge 1/[\sigma_i(\psi_i-1)+1]$ , see Lemma 2i). If C<0, namely if  $\lambda_i < [\sigma_i(\psi_i-K)+K]$ , the solution is shown by equation (11) in the main text. From the solution to the FOC it also follows that:  $\partial p_{i1}/\partial \delta_1 = (1/\sigma_1) \tau (1-\lambda_i K)/[1-\tau+\tau \delta_1 (1-\lambda_1 K)] > 0$ .

**Proof of Lemma 2.** Lemma 2i) has been proven above. Turning to Lemma 2ii), provided that  $0 \le \delta^*_{ij} \le 1$ , totally differentiating the FOC for the optimal degree of VAT evasion, yields

$$\begin{split} \partial \delta^*_{ij} / \partial \sigma_i &= - \Bigg[ \frac{\left(\delta^*_{ij}\right)^2 \left[ \tau(1 - \lambda_i K) / 2 \right] + \delta^*_{ij} (1 - \tau) + \tau(1 - \tau) \lambda_i (\psi_i - K)}{2\delta^*_{ij} A + B} \Bigg] < 0 \\ \partial \delta^*_{ij} / \partial \psi_i &= -\tau \lambda_i \Bigg[ \frac{\delta^*_{ij} \tau(1 - \lambda_i K) + (1 - \tau) \sigma_i}{2\delta^*_{ij} A + B} \Bigg] < 0 \\ \partial \delta^*_{ij} / \partial K &= \tau \lambda_i \Bigg[ \frac{\delta^*_{ij} \left[ \delta^*_{ij} (\sigma_i + 1) / 2 - \tau(1 - \lambda_i \psi_i) \right] + (\sigma_i - 1)(1 - \tau)}{2\delta^*_{ij} A + B} \Bigg] > 0 \\ \partial \delta^*_{ij} / \partial \lambda_i &= -\tau \Bigg[ \frac{\delta^*_{ij} \left[ -\delta^*_{ij} (\sigma_i + 1) K / 2 + \tau K(1 - \lambda_i \psi_i) + \tau(1 - \lambda_i K) \psi_i \right] + (1 - \tau)[\sigma_i (\psi_i - K) + K]}{2\delta^*_{ij} A + B} \Bigg] \\ \partial \delta^*_{ij} / \partial \tau_i &= -\Bigg[ \frac{\left(\delta^*_{ij}\right)^2 (1 - \lambda_i K)(\sigma_i + 1) / 2 - \delta^*_{ij} [\sigma_i + 2\tau(1 - \lambda_i K)(1 - \lambda_i \psi_i)] + (1 - 2\tau)[\sigma_i \lambda_i (\psi_i - K) - (1 - \lambda_i K)]}{2\delta^*_{ij} A + B} \\ \text{Given that} \left[ \delta^*_{ij} (\sigma_i + 1) / 2 - \tau(1 - \lambda_i \psi_i) \right] = \tau(1 - \lambda_i \psi_i) [(\sigma_i - 1) / 2] + \delta^*_{ij} \tau(P_L / p_{ij}) (\sigma_i + 1)(1 - \lambda_i K) / 2 > 0 \end{aligned}$$

using the FOC, it follows that  $\partial \delta *_{ij} / \partial K > 0$ . The sign of the two last derivatives depends on the size of  $\delta *_{ij}$ . The numerical examples of Tables 2.1 and 2.2 show that:  $\partial \delta *_{ij} / \partial \lambda_i < 0$ ,  $\partial \delta *_{ij} / \partial \tau < 0$ , other things being equal. **Proof of Proposition 5**. The real VAT revenue is  $R^{EV}/P^{EV}=R_1^{EV}/P^{EV}+R_2^{EV}/P^{EV}$ ; the nominal revenue is  $R_i^{EV}=\tau(1-\delta_i^*)p_i^{EV}C_i^{EV}$ , i=1,2;  $p_i^{EV}$  and  $C_i^{EV}$  are given by equations (6bis) and (8bis). For given tax rate  $\tau$ , using (7bis) one can derive the effect of internet diffusion on the price index of the industrial sector (thus, using equation 3, on the consumer price index):

 $\partial \ln P_{c}^{EV} / \partial \sigma_{1} = \left[ \frac{1}{2} (1-\beta) \right] \left\{ \left[ \frac{\partial (\sigma_{1}/\sigma_{1}-1)}{\partial \sigma_{1}} \right] / \frac{(\sigma/\sigma-1)}{(\sigma/\sigma-1)} + \frac{(\partial s_{1}^{e}/\partial \sigma_{1})}{(1+s^{e})} \right],$ 

where the derivative has been evaluated at a full symmetric equilibrium with  $\sigma_1 = \sigma_2 = \sigma$  and  $\lambda_1 = \lambda_2 = \lambda$ . The impact on the price index of internet diffusion now comprises two effects. The first term in curly brackets is the standard effect on the price mark-up. It implies that more competition in industry 1 provokes a reduction in equilibrium prices and wages, thus in the price index:  $\left[\frac{\partial(\sigma_1/\sigma_1-1)}{\partial\sigma_1}\right]/(\sigma/\sigma-1) = -1/\sigma(\sigma-1) < 0$  at a full symmetric equilibrium. The second term in curly brackets is a compliance effect, occurring with VAT evasion only. In symmetric equilibrium, using the FOC for the optimal degree of VAT evasion, yields:  $(\partial s_1^e/\partial \sigma_1) = [(1-\tau)(1-t^{e^*})]^{-1} \partial \delta_1^*/\partial \sigma_1 [\tau(1-\lambda K)] [\delta^{*2}/2 + (1-\tau)(1-P_1/p_{ii})] < 0$ , as long as  $\partial \delta_1^*/\partial \sigma_1 < 0$ ,  $(1-\lambda K) > 0$  and  $(1-\tau)(1-t^{e^*})^{-1} = [(1-\tau)(1-t^{e^*})]^{-1} = [(1-\tau)(1-t^{e^*})$  $P_{I}/p_{ii}$ )>0. Thus, the effect of internet diffusion on the consumer price index is surely larger with VAT evasion than with full VAT compliance, or  $\partial \ln P / \partial \sigma_1 = \beta \partial \ln P_c^{EV} / \partial \sigma_1$ . The effect on the nominal VAT revenue  $\text{is: } \partial R^{\text{EV,VAT}} / \partial \sigma_1 = R_1^{\text{EV}} [-(1-\delta_1^{*})^{-1} \partial \delta_1^{*} / \partial \sigma_1 + \partial \ln p_1^{\text{EV,VAT}} C_1^{\text{EV,VAT}} / \partial \sigma_1] + R_2^{\text{EV,VAT}} \partial \ln p_2^{\text{EV,VAT}} C_2^{\text{EV,VAT}} / \partial \sigma_1 > 0,$ as long as  $\partial lnp_2^{EV,VAT}C_2^{EV,VAT}/\partial \sigma_1 = -[(\epsilon-1)/2] \{[1/\sigma(\sigma-1)] - (1+s^*)^{-1} \partial s_1^*/\partial \sigma_1\} \le 0, \ \partial lnp_1^{EV,VAT}C_1^{EV,VAT}/\partial \sigma_1 = -(\epsilon-1)/2 \}$  $1)/[\sigma(\sigma-1)] - \epsilon(1+s^*)^{-1} \partial s_1^* / \partial \sigma_1 + \partial \ln p_2^{EV,VAT} C_2^{EV,VAT} / \partial \sigma_1 > 0, \quad \partial \delta_1^* / \partial \sigma_1 < 0, \quad if evaluating these derivatives at a distribution of the second seco$ symmetric equilibrium with  $\sigma_1 = \sigma_2 = \sigma$ ,  $\lambda_1 = \lambda_2 = \lambda$ , implying  $R_1^{EV} = R_2^{EV}$ ,  $p_1^{EV} C_1^{EV} = p_2^{EV} C_2^{EV}$ ,  $\delta_1^* = \delta_2^*$ . Thus, there is an increase in the nominal VAT revenue, as long as the increase in sector 1's tax base more than outweighs the reduction in sector 2's tax base. The effect on the real VAT revenue is:  $\partial (R^{EV}/P^{EV})/\partial \sigma_1 =$  $(1/P^{EV})\partial R^{EV}/\partial \sigma_1 - (R^{EV}/P^{EV})\partial \ln P^{EV}/\partial \sigma_1 > 0.$ 

Proof of Proposition 6. The optimal VAT rate solves:

$$\frac{\partial \ln R / P}{\partial \tau} = \frac{1}{\tau} - \left[ \frac{\partial \delta^* / \partial \tau}{1 - \delta^*} + \frac{\beta}{1 - \beta} \frac{\partial s}{\partial \tau} \frac{1}{1 + s} \right] = 0$$

The term in square brackets is the marginal cost of a higher tax rate. This term depends on s, whose expression is regime-specific (see equations 7, 10 and 11.2). Evaluating this term in the neighbourhood of the full compliance equilibrium with  $\delta^*=0$  and  $\tau^*=(1-\beta)$ , it follows that

$$\tau^{RST} = \frac{1-\beta}{1+(1-\beta)(1-\lambda\psi)} < \tau^{VAT} = \frac{1-\beta}{1+(1-\beta)(1-2\beta)\left[\sigma\lambda(\psi-1)-(1-\lambda)\right]/\sigma\beta} < \tau^* = (1-\beta)$$

where  $\beta \ge 1/2$ , implying  $\beta \ge \lambda/2$  It also follows that:  $\partial \tau^{VAT} / \partial \sigma > 0$ .

### Appendix 3 Endogenous detection probability under VAT-RST-equivalent taxation

Lemma 1 in the main text has shown that, with an exogenous detection probability, internet diffusion does not influence the optimal degree of evasion if the VAT is equivalent to a RST. This appendix shows that it is possible for internet diffusion to lower VAT evasion under VAT-RST equivalence, if we assume that the probability of detection in each sector is a non-convex function of the average firms' size within each industry, namely if  $\lambda_i(C_i/n_i)$ , with  $\lambda_i > 0$  and  $\lambda_i > 0$ . This assumption implies that the tax authority has more incentives to focus its monitoring efforts in the industry where the larger firms operate. This assumption seems broadly consistent with the common practice, followed for example by EU countries, of focusing control activities on larger firms.<sup>30</sup> One possible explanation for this practice is provided by Keen and Smith (2006). They note that tax authorities have larger administrative costs for monitoring small firms and that the potential tax base is usually concentrated in the largest companies. From Lemma 3, we know that the average firm size in symmetric industrial equilibrium is given by  $C_i/n_i = F(\sigma_i - 1)$ , implying  $\lambda_i(F(\sigma_i - 1))$ . Under this assumption, each single firm perceives the ex ante detection probability  $\lambda_i$  as given, as long as it is small relatively to its industry. Thus, the solution to the firm's problem is still represented by equation (10) in the main text. However, as long as  $\partial \lambda_1 / \partial \sigma_1 < 0$ , internet diffusion in sector 1, namely an increase in  $\sigma_1$ , now lowers tax evasion incentives for active firms in sector 1,  $\delta_1 * \partial \sigma_1 < 0$ . The general equilibrium solution is still represented by equations (6bis)-(8bis). However, there is now a new effect operating through the price externality of internet diffusion: the increased detection probability  $\partial \lambda_1 / \partial \sigma_1 > 0$  implies a higher effective expected tax rate for industry 1 firms,  $\partial t_1^{e^*} / \partial \lambda_1 > 0$ , inducing them to set higher prices. This effect may balance the reduction in prices due to a lower mark-up. Namely, for given tax rate  $\tau$ , using (7bis), it turns out that:  $\partial \ln P_c^{EV} / \partial \sigma_1 = [1/2(1-\beta)] \{ [\partial (\sigma_1/\sigma_1 - 1)/\partial \sigma_1] / (\sigma/\sigma - 1) + (\partial s_1^e/\partial \sigma_1)/(1+s^e) \}, \text{ where } (\partial s_1^e/\partial \sigma_1)/(1+s^e) = [(\tau^2/2)\psi/(1-s^e)] \}$ t<sup>\*</sup>)] $\partial \lambda_1 / \partial \sigma_1 > 0$ . The derivatives have been evaluated at a full symmetric equilibrium with  $\sigma_1 = \sigma_2 = \sigma$  and common  $\lambda$ . To rule out the possibility that internet diffusion leads to price inflation, we impose the following restriction:  $0 < \partial \lambda_1 / \partial \sigma_1 < (1-t^e) / [\sigma(\sigma-1)\psi\tau^2/2]$ , implying  $\partial \ln P / \partial \sigma_1 = \beta \partial \ln P_c^{EV} / \partial \sigma_1 < 0$ . Provided that the consumption and industrial price indexes fall with internet diffusion in sector 1, both nominal and real VAT revenues will increase with it. The effect on nominal VAT revenue is:  $\partial R^{EV}/\partial \sigma_1 = R_1^{EV}[-(1-\delta_1^*)\partial \delta_1^*/\partial \sigma_1 + \partial \sigma_1 + \partial$  $\ln p_1^{EV} C_1^{EV} / \partial \sigma_1 + R_2^{EV} \partial \ln p_2^{EV} C_2^{EV} / \partial \sigma_1$ . Evaluating these derivatives in symmetric equilibrium, with  $\sigma_1 = \sigma_2 = \sigma$ , it follows that  $R_1^{EV} = R_2^{EV}$ ,  $p_1^{EV}C_1^{EV} = p_2^{EV}C_2^{EV}$ ,  $\delta_1^* = \delta_2^*$ ,  $\partial_1^{EV}C_1^{EV}/\partial\sigma_1 = -\partial_1^{EV}C_2^{EV}/\partial\sigma_1 = -\partial_1^{EV$  $1)\Phi\theta^{1/(1-\beta)}[\partial \ln K/\partial\sigma_1]K^{\varepsilon-1}/(1+K^{\varepsilon-1})^2 > 0, \text{ where } K \equiv [(\sigma_1/\sigma_1-1)(1+s_1^e)]/[(\sigma_2/\sigma_2-1)(1+s_2^e)] \text{ and } \partial K/\partial\sigma_1 < 0$ whenever the restriction  $0 < \partial \lambda_1 / \partial \sigma_1 < (1-t^e) / [\sigma(\sigma-1)\psi\tau^2/2]$  holds. Thus, there is a shift in nominal VAT revenue from sector 2 to sector 1, and moreover total nominal VAT revenue increases. It also follows that the real VAT revenue rises.

<sup>&</sup>lt;sup>30</sup> "Almost all Member States claim to give special attention to large enterprises and in most cases there are special departments for their auditing.", European Commission (2005: 22). "In general it can be noted that for large enterprises the Member States try to carry out both desk and on-the-spot audits on a regular basis, which on average implies a control at least every three years. For small and medium-sized taxpayers there are in most cases no rules governing the periodicity of on-site controls. This depends mostly on the outcome of risk assessment, local knowledge, previous controls, etc. Of course, less compliant businesses will normally be visited more frequently", Ibidem: 26. The argument that bigger firms face higher monitoring probabilities from the tax authorities is well established in the public finance literature, see for example Due (1963: 162-163), quoted in Marrelli (1984).

# Appendix 4.

# Data sources and definitions

The sample covers 24 EU countries over the period 2000-2006. The countries are: Austria, Belgium, Czech Republic, Germany, Denmark, Estonia, Spain, Finland, France, Greece, Hungary, Ireland, Italy, Lithuania, Luxembourg, Latvia, Malta, Netherlands, Poland, Portugal, Sweden, Slovenia, Slovakia, UK. There are 168 observations.

**VAT gap share**: Difference between the theoretical amount of VAT that should be paid, based on national accounts data, and the actual VAT receipts divided by the theoretical VAT liability, 2000-2006. <u>Source</u>: Reckon (2009).

**Internet diffusion**: Estimated internet users per 100 inhabitants, 2000-2006. <u>Source</u>: International Trade Union (ITU) World Telecommunication, ICT Indicators Database.

**E-commerce diffusion**: Proportion of respondent to the Flash Eurobarometer question Q1 "Please tell me if you have purchased any goods or services in the past 12 months by distance in (Your country) or elsewhere in any of the following ways...? Via the Internet (website, email, etc.)." Percentage of Yes answers by country, year 2006.

Source: European Commission (2011). Flash Eurobarometer n. 299. Consumers attitudes towards crossborder trade and consumer protection, March 2011.

Standard VAT rate: Official VAT rates, 2000-2006.

<u>Source</u>: European Commission, Taxation and Custom Unions (2011). *Vat rates applied in the member states of the European Union*, Brussels, section VIII "The evolution of the VAT rates applicable in the member states".

**Corruption Control**: World Bank index of corruption control, 2000-2006. The index is measured in units ranging from about -2.5 to 2.5 with higher values corresponding to better corruption control. The corruption control index reflects the statistical compilation of responses on the quality of governance given by a large number of enterprise, citizen and expert survey respondents, as reported by a number of survey institutes, think tanks, non-governmental organizations, and international organizations.

Source: Daniel Kaufmann, Aart Kraay and Massimo Mastruzzi (2010). "The Worldwide Governance Indicators : A summary of methodology, data and analytical issues". *World Bank Policy Research*.

**Real GDP per capita (RGDPCH)**: Constant price GDP per capita, 2005 International dollar per person, 2000-2006.

<u>Source</u>: Alan Heston, Robert Summers and Bettina Aten, *Penn World Table Version 7.0*, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, May 2011.

# Population: In thousands, 2000-2006.

<u>Source</u>: Alan Heston, Robert Summers and Bettina Aten, *Penn World Table Version 7.0*, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, May 2011.

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