# Opportunity-sensitive poverty measurement 

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#### Abstract

This paper characterizes new poverty measures and the respective dominance condition, inspired by the theory of equality of opportunity. Individual poverty is assessed taken into consideration the type the person belongs to. Within the class of poverty measures presented, we propose a measure based on a rank-dependent aggregation of the type-specific poverty levels. Using household data from Brazil in the last decade, we show that when circumstances are included in the measurement of poverty the evolution of poverty is quite distinct from that emerging using traditional poverty tools.


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## 1 Motivation

The formal measurement of (uni-dimensional) poverty has long relied on a number of fundamental axioms: anonymity, monotonicity, and focus. ${ }^{1}$ Simply put, the combination of these axioms results in measures of poverty that aggregate information exclusively on incomes lower than a pre-determined poverty line; that are non-increasing in those incomes; and that are insensitive to the identity of individual income recipients. ${ }^{2}$ In other words, once Sen's (1976) identification problem is solved by choice of an appropriate poverty line $z$, then poverty measurement for a society with an income distribution $F(x)$ is given by some functional:

$$
P(F(x), z)
$$

[^0]The question we ask in this paper is whether the recent philosophical and economic literature on inequality of opportunity has any implications for this time-tested approach to poverty measurement.

Since Rawls's (1971) A Theory of Justice, moral philosophers and social choice theorists have argued that not all income differences are ethically identical. Income differences due to personal choices, or otherwise attributable to personal responsibility are less ethically objectionable than differences due to pre-determined attributes over which individuals have no control. Based on increasingly sophisticated variants of this basic distinction, a number of authors have argued that egalitarian justice ought to focus less and less on the space of outcomes $(F(x)$ in our simple framework), and more on the space of opportunities. See, for instance, Dworkin (1981a, b), Arneson (1989), Cohen (1989), Fleurbaey (2008), Fleurbaey and Maniquet (2010), Elster and Roemer (1993), Roemer (1998).

The theory of inequality of opportunities, and in particular its formalization by John Roemer (1998), has influenced the economic literature on the measurement of inequality. Various authors have proposed ways in which inequality due to economic circumstances could be appropriately quantified, and separated from inequality due to personal efforts. Given the close proximity between the literatures on inequality and poverty measurement, it is interesting that the influence of opportunity egalitarianism on inequality measurement has not, to our knowledge, been extended to the measurement of poverty. After all, if our judgment of overall inequality in society is affected by whether the income differences arise from personal responsibility or pre-determined circumstances, why should our judgment of poverty not be affected in the same way?

To be clear, we do not claim that some poor deserve its condition and we do recognize that poverty itself could be considered a circumstance beyond individual control rather than a joint outcome of effort and circumstance (Fleurbaey, 2007). However, if two societies have, say, the same poverty headcount and poverty gap but different degrees of inequality amongst the poor, then we readily admit that some assessment criteria, such as the poverty gap squared (FGT2), will judge poverty in the more unequal society to be greater. Yet, if two societies have the same poverty headcount and gap (and FGT2, for the sake of argument), but in the first society all members of a certain ethnic minority -or all people whose parents are illiterate- are poor, while in another poverty is uncorrelated with differences in race or family background, our current arsenal of poverty measures does not readily allow us to distinguish them.

We derive a class of opportunity sensitive indices axiomatically. The first set of axioms are commonly imposed to poverty measures; indices in this class are required to be additive in individual poverty, to be insensitive to income changes of non-poor individuals, and to not increase after increments of income among the poor. Other standard axioms are, instead, modified consistently with the theory of equality of opportunity. Individuals are no longer anonymous, but rather, they are characterized by circumstances beyond their control that influence their income. Hence, anonymity is imposed only among individuals sharing the same set of circumstances. The crucial axiom in our construction, inequality of opportunity aversion, is represented by a transfer axiom that involve individuals characterized by different circumstances. Finally, we propose two alternative axioms related to the treatment of individuals with the same circumstances:
the first imposes a strong neutrality with respect to inequality within types; the second imposes weak inequality aversion within types. As a consequence, we identify two families of poverty measures. The first family, characterized by inequality aversion between types and inequality neutrality within types, is new in the literature. The second family, instead, has been already proposed in the literature on poverty dominance for heterogeneous populations (Atkinson and Bourguignon 1987, Jenkins and Lambert 1993) although it is re-interpreted in the current context of opportunity sensitive poverty measurement.

The paper then proceeds by obtaining, for each class, suitable poverty dominance conditions (partial rankings): the first condition involves a types-sequential comparison based on the average income among the poor and the proportions of the poor; the second condition - which has already been proposed by Jenkins and Lambert (1993) - involves a types-sequential comparison of the cumulative distribution functions truncated at the poverty line. Then, we propose a subclass of scalar poverty measures (complete rankings) that are sensitive to inequality of opportunity.

An empirical illustration completes the paper. Using data from Brazil in the last decade, we show that when circumstances are included in the measurement of poverty we are able to uncover aspects of poverty not captured when using traditional poverty measures. We use the yearly Brazilian Pesquisa Nacional por Amostra de Domicilios (PNAD) between 2001 and 2008. We consider two main circumstances: region of birth and race. The empirical exercise concerns both the partial ranking and the complete ranking.

## 2 Opportunity-sensitive poverty measurement: a proposal

Each individual $h$ in our society is completely described by a list of traits, which can be partitioned into two different classes: traits beyond the individual responsibility, called circumstances $\mathbf{c}$, belonging to a finite set $\Omega=\left\{\mathbf{c}_{1}, \ldots, \mathbf{c}_{n}\right\}$; and factors for which the individual is responsible, that can be summarized by a scalar variable called effort, $e \in \mathbb{R}_{+}$. There is no luck, nor random components in our model. Income is generated by a function $g: \Omega \times \mathbb{R}_{+} \rightarrow \mathbb{R}_{+}$.

We partition the total population into types, where a type $\mathbf{T}_{\mathbf{i}} \in \mathcal{T}$ is the set of individuals whose set of circumstances is $\mathbf{c}_{i}$ and the set of types, $\mathcal{T}$, is a partition of the entire population. The income of person $h$ of type $i$ is $x_{h}, h \in \mathbf{T}_{\mathbf{i}}$. Income is distributed according to the type-specific income distribution $F_{i}(x)$, with density function $f_{i}(x)$. The maximum income in type $i$ is denoted $x_{i}^{\text {max }}$, and the population share of type $i$ is denoted $q_{i}^{F} . F(x)=\sum_{i=1}^{n} q_{i}^{F} F_{i}(x)$ is the income distribution of the society, defined as the component-mix distribution of all the type distributions. The maximum income in the population is denoted $x^{\max }$.

The type-specific income distribution represents the set of outcomes which can be achieved - by exerting different degrees of effort - starting from the same circumstance $\mathbf{c}_{i}$. That is to say, the type distribution is a representation of the opportunity set open to any individual endowed with circumstances $\mathbf{c}_{i}$. The support of the distribution represents, so to speak, the options open to individuals in the type; the frequency represents, ex ante, the probability attached
to each option. Hence the cumulative distribution function $F_{i}(x)$ gives a full description of the opportunity set in each type.

We assume that there is a general agreement on an ordering $\succeq$ on the set of types so that, in general, $\mathbf{T}_{i+1} \succeq \mathbf{T}_{i}$ for $i \in\{1, \ldots, n-1\}$. For example, we could rank types according to the poverty rates (measured according to some poverty index) of the types. Notice that the ranking of types in different societies, or even in the same society at different times, can be different. What matters is that there be agreement on the ordering at any particular point in time.

We consider a poverty line $z \in\left[0, x^{\max }\right]$ and we define the set of poor individuals in each type as $\mathbf{T}_{i}^{z}:=\left\{h \in \mathbf{T}_{i} \mid x_{h} \leq z\right\}$. In this paper we work with a unique poverty line for the entire population. We denote by $\mu_{i}$ the mean income of type $i$, and by $\mu_{i}(z)$ the mean income of the poor in type $i . D$ is the set of admissible distributions.

Our aim is to propose a poverty measure that is based on the poverty condition of all individuals in the society but, at the same time, that is sensitive to the type the persons belong to. In the next section we will introduce a class of poverty measures that comply with a set of desirable properties (axioms).

## 3 A class of opportunity sensitive poverty measures

In this section, we present a set of desirable axioms to be imposed on the function $P: D \rightarrow \mathbb{R}$, including new properties specific to inequality of opportunity. From these, we derive a general class of poverty measures and describe a subfamily of measures that satisfy additional desirable properties.

The first property allow us to express aggregate poverty as the sum of individual poverties.

A1 Additivity (ADD). There exist functions $p_{i}: \mathbb{R} \rightarrow \mathbb{R}$, for all $i \in\{1, \ldots, n\}$, assumed to be twice differentiable (almost everywhere) in $x$, such that

$$
P(F(x), z)=\sum_{i=1}^{n} q_{i}^{F} \int_{0}^{x^{\max }} p_{i}(x) f_{i}(x) d x \quad \text { for all } F \in D
$$

Anonymity within types implies that only individuals' income and their type are important for the evaluation of poverty. In other words, the identity of the individuals does not matter within each type, but individuals' poverty are assessed according to their type.

A2 Anonymity within types (ANON): For all $F \in D$, for all $i \in\{1, \ldots, n\}$

$$
P\left(F_{1}(x), \ldots, F_{i}(x), \ldots, F_{n}(x)\right)=P\left(F_{1}(x), \ldots, F_{i}(y), \ldots, F_{n}(x)\right)
$$

whenever $y=\Pi x$ and $\Pi$ is any permutation matrix.
The following axioms are standard in the poverty measurement literature. If an individual sees his income increase, ceteris paribus, overall poverty cannot decrease. In addition, the level of poverty in a society is independent of the income of the non poor (Sen, 1976).

A3 Monotonicity (MON): For all $F \in D$, for all $i \in\{1, \ldots, n\}, P$ is nonincreasing in $x$.

A4 Focus (FOC): For all $F, G \in D$, for all $i \in\{1, \ldots, n\}$,

$$
P(F(x), z)=P(G(x), z) \quad \text { if } F(x)=G(x), \quad \forall x \leq z
$$

The next two axioms are intended to express the equality of opportunity view for the evaluation of the income distribution among the poor. First, we introduce a preference for equality between types. Types are ranked from the poorest $(i=1)$ to the least poor $(i=n)$. Thus, the evaluation of the poverty suffered by individuals with income $x$ is assumed to be decreasing in types. In other words, a transfer from a poor person in the poorer type to a poor person in a better-off type will not decrease the poverty level of the society, irrespective of the income level of the individuals involved in the transfer. ${ }^{3}$

## A5 Inequality of opportunity aversion (IOA):

$$
\frac{\partial P}{\partial x_{h}}\left|h \in \mathbf{T}_{i}^{z} \leq \frac{\partial P}{\partial x_{k}}\right| k \in \mathbf{T}_{i+1}^{z}, \forall i \in\{1, \ldots, n-1\}, \forall h \in \mathbf{T}_{i}^{z}, \forall k \in \mathbf{T}_{i+1}^{z}
$$

The second distributional concern refers to the treatment of individuals in the same type. The following axiom imposes a requirement of no aversion to inequality within types.

## A6 Inequality neutrality within types (INW)

$$
\frac{\partial^{2} P}{\partial x_{h}^{2} \mid h \in \mathbf{T}_{i}^{z}}=0, \forall i \in\{1, \ldots, n\}
$$

This axiom expresses the Natural Reward Principle - specifically of its utilitarian version- whereby income inequalities within types are considered equitable and need not be compensated because they are the result of differences in effort exerted (Peragine, 2004; Fleurbaey, 2008).

Alternatively, we can allow a certain degree of aversion to inequality within type. The next axiom requires that a progressive Pigou-Dalton transfer among the poor within a given type, all else constant, does not increase poverty.

## A7 Inequality aversion within types (IAW)

$$
\frac{\partial^{2} P}{\partial x_{h}^{2} \mid h \in \mathbf{T}_{i}^{z}} \leq 0, \forall i \in\{1, \ldots, n\}
$$

The rationale to allow for a weak aversion to inequality within type is twofold: first, because observed circumstances might not capture the whole set of factor beyond the person's responsibility; second, because even when indeed the specification of the types is complete, one could still prefer a society where the disparities among individuals' income are not extreme.

[^1]We refer to two classes of poverty measures that satisfy the axioms introduced so far as the set of opportunity-sensitive poverty measures. The first class satisfies A1-A6 and is denoted by:

$$
\mathbf{P}^{O}:=\{P: D \rightarrow \mathbb{R} \mid \text { P satisfies ADD, ANON, MON, FOCUS, IOA, INW }\}
$$

The second class satisfies A1-A5 and A7 and is denoted by:

```
\mp@subsup{\mathbf{P}}{}{OI}}:={P:D->\mathbb{R}|P\mathrm{ satisfies ADD, ANON,MON, FOCUS,IOA,IAW }
```

Remark $1 P \in \mathbf{P}^{O}$ if and only if, for all $F \in D$,
$P(F(x), z)=\sum_{i=1}^{n} q_{i}^{F} \int p_{i}(x) f_{i}(x) d x$, where the functions $p_{i}: \mathbb{R} \rightarrow \mathbb{R}$ satisfy the following conditions:

P1. $p_{i}(x)=0, \forall x \geq z$ and $\forall i \in 1, \ldots, n$
P2. $p_{i}(x)>0, \forall x<z$ and $\forall i \in 1, \ldots, n$
P3. $p_{i}^{\prime}(x) \leq p_{i+1}^{\prime}(x) \leq 0 \forall x, \forall i=1, \ldots, n-1$
P4. $p_{i}^{\prime \prime}(x)=0, \forall x$ and $\forall i \in 1, \ldots, n$
Remark $2 P \in \mathbf{P}^{O I}$ if and only if, for all $F \in D$,
$P(F(x), z)=\sum_{i=1}^{n} q_{i}^{F} \int p_{i}(x) f_{i}(x) d x$, where the functions $p_{i}: \mathbb{R} \rightarrow \mathbb{R}$ satisfy conditions P1-P3 and P4':
$\mathrm{p} 4^{\prime} . p_{i}^{\prime \prime}(x) \leq 0, \forall x$ and $\forall i \in 1, \ldots, n$

## 4 Opportunity sensitive poverty dominance

In this section, we identify the poverty dominance conditions corresponding to the families of poverty measures proposed so far. We start with the family $\mathbf{P}^{O}$ the family of poverty measure satisfying all the conditions in Remark 1.

Theorem 1 For all distributions $F(x), G(x) \in D$ and a poverty line $z, P(F(x), z) \geq$ $P(G(x), z) \forall P \in \mathbf{P}^{O}$ if the following conditions are satisfied:
(i) $\sum_{i=1}^{j} q_{i}^{F} \mu\left(F_{i}^{z}\right) \leq \sum_{i=1}^{j} q_{i}^{G} \mu\left(G_{i}^{z}\right), \quad \forall j \in\{1, \ldots, n\}$.
(ii) $\sum_{i=1}^{j} q_{i}^{F} H_{i F} \geq \sum_{i=1}^{j} q_{i}^{G} H_{i G}, \quad \forall j \in\{1, \ldots, n\}$.
where $H_{i F}=F_{i}(z)$ is the headcount poverty ratio of group $i$.
Proof.
See appendix.

The result above says that a sufficient condition for declaring poverty in distribution $F$ higher than poverty in distribution $G$ according to the family $P^{O}$, is that the following sequential conditions are satisfied: $(i)$ the weighted average income among the poor is higher in $G$ than in $F$, at each step of the sequential procedure; (ii) the weighted proportion of poor (the headcount) is higher in $F$ than in $G$ at each step of the sequential procedure.

In these conditions, the sequential procedure, starting from the lowest type, adding the second and so on, expresses our concern for equality of opportunity. Note that these conditions are not necessary. A difference in means can be counterbalanced by a difference in the proportion of poor individuals (weighted by the poverty line). It is possible also to disentangle the effect of demographic composition:

Remark 1.1For all distributions $F(x), G(x) \in D$ and a poverty line $z$, $P(F(x), z) \geq P(G(x), z) \forall P \in \mathbf{P}^{O}$ if the following conditions are satisfied:
(i) $\sum_{i=1}^{j} \mu\left(F_{i}^{z}\right) \leq \sum_{i=1}^{j} \mu\left(G_{i}^{z}\right), \quad \forall j \in\{1, \ldots, n\}$.
(ii) $\sum_{i=1}^{j} H_{i F} \geq \sum_{i=1}^{j} H_{i G}, \quad \forall j \in\{1, \ldots, n\}$.
(iii) $\sum_{i=1}^{j} q_{i}^{F} \geq \sum_{i=1}^{j} q_{i}^{G}, \quad \forall j \in\{1, \ldots, n\}$.

Hence, given a distribution, poverty may decrease according to the family $P^{O}$ because of: $(i)$ an increase the income of the poor; (ii) a move of one person, in the same type, above the poverty line; (iii) a decrease in the population proportion of a poorer type in favor a richer one.

Note, again, that these conditions are not necessary as differences in population shares, in poverty headcount ratios and in average income of the poor interact in determining the difference in opportunity sensitive poverty.

We now turn to the the family of poverty measure $\mathbf{P}^{O I}$. The poverty dominance conditions corresponding to this family have already been identified in the literature, altough they have a different interpretation in the current context.
Theorem 2 (Jenkins and Lambert (1993), Chambaz and Maurin (1998)) For all distributions $F(x), G(x) \in D$ and a poverty line $z, P(F(z), z) \geq P(G(x), z)) \quad \forall P \in$ $P^{O I}$ if and only if the following condition is satisfied:

$$
\begin{equation*}
\sum_{i=1}^{j}\left(q_{i}^{G} G_{i}(x)-q_{i}^{F} F_{i}(x)\right) \geq 0, \quad \forall x \leq z, \quad \text { and } \quad \forall j \in 1, \ldots, n \tag{1}
\end{equation*}
$$

This dominance condition can be rearranged in order to show how changes in poverty can be decomposed in changes in the income distribution and changes in the population compositions:

$$
\begin{equation*}
\sum_{i=1}^{j} q_{i}^{G}\left(G_{i}(x)-F_{i}(x)\right)+\sum_{i=1}^{j}\left(q_{i}^{F}-q_{i}^{G}\right) F_{i}(x) \geq 0 \tag{2}
\end{equation*}
$$

## 5 From dominance to poverty measures

In this section we focus on some specific members of the families of poverty indeces characterized above. First we notice that there is an obvious potential conflict between axioms IOA and IAW. To see this, consider a simple example, with four individuals in three types. Individual A is in the highest type ( $h$ ), individuals B and C are in the intermediate type $(m)$, while individual D is in the lowest type ( $l$ ). Suppose B is poorer than C, and imagine positive transfers from A to C and B to D. Both transfers are progressive across types, but they result in an increase in inequality within type $m$. This is nothing but a manifestation of the tension between the aversion to inequality of opportunity and the aversion to inequality of outcomes. If this conflict were irredeemable, the set $\mathbf{P}^{O I}$ would be empty.

Figure 1: example 1: 3 types 2 transfers


That the set $\mathbf{P}^{O I}$ is not empty can be demonstrated by considering a specific example of individual poverty measures: let $p_{i}(x)=w_{i} p(x)$, such that $w_{i+1}-$ $w_{i}>p(x), \forall x$. In this case, $P(F(x), z)=\sum_{i=1}^{n} q_{i}^{F} \int w_{i} p(x) f_{i}(x) d x$, with a weakly concave $p(x)$ will always satisfy both IOA and IAW. A special member in this class of indices is obtained by using the FGT formulation for $p(x)=\left(\frac{z-x}{z}\right)^{\alpha}$, and selecting inverse ranks as type weights: $w_{i}=n+1-i$.

By selecting these specific $p$ functions and weights, we obtain the family of opportunity-sensitive rank-dependent FGT measure $\mathbf{P}_{F G T}^{O} \subseteq \mathbf{P}^{O I}$ :

$$
\begin{equation*}
P_{F G T}^{O}(F, z)=\sum_{i=1}^{n} q_{i}^{F}(n+1-i) \int_{0}^{z}\left(\frac{z-x}{z}\right)^{\alpha} f_{i}(x) d x \tag{3}
\end{equation*}
$$

where the parameter $\alpha$ expresses a concern for within-type inequality among the poor, and $\alpha \geq 0$.

Properties of $\mathbf{P}_{F G T}^{O I}$
$\odot \mathbf{P}_{F G T}^{O I}$ satisfies both IOA and IAW. This can be seen by noting that, $\forall i \in\{1, \ldots, n-1\}, \forall h \in \mathbf{T}_{i}^{z}, \forall k \in \mathbf{T}_{i+1}^{z}$,

$$
\begin{gathered}
\frac{\partial P}{\partial x_{h}}\left|h \in \mathbf{T}_{i}^{z} \leq \frac{\partial P}{\partial x_{k}}\right| k \in \mathbf{T}_{i+1}^{z} \\
-\frac{\alpha}{z} q_{i}^{F}(n+1-i)\left(\frac{z-x_{h}}{z}\right)^{\alpha-1} f_{i}(x) \leq \\
-\frac{\alpha}{z} q_{i+1}^{F}(n+1-i-1)\left(\frac{z-x_{k}}{z}\right)^{\alpha-1} f_{i+1}(x)
\end{gathered}
$$

The overall effect on poverty of a marginal income transfer from any poor individual $h$ in type $i$ to any poor individual $k$ of a higher type is therefore to increase poverty. The poverty-increasing effect of the outward transfer from type $i$ is larger in absolute value than the poverty-decreasing effect of the inward transfer of the same amount to any higher type, $i+1$. This is a consequence of the fact that rank differences are integers, while $q^{F} \leq 1, \frac{\alpha}{z}\left(\frac{z-x}{z}\right)^{\alpha-1} \leq 1$, $\alpha, z \geq 0 .{ }^{4}$
$\odot$ The lower bound of $\mathbf{P}_{F G T}^{O I}$ is zero, whereas the upper bound is given by $n+1-\left(\sum_{i=1}^{n} q_{i} \quad i\right)$.

The minimum level is achieved when no individual in the society falls below the poverty line $z$. The maximum possible value of any poverty measure in $\mathbf{P}_{F G T}^{O}$ is achieved when all individuals in the society are poor and the ranks are determined according to the population share of the groups, with the smallest group being ranked 1 and the largest $n$. This means that (a) the maximum value of $\mathbf{P}_{F G T}^{O I}$ cannot be defined without knowing the precise population share of each $i$ group; and (b) the upper bound can change across distributions -across time and space.
$\odot$ When there is perfect equality of opportunity, the poverty status is independent of the group to which the person belongs to. In that case, $\mathbf{P}_{F G T}^{O I}$ is given by

$$
P_{F G T}^{O I}(F(x), z)=P\left(n+1-\sum_{i=1}^{n} q_{i} i\right)
$$

where $P$ is the poverty rate common to all types, i.e. $P_{i}=P, \forall i \in\{1, \ldots, n\}$.
On the other hand, if one is not concerned with inequality within types, hence chooses the family $\mathbf{P}^{O}$ instead of $\mathbf{P}^{O I}$, then $\alpha$ can be set equal 1 or 0 .

The opportunity-sensitive poverty gap measure $P_{G}^{O}(F(x, z)) \in \mathbf{P}_{G}^{O} \subseteq \mathbf{P}_{F G T}^{O}$ sets $\alpha=1$ so that $p(x)=\frac{z-x}{z}$ for all $x<z$. Hence,

$$
\begin{equation*}
P_{G}^{O}(F(x), z)=\sum_{i=1}^{n} q_{i}^{F}(n+1-i) \int_{0}^{z} \frac{(z-x)}{z} f_{i}(x) d x \tag{4}
\end{equation*}
$$

[^2]When $\alpha$ equals zero, $p(x)=1$ for all $x<z$, we obtain the opportunitysensitive poverty headcount measure defined as

$$
\begin{equation*}
P_{H}^{O}(F, z)=\sum_{i=1}^{n} q_{i}^{F}(n+1-i) \int_{0}^{z} f_{i}(x) d x=\sum_{i=1}^{n} q_{i}^{F}(n+1-i) H_{i} \tag{5}
\end{equation*}
$$

where for each type $i \in\{1, \ldots, n\}: H_{i}(F, z)=F_{i}(z)$ is the proportion of poor individuals in type $i$. Following Roemer's (1998) proposal, $H_{i}$ can be interpreted as the degree of effort -expressed in percentile terms- necessary to escape poverty if endowed with circumstances $\mathbf{c}_{i}{ }^{5}$.

## 6 Poverty, inequalities and opportunities in Brazil

In this section, we provide an empirical application to the approach to poverty measurement introduced in the paper. Using data from Brazil in the last decade, we show that when circumstances are included in the measurement of poverty we are able to uncover some aspects of poverty not captured when using traditional poverty measures.

We use the yearly Brazilian Pesquisa Nacional por Amostra de Domicilios (PNAD), the national representative household survey, from 2001 to 2008. Income is defined as monthly per capita household income, expressed in current Brazilean real. Household income are computed as the sum of all household members' individual incomes, including earning from all jobs, and all other reported incomes, including those from assets, pensions and transfers. We use national poverty lines for moderate poverty, as reported in Socio-Economic Database for Latin America and the Caribbean (SEDLAC) database. Moderate poverty lines, defined as twice the severe poverty line, are adjusted by regional and urban-rural price differentials. For Sao Paulo city, for instance, the moderate poverty line is set approximately just below US $\$ 4$ dollars a day per person (including PPP conversions).

Given the information available in the survey we can only consider two circumstances: region of birth and race. The former is coded in 5 categories (North, Northeast, Southeast, South, Center-west) and the latter in three (white/east Asians, black/mixed race, indigenous). ${ }^{6}$ Individuals sharing the same region of birth and race form a type. We thus have 15 types. The largest groups are white

[^3]and East Asian born in the southeast (the richest), black and mixed race from the Northeast (the poorest), and black and mixed race born in the Southeastern region (in between) -see Table 1.

We compute the opportunity-sensitive rank dependent FGT measures (3), for three alternative degree of inequality aversion within types, i.e. $\alpha=0,1,2$. Types are ranked accordingly to their poverty headcount rate in each year - see Table 1. To complement the analysis, we also present a measure of inequality of opportunity (and total inequality) for the total population and among the poor -Table 3. We choose to use the non-parametric version of inequality of opportunity index based on the mean log deviation (see Checchi and Peragine, 2005, 2010 and Ferreira and Gignoux, 2008 ). We present both the ex-ante and ex-post measures. These measures represent two alternative approaches to measure inequality of opportunity, each adopting a different definition of compensation between types, but they tend to be highly correlated (Checchi et al, 2011).

Figure 2 presents the main results of the section. ${ }^{7}$ For both sets of measures poverty has decreased between 2001 and 2008, for all degrees of within-type inequality aversion considered. What is more interesting, though, is the evolution of the measures between 2004 and 2006 where they diverge. While according to standard measures poverty has declined steadily throughout this period, opportunity-sensitive poverty measures suggests that poverty went down sharply between 2004 and 2005, and increased slightly in the following year. [A similar discrepancy is observed between 2002 and 2003]


Figure 2: Poverty and opportunity-sensitive poverty in Brazil from 2001 to 2008. Complete sample ( 15 types).
Source: Authors' calculation from PNAD
The discrepancies between the traditional poverty measures and those sensitive to equality across types must be due to at least one of the three components

[^4]of the $P_{F G T}^{O I}$ measure: the type-specific $F G T$, the type population proportion, and the rank of the types. To better understand the differences found in the evolution of poverty, figure 3 presents -in the right panel- the opportunity-sensitive poverty headcount (solid line), and those that would be obtained if each one of the three components were kept fixed at the initial year. From this graph we can see that (1) as expected, the impressive drop in overall poverty is largely driven by changes in the type-specific poverty rates; (2) the re-ranking of types seems to be the largest determinants of the discrepancies between the standard FGT and the opportunity-sensitive FGT. This is because the evolution of $F G T(0)^{O I}$ fixing the ranks of types (opFGT(0)-fixed ranks) mimics quite closely that of the $F G T(0)$, increasing between 2002 and 2003, and steadily falling between 2004 and 2006. Looking at table 1 we see that the position of one of the largest and poorest groups (Black-mixed race from the Northeast) fluctuates between the first and the third position in these key years of discrepancies. ${ }^{8}$


Figure 3: Poverty headcount, opportunity-sensitive poverty headcount and simulations in Brazil from 2001 to 2008. Complete sample ( 15 types).
Source: Authors' calculation from PNAD

The above analysis stresses the importance that the opportunity-sensitive poverty measure gives to the order of types. In this particular exercise, types are ranked by their headcount poverty ratios. If those ratios are predicted with error, as it might be the case when types have few observations, this could have a large impact on the overall measure particularly since it can affect the ranking (and thus type-weight) of types. Therefore, it would be sensible to investigate whether the results found are robust to these sorts of errors. To that end, we compute both sets of poverty measures excluding all the types with indigenous population which, taken together, represent less than $0.2 \%$ of the total population, and each of them they have between 60 and 400 observations.

[^5]As a consequence, the new calculations are done with 10 types, defined according to two categories of race and five categories for region of birth.



Figure 4: Poverty and opportunity-sensitive poverty in Brazil from 2001 to 2008. Reduced sample (10 types).
Source: Authors' calculation from PNAD
Figure 4 presents the overall poverty measures, and table 2 the ranking and poverty headcount of each type. ${ }^{9}$ With the new classification the discrepancies between the standard $F G T$ and the one sensitive to opportunity disappears. Poverty throughout the period has been decreasing consistently, with a slight bump between 2002 and 2003. This is due to the fact that the population proportions change very little in the decade, and there is almost no-re-ranking of types once they small ones are excluded from the analysis (see table 2). Indeed, only two pairs of types switch their position throughout the eight years of analysis (blacks from the south and whites from the north, in the 4th and 5th position, and white from the south and whites from the center-west, in the 8th and 9 th positions). In sum, this second exercise with fewer types and with more stables positions shows a extremely similar evolution of poverty throughout the 2000s.

### 6.1 Sequential test for poverty dominance

To be completed

[^6]| Race | Region of birth | $\begin{gathered} \text { Pop prop } \\ \text { in } 2001 \end{gathered}$ | 2001 |  | 2002 |  | 2003 |  | 2004 |  | 2005 |  | 2006 |  | 2007 |  | 2008 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | rk | FGT0 | $r k$ | FGT0 | rk | FGT0 | $r k$ | FGT0 | $r k$ | FGT0 | rk | FGT0 | rk | FGT0 | $r k$ | FGT0 |
| Indigenous | Center-west | 0.03 | 1 | 0.59 | 4 | 0.49 | 1 | 0.69 | 1 | 0.57 | 2 | 0.49 | 4 | 0.41 | 7 | 0.27 | 4 | 0.30 |
| Black/Mix | Northeast | 22.63 | 2 | 0.58 | 2 | 0.56 | 3 | 0.58 | 2 | 0.55 | 3 | 0.49 | 1 | 0.47 | 1 | 0.43 | 2 | 0.39 |
| Black/Mix | North | 3.76 | 3 | 0.50 | 3 | 0.50 | 4 | 0.50 | 4 | 0.50 | 6 | 0.44 | 2 | 0.42 | 2 | 0.41 | 3 | 0.34 |
| Indigenous | Northeast | 0.04 | 4 | 0.49 | 1 | 0.57 | 2 | 0.59 | 3 | 0.53 | 5 | 0.46 | 3 | 0.41 | 4 | 0.38 | 1 | 0.42 |
| Indigenous | South | 0.02 | 5 | 0.45 | 11 | 0.24 | 6 | 0.40 | 6 | 0.38 | 4 | 0.47 | 8 | 0.27 | 8 | 0.27 | 6 | 0.28 |
| White/EA | Northeast | 10.87 | 6 | 0.40 | 5 | 0.42 | 5 | 0.42 | 5 | 0.41 | 7 | 0.36 | 6 | 0.34 | 5 | 0.32 | 5 | 0.29 |
| Black/Mix | South | 2.53 | 7 | 0.38 | 6 | 0.37 | 8 | 0.37 | 9 | 0.33 | 9 | 0.28 | 9 | 0.27 | 9 | 0.23 | 9 | 0.20 |
| White/EA | North | 1.42 | 8 | 0.35 | 7 | 0.36 | 7 | 0.38 | 8 | 0.34 | 8 | 0.32 | 7 | 0.30 | 6 | 0.29 | 8 | 0.24 |
| Black/Mix | Southeast | 14.30 | 9 | 0.33 | 9 | 0.31 | 10 | 0.32 | 10 | 0.30 | 10 | 0.24 | 10 | 0.23 | 10 | 0.21 | 10 | 0.18 |
| Black/Mix | Center-west | 2.96 | 10 | 0.31 | 10 | 0.31 | 11 | 0.31 | 11 | 0.27 | 11 | 0.23 | 11 | 0.21 | 11 | 0.19 | 12 | 0.16 |
| Indigenous | North | 0.01 | 11 | 0.29 | 8 | 0.33 | 9 | 0.36 | 7 | 0.38 | 1 | 0.55 | 5 | 0.39 | 3 | 0.39 | 7 | 0.26 |
| Indigenous | Southeast | 0.03 | 12 | 0.28 | 12 | 0.23 | 12 | 0.31 | 12 | 0.18 | 12 | 0.20 | 12 | 0.18 | 12 | 0.17 | 11 | 0.17 |
| White/EA | South | 13.30 | 13 | 0.20 | 14 | 0.19 | 14 | 0.18 | 13 | 0.17 | 14 | 0.14 | 13 | 0.13 | 14 | 0.11 | 13 | 0.10 |
| White/EA | Center-west | 2.41 | 14 | 0.19 | 13 | 0.19 | 13 | 0.18 | 14 | 0.16 | 13 | 0.15 | 14 | 0.12 | 13 | 0.12 | 14 | 0.09 |
| White/EA | Southeast | 25.70 | 15 | 0.16 | 15 | 0.16 | 15 | 0.16 | 15 | 0.15 | 15 | 0.12 | 15 | 0.11 | 15 | 0.10 | 15 | 0.08 |
| EA: East Asian |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |


| Race | Region of birth | $\begin{gathered} \text { Pop prop } \\ \text { in } 2001 \end{gathered}$ | 2001 |  | 2002 |  | 2003 |  | 2004 |  | 2005 |  | 2006 |  | 2007 |  | 2008 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | rk | FGT0 | rk | FGT0 | rk | FGT0 | rk | FGT0 | rk | FGT0 | rk | FGT0 | rk | FGT0 | rk | FGT0 |
| Black/Mix | Northeast | 22.66 | 1 | 0.58 | 1 | 0.56 | 1 | 0.58 | 1 | 0.55 | 1 | 0.49 | 1 | 0.47 | 1 | 0.43 | 1 | 0.39 |
| Black/Mix | North | 3.76 | 2 | 0.50 | 2 | 0.50 | 2 | 0.50 | 2 | 0.50 | 2 | 0.44 | 2 | 0.42 | 2 | 0.41 | 2 | 0.34 |
| White/EA | Northeast | 10.88 | 3 | 0.40 | 3 | 0.42 | 3 | 0.42 | 3 | 0.41 | 3 | 0.36 | 3 | 0.34 | 3 | 0.32 | 3 | 0.29 |
| Black/Mix | South | 2.53 | 4 | 0.38 | 4 | 0.37 | 5 | 0.37 | 5 | 0.33 | 5 | 0.28 | 5 | 0.27 | 5 | 0.23 | 5 | 0.20 |
| White/EA | North | 1.42 | 5 | 0.35 | 5 | 0.36 | 4 | 0.38 | 4 | 0.34 | 4 | 0.32 | 4 | 0.30 | 4 | 0.29 | 4 | 0.24 |
| Black/Mix | Southeast | 14.32 | 6 | 0.33 | 6 | 0.31 | 6 | 0.32 | 6 | 0.30 | 6 | 0.24 | 6 | 0.23 | 6 | 0.21 | 6 | 0.18 |
| Black/Mix | Center-west | 2.96 | 7 | 0.31 | 7 | 0.31 | 7 | 0.31 | 7 | 0.27 | 7 | 0.23 | 7 | 0.21 | 7 | 0.19 | 7 | 0.16 |
| White/EA | South | 13.32 | 8 | 0.20 | 9 | 0.19 | 9 | 0.18 | 8 | 0.17 | 9 | 0.14 | 8 | 0.13 | 9 | 0.11 | 8 | 0.10 |
| White/EA | Center-west | 2.41 | 9 | 0.19 | 8 | 0.19 | 8 | 0.18 | 9 | 0.16 | 8 | 0.15 | 9 | 0.12 | 8 | 0.12 | 9 | 0.09 |
| White/EA | Southeast | 25.73 | 10 | 0.16 | 10 | 0.16 | 10 | 0.16 | 10 | 0.15 | 10 | 0.12 | 10 | 0.11 | 10 | 0.10 | 10 | 0.08 |

Table 2: Poverty profiles. Poverty headcount ratio (FGT0) by type in Brazil from 2001 to 2008 (ordered according to poverty headcount ratio in 2001). Reduced sample (10 types).
Source: Authors' calculation from PNAD

| Year | Average <br> income | Gini | mean log <br> deviation | Ex-ante <br> inequality | Ex-post <br> inequality |
| :---: | :---: | :---: | :---: | ---: | ---: |
| 2001 | 347 | 0.597 | 0.728 | 0.147 | 0.230 |
| 2002 | 382 | 0.590 | 0.692 | 0.142 | 0.217 |
| 2003 | 421 | 0.584 | 0.688 | 0.150 | 0.234 |
| 2004 | 455 | 0.574 | 0.653 | 0.142 | 0.227 |
| 2005 | 510 | 0.572 | 0.641 | 0.148 | 0.223 |
| 2006 | 572 | 0.565 | 0.623 | 0.143 | 0.223 |
| 2007 | 616 | 0.558 | 0.631 | 0.135 | 0.244 |
| 2008 | 693 | 0.551 | 0.606 | 0.129 | 0.234 |
|  |  |  |  |  |  |
|  |  |  | Only poor |  |  |
| 2001 | 61 | 0.299 | 0.294 | 0.009 | 0.260 |
| 2002 | 69 | 0.280 | 0.246 | 0.012 | 0.295 |
| 2003 | 79 | 0.288 | 0.271 | 0.012 | 0.292 |
| 2004 | 87 | 0.277 | 0.247 | 0.009 | 0.297 |
| 2005 | 89 | 0.267 | 0.237 | 0.010 | 0.319 |
| 2006 | 99 | 0.267 | 0.236 | 0.008 | 0.304 |
| 2007 | 101 | 0.283 | 0.324 | 0.005 | 0.322 |
| 2008 | 110 | 0.274 | 0.304 | 0.004 | 0.347 |

Table 3: Average income (in current real) and inequality measures in Brazil from 2001 to 2008. Complete sample ( 15 types).
Source: Authors' calculation from PNAD

## Appendix

## Proof - Theorem 1

$P(F(x), z) \geq P(G(x), z) \Longleftrightarrow$

$$
\Delta P=\sum_{i=1}^{j} q_{i}^{F} \int_{0}^{z} p_{i}(x) f_{i}(x) d x-\sum_{i=1}^{j} q_{i}^{G} \int_{0}^{z} p_{i}(x) g_{i}(x) d x \geq 0
$$

Integrating by parts: $\int v d u=u v-\int u d v: v=p_{i}(x), u=F_{i}(x)$ from which:

$$
\int_{0}^{z} p_{i}(x) f_{i}(x)=\left[F_{i}(x) p_{i}(x)\right]_{0}^{z}-\int_{0}^{z} F_{i}(x) p_{i}^{\prime}(x) d x
$$

$\Delta P$ becomes:
$\Delta P=\sum_{i=1}^{j} q_{i}^{F}\left(\left[F_{i}(x) p_{i}(x)\right]_{0}^{z}-\int_{0}^{z} F_{i}(x) p_{i}^{\prime}(x) d x\right)-\sum_{i=1}^{j} q_{i}^{G}\left(\left[G_{i}(x) p_{i}(x)\right]_{0}^{z}-\int_{0}^{z} G_{i}(x) p_{i}^{\prime}(x) d x\right)$
If $F_{i}(0)=0$, then $\left[F_{i}(x) p_{i}(x)\right]_{0}^{z}=\left[G_{i}(x) p_{i}(x)\right]_{0}^{z}=0$, as $p_{i}(z)=0$. Hence

$$
\begin{gathered}
\Delta P=\sum_{i=1}^{j} q_{i}^{F}\left(-\int_{0}^{z} F_{i}(x) p_{i}^{\prime}(x) d x\right)-\sum_{i=1}^{j} q_{i}^{G}\left(-\int_{0}^{z} G_{i}(x) p_{i}^{\prime}(x) d x\right) \\
\Delta P=\sum_{i=1}^{j} q_{i}^{G} \int_{0}^{z} G_{i}(x) p_{i}^{\prime}(x) d x-\sum_{i=1}^{j} q_{i}^{F} \int_{0}^{z} F_{i}(x) p_{i}^{\prime}(x) d x
\end{gathered}
$$

Integrating again by parts: $\int u d v=u v-\int d u v, u=p_{i}^{\prime}(x)$ and $v=\int^{x} F_{i}(y)$

$$
\begin{aligned}
\Delta P & =\sum_{i=1}^{j} q_{i}^{G}\left(\left[p_{i}^{\prime}(x)\right]_{0}^{Z} \int_{0}^{Z} G_{i}(x)-\int_{0}^{z} p_{i}^{\prime \prime}(x) \int^{x} G_{i}(y) d y\right) \\
& -\sum_{i=1}^{j} q_{i}^{F}\left(\left[p_{i}^{\prime}(x)\right]_{0}^{Z} \int_{0}^{Z} F_{i}(x)-\int_{0}^{z} p_{i}^{\prime \prime}(x) \int^{x} F_{i}(y) d y\right)
\end{aligned}
$$

that is

$$
\Delta P=\sum_{i=1}^{j}\left[p_{i}^{\prime}(x)\right]_{0}^{Z}\left(q_{i}^{G} \int_{0}^{Z} G_{i}(x)-q_{i}^{F} \int_{0}^{Z} F_{i}(x)\right)+\int_{0}^{z} p_{i}^{\prime \prime}(x) \int^{x}\left(q_{i}^{F} F_{i}(y)-q_{i}^{G} G_{i}(y)\right) d y
$$

Assuming $p_{i}^{\prime \prime}(x)=0$ the second term disappears.
Now, $\int_{0}^{Z} F_{i}(x)=z F_{i}(z)-\mu\left(F_{i}^{z}\right)$,
where $\mu\left(F_{i}^{Z}\right)$ is the mean of the distribution $F_{i}$ truncated at $z$.
This comes from the fact that:

$$
\mu\left(F_{i}^{z}\right)=\int_{0}^{z} x f(x) d x
$$

integrating by parts one obtains

$$
\mu\left(F_{i}^{z}\right)=[x F(x)]_{0}^{z}-\int_{0}^{z} F(x) d x=z H-\int_{0}^{z} F(x) d x
$$

From which:

$$
\Delta P=\sum_{i=1}^{j} p_{i}^{\prime}(z)\left[q_{i}^{G}\left(z G_{i}(z)-\mu\left(G_{i}^{z}\right)\right)-q_{i}^{F}\left(z F_{i}(z)-\mu\left(F_{i}^{z}\right)\right)\right]
$$

By property 3 and applying Abel lemma, $\Delta P \geq 0$ if and only if

$$
\sum_{i=1}^{j}\left[q_{i}^{G}\left(z G_{i}(z)-\mu\left(G_{i}^{z}\right)\right)-q_{i}^{F}\left(z F_{i}(z)-\mu\left(F_{i}^{z}\right)\right)\right] \leq 0
$$

This can be written in the following ways:

$$
\begin{equation*}
\sum_{i=1}^{j} z\left(q_{i}^{F} F_{i}(z)-q_{i}^{G} G_{i}(z)\right)+\sum_{i=1}^{j}\left(q_{i}^{G} \mu\left(G_{i}^{z}\right)-q_{i}^{F} \mu\left(F_{i}^{z}\right)\right) \geq 0 \tag{7}
\end{equation*}
$$

Alternatively, adding and subtracting $q_{i}^{F} \mu\left(G_{i}^{z}\right)$ and $z q_{i}^{F} G(z)$, as

$$
\begin{gather*}
\sum_{i=1}^{j} z\left(q_{i}^{F} F_{i}(z)-q_{i}^{G} G_{i}(z)-z q_{i}^{F} G_{i}(z)+z q_{i}^{F} G_{i}(z)\right)+ \\
\sum_{i=1}^{j}\left(q_{i}^{G} \mu\left(G_{i}^{z}\right)-q_{i}^{F} \mu\left(G_{i}^{z}\right)-q_{i}^{F} \mu\left(F_{i}^{z}\right)+q_{i}^{F} \mu\left(G_{i}^{z}\right)\right) \geq 0 \\
\left(q_{i}^{F}-q_{i}^{G}\right)\left(z G_{i}(z)-\mu\left(G_{i}^{z}\right)+z q_{i}^{F}\left(F_{i}(z)-G_{i}(z)\right)+q^{F}\left(\mu\left(G_{i}^{z}\right)-\mu\left(F_{i}^{z}\right)\right) \geq 0\right. \tag{8}
\end{gather*}
$$

We get a decomposition of the difference in responsibility sensitive poverty in three terms: the differences in population shares, in headcount poverty ratios, and in average incomes of the poor. The sign of the contribution of each term is positive as they are multiplied by a positive number:

$$
z G_{i}(z)-\mu\left(G_{i}^{z}\right)=\int_{0}^{z} G x d x \geq 0
$$

From (7) we obtain that sufficient conditions for $\Delta P \geq 0$ are:
(i) $\sum_{i=1}^{j} q_{i}^{F} \mu\left(F_{i}^{z}\right) \leq \sum_{i=1}^{j} q_{i}^{G} \mu\left(G_{i}^{z}\right), \quad \forall j \in\{1, \ldots, n\}$.
(ii) $\sum_{i=1}^{j} q_{i}^{F} H_{i F} \geq \sum_{i=1}^{j} q_{i}^{G} H_{i G}, \quad \forall j \in\{1, \ldots, n\}$.

From (8) we obtain that sufficient conditions for $\Delta P \geq 0$ are:
(i) $\sum_{i=1}^{j} \mu\left(F_{i}^{z}\right) \leq \sum_{i=1}^{j} \mu\left(G_{i}^{z}\right), \quad \forall j \in\{1, \ldots, n\}$.
(ii) $\sum_{i=1}^{j} H_{i F} \geq \sum_{i=1}^{j} H_{i G}, \quad \forall j \in\{1, \ldots, n\}$.
(iii) $\sum_{i=1}^{j} q_{i}^{F} \geq \sum_{i=1}^{j} q_{i}^{G}, \quad \forall j \in\{1, \ldots, n\}$.

## Remark 1.2

If $F_{i}(0) \neq 0$
$\Delta P=\sum_{i=1}^{j} q_{i}^{F}\left(\left[F_{i}(x) p_{i}(x)\right]_{0}^{z}-\int_{0}^{z} F_{i}(x) p_{i}^{\prime}(x) d x\right)-\sum_{i=1}^{j} q_{i}^{G}\left(\left[G_{i}(x) p_{i}(x)\right]_{0}^{z}-\int_{0}^{z} G_{i}(x) p_{i}^{\prime}(x) d x\right)$
$\Delta P=\sum_{i=1}^{j}\left[q_{i}^{G} G_{i}(0) p_{i}(0)-q_{i}^{F} F_{i}(0) p_{i}(0)\right]+\left[\sum_{i=1}^{j} q_{i}^{G} \int_{0}^{z} G_{i}(x) p_{i}^{\prime}(x) d x-\sum_{i=1}^{j} q_{i}^{F} \int_{0}^{z} F_{i}(x) p_{i}^{\prime}(x) d x\right]$
we know the sufficient conditions for the second term to be positive, the first term adds a new condition:

$$
\begin{gather*}
\sum_{i=1}^{j}\left[q_{i}^{G} G_{i}(0) p_{i}(0)-q_{i}^{F} F_{i}(0) p_{i}(0)\right] \geq 0 \\
\sum_{i=1}^{j}\left[q_{i}^{G}\left(G_{i}(0)-F_{i}(0)\right)+\left(q_{i}^{G}-q_{i}^{F}\right) F_{i}(0)\right] \geq 0 \\
\sum_{i=1}^{j} G_{i}(0) \geq \sum_{i=1}^{j} F_{i}(0) \tag{9}
\end{gather*}
$$

Where $F_{i}(0), G_{i}(0)$ are the proportions of the individuals in the type $i$ with no income. The sum of the these proportions at each step $i \leq j=1, \ldots, n$ must be larger in $G$ than in $F$.

## Annex

| Year | Population | FGT0 | FGT1 | FGT2 | opFGT0 | opFGT1 | opFGT2 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 2001 | $165,502,656$ | 0.336 | 0.154 | 0.095 | 3.163 | 1.508 | 0.943 |
| 2002 | $168,488,864$ | 0.330 | 0.145 | 0.087 | 3.142 | 1.453 | 0.879 |
| 2003 | $170,719,232$ | 0.337 | 0.152 | 0.092 | 3.007 | 1.420 | 0.876 |
| 2004 | $178,511,392$ | 0.321 | 0.138 | 0.082 | 3.031 | 1.369 | 0.822 |
| 2005 | $181,305,376$ | 0.278 | 0.115 | 0.067 | 2.439 | 1.056 | 0.620 |
| 2006 | $183,821,648$ | 0.261 | 0.108 | 0.063 | 2.661 | 1.145 | 0.670 |
| 2007 | $186,286,400$ | 0.244 | 0.103 | 0.062 | 2.518 | 1.098 | 0.662 |
| 2008 | $186,307,376$ | 0.216 | 0.089 | 0.053 | 2.163 | 0.905 | 0.531 |

Table 4: Poverty and opportunity-sensitive poverty in Brazil 2001-2008.
Source: Authors' calculation from PNAD

| Year | Population | FGT0 | FGT1 | FGT2 | opFGT0 | opFGT1 | opFGT2 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 2001 | 165288624 | 0.336 | 0.154 | 0.095 | 2.331 | 1.108 | 0.691 |
| 2002 | 168183184 | 0.329 | 0.145 | 0.087 | 2.266 | 1.045 | 0.632 |
| 2003 | 170400640 | 0.337 | 0.151 | 0.092 | 2.326 | 1.095 | 0.675 |
| 2004 | 178212944 | 0.321 | 0.138 | 0.082 | 2.262 | 1.015 | 0.608 |
| 2005 | 180964464 | 0.278 | 0.115 | 0.067 | 1.985 | 0.856 | 0.501 |
| 2006 | 183327968 | 0.261 | 0.108 | 0.063 | 1.892 | 0.810 | 0.474 |
| 2007 | 185766256 | 0.243 | 0.103 | 0.062 | 1.760 | 0.764 | 0.460 |
| 2008 | 185792480 | 0.216 | 0.089 | 0.052 | 1.596 | 0.665 | 0.390 |

Table 5: Poverty and opportunity-sensitive poverty in Brazil 2001-2008.
Source: Authors' calculation from PNAD

| Race | Region of | FGT1-Poverty gap |  |  |  |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | birth | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 |
| Indigenous | Center-west | 0.495 | 0.251 | 0.347 | 0.315 | 0.248 | 0.189 | 0.129 | 0.143 |
| Black/Mix | Northeast | 0.289 | 0.274 | 0.286 | 0.260 | 0.222 | 0.209 | 0.196 | 0.168 |
| Black/Mix | North | 0.222 | 0.220 | 0.221 | 0.217 | 0.177 | 0.166 | 0.172 | 0.135 |
| Indigenous | Northeast | 0.297 | 0.333 | 0.350 | 0.305 | 0.241 | 0.200 | 0.203 | 0.239 |
| Indigenous | South | 0.195 | 0.091 | 0.161 | 0.183 | 0.211 | 0.118 | 0.168 | 0.134 |
| White/EA | Northeast | 0.192 | 0.193 | 0.198 | 0.183 | 0.157 | 0.148 | 0.142 | 0.125 |
| Black/Mix | South | 0.168 | 0.151 | 0.151 | 0.131 | 0.110 | 0.102 | 0.091 | 0.076 |
| White/EA | North | 0.150 | 0.150 | 0.159 | 0.135 | 0.123 | 0.117 | 0.118 | 0.093 |
| Black/Mix | Southeast | 0.142 | 0.125 | 0.132 | 0.116 | 0.093 | 0.087 | 0.080 | 0.068 |
| Black/Mix | Center-west | 0.126 | 0.121 | 0.125 | 0.098 | 0.085 | 0.075 | 0.070 | 0.064 |
| Indigenous | North | 0.152 | 0.132 | 0.146 | 0.201 | 0.271 | 0.183 | 0.190 | 0.126 |
| Indigenous | Southeast | 0.115 | 0.109 | 0.117 | 0.062 | 0.082 | 0.086 | 0.104 | 0.066 |
| White/EA | South | 0.080 | 0.070 | 0.068 | 0.063 | 0.050 | 0.045 | 0.041 | 0.036 |
| White/EA | Center-west | 0.074 | 0.073 | 0.067 | 0.057 | 0.053 | 0.042 | 0.043 | 0.037 |
| White/EA | Southeast | 0.063 | 0.060 | 0.063 | 0.057 | 0.042 | 0.038 | 0.040 | 0.034 |

EA: East Asian

Table 6: Poverty profiles. Poverty gap (FGT1) by type in Brazil 2001-2008 (ordered according to poverty headcount ratio in 2001).Source: Authors' calculation from PNAD

| Race | Region of | FGT2 - Poverty |  |  |  |  |  |  | gap |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 2quare |  |  |  |  |  |  |  |  |
|  | birth | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 |
| Indigenous | Center-west | 0.470 | 0.168 | 0.228 | 0.245 | 0.165 | 0.121 | 0.084 | 0.100 |
| Black/Mix | Northeast | 0.184 | 0.169 | 0.180 | 0.160 | 0.133 | 0.124 | 0.119 | 0.099 |
| Black/Mix | North | 0.132 | 0.127 | 0.130 | 0.125 | 0.096 | 0.091 | 0.099 | 0.075 |
| Indigenous | Northeast | 0.222 | 0.240 | 0.261 | 0.220 | 0.181 | 0.125 | 0.150 | 0.163 |
| Indigenous | South | 0.127 | 0.043 | 0.097 | 0.124 | 0.125 | 0.076 | 0.140 | 0.087 |
| White/EA | Northeast | 0.119 | 0.117 | 0.122 | 0.109 | 0.093 | 0.088 | 0.087 | 0.074 |
| Black/Mix | South | 0.103 | 0.085 | 0.087 | 0.075 | 0.062 | 0.057 | 0.052 | 0.042 |
| White/EA | North | 0.088 | 0.085 | 0.091 | 0.076 | 0.067 | 0.064 | 0.070 | 0.052 |
| Black/Mix | Southeast | 0.086 | 0.072 | 0.078 | 0.067 | 0.052 | 0.050 | 0.046 | 0.039 |
| Black/Mix | Center-west | 0.073 | 0.070 | 0.073 | 0.053 | 0.048 | 0.041 | 0.041 | 0.039 |
| Indigenous | North | 0.104 | 0.079 | 0.092 | 0.120 | 0.171 | 0.110 | 0.125 | 0.074 |
| Indigenous | Southeast | 0.071 | 0.066 | 0.059 | 0.032 | 0.046 | 0.055 | 0.084 | 0.045 |
| White/EA | South | 0.046 | 0.038 | 0.038 | 0.035 | 0.027 | 0.024 | 0.023 | 0.021 |
| White/EA | Center-west | 0.043 | 0.043 | 0.037 | 0.031 | 0.030 | 0.024 | 0.026 | 0.023 |
| White/EA | Southeast | 0.039 | 0.035 | 0.037 | 0.034 | 0.024 | 0.022 | 0.026 | 0.022 |

EA: East Asian

Table 7: Poverty profiles. Poverty gap square (FGT2) by type in Brazil 20012008
(ordered according to poverty headcount ratio in 2001). Source: Authors' calculation from PNAD

| Race | Region of | Population propotion |  |  |  |  |  |  |  |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | birth | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 |
| Indigenous | Center-west | 0.03 | 0.03 | 0.02 | 0.02 | 0.03 | 0.03 | 0.04 | 0.04 |
| Black/Mix | Northeast | 22.63 | 22.41 | 22.94 | 22.31 | 22.43 | 22.45 | 22.32 | 22.82 |
| Black/Mix | North | 3.76 | 3.78 | 3.94 | 5.31 | 5.35 | 5.48 | 5.36 | 5.53 |
| Indigenous | Northeast | 0.04 | 0.07 | 0.07 | 0.07 | 0.07 | 0.11 | 0.09 | 0.10 |
| Indigenous | South | 0.02 | 0.02 | 0.04 | 0.02 | 0.03 | 0.04 | 0.04 | 0.05 |
| White/EA | Northeast | 10.87 | 10.97 | 10.49 | 10.50 | 10.25 | 10.07 | 10.11 | 10.23 |
| Black/Mix | South | 2.53 | 2.66 | 2.73 | 2.61 | 2.89 | 3.03 | 3.17 | 3.15 |
| White/EA | North | 1.42 | 1.45 | 1.41 | 1.66 | 1.66 | 1.67 | 1.80 | 1.64 |
| Black/Mix | Southeast | 14.30 | 14.39 | 14.83 | 14.86 | 15.89 | 15.68 | 15.89 | 16.17 |
| Black/Mix | Center-west | 2.96 | 3.00 | 3.05 | 3.11 | 3.06 | 3.13 | 3.24 | 3.21 |
| Indigenous | North | 0.01 | 0.02 | 0.01 | 0.02 | 0.03 | 0.03 | 0.04 | 0.03 |
| Indigenous | Southeast | 0.03 | 0.04 | 0.04 | 0.04 | 0.04 | 0.06 | 0.06 | 0.06 |
| White/EA | South | 13.30 | 12.96 | 12.85 | 12.73 | 12.34 | 12.22 | 12.05 | 11.91 |
| White/EA | Center-west | 2.41 | 2.46 | 2.38 | 2.35 | 2.39 | 2.37 | 2.33 | 2.39 |
| White/EA | Southeast | 25.70 | 25.73 | 25.19 | 24.41 | 23.54 | 23.63 | 23.45 | 22.67 |

EA: East Asian

Table 8: Poverty profiles. Population proportion by type in Brazil 2001-2008 (ordered according to poverty headcount ratio in 2001).
Source: Authors' calculation from PNAD

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[^0]:    ${ }^{1}$ We restrict ourselves to uni-dimensional poverty measurement in this paper. The implications of sensitivity to inequality of opportunity for multidimensional poverty measurement are left for future work. Additivity and aversion to inequality among the poor are also often included in the list of axioms.
    ${ }^{2}$ Differences in needs are of course permissible, but are typically assumed to be taken into account by equivalence scales.

[^1]:    ${ }^{3}$ This requirement is expression of the ex-ante Compensation Principle (see Fleurbaey and Peragine 2009).

[^2]:    ${ }^{4}$ There is also a potential clash between IOA and the standard inequality aversion for the full distribution. It is perfectly possible that, in the above example, $\left(x^{h} \mid h \in \mathbf{T}_{i}^{z}\right)>$ $\left(x_{k} \mid k \in \mathbf{T}_{i+1}^{z}\right)$. There exists a non empty set of possible transfers which are both progressive in the standard Pigou-Dalton sense, and opportunity-regressive in the sense of IOA. The maximum reconciliation between the Pigou-Dalton transfer axiom (with no regard to types) and IOA is given by the within-type inequality aversion (IAW) axiom defined above.

[^3]:    ${ }^{5}$ If $x_{i}^{\text {max }}<z$, type $i$ is said to be 'poverty-trapped' in the sense that, given circumstances $c_{i}$ that define $\mathbf{T}_{\mathbf{i}}$, no amount of effort is sufficient to escape poverty. If one were to take the view that the inevitability of poverty in such a type is qualitatively different from differences in the degree of effort which is required to escape poverty, $F_{i}(z)$, then one could capture such a distinction by means of a different opportunity-sensitive poverty measure (3). For instance, the measure of type-specific poverty inevitability is given by:

    $$
    \begin{equation*}
    U(F(x), z)=\sum_{i=1}^{n} q_{i}^{F} \mathbf{1}\left(x_{i}^{\max }<z\right) \tag{6}
    \end{equation*}
    $$

    where $\mathbf{1}$ is an indicator function. Expression (6) gives the measure of the population that belongs to poverty-trapped types in a particular society. Note that this measure is a partial index of poverty opportunity that is based on different axioms. $U(F(x), z)$ is not based on a rank of types, is based on a focus axiom that identifies poverty as the condition 'belonging to a trapped-type' than simply count the proportion of poor in the population.
    ${ }^{6}$ We exclude individuals who were born abroad and those classified as "other" in the variable race, as the number of observations was too low to make appropriate inferences.

[^4]:    ${ }^{7}$ Table 4 in the Annex presents the numbers associated with this figure.

[^5]:    ${ }^{8}$ In the Annex we include similar tables for poverty gap and squared poverty gap by type for each year, as well as the proportion of the population for each type for the whole period.

[^6]:    ${ }^{9}$ Table 5 in the Annex presents the numbers associated with figure 4.

