GROWTH AND INCOME MOBILITY: AN AXIOMATIC ANALYSIS

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Growth and income mobility: an axiomatic analysis*

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Abstract

In this paper we present an axiomatic framework to measure mobility, as an additional distributional implication of growth. For an individual, we argue that mobility is determined by initial income and both the post-growth income he would have in the absence and in the presence of mobility. We then aggregate this index into a measure of societal mobility, which can be expressed as the weighted average of individual mobility. We also propose a different family of societal mobility, which can be expressed as a weighted average of individual mobility, with weights based on the rank in the initial distribution of income. We argue that our measures can be used to complement standard analysis of pro-poor growth.

1 Introduction

A large body of the literature is involved in the analysis of the distributional implications of growth (see Bourguignon, 2003, 2004; Ferreira, 2010). In particular, a branch defined "pro-poor growth" is focussed on the evaluation of the effect of growth on poverty (see Duclos, 2009; Kakwani and Son, 2008; Essama-Nssah and Lambert, 2009; Zheng, 2010). In this literature the procedure used to analyze a growth process is based on the comparison of the pre-growth and post-growth distribution, which does not account for the possibility that there might be individuals who experience poverty in both periods, while the reshuffling along the distribution allow some individuals to escape poverty and some others to fall into poverty. This is due to the fact that, by imposing the anonymity axiom, these studies ignore the individuals' identity and, with it, the individual mobility along the distribution. One basic tool used in this literature is the growth incidence curve (GIC). GIC measures the quantile-specific rate of

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economic growth between two points in time as a function of each percentile (Ravallion and Chen, 2003).

Bourguignon (2010) and Grimm (2007) suggest to relax anonymity and derive a non-anonymous Growth Incidence Curve, able to encompass mobility as an additional implication of growth. The na-GIC measures the individual-specific rate of economic growth between two points in time, thus comparing the income of individuals which were in the same initial position, independently of the position they acquire in the final distribution of income.

The purpose of this paper is to further explore this issue, with an axiomatic characterization of a measure of mobility, interpreted as an additional implication of growth. For, we deal with two main aspects: the definition of an individual measure of mobility and the definition of an index expressing the overall mobility of a society.

With regard to the first aspect, various interpretations of mobility have given rise to different formalizations. To be more specific, upon agreeing that mobility is a form of income transformation process from time t to time $t+1$, mobility measures can be classified according to a relativistic and an absolutistic approach, and according to complete and partial dominance ordering conditions. As a result, income mobility can be evaluated - for a given function m - ordinally, by looking at distributions ranking, focussing on the partial dominance conditions, or cardinally, by quantifying the amount of the income movement through a well defined index of mobility. In the latter case, a cardinal evaluation of mobility can be expressed in relative terms: $m(\lambda y; \lambda w) = m(y, w)$, for all $\lambda > 0$ and all $y, w \in \Re_{++}$; or in absolute terms: $m(y + \alpha, w + \alpha) = m(y, w)$. A relative mobility measure which is, by definition, scale invariant, means that an equiproportional change in all incomes does not affect a measure of mobility. The term relative mobility might also refer to positional movements. According to this view, an individual experiences relative mobility if and only if he changes his relative position in the income distribution over time (see on this D'Agostino and Dardanoni, 2009a; Jenkins and Van Kerm, 2006).

However, a relative approach is not able to capture particular aspects of mobility. A standard example is the following, consider the processes (3, 5) - (6, 10) and (30, 50) - (60, 100). A linearly homogeneous measure of mobility would classify the second transformation as more mobile, a relative measure would indicate that these two processes are identical. One interpretation of absolute mobility is based on the evaluation of gains and losses of income rather than income shares or positions, such as the concept of directional income movements (Fields and Ok, 1999a). A different interpretation of absolute mobility is encompassed by measures aimed at quantifying the absolute value of income changes, as would be the case in studies of non-directional income movements (D'Agostino and Dardanoni, 2009b; Fields and Ok, 1996, 1999b).

We axiomatize a non-directional income mobility measure. It is useful to stress that, in the development of our framework, we will leave out the assumption of anonymity. Relaxing anonymity entails a widening of the set of information used to construct standard measures of income mobility. First, we employ the information derived from the position in the initial distribution of income. In fact, it is with respect to this position that the individual measure of mobility will be derived. Second, we complement the information contained in the initial level of income with the information retained in two different distributions of post-growth income: one obtained by ordering individuals according to their position in the initial distribution, the other by ordering them according to their final income. The line of reasoning is that, relaxing anonymity implies being able to track the evolution of the economic situation of each single individual, and to unravel the effects of mobility in the overall growth measurement framework. We have to remark that by mobility we mean the reranking over time of individuals across income classes[1]. This procedure represents a new contribution of this work to the income mobility literature, that may bring about a different picture of the mobility of the population under analysis. A justification for this approach can be found in the progressivity literature, where reranking is evaluated as a result of the comparison between post-tax concentration curve and post-tax Lorenz curve[2].

With regard to the second aspect, important aggregation procedures have been recently proposed by Demuynck and Van de Gaer (2010) and Schluter and Van de Gaer (2010). In particular, Demuynck and Van de Gaer (2010) propose a characterization of a rank dependent aggregation procedure. This last step is accomplished by means of two axioms. The first is Weak Decomposability, which is a quite known axiom. The second is the axiom of Decomposability with respect to Highest Mobility (D-HM). That is, the income mobility of the overall society, composed by n individuals, depends on the income mobility of the group of $n-1$ most mobile individuals and the mobility of the least mobile individual. They also define the functional form of the weights, by imposing population invariance, that is, any k-fold reproduction of the society should leave aggregate mobility unchanged. Finally, they derive a measure of aggregate mobility, which increases more when there is an increment of mobility for individuals with lower individual mobility than when the same increment goes to individuals with higher individual mobility, expressing, therefore, aversion toward inequality in the distribution of individual mobility.

We instead propose two alternative aggregation procedures which allow to derive two general families of overall mobility measures.

Concerning the first family, our approach is different form Demuynck and Van de Gaer (2010) since, instead of D-HM, we add the requirement of Recursive Decomposability (RD), which implies that an overall measure of mobility will not be sensitive to the order of aggregation. In our framework, RD is a precondition to derive an expression of overall mobility as a weighted average of each individual mobility.

Concerning the second family, we formalize an aggregate index of mobility, by adopting the requirement of the absence on any joint effect between individual mobilities, on the determination of the value of aggregate mobility, along with

[1] See Fields and Ok (1999b) for a detailed survey on the different meanings of mobility and its measurement.

[2] See on this Lambert (2001). See also Jenkins and Van Kerm (2006) for a similar approach applied to income mobility evaluation.

the requirement of mobility intensity. A rank dependent measure of aggregate mobility[3] is obtained averaging individual mobilities with weights based on the relative position of each individual in the initial distribution of income, whereas, in Demuynck and Van de Gaer (2010) the weights depend on the relative position in the distribution of individual mobilities. For this last family of mobility measures, some dominance conditions will also be provided.

A final remark is that imposing the focus axiom allows to get a measure expressing the extent of mobility in determining the variation of a poverty index between two points in time. Thus, using standard axioms but removing anonymity enables to develop new measures of mobility. The ranking of distributions according to these indices can be used to complement the standard analysis of growth, in order to understand whether the overall poverty variation is determined, not only by growth, but also by the movements of individuals across income classes.

The work is structured as follows. In section 2 we introduce the analytical framework. In section 3 we derive the measure of individual mobility and the first family of aggregate mobility measures. In section 4 we derive the second family of aggregate mobility measures. In section 5 we conclude.

2 The analytical framework

We consider a set of individuals $\{1, ..., n\}$ and the following distributions of income. Let

$$\mathbf{x} = (x_1, ..., x_i, ..., x_n) \in \Re^n_{++}$$

be the initial distribution of income, with total income denoted by $x_+ = \sum_{i=1}^{n} x_i$. We assume that individual incomes are ordered increasingly: $x_1 \leq ... \leq x_i \leq ... \leq x_n$. Thus, x_1 is the income level of the poorest individual before growth. Let

$$\mathbf{w} = (w_1, ..., w_i, ..., w_n) \in \Re^n_{++}$$

be the post-growth distribution of income induced by the growth process, where individuals are ordered according to their position in the initial distribution of income. Therefore, w_i represents the post-growth income of the individual with position i in the initial distribution of income, and w_1 represents the post-growth income level of the poorest individual before growth, who is not necessarily the poorest individual after growth. Let

$$\tilde{\mathbf{w}} = (\tilde{w}_1, ..., \tilde{w}_i, ..., \tilde{w}_n) \in \Re^n_{++}$$

be the post-growth distribution of income where individuals are ordered according to their position in the final distribution of income: $\tilde{w}_1 \leq ... \leq \tilde{w}_i \leq ... \leq \tilde{w}_n$.

[3] See Aaberge and Mogstad (2010) for an alternative derivation of an index of aggregate mobility, which can be considered rank dependent, since it is based on on rank dependent measures of inequality. See also Jenkins and Van Kerm (2011).

Therefore, $\tilde{w}_1$ represents the final income of the poorest individual after growth, who is not necessarily the poorest individual before growth. This observation is relevant, since it is possible to interpret $\tilde{w}_1$ as the level of final income the poorest individual in $\mathbf{x}$ would have if he still were the poorest individual after growth. Thus, $\tilde{\mathbf{w}}$ can be interpreted as the post-growth distribution of income we would obtain in the absence of mobility[4], where, in the context of our analysis, mobility is interpreted as a form of reranking. This implies that, if there is only growth and not reranking, the two distributions of final income, $\mathbf{w}$ and $\tilde{\mathbf{w}}$ coincide.

Let $(x_i, w_i, \tilde{w}_i) \in \Re^3_{++}$ and $m_i(x_i, f_i(w_i, \tilde{w}_i)) \in \Re_+$, then $m_i : \Re^3_{++} \longrightarrow \Re_+$ is a real valued function measuring mobility of the individual with position i in the initial distribution of income, for all $i = 1, ..., n$. By definition, in the standard literature, mobility depends on the initial and final level of income[5]. Here, instead, as explained in the introduction, we widen the class of variables that determine mobility; hence, we assume that the arguments of each individual mobility are: the initial level of income, x_i and a function of two variables, f_i for all $i = 1, ..., n$, explaining the effect of income gains or losses due to re-ranking on individual mobility. Therefore, comparing w_1 to $\tilde{w}_1$ would give a flavour of the impact of reranking on individual income, not in terms of relative position, but in terms of differential income. It can be inferred then that f capture the extent of exchange mobility, while m capture both the extent of exchange mobility and structural mobility. For ease of exposition we will use the following notation: $m_i = m_i(x_i, f(w_i, \tilde{w}_i))$, denoting the amount of mobility of the individuals whose position in $\mathbf{x}$ is i. Then, let $M : \Re^n_{++} \times \Re^n_{++} \times \Re^n_{++} \rightarrow \Re_+$ be a real valued function measuring the aggregate distribution effects due to mobility.

3 Results

3.1 An individual measure of mobility

As explained before, m_i is not only function of the pre- and post-growth income level, but its arguments are the initial level of income and a function relating the post-growth incomes, in the absence and in the presence of mobility. Thus, through m_i, it is possible to evaluate how mobility - in the form of reranking - acts on individual income, once each individual has experienced growth, for a given amount of initial income. That is, the transformation from $\tilde{w}_i$ to w_i due to mobility is not evaluated independently of initial income.

It is necessary to point out that we do not impose anonymity, therefore, the names of income recipients matter for measuring the intensity of mobility. This simply enables to use an additional information with respect to standard measures of mobility, only based on initial and final income. That is, the information derived form the distribution of post-growth income, where individuals

[4] See on this Zheng (2010).

[5] An exception is Jenkins and Ven Kerm (2006).

are ordered according to the relative position in the initial distribution. We focus on the transformation of the income of the $i-th$ individual.

We now begin by imposing some properties, that we use to characterize first the function $f_i(\tilde{w}_i, w_i)$ for all $i = 1, ..., n$, which is only part of the measure of individual mobility.

Axiom 1 *Symmetry (**S**): For $n \geq 1$ and for all $w, \tilde{w} \in \Re_{++}$, $w \neq \tilde{w}$*

$$f(0,1) = f(1,0)$$

S states that what matters is the amount of the impact of mobility and not its direction, that is, a one dollar income gain and a one dollar income loss both are equally valued. We deal with no directional movements, therefore, we do not distinguish between 'good' and 'bad' movements of income, but we are only concerned with its amount. Our aim is to identify the role of mobility, as an additional distribution effect of growth, no matter its direction.

Note that the standard literature on distribution analysis uses anonymity and symmetry indistinctly. In our framework, instead, they do not refer to the same aspect. On the one hand, relaxing anonymity means that we focus on each single individual, who is distinguished by the others through his position in the initial distribution of income. In addition, it suggests that we use not only the information deriving from initial and final income, but we also employ the information deriving from his position in the initial income distribution. On the other hand, symmetry in our framework only implies that we do not discriminate between bad and good mobility.

Example *Consider a society with four individuals, an initial distribution of income: $x = (1,2,3,4)$, a distribution of final income where individuals are ordered according to the level of their post-growth income: $\tilde{w} = (3,4,6,7)$, and a distribution of final income where individuals are ordered according to their position in the initial distribution: $w = (4,6,3,7)$. Furthermore, assume that we measure individual mobility by the absolute difference between the incomes we focus on. Then, assuming S and anonymity implies that we would not distinguish between w and $\tilde{w}$. Therefore, the vector of individual mobilities would be $|\tilde{\mathbf{w}} - \mathbf{x}| = (2,2,3,3)$. Relaxing anonymity and imposing S, we would be able to distinguish between w and $\tilde{w}$; the vector of individual mobilities would be $|\mathbf{w} - \tilde{\mathbf{w}}| = (1,2,3,0)$ which is different from the one that would arise if we impose anonymity. Then, if aggregate mobility is taken to be the unweighted average of the individual mobilities we would end up with different results.*

Axiom 2 *Post-growth Linear Homogeneity (**PGLH**): For $n \geq 1$ and for all $w, \tilde{w} \in \Re_{++}$ and $\lambda > 0$*

$$f(\lambda\tilde{w}, \lambda w) = \lambda f(\tilde{w}, w)$$

Axiom 3 *Post-growth Translation Invariance (**PGTI**): For $n \geq 1$ and for all $w, \tilde{w} \in \Re_{++}$ and $\theta > 0$*

$$f(\tilde{w} + \theta, w + \theta) = f(\tilde{w}, w)$$

These two axioms are standard assumptions in the theory on economic distance and in the literature on mobility measurement (D'Agostino and Dardanoni, 2009a, 2009b; Fields and Ok, 1996). However, in the context of our analysis, their validity is restricted to the post-growth distributions of income. **PGLH** indicates that the function defining the relationship between w and $\tilde{w}$ is scale dependent, that is, an equiproportional change in all income levels, in both the final distributions of income, w and $\tilde{w}$, results in the same percentage change in the function f. **PGTI** states that, the amount of the function f found in going from one distribution to another does not change if the same amount is added to everybody's income in both w and $\tilde{w}$.

The following lemma is an immediate consequence of **PGLH**, **PGTI**, and **S**.

Lemma 1 *f satisfies **PGTI**, **PGLH**, and **S** for all $i = 1, ..., n$ and all $\tilde{w}, w \in \Re_{++}$ if and only if there exists a positive constant ρ such that*

$$f_i = \rho \left| w_i - \tilde{w}_i \right| \tag{1}$$

Proof. The sufficiency can easily be checked, therefore here we prove the necessity part.

PGTI implies the following: $f(w; \tilde{w}) = f(w - \tilde{w}; 0) = f(w - \tilde{w})$ for any $w, \tilde{w} > 0$.

PGLH implies: $f(\lambda\tilde{w}; \lambda w) = \lambda f(\tilde{w}, w)$.

Therefore, by **PGTI** we have

$$f(w, \tilde{w}) = \left\{ \begin{matrix} f(w - \tilde{w}, 0), \text{ if } w > \tilde{w} \\ f(0, \tilde{w} - w), \text{ if } w < \tilde{w} \end{matrix} \right\}$$

by **PGLH**

$$f(w, \tilde{w}) = \left\{ \begin{matrix} |w - \tilde{w}| f(1, 0), \text{ if } w > \tilde{w} \\ |\tilde{w} - w| f(0, 1), \text{ if } w < \tilde{w} \end{matrix} \right\}$$

by **S** $f(1, 0) = f(0, 1)$. The result is

$$f(\tilde{w}, w) = \rho \left| w - \tilde{w} \right| \tag{2}$$

for any $w, \tilde{w} \geq 0$ and ρ being any positive constant (see on this also Aczél (1966) pp. 15-17). ■

We impose the following property in order to characterize m_i, for all $i = 1, ..., n$.

Axiom 4 *Scale invariance with respect to initial income (**R**): For $n \geq 1$ and for all $x, w, \tilde{w} \in \Re_{++}$ and $\lambda > 0$*

$$m(\lambda x, f(\lambda w, \lambda \tilde{w})) = m(x, f(w, \tilde{w}))$$

R reflects the need to evaluate the relationship between w and $\tilde{w}$ relatively to the initial distribution of income. That is, our measure of mobility is scale independent and is not affected by an equiproportional change in the initial and final level of income.

In this framework, we allow for the possibility that translation invariance and scale invariance coexist in the same measure of mobility. It seems quite interesting, since it accommodates the need for a relative measure, which is helpful in particular for interdistribution comparisons, to the need of quantifying mobility in absolute terms, which might be more informative than using a relative measure (see on this Fields and Ok, 1996; Mitra and Ok, 1998).

S, **PGLH**, and **PGTI** are standard axioms, but their validity is restricted to the function f since the aim is to disentangle the effects of mobility from the whole growth; **R** allows to get a measure of mobility in relative terms, which is also informative about the extent of exchange mobility relatively to the extent of growth (or structural mobility).

These axioms give the following result.

Proposition 1 *An individual index of mobility satisfies **PGTI, PGLH, S,** and **R**, for all $i = 1, ..., n$, for all $x, \tilde{w}, w \in \Re_{++}$ if and only if there exists a positive constant ρ such that*

$$m_i = \rho \frac{|w_i - \tilde{w}_i|}{x_i} \tag{3}$$

Proof. The sufficiency can easily be checked, therefore here we prove the necessity part.

By **R**: $m(\lambda x, \lambda f(w; \tilde{w})) = m(x, f(w, \tilde{w}))$; let $\lambda = \frac{1}{x}$, $m(x, f(w, \tilde{w})) = m\left(\frac{x}{x}, \frac{f(w;\tilde{w})}{x}\right)$; that is,

$$m(x, f(w, \tilde{w})) = m\left(1, \frac{f(w; \tilde{w})}{x}\right) \Longrightarrow m(x, f(w, \tilde{w})) = m\left(\frac{f(w; \tilde{w})}{x}\right)$$

for any $w, \tilde{w} \geq 0$.

By Lemma 1, $f(w; \tilde{w}) = |w_i - \tilde{w}_i|$; substituting we get

$$m(x, f(w, \tilde{w})) = m\left(\frac{|w - \tilde{w}|}{x}\right)$$

Since m satisfies **PGLH**, we can apply the result from Aczél (1966) for the functional equation $m(\lambda y) = \lambda^k m(y)$. The solution is $m(y) = \rho y$ for $\rho, y > 0$. Substituting $y = \frac{|w - \tilde{w}|}{x}$ gives the following:

$$m(x, f(\tilde{w}, w)) = \rho \frac{|w - \tilde{w}|}{x}$$

and the statement of the proposition follows. ■

It can be noticed that this index belongs to the general family of distance functions, widely adopted in the income mobility measurement (see, D'Agostino and Dardanoni, 2009b; Fields and OK, 1996; Mitra and Ok, 1998). However, as discussed in the introduction, our measure differs from those since by relaxing anonymity we are able to use a broader set of information. In addition, even though the functional form of m_i, for all $i = 1, ..., n$, is based on distance functions, it measures the impact of reranking on individual income. In fact eq. (3) measures mobility as the differential income the individual would enjoy if he would experience reranking, expressed in percentage of his initial level of income. As a result, m_i, for all $i = 1, ..., n$, is equal to 0 when there is no reranking.

If we further impose focus on the poor, we are able to get a measure which informs us about the effect of reranking on individual poverty, an information which would not arise in a standard pro-poor growth framework.

Axiom 5 *Focus on the poor (**F**): For* $n \geq 1$ *and all* $x, w, \tilde{w} \in \Re_{++}$ *let* $x' = (\min(x_1, z), ..., \min(x_n, z))$, $w' = (\min(w_1, z), ..., \min(w_n, z))$, $\tilde{w}' = (\min(\tilde{w}_1, z), ..., \min(\tilde{w}_n, z))$

$$m(x, f(\tilde{w}, w)) = m(x', f(\tilde{w}', w')).$$

This is a basic axiom in the literature on poverty measurement. In this context, it simply allows to be concerned on the evaluation of the economic status of the poor, but in the context of our analysis it allows for very intuitive and appealing results.

Thus, we can state the following.

Corollary 1 *An individual index of mobility satisfies **F**, **PGTI**, **PGLH**, **S**, and **R**, for all* $i = 1, ..., n$, *for all* $x, \tilde{w}, w \in \Re_{++}$ *if and only if there exists a positive constant* ρ *such that*

$$m_i = \rho \frac{|w_i' - \tilde{w}_i'|}{x_i'} \tag{4}$$

Therefore, an index reflecting **F** can be considered a special case of the more general mobility index introduced with proposition 1.

For both initially poor and non poor individuals, who are poor after growth, that is, $\tilde{w}_i < z$, the index is equal to 0, when there is no reranking, thus their individual poverty is unchanged, this is the case of $w_i = \tilde{w}_i$. It is bigger than 0 when, after growth, reranking generates a variation in their individual poverty both by alleviating or eliminating it, respectively when $w_i > \tilde{w}_i$, up to $w_i = z$, or when $w_i > z$, and by worsening it, when $w_i < \tilde{w}_i$.

For an initially poor individual, who becomes rich after growth, the index is equal to zero if he would still be rich after reranking. This is the case in which the variation of poverty is determined only by growth. It is bigger than 0 if after growth, reranking acts letting him to fall again below the poverty line.

From the point of view of an initially rich individual, who is still rich after growth, the index is equal to 0 if the individual is also rich after reranking. In this case the extent of the reranking that takes place above the poverty line does not matter, since it is irrelevant to the variation of poverty determined by the overall growth process, encompassing mobility. The index is bigger than 0 when reranking after growth allows the initially rich individual to fall down the poverty line, and, therefore, to determine variation in his individual measure of poverty and variation in the aggregate poverty.

In sum, the index is bigger than 0 every time reranking acts by rising or reducing (up to eliminating) individuals' poverty between two points in time.

We want to emphasize that our aim is to identify the role of mobility, as an additional distribution effect of growth, no matter its direction[6]. The special case in eq. (4) can give relevant information when we want to complement standard analysis of pro-poor growth. In fact, when it diverges from 0, we have an additional information which justifies the caution in interpreting the overall variation of poverty due to growth as a Pareto improvement, since it goes side by side with the movements of individuals across the income distribution.

A final observation concerns what kind of mobility this index is able to capture. It is clear that this index is able to capture both aspects of mobility: the function f captures the extent of exchange mobility, while expressing f in terms of x we can capture the extent of structural mobility.

3.2 An aggregate measure of mobility

A second issue arising in the measurement of mobility concerns the aggregation process. The procedure mostly adopted in the literature consists in taking the average of individual mobilities (Fields and Ok, 1996, 1999a, 1999b; Mitra and Ok, 1998; Schluter and Van de Gaer, 2010). We follow closely this strand of the literature; however, combining standard axioms and relaxing anonymity enables to obtain a more general family of aggregate mobility measures.

We proceed by imposing the following axioms in order to formalize the aggregation of the individual measure of mobility.

Axiom 6 *Weak Decomposability (**D**). For $n \geq 2$, for all $x, w, \tilde{w}, x^*, w^*, \tilde{w}^* \in \Re_{++}$, if for all $i = 1, ..., n$, $m_i\left(x_i, f_i\left(\tilde{w}_i, w_i\right)\right) \geq m_i\left(x_i^*, f_i\left(\tilde{w}_i^*, w_i^*\right)\right)$ with a least one inequality holding strictly*

$$M(\mathbf{x}, f\left(\mathbf{w}, \tilde{\mathbf{w}}\right)) > M(\mathbf{x}^*, f\left(\mathbf{w}^*, \tilde{\mathbf{w}}^*\right))$$

D implies that the societal mobility is uniquely determined by m_i, $i = 1, ..., n$. **D** states that aggregate mobility should only depend on the value of individual mobilities. That is, aggregate income mobility is a strictly monotonic function of the individual levels of mobility. **D** is a widely used axiom through which we may restrict ourselves to the ranking of all vectors of individual mobilities, $\mathbf{m} \in \Re_+^n$, where $\mathbf{m} = (m_1, ..., m_n)$, and we recall that, as we have relaxed anonymity, the order of the individual mobilities in $\mathbf{m}$ reflects the order of the initial distribution of income[7]. This implies that, differently from previous

[6] Interesting directional measures have been introducted, *inter alia*, by Fields and Ok (1996, 1999a) and Demuynck and Van de Gaer (2010).

[7] We will not necessarily have $m_1 \leq m_2 ....$ Thus, the distribution is not ordered neither increasingly nor decreasingly.

contributions we do not make any assumption on the symmetry of M, that is anonymity in our context, with respect to the individual mobilities (see Mitra and Ok, 1998). The following Lemma is a direct consequence of imposing **D**.

Lemma 2 *An aggregate index of mobility satisfies* ***D*** *if and only if there exists a continuos and increasing function* $M : \Re_+^n \longrightarrow \Re_+$ *such that for all* $x, w, \tilde{w} \in \Re_{++}^n$ *and for all* $m \in \Re_+^n$

$$M\left(\mathbf{x}, f\left(\mathbf{w}, \tilde{\mathbf{w}}\right)\right) = \mathcal{M}\left(m_1\left(x_1, f_1\left(\tilde{w}_1, w_1\right)\right), ..., m_n\left(x_n, f_n\left(\tilde{w}_n, w_n\right)\right)\right)$$

Lemma 2 shows that we may restrict ourselves to the ranking of all vectors of individual mobilities.

We proceed by imposing the following axioms.

Axiom 7 *Individual Equivalence (**IE**). For* $n \geq 2$, *for all* $x, w, \tilde{w} \in \Re_{++}$, *if* $m_1\left(x_1, f_1\left(\tilde{w}_1, w_1\right)\right) = ... = m_i\left(x_i, f_i\left(\tilde{w}_i, w_i\right)\right) = ... = m_n\left(x_n, f_n\left(\tilde{w}_n, w_n\right)\right)$

$$\mathcal{M}\left(\mathbf{x}, f\left(\tilde{\mathbf{w}}, \mathbf{w}\right)\right) = m_i\left(x_i, f_i\left(\tilde{w}_i, w_i\right)\right)$$

IE states that if all individuals have the same level of mobility, then aggregate mobility can be appropriately represented by that value of individual mobility.

Axiom 8 *Recursive decomposability (**RD**): For all* $n \geq 2$, *and for all* $m \in \Re_+$, *for all* $M : \Re_+^n \to \Re_+$, *for all* $M^a\left(\mathcal{M}\left(m_1, ..., m_{n-1}\right), \mathcal{M}\left(m_1, ..., m_{n-1}\right), ..., m_n\right)$ *and* $M^b\left(m_1, \mathcal{M}\left(m_2, ..., m_n\right), ..., \mathcal{M}\left(m_2, ..., m_n\right)\right)$, *such that* M^a *and* $M^b : \Re_+^{n-1} \longrightarrow \Re_+$

$$\begin{aligned}\mathcal{M}\left(m_1, ..., m_n\right) &= \mathcal{M}\left(\mathcal{M}^a, \mathcal{M}^a, ..., m_n\right) = \\ &= \mathcal{M}\left(m_1, \mathcal{M}^b, ..., \mathcal{M}^b\right)\end{aligned}$$

RD represents a new contribution in the literature on mobility measurement, even if it can appear a strong requirement. It encompasses an independence property, since it implies the following: letting aggregate mobility only depend on the aggregate mobility of the $n-1$ poorest individuals in the initial distribution and on the mobility of the richest individual is equivalent to letting aggregate mobility depend only on the mobility of the poorest individual in x and on the aggregate mobility of the $n-1$ richest individuals[8]. In addition, this property ensures that the income mobility distribution of every group of the initially richest (or equivalently, poorest) members of the society can be evaluated without reference to the mobility of the remaining individuals. In other words, every group of the richest (or poorest) income recipient is strictly separable from the anyone who is poorer (or richer).

[8] It determines a system of functional equations with several unknown functions which can be determined from one equation each.

This property requires an aggregate mobility measure to be strictly recursive (see on this Bossert, 1990). It is clear the difference with the decomposition proposed in Demuynck and Van de Gaer (2010), the Decomposability with respect to highest mobility. First, D-HM applies on ordered distributions of mobility, thus it depends on the position of individuals in distribution $\tilde{\mathbf{m}}$[9], independently from their position in $\mathbf{x}$. Second, it allows to derive an aggregate measure of mobility depending on the lowest mobility and the aggregate mobility of the $n-1$ highest mobilities. Instead, we require aggregate mobility to be sensitive to both the groups of the poorest and of the richest individuals, independently from their position in $\tilde{\mathbf{m}}$. **RD** enables to state that the aggregation process is independent of the order in which it is performed. Thus, aggregating from below should give the same overall mobility as one would obtain aggregating from above.

The previous axioms allow to establish the following Proposition.

Proposition 2 *An aggregate index of mobility satisfies **D**, **RD**, and **IE** if and only if there exist coefficients, $\gamma_1, \gamma_2 ... \gamma_n$, such that, and for all $m \in \Re_+$*

$$M = \sum_{i=1}^{n} \gamma_i m_i \tag{5}$$

where $\sum_{i=1}^{n} \gamma_i = 1$

Proof. The sufficiency can easily be checked, therefore we focus on the necessity part.

D implies that

$$M\left(\mathbf{x}, f\left(\mathbf{w}, \tilde{\mathbf{w}}\right)\right) = \mathcal{M}\left(\mathbf{m}\right)$$

The rest of the proof follows by induction (see Aczél (1966), pp. 237-239). Thus, If eq. (5) is valid in general then,

$$\mathcal{M}^a\left(\mathbf{m}\right) = \frac{a_1 m_1 + ... + a_{n-1} m_{n-1}}{a_1 + ... + a_{n-1}};\ (a_1 + ... + a_{n-1} \neq 0)$$

and

$$\mathcal{M}^b = \frac{b_2 m_2 + ... + b_n m_n}{b_2 + ... + b_n};\ (b_2 + ... + b_n \neq 0)$$

BY **RD**

$$\mathcal{M}\left(\frac{a_1 m_1 + ... + a_{n-1} m_{n-1}}{a_1 + ... + a_{n-1}}, \frac{a_1 m_1 + ... + a_{n-1} m_{n-1}}{a_1 + ... + a_{n-1}}, ..., m_n\right) =$$

$$G\left(a_1 m_1 + ... + a_{n-1} m_{n-1}, m_n\right) =$$

$$\mathcal{M}\left(m_1, \frac{b_2 m_2 + ... + b_n m_n}{b_2 + ... + b_n}, ..., \frac{b_2 m_2 + ... + b_n m_n}{b_2 + ... + b_n}\right)$$

[9] $\tilde{\mathbf{m}}$ is the vector of individual mobilities distributed in ascending order.

with G being any real function.

Thus,

$$\mathcal{M}(\mathbf{m}) = G(a_1 m_1 + ... + a_{n-1}m_{n-1}, m_n) =$$
$$f\left(\frac{b_{n-1}a_1}{a_{n-1}}m_1 + \frac{b_{n-1}a_2}{a_{n-1}}m_2 + ... + b_{n-1}m_{n-1} + b_n m_n\right) =$$
$$f(c_1 m_1 + c_2 m_2 + ... + c_{n-1}m_{n-1} + c_n m_n)$$

($c_k = \frac{b_{n-1}a_k}{a_{n-1}}, k = 1, 2, ..., n-1;\ c_n = b_n$).

By **IE**, $m = \mathcal{M}(m, m, ..., m, m)$, which implies

$$\mathcal{M}_{(n)}(m, m, ..., m, m) = f[(c_1 + ... + c_n)m]$$

that is $f(t) = \frac{t}{c_1 + ... + c_n}$. It follows that

$$\mathcal{M}(m_1, ..., m_n) =$$
$$f(c_1 m_1 + ... + c_n m_n) = \frac{c_1 m_1 + ... + c_n m_n}{c_1 + ... + c_n}$$

It us obvious that $\sum_{i=1}^{n} \frac{c_i}{\sum_{i=1}^{n} c_i} = 1$. Denoting $\sum_{i=1}^{n} \frac{c_i}{\sum_{i=1}^{n} c_i}$ by γ_i for all $i = 1, ..., n$, the statement of the proposition follows. ■

Proposition 2 provides a general family of aggregate mobility measures, which can be expressed as a weighted average of individual mobilities, with generic coefficients summing to 1.

The combination of Proposition 1 and 2 justifies the use of

$$M = \sum_{i=1}^{n} \gamma_i \frac{|w_i - \tilde{w}_i|}{x_i} \tag{6}$$

Imposing **F**, eq. (6) becomes a measure of the impact of mobility on poverty at an aggregate level. As a result, if the variation in the overall poverty between two points in time is equal to 0, but M come out to be bigger that 0, we can state that there has been a flux of individuals crossing from above and from below the poverty line.

Eq. (6) can be interpreted as a component of the overall pro-poor growth which is equal to 0 when there is no reranking. This means that we have an additional term in the pro-poor growth evaluation which captures the impact of mobility. The meaning of this measure is substantial especially when, in aggregate the variation in poverty between two points in time comes out to be equal to 0. In this case a positive value of M informs us about the extent of the flux of individuals in and out of poverty, thus about the fact that even if in aggregate poverty seems to be unchanged, there are relevant variations of poverty at an individual level.

4 A rank dependent aggregate index of mobility

In the previous section, we have proposed a general measure of mobility. The aim of this section is to propose an alternative family of societal mobility. In particular, we derive a rank dependent measure of aggregate mobility that differs from earlier approaches, such as those of Demuynck and Van de Gaer (2010). Demuynck and Van de Gaer (2010) is, to the best of our knowledge, the only other contribution that addresses the issue of decomposable rank dependent measures of mobility axiomatically and provides characterization results[10]. However, in their framework the weights are based on the position in the mobility distribution, where lower mobilities are weighted more than higher one. We believe that this approach might present a drawback. In fact, according to their procedure, one may give more weight to individuals who were initially rich if they experience a lower level of mobility than the mobility experienced by initially poor individuals. We propose an attempt to derive a measure of aggregate mobility, with weights based on the position in the initial distribution of income. The motivation for this attempt relies on the interpretation of mobility as a form of progressive process, that can be used to equalize individuals' income from one period to another, in the same way as progressive taxation reduces inequality.

For, we need to make some simplifying assumptions. We assume that the mobility experienced by each individual in the distribution is measured by a continuous valued function $m_i\left(x_i, w_i, \tilde{w}_i\right)$, where $m_i : \Re^3_{++} \rightarrow \Re_+$, for all $i = 1, ..., n$, and that the societal mobility is measured by a continuous valued function $M\left(\mathbf{x}, \mathbf{w}, \tilde{\mathbf{w}}\right)$, $M : \Re^n_{++} \times \Re^n_{++} \times \Re^n_{++} \rightarrow \Re_+$. All the assumptions we have made about anonymity also hold for this aggregation; therefore, by $m_i\left(x_i, w_i, \tilde{w}_i\right)$ we mean the mobility experienced by the individual ranked i in $\mathbf{x}$, such that $m'_{w_i} \geq 0$, that is an increase in final income after reranking does not decrease the measure of individual mobility. We also assume that $m''_{x_i x_i} = 0$, which can be interpreted as a form of inequality neutrality of the individual measure of mobility with respect to the initial level of income.

In order to propose the rank dependent aggregate measure of mobility we need to impose the following axioms.

Axiom 9 *Weak Decomposability (**D**). For $n \geq 2$, for all $x, w, \tilde{w}, x^*, w^*, \tilde{w}^* \in \Re_{++}$, if for all $i = 1, ..., n$, $m_i\left(x_i, \tilde{w}_i, w_i\right) \geq m_i\left(x^*_i, \tilde{w}^*_i, w^*_i\right)$ with at least one inequality holding strictly*

$$M(\mathbf{x}, \tilde{\mathbf{w}}, \mathbf{w}) > M(\mathbf{x}^*, \tilde{\mathbf{w}}^*, \mathbf{w}^*)$$

D implies that the societal mobility is uniquely determined by $m_i\left(x_i, \tilde{w}_i, w_i\right)$, for all $i = 1, ..., n$. Therefore, aggregate mobility should only depend on the value of individual mobilities. That is, aggregate income mobility is a strictly monotonic function of the individual levels of mobility. The following Lemma is a direct consequence of imposing **D**.

[10] Note that Aaberge and Mogstad (2010) also propose a rank dependent measure of mobility, but it is not decomposable. Also Jenkins and Van Kerm (2011) propose a similar measure but they do not provide characterization results.

Lemma 3 $M(\mathbf{x}, \tilde{\mathbf{w}}, \mathbf{w})$ *satisfies* ***D*** *if and only if, for* $m \in \Re^n_+$*, for all* $x, w, \tilde{w} \in \Re_{++}$*, there exists a continuous and increasing function* g *such that*

$$M = g\left(m_1\left(x_1, \tilde{w}_1, w_1\right), ..., m_n\left(x_n, \tilde{w}_n, w_n\right)\right)$$

Lemma 3 shows that we may restrict ourselves to the ranking of all vectors of individual mobilities.

The following axiom is needed in order to understand in what way these terms jointly determine aggregate mobility. For each individual mobility m_i, we argue that its effect on overall mobility is independent of any other's individual mobility.

Axiom 10 *Individual mobility independence (**IMI**). For* $n \geq 2$*, for all* $m_i \in \Re_+$*, and for all* $i, j = 1, ..., n$*, such that* $i \neq j$

$$\frac{\partial^2 M(\mathbf{x}, \tilde{\mathbf{w}}, \mathbf{w})}{\partial m_i \partial m_j} = 0$$

IMI is a standard axiom used in order to obtain an additive representation of the measure we want to derive. In our context it means that each individual mobility occurs autonomously to affect aggregate mobility, thus there is not any joint effect between individual mobilities. This means that, in the aggregate measure, it is possible to isolate the component due to each individual.

Axiom 11 *Individual Equivalence (**IE**). For* $n \geq 2$*, for all* $x, w, \tilde{w} \in \Re_{++}$*, if* $m_1(x_1, \tilde{w}_1, w_1) = ... = m_i(x_i, \tilde{w}_i, w_i) = ... = m_n(x_n, \tilde{w}_n, w_n)$

$$M(\mathbf{x}, \tilde{\mathbf{w}}, \mathbf{w}) = m_i(x_i, \tilde{w}_i, w_i)$$

IE states that if all individuals have the same level of mobility, then aggregate mobility can be appropriately represented by that value of individual mobility.

Axiom 12 *Mobility Intensity (**MI**). For* $n \geq 2$*, for all* $x, w, \tilde{w} \in \Re_{++}$

$$\frac{\partial^2 M(\mathbf{x}, \tilde{\mathbf{w}}, \mathbf{w})}{\partial x_i^2} \geq 0$$

According to **MI** the effect of an increase in individual mobility is higher the lower is the level of initial income. It is clear the analogy with inequality aversion, that is, the lower is the level of initial income of an individual, the more weight his mobility has in determining aggregate mobility[11], therefore, it expresses a concern toward initially poorer individuals[12].

[11] We have to remark that we are implicitly assuming $\frac{\partial M}{\partial \tilde{x}} < 0$, which means that, mobility increases as the level of initial income decreases, that is, mobility has acted a lot in improving individual final income.

[12] In order to prove Proposition 20 we follow in part Hoy and Zheng (2008).

Proposition 3 *An aggregate index of mobility satisfies* ***D****,* ***IE****,* ***IMI****, and* ***MI*** *if and only if for all* $x, \tilde{w}, w \in \Re_{++}$*, for all* $i = 1, ...n$*,* $\exists g_i : \Re_+ \rightarrow \Re_+$ *such that*

$$M = \sum_{i=1}^{n} g_i(i)\, m_i(x_i, \tilde{w}_i, w_i)$$

with $\sum_{i=1}^{n} g_i(i) = 1$.

Proof. **D** implies that $M(\mathbf{x}, \tilde{\mathbf{w}}, \mathbf{w})$ is an increasing function of only m_i, for all $i = 1, ..., n$; thus, there exists a continuous and increasing function g such that

$$M = g\left(m_1(x_1, \tilde{w}_1, w_1), ..., m_n(x_n, \tilde{w}_n, w_n)\right)$$

IMI implies that $M(\mathbf{x}, \tilde{\mathbf{w}}, \mathbf{w})$ is additively separable in each m_i, for some continuos and positive functions $\hat{g}_1, ..., \hat{g}_i, ..., \hat{g}_n$, that is,

$$M = \hat{g}_1\left(m_1(x_1, \tilde{w}_1, w_1)\right) + ... + \hat{g}_n\left(m_n(x_n, \tilde{w}_n, w_n)\right)$$

Now, suppose that

$$m_1(x_1, \tilde{w}_1, w_1) = ... = m_n(x_n, \tilde{w}_n, w_n) = \beta$$

IE implies $\sum_{i=1}^{n} \hat{g}_i(\beta) = \beta$ for all $\beta \geq 0$. Taking the second derivative of $\sum_{i=1}^{n} \hat{g}_i(\beta) = \beta$ with respect to β gives $\sum_{i=1}^{n} \hat{g}_i''(\beta) = 0$.

However, if $m_1(x_1, \tilde{w}_1, w_1) = ... = m_n(x_n, \tilde{w}_n, w_n)$ this means that we can write the following:

$$M = \hat{g}_i\left(m_i(x_i, \tilde{w}_i, w_i)\right)$$

By **MI** $\frac{\partial^2 M}{\partial x^2} \geq 0$, this implies

$$\hat{g}''\left(m_{x_i}\right)^2 + \hat{g}' m_{x_i x_i} \geq 0$$

By choosing m_i such that $m''_{x_i x_i} = 0$, for all $i = 1, ..., n$, it follows that $\hat{g}'' \geq 0$ for all $i = 1, ..., n$.

Thus, since $\sum_{i=1}^{n} \hat{g}_i''(\beta) = 0$ and $\hat{g}''_{x_i, x_i} \geq 0$, it follows that $\tilde{g}''_{x_i x_i} = 0$ for all $i = 1, ..., n$, or $\hat{g}_i(\beta) = g_i(i)\beta$, for all $i = 1, ..., n$, for some continuous and positive function $g_i(i)$, for all $i = 1, ..., n$.

Substituting $\hat{g}_i()$ in

$$M = \hat{g}_1\left(m_1(x_1, \tilde{w}_1, w_1)\right) + ... +$$

$$\hat{g}_i\left(m_i(x_i, \tilde{w}_i, w_i)\right) + ... + \hat{g}_n\left(m_n(x_n, \tilde{w}_n, w_n)\right)$$

gives

$$M = g_1(1)\, m_1(x_1, \tilde{w}_1, w_1) + ... +$$

$$g_i(i)\, m_i(x_i, \tilde{w}_i, w_i) + ... + g_n(n)\, m_n(x_n, \tilde{w}_n, w_w)$$

and recalling that i refers to the position of the individual in the initial distribution of income completes the proof. ■

Proposition 3 provides a general family of aggregate mobility measures[13], which can be expressed as a weighted average of individual mobilities, with weights based on the relative position in the initial distribution of income. Other than that, no information is available on the functional form of those weights. However, it is reasonable to argue that, from a normative point of view, a concern can be expressed with respect to individuals who were initially poor, for instance, for policy purpose. This concern can be encompassed in an aggregate mobility measure by imposing further restrictions on the weights. With the exception of Jenkins and Van Kerm (2011), this procedure is new in the income mobility literature, but it is quite acknowledged in the inequality measurement literature[14].

We proceed by imposing the following axioms.

Axiom 13 *Inequality Aversion (**IA**). For all $x, w, \tilde{w} \in \Re_{++}$, for all $i = 1, ..., n$, $\exists g_i : \Re_+ \longrightarrow \Re_+$ such that*

$$g_i(i) \geq g_{i+1}(i+1)$$

According to **IA**, initially poor individuals are cause of concern from a social point of view; thus, in the measurement of aggregate mobility more weight is given to poorer individuals, that is the weights are decreasing with the rank. This axiom imposes a kind of social preference for progressive income growth, that is giving more weight to the initially poorer individuals is consistent with a preference for greater equality in post-growth distribution of income.

Axiom 14 *Transfer Sensitivity (**TS**). For all $x, w, \tilde{w} \in \Re_{++}$, for all $i = 1, ..., n$, $\exists g_i : \Re_+ \longrightarrow \Re_+$ such that*

$$g_{i-1}(i-1) - g_i(i) \geq g_i(i) - g_{i+1}(i+1)$$

TS is also standard in the literature of inequality and poverty measurement. It implies that the weights increase at a higher pace the lower is the position in the initial distribution of income. That is, progressive transfer of income mobility increases aggregate mobility more the lower is the part of the distribution it takes place.

The following corollaries are direct consequence of these axioms.

Corollary 2 *An aggregate index of mobility satisfies **D**, **IMI**, **MI**, and **IE** if and only if, for all $x, \tilde{w}, w \in \Re_{++}$ and for all $m_i \in \Re_+$, $\exists g_i : \Re_+ \rightarrow \Re_+$ such*

[13] Note that we cannot apply this aggregation procedure to the individual mobility index derived in section 3.3, since it is not differentiable when $w = \tilde{w}$.

[14] See, *inter alia*, Yaari (1988), Donaldson and Weymark (1980), Bossert (1990), Aaberge (2001).

that

$$M = \sum_{i=1}^{n} g_i(i)\, m_i \tag{7}$$

where $g_i(i) \in \{g_i(i) : g_i(i) \geq 0\}$ *for all* $i = 1, ..., n$

Corollary 3 *An aggregate index of mobility satisfies **D**, **IMI**, **MI**, **IE**, and **IA** if and only if, for all* $x, \tilde{w}, w \in \Re_{++}$ *and for all* $m_i \in \Re_+$, $\exists g_i : \Re_+ \to \Re_+$ *such that*

$$M = \sum_{i=1}^{n} g_i(i)\, m_i \tag{8}$$

where $g_i(i) \in \{g_i(i) : g_i(i) \geq 0;\ \Delta g_i(i) \geq 0\}$ *for all* $i = 1, ..., n$, *with* $\Delta g_i(i) = g_i(i) - g_{i+1}(i+1)$

Corollary 4 *An aggregate index of mobility satisfies **D**, **IMI**, **MI**, **IE**, **IA**, and **TS** if and only if for all* $x, \tilde{w}, w \in \Re_{++}$, *and for all* $m_i \in \Re_+$, $\exists g_i : \Re_+ \to \Re_+$ *such that*

$$M = \sum_{i=1}^{n} g_i(i)\, m_i \tag{9}$$

where $g_i(i) \in \{g_i(i) : g_i(i) \geq 0;\ \Delta g_i(i) \geq 0;\ \Delta(\Delta g_i(i)) \geq 0\}$ *for all* $i = 1, ..., n$, *with* $\Delta(\Delta g_i(i)) = \Delta g_i(i) - \Delta g_{i+1}(i+1) \geq 0$

4.1 A rank dependent mobility ordering

In the previous subsection we have proposed a general class of mobility measures. Consequently, given two different income transformation processes, one could conclude that one process is more mobile than the other whenever $M \geq M^*$, or equivalently whenever

$$\sum_{i=1}^{n} g_i(i)\, m_i \geq \sum_{i=1}^{n} g_i(i)\, m_i^* \tag{10}$$

for a certain choice of $g_i(i)$. We have also explored additional properties that the weights might satisfy, which may depend on the aim of the analysis. However, once these properties have been established, there is still room for arbitrariness in the choice of their appropriate functional form. In fact, it can happen that two aggregate mobility measures, even satisfying the same properties, give rise to different distribution rankings, if different functional forms for the weights are adopted. We would like to know, instead, when we can state that a distribution shows unambiguously more mobility than another for a given class of mobility measures. The problem is similar to the theory of social welfare ordering, when one adopts the Yaari social welfare function (1988). The choice is quite consequential. In fact, it is well known that different assumptions on the behaviour of this class of social welfare functions may generate different distribution rankings. Nevertheless, there is at least one way of making unambiguous social welfare evaluations.

Let define the following binary relation on $\Re^n$ for all $m_i \in \Re_+$, for all $i = 1, ..., n$, $n \geq 1$, $M \geq_M M^*$ if and only if $M_g \geq M_g^*$ for all g satisfying some given conditions. Of course, this way of ordering distributions presents the standard weakness of incompleteness, which is compensated by its robustness to any choice of g. In what follows we will consider, without loss of generality, two distributions with same population size. In fact, the same conditions could also be applied to per capita version of the mobility indices we have characterized in Proposition 3.

We start from the most general case, where the only information available is the positivity of the weights.

Proposition 4 *For all $M, M^* \in \Re_+$ satisfying **D**, **IMI**, **IE**, and **MI**, then*

$$M \geq_M M^*$$

if and only if

$$m_i \geq m_i^*, \text{ for all } i = 1, ..., n \tag{11}$$

for all $g_i(i) \in \{g_i(i) : g_i(i) \geq 0\}$ for all $i = 1, ..., n$

Proof. We want to find a necessary and sufficient condition for

$$M = \sum_{i=1}^{n} g_i(i) m_i \geq \sum_{i=1}^{n} g_i(i) m_i^* = M^* \tag{12}$$

Sufficiency can be checked by considering that $g_i(i) \geq 0, \forall i = 1, 2, ..., n$, therefore, $m_i \geq m_i^*$, for all $i = 1, ..., n$ implies $\Delta M \geq 0$.

For the necessity, suppose that $\Delta M \geq 0$, but there exists an individual $h = 1, ..., n$ such that $m_h - m_h^* \leq 0$. Now select a set of function $\{g_i(i)\}_{i \in \{1,...,n\}}$ such that $g_i(i) \searrow 0$, $\forall i \neq h$, in this case ΔM would reduce to $m_h - m_h^* \leq 0$, a contradiction. ■

The condition obtained in Proposition 4 requires that the mobility of each individual in one distribution to be higher than the mobility of each individual in the other distribution[15]. Thus, given two distributions F and G, we will say that F is unambiguously more mobile than G, for all the aggregate mobility satisfying **D**, **IMI**, **IE**, and **MI**, if and only if each individual in F has higher level of individual mobility than the the same individual in G. This is a dominance condition of the first order, to be checked for each individual, starting from the initially poorest individual and ending with the initially richest individual. Overall mobility dominance requires dominance for each individual taken separately, paying no attention to the inequality in the distribution.

[15] The result in proposition 4 is equivalent to the mobility profile dominance derived in Van Kerm (2006) and Jenkins and Van Kerm (2011), however they do not apply an axiomatic framework.

We proceed in the analysis of classes of aggregate mobility indices consistent with ethically grounded properties. In the next proposition, we identify the distributional condition corresponding to a less broad class of aggregate mobility measures.

Proposition 5 *For all* $M, M^* \in \Re_+$ *satisfying* ***D****,* ***IMI****,* ***IE****,* ***MI****, and* ***IA****, then*

$$M \geq_M M^*$$

if and only if

$$\sum_{i=1}^{k} m_i \geq \sum_{i=1}^{k} m_i^*, \text{ for all } k = 1, ..., n \tag{13}$$

for all $g_i \in \{g_i : g_i \geq 0;\ \Delta g_i(i) \geq 0\}$*, for all* $i = 1, ..., n$

Proof

Before proving this proposition we need to state and prove the following Lemma.

Lemma 4 $\sum_{i=1}^{n} v_k w_k \geq 0$ *for all sets of numbers* $\{v_k\}$ *such that* $v_k \geq v_{k+1} \geq 0$ $\forall k \in \{1, ..., n\}$ *if and only if* $\sum_{i=1}^{k} w_i \geq 0$*,* $\forall k \in \{1, ..., n\}$*.*

Proof. Applying Abel's decomposition:
$\sum_{k=1}^{n} v_k w_k = \sum_{k=1}^{n} (v_k - v_{k+1}) \sum_{i=1}^{k} w_i$. If $\sum_{i=1}^{k} w_i \geq 0$, $\forall k \in \{1, ..., n\}$, then $\sum_{k=1}^{n} v_k w_k \geq 0$.

As for the necessity part, suppose that $\sum_{k=1}^{n} v_k w_k \geq 0$ for all sets of numbers $\{v_k\}$ such that $v_k \geq v_{k+1} \geq 0$, but $\exists j \in \{1, ..., n\} : \sum_{i=1}^{j} w_i < 0$. Consider what happens when $(v_k - v_{k+1}) \searrow 0, \forall k \neq j$. We obtain that $\sum_{i=1}^{n} v_k w_k \longrightarrow (v_j - v_{j+1}) \sum_{i=1}^{j} w_i < 0$ which is the desired contradiction. ∎

We can now prove the proposition. We want to find a sufficient and necessary condition for $\Delta M \geq 0$.

For both conditions, note that $\Delta M \geq 0$ if $\sum_{i=1}^{n} g_i(i) [m_i - m_i^*] \geq 0$ for all $g_i(i)$, such that $g_i(i) \geq g_i(i+1) \geq 0$. Let $w_i = [m_i - m_i^*]$ so that $M =$

$\sum_{i=1}^{n} g_i(i) w_i$. Since $g_i(i) \geq g_i(i+1) \geq 0, \forall i = 1, ..., n-1$, we can apply Lemma 4 and obtain that $\sum_{i=1}^{n} g_i(i) w_i \geq 0$ if and only if $\sum_{i=1}^{k} w_i \geq 0, \forall k = 1, ..., n$. **QED**

The test in Proposition 5 is a second order dominance applied to the distribution of individual income mobilities, ordered according to the position in distribution $\mathbf{x}$ of each individual they represent[16]. We have to check that, at each individual k, the cumulated sum of individual mobilities is higher in one distribution than in the other. Thus, given two distributions F and G, we will say that F is unambiguously more mobile than G, for all the aggregate mobility satisfying **D**, **IMI**, **IE**, **MI**, and **IA**, if and only for each k the cumulated sum of individual mobilities is higher in F than in G.

Proposition 5 presents weaker dominance conditions than the one characterized in Proposition 4; hence, the partial ordering generated by this test will become helpful in cases in which it is not possible to rank distributions according to Proposition 4. It is also possible to identify an even weaker condition, by restricting our focus to the class of admissible aggregate mobility measures satisfying in addition to **IA**, **TS**.

Proposition 6 *For all $M, M^* \in \Re_+$ satisfying **D**, **IMI**, **IE**, **MI**, **IA**, and **TS**, then*

$$M \geq_M M^* \tag{14}$$

if and only if

$$\sum_{j=1}^{k} \sum_{i=1}^{j} m_i \geq \sum_{j=1}^{k} \sum_{i=1}^{j} m_i^*, \text{ for all } k, j = 1, ..., n$$

for all $g_i \in \{g_i : g_i \geq 0; \Delta g_i(i) \geq 0; \Delta(\Delta g_i(i)) \geq 0\}$, for all $i = 1, ..., n$

Proof

We need to find necessary a sufficiency and necessary condition such that $M \geq M^*$, that is,

$$\sum_{i=1}^{n} g_i(i) m_i \geq \sum_{i=1}^{n} g_i(i) m_i^* \tag{15}$$

For the sufficiency, by application of Abel's decomposition, ΔM can be written as

$$\sum_{i=1}^{n} (g_i(i) - g_i(i+1)) \sum_{i=1}^{k} (m_i - m_i^*) \tag{16}$$

Denote $(g_i(i) - g_i(i+1))$ by v_i and $\sum_{i=1}^{k} (m_i - m_i^*)$ by w_i, $\Delta M = \sum_{i=1}^{n} v_i w_i$. By IA, $v_i \geq v_{i+1}$, for all $i = 1, ..., n$. Hence, by Lemma 4, $\sum_{i=1}^{n} v_i w_i \geq 0$ if and

[16] The result in proposition 5 is equivalent to the cumulated mobility profile dominance derived in Van Kerm (2006) and Jenkins and Van Kerm (2011).

only if $\sum_{k=1}^{j} w_k \geq 0$, for all $j = 1, ..., n$, which is equivalent to $\sum_{k=1}^{j}\sum_{i=1}^{k} w_i$, for all $j = 1, ..., n$, or to $\sum_{i=1}^{j}\sum_{k=1}^{i}(m_i - m_i^*)$, for all $j = 1, ..., n$. **QED**

The usefulness of the result in Proposition 6 rests on the fact that it will possibly rank distributions in some cases where the tests in Proposition 4 and 5 fail to do so.

5 Conclusions

The relationship between growth, inequality and poverty is the focus of most distributional analysis, as well as, the measurement of income mobility. Although it is reasonable to believe that growth and mobility may be two interrelated aspects of the income dynamics, their distributional implications have never been analyzed jointly, because standard analysis are based on the anonymity assumption. The main consequence is that the effect on poverty variation of the reshuffling of individuals among income classes is not taken into consideration.

Previous contributions have suggest to solve this problem by relaxing anonymity. In this paper we have further explored this issue from an axiomatic point of view. We have proposed a characterization of a measure of individual mobility and its aggregation procedure, where mobility is interpreted as an additional implication of growth. A special case is obtained by imposing the focus axiom, which gives information about the impact of mobility on individual and aggregate poverty. Its main characteristic is that this measure can be used to complement standard frameworks for the evaluation of the pro-poorness of growth. We have also developed a different measure of aggregate mobility, represented by a weighted average of individual mobility with weights based on the rank in the initial distribution of income. Finally, we have proposed some partial dominance conditions for this class of rank dependent aggregate mobility indices, where mobility is interpreted as a form of progressive process, aimed at reducing income inequality among individuals, between two periods of time.

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