Società italiana di economia pubblica

XX CONFERENZA

ECONOMIA DELLA TASSAZIONE
Sistemi tributari, pressione fiscale, crescita

Pavia, Università, 25 - 26 settembre 2008

LIVING IN A JUNGLE OR TOGETHER AND IN PEACE?

DARIO MAIMONE ANSALDO PATTI AND GIUSEPPE SOBBRIO

pubblicazione internet realizzata con contributo della COMPAGNIA di San Paolo

società italiana di economia pubblica

dipartimento di economia pubblica e territoriale – università di pavia

Living in a Jungle or Together and in Peace?

Dario Maimone Ansaldo Patti
University of Messina and University of Essex

Giuseppe Sobbrio
University of Messina

March, 2008

Abstract

The existing literature on conflicts extensively argues that countries may find a peaceful settlement, in which they renounce fighting. In this paper we try to formalize a model, in which countries focus on a different and more attractive solution for their disputes. More specifically, we the case in which they merge, creating a new entity, instead of simply renouncing fighting. We stress the welfare implications of this choice, as well as the fact that the merging process is not necessarely unchangeable, but it can collpase depending on the parameters of the model. Beyond the literature on conflicts, this paper can also be linked to the one on countries secession/unification.

1 Introduction

In the last decades, economists have devoted a growing attention to study the causes, the consequences and the possible solutions to conflicts. Their interest stemmed from the observations that conflict is in fact an intense form of competition, in which "...contenders try to hamper, disable, or destroy rivals" (Hirshleifer, 1995a p. 167). The welfare implications of such harmful form of competition are immediate: if people need to allocate resources for defending the fruits of their work, less resources are left other activities, such as production. Moreover, the possibility of a violent appropriation from a contender decreases the incentives to produce. Both facts become responsible of lower economic development.

The study of conflicts has been carried out in an anarchic or *state-of-nature* framework, in which contenders' actions are not constrained by any set of rules. On the contrary, the *rule of strength* is in charge and property rights are insecure and cannot be enforced.

The explanations on why countries embark on a costly war are complex. Rationalist theory employes several arguments as causes of war. Among them a prominent role is played by anarchy. This can be easily traced in Waltz's words: "...war occurs because there is nothing to prevent it. [...]Among states as among men there is no automatic adjustment of interests. In the absence of a supreme authority there is then the constant possibility that conflicts will be settled by force." (Waltz, 1959, p. 188)

Notably, Fearon (1995) points out that while anarchy favors the emergence of conflicts among countries, it cannot explain by itself why the lack of a central authority prevents states from negotiating a peaceful agreement. Therefore, he proposes several further reasons as potential rationalist explanations for the war. His observations stem from the fact that even in an anarchic environment there always exists a bargaining range within which countries may negotiate an agreement such that they are better off than after a war. Nonetheless, wars still occur very frequent in the world.

Several contributions validate the anarchic environment as a main characteristic of relations among countries. As Oye (1985) points out, international relations are an appropriate approximation of the Hobbesian state of nature, in which there is not any certainty against aggressions from neighbor countries. More specifically, Gilpin (1981) clarifies that international relations are a "recurring struggle for wealth and power among independent actors in a state of anarchy". (p. 7). It follows that any kind of agreement, which occurs between states must be self-enforcing, since there does not exist a third party, which can supervise and possibly sanction illegal behaviors.[1]

In a more general framework, countries' relations can be encoded in a seven-points scale according to a specific security system, which they choose. The extreme points of the scale are the international state of nature, where the pattern of behavior is characterized by the war of all against all, and the collective security system characterized by a collective action (Cronin, 1999).

Modelling a conflict in such an anarchic environment has become a standard procedure in the economic analysis of war (for instance Skaperdas, 1992; Powell, 1993; Grossman and Kim, 1995; Hirshleifer, 1995b; Grossman and Mendoza, 2001; Hirshleifer, 2001; Muthoo, 2004). All those works try to provide insights about the reasons of a conflict among countries as well as about the determination of the parameter values, which guarantee that states opt for a peaceful settlement of their disputes without going to the war. In all cases the agreement reached by countries is self-enforcing. Moreover, it does not require that states invest in defensive structures in order to deter any future attack. This fact holds for almost all the previous contributions, the most notably exception being Grossman (2004) where a peaceful settlement equilibrium is derived both when countries fortify their borders and also when the latter are left unfortified.

This paper aims at analyzing the behavior of two heterogeneous countries, fighting in an anarchic framework. Heterogeneity is meant to refer to different military and productive skills owned by each country. Incentive to fight comes from the possibility of appropriating some commodity good, which both of them produce separately. This contributions differentiates itself from the others, since we are not interested simply in the existence of a bargaining range within which countries may find a peaceful settlement. Instead, we want to investigate upon the possibility that states negotiate a specific agreement, which, if accepted by both of them, leads them to merge and to create a new political entity.

The results, which we obtain can be summarized as follows. If heterogeneous countries merge, they will improve their own welfare. This quite intuitive result is based

[1] Usually international organization lack of the necessary power to sanction breaks of agreements, even though they may condemn some specific behaviors.

on the fact that in our stylized world countries' decision of merging solves the international dilemma, which every state faces in the allocation of resources between internal and productive ends and external and military purposes. When countries decide to merge, more resources are available for the production, which brings more direct and immediate benefits than if they move to a war (Powell, 1993).

Moreover, what is interesting and worthwhile to stress is that the welfare improvement, which countries enjoy, is such that they are even better off than if they negotiate a peaceful settlement of the dispute without merging. Finally, the long-run consequences of merging are analyzed and, as we will stress, according to some specific parameters values countries may not find convenient anymore to maintain this kind of agreement but they may deviate from the merging path and, consequently, decide to split.

This paper can be considered primarily as a contribution to the theory of conflict resolution. Nonetheless, it shares also some elements with the literature, which deals with the problem of country formation. In particular, the capacity of the model of stressing the conditions, which characterize states' decision of merging, may shed light on the creation of new countries, following a different approach with respect to the one adopted in the main literature on this topic. Moreover, looking at the model from the opposite point of view, it can be employed to evaluate the processes, which lead to secession among countries.

The literature on the country formation is large and covers several aspects regarding the reasons, which drive states toward unification/secession. For instance, Riker (1964) and Gilpin (2001) argue that unification is a useful way to respond to external threats. On the other hand, following North (1990) and Huntington (1996), the opposite process of secession can be the consequence of the lack of common values and interests, inspiring the life of communities. For instance, the secession process involving the former Yugoslavia and the USSR can be explained, among other things, with the lack of those common values and identical cultural endowment. As it will be made clear when the model is fully developed, some of its parameters can be interpret as capturing cultural heterogeneity and, consequently, playing a crucial role in the secession/unification decisions. From the opposite point of view, the Germany unification deserves the same explanation.

For a general introduction to the literature on country formation, Bolton, Gerard and Spolaore (1996) review the main works about this topic,[2] stressing the determinants of the process, which leads to either secession or unification.

The paper is organized as follows. In the next section, we characterize the main features of my model. In section 3, we describe and analyze how countries live in a jungle, where the only accepted rule is that the strongest wins. In section 4 we investigate upon the possibility that countries merge. In section 5 we draw some conclusions.

2 Model Framework

We consider two countries, 1 and 2, owning an initial endowment $e_i, i \in \{1, 2\}$. The latter cannot be consumed directly but is used as an input for the production of a commodity good, y_i, according to the following production function:

$$y_i = a_i r_i \tag{1}$$

In equation (1) r_i is a production input drawn from the initial endowment and $a_i > 1$ is a parameter, which captures the level of the production technology for country i. Clearly, if countries live in peace, the initial endowment is allocated entirely for the production of y_i and $r_i \equiv e_i$.

Unfortunately, countries live in an anarchic world where they struggle for the control of the commodity good. This environment underlines two crucial points. The first one is that the production of the commodity good is insecure, being subject to appropriation. In other words, y_1 and y_2 constitute a common pool and countries fight over its control. The second point, directly linked to the first one, stresses the fact that countries need to allocate their initial endowment between the production of the commodity good (*butter*) and military purposes (*gun*). As Powell (1993) points out, the trade off between those alternative uses represents an international dilemma, which every country faces. Allocation of resources for the war does not have a *direct* benefit, but only an *indirect* one, since it determines the probability of success in a future war but does not have

[2] For more specific references see Wei (1991) and, recently, Bolton and Gerard (1997), Alesina and Spolaore (1997), Casella and Feinstein (2002) and Goyal and Staal (2004). All of them link the decision of merging/secession to the political process, which characterizes the behavior of citizens in the countires, involved in such a process.

an immediate impact on the amount of commodity good enjoyed by each country. On the contrary, using the initial endowment for producing the commodity good gives an immediate and direct advantage, although it weakens state's chance of success in the war.

The victory in the fight depends on two factors: the amount of resources allocated for the war and each state's own military technology. Those elements are combined into a standard contest function, which ascribes a positive probability of success to each country,[3] i.e.:

$$s_i = \frac{\theta_i w_i}{\theta_i w_i + \theta_j w_j} \tag{2}$$

where $w_i \in \left[0, \frac{e_i}{2}\right]$, $w_j \in \left[0, \frac{e_j}{2}\right]$ and θ_i and θ_j are positive parameters, referring to countries' military technology.[4] From a different point of view, θ_i and θ_j capture states' "....productivity in transforming fighting resources into effective units of weaponry" (Anderton, 2000 p. 824).[5]

While we do not introduce any parameter capturing the cost of the war, it should be noted that, implicitely, the allocation of a part of the initial endowment for the war measures the cost of living in a jungle and, eventually, the cost of the war in terms of a lower production of y with respect to the amount, which would be realized in a peaceful world.

While the war indicates the default tool for settling dispute between countries, we focus on the possibility that a different solution can be implemented. More specifically, we analyze the case that countries meet before the war and decide to leave the jungle and to form a new political entity by merging. Merger implies that countries jointly produce the commodity good and share their knowledge. In turn, this fact entails the key assumption that after merging countries adopt the best productive technology, as well as that they share the control over the military technology available in the new political entity. After the commodity good is produced, it is divided according to a specific rule.

The most important issue connected to such a solution is that the agreement between states must be self-enforcing, since there does not exist a third party, which

[3] See Tullock (1980) and Hirshleifer (2001) for instance.

[4] The employed contest function is a standard ratio form with exponents equal to 1.

[5] Differently from other contributions, for example Grossman and Kim (1995) and Grossman (2004), we do not distinguish between defensive and attacking technologies.

supervises countries' behaviors and, if necessary, detects and sanctions illegal actions.

Within this framework each country's set of strategy consists of several elements. More specifically, they need to decide:

- the allocation of the initial endowment between *butter* and *gun*;
- which sharing rule has to be set in the event of a merger;
- whether or not to break the agreement which has been reached.

Following Grossman (2004), in order to maximize the amount of commodity good, each country chooses the element of its own strategy set, which is relevant for the specific situation it faces. For instance, when living in the jungle and struggling over the control of the common pool, countries need to allocate optimally their endowment between *butter* and *gun*. On the contrary, if they are facing the possibility of merging, they will need to choose the optimal sharing rule, which fairly distributes the joint production between countries, making both of them accept the agreement.

We introduce some assumptions in order to keep the model as simple as it is possible.

Assumption 1 $e_1 \geq e_2$*;* $e_i \geq 2w_i$ $\forall i = 1, 2$*;* $a_1 > a_2$ *and* $\theta_1 < \theta_2$*.*

Assumption 1 specifies that country 1 is (weakly) richer than its opponent and owns the best productive technology, while, on the contrary, country 2 displays superior military skills. While this fact does not imply that one country specializes in a particular activity, nonetheless it stresses that country 1 finds marginally more convenient to allocate resources for the production of the commodity good (*direct benefit* from the use of the endowment) rather than for the war (*indirect benefit*). Finally the second inequality, $e_i \geq 2w_i$, guarantees that countries always allocate something for both the production and the war.

Assumption 2 *Countries behave as unitary agents.*

Previous assumption implies that the decisions about how to allocate the endowment, as well as the other decisions in the different situations, are taken unanimously.

This simplifying assumption is not rare in international relations theory, although several approaches take the different perspective of investigating upon the aggregation political process, which embodies the formation of preferences.[6]

Assumption 3 *All the relevant parameters of the model are common knowledge.*

Differently from other contributions (for instance, Garfinkel and Skaperdas, 2000) we assume that countries do not have any private information.

3 Living in the Jungle

3.1 The Equilibrium

When living in a jungle, countries allocate their endowment for the production of the commodity good and the war. We start our analysis from the behavior of states in the jungle, since, as recalled earlier, we consider the war as the default option to settle their dispute.

We model this scenario as a simultaneous move game employing the Nash equilibrium as solution concept. Each country chooses the amount of endowment to employ for the war, taking as given the choice operated by its opponent.

Before deriving the equilibrium for this game, we state in detalis the objective function for the contestants. Given the characteristics of the anarchic world, the amount of commodity good, which country i expects to enjoy, depends on the quantity produced by itself, the amount of good, which it obtains from the fight, and the output which it loses. Putting those elements together, the objective function becomes:

$$y_i = a_i r_i + s_i a_j r_j - s_j a_i r_i$$

Using the fact that $s_i + s_j = 1$, the previous equation can be re-written as follows:

$$y_i = s_i \left(a_i r_i + a_j r_j \right) \tag{3}$$

Within this framework, countries maximize equation (1.3) under the following constraint:

$$r_i + w_i \leq e_i \tag{4}$$

[6]See, for instance, Casella (1992), Alesina and Spolaore (1997, 2003, 2005), Casella and Feldestein (2002) and Goyal and Kaal (2004) for the opposite point of view.

Assuming that the latter holds with equality and substituting it in (3) for r_i, the maximization problem yields the following result:

Proposition 1 *The unique Nash equilibrium* $(\tilde{w}_1, \tilde{w}_2)$ *in the jungle is the solution to the following pair of equations:*

$$\tilde{w}_1(w_2) = \frac{y\theta_2 w_2}{a_1(\theta_1 w_1 + \theta_2 w_2)} \tag{5}$$

$$\tilde{w}_2(w_1) = \frac{y\theta_1 w_1}{a_2(\theta_1 w_1 + \theta_2 w_2)} \tag{6}$$

where $y = [a_1(e_1 - w_1) + a_2(e_2 - w_2)]$ *measures the extension of the common pool.*[7]

Equations (5) and (6) characterize the reaction functions of each country in terms of the optimal choice of its opponent. Differentiating $\tilde{w}_i$ i with respect to w_j yields:

$$\frac{\partial \tilde{w}_i(w_j)}{\partial w_j} = \frac{\theta_j\left[\theta_i w_i (y - a_j w_j) - a_j \theta_j w_j^2\right]}{a_i(\theta_i w_i + \theta_j w_j)^2}$$

Given the difficulty of deriving an explicit solution for the equilibrium values, we cannot establish unambiguously how each of them reacts to a change in the opponent's choice of the amount of resources employed in the fight. As a matter of fact, after rearranging the sign of the above derivative depends on the following inequality:

$$\frac{\partial \tilde{w}_i(w_j)}{\partial w_j} \gtrless 0 \Longleftrightarrow s_i y \gtrless a_j w_j$$

Although we have not enough information, we can observe that if the probability of success for country i is sufficiently large (small) the second inequaliy is positive (negative) and so the first one is.

This result can be explained in the following way. If a country is sufficiently weak compared to its opponent and the latter increases the portion of resources devoted to the fight, the former prefers to reduce its effort in the fight in order to enhance the production of the commodity good. In other words, the war becomes the less attactive option. Differently, if country i is sufficiently strong, any increase in w_j leads to the decision of mobilizing more resource for the fight, since the latter is the most profitable activity and the main source of gain. This point may suggest that if a weak country

[7]Through the paper we use the symbol "$\sim$" to mark the equilibrum values, which are derived in the war game.

augments the portion of resources employed in the war, its decision may give rise to an arms race, since also its opponent will do the same. That result can be interpreted in the light of the confrontation between a strong country and a new rising power and suggests that the incumbent power is not incline to accept the new status of its opponent. Consequently, it allocates more resources for the fight in response to the same action taken by its opponent.

While it is not possible to solve explicitly equations (5) and (6) for $\tilde{w}_1$ and $\tilde{w}_2$, we may apply the implicit function theorem to establish the relation between the equilibrium values and the parameters of the model.

It becomes useful to rewrite the reaction functions as follows:

$$\tilde{H}(\theta_1, \theta_2, a_1, a_2) = w_1 - \frac{y\theta_2 w_2}{a_1(\theta_1 w_1 + \theta_2 w_2)} \tag{7}$$

$$\tilde{K}(\theta_1, \theta_2, a_1, a_2) = w_2 - \frac{y\theta_1 w_1}{a_1(\theta_1 w_1 + \theta_2 w_2)} \tag{8}$$

Our first objective is to evaluate the effect of a change in the military technologies on the equilibrium values. By totally differentiating equations (7) and (8) we obtain:

$$\begin{aligned} \tilde{H}_{w_i}\partial\tilde{w}_i + \tilde{H}_{w_j}\partial\tilde{w}_j &= -\tilde{H}_{\theta_i}\partial\theta_i \\ \tilde{K}_{w_i}\partial\tilde{w}_i + \tilde{K}_{w_j}\partial\tilde{w}_j &= -\tilde{K}_{\theta_i}\partial\theta_i \end{aligned}$$

which in matrix notation become:

$$\tilde{\mathcal{J}} \begin{bmatrix} \frac{\partial\tilde{w}_i}{\partial\theta_i} \\ \frac{\partial\tilde{w}_j}{\partial\theta_i} \end{bmatrix} = \begin{bmatrix} -\tilde{H}_{\theta_i} \\ -\tilde{K}_{\theta_i} \end{bmatrix} \tag{9}$$

$\tilde{\mathcal{J}}$ is the Jacobian matrix of the first order partial derivatives of the implicit functions $\tilde{H}$ and $\tilde{K}$. Before moving to assess the relations between $\tilde{w}_1$ and $\tilde{w}_2$ and the military technology parameters, we need to be sure that the determinant of the Jacobian matrix is different from 0. The following lemma clarifies this point:

Lemma 1 *The determinat of the Jacobian matrix,* $\left|\tilde{\mathcal{J}}\right|$*, is positive.*

Proof. In the Appendix. ■

Using Lemma 1, we can now derive the following result:

Proposition 2 *If country i develops a more effective military technology, it increases (reduces) the amount of resources mobilized for the war if it is sufficiently strong (weak). Instead, country j always lowers the resouces employed in the war, i.e.*

$$\frac{\partial \tilde{w}_i}{\partial \theta_i} \lesseqgtr 0 \quad and \quad \frac{\partial \tilde{w}_j}{\partial \theta_i} < 0$$

Proof. In the appendix. ■

The result in Proposition 2 is consistent with the response of one country to a change in the optimal allocation of resources allocated by its opponent. Also in the case stressed in Proposition 2, if a country is sufficiently strong, a more effective military technology leads to a larger allocation of resources for the fight. The reason for this choice may rely on the fact that it finds more convenient the war than the production for gaining larger amounts of output. However the most interesting result emerges if country i is sufficiently weak. In this case, if it develops a more effective military technology, both countries reduce the resources for the fight. In other words, an increase in θ_i when country i is sufficiently weak solves the international dilemma in favor of the production of the commodity good, which, as recalled in Powell (1993), represents the *direct* benefit from the allocation of each country's endowment in an anarchic context. This is due to the fact that any improvement in the fighting skills of the weak player balances the strenght of contestants. Consequently, if the military technology of the weak country is large enough, contestants would prefer to allocate small amounts of resources for the fight, since the war is a less appealing activity. In turn, this might lead to a situation in which they opt for a peaceful-but-armed solution rather than for the war.

An opposite result emerges if we consider a change in the productive technology. The result is contained in the following proposition:

Proposition 3 *If country i improves its productive technology, it allocates less resources for the war, while its opponent does the opposite. Formally:*

$$\frac{\partial \tilde{w}_i}{\partial a_i} < 0 \quad and \quad \frac{\partial \tilde{w}_j}{\partial a_i} > 0$$

Proof. In the appendix. ■

The rationale behind this result can be put as follows. When a_i raises, country i has a larger incentive to allocate more resources for the production of the commodity good. The peaceful activity is now more attractive, since it becomes comparatively more profitable than war. At the same time, the increase in a_i has the effect of enlarging the common pool. Therefore, as far as its opponent concerns, the enlargement of the common pool creates an incentive to mobilize more resources for the war, since the potential gain is now large than before.

If we interpret together the content of proposition 2 and 3, we may state the following result:

Corollary 1 *If the weak country improves its military technology, the war becomes less attractive and a peaceful-but-armed situation may prevail. On the other hand, if either the strong country develops the most effective military technology or if one of the contestants improves its productive skills, at least one of them prefers the war to the production. Hence, the former becomes a more likely outcome in the anarchic world.*

Previous corollary is based on the reasonable idea that if a country increases the portion of resources for the war, then it (comparatively) prefers this activity to the production of the commodity good. Consequently, it is interesting to note that the only possibility to lower the risk of a war is that the *less military skillful country improves its abilities.*

3.2 Welfare Implication of the War

Any change in the parameters of the model affects also the amount of commodity good, that each country expects to obtain. This occurs not only directly, through the impact of parameters on equation (3), but also indirectly via the changes in the equilibrium values, $\tilde{w}_1$ and $\tilde{w}_2$.

It is important to stress preliminarly that the war has always a negative impact on the global welfare of countries. Indeed, it is not difficult to observe that the total amount of consumption good, $\tilde{y}$, is smaller than the one which would be produced in peace, since in this case all countries' endowments are used for the production.

In the equilibrium, the value of y, which every country obtain is:

$$\tilde{y}_i = \tilde{s}_i \left[a_i \tilde{r}_i + a_j \tilde{r}_j \right] \tag{10}$$

We first analyze how a changes in the military technology affects $\tilde{y}_i$. Totally differentiating equation (10) with respect to θ_i yields:

$$\frac{\partial \tilde{y}_i}{\partial \theta_i} = \frac{\partial \tilde{y}_i}{\partial \tilde{w}_i}\frac{\partial \tilde{w}_i}{\partial \theta_i} + \frac{\partial \tilde{y}_i}{\partial \tilde{w}_j}\frac{\partial \tilde{w}_j}{\partial \theta_i} + \frac{\partial \tilde{y}_i}{\partial \theta_i} \tag{11}$$

It is crucial to note that the first term in equation (11) corresponds to the first order condition. However, as Nti (1997) clarifies, envelope theorem cannot be applied in this case. The explanation relies on the fact that in equilibrium any increase in the amount of resources for the war decided by one country displays its effects on resources ' allocation of its opponent. This fact should be contrasted with the first order condition $\partial y_i / \partial w_i = 0$, in which the allocation of the other contestant, w_j, is taken as fixed and, specifically, is assumed to be the optimal one.

In order to interpret correctly the sign of equation (11) we substitute the corresponding values for each derivative that appears in it, including $\partial \tilde{w}_i / \partial \theta_i$ and $\partial \tilde{w}_j / \partial \theta_i$. The findings of our analysis characterize the content of the following proposition:

Proposition 4 *Country i enjoys an increase in the expected output if $\frac{\partial \tilde{w}_i}{\partial \theta_i} > 0$, while the expected output for its opponent decreases. The result is ambiguous if $\frac{\partial \tilde{w}_i}{\partial \theta_i} < 0$. Therefore:*

$$\begin{aligned} \frac{\partial \tilde{y}_i}{\partial \theta_i} &> 0 \quad \text{and} \ \frac{\partial \tilde{y}_j}{\partial \theta_i} < 0 \quad \text{if} \ \frac{\partial \tilde{w}_i}{\partial \theta_i} > 0 \\ \frac{\partial \tilde{y}_i}{\partial \theta_i} &\gtrless 0 \quad \text{and} \ \frac{\partial \tilde{y}_j}{\partial \theta_i} \lesseqgtr 0 \quad \text{if} \ \frac{\partial \tilde{w}_i}{\partial \theta_i} < 0 \end{aligned}$$

Proof. In the appendix. ■

The content of the above proposition is consistent with Proposition 2. According to the latter it is always true that $\frac{\partial \tilde{w}_j}{\partial \theta_j} < 0$. Therefore, if $\frac{\partial \tilde{w}_i}{\partial \theta_i} > 0$ this implies that country i has (in relative terms) a larger probability of success in the fight than its opponent. Consequently, the expected amount of consumption good increases for country i while it decreases for the other contestant.

Instead, the ambiguous result of the second part of the Proposition depends on countries' allocation decisions. If the decrease in $\tilde{w}_i$ generated by a change in θ_i is larger (smaller) than the one induced in $\tilde{w}_j$, then country i would expect a smaller (larger) amount of good than its opponent

This result is partially consistent with the one found in Muthoo (2004). In this contribution it is shown that any positive change in players' fighting skills produces an increase in players payoffs if and only if they are sufficiently strong. This applies also to the result of Proposition 4. Country i's expected output from the fight increases, since it becomes (in relative terms) stronger than its opponent.

In the remaining part of this section, we evaluate how a change in the productive technology of country i affects the expected ammount of commodity good for both contestants. Given the lack of an explicit solution for $\tilde{w}_i$ and $\tilde{w}_j$, it becomes difficult to evaluate this relation.

Totally differentiating equation (10) yields:

$$\frac{\partial \tilde{y}_i}{\partial a_i} = \overset{+}{\frac{\partial \tilde{y}_i}{\partial \tilde{w}_i}} \overset{-}{\frac{\partial \tilde{w}_i}{\partial a_i}} + \overset{-}{\frac{\partial \tilde{y}_i}{\partial \tilde{w}_j}} \overset{+}{\frac{\partial \tilde{w}_j}{\partial a_i}} + \overset{+}{\frac{\partial \tilde{y}_i}{\partial a_i}} \lesseqgtr 0 \tag{12}$$

$$\frac{\partial \tilde{y}_j}{\partial a_i} = \overset{-}{\frac{\partial \tilde{y}_j}{\partial \tilde{w}_i}} \overset{-}{\frac{\partial \tilde{w}_i}{\partial a_i}} + \overset{+}{\frac{\partial \tilde{y}_j}{\partial \tilde{w}_j}} \overset{+}{\frac{\partial \tilde{w}_j}{\partial a_i}} + \overset{+}{\frac{\partial \tilde{y}_j}{\partial a_i}} > 0 \tag{13}$$

It is ambiguous if country i can take advantage from an improvement in its productive technology. This depends on whether the direct effect of an increase in a_i on the expected output is able to offset the indirect one. The former is measured by the last term in equation (12) and refers to the enlargement of the common pool, determined by the improvement of country i's productive skills. Instead, the indirect effect depends on the changes aroused in the equilibrium values $\tilde{w}_i$ and $\tilde{w}_j$ and is measured by the first two terms of equation (12). Those changes affect the probabilities of success for both countries decreasing $\tilde{s}_i$ and augmenting $\tilde{s}_j$. Depending on whether the direct effect is bigger than the indirect one, country i can enjoy a larger expected output even though its probability of success decreases.

On the contrary, it clearly appears that an increase in a_i enlarges the amount of output for country j. A similar explanation to the one given for country i can be applied to its opponent. As a matter of fact, it enjoys the positive effect of both the enlargement of the common pool and the improvement of its probability of success.

Equations (12) and (13) allow us to characterize the following result:

Proposition 5 *When country i improves its military technology, its expected output from the fight increase or not depending on whether the positive and direct effect of the enlargement of the common pool provoked by a_i offsets the negative and indirect effect of the reduction in its probabiliy of success. Instead its opponent always benefits of an increase in the expected output.*

4 Living Together and in Peace

In this section we focus on the idea that, before fighting, countries discuss about the possibility of finding an agreement, which may lead them to merge.

As pointed out earlier, this choice has several consequences. The most important one refers to the fact that if states merge, they will share their technologies. However, if countries do not reach an agreement, the only way to sort out their dispute is fighting a war.

That scenario is modelled as a two-stage game, its timing being as follows:

1: At the first stage countries set the sharing rule which should be implemented;

2: At the second stage:

- **2 a:** If they are unable to agree on a sharing rule, then the war is the only way to settle the dispute and countries never come out of the jungle;
- **2 b:** If, on the contrary, they negotiate an agreement at the first stage, which is accepted by both of them, they start to produce jointly the commodity good, which is divided according to the sharing rule.

Clearly, the two second stages are alternative. Moreover, if countries are unable to negotiate an agreement and decide to fight, the equilibrium, which emerges is the one in the jungle.

We divide this section into two parts. In the first one, we carry out a one-shot analysis, assuming that the decision of jointly produce the commodity good lasts only for one period. From a realistic point of view this choice can be consistently acknowledged as an attempt made by both countries to assess whether it is really fruitful for them to merge or not. Moreover, the one-shot analysis allows us to consider and evaluate the role of some specific parameters of the model. In this context, neither we have any military technology transfer nor we observe any learning process in the use of the most effective productive technology from the less productive country.

In the second part of the section we assess the behavior of the states in the long run, also deriving the conditions which make the merger a self-enforcing agreement. In this context, the transfer of technologies takes place and we may evaluate the enrichment of the model due to this fact.

4.1 One-Shot Analysis

It is worthwhile to stress that countries accept to merge if and only if this choice makes them strictly better off. The model is solved backward.

Second Stage

If countries cannot reach any agreement at the first stage, the second one is charcterized by the equilibrium in the anarchic world, since countries solve their dispute by fighting a war. Instead if they are able to strike an agreement in the first stage, in the second one they produce the commodity good using only the best productive technology. Obviously they use all their endowments in the production, since now they do not need to mobilize resources for the war. Therefore, the amount of commodity good is given by the following straightforward equation:[8]

$$y^{**} = a_1 \left(e_1 + e_2 \right) \tag{14}$$

An important issue concerning the equilibrium output after merging is the existence of a comparative advantages with respect to the previous situation. More specifically, it is required that the quantity y^{**} is strictly larger than $\tilde{y}$. On the contrary, there would be no incentive for countries to merge. The following lemma clarifies this point:

Lemma 2 *The amount of jointly produced commodity good is always larger than the common pool in the jungle. Moreover they are better off than in the case in which they live peacefully but do not merge.*

If countries neither fought nor decided to merge, each of them would enjoy an amount of commodity good equal to $a_i e_i$. Using this fact, it is easy to prove the previous lemma through the following inequality:

$$[a_1(e_1 - \tilde{w}_1) + a_2(e_2 - \tilde{w}_2)] < (a_1 e_1 + a_2 e_2) < a_1(e_1 + e_2) \tag{15}$$

The lhs of (15) measure the extension of the pool in the jungle. The term in the middle represents the sum of the commodity goods produced by countries, if they neither fight nor merge. Finally, the rhs of previous inequality is equal to y^{**}, which is larger than the last two because a_1 is the most efficient production technology.

[8] I use the symbol "**" to denote the equilibrium values in the merge scenario.

Inequality (15) guarantees the existence of a comparative advantage for countries, if they merge. Moreover, it also stresses an important issue, namely that merger is more efficient than not fighting but not merging.

First Stage

At the first stage of the game, countries set the sharing rule, x, to divide the jointly produced output. At the second one, we derive it by implementing a Nash bargaining problem. Choosing this solution concept brings some advantages. Among others, it takes also into consideration what happens if countries cannot negotiate a peaceful environment.[9] Therefore, the expected outcome from the war characterizes the set of the disagreement points in the Nash problem.

The optimal sharing rule is derived by maximizing the following Nash product:

$$\max_x \left[xy^{**} - \tilde{y}_1\right]^\gamma \left[(1-x)\, y^{**} - \tilde{y}_2\right]^{1-\gamma} \tag{16}$$

where γ measures the bargaining power of country 1, while $(1-\gamma)$ is the measure of the bargaining power of its opponent.

Maximization yields the following value for the sharing rule:

$$x^{**} = \gamma + \frac{(1-\gamma)\,\tilde{y}_1 - \gamma\tilde{y}_2}{y^{**}} \tag{17}$$

Making use of (17), the following result can be established:

Proposition 6 *In the merger equilibrium countries consume the following amounts of commodity good:*

$$\begin{aligned} y_1^{**} &= s_1\tilde{y} + \gamma\left(y^{**} - \tilde{y}\right) \\ y_2^{**} &= y^{**} - s_1\tilde{y} - \gamma\left(y^{**} - \tilde{y}\right) \end{aligned} \tag{18}$$

Equations (18) have a simple interpretation. In equilibrium country 1 obtains the amount of commodity good, which it would obtain after the fight, augmented by a fraction of the difference between the merger production and $\tilde{y}$. That fraction is determined by its bargaining power. Consequently, country 2 receives the remaining part of the joint production.

[9] Assuming that if countries cannot reach an agreement they fight forever is reasonable, since if their representatives fail to strike an agreement when seated at the same table, it is quite unlikely that they can reach an agreement otherwise.

As expected, the distribution of bargaining power between countries is a key determinant of the bargaining outcome. As a matter of fact, whenever γ goes to 0, the previous set of equations reduces to:

$$y_1^{**} = s_1\tilde{y}$$
$$y_2^{**} = y^{**} - s_1\tilde{y}$$

In this case country 1 obtains precisely the same output, which it would get in the jungle, while its opponent receives the remaining part. The situation is reverted when γ tends to 1, since in this case we have:

$$\begin{aligned} y_1^{**} &= y^{**} - s_2\tilde{y} \\ y_2^{**} &= s_2\tilde{y} \end{aligned} \tag{19}$$

Therefore, we can state the following corollary:

Corollary 2 *The distribution of gains from merging decreases (increases) for country* 1 *as* γ *tends to* 0 *(*1*). The opposite holds for its opponent.*

This point is interesting, since the distribution of gains from merging eventually affects the decision of taking this step. As recalled before, countries agree to merge if and only if this choice has a positive impact on their welfare. Since, country 1's gain from merging decreases as $y \to 0$, we may infer that the smaller is its bargaining power, the smaller is its gain from merger and, consequently, the less incline it is regarding this solution. Eventually, if its bargaining power is 0, it will decide not to merge but to remain into the anarchic world. The same applies to country 2 when $\gamma \to 1$.

While γ is usually considered as the parameter for the baragaining power of countries, it can be interpreted in a deeper way. For instance Binmore, Rubinstein and Wolinsky (1986) show that the asymmetric Nash bargaining solution is the unique perfect equilibrium of a negotiation process between two parties with asymmetric beliefs about the likelihood of a break-down, when the length of each bargaining period becomes infinitely small. In other words, the higher is country 1's estimate of the probability of breakdown, the lower is γ. Therefore, the bargaining power of each country can be interpreted as a direct measure of the probability that they attaches to the possibility that the bargaining process is successful. This exaplanation is perfectly in line with the content of corollary 2, since a small value of γ implies a small gain from merging. In the limit, when $\gamma = 0$, country 1 gains nothing from the merger and it prefers the anarchy to any other solution, the probability of success that it is ascribed to the bargaining process being 0.

Another interesting interpretation of γ comes from the institutional and applied researches view.[10] A formalization of that interpretation can be found in Svejnar (1986) who argues that the bargaining power can be "...influenced by institutional, economic, and other variables..." (p.1061). More specifically, he brings into the theory of bargaining the view that γ is able to capture other characteristics of parties involved in the bargaining process, which do not enter directly into their utility functions as a subject of bargaining.[11] The above interpretation is quite appealing in this context since it can bring into the model a further heterogeneity, which is not included into the utility functions in the bargaining problem. While it is beyond the objectives of our work to establish which variables may affect γ, we may suggest some ways to interpret it and to observe how other elements may affect bargaining outcome. For instance, Assumption 2 of the model states that countries behave as unitary agents. This allows us to neglect the problem of social choices formation. However one of the determinant of the bargaining power of a country can be exactly the degree of homogeneity in the preferences across the population. A larger fractionalization would reduces the bargaining power of a country and, eventually, this would reduce the possibility of reaching an agreement. While this aspect does not enter directly into the maximization problem, although it can affect the bargaining outcome, it can be included in the exogenously determined parameter γ.[12]

4.2 A Long Run Analysis

The decision of merging implies that countries should be able to maintain that choice through time. In this section we investigate upon the possibility this possibility, deriv-

[10] See for instance Chamberlain and Kuhn (1965) and Kochan (1980).

[11] While there have already been some empirical attempts to define the determinants of the bargaining power, for instance Ashenfelter, Johnson an Pencavel (1972), Svejanar's contribution theoretically defines the role of bargaining power into the bargaining process.

[12] A further and more fascinating idea would be to link the parameter γ with the degree of cultural homogeneity. According to this interpretation, countries with similar cultural endowment will have a larger chance of following the merger path. This idea relies on Huntington (1996)'s theory that the geopolitical appearance of the world is likely to change on the basis of cultural affinity.

ing the incentive compatibility requirements, which make a long run merger feasible.

It is worthwhile to recall that merger in the long run has some consequences about the allocation of technologies between countries. On one hand, country 2 partially gives up the control of its military technology, while, on the other country 1 allows its opponent to use its most productive tecnology. Moreover, while we assume that country 2 cannot totally acquire the best productive technology, however it can learn at least a part of it.

Those facts have a particular relevance when the analysis focuses on the possibility that countries my deviate from the merger path. As a matter of fact, if countries decide to split, we have a different distribution of technologies between them with respect to the one before merging.

Given a parameter $0 \leq \alpha \leq 1$, the extension of control over the military technology of each country is:[13]

$$\hat{\theta}_1 = \alpha [\theta_1 + \theta_2] \tag{20}$$

$$\hat{\theta}_2 = (1 - \alpha) [\theta_1 + \theta_2] \tag{21}$$

As far as the productive technology concerns, in the event of a secession, countries enjoy respectively:

$$\hat{a}_1 = a_1 \tag{22}$$

$$\hat{a}_2 = a_2 + \lambda a_1 \tag{23}$$

While country 1 maintains its own technology, country 2 will benefit of an increase in it. In particular the parameter $\lambda < 1$ specifies how good country 2 has been in learning the new technology. Nonetheless, we make the assumption that, if they split, country 1 still maintains the best productive technology. In other words:

$$a_2 + \lambda a_1 < a_1 \tag{24}$$

Equation (24) implies that

$$\lambda < \frac{a_1 - a_2}{a_1}$$

[13] I use the symbol "ˆ" to denote the parameters and the equilibrium values which refer to this modified version of the anarchich world.

In order to satisfy the previous inequality, we assume that $\lambda = \frac{a_1 - a_2}{\kappa a_1}$, where $\kappa > 1$ measures country 2's error in learning the new technology. Substituting this value back into equation (23) yields:

$$\hat{a}_2 = \frac{a_1 + (\kappa - 1)\, a_2}{\kappa} \tag{25}$$

The production technology enjoyed by country 2 is decreasing in κ: the larger is its error in the learning process, the less efficient is the technology it owns after the secession.

Using the elements above, we now investigate upon the conditions, which make a country decide to maintain the merger instead of deviating when it repetitively interact with its opponent. This new scenario requires a better specification of how countries' life evolves after merger. At the beginning of each day they jointly produce the commodity good. Then, when the day expires, they share it according to the sharing rule, which they have agreed on. Nonetheless, before the division takes place, one of them may decide to steal some of the output. If this event happens, we assume that the other country would move back to anarchy. In other words, countries adopt a trigger strategy: they accept to maintain merger unless one of them decides to steal some of the commodity good and splits, this giving rise to the anarchy equilibrium, characterized by the new distribution of military and productive technologies.

Let δ be the common rate at which each country discounts its future amount of output y. Country i does not have an incentive to deviate if the following inequality is satisfied:

$$y^{**} + \sum_{k=1}^{\infty} \delta^k \hat{y}_i \leq \sum_{k=0}^{\infty} \delta^k y_i^{**} \tag{26}$$

The incentive-compatibility constraint (26) can be interpreted in the following way. If country i deviates from the merger path, it will steal all the output produced in one day (y^{**}) but from the next day onwards it will obtain the discounted amount of the commodity good from the fight ($\sum_{k=1}^{\infty} \delta^k \hat{y}_i$). It is useful to remark that if countries split the amount of commodity good obtained by countries in the anarchy is not $\tilde{y}_i$, but $\hat{y}_i$, since now they enjoy the new distribution of technologies.

Solving the incentive-compatibility constraint for δ yields the pair of critical discount factors, which define the range of the parameters value, which guarantee that countries do not deviate:

Proposition 7 *Countries would not deviate from the merger path if and only if*

$$\delta^{**} \geq \max\left\{\underline{\delta}_i, \underline{\delta}_j\right\} \;\; \forall i \neq j = 1, 2$$

where

$$\underline{\delta}_i = \frac{y_j^{**}}{y^{**} - \hat{y}_i} \tag{27}$$

Proof. In the appendix. ■

The content of the previous proposition can be explained as follows. Country i (j) would not deviate from the merger path if the common discount factor is at least as larger than $\underline{\delta}_i$ $(\underline{\delta}_j)$. It follows that δ^{**} needs to be at least larger as the highest of the critical discount factor in order to avoid for both countries any incentive to deviate.

It is interesting to note that by the same reasons, highlighted early, the likelihood that countries follow the merger path decreases when the value of the parameter γ is either very high or very low. More specifically, if $\gamma \to 0$, $\underline{\delta}_1 = 1$ and it is larger than $\underline{\delta}_2$. On the other hand, if $\gamma \to 1$, then $\underline{\delta}_2 = 1$ and it is larger than $\underline{\delta}_1$. Hence, an inequal distribution of the bargaining power enlarges the distance between $\underline{\delta}_1$ and $\underline{\delta}_2$ and, when γ takes its extreme values, the smallest discount factor needed to avoid any deviation is 1. Therefore, the following corollary can be established:

Corollary 3 *The possibility of reamining merged through time may be compromised by an unequal distribution of the bargaining power.*

An unequal distribution of the bargaining power makes the perspective of maintaining the merger less appealing for one country with respect to the other. This is a necessary consequence of the small quantity of the commodity good, which it would obtain, if it accepts to merge with its opponent. Even though the decision of merger is taken, it is quite unlikely that it can be maintained for a long time.

Unfortunately we cannot establish whether either $\underline{\delta}_1$ is larger than $\underline{\delta}_2$ or the opposite holds, due to the lack of the explicit solutions for the equilibrium values in the anarchic world. Nonetheless, establishing that result would be uselful in order to evaluate if any change in the main parameters of the model can compromise or support the merger between countries, as we will make clear in the next pages. However, while we cannot state unambiguousy which of those critical discount factors is larger, we can show at least one case in which $\underline{\delta}_1 > \underline{\delta}_2$:

Corollary 4 *If country* 2*'s probability of success is sufficiently large and country* 1*'s bargaining power small enough,* $\underline{\delta}_1 > \underline{\delta}_2$.

Proof. In the Appendix. ■

From proposition 7 emerges that the value of the discount factor δ^{**}, which is crucial for mantaining merger through time, depends upon the parameters α and κ since they affects $\underline{\delta}_1$ and $\underline{\delta}_2$, enlarging or reducing their distance. Therefore, in the remaining part of this section we focus on the relation between the critical discount factors $\underline{\delta}_1$ and $\underline{\delta}_2$ and α and κ.

4.3 Relation Between α and The Critical Discount Factors

In order to analyze the relation between the distribution of the control over the military technology and the critical discount factors, we define how a change in α affect the allocation of resources between *butter* and *gun* in the event of a fight.

Lemma 3 *If* α *increases and country* 1 *is assigned a larger control over the military technology in the merger scenario, it allocates less resouces for the war, if merger breaks down and countries move back to anarchy. The opposite holds for its opponent, i.e.:*

$$\frac{\partial \hat{w}_1}{\partial \alpha} < 0 \quad \text{and} \quad \frac{\partial \hat{w}_2}{\partial \alpha} > 0 \tag{28}$$

Moreover, its probability of being success in the future war may raise or decline depending on whether the direct effect of an increase in α *is offset by the indirect effect caused by the new allocation of resources.*

Proof. In the appendix. ■

The content of the previous lemma has an easy interpretation. Any increase in α produces a substitution effect for country 1. Therefore it allocates less for the war, relying on the larger control over the military technology, and raises the production of the commodity good. As a final result, its probability of success in the fight might not be affected, since the decrese in $\hat{w}_1$ can be compensated by a larger value of α. As far as country 2 concerns, the reduction over $[\theta_1 + \theta_2]$ may be compensated by an increase in the amout of resources allocated for the fight.

It is interesting to contrast the result of Lemma 3 with the one in Proposition 2. The main difference between those results refers to the fact that in the latter case the reaction of country 2 to a change in θ_1 is opposite to the one in Lemma 3. This discrepancy depends on the fact that in the modified version of anarchy, the control over the military technology is now interdependent, therefore any increase in the control for one country translates in a reduction for the other. Consequently, if α raises, country 2 needs to allocate more resources for the war if it wants to compensate the smaller control over the military technology.

Whether countries' reactions to a change in α are sufficient to keep unaffected their own probabilities of success in the fight cannot be established unambiguously. As a matter of fact, if we differentiate the contest success function for country 1, we obtain:

$$\frac{\partial \hat{s}_1}{\partial \alpha} = \frac{\hat{w}_1 \hat{w}_2 + \alpha (1-\alpha) \left[\hat{w}_2 \frac{\partial \hat{w}_1}{\partial \alpha} - \hat{w}_1 \frac{\partial \hat{w}_2}{\partial \alpha} \right]}{\left[\alpha \hat{w}_1 + (1-\alpha) \hat{w}_2 \right]^2} \tag{29}$$

The first term in the numerator, $\hat{w}_1 \hat{w}_2$, measures the direct effect of a change in the contest success function, when α varies and it is clearly positive. Instead, the second term, $\alpha (1-\alpha) \left[\hat{w}_2 \frac{\partial \hat{w}_1}{\partial \alpha} - \hat{w}_1 \frac{\partial \hat{w}_2}{\partial \alpha} \right]$, is negative and measures the indirect effect of α on $\hat{s}_1$ through the changes in $\hat{w}_1$ and $\hat{w}_2$. Depending on which term is larger the probability of success for country 1 is either positive or negative. It should be acknowledged the opposite behavior of country 2's probability of success to a change in α. In other words we have that $\frac{\partial \hat{s}_1}{\partial \alpha} = -\frac{\partial \hat{s}_2}{\partial \alpha}$.

Making use of equations (28) and (29), we can establish the relation between α and the critical discount factors.

Proposition 8 *When country* 1 *has a larger control over the military technology, its incentive-compatible discount factor increases (decreases) if its probability of success in the fight declines (raises). The opposite holds for its opponent. Therefore:*

$$\frac{\partial \underline{\delta}_1}{\partial \alpha} \gtrless 0 \quad \textit{and} \quad \frac{\partial \underline{\delta}_2}{\partial \alpha} \lesseqgtr 0 \quad \Longleftrightarrow \quad \frac{\partial \hat{s}_1}{\partial \alpha} \lesseqgtr 0$$

Proof. See the Appendix. ■

The most interesting result of Proposition 8 occurs when $\frac{\partial \hat{s}_1}{\partial \alpha} < 0$. In this case, when country 1 controls a larger portion of military technology, its critical discount factor increases, while $\underline{\delta}_2$ decreases. More specifically, it is peculiar to understand why country 1's critical discount factor raises making more difficult to maintain merger, even if its probability of success goes down. A possible interpretation for this result could be based on some miscalculation from country 1: having more control over the military technology could generate a wrong idea of strenght, which cannot be supported in real terms. In other words, enjoying a larger value of α could lead country 1 to overestimate its own probability of success and this can create an incentive to deviate from the merger path. However, it should be noted that the increase in $\underline{\delta}_1$ does not necessarily threaten the possibility of maintain merger, since this can happen only if $\underline{\delta}_1 > \underline{\delta}_2$. If this is not the case, when α varies, the distance between the critical discount factors shrinks and it becomes easier to stay merged.

Corollary 5 *Maintaining merger may become difficult if α increases, depending on which discount factors between $\underline{\delta}_1$ and $\underline{\delta}_2$ is larger.*

The content of Corollary 5 underlines only one of the possible cases, when staying merged becomes difficult when the shares of control over the military technology varies. However, it is useful to highlight that case, since it stresses the fact that the long run merger is feasible according to whether any change in the parameters of the model may enlarge or shrink the distance between the critical discount factors.

4.4 Relation Between κ and the Critical Discount Factors

As we have done in the previous section, a preliminary step for the analysis of the relation between κ and $\underline{\delta}_1$ and $\underline{\delta}_2$ consists of investigating how the learning error parameter affect the distribution of the resources in the case of a conflict.

Lemma 4 *If country 2 shows poorer skills in learning the most effective productive technology, both countries allocate less resources for the war i.e.:*

$$\frac{\partial \hat{w}_1}{\partial \kappa} < 0 \quad \text{and} \quad \frac{\partial \hat{w}_2}{\partial \kappa} < 0 \tag{30}$$

Derivatives (38) clarify that both countries would allocate less resources for the fight. The explanation behind this result can be put as follows. If country 2 is not able to learn and to use the new productive technology, it has to allocate more resources for the production of the commodity good. If this is the case, country 1 may follow the same choice of its opponent, since it feels more secure.

Clearly, the reduction in $\hat{w}_1$ and $\hat{w}_2$ brings some consequences on the probabilities of success for both countries. More specifically, how they change depends on which one between $\hat{w}_1$ and $\hat{w}_2$ decreases more. Differentiating $\hat{s}_1$ for κ yields:

$$\frac{\partial \hat{s}_1}{\partial \kappa} = \frac{\alpha (1-\alpha) \left[\hat{w}_2 \frac{\partial \hat{w}_1}{\partial \kappa} - \hat{w}_1 \frac{\partial \hat{w}_2}{\partial \kappa}\right]}{\left[\alpha \hat{w}_1 + (1-\alpha) \hat{w}_2\right]^2} \lesseqgtr 0 \tag{31}$$

The relation between $\hat{s}_1$ and κ depends on the value in the square brackets, which measures the impact of κ on the equilibrium allocation of endowments.

Making use of Lemma 4 and equation (31) we can establish the following result:

Proposition 9 *If country 2's learning error becomes sufficiently large and γ is sufficiently high, $\underline{\delta}_1$ decreases if $\frac{\partial \hat{s}_1}{\partial \kappa} > 0$, while $\underline{\delta}_2$ either reduces or raises. Instead, if $\frac{\partial \hat{s}_1}{\partial \kappa} > 0$ a specular situation occurs. More specifically:*

$$\begin{array}{lllll} \frac{\partial \underline{\delta}_1}{\partial \kappa} & < & 0 \quad \textit{and} \quad \frac{\partial \underline{\delta}_2}{\partial \kappa} \gtreqless 0 & \textit{if} & \frac{\partial \hat{s}_1}{\partial \kappa} < 0 \\ \frac{\partial \underline{\delta}_1}{\partial \kappa} & \gtreqless & 0 \quad \textit{and} \quad \frac{\partial \underline{\delta}_2}{\partial \kappa} < 0 & \textit{if} & \frac{\partial \hat{s}_1}{\partial \kappa} > 0 \end{array}$$

The result contained in the previous Proposition follows immediately from differentiating the critical discount factors with respect to κ. Calculation yields:

$$\frac{\partial \underline{\delta}_1}{\partial \kappa} = -\frac{\frac{\partial \hat{s}_1}{\partial \kappa} \hat{y} \left(y_1^{**} - \hat{y}_1\right) + \left[\left(\hat{s}_1 - \gamma\right)\left(y^{**} - \hat{y}_1\right) - \hat{s}_1 y_2^{**}\right] \left[Z - \frac{\partial \hat{a}_2}{\partial \kappa}\left(e_2 - \hat{w}_2\right)\right]}{\left(y^{**} - \hat{y}_1\right)^2} \tag{32}$$

$$\frac{\partial \underline{\delta}_2}{\partial \kappa} = \frac{\frac{\partial \hat{s}_1}{\partial \kappa} \hat{y} \left(y_2^{**} - \hat{y}_2\right) - \left[\left(\hat{s}_1 - \gamma\right)\left(y^{**} - \hat{y}_2\right) - \hat{s}_1 y_1^{**}\right] \left[Z - \frac{\partial \hat{a}_2}{\partial \kappa}\left(e_2 - \hat{w}_2\right)\right]}{\left(y^{**} - \hat{y}_2\right)^2} \tag{33}$$

where

$$Z = \left[\hat{a}_1 \frac{\partial \hat{w}_1}{\partial \kappa} + \hat{a}_2 \frac{\partial \hat{w}_2}{\partial \kappa}\right] \tag{34}$$

The terms in the last square backets in both equations measure the change in the size of the common pool as κ varies. For sufficiently large values of the latter parameter, that value can be negative Also it is easy to show that for sufficiently large values of γ, such that $\gamma \geq \hat{s}_1$, the value in the first square brackets of both equation is is negative as well. Therefore, whether countries' critical discount factors increase or not, as κ varies, depends on the the relation between the latter and $\hat{s}_1$.

Equations (32) and (33) are able to characterize several situations. Among all, we would like to pay attention to two of them, which stress when merger is surely the preferred outcome for both countries and there is not any incentive to deviate.

First, we consider the case in which $\frac{\partial \hat{s}_1}{\partial \kappa} < 0$. While $\underline{\delta}_2$ unambiguosly declines as κ becomes larger, $\underline{\delta}_1$ behaves in the same situations only if country 1's probability of success decreases more than the reduction in the size of the common pool. If this hypothesis prevails, along with the reduction of the common pool, country 1 suffers from the consistent decrease of its probability of success. Therefore, the expected output from the fight declines consistently. As far as country 2 concerns, it does not have any incentive to deviate since a large value of κ means that it is not able to use the most effective technology and, consequently, in the anarchic world it would not be able to exploit it. This situation is particular appealing because it reduces both critical discount factors and makes merger easier to be achieved, no matter which of them is larger.

A similar result can be obtained when $\frac{\partial \hat{s}_1}{\partial \kappa} > 0$. In this case for both countries is determinant the fact that the size of the common pool shrinks as κ becomes larger. Therefore, moving back to the anarchy is not a good choice, expecially for country 2, since now its opponent is also stronger than before in relative terms.

5 Concluding Remarks

In this paper, we characterize an anarchic world in which two countries struggle over the control of a valuable good. As we made clear through the paper, our model can mimic the behavior of countries and the way in which they act in the international relations, particularly when they are experiencing episodes of violence.

While the war is the default option to settle their dispute, we proposed an alternative solution by investigating upon the possibility that they merge and form a new political entity. To our best knowledge, the merger solution has not been analyzed in the literature so far. Instead, existing contributions prefer to derive the conditions under which countries renounce fighting, living in peace but as two separate political entities.

We achieve several interesting results. The first one refers to countries' welfare improvement after merging. More specifically, the amounts of commodity good, which they enjoy under this solution, are larger than the one after the fight. Moreover, they are also larger than the ones they will obtain if they do not fight but do not merge. This fact makes our proposed solution more efficient than the one introduced by the literature so far.

Further, we derived the conditions which guarantee that merger can be preserved in the long-run. The most important feature of this possibility is that the agreement, which countries strike, does not need any third party since it is self-enforcing.

While in our model we indicate the conditions, which guarantee that merger can be a long run equilibrium, from a dual perspective, we show that merger, in fact, may collapse. In particular, this may happen depending on the value taken by some specific parameters. In this case, on one hand, merger cannot be achieved at all, while, on the other, if countries moved on to the merger solution, quite likely it breaks down and they go back to the anarchy.

As recalled in the introduction, this work can be considered not only as a contribution to the literature on conflicts and their solution, but it can be also used as a starting point for interpreting and assessing several merger/secession processes in a fashion which has been poorly investigated by the specific literature on the formation of countries. Instead, as it has been pointed out, international relations are a valid approximation of the Hobbesian state of nature (Oye, 1985) and it is logically consistent to study the merger/secession topic within this framework.

Several extension of the model can be conceived. For instance, one of the most interesting could be related to the introduction of a third countries, allowing for a coalition formation between two of them against the third one. Possibly, the coalition formation is supported by the specialization in a specific activity by each country involved.

This point as well as other extensions of the model will be left for further research.

References

[1] Alesina, A., and E. Spolaore (1997): "On the Number and Size of Nations," Quarterly Journal of Economics, 112(4), 1027–1056.

[2] Alesina, A., and E. Spolaore (2003): The Size of Nations, MIT Press, Cambridge.

[3] Alesina, A., and E. Spolaore (2005): "War, Peace, and the Size of Countries," Journal of Public Economics, 89 (7), 1333–1354.

[4] Anderton, C.H. (2000): "An Insecure Economy under Ratio and Logistic Conflict Technologies, " Journal of Conflict Resolution, 44(6), 823–838.

[5] Ashenfelter, O.C, G.E. Johnson, and J.H. Pencavel (1972): "Trade Union and the Rate of Change of Money Wages in United States Manufacturing Industry," Review of Economic Studies, 39, 27–54.

[6] Binmore, K., Rubinstein, A. and A. Wolinsky (1986): "The Nash Bargaining Solution in Economic Modelling," The RAND Journal of Economics, 17(2), 176–188.

[7] Bolton, P., and R. Gerard (1997): "The Break-Up of Nations: A Political Economy analysis," Quarterly Journal of Economics, 112(4), 1057–1089.

[8] Bolton, P., R. Gerard, and E. Spolaore (1996): "Economic Theories of the Break-Up and Integration of Nations," European Economic Review, 40 (3-5), 697–705.

[9] Casella, A. (1992): "On Markets and Clubs. Economic and Political Integration of Regions with Unequal Productivity," American Economic Review, 82(2), 115–121.

[10] Casella, A. and J. Feinstein (2002): "Public Goods in Trade: On the Formation of Markets and Jurisdictions," International Economic Review, 43, 437–462.

[11] Chamberlain, N.W. and J.W. Kuhn (1965): Collective Bargaining. McGraw-Hill, new York.

[12] Cronin, B. (1999): Community Under Anarchy: Transnational Identities and the Evolution of Cooperation. Columbia University Press, New York.

[13] Fearon. J.D. (1995): "Rationalist Explanations for War," International Organization, 49(3), 379–414.

[14] Friedman, J.W. (1990): Game Theory with Applications to Economics, 2nd ed. New York: Oxford University Press.

[15] Gilpin, R. (1981): War and Change in World Politics. Cambridge University Press, Cambridge.

[16] Gilpin, R. (2001): Global Political Economy. Understanding the International Economic Order. Princeton University Press, Princeton.

[17] Goyal, S., and K. Staal (2004): "The Political Economy of Regionalism," European Economic Review, 48, 563–593.

[18] Grossman, H., (2004): "Peace and War in Territorial Disputes," Brown University Working Paper Series, # 7.

[19] Grossman, H., and M. Kim (1995): "Swords or Plowshares? A Theory of the Security of Claims to Property," Journal of Political Economy, 103(6), 1275–1288.

[20] Grossman, H., and J. Mendoza (2001): "Butter and Guns: Complementarity between Economic and Military Competition," Economics of Governance, 2(1), 25-33.

[21] Hirshleifer, J. (1995a): "Theorizing about conflict". In Handbook of defense economics, Vol. 1, edited by Keith Hartley and Todd Sandler, 165-92. Amsterdam: North-Holland.

[22] Hirshleifer, J. (1995b): "Anarchy and its breakdown," Journal of Political Economy, 103(1), 26–52.

[23] Hirshleifer, J. (2001): The Dark Side of the Force. Cambridge University Press, Cambridge.

[24] Huntington, S. P. (1996): The clash of civilizations and the remaking of world order. Simon & Schuster, New York.

[25] Kochan, T.A. (1980): Collective Bargaining and Industrial Relations: From Theory to Policy and Practise.Irwin, Homewood.

[26] Muthoo, A. (2004): "A Model of Origins of the Basic Property Rights," Games and Economic Behavior, 49(2), 288–312.

[27] North, D. C. (1990): Institutions, institutional change, and economic performance. Cambridge University Press, Cambridge.

[28] Nti, K. O. (1997): "Comparative Statics of Contests and Rent-Seeking Games," International Economic Review, 38(1), 43–59.

[29] Oye, K. A. (1985): "Explaining Cooperation under Anarchy: Hypotheses and Strategies," World Politics, 38(1), 1–24.

[30] Powell, R. (1993): "Butter, Gun and Anarchy," American Political Science Review, 87(1), 115–132.

[31] Riker, W. (1964): Federalism, Little Brown, New York.

[32] Skaperdas, S. (1992): "Cooperation, Conflict and Power in the Absence of Property Rights," American Economic Review Papers and Proceedings, 82, 720–732.

[33] Svejnar, J. (1986): "Bargaining Power, Fear of Disagreement, and Wage Settlements: Theory and Evidence from U.S. Industry," Econometrica, 54(5), 1055–1078.

[34] Tullock, G. (1980): "Efficient Rent Seeking," in Toward a Theory of the Rent-Seeking Society, ed. by J. M. Buchanan, R. D. Tollison, and G. Tullock. Texas A&M University Press, College Station.

[35] Waltz, K. (1959): Man, the State, and War: A Theoretical Analysis. Columbia University Press, New York.

[36] Wei, S.-J. (1991): "To Divide or to Unite: A theory of Secessions," Mimeo, University of California at Berkeley.

Appendix

Proof of Proposition 1

Differentiating equation (3) for both w_1 and w_2 and setting the first order conditions equal to 0 yields:

$$\begin{aligned}\frac{\partial y_1}{\partial w_1} &= \frac{\theta_1}{\theta_1 w_1 + \theta_2 w_2}\left[\theta_2 w_2 y - w_1 a_1 \left(\theta_1 w_1 + \theta_2 w_2\right)\right] = 0 \\ \frac{\partial y_2}{\partial w_2} &= \frac{\theta_2}{\theta_1 w_1 + \theta_2 w_2}\left[\theta_1 w_1 y - w_2 a_2 \left(\theta_1 w_1 + \theta_2 w_2\right)\right] = 0\end{aligned}$$

Solving the first equation for w_1 and the second for w_2 yields the result stated in the proposition.

As far as the uniqueness of the equilibrium concerns, following Friedman (1990, p.86) the above reactions functions characterize the unique Nash equilibrium, if and only if the Hessian matrix of their second order partial derivatives is negative quasi-definite for all strategy profiles. In the remaining part of the proof, we show that this is the case.

Let $\mathcal{H}$ be the Hessian matrix, i.e.:

$$\mathcal{H} = \begin{bmatrix} \frac{\partial^2 y_i}{\partial w_i^2} & \frac{\partial^2 y_i}{\partial w_j \partial w_j} \\ \frac{\partial^2 y_j}{\partial w_j \partial w_j} & \frac{\partial^2 y_j}{\partial w_j^2} \end{bmatrix} \tag{A1}$$

Negative quasi-definiteness is established if the following inequality holds for all $\forall i = 1, 2$:

$$\frac{\partial^2 y_i}{\partial w_i^2} < 0 \tag{A2}$$

$$\frac{\partial^2 y_i}{\partial w_i^2}\frac{\partial^2 y_j}{\partial w_j^2} - \frac{\partial^2 y_i}{\partial w_i \partial w_j}\frac{\partial^2 y_j}{\partial w_i \partial w_j} > 0 \tag{A3}$$

Twice differentiating reactions functions with respect to w_i and w_j yields the following set of derivatives, after rearranging and simplifying:

$$\frac{\partial^2 y_i}{\partial w_i^2} = -\frac{2\theta_i\theta_j w_j}{(\theta_i w_i + \theta_j w_j)^2}\left\{\frac{[a_i(\theta_i w_i + \theta_j w_j) + y\theta_i]}{(\theta_i w_i + \theta_j w_j)}\right\} \tag{A4}$$

$$\frac{\partial^2 y_i}{\partial w_i \partial w_j} = \frac{\theta_i\theta_j[(y - a_j w_j)(\theta_i w_i - \theta_j w_j) + w_i a_i(\theta_i w_i + \theta_j w_j)]}{(\theta_i w_i + \theta_j w_j)^3} \tag{A5}$$

$$\frac{\partial^2 y_j}{\partial w_j^2} = -\frac{2\theta_i\theta_j w_i}{(\theta_i w_i + \theta_j w_j)^2}\left\{\frac{[a_j(\theta_i w_i + \theta_j w_j) + y\theta_j]}{(\theta_i w_i + \theta_j w_j)}\right\} \tag{A6}$$

$$\frac{\partial^2 y_j}{\partial w_i \partial w_j} = \frac{\theta_i\theta_j[(y - a_i w_i)(\theta_j w_j - \theta_i w_i) + w_j a_j(\theta_i w_i + \theta_j w_j)]}{(\theta_i w_i + \theta_j w_j)^3} \tag{A7}$$

The first derivative is unambiguously negative and this satisfies the first requirement for negative quasi-definiteness.

As far as the second requirement concerns, we substitute the corresponding values for the derivatives in equation (A3) and after simplification we obtain:

$$\begin{aligned} 4w_i w_j [a_i(\theta_i w_i + \theta_j w_j) + y\theta_i][a_j(\theta_i w_i + \theta_j w_j) + y\theta_j] > \\ \{[(y - a_j w_j)(\theta_i w_i - \theta_j w_j) + w_i a_i(\theta_i w_i + \theta_j w_j)] \\ [(y - a_i w_i)(\theta_j w_j - \theta_i w_i) + w_j a_j(\theta_i w_i + \theta_j w_j)]\} \end{aligned} \tag{A8}$$

We can show that the above inequality always holds by examining it by parts. First we show that the following inequality holds:

$$w_i[a_i(\theta_i w_i + \theta_j w_j) + y\theta_i] > [(y - a_j w_j)(\theta_i w_i - \theta_j w_j) + w_i a_i(\theta_i w_i + \theta_j w_j)]$$

After simplification, it reduces to:

$$y\theta_j > a_j(\theta_j w_j - \theta_i w_i)$$

Moreover, expanding the term y yields:

$$a_i(e_i - w_i)\theta_j + a_j\theta_j e_j + a_j\theta_i w_i > 2a_j\theta_j w_j$$

By Assumption 1 it can be noted that $a_j\theta_j e_j \geq 2a_j\theta_j w_j$, which implies that the above inequality is always satisfied. In the same fashion, it can be proved that the remaining term in the lhs of inequality (A8) is lager than the term in the second square brackets in the rhs.

This satisfies also the second requirement for negative quasi-definiteness, proving that the above reaction functions characterize the unique Nash equilibrium in the anarchic contest.

Proof of Lemma 1

The analysis of the relation between the equilibrium values when countries fight and the main parameters of the model requires that the determinant of the Jacobian matrix, $\det \tilde{\mathcal{J}}$, is different from 0. Therefore, we differentiate $\tilde{H}$ and $\tilde{K}$ with respect to w_1 and w_2, obtaining the following set of derivatives:

$$\tilde{H}_{w_1} = \frac{a_1(\theta_1 w_1 + \theta_2 w_2)[\theta_1 w_1 + 2\theta_2 w_2] + y\theta_1\theta_2 w_2}{a_1(\theta_1 w_1 + \theta_2 w_2)^2} \tag{A9}$$

$$\tilde{H}_{w_2} = -\frac{y\theta_2\theta_1 w_1 - a_2\theta_2 w_2(\theta_1 w_1 + \theta_2 w_2)}{a_1(\theta_1 w_1 + \theta_2 w_2)^2} \tag{A10}$$

$$\tilde{K}_{w_1} = -\frac{y\theta_1\theta_2 w_2 - a_1\theta_1 w_1(\theta_1 w_1 + \theta_2 w_2)}{a_2(\theta_1 w_1 + \theta_2 w_2)^2} \tag{A11}$$

$$\tilde{K}_{w_2} = \frac{a_2(\theta_1 w_1 + \theta_2 w_2)[\theta_2 w_2 + 2\theta_1 w_1] + y\theta_1\theta_2 w_1}{a_2(\theta_1 w_1 + \theta_2 w_2)^2} \tag{A12}$$

The determinant of the Jacobian matix is calculated as follows:

$$\det \tilde{\mathcal{J}} = \tilde{H}_{w_1}\tilde{K}_{w_2} - \tilde{H}_{w_2}\tilde{K}_{w_1}$$

After substitution and simplification, we obtain:

$$\begin{gathered}\langle\{a_1(\theta_1 w_1 + \theta_2 w_2)[\theta_1 w_1 + 2\theta_2 w_2] + y\theta_1\theta_2 w\} \times \\ \{a_2(\theta_1 w_1 + \theta_2 w_2)[\theta_2 w_2 + 2\theta_1 w_1] + y\theta_1\theta_2 w_1\}\rangle > \\ \langle[y\theta_2\theta_1 w_1 - a_2\theta_2 w_2(\theta_1 w_1 + \theta_2 w_2)] \times \\ [y\theta_1\theta_2 w_2 - a_1\theta_1 w_1(\theta_1 w_1 + \theta_2 w_2)]\rangle\end{gathered}$$

We can prove that the previous inequality holds analysing it by parts. First we contrast the first term in the lhs of inequality with the second one in the rhs and show that the first one is larger than the second, i.e.:

$$\begin{gathered}\{a_1(\theta_1 w_1 + \theta_2 w_2)[\theta_1 w_1 + 2\theta_2 w_2] + y\theta_1\theta_2 w_2\} > \\ \{y\theta_1\theta_2 w_2 - a_1\theta_1 w_1(\theta_1 w_1 + \theta_2 w_2)\}\end{gathered} \tag{A13}$$

Rearranging and simplifying inequality (A13) lead to the following result:

$$2a_1[\theta_1 w_1 + \theta_2 w_2](\theta_1 w_1 + \theta_2 w_2) > 0$$

In the same way, we can establish that the second term in the lhs of inequality is larger than the first term. Therefore:

$$\det \tilde{\mathcal{J}} = \tilde{H}_{w_1}\tilde{K}_{w_2} - \tilde{H}_{w_2}\tilde{K}_{w_1} > 0$$

Proof of Proposition 2

In order to prove the content of proposition 2 we calculate the derivatives of $\tilde{H}$ and $\tilde{K}$ with respect to θ_1. Actually, for keeping things as simple as possible, in this proof as well as in the following ones, we focus on the relations between the equilibrium values and the parameters referring to country 1. It is not difficult to note that the same results applies whether we consider a change in the country 2's productive and military skills.

Differentiating the implici function with respect to θ_1 yields:

$$\tilde{H}_{\theta_1} = \frac{y\theta_1 w_1 w_2}{a_1 \left(\theta_1 w_1 + \theta_2 w_2\right)^2} \tag{A14}$$

$$\tilde{K}_{\theta_1} = -\frac{y\theta_2 w_1 w_2}{a_2 \left(\theta_1 w_1 + \theta_2 w_2\right)^2} \tag{A15}$$

Applying Cramer rule to the system (10) in the text yields:

$$\frac{\partial \tilde{w}_1}{\partial \theta_1} = \frac{\begin{vmatrix} -\tilde{H}_{\theta_1} & \tilde{H}_{w_2} \\ -\tilde{K}_{\theta_1} & \tilde{K}_{w_2} \end{vmatrix}}{\det \tilde{\mathcal{J}}}$$

Previous equation can be rewritten as follows:

$$\frac{\partial \tilde{w}_1}{\partial \theta_1} = \frac{-\tilde{H}_{\theta_1}\tilde{K}_{w_2} + \tilde{H}_{w_2}\tilde{K}_{\theta_1}}{\det \tilde{\mathcal{J}}} \gtrless 0 \tag{A16}$$

Clearly the sign of the above derivative depends on the numerator. Substituting the corresponding values, we obtain:

$$-\tilde{H}_{\theta_1}\tilde{K}_{w_2} = -\frac{y\theta_1 w_1 w_2}{a_1 \left(\theta_1 w_1 + \theta_2 w_2\right)^2} \times \frac{a_2 \left(\theta_1 w_1 + \theta_2 w_2\right)\left[\theta_2 w_2 + 2\theta_1 w_1\right] + y\theta_1\theta_2 w_1}{a_2 \left(\theta_1 w_1 + \theta_2 w_2\right)^2}$$

$$\tilde{H}_{w_2}\tilde{K}_{\theta_1} = \frac{y\theta_2 w_1 w_2}{a_2 \left(\theta_1 w_1 + \theta_2 w_2\right)^2}\frac{y\theta_2\theta_1 w_1 - a_2\theta_2 w_2 \left(\theta_1 w_1 + \theta_2 w_2\right)}{a_1 \left(\theta_1 w_1 + \theta_2 w_2\right)^2}$$

The sign of equation (A16) depends on which of the previous terms is larger. Rearranging and simplifying yields:

$$\begin{gathered} y\theta_2 w_1 w_2 \left\{y\theta_2\theta_1 w_1 - a_2\theta_2 w_2 \left(\theta_1 w_1 + \theta_2 w_2\right)\right\} \gtrless \\ y\theta_1 w_1 w_2 \left\{a_2 \left(\theta_1 w_1 + \theta_2 w_2\right)\left[\theta_2 w_2 + 2\theta_1 w_1\right] + y\theta_1\theta_2 w_1\right\} \end{gathered}$$

The latter inequality can be reduced to:

$$y\theta_2 s_1 \left(\theta_2 - \theta_1\right) \gtrless a_2 \left[\theta_1 \left(\theta_2 w_2 + 2\theta_1 w_1\right) + \theta_2^2 w_2\right] \tag{A17}$$

We now move to the second part of Proposition 2. We apply the same procedure to derive the sign of the relation between $\tilde{w}_2$ and θ_1. Making the appropriate changes we obtain:

$$\frac{\partial \tilde{w}_2}{\partial \theta_1} = \frac{\begin{vmatrix} \tilde{H}_{w_1} & -\tilde{H}_{\theta_1} \\ \tilde{K}_{w_1} & -\tilde{K}_{\theta_1} \end{vmatrix}}{\tilde{\mathcal{J}}}$$

or, in a different form:

$$\frac{\partial \tilde{w}_2}{\partial \theta_1} = \frac{-\tilde{K}_{\theta_1}\tilde{H}_{w_1} + \tilde{H}_{\theta_1}\tilde{K}_{w_1}}{\tilde{\mathcal{J}}} < 0 \tag{A18}$$

Again the sign is determined by the numerator. Substituting the corresponding values, we obtain:

$$\begin{gathered} \{-a_1 (\theta_1 w_1 + \theta_2 w_2) [\theta_1 w_1 + 2\theta_2 w_2] - y\theta_1\theta_2 w_2\} \, y\theta_2 w_1 w_2 < \\ y\theta_1 w_1 w_2 [-y\theta_1\theta_2 w_2 + a_1\theta_1 w_1 (\theta_1 w_1 + \theta_2 w_2)] \end{gathered}$$

Moreover, easy algebraic manipulation shows:

$$-a_1 (\theta_1 w_1 + \theta_2 w_2) \left[\theta_1 w_1 (\theta_1 + \theta_2) + 2\theta_2^2 w_2\right] - y\theta_1\theta_2 w_2 (\theta_2 - \theta_1) < 0$$

Therefore, as required:

$$\frac{\partial \tilde{w}_2}{\partial \theta_1} < 0 \tag{A19}$$

Proof of Proposition 3

In order to prove the content of proposition 3, we need the following set of derivatives:

$$\tilde{H}_{a_1} = \frac{a_2 (e_2 - w_2) \theta_2 w_2}{a_1^2 (\theta_1 w_1 + \theta_2 w_2)} \tag{A20}$$

$$\tilde{K}_{a_1} = -\frac{(e_1 - w_1) \theta_1 w_1}{a_2 (\theta_1 w_1 + \theta_2 w_2)} \tag{A21}$$

Using the above derivatives, we can establish the relationship between $\tilde{w}_1$ and a_1 in the following manner:

$$\frac{\partial \tilde{w}_1}{\partial a_1} = \frac{\begin{vmatrix} -\tilde{H}_{a_1} & \tilde{H}_{w_2} \\ -\tilde{K}_{a_1} & \tilde{K}_{w_2} \end{vmatrix}}{\det \tilde{\mathcal{J}}}$$

Previous equation can be rewritten as follows:

$$\frac{\partial \tilde{w}_1}{\partial a_1} = \frac{-\tilde{H}_{a_1}\tilde{K}_{w_2} + \tilde{H}_{w_2}\tilde{K}_{a_1}}{\det \tilde{\mathcal{J}}} < 0 \tag{A22}$$

Again the sign of the derivative is determined by the numerator. Making the appropriate substitutions yields:

$$\begin{aligned}
-\tilde{H}_{a_1}\tilde{K}_{w_2} &= -\frac{a_2\left(e_2-w_2\right)\theta_2 w_2}{a_1^2\left(\theta_1 w_1+\theta_2 w_2\right)}\frac{a_2\left(\theta_1 w_1+\theta_2 w_2\right)\left[\theta_2 w_2+2\theta_1 w_1\right]+y\theta_1\theta_2 w_1}{a_2\left(\theta_1 w_1+\theta_2 w_2\right)^2} \\
\tilde{H}_{w_2}\tilde{K}_{a_1} &= \left[-\frac{y\theta_2\theta_1 w_1-a_2\theta_2 w_2\left(\theta_1 w_1+\theta_2 w_2\right)}{a_1\left(\theta_1 w_1+\theta_2 w_2\right)^2}\right]\left[-\frac{\left(e_1-w_1\right)\theta_1 w_1}{a_2\left(\theta_1 w_1+\theta_2 w_2\right)}\right]
\end{aligned}$$

Contrasting the first term with the second term, after simplifying and rearranging, we obtain:

$$\begin{gathered}
a_2 w_2\left(\theta_1 w_1+\theta_2 w_2\right)\left\{a_1\left(e_1-w_1\right)\theta_1 w_1-a_2\left(e_2-w_2\right)\left[\theta_2 w_2+2\theta_1 w_1\right]\right\}< \\
y\theta_1 w_1\left[a_1\left(e_1-w_1\right)\theta_1 w_1+a_2\left(e_2-w_2\right)\theta_2 w_2\right]
\end{gathered} \tag{A23}$$

We analyse the lhs and the rhs of inequality (A23) by parts. First, it is easy to note that:

$$\begin{gathered}
\left\{a_1\left(e_1-w_1\right)\theta_1 w_1-a_2\left(e_2-w_2\right)\left[\theta_2 w_2+2\theta_1 w_1\right]\right\}< \\
\left[a_1\left(e_1-w_1\right)\theta_1 w_1+a_2\left(e_2-w_2\right)\theta_2 w_2\right]
\end{gathered}$$

As a matter of fact previous inequality is always satisfied, since, after rearrangement, we obtain:

$$0<2a_2\left(e_2-w_2\right)\left[\theta_2 w_2+\theta_1 w_1\right]$$

Moreover as far as the last two terms concerns, it can be shown that:

$$a_2 w_2\left(\theta_1 w_1+\theta_2 w_2\right)\leq y\theta_1 w_1$$

Expanding the term y and rearranging we obtain:

$$a_2 w_2\left(\theta_1 w_1+\theta_2 w_2\right)\leq\left[a_1\left(e_1-w_1\right)+a_2\left(e_2-w_2\right)\right]\theta_1 w_1$$

It can be noted that for sufficiently large values of e_1 and e_2 the above inequality is satisifed. This complete the first part of the proof. Therefore the numerator of equation (A22) is negative and so it is the relation between $\tilde{w}_1$ and a_1. .

As far as the second part concerns, we proceed in the same way. Hence, we have:

$$\frac{\partial\tilde{w}_2}{\partial a_1}=\frac{-\tilde{K}_{a_1}\tilde{H}_{w_1}+\tilde{H}_{a_1}\tilde{K}_{w_1}}{\det\tilde{\mathcal{J}}}>0 \tag{A24}$$

The sign of the derivative is determined by the numerator. Making the appropriate substitutions yields:

$$\begin{aligned}
-\tilde{K}_{a_1}\tilde{H}_{w_1} &= -\left[-\frac{\left(e_1-w_1\right)\theta_1 w_1}{a_2\left(\theta_1 w_1+\theta_2 w_2\right)}\right]\frac{a_1\left(\theta_1 w_1+\theta_2 w_2\right)\left[\theta_1 w_1+2\theta_2 w_2\right]+y\theta_1\theta_2 w_2}{a_1\left(\theta_1 w_1+\theta_2 w_2\right)^2} \\
\tilde{H}_{a_1}\tilde{K}_{w_1} &= \frac{a_2\left(e_2-w_2\right)\theta_2 w_2}{a_1^2\left(\theta_1 w_1+\theta_2 w_2\right)}\left[-\frac{y\theta_1\theta_2 w_2-a_1\theta_1 w_1\left(\theta_1 w_1+\theta_2 w_2\right)}{a_2\left(\theta_1 w_1+\theta_2 w_2\right)^2}\right]
\end{aligned}$$

Simplification yields:

$$\begin{aligned} & a_1 (e_1 - w_1) \theta_1 w_1 \{a_1 (\theta_1 w_1 + \theta_2 w_2) [\theta_1 w_1 + 2\theta_2 w_2] + y\theta_1\theta_2 w_2\} > \\ & a_2 (e_2 - w_2) \theta_2 w_2 [-y\theta_1\theta_2 w_2 + a_1\theta_1 w_1 (\theta_1 w_1 + \theta_2 w_2)] \end{aligned} \tag{A25}$$

Rearranging the previous inequality, we obtain:

$$\begin{aligned} & y\theta_2 w_2 [a_1 (e_1 - w_1) \theta_1 w_1 + a_2 (e_2 - w_2) \theta_2 w_2] > \\ & a_1 w_1 (\theta_1 w_1 + \theta_2 w_2) [a_2 (e_2 - w_2) \theta_2 w_2 - a_1 (e_1 - w_1) (\theta_1 w_1 + 2\theta_2 w_2)] \end{aligned}$$

As in the previous case, we analyse the above inequality by parts. First it is easy to note that the second terms in the lhs is larger than the second one in the rhs of inequality (i.e. the terms in the square brackes). As a matter of fact, after simplification and rarranging we obtain:

$$2a_1 (e_1 - w_1) (\theta_1 w_1 + \theta_2 w_2) > 0$$

As far as the remaining values concerns, comparision yields to the following result:

$$[a_1 (e_1 - 2w_1) + a_2 (e_2 - w_2)] \theta_2 w_2 \geq a_1 w_1 (\theta_1 w_1 + \theta_2 w_2)$$

As before, previous inequality is satisfied for sufficiently large values of e_1 and e_2. It follows that:

$$\frac{\partial \tilde{w}_2}{\partial a_1} > 0$$

Proof of Proposition 4

First let us consider the following set of derivatives for $\forall i, j = 1, 2$ and $i \neq j$:

$$\frac{\partial y_i}{\partial \theta_i} = \frac{\theta_j w_i w_j y}{(\theta_i w_i + \theta_j w_j)^2} > 0 \tag{A26}$$

$$\frac{\partial y_i}{\partial \theta_j} = -\frac{\theta_j w_i w_j y}{(\theta_i w_i + \theta_j w_j)^2} < 0 \tag{A27}$$

Also we need to evaluate the sign of the following derivatives:

$$\frac{\partial y_i}{\partial w_i} = \frac{\theta_i}{\theta_i w_i + \theta_j w_j} [\theta_j w_j y - w_i a_i (\theta_i w_i + \theta_j w_j)] > 0 \tag{A28}$$

$$\frac{\partial y_i}{\partial w_j} = -\frac{\theta_i w_i}{\theta_i w_i + \theta_j w_j} [\theta_j y + a_j (\theta_i w_i + \theta_j w_j)] < 0 \tag{A29}$$

In order to evaluate the impact of a change in the level of military technology on the amount of commodity good, that each country expects to gain from the fight, we totally differentiate the wealth functions with respect to θ_1 and θ_2. We show that the content of proposition is satisfied with respect to a change in θ_1, since by the same arguments it can be shown the results with respect to a change in θ_2.

Totally differentiating yields:

$$\frac{\partial \tilde{y}_1}{\partial \theta_1} = \frac{\partial \tilde{y}_1}{\partial \tilde{w}_1}\frac{\partial \tilde{w}_1}{\partial \theta_1} + \frac{\partial \tilde{y}_1}{\partial \tilde{w}_2}\frac{\partial \tilde{w}_2}{\partial \theta_1} + \frac{\partial \tilde{y}_1}{\partial \theta_1} \tag{A30}$$

It is easy to note that if $\frac{\partial \tilde{w}_1}{\partial \theta_1} > 0$, then an increase in θ_1 leads to an increase in the amount of expected good from the fight. On the other hand, if $\frac{\partial \tilde{w}_1}{\partial \theta_1} < 0$, the sign of the derivative is ambiguous.

We now consider the effect of a change of θ_1 on y_2:

$$\frac{\partial \tilde{y}_2}{\partial \theta_1} = \frac{\partial \tilde{y}_2}{\partial \tilde{w}_1}\frac{\partial \tilde{w}_1}{\partial \theta_1} + \frac{\partial \tilde{y}_2}{\partial \tilde{w}_2}\frac{\partial \tilde{w}_2}{\partial \theta_1} + \frac{\partial \tilde{y}_2}{\partial \theta_1} \tag{A31}$$

So we have:

$$\frac{\partial \tilde{y}_1}{\partial \theta_1} > 0 \text{ and } \frac{\partial \tilde{y}_2}{\partial \theta_1} < 0 \text{ if } \frac{\partial \tilde{w}_1}{\partial \theta_1} > 0 \tag{A32}$$

$$\frac{\partial \tilde{y}_1}{\partial \theta_1} \gtrless 0 \text{ and } \frac{\partial \tilde{y}_2}{\partial \theta_1} \lesseqgtr 0 \text{ if } \frac{\partial \tilde{w}_1}{\partial \theta_1} < 0 \tag{A33}$$

Proof of Proposition 7

The pair of critical discount factors indicated in equation (27) is easily derived by solving it for δ. First, it should be noted that the latter can be rewritten as follows:

$$y^{**} + \frac{\delta}{1-\delta}\hat{y}_i \leq \frac{1}{1-\delta}y_i^{**} \tag{A34}$$

Solving inequality (A34) for δ yields the critical discount factors $\underline{\delta}_1$ and $\underline{\delta}_2$.

Proof of Corollary 4

In order the content of Corollary 4, it is useful to rewrite the critical discount factors in the followin way:

$$\underline{\delta}_1 = 1 - \frac{\gamma\left(y^{**} - \hat{y}\right)}{y^{**} - \hat{s}_1\hat{y}} \tag{A35}$$

$$\underline{\delta}_2 = \frac{\hat{s}_1\hat{y} + \gamma\left(y^{**} - \hat{y}\right)}{y^{**} - \hat{s}_2\hat{y}} \tag{A36}$$

The above values can be rearranged in the following inequality:

$$1 > \frac{\left[\hat{s}_1\hat{y} + \gamma\left(y^{**} - \hat{y}\right)\right]\left[y^{**} - \left(1 - \hat{s}_2\right)\hat{y}_1\right] + \gamma\left(y^{**} - \hat{y}\right)\left(y^{**} - \hat{s}_2\hat{y}_1\right)}{\left(y^{**} - \hat{s}_1\hat{y}\right)\left(y^{**} - \hat{s}_2\hat{y}\right)}$$

Simplification yields:

$$1 > \frac{\hat{s}_1\hat{y}\left(y^{**} - \hat{s}_1\hat{y}\right) + \gamma\left(2y^{**} - \hat{y}\right)\left(y^{**} - \hat{y}\right)}{\left(y^{**} - \hat{s}_1\hat{y}\right)\left(y^{**} - \hat{s}_2\hat{y}\right)}$$

The latter inequality can be rewritten in the following way after simplification:

$$\left(y^{**} - \hat{s}_1\hat{y}\right) > \gamma\left(2y^{**} - \hat{y}\right)$$

Moreover, expanding the term in the rhs and rearranging yields:

$$y^{**}\left(1 - 2\gamma\right) > \hat{y}\left(\gamma + \hat{s}_1\right) \tag{A37}$$

After further rearrangement we eventually obtain:

$$\frac{y^{**}}{\hat{y}} > \frac{\gamma + \hat{s}_1}{1 - 2\gamma} \tag{A38}$$

Clearly, the lhs of (A38) is larger than 1. The above inequality is satisfied if:

$$1 < \frac{\gamma + \hat{s}_1}{1 - 2\gamma} \tag{A39}$$

The latter is satisfied if:

$$\hat{s}_2 > 3\gamma \tag{A40}$$

It is important to note that the last inequality requires that $\gamma \ll \frac{1}{3}$.

Proof of Lemma 3

In order to show the content of this lemma, we proceed as before by applying implicit function theorem. First, we rewrite equations (7) and (8) in the following way:

$$\hat{H}\left(\alpha, w_1, w_2\right) = w_1 - \frac{\left(1 - \alpha\right) w_2 y}{\hat{a}_1\left[\alpha w_1 + \left(1 - \alpha\right) w_2\right]} \tag{A41}$$

$$\hat{K}\left(\alpha, w_1, w_2\right) = w_2 - \frac{\alpha w_1 y}{\hat{a}_2\left[\alpha w_1 + \left(1 - \alpha\right) w_2\right]} \tag{A42}$$

It should be noted that they differ from before because of the use of the new distribution of technologies. Using (A41) and (A42), we obtain the following sets of derivatives:

$$\hat{H}_{w_1} = \frac{\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right] \left[\alpha w_1 + 2(1-\alpha) w_2\right] + (1-\alpha) ayw_2}{\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \tag{A43}$$

$$\hat{H}_{w_2} = -\frac{(1-\alpha)\left\{\alpha y w_1 - \hat{a}_2 w_2 \left[\alpha w_1 + (1-\alpha) w_2\right]\right\}}{\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \tag{A44}$$

$$\hat{K}_{w_1} = -\frac{\alpha \left\{(1-\alpha) y w_2 - \hat{a}_1 w_1 \left[\alpha w_1 + (1-\alpha) w_2\right]\right\}}{\hat{a}_2 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \tag{A45}$$

$$\hat{K}_{w_2} = \frac{\hat{a}_2 \left[\alpha w_1 + (1-\alpha) w_2\right] \left[(1-\alpha) w_2 + 2\alpha w_1\right] + (1-\alpha) ayw_1}{\hat{a}_2 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \tag{A46}$$

$$\hat{H}_{\alpha} = \frac{y w_1 w_2}{\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \tag{A47}$$

$$\hat{K}_{\alpha} = -\frac{y w_1 w_2}{\hat{a}_2 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \tag{A48}$$

We use the first four derivatives of the above set to show that the determinat of the Jacobian matrix in this modified anarchic world, $\widehat{\mathcal{J}}$, is different from zero. More specifically, we show that:

$$\det \widehat{\mathcal{J}} = \hat{H}_{w_1} \hat{K}_{w_2} - \hat{H}_{w_2} \hat{K}_{w_1} > 0 \tag{A49}$$

Making the appropriate substitutions we obtain:

$$\begin{aligned} \hat{H}_{w_1} \hat{K}_{w_2} &= \frac{\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right] \left[\alpha w_1 + 2(1-\alpha) w_2\right] + (1-\alpha) ayw_2}{\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \times \\ &\quad \frac{\hat{a}_2 \left[\alpha w_1 + (1-\alpha) w_2\right] \left[(1-\alpha) w_2 + 2\alpha w_1\right] + (1-\alpha) ayw_1}{\hat{a}_2 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \\ \hat{H}_{w_2} \hat{K}_{w_1} &= \frac{(1-\alpha)\left\{\alpha y w_1 - \hat{a}_2 w_2 \left[\alpha w_1 + (1-\alpha) w_2\right]\right\}}{\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \times \\ &\quad \frac{\alpha \left\{(1-\alpha) y w_2 - \hat{a}_1 w_1 \left[\alpha w_1 + (1-\alpha) w_2\right]\right\}}{\hat{a}_2 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \end{aligned}$$

We need to show that $\hat{H}_{w_1} \hat{K}_{w_2} > \hat{H}_{w_2} \hat{K}_{w_1}$. After simplification, it yields:

$$\begin{gathered} \left\{\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right] \left[\alpha w_1 + 2(1-\alpha) w_2\right] + (1-\alpha) ayw_2\right\} \times \\ \left\{\hat{a}_2 \left[\alpha w_1 + (1-\alpha) w_2\right] \left[(1-\alpha) w_2 + 2\alpha w_1\right] + (1-\alpha) ayw_1\right\} > \\ (1-\alpha)\left\{\alpha y w_1 - \hat{a}_2 w_2 \left[\alpha w_1 + (1-\alpha) w_2\right]\right\} \times \\ \alpha \left\{(1-\alpha) y w_2 - \hat{a}_1 w_1 \left[\alpha w_1 + (1-\alpha) w_2\right]\right\} \end{gathered} \tag{A50}$$

Let us first consider the first term in the lhs and the latter one in the rhs. It is easy to show that:

$$\begin{gathered} \left\{\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right] \left[\alpha w_1 + 2(1-\alpha) w_2\right] + (1-\alpha) ayw_2\right\} > \\ \alpha \left\{(1-\alpha) y w_2 - \hat{a}_1 w_1 \left[\alpha w_1 + (1-\alpha) w_2\right]\right\} \end{gathered}$$

After simplification it yields:

$$2\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right]^2 > 0 \tag{A51}$$

It is easy to prove in the same way that the second term in the lhs is larger than the first term in the rhs of inequality (A50).

We have now all the elements to prove the content of Lemma 3. We first check the sign of the following derivative:

$$\frac{\partial \hat{w}_1}{\partial \alpha} = \frac{-\hat{H}_\alpha \hat{K}_{w_2} + \hat{H}_{w_2}\hat{K}_\alpha}{\det \widehat{\mathcal{J}}} < 0 \tag{A52}$$

Clearly the sign of the derivative depends on the numerator. Making the appropriate substitutions yields:

$$\begin{aligned} -\hat{H}_\alpha \hat{K}_{w_2} &= -\left\{ \frac{y w_1 w_2}{\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \right. \\ &\quad \left. \times \frac{\hat{a}_2 \left[\alpha w_1 + (1-\alpha) w_2\right]\left[(1-\alpha) w_2 + 2\alpha w_1\right] + (1-\alpha) a y w_1}{\hat{a}_2 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \right\} \\ \hat{H}_{w_2}\hat{K}_\alpha &= \frac{(1-\alpha)\left\{\alpha y w_1 - \hat{a}_2 w_2 \left[\alpha w_1 + (1-\alpha) w_2\right]\right\}}{\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \times \frac{y w_1 w_2}{\hat{a}_2 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \end{aligned}$$

Therefore:

$$\begin{aligned} -\hat{H}_\alpha \hat{K}_{w_2} + \hat{H}_{w_2}\hat{K}_\alpha &< 0 \\ \hat{H}_{w_2}\hat{K}_\alpha &< \hat{H}_\alpha \hat{K}_{w_2} \end{aligned}$$

We can substitute the corresponding values for each term in the last inequality and, after simplification, we obtain:

$$(1-\alpha)\left\{\alpha y w_1 - \hat{a}_2 w_2 \left[\alpha w_1 + (1-\alpha) w_2\right]\right\} \tag{A53}$$

$$< \hat{a}_2 \left[\alpha w_1 + (1-\alpha) w_2\right]\left[(1-\alpha) w_2 + 2\alpha w_1\right] + (1-\alpha) a y w_1 \tag{35}$$

The above inequality is always satisified since after easy algebraic manipulation we obtain:

$$0 < 2\hat{a}_2 \left[\alpha w_1 + (1-\alpha) w_2\right]^2$$

In the same way we can show:

$$\frac{\partial \hat{w}_2}{\partial \alpha} = \frac{-\hat{K}_\alpha \hat{H}_{w_1} + \hat{H}_\alpha \hat{K}_{w_1}}{\det \widehat{\mathcal{J}}} > 0 \tag{A54}$$

Again, subtituiting the corresponding values for the terms in the numerator yields:

$$\begin{aligned}
-\hat{K}_\alpha \hat{H}_{w_1} &= \left\{ \frac{y w_1 w_2}{\hat{a}_2 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \right. \\
&\quad \left. \times \frac{\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right] \left[\alpha w_1 + 2(1-\alpha) w_2\right] + (1-\alpha) a y w_2}{\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \right\} \\
\hat{H}_\alpha \hat{K}_{w_1} &= \frac{y w_1 w_2}{\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \times \left\langle -\frac{\alpha \left\{(1-\alpha) y w_2 - \hat{a}_1 w_1 \left[\alpha w_1 + (1-\alpha) w_2\right]\right\}}{\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right]^2} \right\rangle
\end{aligned}$$

We can show that

$$-\hat{K}_\alpha \hat{H}_{w_1} > \hat{H}_\alpha \hat{K}_{w_1}$$

After simplification the latter inequality implies:

$$\begin{aligned}
& \hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right] \left[\alpha w_1 + 2(1-\alpha) w_2\right] + (1-\alpha) a y w_2 && \text{(A55)} \\
> \; & \alpha \left\{(1-\alpha) y w_2 - \hat{a}_1 w_1 \left[\alpha w_1 + (1-\alpha) w_2\right]\right\} && \text{(36)}
\end{aligned}$$

Simplification yields:

$$2\hat{a}_1 \left[\alpha w_1 + (1-\alpha) w_2\right]^2 > 0$$

Proof of Proposition 8

Differentiating the critical discount factors with respect to α yields:

$$\frac{\partial \underline{\delta}_1}{\partial \alpha} = -\frac{\gamma R (y^{**} - \hat{y}_1) - \gamma (y^{**} - \hat{y}) \left\{-\frac{\partial \hat{s}_1}{\partial \alpha} \hat{y} + \hat{s}_1 R\right\}}{(y^{**} - \hat{y}_1)^2} \tag{A56}$$

$$\frac{\partial \underline{\delta}_2}{\partial \alpha} = -\frac{\left\{\frac{\partial \hat{s}_1}{\partial \alpha} \hat{y} - (\hat{s}_1 - \gamma) R\right\} (y^{**} - \hat{y}_2) + y_1^{**} \left\{\frac{\partial \hat{s}_2}{\partial \alpha} \hat{y} - \hat{s}_2 R\right\}}{(y^{**} - \hat{y}_1)^2} \tag{A57}$$

where

$$R = \left[\hat{a}_1 \frac{\partial \hat{w}_1}{\partial \alpha} + \hat{a}_2 \frac{\partial \hat{w}_2}{\partial \alpha}\right]$$

It is important to note that the two terms in the squares brackets compensate each other. Therefore $R = 0$ and equations (A56) and (A57) can be rewritten after an easy algebraic manipulation as follows:

$$\frac{\partial \underline{\delta}_1}{\partial \alpha} = -\frac{\gamma \frac{\partial \hat{s}_1}{\partial \alpha} \hat{y} (y^{**} - \hat{y})}{(y^{**} - \hat{y}_1)^2} \tag{A58}$$

$$\frac{\partial \underline{\delta}_2}{\partial \alpha} = \frac{(1-\gamma) \frac{\partial \hat{s}_1}{\partial \alpha} \hat{y} (y^{**} - \hat{y})}{(y^{**} - \hat{y}_2)^2} \tag{A59}$$

Therefore, the sign of both derivatives depends on whether $\frac{\partial \hat{s}_1}{\partial \alpha}$ is positive or negative.

Proof of Lemma 4

In order to prove the content of Lemma 4 we make use of equations (A43) - (A46) and of the following ones:

$$\hat{H}_\kappa = \frac{(a_1 - a_2)(1-\alpha) w_2 (e_2 - w_2)}{\kappa^2 [\alpha w_1 + (1-\alpha) w_2]} \tag{A60}$$

$$\hat{K}_\kappa = \frac{(a_1 - a_2) \alpha w_1 (e_2 - w_2)}{\kappa^2 [\alpha w_1 + (1-\alpha) w_2]} \tag{A61}$$

We need to show that:

$$\frac{\partial \hat{w}_1}{\partial \kappa} = \frac{-\hat{H}_\kappa \hat{K}_{w_2} + \hat{H}_{w_2} \hat{K}_\kappa}{\det \widehat{\mathcal{J}}} < 0 \tag{A62}$$

Substituting for the corresponding values in the numeratore yields:

$$\begin{aligned} -\hat{H}_\kappa \hat{K}_{w_2} &= -\left\{ \frac{(\hat{a}_1 - \hat{a}_2)(1-\alpha) w_2 (e_2 - w_2)}{\kappa^2 [\alpha w_1 + (1-\alpha) w_2]} \right. \\ &\quad \left. \times \frac{\hat{a}_2 [\alpha w_1 + (1-\alpha) w_2] [(1-\alpha) w_2 + 2\alpha w_1] + (1-\alpha) a y w_1}{\hat{a}_2 [\alpha w_1 + (1-\alpha) w_2]^2} \right\} \\ \hat{H}_{w_2} \hat{K}_\kappa &= -\frac{(1-\alpha) \{\alpha y w_1 - \hat{a}_2 w_2 [\alpha w_1 + (1-\alpha) w_2]\}}{\hat{a}_1 [\alpha w_1 + (1-\alpha) w_2]^2} \times \frac{(\hat{a}_1 - \hat{a}_2) \alpha w_1 (e_2 - w_2)}{\kappa^2 [\alpha w_1 + (1-\alpha) w_2]} \end{aligned}$$

After simplification we obtain:

$$\frac{\partial w_1}{\partial \kappa} = -\Omega_1 \left\{ \hat{a}_2 (1-\alpha) w_2^2 + \alpha w_1 [\hat{a}_1 (e_1 - w_1) + \hat{a}_2 e_2] \right\} < 0 \tag{A63}$$

where

$$\Omega_1 = \frac{[\alpha w_1 + (1-\alpha) w_2] (\hat{a}_1 - \hat{a}_2)(1-\alpha)(e_2 - w_2)}{\left\{ \hat{a}_1 \hat{a}_2 \kappa^4 [\alpha w_1 + (1-\alpha) w_2]^3 \right\} \det \widehat{\mathcal{J}}}$$

As far as country 2 concerns, we have:

$$\frac{\partial \hat{w}_2}{\partial \kappa} = \frac{-\hat{K}_\kappa \hat{H}_{w_1} + \hat{H}_\kappa \hat{K}_{w_1}}{\det \widehat{\mathcal{J}}} < 0 \tag{A64}$$

Following the sama procedure applied above, we obtain:

$$\begin{aligned} -\hat{K}_\kappa \hat{H}_{w_1} &= -\left\{ \frac{(a_1 - a_2) \alpha w_1 (e_2 - w_2)}{\kappa^2 [\alpha w_1 + (1-\alpha) w_2]} \right. \\ &\quad \left. \times \frac{\hat{a}_1 [\alpha w_1 + (1-\alpha) w_2] [\alpha w_1 + 2(1-\alpha) w_2] + (1-\alpha) a y w_2}{\hat{a}_1 [\alpha w_1 + (1-\alpha) w_2]^2} \right\} \\ \hat{H}_\kappa \hat{K}_{w_1} &= -\left\{ \frac{(a_1 - a_2)(1-\alpha) w_2 (e_2 - w_2)}{\kappa^2 [\alpha w_1 + (1-\alpha) w_2]} \right. \\ &\quad \left. \times \frac{\alpha \{(1-\alpha) y w_2 - \hat{a}_1 w_1 [\alpha w_1 + (1-\alpha) w_2]\}}{\hat{a}_2 [\alpha w_1 + (1-\alpha) w_2]^2} \right\} \end{aligned}$$

After simplification, we obtain:

$$\frac{\partial \hat{w}_2}{\partial \kappa} = -\Omega_2 \left\{ \hat{a}_1 \alpha w_1^2 + (1-\alpha) w_2 \left[\hat{a}_1 e_1 + \hat{a}_2 (e_2 - w_2) \right] \right\}$$

where

$$\Omega_2 = \frac{(a_1 - a_2) \alpha (e_2 - w_2) [\alpha w_1 + (1-\alpha) w_2]}{\det \widehat{\mathcal{J}} \left\{ \hat{a}_1 \hat{a}_2 \kappa^4 [\alpha w_1 + (1-\alpha) w_2]^3 \right\}}$$