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Public Provision of Private Goods and Nondistortionary Marginal Tax Rates

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Abstract
The incidence and efficiency losses of taxes have usually been analysed in isolation from public expenditures. This negligence of the expenditure side may imply a serious misperception of the effects of marginal tax rates. The reason is that part of the marginal tax may in fact be payment for publicly provided goods and reflects a cost that the consumers should bear in order to face the proper incentives. Hence, part of the marginal tax serves the same role as a market price in the sense that it conveys information about a real social cost of working more hours.

We develop this idea formally by studying an optimal income tax model in combination with a type of public provision scheme not analyzed before; the provision level is individualized and positively associated with the individual's labor supply. As examples we discuss child care, elderly care, primary education and health care. We show that there is a gain in efficiency where public provision of such a service replaces market purchases. We also show that it is necessary for efficiency that marginal income tax rates are higher than in economies where the services are purchased in the market. This is because the optimal tax should be designed so as to face the taxpayers with the real cost of providing the services. Hence, it might very well be that economies with higher marginal tax rates have less severe distortions than economies with lower marginal tax rates.

Keywords: Nonlinear income taxation; Marginal income tax rates; Public provision of private goods; In-kind transfers

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1. Introduction

Why does the Bumble Bee fly? It is a common saying that according to the laws of aero dynamics the Bumble Bee can't fly. The wings are too small in relation to the weight of the body. Still it does fly! The parallel in economics might be: Why does the Swedish economy function? It has the highest marginal tax rates in the world, and, according to standard economic theory, the distortions would severely hamper the economy. Some would say that the Swedish economy ought to collapse because of the high taxes. Still, the Swedish economy functions very well and outperforms many economies nurturing substantially lower marginal tax rates.

Clearly, if a version of aero-dynamic theory predicts that the Bumble Bee can't fly, something important is missing in that theory. Likewise, if a version of economic theory predicts that high marginal taxes necessarily imply damaging distortions and poor economic performance, something is missing in that theory. What we will argue in this article is that if there is public provision of private goods, then, for reasons explained below, a substantial part of the marginal income tax will be nondistortionary.

There is a long standing interest in quantifying the deadweight losses of taxation. Harberger's work in the sixties laid the foundations for a first generation of empirical studies.[4] A second generation of empirical work was inspired by Feldstein in the mid nineties.[5] More recently, Prescott (2002, 2004) has argued that high (marginal) taxes severely affect the performance of an economy. The common view seems to be that marginal income taxes are purely distortive. However, as shown below, under certain conditions, a substantial portion of the marginal income tax faced by individuals is nondistortive. This part of the marginal income tax should not enter the calculations when computing the deadweight loss of a tax. We believe one important reason why the Swedish economy performs so well although it has very high marginal tax rates is that a substantial portion of those tax rates is nondistortionary.

The reason why part of the marginal tax is nondistortive is that it is a payment for publicly provided goods/services and reflects a cost that the consumers should bear in order to face the right incentives. That is, marginal tax rates sometimes play the same role as prices in

[4] See for example Harberger (1962, 1964). In many cases the estimated welfare losses were surprisingly small.

[5] See Feldstein (1995, 1999). Feldstein argued that previous studies had neglected many important margins that are distorted by taxes. By estimating how total taxable income reacts to changes in the marginal tax one would be able to capture distortions of all relevant margins. Feldstein's own estimates indicated large welfare losses whereas many later studies arrived at estimates of the welfare loss that was larger than those obtained in pre-Feldstein studies, but considerably lower than the estimates obtained by Feldstein (Gruber and Saez, 2002, Saez, 2003, Kopczuk, 2005). See also Chetty (2008) for a recent re-assessment of the taxable income elasticity as the correct measure of excess burden in the presence of evasion and avoidance.

the sense that they convey information on resource costs. The part of the tax that reflects a real cost of working is nondistortionary.

Public provision of private goods is common in all developed countries and often is of the order of 20-30% of GDP. Previous contributions have usually considered public provision schemes that furnish each consumer with the same fixed quantity.[6] In this paper we address another type of public provision scheme not analyzed before, although being empirically important.[7] The provision level is individualized and positively associated with the individual's hours of work. In section 6, where we discuss specific examples, we will argue that several important public provision schemes are of a form such that provision levels are individualized and positively related to hours of work.

There is a small, related literature addressing how taxes and public spending affect labor supply (Ragan, 2006, Rogerson, 2007). Rogerson uses a labor supply model with taxes and public expenditures to explain differences in market work across the US, Continental Europe and Scandinavia. He argues that differences in the spending patterns of governments can account for the large labor supply in Scandinavia in spite of high taxes. In Scandinavia a larger portion of public expenditures is devoted to provision of family services, child care, elderly care, or, in general, transfers that are conditional on working. While Rogerson and we both emphasise the need to consider how tax revenues are being spent, our concern is the extent to which marginal taxes are distortionary whereas Rogerson focuses on explaining labor supply. Ragan's message is very similar to that of Rogerson but she uses more detailed data for a larger number of countries to show the combined effects of taxes and public spending on labor supply and welfare.

Inspired by the distinction originally made by Olson (1982) between "encompassing organizations" and "narrow distributional coalitions", Summers et al. (1993) put forward a different explanation why labor taxes may be less distortionary and therefore higher in some countries, including Scandinavia. Their argument is that in these countries labor supply is to a larger extent determined collectively in settings where the decision makers internalize the labor supply effects on government revenue. While the argument may be of some interest, we believe that it tends to overstate the "corporatist" nature and understate the flexibility of the

[6] See, for example, Blomquist and Christiansen (1995), Boadway and Marchand (1995), Cremer and Gahvari (1997), Balestrino (2000) and Pirttilä and Tuomala (2002).

[7] We want to emphasize that what we study in this paper is public provision, i.e. publicly *financed* goods. Whether the goods are privately or publicly produced does not matter for our analysis.

Scandinavian labor markets, but a further discussion is beyond the scope of the present paper.[8]

As a vehicle for our analysis we will use an extension of the Stern (1982) and Stiglitz (1982) two-type version of Mirrlees' optimal income tax model (Mirrlees (1971)). A non-linear redistributive income tax is imposed under the assumption that knowledge of who is high-skilled and who is low-skilled is private information not available to the government. The tax schedule must then be designed subject to the self-selection constraint ensuring that a high-skilled person does not select an income point intended for a low-skilled person. If he were to, we would refer to his behaviour as mimicking. If the high-skilled person were to *mimic*, he would obtain more leisure than the low-skilled person with the same income as, being more productive, the high-skilled person could earn the same income in less time. However, if some of the transfer is given in kind, it will be of less value to the mimicker than to the genuine low-skilled type if the good being transferred is less beneficial to someone who has more leisure time. Shifting to a transfer in kind may therefore make mimicking less appealing, and thus alleviate the self-selection constraint and enhance welfare. Given the particular type of provision scheme we study here, it will also be the case that the marginal tax should reflect the real social cost of additional hours of work. That is, part of the marginal tax serves the same role as a market price in the sense that it conveys information about a real social cost of working longer hours, but the tax is on balance more efficient as it also discourages mimicking.

Before introducing our main model we in section 2 present a simple, preliminary case without any heterogeneity in order to highlight the key role for taxes in our analysis. As our next step we set up the Mirrlees type tax model where, in order to obtain sharp results, we assume that the need for the publicly provided good is a strictly positive monotone function of hours of work. In section 3 we show how a strict Pareto improvement can be achieved by supplementing the optimal tax with a publicly provided private good. In section 4 we characterize the optimal tax/public provision scheme and show that the real social cost of

[8] The distinction between "encompassing organizations" and "narrow distributional coalitions" was used by Olson (1990) himself, together with that between "explicit" and "implicit" redistribution, to provide a key for the understanding of the success of the Swedish economy. According to him, by exploiting the rational ignorance of the typical citizen, narrow distributional coalitions, which usually represent well off people who would not have been able to persuade the electorate to give them a transfer on altruistic grounds, have an incentive to seek redistribution in implicit forms, namely in forms that bypass the public treasury (as for instance protectionist measures or restrictions on competition). For a variety of reasons it can be maintained that the distortions and social costs associated with implicit redistribution far exceed those associated with explicit redistribution and are especially detrimental for growth. Olson claimed that part of the success of the Swedish economy was due to the fact that, if compared with many other countries, the degree of implicit redistribution was relatively low.

providing the private good should be reflected in the individuals' marginal tax rates. The model used in sections 3 and 4 is purposely simple and highly stylized since it is meant to capture important common features characterizing several important publicly provided services. These services are discussed in section 6. For each of them, however, one could build a more specific model which is tailored to fit that particular service and, because of that, uses less restrictive assumptions than those made in sections 3 and 4. To save space we only perform such an extension for one particular service, namely child care. This is done in section 5. Finally, section 7 concludes.

2. A Simple, Preliminary Case.

The fundamental message of our paper is twofold. On one hand we claim that, in the presence of public provision of private goods, the distortionary part of a marginal tax rate does not necessarily coincide with its face value. On the other hand we also claim that economies with higher marginal tax rates might actually be less distortionary than economies with lower marginal tax rates. To illustrate the first claim, we will start by presenting the basics within a model which is stripped down to a bare minimum. There is a large population of identical individuals each of whom is a parent with a single child, and initially there is no public sector. Denote by w and h the wage rate and the working hours of the representative agent, respectively. We assume that the wage rate reflects the true productivity of the worker. Let p be the cost per hour of child care, and denote by C the consumption of the agent. The agent has preferences for consumption and labor expressed by the utility function $u(C,h)$. According to the budget constraint of the agent $C=wh-ph$. Along the budget line, $dC/dh=w-p$. The net income obtained from an hour of work is the wage rate minus the cost of working, which is the price paid for child care. This is the net social income, and where the agent faces no taxes and buys child care in the market, the net private income is equal to the social one. There is no distortion. The agent will maximize utility by setting the marginal disbenefit from working equal to the net marginal income, and the demand for child care is determined by the hours of work.

Assume now that there is a government providing child care free of charge and satisfying any demand for child care required in order to work. The child care is financed by a lump sum tax on each agent. The social gain from an hour of work is of course unaffected, as nothing happens to productivity and the need for someone to look after the child while working remains the same. However, the parent will now behave as if child care is a free good. There is no child care fee, and, from the perspective of each single agent, the increase in

the lump sum tax caused by that agent's separate working decision is negligible, as all lump sum taxes will increase and each one only marginally. The private trade off will be based on *dC/dh=w>w-p,* and there is a distortion. While as such the lump sum tax is nondistortionary, returning it as a subsidy will conceal the true cost of working and cause an upward distortion of labor supply.

Now suppose that rather than levying a lump sum tax, the government imposes an income tax. Denote by τ the income tax rate. An individual's budget constraint will be $C = w(1-\tau)h$ and the private trade off will be based on $dC/dh = w(1-\tau)$. There is a tax wedge between the social gain and the private gain equal to $w - p - (w - w\tau) = w\tau - p$. This wedge will vanish when one sets $\tau = p/w$, which also happens to be the tax rate required for fully funding the child care. Thus, funding the child care through an income tax is nondistortionary. The income tax simply replaces the market price in facing the agent with the true social cost of working. Where a higher tax is imposed it is only the part of the tax exceeding the cost of child care which constitutes a tax wedge.

3. The Model

We are now ready to set up the model we will use to illustrate both the desirability of public provision of private goods and the fact that taxes used to finance these goods are nondistortionary. We will build on the discrete type version of the Mirrlees model.

Contrary to previous contributions considering public provision of private goods in an optimal taxation setting, the public provision system on which we focus in this paper, i.e. a system where agents get as much as they want of the publicly provided good, allows to mitigate all the binding self-selection constraints thwarting the government from achieving its redistributive goals. To highlight this feature, we need a model with more than two types. Hence, in this section we consider a model with three skill levels reflected by wage rates w^1, w^2, and w^3; $w^1 < w^2 < w^3$. For simplicity we normalize the population size of each type to unity. We let $Y(=wh)$ denote the before tax labor income. We also make the usual assumption that the policy maker can observe Y but not w or h separately. Each agent chooses how much labor to supply and the corresponding consumption level, which also depends on the tax liability. There is a private commodity which is a candidate for public provision. The demand for this good, which we in the following will call the *x*-good, is strictly related to the hours of work, i.e. $x = f(h) = f(Y/w)$. (The case considered in section 2 is the one where $f(h) = h = Y/w$). The *x*-good doesn't enter the utility function directly. It's instead

a commodity one must acquire in order to work. Hence, it entails a cost of working. The best example is probably child care as in the case considered in the previous section. We will discuss further examples in section 6.

The various skill types have identical preferences over leisure and consumption, which is equal to disposable income net of the amount spent on the *x*-good. Since every person has the same time endowment, there is a unique relationship between working time and leisure and it is a matter of convenience whether preferences are expressed as preferences for leisure or for work. We opt for the latter alternative and write the direct utility function as $u(C,h)$, where *C* is consumption net of expenditures on the *x*-good.

We let Y^i and B^i denote respectively the before and after tax income of a person of type *i*, with a higher index indicating a higher ability. The labor supply of type *i* can then be expressed as Y^i / w^i. We denote the per unit resource cost of the *x*-good by *p*, which would be the price in a competitive market. In a situation without public provision of *x* the consumption level of type i is then $C^i = B^i - pf(Y^i / w^i)$, and we can write the utility function as $u(B^i - pf(Y^i / w^i), Y^i / w^i)$. Following the convention of suppressing w^i in the latter argument, we write the utility function as $U^i(B^i - pf(Y^i / w^i), Y^i)$. If the *x*-good is publicly provided free of charge, the relevant utility function is $U^i(B^i, Y^i) \equiv u(B^i, Y^i / w^i)$.

We note that in the situation without any taxes or public intervention the individual's budget constraint (BC) would be $C = wh - pf(h)$ and the *marginal net wage rate* would be given by $(dC/dh)_{BC} = w - pf'(h)$. In the (Y,B)-space it would read $(dC/dY)_{BC} = 1-(p/w)f'(Y/w)$. The utility function would take the form $U^i(Y^i - pf(Y^i / w^i), Y^i)$ with first order condition: $-U_Y / U_C = (dC/dY)_{\bar{U}} = 1-(p/w)f'(Y/w)$, i.e. the marginal rate of substitution between income and consumption should be equal to the corresponding marginal rate of transformation. This is the condition that holds when there are no distorsions. As we will see below, when the private good is publicly provided and the real cost of the x-good is not given by a market price, the optimal marginal income tax should mirror the cost $(p/w)f'(Y/w)$.

We assume the standard single crossing property that, for any given point in the (Y,B)-space, the indifference curve of a lower ability type is steeper than that of a higher ability type.

A Pareto Improving Public Provision Scheme

The way to show how public provision of the *x*-good yields a Pareto improvement is similar to the approach taken in earlier studies. However, to make the paper self-contained we briefly describe the mechanism. As in the conventional two-type model, we assume that the government imposes a positive net tax on the high-skilled agent and makes a transfer to the lower skilled types. The conditions for the optimal income tax are standard and are not derived here. Let us just recall that the income tax must be designed subject to the self-selection constraints that a type *3* agent should not mimic type *2* agents by choosing the income point (*Y*,*B*-bundle) intended for the latter, and likewise type *2* agents should not mimic those of type *1*.[9] The implication of these constraints is that the leisure-consumption choices of the two lower skill groups must be distorted so that they work too little as compared to the first best. Relaxation of any of these self-selection constraints will allow a Pareto improving tax reform entailing smaller distortions for the lower skill types and allowing them to work more, have larger consumption and higher utility.

To show how a strict Pareto improvement can be obtained by introducing public provision of the *x*-good being financed by (increased) taxes, notice first that, since there is satiation (conditional on labor supply), the public sector can offer any amount free of charge. Conditional on his labor supply, each person will then choose the amount that he needs.[10] The demand for the *x*-good is given by $x^i = f(Y^i / w^i)$, *i=1,2,3,* for the actual types, whereas for mimickers we would have $f\left(Y^i / w^{i+1}\right)$, *i=1,2*. It is evident that $Y^i / w^{i+1} < Y^i / w^i$ as the mimicked person has a lower wage rate than the mimicker. This simply means that a mimicker, being more productive, would earn the same income in less time and hence demand less of the *x*-good. If we let the actual individuals get the amount of *x* they want and decrease their after-tax incomes by $pf(Y^i / w^i)$, *i=1,2,3,* the situation for the actual persons is unchanged. However, mimickers are forced to pay, via taxes, for more of the *x*-good than they need (the extra expenditure being equal to $p\left[f(Y^i / w^i) - f(Y^i / w^{i+1}) \right]$, *i*=1,2) and hence suffer a loss of utility, implying that the self-selection constraints will no longer bind. This means that we can offer the medium and low-skilled individuals less distorted consumption-

[9] Other self-selection constraints can be neglected. In a finite-class economy, when the government wishes to redistribute from the higher ability types to the lower ability types, an optimal allocation results in a so-called simple monotonic chain to the left (see Guesnerie and Seade, 1982), meaning that only downward adjacent self-selection constraints will be binding.

[10] Without satiation, at a reasonable level, it will not be possible to offer any amount free of charge as each agent would then expand his consumption beyond any reasonable limit.

leisure bundles where they work more and enjoy larger consumption. Hence, we can improve welfare for the lower skill persons without hurting the most able people.

4. Characterization of the Optimal Tax-Public Provision Optimum

To characterize the (information constrained) Pareto optimal policy we assume that the policy maker maximizes the utility of the lowest-skill group subject to pre-set minimum utility levels ($\bar{U}^2$ and $\bar{U}^3$, respectively) for the medium and high-skill group, and subject to the relevant self-selection constraints and the government budget constraint. Using the notation that a double superscript *ij* indicates type *i* when mimicking type *j*, the Lagrange function of this optimization problem will take the form:

$$\Lambda = U^1(B^1,Y^1) + \lambda_2(U^2(B^2,Y^2) - \bar{U}^2) + \lambda_3(U^3(B^3,Y^3) - \bar{U}^3) + \beta_2(U^2(B^2,Y^2) - U^{21}(B^1,Y^1))$$
$$+ \beta_3(U^3(B^3,Y^3) - U^{32}(B^2,Y^2)) + \mu\sum_{i=1}^{3}\left(Y^i - B^i - pf\left(Y^i / w^i\right)\right). \quad (1)$$

The first and the second constraint (with Lagrange multipliers λ_2 and λ_3) are the minimum utility requirements for the medium- and high-skilled individuals. The third restriction (with multiplier β_2) is the self-selection constraint requiring that the medium-skilled person is not better off by mimicking the low-ability person than by accepting the bundle intended for him. The next constraint (with multiplier β_3) is the corresponding self-selection constraint for the high-skilled person. The last constraint (with shadow price μ) is the government budget constraint.

The first order conditions are derived in the appendix. Invoking those results we obtain from (a13)

$$MRS^3 = \frac{-U_Y^3}{U_C^3} = 1 - \frac{p}{w^3} f'\left(\frac{Y^3}{w^3}\right), \quad (2)$$

whereas from (a11) and (a12) we obtain respectively

$$MRS^1 = \rho_2(MRS^{21} - MRS^1) + 1 - \frac{p}{w^1} f'\left(\frac{Y^1}{w^1}\right) < 1 - \frac{p}{w^1} f'\left(\frac{Y^1}{w^1}\right), \quad (3)$$

$$MRS^2 = \rho_3(MRS^{32} - MRS^2) + 1 - \frac{p}{w^2} f'\left(\frac{Y^2}{w^2}\right) < 1 - \frac{p}{w^2} f'\left(\frac{Y^2}{w^2}\right), \quad (4)$$

where $MRS = -U_Y / U_C$, $\rho_2 = \beta_2 U_C^{21} / \mu > 0$, $\rho_3 = \beta_3 U_C^{32} / \mu > 0$, and the inequalities follow because $MRS^{ij} < MRS^j$ for $i>j$, which is the standard assumption that a more able type of individual has the flatter indifference curve through any given point in the (Y, B)-space.

We saw above that an undistorted optimum for an individual is characterized by $MRS = 1-(p/w)f'(Y/w)$. Hence, according to condition (2), the labor supply of agents of type 3 is undistorted. Since resources must be allocated to the *x*-good, an additional hour of work from an agent of type 3 inflicts a real cost of $pf'(Y^3/w^3)$ on society, which should be reflected in the worker's budget constraint to provide the right incentives for labor supply and make sure that the marginal rate of substitution between leisure and consumption is equalized to the corresponding marginal rate of transformation. This is exactly the content of condition (2). The marginal rate of substitution (*MRS*) between gross and net income on the left hand side expresses the compensation in terms of consumption required to offset the disutility from earning an additional unit of gross income. In other words, this is a money measure of the marginal disutility of acquiring further income. According to (2) this marginal disbenefit should be equated to the extra income actually generated after deducting the marginal cost of the *x*-good. Since the latter is a necessary social cost, the right hand side does in fact express the *net* social income generated by work effort at the margin. Thus, the condition is equivalent to the first best efficiency condition $MRS = MRT$, and there is no distortion.

It is of interest to interpret these findings further in terms of marginal tax rates. For this purpose it is helpful to distinguish between a gross and a net tax concept, where the latter is defined net of transfers to the consumers in terms of *x*-good provision. The rationale is that a transfer can be perceived as a negative tax. If we write the gross tax function as $T(Y) = Y - B(Y)$ and the tax net of the public provision of the *x*-good as $\tau(Y) = T(Y) - pf(Y/w)$, the corresponding marginal tax rates are $T'(Y)$ and $\tau'(Y)$, where $\tau'(Y) = T'(Y) - (p/w)f'(Y/w)$. Employing the usual measure of marginal tax rates in the Mirrlees-Stern-Stiglitz tradition, we can define $T'(Y)$ as $1+U_Y/U_B = 1-MRS$. As observed from the optimality condition (2), the marginal gross tax rate for the highest skill type becomes:

$$T'(Y^3) = (p/w^3)f'(Y^3/w^3) > 0, \tag{5}$$

whereas the marginal net tax rate becomes:

$$\tau'(Y^3) = T'(Y^3) - (p/w^3)f'(Y^3/w^3) = 0. \tag{6}$$

That is, the marginal income tax should not be zero but should equal the marginal cost of the *x*-good incurred when an additional unit of gross income is earned. The implication is that type 3 agents face the same marginal prices as in a situation with no public provision of *x*. The

rationale for this result is exactly the one presented in section 2 above. Even if true that the individual obtains the *x*-good "for free" from the public sector, it is still the case that the individual acts as if he were facing the real cost of purchasing the *x*-good. He simply pays for it via the tax bill. Hence, the optimal tax/public provision scheme faces the high-skilled individual, locally at the individual's optimum point, with exactly the same budget constraint as in the system where the *x*-good is bought in the market.

One may then wonder what difference it makes whether the *x*-good is publicly or privately funded. The answer is that is makes a crucial difference with respect to mimicking. If a person of type 3 were to mimic, his tax payment would cover the cost of the amount of *x*-good used by the mimicked person, which would exceed his own need. While it is true that the tax pays for the amount of *x* that actual people need at their optimum income points, it pays for a larger amount than the one needed by mimickers, which imposes a cost to mimicking.[11]

Turning our attention to the agents of type 1 and 2, we can see that the consumption-leisure bundle of the lower skill groups must be distorted in order to prevent these agents from being mimicked. The marginal income tax of type 1 is:

$$T'(Y^1) = \rho_2(MRS^1 - MRS^{21}) + (p/w^1) f'\left(Y^1/w^1\right) \tag{7}$$

and the marginal tax net of the cost of the *x*-good is:

$$\tau'(Y^1) = T'(Y^1) - (p/w^1) f'\left(Y^1/w^1\right) = \rho_2(MRS^1 - MRS^{21}) > 0. \tag{8}$$

Similarly, for type 2, we have that

$$T'(Y^2) = \rho_3(MRS^2 - MRS^{32}) + (p/w^2) f'\left(Y^2/w^2\right) \tag{9}$$

and

$$\tau'(Y^2) = T'(Y^2) - \left(p/w^2\right) f'\left(Y^2/w^2\right) = \rho_3(MRS^2 - MRS^{32}) > 0. \tag{10}$$

We note that the marginal gross tax is made up of two terms – one reflecting the social marginal cost of the *x*-good and the other being a distortionary term required to deter mimicking. The part that reflects the social marginal cost is corrective and nondistortionary,

[11] Note also that agents of type 3 are likely to complain and argue that they would prefer a system where one does not pay for the x-good via the taxes. They would like a system with lower tax payments and all of the x-good purchased in the market. The reason is that in such a system they could gain by mimicking, obtaining a tax relief exceeding their cost of the x-good as mimickers. However, if the political majority is adamant to retain the previous redistribution, steps would have to be taken to prevent such behavior, and a Pareto-inferior situation would arise as compared to the regime with public provision of x. Hence, a public provision system, improving upon the market solution, is likely to have the property that some people would like to change it in the hope of escaping the burden of redistribution.

and serves the same role as a market price as it conveys information about the cost of working an additional hour. This is a crucial insight. Just taking the marginal tax rates at face value, one is easily led to exaggerate the distortionary effect as one may easily overlook that part of the marginal tax is indeed a payment for a true social cost. The second part, which appears on its own in the net marginal tax, is truly distortionary. It follows from the expression for the net tax rate that the labor supply of the medium- and low-skilled types is distorted downwards. However, it is important to realize that the distortion is *smaller* than it would be in an optimal taxation setting without public provision, where individuals would buy the x-good in the market. As pointed out in section 3, the reason is that the public provision scheme opens the way for the government to achieve a Pareto improvement upon the optimum without public provision. This in turn allows the government to offer agents less distorted bundles.

What we have previously labelled the net marginal tax rate, i.e. the marginal tax after taking into account the transfer in kind, is of the same form as the marginal income tax in a pure income tax system. However, now there is an additional term, $\left(p/w^i\right) f'\left(Y^i/w^i\right)$, in the expression for the gross marginal income tax. For all actual persons the tax financed public provision makes the cost of obtaining *x* similar to when it is purchased in the market. In either case the consumers face the real marginal cost of the *x*-good, as is socially desirable since the cost of *x* is a true cost of supplying labor. The market price and the extra marginal tax, perceived as earmarked for the *x*-good, yield the same labor market disincentives. Only if a person were to mimic, would the public scheme be different than a market system as a mimicker, via the tax system, would be forced to pay for an amount of *x* equal to that demanded by the person with lower skill, and exceeding the need of the mimicker. This is the crucial property of the public provision system as it is indeed the capability of alleviating the self-selection constraint that is the very justification for the public provision in our context.

5. Child Care and Two Types of Leisure

In the model studied above the demand for the x-good was only dependent on hours or work. This might in some cases be considered too restrictive an assumption. We therefore in this section generalize the model for the case where the publicly provided private good is child care, which represents one important application of the model above. As we will see, the major result that part of the marginal income tax reflects society's cost of providing the *x*-good, and is therefore nondistortionary, still goes through.

In the previous section with three skill types we showed how all self-selection constraints are mitigated if the public provision scheme is individualized and individuals get the amounts they want. This also holds true for the model that we are going to present here. However, since the general result already has been shown, we consider now for simplicity a two type model with high- and low-skilled agents.

Let the utility function be described by $U(C,l_g,l_k)$, where C denotes consumption and l_k is time spent together with the kid(s). The remaining leisure time can be spent in many different ways. As a shorthand, we will in the following call it "golfing" and denote it by l_g. All goods in the utility function are assumed to be normal. There are two time constraints. For the parent there is the constraint

$$\theta = l_g + l_k + h, \tag{11}$$

where θ is the total endowment of time. For the child there is the constraint

$$\theta = l_k + x, \tag{12}$$

where *x* represents hours of child care. The child must be looked after all the time and this can either be done by the parent or by professional child carers. The two time constraints can be combined to obtain $x = h + l_g$. Child care is bought either for time devoted to work (h) or for golfing (l_g). Before analyzing the public provision schemes we characterize the conditions for no distortions.

Conditions for no distortions

To establish the conditions that characterize a situation with no distortions we study the individual's optimization problem when there are no taxes and no public provision. In this case the budget constraint for the individual becomes

$$C = w(\theta - l_k - l_g) - p(\theta - l_k). \tag{13}$$

The marginal rates of transformation are obtained from the budget constraint (BC): $(dC/dl_k)_{BC} = -(w-p)$, $(dC/dl_g)_{BC} = -w$ and $(dl_k/dl_g) = -w/(w-p)$. The characterization of an undistorted utility optimum is given by the conditions that the various marginal rates of substitution are equated to the corresponding marginal rates of transformation:

$$U_{l_k}/U_C = w - p, \qquad U_{l_g}/U_C = w \quad \text{and} \quad U_{l_g}/U_{l_k} = w/(w-p). \tag{14 a-c}$$

Public provision and taxes

We will study two different public provision schemes. Under the first we assume that free child care is only provided for hours of work and not for time spent golfing. This is basically the type of system in force in Sweden.[12] For such a system to be feasible it must be the case that the child care centres can observe whether a parent goes to work or goes golfing. As a matter of fact it has never been an issue in Sweden that parents would cheat and go golfing instead of using the free child care for work.

To facilitate the analysis of the optimal taxation problem, we will solve the individual's optimization problem in two stages. In the first stage the individual maximizes utility for a given pair of before- and after-tax income. Given that the child care is provided free of charge for hours of work, the individual's problem becomes:

$$\underset{l_g}{Max} \;\; U\left(C, l_g, \theta - l_g - Y/w^i\right) \tag{15}$$

$$s.t. \;\; C = B - pl_g \,. \tag{16}$$

The first order condition for l_g is given by: $-pU_C + U_{l_g} - U_{l_k} = 0$. This first order condition defines a conditional demand function $l_g\left(B, Y, w^i, p\right)$. Sticking this back into the direct conditional utility function we obtain the indirect conditional utility function

$$V^i\left(B,Y\right) \equiv U\left(B - pl_g\left(B,Y,w^i,p\right), l_g\left(B,Y,w^i,p\right), \theta - l_g\left(B,Y,w^i,p\right) - Y/w^i\right). \tag{17}$$

In the second stage the individual maximizes $V^i\left(B,Y\right)$ subject to $B = Y - T(Y)$, giving the condition $T'(Y^i) = 1 + V_Y^i / V_B^i = 1 - MRS$.

We can write the government's optimal taxation problem as:

$$\underset{B^1,Y^1,B^2,Y^2}{Max} \;\; V^1(B^1,Y^1)$$

$$s.t. \;\; V^2(B^2,Y^2) \geq \bar{V}^2 \qquad (\lambda)$$

$$V^2(B^2,Y^2) \geq V^{21}(B^1,Y^1) \qquad (\beta)$$

$$Y^1 - B^1 + Y^2 - B^2 - p\left(\frac{Y^1}{w^1} + \frac{Y^2}{w^2}\right) \geq 0 \,. \qquad (\mu)$$

Here λ is the Lagrange multiplier for the constraint requiring a minimum utility level for the agent of type 2 (the high-skilled), β is the Lagrange multiplier for the self-selection

[12] Child care in Sweden is not completely free. Parents pay a fee that amounts to about 10-15% of the production cost of child care.

constraint and μ the Lagrange multiplier for the government's budget constraint. To economize on analysis we here only study the extent to which the income tax distorts the allocation of the high-skilled agent. The conditions for the low-skilled (agent of type 1) would be similar but also include a self-selection term.

The first order conditions with respect to the tax parameters for the high-skilled agent are:

$$(\lambda+\beta)V_Y^2+\mu\left(1-\frac{p}{w^2}\right)=0 \quad \text{and} \quad (\lambda+\beta)V_B^2-\mu=0, \text{ implying } -V_Y^2/V_B^2=1-p/w^2 \quad \text{and}$$

$$T'(Y^2)=1+V_Y^2/V_B^2=p/w^2. \tag{18}$$

Since the envelope theorem implies that $V_Y^i=-U_{l_k}/w^i$ and $V_B^i=U_C$, we can notice that the first order conditions also imply $U_{l_k}^2/U_C^2=w^2-p$. Since the individual's first order condition for l_g gives $-pU_C^2+U_{l_g}^2-U_{l_k}^2=0$, we find that $U_{l_g}^2/U_C^2=w^2$ and $U_{l_g}^2/U_{l_k}^2=w^2/(w^2-p)$. That is, the conditions (14a-c) are satisfied implying that the high-skilled agent is undistorted on all margins. In particular, the marginal income tax that he faces, $T'(Y^2)=p/w^2$, is nondistortionary.

Eq. (18) illustrates once again the general principle according to which, when judging how distortive an income tax is, pre-existing distortions should be taken into account.[13] In this case a pre-existing distortion is associated with the free provision of child care for the hours of work. That in itself distorts the individual's choice between hours of work and consumption, so that the individual is over-incentivized to work. If the marginal income tax were set to zero, the distortion stemming from the free provision of child care would prevail. However, if the marginal income tax is set according to eq. (18), the income tax exactly corrects for the distortion caused by the free provision of child care. Hence, the income tax is in the present case corrective and not distortive.

If for some reason, like an informational constraint, the public provision scheme is designed so that also child care for going golfing is provided for free, the analysis is slightly different. The first stage of the individual's optimization problem now becomes

[13] See Kaplow (1998).

$$\underset{l_g}{Max} \;\; U\left(C, l_g, \theta - l_g - Y/w^i\right) \tag{19}$$

$$s.t. \quad C = B. \tag{20}$$

The first order condition for l_g is given by $U_{l_g} = U_{l_k}$. From this first order condition we can derive a conditional demand function $l_g\left(B, Y, w^i\right)$. Substituting it into the conditional direct utility function we get

$$V^i(B,Y) = U(B, l_g(B,Y,w^i), \theta - l_g(B,Y,w^i) - Y/w^i). \tag{21}$$

In the second stage the individual maximizes utility subject to the constraint $B = Y - T(Y)$ and we obtain, as usual, $T'(Y^i) = 1 + V_Y^i / V_B^i = 1 - MRS$.

The government's optimal taxation problem has the same form as above except for the budget constraint which now takes the form

$$Y^1 - B^1 + Y^2 - B^2 - p\left(\frac{Y^1}{w^1} + l_g^1 + \frac{Y^2}{w^2} + l_g^2\right) \geq 0. \qquad (\mu)$$

As in the previous case we only discuss here the conditions pertaining to the allocation of the high-skilled agents. The first order conditions with respect to the relevant tax parameters are

$$(\lambda + \beta)V_Y^2 + \mu\left[1 - p\left(\frac{1}{w^2} + \frac{\partial l_g}{\partial Y^2}\right)\right] = 0 \quad \text{and} \quad (\lambda + \beta)V_B^2 - \mu\left[1 + p\frac{\partial l_g}{\partial B^2}\right] = 0.$$

Introducing the definition $\left(dl_g / dY\right)_{dV^2=0} = dl_g / dY + MRS_{BY}^2 (dl_g / dB)$, these conditions imply that the marginal income tax can be written as

$$T'(Y^2) = 1 + \frac{V_Y^2}{V_B^2} = \frac{p}{w^2} + p\left(\frac{dl_g}{dY}\right)_{dV^2=0} = p\left[\frac{1}{w^2} + \left(\frac{dl_g}{dY}\right)_{dV^2=0}\right]. \tag{22}$$

The two remarks we made about eq. (18) are valid also for eq. (22). In particular, the income tax according to eq. (22) is not a distortive, but a corrective tax. However, a difference between the case represented by eq. (18) and the case represented by eq. (22) is that in the former the income tax *fully* corrects for the distortion created by the free provision of child care. Instead, in the case represented by eq. (22), the income tax is a corrective tax, but it does not fully correct for the distortions caused by the free provision of child care. We elaborate on these features below.

The fact that child care services are publicly provided free of charge to households implies that individuals choose l_g and l_k such that U_{l_g}/U_{l_k}, the marginal rate of substitution between l_g and l_k, equals 1. Without public provision U_{l_g}/U_{l_k} would instead be equal to $w/(w-p)>1$.[14] Thus, public provision of child care services entails an implicit subsidy on l_g, distorting the choice between l_g and l_k in favour of the former. More generally, it entails an implicit subsidy on all the uses of time which require the consumption of child care services. Since in our example both the time devoted to labor in the market and the time devoted to golfing require the consumption of child care services, the public provision of child care does not distort the choice between l_g and h (and consequently it does not distort the choice between l_g and the consumption of the composite commodity C). It does however distort the choice between l_k and h. Therefore, to correct for the distortions created by the free provision of child care, the marginal income tax rate on high-skilled households should depend on how the increase in labor supply, needed to marginally increase the pre-tax income, is made available: whether through a reduction in l_g or through a reduction in l_k.

Suppose for instance that, moving up along an indifference curve in the (Y,B)-space, the increase in h is matched by a reduction in l_g, leaving l_k unaffected. Then, since there is no pre-existing distortion due to the public provision of child care in the household's choice between l_g and h, the marginal tax rate on labor income should be equal to zero in order to be nondistortionary. This is exactly what is prescribed by eq. (22) above, since in this case $\left(dl_g/dY\right)_{dV^2=0}=-1/w^2$.

Suppose instead that the increase in h is accomplished through a reduction in l_k, leaving l_g unaffected. In this case, since due to the public provision of child care there is a pre-existing distortion in the household's choice between l_k and h (a distortion inducing to over-supply labor), to be nondistortionary the marginal tax rate on labor income should be equal to p/w^2. This would imply that the household's after tax marginal rate of substitution between l_k and C is still given by $U_{l_k}/U_C=w^2-p$ as in the (no tax, no public provision)

[14] Since the envelope theorem implies that $V_B^i=U_C$ and $V_Y^i=-U_{l_k}/w^i$, we can notice that the first order conditions for the optimal taxation problem also imply that $U_{l_k}/U_C=U_{l_g}/U_C=\left(w-p-p\partial l_g/\partial h\right)/\left(1+p\partial l_g/\partial B\right)$. Hence, the individual is distorted on all margins.

undistorted setting. Once again, this is precisely what is prescribed by the eq. (22) above, since in this case $\left(dl_g / dY\right)_{dV^2=0} = 0$.

More generally, if the increase in h is accompanied by varying both l_g and l_k, the same logic applied above requires that, to be nondistortionary, the marginal income tax rate should be proportional (at a rate p) to the reduction in l_k. That this is precisely what is prescribed by eq. (22) can be easily ascertained rewriting eq. (22) using the parent's time constraint $h = \theta - l_g - l_k$. Differentiating the time constraint gives $dh = -dl_g - dl_k$. Since $dh/dY = 1/w$, we also have:

$$1/w = -(dl_g / dY) - (dl_k / dY) \quad \Rightarrow \quad 1/w + (dl_g / dY) = -dl_k / dY . \tag{23}$$

Using (23), eq. (22) can then be equivalently rewritten as:

$$T'(Y^2) = -p\left(dl_k / dY\right)_{dV^2=0} . \tag{24}$$

When child care is provided for free only for hours of work we have seen that the marginal income tax fully corrects for the distortion introduced by the free provision. Since the free provision helps mitigating the self-selection constraint, it is apparent that the introduction of a system of free child care provision financed by increased taxes yields a Pareto improvement over the tax optimum without public provision. In the case where child care is for free also for golfing time, instead, the marginal income tax is nondistortive but not fully corrective. In that framework it is in the end an empirical question whether the benefit of slackened self-selection constraints outweighs the cost in terms of remaining distortions.[15]

We have seen that the result from section 4 that the marginal income tax for the high-skilled is nondistortive goes through also in the present more general framework. If we were to write out the corresponding equation for the low-skilled we would find as before that the marginal income tax consists of a distortive term, originating from the self-selection constraint, and a term that, as for the high-skilled, corrects (fully or partially) for the distortions associated with the free of charge provision of child care services.

6. Further examples

In the basic case considered in sections 3 and 4, a crucial assumption about the publicly provided private good was that the demanded amount is a positive monotone function of

[15] Notice that, when child care is provided for free also during golfing time, a necessary condition for public provision to be a welfare-enhancing policy instrument is $Y^1 / w^1 + l_g^1 > Y^1 / w^2 + l_g^{21}$, namely that the sum of working time and golfing time is larger for the true low skilled than for the mimicker. When, as we have assumed earlier, all goods in the agent's utility function are normal, this condition is satisfied.

hours of work. As we have seen, the public provision system that we consider in this paper works best either if, conditional on hours of work, there is satiation, or if the agency that delivers the service can easily control that the provided service is not used for any other purpose than work. Here we will discuss four examples of services that, to various extents, satisfy this assumption.

6.1 Child care

The commodity, to be publicly provided, that we believe best fits our model is child care, which is a case already repeatedly used as our major illustration. It would be of interest to get a rough idea of the extent to which the real cost of child care can contribute to the marginal income tax. We use stylized Swedish figures to do a "back of the envelope calculation". Since there is a clear mode in the distribution of wage rates at the bottom part of the distribution, we think this mode can be used as an empirical value for the wage of the low-skilled. Using this mode and assuming that one child is in child care, a simple back of the envelope calculation indicates that, for low income individuals, p/w^1 amounts to something like 30-35%. Hence, up to values of about 30-35%, the marginal tax rate faced by low-skilled agents can be interpreted as a payment for child care. Beyond this nondistortionary component there is, according to equation (7), an additional component, $\rho_2(MRS^1 - MRS^{21})$, reflecting the distortion that is required to deter mimicking. If there are more children in the household the marginal cost of working longer hours is larger as there are more children to pay for in child care. This means that an even larger share of the marginal tax rate reflects a cost of child care rather than a distortion, indicating that our "back of the envelope figure" above is on the lower side.

Public provision of child care has strong implications for the optimal income tax schedule that should apply to individuals with children in child care. This of course also implies that families with children should face other income tax schedules than families without children. Akerlof (1978) argued that the income tax should be differentiated between groups with different observable characteristics. Our analysis provides a further argument for the usefulness of having different tax schedules for different groups of people.[16]

It can also be of interest to consider the quantitative relevance of child care services in terms of how many children who are in some form of child care. As the labor force

[16] With households supporting different numbers of children one might in general have different tax schedules contingent on the number of children, but we shall not pursue the more general issue of household taxation (see e.g. Cremer et al. (2003) and Schroyen (2003)).

participation of females has increased in many countries so has the number of children in child care. For example, in Sweden 85% of children in ages 2-5 are in child care.[17] To a substantial part (80-85%) this child care is publicly financed. In US child care is also common. According to Blau and Currie (2006): "For good or ill, the majority of children in the U.S. and many other high-income nations are now cared for many hours per week by adults other than their parents and school teachers". Hence, child care is already of great importance in many countries. It also seems that over time its importance is growing in more and more countries.

6.2 Elderly care

Another good candidate for our model of public provision is represented by elderly care services. In our view elderly care has strong similarities to child care. In many countries an elderly person is cared for by a near relative, like a daughter, a son or a (younger) spouse. In Sweden in the past, before the system of publicly provided elderly care was as common as it is nowadays, this was quite usual. Anecdotal evidence suggests that it has been and still is fairly common in the US. In Germany children must by law contribute to the care of their parents.

If a relative is responsible for the care of an elderly person, the relative can do the care himself, buy care in the market, or rely on publicly provided care. The demand for elderly care for a near relative ought to increase as the potential carer works more.

There also seems to be satiation. You do not want the carer to visit your elderly relative in need of care more than needed. If visits twice a day are sufficient, you do not want six visits a day.

In Sweden, as an example, the provision system is such that you have to accept the quality of the publicly provided elderly care.[18] With respect to quantity individuals get according to their needs. In the case an elderly person needs many visits a day from the carer, he gets it. If the need is large, the person is cared for in a nursing home. Hence, elderly care and the way it is provided in Sweden fit the assumptions of our model quite well.

Elderly care is already of large quantitative importance and, because of the coming demographic changes, will become even more important.

[17] Calculated from tables 62 and 500 in Statistisk Årsbok 2003.
[18] Sometimes the production is private, but the care is paid for with public money.

6.3 Primary education

Primary education is yet another example fitting our model. Children in ages for primary education can get their education either as home schooling, education in a private school or in a public sector school. In the absence of publicly provided primary education parents would have to undertake home schooling or buy private education. In some countries like US and UK parents have a legal right to educate their children at home and this is a right that is used by some parents.[19] Like for child care the demand for schooling for children would be an increasing function of the parents' hours of work as parents would have less time for teaching their children when they work more. Also, most parents would still like that their children had time for activities such as playing, rest, and social activities besides being educated. Thus there would be satiation in the demand for hours of primary education.

Primary education has the necessary properties to make it suitable for the type of public provision studied in this paper. This means that one might conceivably use public provision of primary education as a screening device to mitigate the informational deficiencies encountered by the policy maker in the design of an optimal tax system. However, in practice education policy is governed by other concerns. In most countries the provision system is not designed so that parents can choose how many hours per week they want to use the public school. Rather, there is a requirement that the children should go to either a private or a public sector school for a certain number of hours per week.

6.4 Health care

The last example that we consider is given by health care. Even if many or most health problems are not related to work, there can be little doubt that several health problems due to diseases and accidents are related to occupation and the number of working hours. The work-related *expected* need for health care is likely to increase roughly in proportion to the hours of work, and the demand for this part of health care would thus be work-induced. It is also fair to say that there is satiation in the sense that (most) individuals only want to be treated for their actual health problems. If a person has hurt his right knee, he wants that injury to be cured. He does not want his unhurt left knee or his eye to be operated, even if he would get it for free.

The aim of public health care is to furnish people with care and treatment according to need. This is obviously not a sharply defined concept. The waiting time for treatment may

[19] For US it is estimated that something like 2-4% of children in relevant ages are educated in home schooling and that the percentage is on the rise. For more information see http://nces.ed.gov/pubs2001/Homeschool/background.asp and http://www.reason.com/news/show/36591.html.

vary and one may choose quality levels with different probabilities of successful cure or prospects for speedy recovery. In practice it is the doctors that define what represents an adequate treatment, and satiation may be defined by the standards that are actually set.

Health care amounts to around 9% of GDP in Sweden and 15% in US. So health care is quantitatively of large importance. Even if only a fraction, but presumably a significant fraction, is work-related, we have another quantitatively important good that fits the assumptions of our model fairly well.

7. Conclusions and Discussion

It is well known since long that public provision of certain private goods can mitigate the self-selection constraint. An important and new insight from the analysis of this paper is that there can be provision systems where provision levels are individualized and positively related to an individual's hours of work. For such systems it is essential that the true marginal social cost of providing the good is mirrored in marginal income taxes. That part of the marginal tax that reflects the marginal social cost generates no deadweight loss and plays the same part as a market price in the sense that it conveys information on resource costs.

In some countries public provision of the type of services listed above amounts to something like 15% of GDP. A substantial part of the marginal taxes individuals face in such countries might therefore be nondistortionary. It would be an important future research task to empirically estimate, for various countries, how large part of the marginal tax that is nondistortionary. Such work might require some further theoretical analysis. Almost all previous work on public provision of private goods has used models where all individuals have the same preferences for the publicly provided private good. In real life the need for child care, elderly care etc. is not the same for everyone. It still remains to characterize how the tax schedule should be designed for an economy where the need for the publicly provided private goods varies between individuals.

Let us end by emphasizing some of the most important conclusions. Going from a situation where individuals buy the service in question in the market to a situation where the good is publicly provided, total and marginal taxes will increase. However, part of the marginal taxes will be nondistortionary and total distortions will decrease while labor supply and welfare will increase. An implication of this is that if one compares the tax systems in two countries it may very well be that it is the country with higher marginal tax rates that has the less severe distortions. One cannot judge the distortions generated by a tax system in isolation, one must also consider the expenditure side.

Appendix

From the Lagrange function (1) we can derive the following first order conditions:

$$\Lambda_{B^1} = U_C^1 + \beta_2 U_C^{21} - \mu = 0, \tag{a1}$$

$$\Lambda_{Y^1} = U_Y^1 + \beta_2 U_Y^{21} + \mu\left[1 - \frac{p}{w^1} f'\left(\frac{Y^1}{w^1}\right)\right] = 0, \tag{a2}$$

$$\Lambda_{B^2} = (\lambda_2 + \beta_2) U_C^2 - \beta_3 U_C^{32} - \mu = 0, \tag{a3}$$

$$\Lambda_{Y^2} = (\lambda_2 + \beta_2) U_Y^2 - \beta_3 U_Y^{32} + \mu\left[1 - \frac{p}{w^2} f'\left(\frac{Y^2}{w^2}\right)\right] = 0, \tag{a4}$$

$$\Lambda_{B^3} = (\lambda_3 + \beta_3) U_C^3 - \mu = 0, \tag{a5}$$

$$\Lambda_{Y^3} = (\lambda_3 + \beta_3) U_Y^3 + \mu\left[1 - \frac{p}{w^3} f'\left(\frac{Y^3}{w^3}\right)\right] = 0. \tag{a6}$$

The first order conditions can be manipulated to obtain further economic insights. Solving (a1) and (a2) for $-U_Y^1$ and U_C^1, and dividing, we obtain

$$\frac{-U_Y^1}{U_C^1} = \frac{-\beta_2 U_Y^{21} + \mu\left[1 - \frac{p}{w^1} f'\left(\frac{Y^1}{w^1}\right)\right]}{\beta_2 U_C^{21} + \mu}, \tag{a7}$$

which can be reformulated as

$$\frac{-U_Y^1}{U_C^1}(\beta_2 U_C^{21} + \mu) = -\beta_2 U_Y^{21} + \mu\left[1 - \frac{p}{w^1} f'\left(\frac{Y^1}{w^1}\right)\right]. \tag{a8}$$

Straightforward manipulations yield the expression

$$\frac{-U_Y^1}{U_C^1}\left(1 + \frac{\mu}{\beta_2 U_C^{21}}\right) = -\frac{\beta_2 U_Y^{21}}{\beta_2 U_C^{21}} + \frac{\mu\left[1 - \frac{p}{w^1} f'\left(\frac{Y^1}{w^1}\right)\right]}{\beta_2 U_C^{21}}. \tag{a9}$$

Or, equivalently

$$\frac{-U_Y^1}{U_C^1}\left(\frac{\mu}{\beta_2 U_C^{21}}\right) = \frac{-U_Y^{21}}{U_C^{21}} - \frac{-U_Y^1}{U_C^1} + \frac{\mu\left[1 - \frac{p}{w^1} f'\left(\frac{Y^1}{w^1}\right)\right]}{\beta_2 U_C^{21}}. \tag{a10}$$

Multiplying on both sides, we get

$$\frac{-U_Y^1}{U_C^1} = \frac{\beta_2 U_C^{21}}{\mu}\left(\frac{-U_Y^{21}}{U_C^{21}} - \frac{-U_Y^1}{U_C^1}\right) + 1 - \frac{p}{w^1} f'\left(\frac{Y^1}{w^1}\right). \quad \text{(a11)}$$

Then, making use of (a3) and (a4) and performing manipulations analogous to those above, we obtain

$$\frac{-U_Y^2}{U_C^2} = \frac{\beta_3 U_C^{32}}{\mu}\left(\frac{-U_Y^{32}}{U_C^{32}} - \frac{-U_Y^2}{U_C^2}\right) + 1 - \frac{p}{w^2} f'\left(\frac{Y^2}{w^2}\right). \quad \text{(a12)}$$

(a5) and (a6) readily imply that

$$\frac{-U_Y^3}{U_C^3} = 1 - \frac{p}{w^3} f'\left(\frac{Y^3}{w^3}\right). \quad \text{(a13)}$$

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