Società italiana di economia pubblica

XX CONFERENZA

ECONOMIA DELLA TASSAZIONE
Sistemi tributari, pressione fiscale, crescita

Pavia, Università, 25 - 26 settembre 2008

DISCRIMINATORY PRICES, ENDOGENOUS LOCATIONS AND THE PRISONER DILEMMA PROBLEM

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pubblicazione internet realizzata con contributo della COMPAGNIA di San Paolo

Discriminatory prices, endogenous locations and the Prisoner Dilemma problem

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Abstract

In the Hotelling framework, the equilibrium first-degree discriminatory prices are all lower than the equilibrium uniform price. When firms' locations are fixed, price discrimination emerges as the unique equilibrium in a game in which every firm may commit not to discriminate before setting the price schedule. This paper assumes endogenous locations and shows that uniform pricing emerges as the unique equilibrium in a game in which every firm may commit not to discriminate before choosing where to locate in the market. Price discrimination still is the unique equilibrium outcome when firms may commit only after the location choice.

JEL codes: D43; L11

Keywords: Price discrimination; Commitment; Location.

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1. Introduction

Price discrimination is a widely used business practice. However, in oligopoly it may be possible that the equilibrium discriminatory prices are all lower than the equilibrium uniform price. This phenomenon is called *all-out competition*[1]. When *all-out competition* occurs, equilibrium profits under the discriminatory price regime are lower than the equilibrium profits under the uniform price regime.

All-out competition typically emerges in the Hotelling (1929) framework with linear or quadratic transportation costs[2]. Thisse and Vives (1988) have studied the case of first-degree price discrimination within the Hotelling's (1929) model assuming that the firms are exogenously located at the endpoints of the market. First, they show that when firms can perfectly price discriminate and simultaneously choose the price schedule, uniform pricing is never an equilibrium. Then, Thisse and Vives (1988) assume a two-stage game, where in the first stage each firm has the possibility to announce that it will not price discriminate, while in the second stage the price schedules are effectively set. For example, a firm may announce in the first stage that it would not hold sales or would not issue coupons. Of course, such announcements have to be credible. There are a lot of practices that make these announcements credible: the *most-favoured nation clause*[3] is one of these practices. Thisse and Vives (1988) show that even when every firm may credible commit not to discriminate before setting the price schedule, the discriminatory prices still arise in equilibrium, since no-commitment is the dominant strategy for each firm in the first stage of the game conditioned on the equilibrium path in the second stage of the game. This situation gives rise to a typical Prisoner Dilemma: both firms would be better off setting uniform prices, but the dominant strategy of each firm induces the discriminatory equilibrium, that in turn yields lower profits.

The aim of this paper is to test whether Thisse and Vives' (1988) result still holds when the locations of the firms are endogenous. First of all, allowing firms to choose where to locate in the market, we obtain that the location-price equilibrium with perfect price discrimination is characterized by a less than maximal differentiation degree: in

[1] Corts (1998).
[2] Ulph and Vulkan (2000).
[3] The *most-favoured nation clause* engages a firm to offer a consumer the same price as its other consumers: if the clause is not respected, the firm must pay back the consumer the difference between the price he effectively paid and the lowest price fixed by the firm.

fact, firms locate at 1/4 and 3/4. *All-out competition* occurs. A second step of the analysis consists in supposing a three-stage game. In the first stage of the game, firms simultaneously choose where to locate in the market. In the second stage of the game each firm decides whether to commit not to price discriminate or not to commit: if a firm has committed, it is obliged to set the same price for all consumers when the competition in price arises; if a firm has not committed, it competes with unrestricted price schedules. In the third stage the firms set simultaneously the price schedules. We show that there exists a unique sub-game perfect equilibrium, which is characterized by price discrimination. Therefore, even with endogenous locations, discriminatory prices arise in equilibrium. Finally, we suppose a change in the timing of the three-stage game. That is, in the first stage of the game, each firm commits or does not commit; in the second stage the firms simultaneously choose the location in the market; in the third stage the firms set simultaneously the price schedules. Interestingly, in this case the (unique) sub-game perfect equilibrium is characterized by uniform pricing: both firms commit in the first stage, maximally differentiate in the second stage and set uniform prices in the third stage. No Prisoner Dilemma is present. The intuition behind this result is the following. When the commitment decision is taken for given locations of the firms (first timing), the only effect of commitment is to reduce the firm's flexibility on setting prices. Therefore, the dominant strategy for each firm is no-commitment, and the Thisse and Vives' (1988) result still is valid. Instead, when the commitment decision affects not only the price decision but also the location decision (second timing), each firm anticipates that its decision to commit will induce a higher equilibrium differentiation degree, and this makes profitable for each firm to commit, even if the flexibility in setting price is reduced: in this case, the Thisse and Vives (1988) result does not hold.

This paper is organized as follows. In Section 2 we describe the model and we briefly recall the well-known location-price equilibrium under the hypothesis of uniform price regime. In section 3 we analyse the location-price equilibrium when the firms can perfectly price discriminate. In section 4 we analyze the three-stage game with the two different timings. Section 5 concludes. In the appendix, we extend the model to consider third-degree price discrimination: the main results do not change.

2. Uniform price

Assume a linear market of length 1. Consumers are uniformly distributed along the segment. Define with $x \in [0,1]$ the location of each consumer. Each point in the linear market represents a certain variety of a given good. For a consumer positioned at a given point, the preferred variety is represented by the point in which the consumer is located: the more the variety is far from the point in which the consumer is located, the less it is appreciated by the consumer. Each consumer consumes no more than 1 unit of the good. Define with v the maximum price that a consumer is willing to pay for buying his preferred variety. Suppose that v is equal for all consumers. Suppose further that v is large enough to guarantee that each consumer always buys the good.

There are two firms, *A* and *B*, competing in the market. Both firms have identical constant marginal and fixed costs, both normalized to zero. The firms' decision concerning where to locate coincides with the decision of which variety to produce. Define with a the location chosen by firm *A* and with b the location chosen by firm *B*. Without loss of generality, assume: $0 \leq a \leq b \leq 1$. Define with $\overline{p}^A$ the uniform price set by firm *A* and with $\overline{p}^B$ the uniform price set by firm *B* [4].

The utility of a consumer depends on v, on the price set by the firm from which he buys, and on the distance between his preferred variety and the variety produced by the firm. We assume quadratic transportation costs. Define with t, equal for all consumers, the importance attributed by the consumer to the distance between his preferred variety and the variety offered by the firm. The utility of a consumer located at x when he buys from firm *A* is given by: $u_x = v - \overline{p}^A - t(x-a)^2$, while the utility of a consumer located at x when he buys from firm *B* is given by: $u_x = v - \overline{p}^B - t(x-b)^2$. Define with x^* the consumer which is indifferent between buying from firm *A* or from firm *B* for a given

[4] Given the interpretation of the linear market that we are adopting, the "transportation costs" are necessarily sustained by the consumers: therefore, prices are *f.o.b.*. However, the linear market can also have a "spatial" interpretation: in this case each point of the segment represents a point in the physical space. Since the distance between a consumer and the firm implies now effective transportation costs, two pricing methods are possible: *f.o.b.* prices, when the transportation costs are sustained by the consumer which goes and takes up the product at the firm's mill, and *delivered* prices, when the transportation costs are sustained by the firm that carries the product from the mill to the consumer. Thisse and Vives (1988) adopt a spatial interpretation of the market and assume *delivered* prices.

couple of locations, a and b, and for a given couple of uniform prices, $\overline{p}^A$ and $\overline{p}^B$. Equating the utility in the two cases and solving for x it follows:

$$x^* = \frac{a+b}{2} + \frac{\overline{p}^B - \overline{p}^A}{2t(b-a)}$$

Given the uniform distribution of the consumers, x^* is the demand function of firm A and $1-x^*$ is the demand function of firm B. It is well known that in a two-stage game in which firms first choose locations and then choose the uniform price, the unique sub-game perfect equilibrium implies maximal differentiation, as the following proposition indicates:

Proposition 1 (D'Aspremont *et al*. 1979): *in a two-stage game in which the firms first simultaneously decide where to locate and then simultaneously decide the* [uniform] *price, there is a unique sub-game perfect equilibrium, defined by* $a^* = 0$ *and* $b^* = 1$*, and* $\overline{p}^A{}^* = \overline{p}^B{}^* = t$.

Given the equilibrium locations and the equilibrium prices, the equilibrium profits for each firm are: $\Pi^A = \Pi^B = t/2$.

3. Perfect discriminatory prices

We study now the location-price equilibrium when both firms can perfectly price discriminate between consumers. We suppose a two-stage game, in which the firms first decide where to locate and then compete on prices. Before to start, note that the fact that the firms have the possibility to price discriminate does not imply that the firms effectively price discriminate: a firm may decide to price uniformly even if it can price discriminate. In the following we show that when firms can price discriminate, they do it. Consider a consumer located in x. Define with p_x^J the price charged by firm

$J = A, B$ to the consumer x. The utility of that consumer when he buys from firm A is given by: $u_x = v - p_x^A - t(x - a)^2$, while his utility when he buys from firm B is given by: $u_x = v - p_x^B - t(x - b)^2$. The consumer buys from the firm which gives him the higher utility. If the utility of the consumer is the same when he buys from firm A and when he buys from firm B, we suppose that he buys from the nearer firm[5]. Suppose that consumer x is nearer to firm A than to firm B. For a given couple of firms' locations and for a given price set by firm B, the best thing firm A can do is setting a price that gives the consumer the same utility he receives from firm B: this is the highest possible price that guarantees that consumer x buys from A. Suppose instead that the consumer x is nearer to firm B. For a given couple of firms' locations and for a given price set by firm B, in order to serve consumer x the best thing firm A can do is giving him a slightly higher utility than the utility provided to him by firm B. Of course, an analogous reasoning holds for firm B.

The following proposition defines the equilibrium price schedule for any couple of locations.

Proposition 2: *when the firms can perfectly price discriminate between the consumers, the equilibrium prices in the second stage of the game are the following*:

$$p_x^A*(a,b) = t(x-b)^2 - t(x-a)^2 \quad \text{if} \quad x \le (a+b)/2 \quad \text{and}$$
$$p_x^A*(a,b) = 0 \quad \text{if} \quad x \ge (a+b)/2$$
$$p_x^B*(a,b) = t(x-a)^2 - t(x-b)^2 \quad \text{if} \quad x \ge (a+b)/2 \quad \text{and}$$
$$p_x^B*(a,b) = 0 \quad \text{if} \quad x \le (a+b)/2$$

Proof. Suppose that x is near to firm A, that is, $x < (a+b)/2$. Consider firm B. First, we show that $p_x^B > 0$ cannot be an equilibrium. When $p_x^B > 0$, the best-reply of firm A

[5] This assumption is common in spatial models, and it is necessary to avoid the technicality of ε-equilibrium concepts when both firms price discriminate. For more details about this assumption, see among the others Hurter and Lederer (1985), Lederer and Hurter (1986), Thisse and Vives (1988), Hamilton *et al.* (1989), Hamilton and Thisse (1992).

consists in setting: $p_x^A = p_x^B + t(x-b)^2 - t(x-a)^2$: the consumer x obtains the same utility and buys from firm A. But firm B has now the incentive to undercut firm A by setting a price equal to: $p_x^{B}{'} = p_x^B - \varepsilon$, where ε is a positive and infinitely small number. Since p_x^B is higher than 0 by hypothesis and ε is a positive and infinitely small number by definition, $p_x^{B}{'}$ is higher than 0. Therefore, $p_x^B > 0$ cannot be an equilibrium, because firm B would obtain higher profits by setting $p_x^{B}{'}$. We show instead that $p_x^B = 0$ is an equilibrium. The best-reply of firm A is: $p_x^A = t(x-b)^2 - t(x-a)^2$. With such a price firm B obtains zero profits from consumer x, which buys from firm A, but it has no incentive to change the price, because increasing the price it would continue to obtain zero profits, and setting a price lower than the marginal costs would entail a loss. It follows that $p_x^A = t(x-b)^2 - t(x-a)^2$ and $p_x^B = 0$ represents the (unique) price equilibrium. The proof for $x > (a+b)/2$ is symmetric to the proof for $x < (a+b)/2$. Finally, when the consumer is at the same distance from the two firms, that is $x = (a+b)/2$, the standard Bertrand's result holds: the unique price equilibrium when two undifferentiated firms compete on price is represented by both firms setting a price equal to the marginal cost. ■

The equilibrium locations in the first stage of the game are defined in the next proposition:

Proposition 3: *in the first stage of the game the unique Nash equilibrium is given by* $a^* = 1/4$ *and* $b^* = 3/4$.

Proof. Using Proposition (2), the firms' profits can be written directly as functions of a and b. Then:

$$\Pi^A(a,b) = t(b-a)(a+b)^2/4 \qquad (1)$$

$$\Pi^B(a,b) = t(b-a)(2-a-b)^2/4 \qquad (2)$$

Maximizing them with respect to *a* and *b* it follows:

$$\partial \Pi^A / \partial a = t(b^2 - 3a^2 - 2ab)/4 = 0 \quad (3)$$

$$\partial \Pi^B / \partial b = t(3b^2 - a^2 + 2ab - 8b + 4)/4 = 0 \quad (4)$$

Consider equation (3) as a function of *b*. This equation has two solutions: $b = 3a$ and $b = -a$. The second solution is impossible, since neither *a* or *b* can be negative, and $a = b = 0$ does not solve equation (4). Therefore it must be: $b = 3a$. Substituting it in equation (4) and solving with respect to *a* we obtain two solutions: $a = 1/4$ and $a = 1/2$. The second solution is impossible, since we have $b = 3a = 3/2 > 1$, which is impossible. Therefore, the only admissible values which solve the system defined by equations (3) and (4) are $a^* = 1/4$ and $b^* = 3/4$. ■

The following proposition compares the location-price equilibrium when perfect price discrimination is possible with the location-price equilibrium under the uniform price regime:

Proposition 4:

a) *All prices are lower under perfect price discrimination than under uniform price. Therefore, profits are lower under perfect price discrimination than under uniform price.*

b) *The surplus of each consumer is higher under perfect price discrimination than under uniform price, and the more the consumer is located near to the middle the higher is the difference.*

c) *The equilibrium locations under perfect price discrimination maximize total welfare*[6].

Proof.

a) Substituting $a = 1/4$ and $b = 3/4$ into the equilibrium discriminatory price schedules, it follows: $p_x^A{}^* = t(1/2 - x), \forall x \in [0,1/2]$ and $p_x^B{}^* = t(x - 1/2)$, $\forall x \in [1/2,1]$.

[6] Lederer and Hurter (1986) obtain the same result assuming *delivered* instead of *f.o.b.* prices.

It follows that $\forall x \in [0,1/2]$ we get $\overline{p}^A * = \overline{p}^B * = t > t(1/2 - x) = p_x^A *$, and $\forall x \in [1/2,1]$ we get $\overline{p}^A * = \overline{p}^B * = t > t(x - 1/2) = p_x^B *$. Under price discrimination total profits are: $\Pi^D = t/4$, while under uniform price they are: $\Pi^U = t$. Then: $\Delta\Pi = \Pi^D - \Pi^U = -3t/4 < 0$.

b) Under price discrimination, the surplus of a consumer located at $x \in [0,1/2]$ is given by: $CS_x^D = v - p_x^A * - t(a^D * - x)^2 = v - t(1/2 - x) - t(1/4 - x)^2$, while the surplus of a consumer located at $x \in [1/2,1]$ is given by: $CS_x^D = v - p_x^B * - t(b^D * - x)^2 =$ $= v - t(x - 1/2) - t(3/4 - x)^2$. Under uniform price, the surplus of a consumer located at $x \in [0,1/2]$ is: $CS_x^U = v - \overline{p}^A * - t(a^U * - x)^2 = v - t - tx^2$, while the surplus of a consumer located at $x \in [1/2,1]$ is: $CS_x^U = v - \overline{p}^B * - t(b^U * - x)^2 = v - t - t(1 - x)^2$. Define: $\Delta CS \equiv CS_x^D - CS_x^U$. It follows that: $\Delta CS = t(x/2 + 15/16) > 0$, $\forall x \in [0,1/2]$, and $\Delta CS = t(-3x/2 + 31/16) > 0, \forall x \in [1/2,1]$. Moreover, $\partial\Delta CS/\partial x > 0 \ \forall x \in [0,1/2]$ and $\partial\Delta CS/\partial x < 0 \ \forall x \in [1/2,1]$.

c) Since the output is the same under the uniform price regime and the discriminatory price regime and the prices have only a redistributive effect, total welfare depends only on transportation costs, which in turn are determined by the equilibrium locations. Define with $\hat{a}$ and $\hat{b}$ the optimal locations from the total welfare point of view. They are simply: $(\hat{a},\hat{b}) = \arg\min CT = \ = \arg\min\left\{\int_0^x (t(z-a)^2 dz + \int_x^1 (t(z-b)^2 dz\right\}$, where the bracketed expression indicates the total transportation costs. The proof has two steps: first we calculate the optimal sharing of consumers, and then we calculate the optimal values of a and b.

$$1)\frac{\partial CT}{\partial x} = \frac{\partial\left\{\int_0^x t(z-a)^2 dz + \int_x^1 t(z-b)^2 dz\right\}}{\partial x} = a^2 - 2ax + 2bx - b^2 = 0 \quad \rightarrow \quad x\text{^} = \frac{a+b}{2}$$

$$2) CT(a,b) = \int_0^{\frac{a+b}{2}} t(x-a)^2 dx + \int_{\frac{a+b}{2}}^1 t(x-b)^2 dx = \frac{t(a^3 - ab^2 - b^3 + a^2b + 4/3 + 4b^2 - 4b)}{4}$$

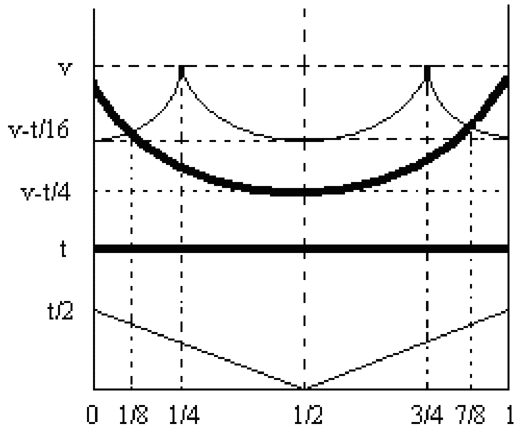
$$\partial CT/\partial a = t(3a^2 - b^2 + 2ab)/4 = 0 \tag{5}$$

$$\partial CT/\partial b = t(-2ab - 3b^2 + a^2 + 8b - 4)/4 = 0 \tag{6}$$

Since equations (5) and (6) coincide respectively with equations (3) and (4), the optimal locations $\hat{a}$ and $\hat{b}$ coincide with the equilibrium locations $a^* = 1/4$ and $b^* = 3/4$. ■

The characteristics of the location-price equilibrium under the two pricing regimes are summarized in the following figure:

Figure 1: Illustration of Proposition 4



The thin and slopped lines in the bottom part of the graph represent the equilibrium prices set by the firms to each consumer under perfect price discrimination, while the bold and flat line represents the equilibrium prices under uniform price. It is immediate to see that all equilibrium discriminatory prices lay below the price line under the uniform price regime (*all-out competition*), and that the discriminatory prices decrease moving from consumers located at the endpoints to consumers located at the middle.

From the consumers' point of view, the surplus depends on the price paid and on the transportation costs sustained. The curves in the upper part of the graph describe the surplus, gross of the price, of each consumer: the bold curve refers to the uniform price regime while the thin curve refers to the discriminatory price regime. Under the discriminatory price regime, the gross consumer surplus is maximum for consumers located at $1/4$ and $3/4$, where the firms are located, and decreases the more the consumers are distant from these points. The minimum gross consumer surplus is at points 0, $1/2$ and 1. Under the uniform price regime the gross consumer surplus is maximum at points 0 and 1 since firms are located at the endpoints of the segment, and it is minimum at $1/2$. The net consumer surplus is given by the difference between the upper curves and the price lines. In Proposition 4 we state that the surplus of each consumer is higher under price discrimination than under the uniform price regime. For consumers located between $1/8$ and $7/8$ this is immediate, since both the transportation costs and the prices decrease passing from the uniform price regime to the discriminatory price regime. For the other consumers we observe two opposite effects: the transportation costs increase under price discrimination (since the firms now are farther from these consumers) but the equilibrium prices decrease. In order to prove that even for these consumers the surplus is higher under the discriminatory price regime than under the uniform price regime it is sufficient to compare the surplus of the most external consumers in the two cases, since the consumers located at point 0 and 1 are the best-positioned consumers under the uniform price regime and the worst-positioned consumers under the discriminatory price regime. Under uniform pricing, the surplus of the consumers located at points 0 and 1 is equal to $v - t$; under perfect price discrimination, the same consumers obtain a surplus which is equal to $v - 9t/16$. Since the surplus of these consumers increases passing from the uniform price regime to the discriminatory price regime, the same must be true for all other consumers.

Finally, in Proposition 4 we state that total welfare is maximized under price discrimination. Since the total output is the same under the uniform price regime and the discriminatory price regime and since prices have only a redistributive function, total welfare depends only on the equilibrium locations which determine the total transportation costs sustained by the consumers: the equilibrium locations under price discrimination, $1/4$ and $3/4$, minimize the total transportation costs and therefore maximize total welfare.

4. A three-stage model

In section 3 we have shown that perfect price discrimination yields lower profits then uniform pricing. Now, suppose that each firm can announce its intention not to price discriminate before setting the price schedule. This commitment can be made credible by the adoption of business practices like the most-favoured nation clause. When firms' locations are fixed, Thisse and Vives (1988) show that the possibility to commit before competing on prices does not alter the fundamental result: uniform pricing does not emerge in equilibrium.

In this section we ask whether this result is still valid when locations are endogenous instead of exogenous. Therefore, we need to move from a two-stage model to a three-stage model. Two timings are possible. Until now we have assumed that the final decision of the firms regards the price schedule to be applied, and we maintain this hypothesis. However, the decision regarding the commitment to uniform pricing may precede the decision on price and come after the decision on location, or it may precede both the decision on price and the decision on location: these two alternatives generate two different timings of the game. In what follows we solve the game in both cases. We show that when the decision on commitment is taken after the decision on location there exists a unique sub-game perfect equilibrium in which both firms price discriminate, while when the decision on commitment is taken before the decision on location there exists a unique sub-game perfect equilibrium in which both firms set a uniform price.

Game 1

Timing: at time 1, both firms simultaneously choose the location along the market; at time 2 both firms simultaneously decide whether to commit (U) or not (D); at time 3 both firms simultaneously choose the price schedule.

We solve the game by backward induction. Consider the third stage of the game. We need to calculate the equilibrium prices when one firm has committed while the other has not committed. Suppose that firm *A* has committed at stage 2 while firm *B* has not committed. Consider a generic consumer *x*. If firm *A* sets a uniform price such that $\overline{p}^A > t(x-b)^2 - t(x-a)^2$, firm *B* can always serve the consumer *x* by undercutting the uniform price set by firm *A* without pricing below the marginal cost: therefore consumer *x* will always buy from firm *B* and firm *A* will obtain zero profits. In order to have a positive demand, firm A must set a uniform price such that: $\overline{p}^{A} \leq t(x-b)^2 - t(x-a)^2$, which cannot be undercut by firm B without setting a price lower than the marginal cost. Therefore, the highest uniform price that firm B cannot undercut is given by: $\overline{p}^{A} = t(x-b)^2 - t(x-a)^2$. Solving for x, we obtain the most at the right consumer served by firm A: $x^{*\prime} = (a+b)/2 - \overline{p}^{A} / 2t(b-a)$. It follows that the demand of firm A is given by $x^{*\prime}$, while the demand of firm B is given by $1 - x^{*\prime}$.

We state the following proposition:

Proposition 5: *if firm A has committed and firm B has not committed, the equilibrium prices in the third stage of the game are the following*[7]*:*

$$\overline{p}^{A} * (a,b) = t(a+b)(b-a)/2$$

$$p_x^B * (a,b) = \begin{cases} -t(a+b)(b-a)/2 + 2tx(b-a) & \textit{if} \quad x \geq (a+b)/2 \\ -t(a+b)(b-a)/2 + 2tx(b-a) - \varepsilon & \textit{if} \quad (a+b)/4 \leq x \leq (a+b)/2 \\ 0 & \textit{if} \quad x \leq (a+b)/4 \end{cases}$$

If firm A has not committed and firm B has committed, the equilibrium prices in the third stage of the game are the following:

$$p_x^A * (a,b) = \begin{cases} t(4+a+b)(b-a)/2 - 2tx(b-a) & \textit{if} \quad x \leq (a+b)/2 \\ t(4+a+b)(b-a)/2 - 2tx(b-a) - \varepsilon & \textit{if} \quad (a+b)/2 \leq x \leq (a+b)/4 \\ 0 & \textit{if} \quad x \geq (a+b)/4 \end{cases}$$

$$\overline{p}^{B} * (a,b) = t(2-a-b)(b-a)/2$$

[7] Unfortunately, ε -equilibrium cannot be avoided for a subset of consumers when one firm sets a uniform price and the other firm can perfectly price discriminate.

Proof. Consider $\Pi^A = \overline{p}^A x^{*\prime} = \overline{p}^A [\frac{(c - \overline{p}^A)}{2t(b-a)} + \frac{a+b}{2}]$. Maximize it with respect to $\overline{p}^A$. It follows: $\overline{p}^A * = t(b^2 - a^2)/2$. The discriminatory firm in turn sets for each consumer the highest possible discriminatory price which allows it to serve the consumer. The demonstration of the second part of Proposition 5 proceeds in the same way. ■

We can write the firms' profits directly as functions of *a* and *b* in the four possible cases: (U,U), (U,D), (D,U) and (D,D)[8]. We do it in the following table:

Table 1

Π^B / Π^A	U	D
U	$t(b-a)(2+a+b)^2/18$; $t(b-a)(4-a-b)^2/18$	$t(b-a)(a+b)^2/8$; $t(b-a)(4-a-b)^2/16$
D	$t(b-a)(2+a+b)^2/16$; $t(b-a)(2-a-b)^2/8$	$t(b-a)(a+b)^2/4$; $t(b-a)(2-a-b)^2/4$

It is immediate to see that, for any couple of locations, the dominant strategy of each firm is D. Given that at the second stage both firms do not commit, in the third stage they price discriminate and the equilibrium locations are given by Proposition 3.

The following proposition summarizes and defines the unique sub-game perfect equilibrium:

Proposition 6: *in game 1, the unique sub-game perfect equilibrium is given by* $a^* = 1/4$ *and* $b^* = 3/4$, *(D,D),* $p_x^A * = t(1/2 - x)$ *and* $p_x^B * = 0$ *for* $x \le 1/2$, *and* $p_x^A * = 0$ *and* $p_x^B * = t(x - 1/2)$ *for* $x \ge 1/2$.

Proof. Consider Table 1. If firm *A* chooses U, then firm *B* chooses D for any *a* and *b*, since $1/16 > 1/18$. When firm *A* chooses D, firm *B* chooses D for any *a* and *b*, since $1/4 > 1/8$. Then, D is the dominant strategy for firm *B*. The same is true for firm *A*. It

[8] The profit functions in (D,D) are simply the functions (1) and (2); the profit functions in (U,D) and (D,U) come from Proposition 5 (disregarding the ε's); the profit functions in (U,U) can be obtained by standard calculations (see, for example, Tirole, 1988, pag. 281).

follows that in the second stage of the game the equilibrium is given by both firms choosing D. The rest of the Proposition follows from Propositions 2 and 3. ■

Proposition 6 shows that the Prisoner Dilemma is present in game 1, since both firms do not commit even if this strategy is conducive to lower equilibrium profits. That is, assuming endogenous choice of the locations before the commitment decision does not alter the Thisse and Vives' (1988) result: firms price discriminate in equilibrium.

Game 2

Timing: at time 1 both firms simultaneously decide whether to commit or not; at time 2 both firms simultaneously choose the location along the market; at time 3 both firms simultaneously choose the price schedule.

As usual, in order to solve the game we start from the last stage. We already have the equilibrium prices and locations when both firms set a uniform price (Proposition 1) and when both price discriminate (Propositions 2 and 3). Moreover, we already know the equilibrium prices when one firm has committed and the other has not (Proposition 5). Therefore, it remains to calculate the equilibrium locations in the sub-game that arises when only one firm has committed in the first stage. Equilibrium locations in this sub-game are defined by the following proposition:

Proposition 7: *if at the first stage firm A has chosen* U *and firm B has chosen* D*, the equilibrium locations at the second stage are given by* $a^* = 1/3$ and $b^* = 1$*; if at the first stage firm A has chosen* D *and firm B has chosen* U*, the equilibrium locations at the second stage are given by* $a^* = 0$ and $b^* = 2/3$.

Proof. Maximize the profit functions in (U,D) of Table 1. It follows: $\partial \Pi^A / \partial a = t(b^2 - 3a^2 - 2ab)/8$ and $\partial \Pi^B / \partial b = t(4 - a - b)(4 + a - 3b)/16$. Consider the latter equation. Since it is always positive, firm B locates at the right extremity of the market: that is, $b = 1$. Substitute it into the first equation and solve. There are two solutions: $a = 1/3$ and $a = -1$. Since the latter solution is impossible, the equilibrium locations are $a^* = 1/3$ and $b^* = 1$. The second part of Proposition is demonstrated in the same way. Maximize the profit functions in (D,U) of Table 1. It follows: $\partial\Pi^A/\partial a = t(2+a+b)(-2-3a+b)/16$ and $\partial\Pi^B/\partial b = t(2-a-b)(2+a-3b)/8$. The first equation is always negative: therefore, firm *A* has always the incentive to move to the left, that is, $a = 0$. Substitute into the second equation and solve. There are two solutions: $b = 2/3$ and $b = 2$. Since the second solution is impossible (*b* cannot be higher than 1) the unique equilibrium locations are $a^* = 0$ and $b^* = 2/3$. ■

Since we have the equilibrium prices (third stage) and the equilibrium locations (second stage) in all possible cases, we can write the equilibrium profits of each firm directly as functions of the decision whether to commit or not taken at the first stage of the game. The equilibrium profits are summarised in the following table:

Table 2

Π^B / Π^A	U	D
U	$t/2\,;t/2$	$4t/27\,;8t/27$
D	$8t/27\,;4t/27$	$t/8\,;t/8$

The next proposition follows directly from Table 2:

Proposition 8: *in game 2, the unique sub-game perfect equilibrium is given by* (U,U), $a^* = 0$ *and* $b^* = 1$, *and* $\overline{p}^A * = \overline{p}^B * = t$.

Perhaps surprisingly, if the location decision is taken once the decisions regarding the commitment have been already taken, there exists a (unique) sub-game perfect equilibrium in which both firms commit. On the contrary, when the decision whether to commit or not is taken after the location decision, the equilibrium is characterized by price discrimination by both firms (and, consequently, by lower profits). However, if one takes into account the different forces working in the two games, such a result has an intuitive explanation. The main difference between a commitment strategy and a non-commitment strategy is that the former reduces the flexibility of a firm in setting prices: when a firm has committed, it can choose its price schedule only from a subset of the complete price schedules set (namely, the subset composed by the uniform price schedules). Therefore, there is no reason for a firm to choose to commit *if the only consequence of the commitment is to reduce its own flexibility in setting prices.* This is exactly what happens in game 1. The decision whether to commit or not affects only the decision regarding the price(s) to be set. When each firm announces that in the future it will not price discriminate, the locations have been already fixed, and therefore they cannot be modified by any commitment decision. The only consequence for a firm that decides to commit is to reduce its own ability to undercut the rival for each consumer: it is obvious that no firm would find it convenient, and the dominant strategy for each firm necessarily is no-commitment. This is what has been obtained also by Thisse and Vives (1988) in their two-stage model with exogenous locations: *"there is a robust tendency for a firm to choose the discriminatory policy since it is more flexible and does better against any generic strategy of the rival, although…firms may end up worse off than if they choose to price uniformly"* (pag. 134).

Why this does not occur in game 2? The timing is different: in this case the location is set after the decision regarding the commitment. This implies that the decision regarding whether to commit or not has an impact on the locations chosen by the firms. The locations of the firms in turn determine the equilibrium prices and the equilibrium profits. The more the firms are differentiated, the higher is their market power, and, consequently, the higher is the equilibrium price. It is precisely the impact of the commitment decision on the equilibrium degree of differentiation that makes the commitment decision more profitable for each firm, even if it reduces the flexibility in setting prices. Each firm anticipates that its own commitment not to price discriminate induces an higher equilibrium degree of differentiation in any case: when the rival chooses to commit, deciding to commit too allows to obtain the maximum degree of differentiation; if the rival chooses not to commit, deciding to commit allows to obtain an higher degree of differentiation than in the situation in which both firms do not commit[9]. Inducing higher differentiation is profitable for both firms, even if such higher

[9] When both firms commit, the equilibrium distance between the firms is 1 (Proposition 1); if one firm commits not to price discriminate while the other does not, the equilibrium distance is $2/3$ (Proposition 7); when both firms do not commit, the equilibrium distance is $1/2$ (Proposition 3).

differentiation is obtained at the cost of losing the flexibility in setting prices guaranteed by the no-commitment strategy. In this model the benefits from the higher differentiation outweigh the costs from the reduced flexibility, and therefore *commitment is convenient for each firm* and for any possible decision by the rival. That is, the dominant strategy of each firm is committing to uniform pricing. This in turn induces a uniform price equilibrium, which is characterized by higher profits.

Summing up, in game 2 the decision whether to commit or not determines the equilibrium differentiation between the firms. Taking no commitment induces lower differentiation, which in turn damages both firms through lower profits. Anticipating this fact, each firm has the incentive to commit in the first stage of the game, and the equilibrium is characterized by no discrimination. On the contrary, when the commitment is decided after the location stage, the degree of differentiation between the firms is given. Therefore, the incentive to commit disappears, while is still present the incentive to not commit, linked to the competition on prices at the last stage of the game: no commitment and price discrimination by both firms follow.

5. Conclusion

Using the Hotelling's model (1929) with endogenous locations, we study the location-price equilibrium when firms can perfectly price discriminate. If firms cannot commit not to price discriminate before competing on price, price discrimination emerges as the unique sub-game perfect equilibrium and firms locate respectively at 1/4 and 3/4 (Propositions 2 and 3). Equilibrium first-degree discriminatory prices are all lower than the equilibrium uniform price of a two-stage location-price game where price discrimination is impossible (Proposition 4). If firms can commit not to price discriminate before competing on price but after locating in the market, the unique equilibrium is characterized by price discrimination (Proposition 6). On the contrary, if firms can commit not to price discriminate before competing on price and before locating in the market, the unique equilibrium is characterized by uniform pricing (Proposition 8).

Appendix

In this appendix we extend the model in order to analyse third-degree price discrimination. Using the framework developed by Liu and Serfes (2004), we show that uniform pricing is the unique sub-game perfect equilibrium when the commitment decision is taken before the location decision even if price discrimination is not perfect. Other results regard equilibrium locations, firms' profits, consumer surplus and total welfare: when firms can imperfectly price discriminate, they locate very closely to the locations that maximize total welfare; firms are damaged by the possibility to price discriminate; imperfect price discrimination unambiguously increases consumer surplus and total welfare.

A.1 The model

As in Liu and Serfes (2004), we suppose that there is an information technology which allows firms to partition the consumers into different groups. We assume that the technology partitions the linear market into *n* sub-segments indexed by *m*, with $m = 1,...,n$. Each sub-segment is of equal length, $1/n$. It follows that sub-segment *m* can be expressed as the interval $\left[\frac{m-1}{n};\frac{m}{n}\right]$. A firm can price discriminate between consumers belonging to different sub-segments, but not between the consumers belonging to the same sub-segment. The cost of using the information technology is zero. Define with p_m^J the price set by firm *J*=*A*,*B* on consumers belonging to sub-segment *m*. Clearly, when firm *J* cannot price discriminate, it must be $p_m^J = p_{m'}^J, \ \forall \ m, m'$. Finally, assume that $n = 2^k$, with $k = 1,2,3,4...$ Therefore, *n* measures the precision of consumer information: an higher *n* means an higher information precision. At the limit,

$n \to \infty$ implies perfect price discrimination, since firms are able to distinguish between each consumer[10].

Differently from Liu and Serfes (2004), we do not assume that firms are exogenously located at the endpoints of the market: as in the previous part of the paper, we are interested in the location-price equilibria, so we allow for endogenous locations of the firms.

A2. Imperfect discriminatory prices

This section extends the analysis developed in section 3 of this paper to the case of third degree price discrimination. A two-stage game is supposed: the firms first decide where to locate and then compete on prices. Here we assume that both firms can (imperfectly) price discriminate. The utility of the consumer *x* belonging to sub-segment *m* when he buys from firm *A* is therefore: $u_x = v - p_m^A - t(x-a)^2$, while his utility when he buys from firm *B* is given by: $u_x = v - p_m^B - t(x-b)^2$. Consider segment *m*. Define x_m* as the consumer on segment *m* which is indifferent between buying from firm *A* or from firm *B* for a given couple of locations, *a* and *b*, and for a given couple of discriminatory prices, p_m^A and p_m^B. Equating the utility in the two cases and solving for *x* it follows:

$$x_m* = \frac{a+b}{2} + \frac{p_m^B - p_m^A}{2t(b-a)}$$

Therefore, the demands of firm *A* and firm *B* on sub-segment *m* are respectively:

$$d_m^A = \frac{a+b}{2} + \frac{p_m^B - p_m^A}{2t(b-a)} - \frac{m-1}{n} \tag{7}$$

[10] For an extensive discussion of the pros and cons of the partition technology assumed in the model, see Liu and Serfes (2004).

$$d_m^B = \frac{m}{n} - \frac{a+b}{2} - \frac{p_m^B - p_m^A}{2t(b-a)} \tag{8}$$

It follows that the profits of firm *A* and firm *B* on sub-segment *m* are respectively:

$$\Pi_m^A = p_m^A d_m^A = p_m^A [\frac{a+b}{2} + \frac{p_m^B - p_m^A}{2t(b-a)} - \frac{m-1}{n}]$$

$$\Pi_m^B = p_m^B d_m^B = p_m^B [\frac{m}{n} - \frac{a+b}{2} - \frac{p_m^B - p_m^A}{2t(b-a)}]$$

The next proposition, which follows very closely Proposition 1 in Liu and Serfes (2004), defines the equilibrium price schedules for any couple of locations:

Proposition 9: *Define* $m_A \equiv \frac{n(a+b)}{2} - 1$ *and* $m_B \equiv \frac{n(a+b)}{2} + 2$. *Then:*

- $m_A < m < m_B$

$$p_m^A * = \frac{t(b-a)}{3}(\frac{4-2m}{n} + a + b), \quad d_m^A * = \frac{a+b}{6} + \frac{2-m}{3n}$$

$$p_m^B * = \frac{t(b-a)}{3}(\frac{2+2m}{n} - a - b), \quad d_m^B * = \frac{m+1}{3n} - \frac{a+b}{6}$$

- $m \leq m_A < m_B$

$$p_m^A * = t(b-a)(a+b-2\frac{m}{n}), \quad d_m^A * = \frac{1}{n}$$

$$p_m^B * = 0, \quad d_m^B * = 0$$

- $m_A < m_B \leq m$

$$p_m^A * = 0, \quad d_m^A * = 0$$

$$p_m^B* = t(b-a)(\frac{2m-2}{n} - a - b)\,,\; d_m^B* = \frac{1}{n}$$

Proof. We refer directly to the proof provided by Liu and Serfes (2004) for their Proposition 1. The only difference is that firm *A* is located at *a* instead of 0, and firm *B* is located at *b* instead of 1. ■

Using Proposition 9, the firms' profits can be written directly as functions of *a* and *b* (the subscript indicates that both firms are price discriminating). They are:

$$\Pi_{DD}^A* = \sum_{m=1}^{m=m_A} \frac{t(b-a)}{n}(a+b-2\frac{m}{n}) + \sum_{m=m_A+1}^{m=m_B-1} \frac{t(b-a)}{3}(\frac{4-2m}{n} + a + b)(\frac{a+b}{6} + \frac{2-m}{3n}) =$$

$$= \frac{t(b-a)[9n^2(a+b)^2 - 18n(a+b) + 40]}{36n^2}$$

$$\Pi_{DD}^B* = \sum_{m=m_A+1}^{m=m_B-1} \frac{t(b-a)}{3}(\frac{2+2m}{n} - a - b)(\frac{m+1}{3n} - \frac{a+b}{6}) + \sum_{m=m_B}^{n} \frac{t(b-a)}{n}(\frac{2m-2}{n} - a - b) =$$

$$= \frac{t(b-a)[9n^2(2-a-b)^2 - 18n(2-a-b) + 40]}{36n^2}$$

The equilibrium locations in the first stage of the game are defined in the next proposition:

Proposition 10: *in the first stage of the game the unique Nash equilibrium is given by:*

- $a* = 0$ and $b* = 1$, *if* $n = 2$
- $a* = \frac{9n^2 - 40}{36n^2 - 36n}$ and $b* = 1 - a*$, *if* $n \geq 4$

Proof. The equilibrium locations come from the solution of $\begin{cases} \partial \Pi_{DD}^{A*} / \partial a = 0 \\ \partial \Pi_{DD}^{B*} / \partial b = 0 \end{cases}$. Note that when $n = 2$, $\partial \Pi^{A} / \partial a$ is always negative and $\partial \Pi^{B} / \partial b$ is always positive: the corner solution follows. For $n \geq 4$ an interior solution exists. ■

The following figure illustrates the equilibrium location of firm *A* (firm *B* is symmetric) for $n \geq 4$[11]. Firm *A* locates just below 1/4 when the market is partitioned in 4 sub-segments; it locates just above 1/4 when the market is partitioned in 8 sub-segments and afterwards the equilibrium location decreases monotonically with *n* and converges to 1/4 when $n \to \infty$. Maximal differentiation emerges in equilibrium just for a very low-quality information technology ($n = 2$): in this case the firms locate as in the absence of price discrimination.

Figure 2: Illustration of Proposition 10

The following propositions compare the location-price equilibrium when price discrimination is possible with the location-price equilibrium under the uniform price regime (section 2).

[11] Clearly, *n* does not take all values, but only 4,8,16,32,64… In order to better illustrate the pattern of the equilibrium locations as *n* increases we draw a continuous line.

Proposition 11. *All equilibrium prices are lower under imperfect price discrimination than under uniform price. Therefore, profits are lower under imperfect price discrimination than under uniform price.*

Proof. Consider first the case with $n \geq 4$. Look at firm *A*. By Proposition 10, we have that: $a^* + b^* = 1$. Therefore, the equilibrium prices (Proposition 9) can be written as follows:

$$p_m^A{}^* = \begin{cases} \dfrac{t(1-2a^*)}{3}\left(\dfrac{4-2m}{n}+1\right) & if \quad m_A < m < m_B \\ t(1-2a^*)\left(1-2\dfrac{m}{n}\right) & if \quad m \leq m_A < m_B \end{cases},$$

First, note that the equilibrium price for $m \leq m_A < m_B$ is always larger than the equilibrium price for $m_A < m < m_B$. In fact, the lower equilibrium price for $m \leq m_A < m_B$ occurs when $m = m_A$, while the higher equilibrium price for $m_A < m < m_B$ occurs for $m = m_A + 1$. Substituting a^* and b^* into m_A, and then substituting m_A into the equilibrium price, one obtains that: $p^A_{m_A(a^*,b^*)}{}^* = \dfrac{2t(1-2a^*)}{n} > \dfrac{4t(1-2a^*)}{3n} = p^A_{m_A(a^*,b^*)+1}{}^*$.

Therefore, the comparison between the uniform equilibrium price, $\overline{p}^A{}^* = t$, and the discriminatory prices can be limited to the comparison between $\overline{p}^A{}^*$ and the highest discriminatory price for $m \leq m_A < m_B$. Since $p_m^A{}^*$ is decreasing in *m*, the highest discriminatory price occurs when $m = 1$. Substituting $m = 1$ into $p_m^A{}^*$, it follows $t(1-2a^*)(1-\dfrac{2}{n})$, which is always lower than *t* given that both terms in the round brackets are lower than 1.

Consider now the case with $n = 2$. In this case, both firms have a positive demand in both sub-segments. Consider firm *A*. Its equilibrium prices in sub-segment 1 and sub-segment 2 are respectively: $p_1^A{}^* = \dfrac{2}{3}t$ and $p_2^A{}^* = \dfrac{t}{3}$. Clearly: $\overline{p}^A{}^* > p_1^A{}^* > p_2^A{}^*$.

Therefore, *all-out competition* occurs for any *n*. Since the output is constant, equilibrium profits under price discrimination are necessarily lower than equilibrium profits under uniform price for any *n*. ■

Proposition 12: *Consumer surplus and total welfare are higher under imperfect price discrimination than under uniform price.*

Proof. Note that the consumer surplus and the total welfare can be written respectively as:

$CS = v - \Pi^{T} - TC$

$W = \Pi^{T} + CS = \Pi^{T} + v - \Pi^{T} - TC = v - TC$

where $\Pi^{T} \equiv \Pi^{A}_{DD} + \Pi^{B}_{DD}$ is the sum of the profits of each firm, and *TC* are the transportation costs. From Proposition 11 we know that profits are lower under price discrimination than under uniform price. Moreover, transportation costs are lower under price discrimination than under uniform price (apart from the case of $n = 2$, where transportation costs are the same than under the uniform price regime), since firms locate near to the socially optimal positions, 1/4 and 3/4. It follows that both consumer surplus and total welfare increase passing from the uniform price regime to the discriminatory price regime. ■

A3. The three-stage model

Suppose now the following three-stage model (see *Game* 2 in section 4)[12], in which firms commit not to price discriminate before choosing the location. The timing of the game is the following: at the first stage of the game the firms simultaneously decide whether to commit (U) or not (D); at the second stage of the game the firms simultaneously choose where to locate in the market; at the third stage of the game the firms simultaneously set the price schedules. We solve the game by backward

[12] The correspondent third-degree version of *Game* 1 in section 4 cannot be solved by backward induction. However it is possible to show that, *for any possible couple of exogenous locations*, it never occurs that both firms choose to commit not to discriminate, apart from the case of a very low-quality information technology ($n = 2$ and $n = 4$).

induction. At the third stage firms compete on prices, given the locations and the commitment decision. We need to calculate the equilibrium prices when one firm has committed and the other has not committed. The following proposition defines the equilibrium prices in such case:

Proposition 13: *if firm A has committed and firm B has not committed, the equilibrium prices in the third stage of the game are the following*:

$$\bar{p}^{A}* = \frac{t(b-a)(a+b)}{2} + \frac{t(b-a)}{2n}$$

$$p_m^B* = \begin{cases} \dfrac{t(b-a)}{n} & if \quad m = m^\wedge - 1 \\ \dfrac{t(b-a)}{2}\left(\dfrac{4m-3}{n} - a - b\right) & if \quad m \geq m^\wedge \end{cases}$$

where $m^\wedge = \dfrac{n(a+b)+7}{4}$

If firm A has not committed and firm B has committed, the equilibrium prices in the third stage of the game are the following:

$$p_m^A* = \begin{cases} \dfrac{t(b-a)}{n} & if \quad m = m^\circ + 1 \\ \dfrac{t(b-a)}{2}\left(2 + a + b + \dfrac{1-4m}{n}\right) & if \quad m \leq m^\circ \end{cases}$$

$$\bar{p}^{B}* = \frac{t(b-a)(2-a-b)}{2} + \frac{t(b-a)}{2n}$$

where $m^\circ = \dfrac{n(2+a+b)-3}{4}$

Proof. Suppose firm *A* has committed while firm *B* has not committed. Consider sub-segment *m*. The demand of firm *B* is:

$$d_m^B = \frac{m}{n} - \frac{a+b}{2} - \frac{p_m^B - \overline{p}^A}{2t(b-a)} \tag{9}$$

The profits obtained by firm *B* from sub-segment *m* are therefore:

$$\Pi_m^B = p_m^B\left[\frac{m}{n} - \frac{a+b}{2} - \frac{p_m^B - \overline{p}^A}{2t(b-a)}\right] \tag{10}$$

Maximizing equation (10) with respect to p_m^B, we obtain the optimal discriminatory price set in sub-segment *m* given the price set by the non-discriminating firm and given the locations of the firms. We get:

$$p_m^B = t(b-a)\left(\frac{m}{n} - \frac{a+b}{2}\right) + \frac{\overline{p}^A}{2} \tag{11}$$

Inserting equation (11) into equation (7), we obtain the demand of firm *A* in each segment:

$$d_A^m = \frac{a+b}{4} - \frac{m}{2n} - \frac{\overline{p}_A}{4t(b-a)} + \frac{1}{n} \tag{12}$$

The demand of firm *A* is zero in the most at the right sub-segments. More precisely:

$$d_A^m \leq 0 \;\; \rightarrow \;\; m \geq m\hat{} \equiv n\left[\frac{a+b}{2} - \frac{\overline{p}^A}{2t(b-a)} + \frac{2}{n}\right]$$

The demand of firm *A* is $1/n$ in the most at the left sub-segments. More precisely:

$$d_A^m \geq 1/n \;\; \rightarrow \;\; m \leq m\hat{}\hat{} \equiv n\left[\frac{a+b}{2} - \frac{\overline{p}^A}{2t(b-a)}\right]$$

Since $\hat{m} - \hat{\hat{m}} = 2$, it follows that only in sub-segment $\hat{m} - 1$ both firms have a positive demand. The profits of firm *A* are therefore defined in the following equation:

$$\Pi^A_{UD} = \overline{p}^A \sum_{m=1}^{m=\hat{\hat{m}}} \frac{1}{n} + \overline{p}^A [\frac{a+b}{4} - \frac{\hat{m}-1}{2n} - \frac{\overline{p}^A}{4t(b-a)} + \frac{1}{n}] = \overline{p}^A [\frac{a+b}{2} - \frac{\overline{p}^A}{2t(b-a)} + \frac{1}{2n}] \quad (13)$$

Maximizing equation (13) with respect to $\overline{p}^A$ we obtain the optimal uniform price set by the non-discriminating firm:

$$\overline{p}^A * = \frac{t(b-a)(a+b)}{2} + \frac{t(b-a)}{2n} \quad (14)$$

Inserting equation (14) in equation (11), and substituting *m* with $\hat{m} - 1$ (in which we insert equation (14) again), we obtain the optimal discriminatory price in the only sub-segment in which both firms sell a positive amount. That is:

$$p^B_{\hat{m}-1} = \frac{t(b-a)}{n} \quad (15)$$

The demand of firm *B* in sub-segment $\hat{m} - 1$ is obtained inserting equation (15) in equation (9). It follows:

$$d^B_{\hat{m}-1} = \frac{1}{2n}$$

The optimal discriminatory prices in sub-segments $m \geq \hat{m}$ are obtained by solving: $d^B_m(\overline{p}^A *) = \frac{1}{n}$. It follows:

$$p^B_m * = \frac{t(b-a)}{2}(\frac{4m-3}{n} - a - b).$$

The proof of the second part of the proposition proceeds in the same way, and therefore it is omitted. ■

By Proposition 13, the profits of the two firms when one sets a uniform price while the other discriminates follow immediately.

Corollary of Proposition 13: *if firm A has committed and firm B has not committed the equilibrium profits are:*

$$\Pi_{UD}^{A}* = \frac{t(b-a)}{8n^2}[n^2(a+b)^2 + 2n(a+b)+1]$$

$$\Pi_{UD}^{B}* = \frac{t(b-a)}{16n^2}[n^2(4-a-b)^2 - 2n(4-a-b)+5]$$

If firm A has not committed and firm B has committed the equilibrium profits are:

$$\Pi_{DU}^{A}* = \frac{t(b-a)}{16n^2}[n^2(2+a+b)^2 - 2n(2+a+b)+5]$$

$$\Pi_{DU}^{B}* = \frac{t(b-a)}{8n^2}[n^2(2-a-b)^2 + 2n(2-a-b)+1]$$

Now we move to the second stage of the game. We already calculated the equilibrium locations when both firms set a uniform price (Proposition 1) and when both firms set discriminatory prices (Proposition 10). It remains to calculate the equilibrium locations when one firm has committed and the other has not committed. The following Proposition defines the equilibrium locations:

Proposition 14: *if firm A has chosen* U *and firm B has chosen* D*, the equilibrium locations at the second stage of the game are given by* $a* = \frac{1}{3} - \frac{1}{3n}$ and $b* = 1$.

If firm A has chosen D *and firm B has chosen* U*, the equilibrium locations at the second stage of the game are given by* $a^* = 0$ and $b^* = \frac{2}{3} + \frac{1}{3n}$.

Proof. Suppose that firm *A* has chosen U and firm *B* has chosen D in the first stage of the game. Then, take the derivative of Π_{UD}^{B*} with respect to *b*. It results: $\partial \Pi_{UD}^{B*}/\partial b = t[5 + 4n(b-2) + n^2(16 - 16b - a^2 + 2ab + 3b^2)]/16n^2$, which is always positive. Therefore the discriminating firm, *B*, locates at the right endpoint of the market. Substituting $b^* = 1$ into Π_{UD}^{A*} and maximizing it with respect to *a* gives: $a^* = 1/3 - 1/3n$, which completes the proof. The second part of the proposition can be proved in the same way. ■

Since we have the equilibrium prices (third stage) and the equilibrium locations (second stage) in all possible cases, we can write the equilibrium profits of each firm directly as functions of the decision whether to commit or not taken at the first stage of the game. The equilibrium profits are summarised in the following table[13]:

Table 3

Π^B / Π^A	U	D
U	$\frac{t}{2}; \frac{t}{2}$	$\frac{t(1+2n)^3}{54n^3}; \frac{t(1+2n)(10-9n+16n^2)}{108n^3}$
D	$\frac{t(1+2n)(10-9n+16n^2)}{108n^3}; \frac{t(1+2n)^3}{54n^3}$	$\frac{t(9n^2-18n+40)^2}{648n^3(n-1)}; \frac{t(9n^2-18n+40)^2}{648n^3(n-1)}$

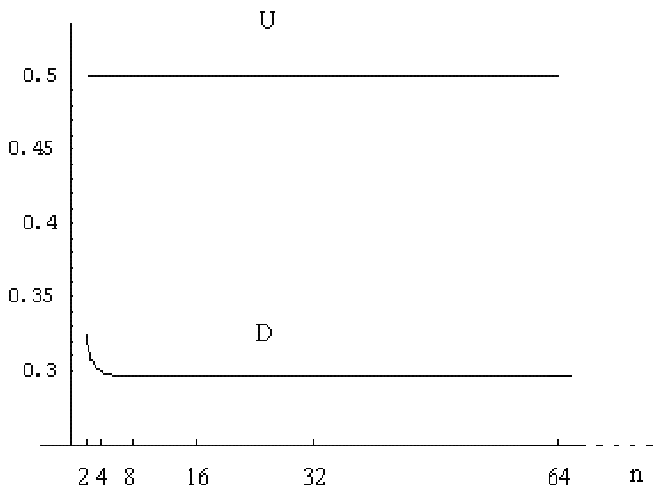
We state the following proposition:

Proposition 15: *the (unique) sub-game perfect Nash equilibrium entails uniform pricing by both firms.*

[13] In Table 3 we only consider $n \geq 4$. If *n*=2, firms maximally differentiate even when both price discriminate (Proposition 10), and equilibrium profits in DD are 0.27*t*.

Proof. Suppose that firm *A* chooses U. The pattern of the profits of firm *B* as function of *n* when it chooses U and when it chooses D is described in the following picture:

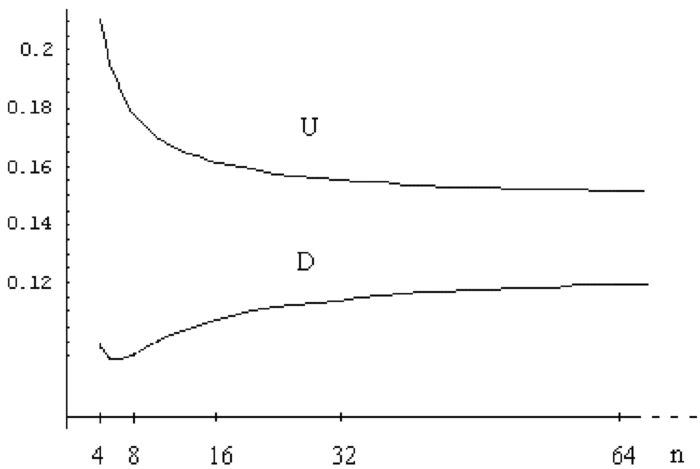
Figure 3



Then, firm *B* always chooses U when firm *A* chooses U.

Suppose now that firm *A* chooses D. The pattern of the profits of firm *B* as function of *n* (with $n \geq 4$) when it chooses U and when it chooses D is described in the following picture:

Figure 4



It is immediate to see that firm B always prefers to commit when firm A chooses D. When n=2, direct calculations show that firm B obtains profits equal to $0.29t$ by choosing U and profits equal to $0.27t$ by choosing D (see note 13). Therefore the dominant strategy of firm A is U. The same reasoning is valid for firm B, and this completes the proof. ■

Therefore, when the commitment decision occurs before the location decision, discriminatory pricing does not arise in equilibrium, independently on the numbers of partitions of the consumers.

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