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DECENTRALISATION VS FISCAL FEDERALISM IN THE PRESENCE OF IMPURE PUBLIC GOODS

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Abstract

The traditional theory for fiscal federalism assumes that the lower tier is more efficient in producing local public goods because of information asymmetry, while on the finance side Central Government might be more efficient in raising resources that can be redistributed through grants-in-aid. This scheme does not take into account that services produced at local level are usually impure public goods. The model developed in this paper allows to derive grants-in aid distribution formulae in this environment and a set of rules that allows to establish when fiscal federalism is a superior alternative to decentralisation.

Keywords: Decentralisation, fiscal federalism, impure public goods **J.E.L. n**. H71, H72

1 Introduction

Policy implementation often requires delegated choices in which either a government agency (acting as an agent), or an autonomous government level is charged with the responsibility of supplying a specific service. The relationships between the agents are different, but their common denominator is that Central Government's problem can be defined in terms of finding the best trade off between autonomy and control. The traditional literature on fiscal federalism¹ suggests that the allocation of functions to local government should follow efficiency principles. The choice of the quantity to be produced should be left to the tier which is better informed on local preferences, while grants might be used for equity and efficiency reasons. A second generation models² suggest that the success of fiscal federalism depends on the information the agents posess about: a) specific parameters (Levaggi and Smith, 1992; Levaggi, 2006; Akay and Mikami, 2006; Snoddon and Wen, 2003); b) the behaviour of other agents (Petretto, 2000) and the effects of their decisions on total welfare (Wildasin, 2001; Crivelli and Staal, 2006). The first issue has been widely studied in the literature and suggests a trade off between autonomy and control: the local level is better informed than the centre on the relevant parameters that affect welfare and it can use its information strategically. Central Government should then balance the improvement in welfare with the cost deriving from asymmetry of information. The last two issues are related since the need for coordination often arises from the presence of spillovers (Besley and Coate, 1997; Wildasin and Ogawa, 2007)

However sophisticated in its modelling approach, most of this literature assumes that the good is a local public good which may produce spillovers on the other local authorites. In actual fact most of the services produced at local level are either impure public goods or merit goods. The former, having the double nature of private and public goods, enter the utility function of each individual twice: as a private good for the quantity actually bought and as public good for the entire amount produced; the latter are private goods whose consumption is financed by the Government for equity/redistribution reasons.

In this article we want to study decentralisation versus fiscal federalism in a context where the good to be produced is an impure public good and information is incomplete. We suggest that in this case fiscal federalism is

¹. See Oates (1972) and King (1984). for a more detailed review see Oates (2005).

²See Oates (2005) for a review.

a more complicated game than what traditional theory suggests and that decentralisation or a limited form of fiscal federalism might be a better option. The organisation of the paper will be as follows: in the following section the main features of the model are presented; in section three the decentralised solution is analyzed while in section four fiscal federalism are introduced; and finally in section five the conclusions are drawn.

2 The model

A benevolent decision-maker (Central Government in our case) has to finance the provision of an impure public good, using a linear income tax. The community is made of S individuals, normalised to one. Each individual has an exogenous money income, M^i in the range $(\underline{M}, \overline{M})$ and is indexed by a taste parameter α in the range $(0, \beta)$. The distribution of income and the taste parameter is uniform and the two distributions are independent. To concentrate on the problems deriving from asymmetry of information, we assume that the individuals live in 2 separate local authorities, equal in everything but the preferences for the impure public good. Income is used to buy private commodities and one or zero unit of an impure public good y whose price is equal to p. Such good has a different level of utility θ ; the latter is equal to one if it is produced by Central Government or through one of its agencies ($\theta_C = 1$) and it is greater than one when it is produced by an autonomous lower government tier ($\theta_L = \theta > 1$). The utility function for a representative individual is written as:

$$U(M-p;\alpha;\phi(Q)) = M + \max(\alpha\theta_i - p; 0) + \phi_i(\theta_i Q_i + k\theta_i Q_{-i}) \qquad i = C, L$$
(1)

The representative individual receives utility from the quantity of impure public goods that is collectively bought, but it does not perceive such utility when he decides to buy it. His decision will simply depend on the sign of $\alpha \theta_i - p$ while the correct decision should also take account of the positive externality that such consumption creates to the consumer and to the community.

The nature of the impure public good is captured by $\phi_i(.)$ where Q is the total quantity of good produced and $0 \le k \le 1$. The latter parameter allows to differentiate the utility generated by the public good according to where it is produced. In particular, if k = 1, the good is an national impure public good; for k = 0 we have a local impure public good and 0 < k < 1 we have a local impure public good with spillovers. To introduce fiscal federalism we assume that preferences for the impure public good are linear and homogeneous within each local authority, but are specific to each of them, i.e. $\phi_i(.) = z_i(\theta_i Q_i + k\theta_i Q_{-i})$ i = 1, 2

In this environment, the decision maker has to internalise the externality caused by the consumption of y through a subsidy that is financed using a linear income tax. To concentrate on the problems deriving from coordination and asymmetry of information, we assume that M > p, i.e. the income of the individual at the low end of the distribution is greater than the price that should be paid to get the impure public good. In this environment, given that the good produced at the lower tier level is more productive in terms of utility, Central Government should delegate any decision to the lower level and to induce the latter to maximise total welfare through the use of grants-in-aid. Such a process is complicated in this environmen because the lower tier is better informed than Central Governments on local needs and because there exists a coordination problem between the decisions of the different tiers. Such problem is well known in the literature (Petretto, 2000). For this reason, fiscal federalism may not be an optimal solution in this context. To show the effects of coordination and asymmetry of information, we will first present a model of simple decentralisation.

3 Decentralisation

Let us examine the case in which Central Government delegates the production to a government agency. The solution will be used as a benchmark to evaluate the benefits of fiscal federalism. We assume that the agency is a perfect agent for the Central Government or that alternatively it has a full control of the latter. In this context it is also assumed that, since the good is sold in each region by a different agency, the subsidy can be tailor made to the region. However, the Government agency can can only produce the good with lower productivity ($\theta = 1$). The decentralisation process is assumed to follow this process: 1) the agency observes z_i ; 2) it informs Central Government which will then set the grant, tailor made to each region; 3) the agency produces the good and sells it³.

Central government has to set the matching grant that maximises welfare. We then assume that a fraction $(1 - \rho_i)p$ of the price of the impure public good is financed through a linear income tax. Given that all the local authorities are equal Central Government problem consists of finding the

³Alternatively we can think that the agency susbidize the production of that good.

matching rate that maximise total welfare. The problem can be written as:

$$Max_{\rho_{i}} W = \sum_{i=1}^{2} \left(\int_{\underline{M}}^{\overline{M}} \left(\begin{array}{c} M\left(1-t\right) + \int_{\frac{\rho_{i}p}{\theta}}^{\beta} \left(\alpha-\rho p\right) \frac{1}{\beta} d\alpha + \\ z_{i}\theta\left(\int_{\frac{\rho_{i}p}{\theta}}^{\beta} \frac{1}{2\beta} d\alpha + k \int_{\frac{\rho_{i}p}{\theta}}^{\beta} \frac{1}{2\beta} d\alpha \right) \end{array} \right) \frac{1}{2\left(\overline{M}-\underline{M}\right)} dM \right)$$

$$j = 1, 2 \quad i \neq j$$

$$s.t.$$

$$t = \frac{\sum_{i=1}^{2} \left(\left(1-\rho_{i}\right) \int_{\frac{\rho_{i}p}{\theta}}^{\beta} \frac{1}{2} d\alpha \right) }{\int_{\underline{M}}^{\overline{M}} \frac{M}{\left(\overline{M}-\underline{M}\right)} dM}$$

$$(2)$$

The maximisation under the assumption that Central Government may subsidize the more productive good allows to derive the optimal matching rate for decentralisation and a first best solution. The optimal matching rate is derived in appendix one and presented in table 1 where we compare the results for decentralisation with the First Best (FB):

First best Decentralisation
$$(\theta = 1)$$

 $(1 - \rho_i^*) = \frac{z_i\theta}{2p} + \frac{k\theta z_j}{2p}$ $(1 - \rho_i^d) = \frac{z_i}{2p} + \frac{kz_j}{2p}$
 $y = \beta - \frac{p}{\theta} + \frac{1}{4}(1+k)\sum_{i=1}^2 z_i$ $y = \beta - p + \frac{1}{4}(1+k)\sum_{i=1}^2 z_i$
 $i = 1, 2; \quad j = 1, 2 \quad i \neq j$

Table 1: The optimal grant: decentralised solution vs First Best

Substituting such result in the welfare function we can obtain the level achieved with decentralisation and compare it with a First Best allocation:

$$\begin{aligned} \mathbf{First \ Best} \\ & \frac{1}{2} \left(\overline{M} - \underline{M} \right) + \frac{1}{2} \beta \theta + \frac{1}{4} \left(z_A + z_B \right) (1+k) \theta - p \\ & + \frac{1}{2\beta} \left(\frac{1}{8} \left((z_A + z_B k)^2 + (z_B + z_A k)^2 \right) \theta - \frac{1}{2} p \left(z_A + z_B \right) (1+k) + \frac{p^2}{\theta} \right) \\ & \mathbf{Decentralisation} \\ & \frac{1}{2} \left(\overline{M} - \underline{M} \right) + \frac{1}{2} \beta + \frac{1}{4} \left(z_A + z_B \right) (1+k) - p \\ & + \frac{1}{2\beta} \left(\frac{1}{8} \left((z_A + z_B k)^2 + (z_B + z_A k)^2 \right) - \frac{1}{2} p \left(z_A + z_B \right) (1+k) + p^2 \right) \end{aligned}$$

The decentralised solution is not efficient because it does not allow to reach the same level of welfare as in the first best. For this reason, Central Government may want to devolve the production of good y to local governments. In this decision Central Government faces a trade off between an increase in utility derived from making available a good that produces more utility and the cost arising from mistakes in anticipating local governments' reaction to Central Government policies.

4 Fiscal federalism

If the good produced at local level was a public good, the solution obtained through decentralisation would always be improved upon by fiscal federalism, i.e. by devolving the production to a local government level. In this case, in fact with the same resources it would be possible to produce a good that accrues utility by θ . In this environment where the good produced is an impure public good, fiscal federalism might not always be welfare improving. This is due to two main factors:

- coordination and spillover effects. In maximising its welfare function each local government has to foresee the behaviour of the other local authorities and the impact that its choices will have on welfare;
- asymmetry of information on local preferences which means that the centre cannot determine neither the optimal quantity of impure public good to be sold locally nor the reaction function of the local government.

To show the effects of fiscal illusion we start by assuming that information is symmetric, i.e. Central Government observes preferences for each single local government.

True fiscal federalism is made operation if Central Government allows the local authority to choose its expenditure and tax revenue. For this reason we assume that Central Government uses the tax revenue to finance the provision of good y supplied by a local government that receives resources in the form of grants-in-aid. The grant can either be a lump sum G_i or a matching grant at rate $(1-\rho_i)p$. The theory on fiscal federalism show that a matching grant is a superior instrument (Oates, 1972; Tresh, 1981), although it might stimulate more expenditure. In our context, given the nature of the good produced, we will use both instrument to assess if this result holds. We assume that Central Government is a Stackleberg leader, i.e. it sets the grant after observing (predicting) Local Government's reaction function.

This means that the problem can be solved using backward induction. In the next section we show the problem that may prevent Central Government to replicate the first best solution.

4.1 Coordination problems

Central Government can replicate the first best solution using fiscal federalism only if it can perfectly observe the reaction function of each local authority. This means that the former should observe both preferences and behaviour. In this section we assume that Central Government may observe local preferences and we concentrate on the coordination problem. We assume that local government maximises the utility function and it does not take into account of the spillover effects its production is causing on the other local communities. This first coordination problem may be solved using a matching grant as the traditional literature on fiscal federalism has long shown (Oates, 1972; King, 1994). A second a more important source of coordination arises from the effects that Local Government decision may have on total welfare. If Central Government can observe local government's reaction it may be possible to improve welfare as compared with a decentralised solution, but the first best allocation may not be reached as shown in the following section.

4.1.1 Local government reaction function

Central Government uses a matching grant at rate $(1 - \rho_i)$ to subsidize the price of good y. If the local government thinks that the production of the impure public good should be further incentivated, it can introduce a supplementary matching grant at rate ρ_L^i which will be financed using a proportional tax on local income at rate t_L^i .

$$t_L^i = \frac{\rho_L^i \int\limits_{\frac{p^{\rho_i - \rho_L^i}}{\theta}}^{1} \frac{1}{2} d\alpha}{\int_{\underline{M}}^{\overline{M}} M \frac{1}{2(\overline{M} - M)} dM}$$
(3)

Given the matching form of the grant to local government, the choice to increase local production affects the budget at national level ,hence t. Three are the basic expectations any local government might possess as concerns the behaviour of the other local government:

• it might think that the other will use its same strategy (full coordination - FC), i.e. they will increase the local production by its same amount using a grant equal to ρ_L^i . In this case the national tax rate t will be determined according to the following formula:

$$t = \overline{t} + \frac{(1-\rho_i)p \int_{\theta}^{\frac{\rho_i p}{\theta}} \frac{1}{2\beta} d\alpha + (1-\rho_i)p \int_{p\frac{\rho_i - \rho_L^i}{\theta}}^{\frac{\rho_i p}{\theta}} \frac{1}{2\beta} d\alpha}{\int_{\underline{M}}^{\overline{M}} \frac{M}{(\overline{M} - \underline{M})} dM}$$
(4)

• it might think that the other local government will not follow its policy so that the increase in expenditure in local authority *i* will be borne by the entire community (partial coordination - PC):

$$t = \bar{t} + \frac{p_i p_{\theta_i}}{\int_{\underline{M}}^{\underline{M}} \frac{M}{(\overline{M} - \underline{M})} dM}$$
(5)

• it might think that the contribution to the setting of the tax rate is marginal, so that its decision does not influence the rate, i.e. $t = \overline{t}$ (free rider, FR)

The latter hypothesis may not be reasonable in the context of this model with two local authorites. However, in a more general setting where the number of local government is rather large and some of them may be "little" with reference to the number of people or their tax base, this behaviour may quite plausible.

On the expenditure side, the local government may think the other local authorities will do exactly the same, in this case it will foresee an increase in Q-j exactly equal to the increase in yi (full coordination); or it may think to be the only local authority to follow this policy (partial coordination and free rider):

In its general form the problem for each local Government can be written as:

$$Max_{\rho_{L}^{i}} \int_{\underline{M}}^{\overline{M}} \begin{pmatrix} M\left(1-\overline{t}-t_{L}\right) + \int_{p\frac{\rho_{i}-\rho_{L}^{i}}{\theta}}^{\beta} \left(\alpha\theta - (\rho_{i}-\rho_{L}^{i})p\right)\frac{1}{\beta}d\alpha + \\ z_{i}\theta\left(k\int_{p\frac{\rho_{j}}{\theta}}^{\beta}\frac{1}{2\beta}d\alpha + sk\int_{p\frac{\rho_{j}-\rho_{L}^{i}}{\theta}}^{p\frac{\rho_{j}}{\theta}}\frac{1}{2\beta}d\alpha + \int_{p\frac{\rho_{i}-\rho_{L}^{i}}{\theta}}^{\beta}\frac{1}{2\beta}d\alpha \end{pmatrix} \end{pmatrix} \frac{1}{2\left(\overline{M}-\underline{M}\right)}dM$$

$$t = \overline{t}; 4; 5 \qquad (6)$$

$$t_{L}^{i} = \frac{\rho_{L}^{i}}{\int_{\underline{M}}^{\overline{M}}M\frac{1}{2(\overline{M}-\underline{M})}dM}{\int_{\underline{M}}^{\overline{M}}M\frac{1}{2(\overline{M}-\underline{M})}dM}$$

$$s = 0 (PC \text{ and FR}), 1(FC)$$

The maximisation of equation 6 leads to three different conditions that reflect the differences in the expectations of the local government about the behaviour of other local authorities that can be also interpreted in terms of reaction function to Central Government policies:

$$\begin{array}{ll} \mathrm{FC} & \rho_L^i = \frac{\theta z_i(1+k)}{2p} - (1 - \frac{\rho_1 + \rho_2}{2}) \\ \mathrm{PC} & \rho_L^i = \frac{z\theta}{2p} - \frac{1 - \rho_i}{2} \\ \mathrm{FR} & \rho_L^i = \frac{z_i\theta}{2p} \end{array}$$

Table two: Local government reaction function

The reaction function of local government depends on the assumption it makes about the behaviour of the other local governments. In the first case, every local authority pays for its extra provision under the assumption that the other increases expenditure in the same proportion; in the second case the local authority assumes to be the only one to change the level set by Central Government; in the third case the local government assumes to be able to be a free rider on the extra revenue needed. It is interesting to note that when Local Government takes into account the effects of its policy on expenditure and taxation (the full coordination case), Central Government cannot use grants-in-aid to induce Local Government to supply the first best quantity. The impact on total welfare is not correctly estimated because each local government evaluates the effects on welfare using his own preferences (z_i), and Central Government cannot correct this problem because the each local authority reacts to the average grants, not the one it receives. This is the reason why, even in the full coordination case the grant Central Government would give to a local agency $(1 - \rho_i)$ is not sufficient to reach the first best allocation. Such problem is more important the greater the value of k, the spillover parameter.

4.1.2 Central Government's choice

Having predicted local government's reaction function, the Centre can set the grant in order to maximise total welfare. The problem can be written as:

$$\begin{aligned} Max_{\rho_{1},\rho_{2}}W &= \sum_{i=1}^{2} \left(\int_{\underline{M}}^{\overline{M}} \left(\begin{array}{c} M\left(1-t-t_{L}^{i}\right)+\int_{\frac{\left(\rho_{1}-\rho_{L}^{i}\right)p}{\theta}}^{\beta}\left(\alpha\theta-\left(\rho_{1}-\rho_{L}^{i}\right)p\right)\frac{1}{\beta}d\alpha}{\left(\beta\right)}\right) + z_{i}\theta \left(\begin{array}{c} \int_{\underline{M}}^{\beta} \frac{1}{2\beta}d\alpha+\int_{\frac{\left(\rho_{1}-\rho_{L}^{i}\right)p}{\theta}}^{\beta}\frac{1}{2\beta}d\alpha}{\left(\beta\right)}\right) \right) \\ t &= \frac{\left(1-\rho_{1}\right)p\frac{\beta\theta-\rho_{1}p}{\beta\theta}+\left(1-\rho_{2}\right)p\frac{\beta\theta-\rho_{2}p}{\beta\theta}}{\left(\overline{M}-\underline{M}\right)} + \frac{\left(1-\rho_{1}\right)p^{2}\frac{\rho_{L}^{i}}{\beta\theta}+\left(1-\rho_{1}\right)p^{2}\frac{\rho_{L}^{2}}{\beta\theta}}{\left(\overline{M}-\underline{M}\right)} \\ t_{L}^{i} &= 2\rho_{L}^{i}p\frac{\beta\theta-p\rho_{1}+p\rho_{L}^{i}}{\beta\theta D}}{\beta\theta D} \end{aligned}$$
(7)
$$\rho_{L}^{i} &= \text{ see table two} \end{aligned}$$

The results are obtaine in appendix 2 and summarized in table 3

Table three: The coordination problem

If Central Government is able to observe or predict the behaviour of each local authority, the first best solution can be replicate only in some cases; if this is not the case the result obtained with fiscal federalism will always be a second best.

This point is very important and shows that asymmetry on preferences is not the only source of failures in fiscal federalism. In the first case ("full coordination"), the local authority expects the other governments to change expenditure, but it uses its preferences to evaluate the effects on welfare. In this case, Central Government cannot use the grant to reach a first best allocation. The total quantity of impure public good is the optimal one, but its local distribution is not correct and total welfare is not maximised. Such difference is more important the greater k and $(z_1 - z_2)$. In general, however, predicting local government behaviour may be very difficult because the perception of how the policy at local level affects total welfare depends on the relative size of each local authority, both in terms of people and income. If this is the case, the grant plays a very important role. For example, if the Centre thinks local authority reacts according to partial coordination while in fact the reaction function is free rider, the matching grant given to the local authority will be twice as much the optimal one, expenditure will be soaring up and welfare will be decreasing. In the next section we show the combined effects of asymmetry of information on fiscal federalism.

4.2 Asymmetry of information

The new environment is characterised by the following assumptions: y is produced by the local authority that have full information on the distribution the preferences al local level while Central Government can observe it with an error. This assumption has two important implications for Central Government policy: it cannot define the optimal aggreate level and it cannot predict local government's reaction function.

In this paper we concentrate on the first aspect by assuming that Central Government cannot observe the true z_i . The grant is distributed using an estimate z of this parameter, equal for both local authorities.

In this case, while the true reaction function for Central Government is the one presented in table two, Central Government allocates grant according to the perceived reaction function as shown in table four

$$\begin{array}{cccccc} \text{Perceived reaction} & \text{Grant} & \text{True reaction} & \text{Q} \\ \rho_L^i = \rho_L^j = \frac{\theta z(1+k)}{2p} - (1-\rho) & 0 & \rho_L^i = \frac{\theta z_i(1+k)}{2p} - (1-\frac{\rho_1+\rho_2}{2}) & \beta - \frac{p}{\theta} + \frac{1}{4}(z_1+z_2)(1+k) \\ \rho_L^i = \rho_L^j = \frac{z\theta}{2p} - \frac{1-\rho}{2} & \frac{z\theta k}{p} & \rho_L^i = \frac{z_i\theta}{2p} - \frac{1-\rho_i}{2} & \beta - \frac{p}{\theta} + \frac{1}{4}(z_1+z_2+zkz) \\ \rho_L^i = \rho_L^j = \frac{z\theta}{2p} & \frac{z\theta k}{2p} & \rho_L^i = \frac{z_i\theta}{2p} & \beta - \frac{p}{\theta} + \frac{1}{4}(z_1+z_2+2kz) \end{array}$$

Table four: Asymmetry of information on preferences

For the full coordination case, information asymmetry does not play any role because each local authority pays for its own provision. In this case, as shown in section 4.1, the best action for Central Government is to leave each local authority to set its own matching grant since Central Government cannot correct local preferences. In the other two cases, Central Government will give the local authority a higher (lower) grant than the optimal one; the difference depending on whether $z \leq z$. In aggregate, given the assumption of linear preferences, the total quantity depends on how good is the estimate of average local preferences (i.e on the difference $z - E(z_i)$). However, as in the previous section, even if total quantity is the optimal one, its distribution across local authorities is not the First Best and welfare is not maximised.

4.3 Using a lump sum grant

The traditional theory for fiscal federalism shows that a matching grant is more effective in stimulating expenditure than a lump-sum grant (King, 1984) and for this reason it should be used when Central Government wants to correct for externalites and spillover. However, a matching grant in the presence of asymmetry of information may become a second best instrument because of its incentive to increase expenditure. In the presence of an impure public good, the hypothesis that a lump-sum grant stimulate less expenditure than a matching grant should be verifed.

To show this, we present local government's reaction function to a lump sum grant. To compare the results with the previous sections, we assume that Central Government supplies each local authority with a lump sum grant equal to G_i . The local authority will then provide a grant a rate $(1 - \rho_L^i)$ to those that want to buy y. The budget constraint for the local authority can be written as:

$$t_L^i = \frac{(1-\rho_L^i)p}{\int_{\underline{M}}^{\overline{M}} M \frac{1}{N(\overline{M}-\underline{M})} dM} dM$$

Also in this case, each Local Government has to foresee what the other local authorities will do and the impact of its decisions on welfare. The use of a lump-sum grant reduces the problems as concerns the financial side: the local authority in fact knows that it will have to borne the full amount of its expenditure, the only prediction that has to be made concerns the level of y that will be supplied in the other local authorities. Two are the relevant assumptions that may be made in this context. Each local authority may take the provision in the other local authority fixed at a specific level (A) or it may think that the other local authority, having the same preferences for the impure public good, will fix the level in the same way.

The problem for each Local Government can be written as:

$$Max_{\rho_{L}^{i}}\int_{\underline{M}}^{\overline{M}} \begin{pmatrix} M\left(1-\overline{t}-t_{L}^{i}\right)+\int_{p\frac{\rho_{L}^{i}}{\theta}}^{\beta}\left(\alpha\theta-\rho_{L}^{i}p\right)\frac{1}{\beta}d\alpha\\+z_{i}\theta\left(kA+\int_{p\frac{\rho_{L}^{i}}{\theta}}^{\beta}\frac{1}{2\beta}d\alpha\right) \end{pmatrix} \frac{1}{2\left(\overline{M}-\underline{M}\right)}dM$$
s.t.
$$(1-\rho_{L}^{i})p\int_{\frac{\rho_{L}^{i}}{\theta}}^{\beta}\frac{1}{2\beta}d\alpha-G\\\frac{t_{L}^{i}}{\int_{\underline{M}}^{\overline{M}}M\frac{1}{2\left(\overline{M}-\underline{M}\right)}}dM$$

$$A = \overline{y}; \int_{p\frac{p_{L}^{i}}{\theta}}^{\beta}\frac{1}{2\beta}d\alpha$$

$$(8)$$

The results are presented in appendix four and summarized in table five:

Subsidy Total quantity
FC
$$\rho_L^i = 1 - \frac{\theta z_i(1+k)}{2p}$$
 $\beta - \frac{p}{\theta} + \frac{1}{4}(z_1+z_2)(1+k)$
PC-FR $\rho_L = 1 - \frac{\theta z_i}{2p}$ $\beta - \frac{p}{\theta} + \frac{1}{4}(z_1+z_2)$

Table five: Local government reaction to a lump-sum grant

The results presented in table five show that in the provision of the impure public good, the reaction of the local authority to the grant mostly depends on the assumption the local authority does about the behaviour of the other local authorities. The lump-sum grant does not necessarily stimulate expenditure less than a matching grant, but it is not able to correct for the spillover effect. If the spillover effect is limited (k close to 0) the use of a lump-sum grant may be a valid alternative because in this way Central Government can predict its budget and have a better control over expenditure.

The use of a lump-sum grant is even more important in the presence of spillovers and an heterogeneity in the income distribution. In the model presented here there is no need of an equalisation grant because income is evenly distributed and because utility is linear in income. In the actual world both conditions may not be verified and the centre may have to distribute resources for equalisation purposes. In this case, the literature suggests that a lump sum grant is less distorsive (Smart), but it might costly in terms of expenditure it is able to stimulate. For an impure public good such problem. This result has important policy implications: several public health care systems have chosen to allocate resources using lump-sum grants and such choice seems to be justified on a theoretical ground.

5 Is fiscal federalism always a better option?

In the previous section we have shown that several problems may lead to a second best solution in the presence of decentralised production of an impure public good. In this section we will try to answer the following question: is fiscal federalism welfare improving in our environment?

To answer this question, we have to combine the results of section and and evaluate the welfare function in the different cases. Table six summarizes the results.

Table six: Welfare comparision

Welfare has been evaluated under the assumption of asimmetry on preferences and behaviour. Central Government uses z as an estimate of local preferences and the first two columns of table six show expected and true behaviour of the local authority. For the full coordination case, asymmetry of information is not important since the optimal grant is zero; it is interesting to note that the first three lines of table represents welfare for the case where Central Government gives a lump-sum to the local authority. In general, it is not possible to establish whether fiscal federalism is better than decentralisation. If Central Government is able to predict the reaction function of the local authority, the error is going to make depens on two factors: its ability to predict z_i and k, the spillover effect. If cannot predict local government behaviour, the mistake will have important effects on welfare. Expenditure may be above or below the optimal one and it may not be well distributed. In general, if θ is close to one and the asymptotic information is quite important, the Centre may find it convenient to use decentralisation. However, if k is very small fiscal federalism and a lump-sum grant may be a better choice for welfare maximisation. Fiscal federalism in combination with a lump-sum grant may be a better solution when the Centre is fairly sure to predict the behaviour of local authorities, k is close to one, θ rather high, but the preferences for the local public good (z) have a low variance and can be estimated with a small error.

6 Conclusions

This paper studies the allocation of grants to local authorities in a context where goods and services produced at local level are impure public goods. The model shows that when goods and services produced at local level are impure public goods it might not always be optimal to introduce fiscal federalism, a simple decentralisation might be more effective. Furthermore, in the presence of an impure public good, the lump-sum grant may not necessarily stimulate less expenditure than the matching grant as the traditional literature on fiscal federalism suggests.

These results depend on two main problems: coordination of policies among local governments and asymmetry of information. As concerns the first point, our results are in line with the most recent literature (Petretto, 2000; Grazzini and Petretto, 2005) that shows that for health care fiscal federalism improves welfare only if the local government takes account of the effects of its policies both at local and at national level. In this paper we characterize this result by showing that Local government has to take account of such consequences using the appropriate utility function. It is in fact not sufficient that the local authority predicts the impact of its decisions on expenditure and taxation, it has also to take account of the effects on the welfare function. In our model we show that if Central Government can observe the right parameters, it may be able to correct preferences using the matching grant, but this policy may not always be used. The model presented in this paper might explain why some countries, as the UK, have chosen a process of decentralisation rather than fiscal federalism.

The work presented in this paper could be extended in several direction: first of all, redistribution policies could be considered by introducing a specific income distribution at national and at local level, secondly political consideration could be explicated by assuming that political parties compete for votes on different variables and in a different setting at national and at local level.

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APPENDIX 1: First best and decentralisation

The problem can be written as:

$$\begin{split} Max_{\rho_{i}}W &= \sum_{i=1}^{2} \left(\int_{\underline{M}}^{\overline{M}} \left(\begin{array}{c} M\left(1-t\right) \\ +z_{i}\theta\left(\int_{\underline{\rho_{i}p}}^{\beta} \frac{1}{2\beta}d\alpha + k\int_{\underline{\rho_{j}p}}^{\beta} \frac{1}{2\beta}d\alpha\right) \\ +\int_{\underline{\rho_{i}p}}^{\beta} \left(\alpha - \rho p\right) \frac{1}{\beta}d\alpha \end{array} \right) \frac{1}{2\left(\overline{M} - \underline{M}\right)} dM \end{split} \\ j &= 1, 2 \quad i \neq j \\ s.t \\ t &= \frac{\sum_{i=1}^{2} \left(\left(1 - \rho_{i}\right) \int_{\underline{\rho_{i}p}}^{\beta} \frac{1}{2}d\alpha\right) \\ \int_{\underline{M}}^{\overline{M}} \frac{M}{\left(\overline{M - M}\right)} dM \end{split}$$

 $t~{\rm can}$ be substituted in the objective function and the problem becomes an unconstrained maximisation. The FOC can be written as:

$$\frac{\frac{1}{4}p\frac{-2p+z_1+2\rho_1p+kz_2}{\beta} = 0}{-\frac{1}{4}p\frac{-2p+2\rho_2p+kz_1+z_2}{\beta} = 0}$$

which gives the solution in the text. Substituting the optimal ρ_i in the welfare function we obtain the results presented in table one

A APPENDIX 2: Fiscal federalism: coordination

problem

A.1 Local government's reaction

The constraints can be substituted in the welfare function that. For the first local authority and the most generic behaviour can be written as:

$$\begin{split} Max_{\rho_{L}^{i}} \int_{\underline{M}}^{\overline{M}} \begin{pmatrix} M\left(1-\bar{t}-t_{L}\right) + \int_{p^{\underline{\rho_{i}-\rho_{L}^{i}}}}^{\beta} \left(\alpha\theta - \left(\rho_{i}-\rho_{L}^{i}\right)p\right)\frac{1}{\beta}d\alpha + \\ z_{i}\theta\left(k\int_{p^{\underline{\rho_{j}}}}^{\beta}\frac{1}{2\beta}d\alpha + sk\int_{p^{\underline{\rho_{j}^{i}-\rho_{L}^{i}}}}^{p^{\underline{\rho_{j}^{i}-\rho_{L}^{i}}}\frac{1}{2\beta}d\alpha + \int_{p^{\underline{\rho_{i}-\rho_{L}^{i}}}}^{\beta}\frac{1}{2\beta}d\alpha \end{pmatrix} \end{pmatrix} \frac{1}{2\left(\overline{M}-\underline{M}\right)}dM \\ t = \frac{(1-\rho_{1})p\frac{\beta\theta-\rho_{1}p}{\beta\theta} + (1-\rho_{2})p\frac{\beta\theta-\rho_{2}p}{\beta\theta}}{\left(\overline{M}-\underline{M}\right)} + m\frac{(1-\rho_{1})p^{2}\frac{\rho_{L}^{1}}{\beta\theta} + r\left(1-\rho_{1}\right)p^{2}\frac{\rho_{L}^{2}}{\beta\theta}}}{\left(\overline{M}-\underline{M}\right)} \\ t_{L}^{i} = \frac{p_{L}^{i}\int_{-\frac{\beta}{\theta}}^{1}\frac{1}{2}d\alpha}{\int_{\underline{M}}^{\overline{M}}M\frac{1}{2\left(\overline{M}-\underline{M}\right)}}dM \\ \\ \text{The F.O.C can be written as:} \\ \frac{1}{4}p\frac{-mp+mp\rho_{i}-mpr+mpr\rho_{j}-2p\rho_{L}^{i}+z_{i}\theta ks+z_{i}\theta}{\beta\theta} \\ \text{and the solution is:} \\ \rho_{L}^{i} = \frac{1}{2p}\left(-mp+mp\rho_{i}-mpr+mpr\rho_{j}+z_{i}\theta ks+z_{i}\theta\right) \\ \text{Subtivuting the radius for merginal partial parti$$

Substituting the values for m, r, s it is possible to obtain the values in table...

A.2 Central Government's grant setting

Central Governments' problem is written in terms of maximisation of the total welfare function expecting to observe the reaction of Local Government. In general the problem can be written as:

$$\begin{split} Max_{\rho_{1},\rho_{2}}W &= \sum_{i=1}^{2} \left(\int_{\underline{M}}^{\overline{M}} \left(\begin{array}{c} M\left(1-t-t_{L}^{i}\right) + \int_{\frac{\left(\rho_{1}-\rho_{L}^{i}\right)p}{\theta}}^{\beta}\left(\alpha\theta-\left(\rho_{1}-\rho_{L}^{i}\right)p\right)\frac{1}{\beta}d\alpha}{\mu^{2}} \right) \right) \frac{1}{2\left(\overline{M}-\underline{M}\right)} dM \\ &+ z_{i}\theta\left(k\int_{\frac{\left(\rho_{j}-\rho_{L}^{i}\right)p}{\theta}}^{\beta}\frac{1}{2\frac{1}{\beta}}d\alpha + \int_{\frac{\left(\rho_{1}-\rho_{L}^{i}\right)p}{\theta}}^{\beta}\frac{1}{2\frac{1}{\beta}}d\alpha}{\mu^{2}} \right) \right) \frac{1}{2\left(\overline{M}-\underline{M}\right)} dM \\ &t = \frac{\left(1-\rho_{1}\right)p\frac{\beta\theta-\rho_{1}p}{\beta\theta} + \left(1-\rho_{2}\right)p\frac{\beta\theta-\rho_{2}p}{\beta\theta}}{\left(\overline{M}-\underline{M}\right)} + \frac{\left(1-\rho_{1}\right)p^{2}\frac{\rho_{L}^{1}}{\beta\theta} + \left(1-\rho_{1}\right)p^{2}\frac{\rho_{L}^{2}}{\beta\theta}}{\left(\overline{M}-\underline{M}\right)} \\ &t_{L}^{i} = 2\rho_{L}^{i}p\frac{\beta\theta-p\rho_{1}+p\rho_{L}^{i}}{\beta\theta D} \\ &\rho_{L}^{i} = \frac{1}{2p}\left(-mp+mp\rho_{i}-mpr+mpr\rho_{j}+z_{i}\theta ks+z_{i}\theta\right) \end{split}$$

The F.O.C can be written as:

$$-\frac{1}{8}p\frac{4\rho_{1}p-4p-2z_{1}\theta ks+m^{2}p\rho_{1}+m^{2}r^{2}p\rho_{1}+2z_{2}\theta k+4mrp-4mrp\rho_{2}+2m^{2}rp\rho_{2}-mz_{2}\theta k+4mp-4mp\rho_{1}-2m^{2}rp-m^{2}p+mz_{1}\theta ks-\frac{\beta\theta}{\beta\theta}}{-\frac{1}{8}p\frac{4\rho_{2}p-4p+m^{2}p\rho_{2}+2kz_{1}\theta-2z_{2}\theta ks+4mrp-mkz_{1}\theta+4mp-2m^{2}rp-4m\rho_{2}p-m^{2}p+mz_{2}\theta ks+2m^{2}rp\rho_{1}+m^{2}r^{2}p\rho_{2}-4rmp\rho_{1}-\frac{\beta\theta}{\beta\theta}}$$

The general solution can be written as: $\rho_1 = \frac{m^2 r^2 p - m^2 p - 4mp - mz_2 \theta k + mz_1 \theta k s - mrz_2 \theta k s + mrkz_1 \theta - 4p - 2z_1 \theta k s + 2z_2 \theta k}{p(-4+4m-m^2+m^2r^2)}$ $\rho_2 = \frac{m^2 r^2 p - m^2 p - mrz_1 \theta k s + mrz_2 \theta k + mz_2 \theta k s + 4mp - mkz_1 \theta - 4p + 2kz_1 \theta - 2z_2 \theta k s}{(-4+4m-m^2+m^2r^2)p}$ Substituting the values for m,r,s it is possible to obtain the values in

Substituting the values for m,r,s it is possible to obtain the values in table...

B Appendix 3: Asymmetry of information.

The problem faced by Central Government is the same as in appendix 2, but in this case it has to use its estimate z of local preferences both in the maximisation process and in the reaction function:

$$\begin{split} Max_{\rho_{1},\rho_{2}}W &= \sum_{i=1}^{2} \left(\int_{\underline{M}}^{\overline{M}} \left(\begin{array}{c} M\left(1-t-t_{L}^{i}\right) + \int_{\frac{\left(\rho_{1}-\rho_{L}^{i}\right)p}{\theta}}^{\beta}\left(\alpha\theta-\left(\rho_{1}-\rho_{L}^{i}\right)p\right)\frac{1}{\beta}d\alpha}{\mu^{2}}\right) + z\theta\left(k\int_{\frac{\left(\rho_{j}-\rho_{L}^{i}\right)p}{\theta}}^{\beta}\frac{1}{2}\frac{1}{\beta}d\alpha + \int_{\frac{\left(\rho_{1}-\rho_{L}^{i}\right)p}{\theta}}^{\beta}\frac{1}{2}\frac{1}{\beta}d\alpha}\right) \right) \frac{1}{2\left(\overline{M}-\underline{M}\right)} dM \end{split}$$

$$\begin{split} t &= \frac{\left(1-\rho_{1}\right)p\frac{\beta\theta-\rho_{1}p}{\beta\theta} + \left(1-\rho_{2}\right)p\frac{\beta\theta-\rho_{2}p}{\beta\theta}}{(\overline{M}-\underline{M})} + \frac{\left(1-\rho_{1}\right)p^{2}\frac{\rho_{L}^{1}}{\beta\theta} + \left(1-\rho_{1}\right)p^{2}\frac{\rho_{L}^{2}}{\beta\theta}}{(\overline{M}-\underline{M})} \\ t_{L}^{i} &= 2\rho_{L}^{i}p\frac{\beta\theta-p\rho_{1}+p\rho_{L}^{i}}{\beta\theta D} \\ \rho_{L}^{i} &= \frac{1}{2p}\left(-mp+mp\rho_{i}-mpr+mpr\rho_{j}+z\theta\left(1+ks\right)\right) \end{split}$$

The FOC can be written as: $-\frac{1}{8}p\frac{4\rho_{1}p-4p+m^{2}p\rho_{1}-2z\theta ks+4mrp-4mrp\rho_{2}+2m^{2}rp\rho_{2}+m^{2}r^{2}p\rho_{1}+4mp-4mp\rho_{1}-2m^{2}rp-m^{2}p+mz\theta ks+mrw\theta ks-m^{2}r^{2}p-\rho_{2}}{\beta\theta}$ $-\frac{1}{8}p\frac{4\rho_{2}p-4p+4mrp+2z\theta k-2w\theta ks-4mrp\rho_{1}-mz\theta k+2m^{2}rp\rho_{1}+m^{2}r^{2}p\rho_{2}+4mp-4mp\rho_{2}-2m^{2}rp-m^{2}p-m^{2}r^{2}p+m^{2}p\rho_{2}+mrz}{\beta\theta}$

and the optimal solution can be written as: $\rho_2 = \frac{m^2 r^2 p + mrw\theta k - mrz\theta ks - m^2 p + mw\theta ks + 4mp - mz\theta k + 2z\theta k - 4p - 2w\theta ks}{(-4 + 4m - m^2 + m^2 r^2)p}$ $\rho_1 = \frac{m^2 r^2 p - m^2 p + mz\theta ks + 4mp + mrz\theta k - mw\theta k - mrw\theta ks - 2z\theta ks + 2w\theta k - 4p}{p(-4 + 4m - m^2 + m^2 r^2)}$

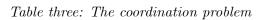
C Appendix four: Local government's reaction to a matching grant

The problem can be written as:

$$\begin{split} Max_{\rho_{L}^{i}} & \int_{\underline{M}}^{\overline{M}} \left(\begin{array}{c} M\left(1-\overline{t}-t_{L}^{i}\right)+\int_{p\frac{\rho_{L}^{i}}{\theta}}^{\beta}\left(\alpha\theta-\rho_{L}^{i}p\right)\frac{1}{\beta}d\alpha}{\mu^{2}}\right) \\ & +z_{i}\theta\left(kA+\int_{p\frac{\rho_{L}^{i}}{\theta}}^{\beta}\frac{1}{2\beta}d\alpha\right) \end{array} \right) \frac{1}{2\left(\overline{M}-\underline{M}\right)} dM \\ & s.t. \\ (1-\rho_{L}^{i})p\int_{\frac{\rho_{L}^{i}}{\theta}}^{\beta}\frac{1}{2\beta}d\alpha-G \\ & \frac{\frac{\rho_{L}^{i}}{\theta}p}{\int_{\underline{M}}^{\overline{M}}M\frac{1}{2(\overline{M}-\underline{M})}dM} \\ A & = \overline{y}; \int_{p\frac{p_{L}^{i}}{\theta}}^{\beta}\frac{1}{2\beta}d\alpha, \text{ the FOC can be written as:} \\ & -\frac{1}{4}p\frac{-2p+2\rho_{L}p+z_{A}\theta k+z_{A}\theta}{\beta\theta} \end{split}$$

from which it is possible to derive the solution in the text For A= \overline{y} , the FOC can be written as: $-\frac{1}{4}p\frac{-2p+2\rho_L p+z_A \theta}{\beta \theta}$ from which it is possible to derive the solution presented in the text

$$\begin{array}{cccc} \text{LG grant} & \text{CG grant} & \text{Q} \\ \text{FC} & \frac{\theta z_i(1+k)}{2p} - \left(1 - \frac{\rho_1 + \rho_2}{2}\right) & 0 & \beta - \frac{p}{\theta} + \frac{1}{4}(1+k)\sum_{i=1}^2 z_i \\ \text{PC} & \frac{z_i\theta}{2p} - \frac{1-\rho_i}{2} & \frac{z_j\theta k}{p} & \beta - \frac{p}{\theta} + \frac{1}{4}(1+k)\sum_{i=1}^2 z_i \\ \text{FR} & \frac{z_i\theta}{2p} & \frac{z_j\theta k}{2p} & \beta - \frac{p}{\theta} + \frac{1}{4}(1+k)\sum_{i=1}^2 z_i \end{array}$$



CG True Welfare

$$FC FC = FC = \frac{\frac{1}{2}D + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2\beta}\left((1 + k)\left(\left(z_1\frac{z_1 + z_2k}{4} + z_2\frac{z_2 + z_1k}{4} - \frac{1}{8}(1 + k)(z_1^2 + z_2^2)\right)\theta - \frac{p}{2}(z_1 + z_2)\right) + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta - p + \frac{1}{2}\beta\theta + \frac{1}{4}(z_1 + z_2)(1 + k)\theta + \frac{1}{2}\beta\theta + \frac{1}{2}\beta\theta$$

$$FC \quad PC \\ +\frac{1}{2\beta} \left(\left(\frac{1}{8} \left(z_1 \frac{z_1 + z_2 k}{4} + z_2 \frac{z_2 + z_1 k}{4} \right) - \frac{1}{8} \left(\frac{z_1^2 + z_2^2}{\beta} \right) \right) \theta - \frac{p}{2} \left(z_1 + z_2 \frac{z_2 + z_1 k}{4} \right) + \frac{1}{2\beta} \left(\frac{z_1^2 + z_2^2}{\beta} \right) \theta + \frac{p}{2} \left(z_1 + z_2 \frac{z_2 + z_1 k}{4} \right) + \frac{1}{2\beta} \left(\frac{z_1^2 + z_2 k}{4} + z_2 \frac{z_2 + z_1 k}{4} \right) + \frac{1}{2\beta} \left(\frac{z_1^2 + z_2 k}{4} + z_2 \frac{z_2 + z_1 k}{4} \right) + \frac{1}{2\beta} \left(\frac{z_1^2 + z_2 k}{4} + z_2 \frac{z_2 + z_1 k}{4} \right) + \frac{1}{2\beta} \left(\frac{z_1^2 + z_2 k}{4} + z_2 \frac{z_2 + z_1 k}{4} \right) + \frac{1}{2\beta} \left(\frac{z_1^2 + z_2 k}{4} \right) + \frac{1}{2\beta} \left(\frac{z_1^2 + z_2 k}{4} + z_2 \frac{z_2 + z_1 k}{4} \right) + \frac{1}{2\beta} \left(\frac{z_1^2 + z_2 k}{4} + z_2 \frac{z_2 + z_1 k}{4} \right) + \frac{1}{2\beta} \left(\frac{z_1^2 + z_2 k}{4} \right) + \frac{1}{2\beta} \left(\frac{z_1^2 + z_2 k}{4} + z_2 \frac{z_2 + z_1 k}{4} \right) + \frac{1}{2\beta} \left(\frac{z_1^2 + z_2 k}{4} \right) + \frac{1}{2\beta} \left(\frac{z_1^2 + z_2 k}{4} + z_2 \frac{z_2 + z_1 k}{4} \right) + \frac{1}{2\beta} \left(\frac{z_1^2 + z_2 k}{4} \right) + \frac{1}{$$

$$\begin{array}{rcl} FC & FR \\ PC & FC \\ PC & FC \\ \end{array}$$

$$\begin{array}{rcl} \frac{1}{2}D + \frac{1}{2}\beta\theta + \frac{1}{4}\left(z_{1} + z_{2}\right)\left(1 + k\right)\theta - p + \\ \frac{1}{2\beta}\left(\frac{p^{2}}{\theta} + \frac{z_{1} + z_{2}k}{4}\left(\left(kz + z_{1}\right)\theta - 2p\right) + \frac{z_{2} + z_{1}k}{4}\left(\left(2kz + z_{2}\right)\theta - 2p\right) - \theta\left(\frac{5}{8}k^{2}z^{2} + \frac{z_{1}^{2}}{4}\right) \\ \frac{1}{2}D + \frac{1}{2}\beta\theta + \frac{1}{4}\left(z_{1} + z_{2}\right)\left(1 + k\right)\theta - p + \\ PC & FR \\ \frac{1}{2\beta}\left(\frac{p^{2}}{\theta} + \frac{z_{1} + z_{2}k}{4}\left(\left(2kz + z_{1}\right)\theta - 2p\right) + \frac{z_{2} + z_{1}k}{4}\left(\left(2kz + z_{2}\right)\theta - 2p\right) - \theta\left(k^{2}z^{2} + \frac{z_{1}^{2}}{4}\right) \\ FR & FC \\ \frac{1}{2}D + \frac{1}{2}\beta\theta + \frac{1}{4}\left(z_{1} + z_{2}\right)\left(1 + k\right)\theta - p + \\ \frac{1}{2\beta}\left(\frac{p^{2}}{\theta} + \frac{z_{1} + z_{2}k}{4}\left(\left(\frac{k}{2}z + z_{1}\right)\theta - 2p\right) + \frac{z_{2} + z_{1}k}{4}\left(\left(\frac{k}{2}z + z_{2}\right)\theta - 2p\right) - \theta\left(\frac{k^{2}z^{2}}{16} + \frac{z_{1}^{2}}{16}\right) \\ FR & FD \\ \end{array}$$

Table six: Welfare

FR FR

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