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ON ECONOMIC GROWTH WITH PUBLIC EXPENDITURE, MALTHUSIAN TYPE POPULATION CHANGE AND TECHNOLOGICAL PROGRESS

A.BUCCI, M.FLORIO, D. LA TORRE

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On economic growth with public expenditure, Malthusian type population change and technological progress

A.Bucci, M.Florio, D. La Torre

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Department of Economics, Business and Statistics, University of Milan, Italy.

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Abstract

In this paper we combine within the same framework two different research lines that have been analyzed separately in the recent past. The two research lines we aim at joining together concern, respectively, the effects of government spending on economic growth and the analysis of the properties of (asymptotically) balanced growth paths in the presence of a non-constant time preference rate. We assume that both the level of technology and the size of population increase exogenously over time and we postulate that both population and the ratio of public expenditure to GDP follow a logistic behavior over time. Within this framework we are interested in analyzing the dynamics of such variables (both in aggregate and in per-capita terms) as consumption, output and physical capital and their growth rates. Through numerical simulations we prove the existence of an asymptotically balanced growth path (ABGP) equilibrium.

Keywords: Economic growth, physical capital accumulation, Malthusian population change, public expenditure, technological progress.
JEL numbers: O40, O41, H50.

1 Introduction

In recent years there has been a renewed interest in uncovering the ultimate determinants of economic growth. Overall, a large consensus has been reached on the fact that such factors as technological progress, knowledge spill-overs, accumulation of capital (in a broad sense), government's purchases of goods and services and population dynamics might be highly influential in increasing people's wealth and living standards in the long run [1]. The main objective of this paper is to combine within the same framework two different research lines that, for the most part, have been analyzed separately in the recent past. The two research lines we aim at joining together in this paper are, respectively, the one studying the effects of government spending on economic growth and that analyzing the properties of (asymptotically) balanced growth paths in the presence of a non-constant time preference rate. Since the pioneering work of Barro (1990), it is widely recognized that government spending (more precisely, government's purchases of goods and services) plays a major role in affecting economic growth. Indeed, it is found[2] that in the very long-run consumption, physical capital and output all grow at a common (constant) rate determined, among others, by the constant level of technology and of the labor force (strong scale effect)[3] and by the constant ratio of public spending to GDP. With respect to this strand of the literature our paper introduces two important differences. The first consists in assuming that both the level of technology and the size of population (labor force) increase exogenously over time (i.e., we consider the case of positive technological progress and demographic dynamics). The second consists in postulating that both population and the ratio of public expenditure to GDP follow a logistic behavior over time. Within this framework we are interested in analyzing the dynamics of such variables (both in aggregate and in per-capita terms) as consumption, output and physical capital and their growth rates. While the use of a logistic-like function for population change is not new[4], the use of such a function in stating the behavior over time of the public expenditure to the GDP ratio deserves some further comment since it represents one of the main novelties of this contribution. The logistic model assumption to describe the dynamics of the ratio of public expenditure to income is due to Florio and Colautti (2005). However in their paper they did not include it in a fully developed growth model which is the aim of the present work. As mentioned above our paper is also related to the literature on endogenous time-preference rates and the existence of balanced growth path equilibria. Using the preference setup introduced by Uzawa (1968) and later on extended by Epstein and Hynes (1983), Epstein (1987), Obstfeld (1990), Shi and Epstein (1993) and

[1]Elegant surveys of the different approaches to the so-called Endogenous Growth Theory are represented by, among others, the books by Barro and Sala-i-Martin (2004) and Aghion and Howitt (1998).

[2]See Barro and Sala-i-Martin (2004), p.221, equation (4.42).

[3]Empirical evidence (Jones, 2005) rejects the strong scale effect.

[4]Exponential population growth is a special case of logistic population growth. By employing a logistic-like process for population change one easily arrives at the Malthusian result that the size of the population is asymptotically constant (population growth is asymptotically equal to zero).

Drugeon (1998), Palivos et al. (1997) establish (necessary and sufficient) conditions for the existence of balanced growth and asymptotic balanced growth paths. Under a linear technology, they show that the constancy of the elasticity of marginal utility and of the discount rate are necessary and sufficient conditions for the existence of a balanced growth equilibrium path. Similarly, they also show that asymptotically constant elasticity of marginal utility and discount rate are necessary and sufficient for the existence of a unique asymptotically balanced growth equilibrium path, defined as a solution to an optimal growth problem such that al variables grow at asymptotically constant growth rates. While our model shares this property of asymptotically balanced growth equilibrium path (namely that in the very long-run, i.e., when , variables depending on time approach a given constant, endogenously determined), with respect to the strand of literature mentioned above, the novelty we introduce in our paper consists in linking the non-constancy of the time preference rate to the non-constancy of population growth. The remainder of the paper is organized as follows. In section 2 we describe the model economy and discuss in more detail our assumption of a logistic-type function for the evolution over time of the ratio of aggregate public expenditure to total GDP. In Section 3 we introduce some mathematical preliminaries, while in Section 4 we solve for the model's asymptotically balanced growth equilibrium path. In Section 5 we perform numerical analysis in order to study the dynamics of some key-variables of the model over time and Section 6 concludes the paper.

2 The model

We extend here the model of Barro (1990) in which the government's purchases of goods and services (G) enter into the production function. In more detail we consider a closed economy where a homogeneous final good (Y) is produced competitively by employing four different inputs: physical capital (K), labor (L), public goods (G) and ideas (a proxy for the available level of technology, A). If the production function takes the Cobb-Douglas form, its specification for a generic representative firm producing final goods is supposed to be:

$$Y(t) = A(t)^{\eta} K(t)^{\alpha} L(t)^{\theta} G(t)^{1-\alpha-\theta-\eta} \tag{1}$$

where $\eta, \alpha, \theta \in (0,1)$. A peculiarity of equation (1) is that it displays constant returns to scale to the four factor-inputs (A, K, L and G), jointly considered. Since all these factors are reproducible, this property of the aggregate production function represents a sufficient condition for endogenous growth. Once produced, output (Y) can be either consumed (C), or used as a public good (G) or invested in (private) physical capital. For the sake of simplicity we assume that physical capital does not depreciate over time (its instantaneous depreciation rate is always equal to zero). Thus, the law of motion of the aggregate physical capital stock is given by:

$$\dot{K}(t) = Y(t) - \beta_K K(t) - C(t) - G(t) \tag{2}$$

We depart from the basic Barro's (1990) model in three fundamental respects. The first two concern the fact that we assume both (exogenous) technological progress and population change. As for the level of technology, we postulate a simple exponential growth process:

$$\dot{A}(t) = g_A A(t) \tag{3}$$

whereas we posit a logistic-like function for the change of population over time:

$$\dot{L}(t) = nL(t)dt \tag{4}$$

By using the first law of motion (the one for A_t) we assume that the level of technology can grow over time without bounds. On the other hand, the law of motion of L_t allows us replicating the Malthusian result that the size of the population becomes asymptotically constant - i.e., when $t \to \infty$ population growth goes smoothly to zero. The last, and probably the most important, difference with respect to the basic Barro's (1990) model concerns our assumption on the evolution over time of the ratio of aggregate public expenditure to total income ($\frac{G}{Y}$). While that paper takes such a ratio as constant, we consider the case where the derivative of $\frac{G}{Y}$ with respect to time follows a logistic-type trajectory. The logistic model assumption to describe the dynamics of the ratio of public expenditures and income is due to Florio and Colautti (2005). However in their paper they did not include it in a fully developed growth model. Before discussing this assumption, we briefly mention some earlier attempts to consider public expenditure in economic growth theory and how this approach is innovative and different from the ones in the literature. Government spending G appears in standard exogenous growth models as purchases of goods and services (see Barro and Sala-i-Martin, 2004, 143 and ff). In a variation of the above mentioned model, when G is included in the production function, the $\frac{G}{Y}$ ratio is supposed to be constant along the dynamic path of the economy. This feature is unattractive since over the last one hundred years we have observed a dynamics of this ratio. For instance it was well below 0.10 in the US around 1900-1910, and above 0.30 around 1990; the ratio increased in a similar way in many other countries over the twentieth century. To explain why there is a sustained demand for government services, we consider public expenditures as interacting with other production factors. Barro and Sala-i-Martin(2004, pp 220 and ff) consider an AK-type model where capital is augmented with public services. Given our definition of productive public expenditure, we prefer to relate their effects to all production factors. After all, most of public consumption is related to education, health and welfare. Moreover, the Barro and Sala-i-Martin model of endogenous growth still assumes that the government chooses a constant $\frac{G}{Y}$ ratio. This is clearly and ad hoc assumption.

One of the earliest contribution to the study of the long term trend of public expenditure was Wagner (1894), who prompted a huge flow of literature, reviewed by Peacock and Scott (2000). Under the 'Wagner's law', public services are considered as a bundle of goods with elasticity to income greater than one. Focussing on logarithmic derivatives of public expenditure (g) and national income (y), it is easy to see that $\frac{d}{dt}\frac{G}{Y} = h\frac{G}{Y}$ where $h = g - y$, or the difference in growth rates of government and the economy. In this context, h can be seen also as the product of g and income elasticity of public expenditure minus one: thus for any positive value of g, income elasticity should be more than one to generate positive h, which is consistent with the intuition by Wagner and several recent contributions to the public expenditure literature. More interestingly, for a constant h to hold, it is possible to combine different offsetting changes of growth and income elasticity. However the resulting process is exponential, and this is very implausible in the long term (at least if a country cannot incur in public debt without any limit). As government becomes bigger, fiscal resistance increases, and this acts as a brake to Wagner's law: the importance of this factor may vary among countries, as the income elasticity of public expenditures do, but despite different country specificities, it is helpful to model a common underlying process. If fiscal resistance depends upon the deadweight loss of distorted taxation, under the assumption of balanced budget, Wagner's law is replaced by a more complex pattern. The excess burden of taxation (the Pigou's effect) by a standard approximation in welfare economics literature depends upon the price elasticity of private goods and the average tax rate. Under balanced budget, the ratio of the excess burden of taxation to Y, is simply one half of the price elasticity k multiplied by the square of $\frac{G}{Y} = \frac{T}{Y}$. The combined action of the Wagner Law and the Pigou's Effect generates a S-shaped trajectory. Because the excess burden of taxation is quadratic, when we assume long-run balanced budget, the resulting differential non-linear equation is a Bernoulli in $\frac{G}{Y}$ and can be integrated as a logistic.

Modeling the dynamics of government as a logistic process has a number of attractive features. The two parameters in a logistic process, h and k, have some nice properties that are easy to interpret: $\frac{h}{k}$ is the upper limit of $\frac{G}{Y}$, thus if we can estimate the parameters (and a set of country specific variables) we can predict where and when $\frac{G}{Y}$ increase stops, so that we get a steady state of the ratio of public expenditure to income. A saddle point occurs at $t = \frac{1}{h}\ln(B)$, where B is a constant and there $\frac{G}{Y}$ is half its upper limit. At the saddle point year, Wagner's law is reversed and Pigou's effect starts to prevail. Florio and Colautti (2005) provide an empirical test of the logistic dynamics of $\frac{G}{Y}$ for around one century data (1870-1990) and five

countries (US, Germany, France, UK and Italy) and figure out that a logistic process fits the data better than an exponential process (perhaps except for Italy, a country with notoriously high public debt). As mentioned above, the empirical model is however not integrated in a growth model, and in this paper we go beyond it into two ways: first, we study the interaction of public expenditure with income growth through its interaction with population change, and - second - we explore the mutual consequences on public expenditure on growth of a dynamic setting where there are deterministic constraints on demographic trends and technological progress. As a first step the differential equation for the $\frac{G}{Y}$ dynamics we consider here is fully deterministic and it replaces the $\frac{G}{Y}$ constant assumption of earlier contributions.
Under the assumptions stated above in this section and with a logarithmic individual utility function, the problem of a representative infinitely-lived household seeking to maximize under constraints its lifetime discounted utility can be written as:

$$\max_{C(t)} \int_0^{+\infty} \ln(c(t))L(t)e^{-\rho t}dt \tag{5}$$

subject to:

$$\begin{cases} \dot{K}(t) = Y(t) - \beta_K K(t) - C(t) - G(t) \\ \dot{A}(t) = g_A A(t)dt + \sigma_A A(t)dW(t) \\ \dot{L}(t) = nL(t) - dL^2(t) \\ \frac{d}{dt}\left(\frac{G(t)}{Y(t)}\right) = \mu\left(\frac{G(t)}{Y(t)}\right) - \gamma\left(\frac{G(t)}{Y(t)}\right)^2 \end{cases}$$

where $c(t) \equiv \frac{C(t)}{L(t)}$ and $K(0) = K_0$, $A(0) = A_0$, $L(0) = L_0$ and $G(0) = G_0$.

3 Mathematical preliminaries

By easy calculations, the objective function can be rewritten as

$$\begin{aligned} \int_0^{+\infty} \ln(c(t))L(t)e^{-\rho t}dt &= \int_0^{+\infty} (\ln(C(t)) - \ln(L(t)))L(t)e^{-\rho t}dt \\ &= \int_0^{+\infty} \ln(C(t))L(t)e^{-\rho t}dt - \int_0^{+\infty} \ln(L(t))L(t)e^{-\rho t}dt \\ &= \int_0^{+\infty} \ln(C(t))L(t)e^{-\rho t}dt + \Omega \end{aligned} \tag{6}$$

where

$$\Omega = -\int_0^{+\infty} \ln(L(t))L(t)e^{-\rho t}dt \tag{7}$$

is a constant. Since a constant does not have any influence on the maximization, the model can be rewritten as

$$\max_{C(t)} \int_0^{+\infty} \ln(C(t))L(t)e^{-\rho t}dt \tag{8}$$

subject to

$$\begin{cases} \dot{K}(t) = Y(t) - \beta_K K(t) - C(t) - G(t) \\ \dot{A}(t) = g_A A(t) dt \\ \dot{L}(t) = nL(t) - dL^2(t) \\ \frac{d}{dt}\left(\frac{G(t)}{Y(t)}\right) = \mu \left(\frac{G(t)}{Y(t)}\right) - \gamma \left(\frac{G(t)}{Y(t)}\right)^2 \end{cases}$$

The third and the fourth differential equations are Bernoulli-type equations which can be solved in a closed form. For the third one, by easy calculations one can prove

$$L(t) = \frac{n}{d + (\frac{n}{L_0} - d)e^{-nt}} \tag{9}$$

On the other hand for the fourth differential equation we get

$$\frac{G(t)}{Y(t)} = \frac{\mu}{\gamma + (\frac{\mu Y_0}{G_0} - \gamma)e^{-\mu t}} \tag{10}$$

The last expression implies that G is a function of A, L and K since

$$G(t) = \frac{\mu}{\gamma + (\frac{\mu Y_0}{G_0} - \gamma)e^{-\mu t}} A(t)^\eta K(t)^\alpha L(t)^\theta G(t)^{1-\alpha-\theta-\eta} \tag{11}$$

and then

$$G(t) = \left(\frac{\mu}{\gamma + (\frac{\mu Y_0}{G_0} - \gamma)e^{-\mu t}} A(t)^\eta K(t)^\alpha L(t)^\theta\right)^{\frac{1}{\alpha+\theta+\eta}} \tag{12}$$

If we replace this expression into 8 the problem can be reduced to

$$\max_{C(t)} \int_0^{+\infty} \ln(C(t)) L(t) e^{-\rho t} dt \tag{13}$$

subject to

$$\begin{cases} \dot{K}(t) = \left(1 - \frac{\mu}{\gamma + (\frac{\mu Y_0}{G_0} - \gamma)e^{-\mu t}}\right) Y(t) - \beta_K K(t) - C(t) \\ \dot{A}(t) = g_A A(t) \\ \dot{L}(t) = nL(t) - dL^2(t) \\ G(t) = \left(\frac{\mu}{\gamma + (\frac{\mu Y_0}{G_0} - \gamma)e^{-\mu t}} A(t)^\eta K(t)^\alpha L(t)^\theta\right)^{\frac{1}{\alpha+\theta+\eta}} \end{cases}$$

and initial conditions K_0, L_0 and A_0. By substituting the last expression in the production function Y, the optimization problem can be rewritten as

$$\max_{C(t)} \int_0^{+\infty} \ln(C(t)) L(t) e^{-\rho t} dt \tag{14}$$

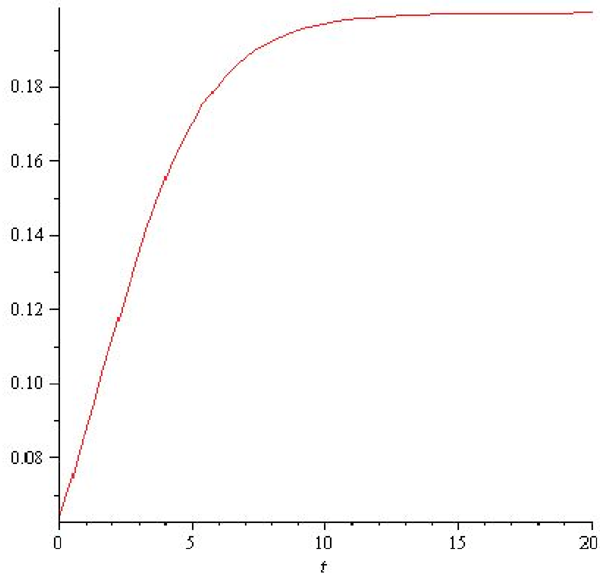

Figure 1: $\frac{G_t}{Y_t}$

subject to

$$
\begin{aligned}
\dot{K}(t) &= \left(1 - \frac{\mu}{\gamma + (\frac{\mu Y_0}{G_0} - \gamma)e^{-\mu t}}\right)\left(\frac{\mu}{\gamma + (\frac{\mu Y_0}{G_0} - \gamma)e^{-\mu t}}\right)^{1-\alpha-\theta-\eta} \\
&\quad A(t)^{\frac{\eta}{\alpha+\theta+\eta}} K(t)^{\frac{\alpha}{\alpha+\theta+\eta}} L(t)^{\frac{\theta}{\alpha+\theta+\eta}} - \beta_K K(t) - C(t) \\
\dot{A}(t) &= g_A A(t) \\
\dot{L}(t) &= nL(t) - dL^2(t)
\end{aligned}
$$

By defining

$$
\begin{aligned}
\xi(t) &= \left(1 - \frac{\mu}{\gamma + (\frac{\mu Y_0}{G_0} - \gamma)e^{-\mu t}}\right)\left(\frac{\mu}{\gamma + (\frac{\mu Y_0}{G_0} - \gamma)e^{-\mu t}}\right)^{1-\alpha-\theta-\eta} \\
&= \left(1 - \frac{G(t)}{Y(t)}\right)\left(\frac{G(t)}{Y(t)}\right)^{1-\alpha-\theta-\eta} \qquad (15)
\end{aligned}
$$

we get

$$\max_{C(t)} \int_0^{+\infty} \ln(C(t))L(t)e^{-\rho t}dt \qquad (16)$$

subject to

$$\dot{K}(t) = \xi(t)A(t)^{\frac{\eta}{\alpha+\theta+\eta}} K(t)^{\frac{\alpha}{\alpha+\theta+\eta}} L(t)^{\frac{\theta}{\alpha+\theta+\eta}} - \beta_K K(t) - C(t)$$

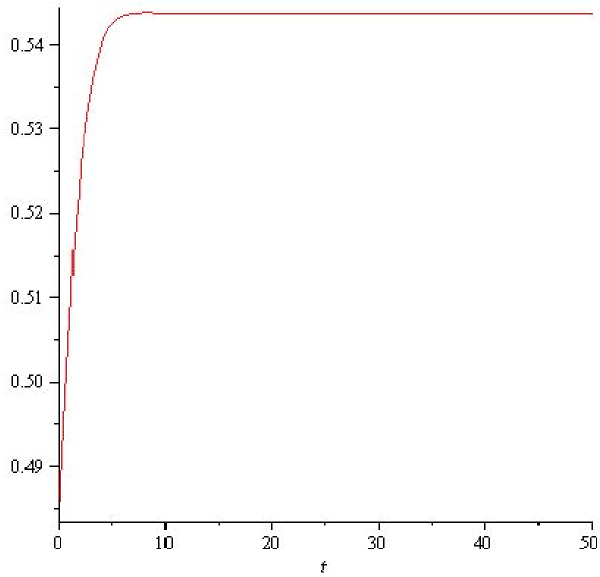

Figure 2: $\xi(t)$

$$\begin{array}{rcl}\dot{A}(t) & = & g_A A(t)\\ \dot{L}(t) & = & nL(t) - dL^2(t)\end{array}$$

4 The (asymptotically) balanced growth path equilibrium of the model

The Hamilton function associated with this problem is the following

$$\mathcal{H}(t) = \ln(C(t))L(t)e^{-\rho t} + \lambda_K(t)\dot{K}(t) \tag{17}$$

A necessary and sufficient condition for the optimality is stated by the first order conditions (FOCs) which read as

$$\begin{cases} \frac{\partial \mathcal{H}}{\partial C} = \frac{L(t)e^{-\rho t}}{C(t)} - \lambda_K(t) = 0 \\ \frac{\partial \mathcal{H}}{\partial K} = \lambda_K(\frac{\alpha}{\alpha+\theta+\eta}\xi(t)A(t)^{\frac{\eta}{\alpha+\theta+\eta}}K(t)^{\frac{\alpha}{\alpha+\theta+\eta}-1}L(t)^{\frac{\theta}{\alpha+\theta+\eta}} - \beta_K) = -\dot{\lambda_K}(t) \end{cases}$$

The first condition of the above system implies that

$$\ln\left(\frac{L(t)e^{-\rho t}}{C(t)}\right) = \ln(\lambda_K(t)) \tag{18}$$

and by differentiating both sides with respect to t, we have

$$\frac{\dot{L}(t)}{L(t)} - \rho - \frac{\dot{C}(t)}{C(t)} = \frac{\dot{\lambda_K}(t)}{\lambda_K(t)} \tag{19}$$

which implies

$$\frac{\dot{C}(t)}{C(t)} = \frac{\dot{L}(t)}{L(t)} - \rho + \frac{\alpha}{\alpha+\theta+\eta}\xi(t)A(t)^{\frac{\eta}{\alpha+\theta+\eta}}K(t)^{\frac{\alpha}{\alpha+\theta+\eta}-1}L(t)^{\frac{\theta}{\alpha+\theta+\eta}} - \beta_K \quad (20)$$

If we substitute the constraint involving K into 20 we get

$$\frac{\dot{C}(t)}{C(t)} = \frac{\dot{L}(t)}{L(t)} - \rho + \frac{\alpha}{\alpha+\theta+\eta}\left(\frac{\dot{K}(t)}{K(t)} + \beta_K + \frac{C(t)}{K(t)}\right) - \beta_K \quad (21)$$

We now introduce the definitions of Balanced Growth Path (BGP) and Asymptotically Balanced Growth Path (ABGP) equilibrium.

Definition 4.1. *[4] A BGP equilibrium is a long-run equilibrium where all variables depending on time grow at constant (possibly positive) exponential rates. A BGP equilibrium is said to be nondegenerate if all growth rates are strictly positive.*

Definition 4.2. *[21] An ABGP equilibrium is a long-run equilibrium where the growth rates of all variables admit a finite limit when $t \to \infty$. An ABGP equilibrim is said to be nondegenerate if all these limits are strictly positive.*

Next section will be devoted to the simulation of the dynamics of all variables involved in this model. Through numerical simulations we will show that this model admits an Asymptotically Balanced Growth Path (ABGP) equilibrium.

5 Numerical simulations

In this section we consider the following values for the unknown parameters: $\alpha = 0.36$, $\eta = 0.21$, $\theta = 0.19$, $g_A = 6.4\%$, $n = 0.0144$, $\rho = 0.01$, $\frac{G_0}{Y_0} = 20\%$. Moreover, we consider the following initial conditions: $L(0) = 1$, $A(0) = 1$, $\beta_K = 0$, $G(0) = 4$, $K(0) = 10^{3.33}$. As for the capital share (α), Blanchard (1997) emphasizes that capital shares in France, Germany, Italy and Spain exhibited large increases starting in the early 1980s and continuing through the 1990s. The magnitude of the increase is approximately from 0.32 to 0.40. According to Jones (2003) the same can be also said, as an example, for Denmark as well. Denmark's capital share, indeed, has risen from about 0.3 to about 0.4 over the last quarter of the century. Based on the behaviour of the capital share in OECD countries over the period 1950-1997 (Jones, 2003, Figure 1, p.8) we take for the capital share (α) a value of 0.36 - see also below. The labor-force growth rate ($n = 0.0144$) and the values for g_A (6.4%) and η (0.19) are taken from Jones and Williams (2000, Table 1, p.73). The first parameter (n) represents the average growth rate of the labor-force in the U.S. private business sector over the period 1948-1997.

We take this value for the growth rate coefficient in the logistic-like function giving the change of population size over time. The other two parameters (g_A and η) are obtained in the following way. Jones and Williams (2000) consider a model where a homogeneous final output (Y) is produced by:

$$Y_t = L_t^{\alpha} \left(\sum_{i=1}^{A_t} x_{it}^{\rho(1-\alpha)} \right)^{\frac{1}{\rho}} \tag{22}$$

where L is the labor-force, α is the labor-share, A is the level of technology (approximated by the number of varieties of intermediate capital goods existing at time t), x_i is the amount of the i-th intermediate capital good employed at time t in final output production and ρ is a parameter determining the elasticity of substitution between intermediate capital goods. In a symmetric equilibrium where each producer of intermediate capital goods produce the same amount of output $\left(x_{it} = x_t = \frac{K_t}{A_t} \right)$, with K_t representing the total quantity of intermediate capital goods being produced at time t, the aggregate production function above can be recast as:

$$Y_t = A_t^{\frac{1}{\rho}-(1-\alpha)} L_t^{\alpha} K_t^{1-\alpha} \tag{23}$$

With a labor-share of 0.64 - thus, a capital share $(1-\alpha)$ of 0.36 - and $\rho = 1.8$ the exponent of A_t, that we denoted by η in our model, is approximately equal to 0.196 (see Jones and Williams, Table 2, p.75). Given an estimate of 0.0125 for the average TFP growth rate in the U.S. private business sector (σg_A) over the period 1948-1997 and with this leads to a value of g_A close to 6.4We give a value of 0.21 to θ. The reason is as follows. If technological progress were embodied in humans (i.e., $A_t = h_t$, with h denoting the average quality of population or per-capita human capital), then $A_t L_t$ would give the aggregate stock of human capital. Estimates of Jorgenson et al. (1987) suggest that with an aggregate Cobb Douglas-type production function the human capital share (in our case the sum of η and θ) is between 0.4 and 0.5. If we fix such a share to 0.4 (Barro and Sala-i-Martin, 2004, p.60), and with $\eta = 0.19$, we easily get a value for θ equal to 0.21. With these parameter values, and in particular with $\eta = 0.19$, $\theta = 0.21$ and $\alpha = 0.36$, it follows that $1 - \alpha - \theta - \eta = 0.24$. This number is in line with the main findings of Aschauer (1989; 1990) on the influence of core infrastructure investment spending on total output. Following Oliver (1982), who estimates the coefficients of the logistic curve for the population of Great Britain from 1801 to 1971, we take a value of 10 millions for the starting population size (L_0) and a value of 63 millions for the population saturation level (L_s). Accordingly, with these values and with $n = 0.0144$, the last parameter of interest in the logistic curve for population (d) is taken as equal to 0.0144/63 millions. Concerning the long-run discount rate, the value we are considering in the simulation is $\rho = 1\%$ which can be found in The Green Book, H.H. Treasury ([12]).

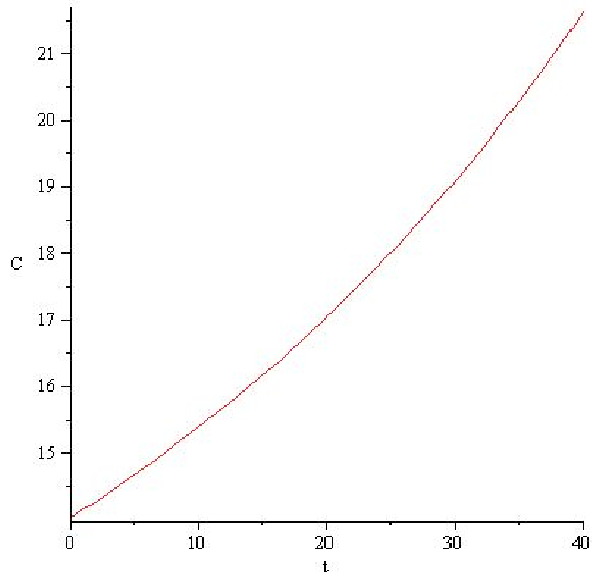

Figure 3: C_t

6 Concluding remarks

We have considered an economic growth model involving physical capital, technological progress, population dynamics and public expenditure. We assume logistic evolution equations for both population and public expenditure. While the use of a logistic-like function for population change is not new, the use of such a function in stating the behavior over time of the public expenditure to the GDP ratio deserves analysis since it represents the main novelty of this contribution. The use of the logistic model for describing the dynamics of the ratio of public expenditures and income is due to Florio and Colautti (2005). Within this framework we are interested in analyzing the dynamics of such variables (both in aggregate and in per-capita terms) as consumption, output and physical capital and their growth rates. Through numerical simulations, we show that this model admits an (asymptotically) balanced growth path equilibrium.

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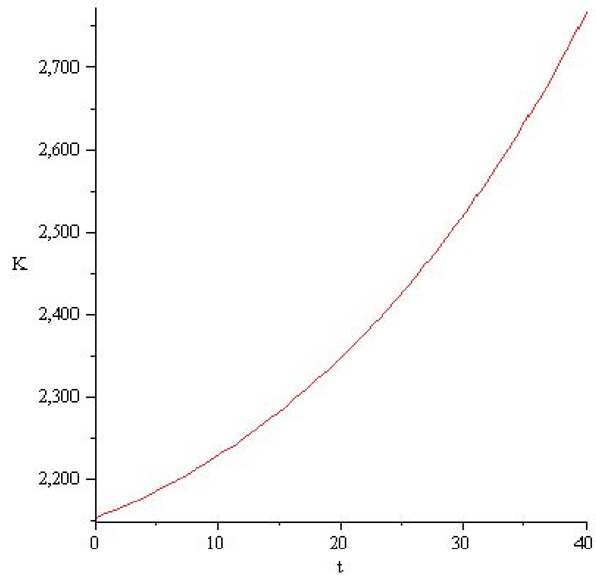

Figure 4: K_t

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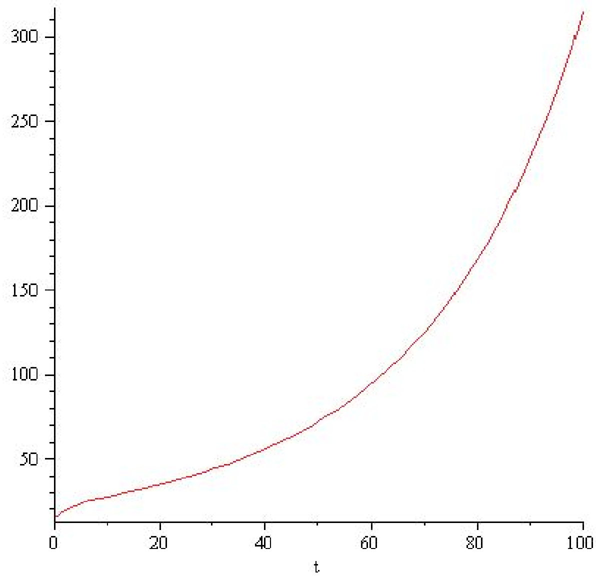

Figure 5: Y_t

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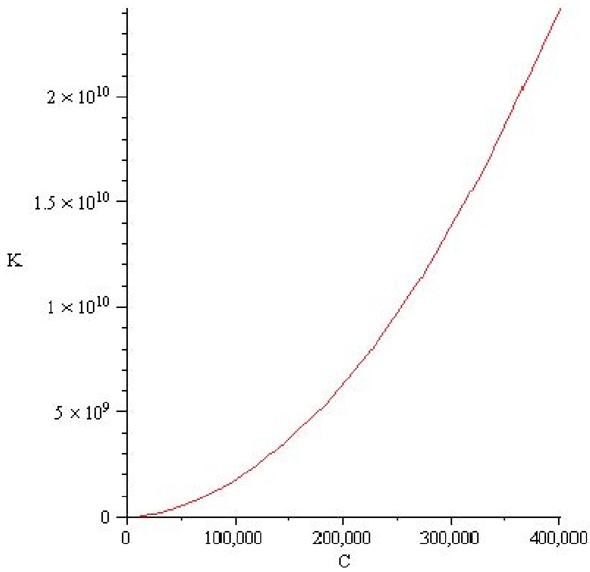

Figure 6: $\frac{C_t}{K_t}$

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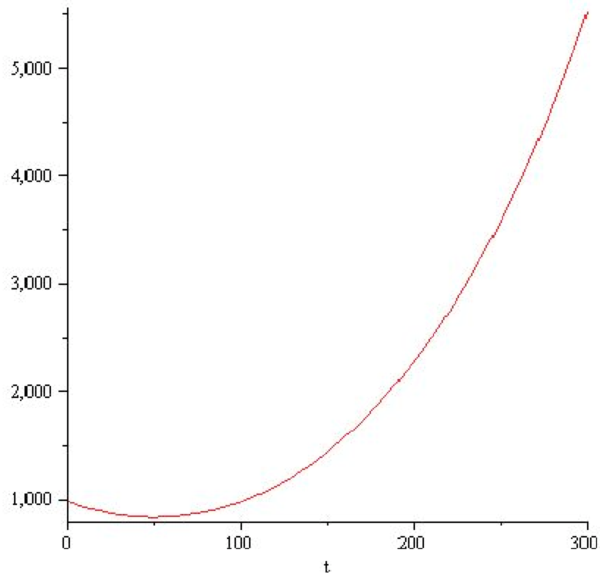

Figure 7: $\frac{K_t}{L_t}$

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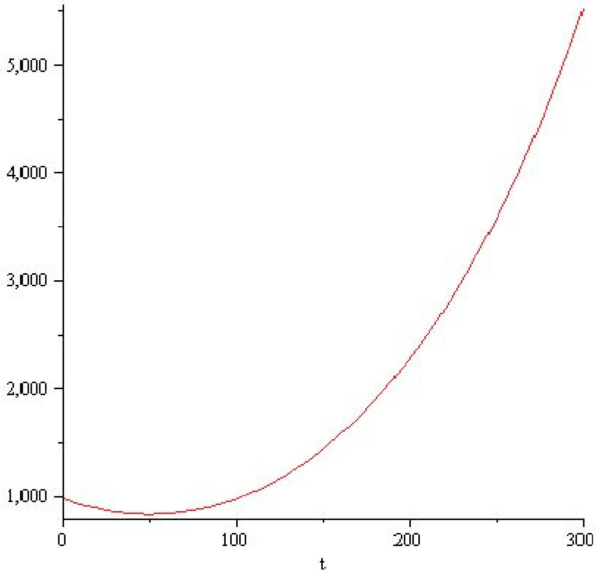

Figure 8: $\frac{C_t}{L_t}$

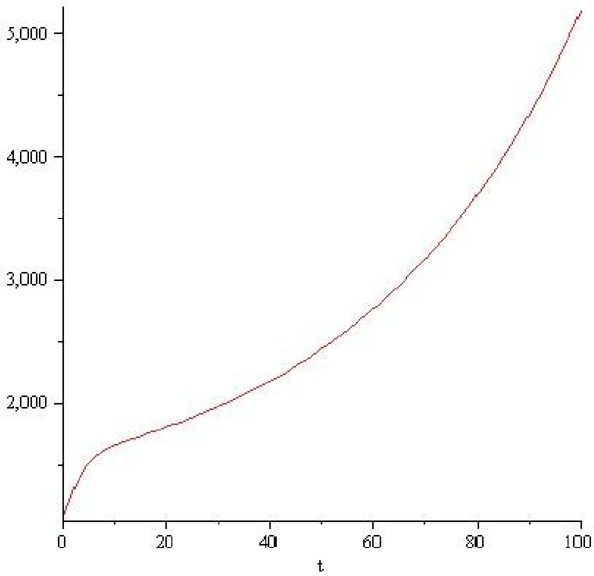

Figure 9: $\frac{Y_t}{L_t}$

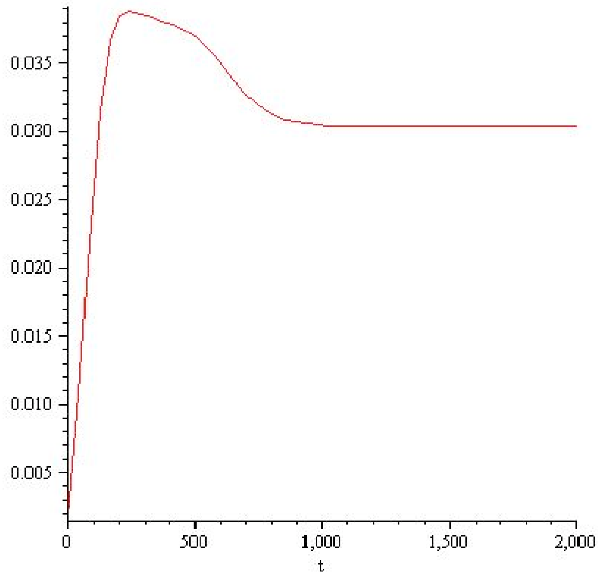

Figure 10: $\frac{\dot{K}_t}{K_t}$

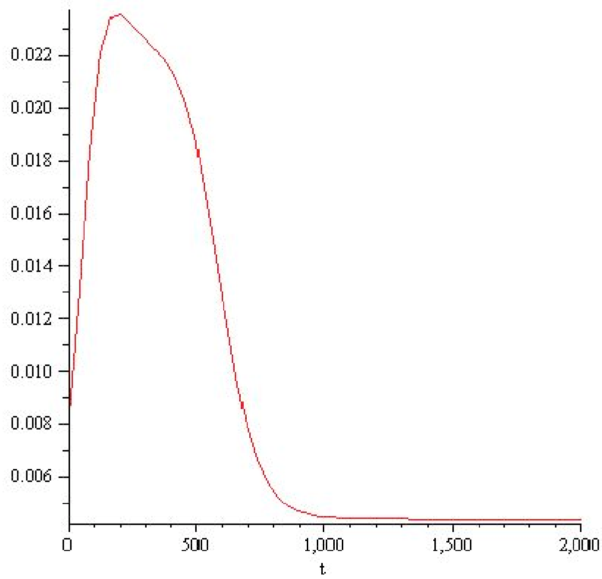

Figure 11: $\frac{\dot{C}_t}{C_t}$

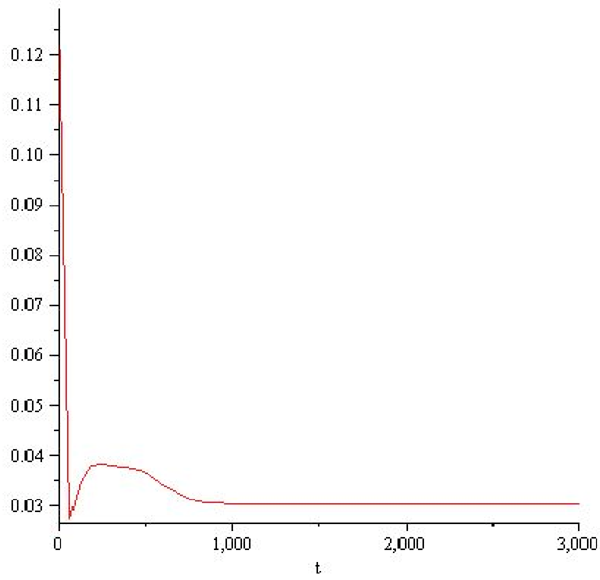

Figure 12: $\frac{\dot{Y}_t}{Y_t}$

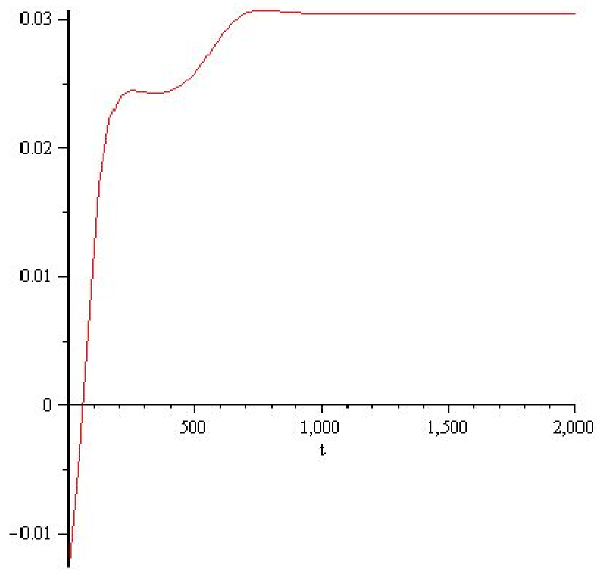

Figure 13: $\frac{\left(\frac{K_t}{L_t}\right)'}{\frac{K_t}{L_t}}$

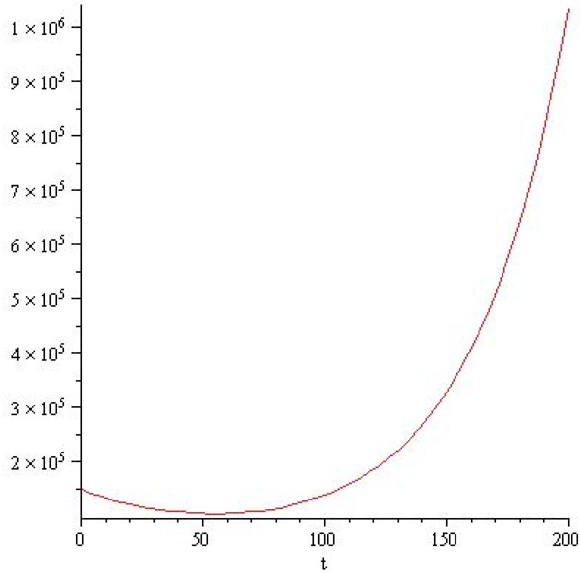

Figure 14: : $\frac{\left(\frac{C_t}{L_t}\right)'}{\frac{C_t}{L_t}}$

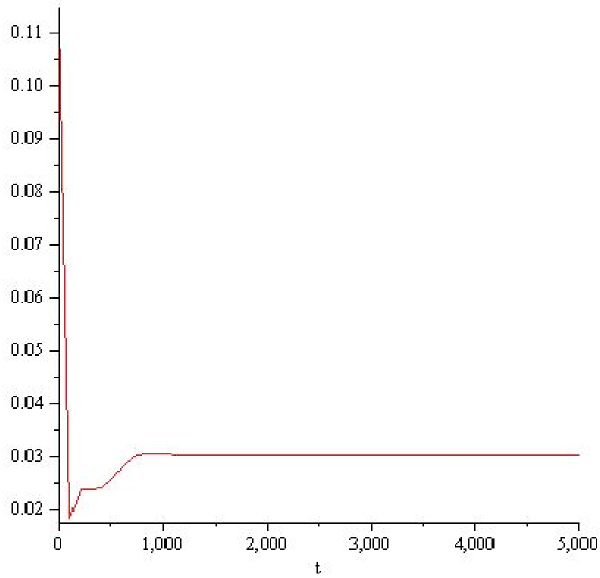

Figure 15: : $\frac{\left(\frac{Y_t}{L_t}\right)'}{\frac{Y_t}{L_t}}$