Società italiana di economia pubblica

XX CONFERENZA

ECONOMIA DELLA TASSAZIONE
Sistemi tributari, pressione fiscale, crescita

Pavia, Università, 25 - 26 settembre 2008

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pubblicazione internet realizzata con contributo della COMPAGNIA di San Paolo

società italiana di economia pubblica

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ON DETERMINING "CLOSE EQUALS GROUPS" IN DECOMPOSING REDISTRIBUTIVE AND RERANKING EFFECTS

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15/6/2008

Abstract

Recently van De Van, Creedy and Lambert (2001) and Lambert and Urban (2005) have reconsidered the original Aronson, Johnson and Lambert (1994) decomposition of the redistributive effect in order to individuate the optimal bandwidth that should be used in decomposing the redistributive effect when groups with close pre-tax incomes are considered. The methodology proposed by van De Van, Creedy and Lambert (2001) suggests choosing as the optimal bandwidth the one which maximizes the ratio between the potential effect V (which depends on the bandwidth) and the actual redistributive effect RE (which is invariant). Lambert and Urban (2005) discuss a set of further possible decompositions of the redistributive effect together with a decomposition of the Atkinson-Plotnick-Kakwani index into three terms. In this paper we want to contribute to throw some more light on the behavior of three of the main decompositions analyzed by Lambert and Urban (2005) in order to look for criteria for the choice of a bandwidth which allows the three different definitions of potential redistributive effect to assume as coherent as possible values and, in the meanwhile, to catch as much as possible of the potential vertical effect. We suggest looking for the bandwidth (or for a set of bandwidths) where the maximum distance among the different potential vertical effects is minimum, provided that the greatest of the three indexes is not lower than the global maximum assumed by the lowest of them, over the whole income distribution range.

JEL Classification Numbers: H23; H24.
Keywords: Personal Income Tax, Redistributive Effect, Horizontal Inequity, Reranking.

1. Introduction

Decomposing redistributive effect across groups of pre-tax equals into vertical, horizontal and reranking effect has been intensively studied in the last years. The original work by Aronson, Johnson and Lambert (1994), henceforth AJL, considers exact pre-tax equals in portioning the pre-tax income distribution.

As van de Ven, Creedy and Lambert (2001), henceforth VCL, pointed out, in the real word taxation this is not the case: only groups with close pre-tax incomes can be considered. They got through this problem in order to individuate the optimal bandwidth that should be used in decomposing the redistributive effect. Therefore, VCL methodology suggests choosing as the optimal bandwidth the one which maximizes the ratio between the potential effect (which depends on the bandwidth) and the actual redistributive effect (which is invariant). Here a problem arises as this ratio may have more than one relative maximum and presents a layout which may be irregular, and somewhere quite irregular; as a consequence, identifying univocally the maximum is not so obvious (Vernizzi and Pellegrino 2007).

Lambert and Urban (2005), henceforth LU, present an exhaustive discussion on a complete set of possible redistributive effect decompositions, and introducing new indexes based on the taxation of close equals by their average tax rate.

In this work we desire to contribute to VCL and LU discussion with some suggestions about the choice of a convenient bandwidth, by intensively looking to the empirical analysis. We would conclude to look for the bandwidth (or for a set of bandwidths) where the maximum distance among the considered possible definitions of potential vertical effect is minimum, provided that the greatest of the three indexes is not lower than the global maximum assumed by the lowest of the three indexes over the whole income distribution range.

The structure of the paper is as follows. In Section 2 we recall how the original AJL decomposition should be applied in the real world where strict equals groups are rare and, consequently, they must be replaced by "close equals" groups; to overcome this problem, according to LU's suggestions, alternative *RE* decompositions are introduced together with the decomposition of the Atkinson-Plotnick-Kakwani index (henceforth R^{APK})[1]. In section 3 we report the values the indexes assume at bandwidth limits, that is either when the bandwidth tend to zero or when it cover the whole income distribution range; then we sketch some preliminary a priori considerations about some aspects of their behavior. The empirical behavior of indexes is analyzed in Section 4. Section 5 discusses whenever a bandwidth with "optimal" or at least "desirable" properties can be identified. Section 6 concludes.

2. Redistribution and reranking indexes

Let G_y and G_{y-T} be the Gini index on the gross and net incomes respectively. The redistributive index *RE* is equal to $RE = G_y - G_{y-T}$. It is well-known that the Gini coefficient fails to decompose across subgroups into between and within group inequality components in case subgroup income ranges overlap. When considering the pre-tax income parade, if groups are selected in a sequential order, such that pooling all groups incomes are in a non decreasing order, we have that $G_y = G_y^B + G_y^W$, where G_y^B

[1] The decomposition of R^{APK} is described and discussed in Urban and Lambert (2005); for further details see Vernizzi (2007).

is the between-group Gini pre-tax index and G_y^W is the within-group Gini pre-tax index[2]. However, if post-tax income groups contain the same subjects they did before taxation, it is no longer granted that the after tax maximum value in the *i*-th group is not greater than the minimum value in the $i+1$-th group and that no intersection (or overlapping) effect appears among groups.

If taxation induces overlapping among groups, the post-tax Gini index becomes $G_{y-T} = G_{y-T}^B + G_{y-T}^W + G_{y-T}^t$, where [3] $G_{y-T}^t = R^{AJL} = G_{y-T} - \left(G_{y-T}^B + G_{y-T}^W\right)$.

When exact equals are considered

In their seminal paper, AJL not only organize groups so that no overlapping effect exists for pre-tax groups, but also implicitly assume that for the after-tax income parade (*i*) the group averages maintain the same ranking as before taxation and (*ii*) the within group orderings remain the same as before taxation. If this is the case, the post-tax concentration index (evaluated when post-tax incomes are ordered according to the order they had before taxation) is $\left(G_{y-T}^B + G_{y-T}^W\right)$, so that $R^{AJL} = R^{APK}$, being R^{APK} the Atkinson-Plotnick-Kakwani reranking index[4].

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[2] G_y^B is the Gini index for pre-tax incomes when within each group all incomes are substituted by their group average; $G_y^W = \sum_k a_{k,y} G_{k,y}$, where $G_{k,y}$ is the Gini index for the *k*-th group and $a_{k,y}$ is the product of the *k*-th group population share and pre-tax income share.

[3] G_{y-T}^B and $G_{y-T}^W = \sum_k a_{k,y-T} G_{k,y-T}$ are the analog forms for G_y^B and G_y^W when incomes have been taxed; in particular $a_{k,y-T}$ is the product of the *k*-th group population share and post-tax income share. G_{y-T}^t is what Dagum (1997) calls "the transvariation term". In UL notation $R^{AJL} = G_{y-T} - D_4$, where $D_4 = G_{y-T}^B + G_{y-T}^W$ is the concentration index for the after tax income parade, ordered according to non decreasing group averages and, within each group, in a non decreasing order. The relations which involves Gini and concentration indexes components are analyzed, e.g., in Vernizzi (2007).

[4] In UL notation D_1 is the concentration index for after-tax incomes, when ordered according to the before taxation ranking. D_1 may be different from $D_4 = G_{y-T}^B + G_{y-T}^W$ and, in general, it is. In our notation D_1 is D_{y-T}.

If we split G_y and G_{y-T} into the above described components, as AJL do, the redistributive effect can be written as $RE = \left(G_y^B - G_{y-T}^B\right) - \left(G_{y-T}^W - G_y^W\right) - R^{APK}$. A further simplification can be applied when the analysis is limited to the case in which the population groups contain exact pre-tax equals, which implies $G_y^W = 0$ and $G_y^B = G_y$. In this case the redistributive effect can be expressed as $RE = \left(G_y - G_{y-T}^B\right) - G_{y-T}^W - R^{APK}$.

AJL name $\left(G_y - G_{y-T}^B\right)$ the vertical potential redistributive which looses part of its potentiality whenever either the within-group inequality index G_{y-T}^W or the group overlapping index $R^{AJL} = G_{y-T}^t = G_{y-T} - \left(G_{y-T}^B + G_{y-T}^W\right) = R^{APK}$ becomes different from zero after taxation.

When close equals are considered

However, as observed before, this decomposition can be correctly applied provided that each group is composed by subjects with the same pre-tax income and taxation does not modify either the ranking among group averages or the within-group rankings (van de Ven, Creedy and Lambert, 2001; Urban and Lambert, 2005; Vernizzi, 2006).

In the real world, even for gross incomes, the within-group Gini index, G_y^W, is generally different from zero, as only groups with close pre-tax incomes can be considered. As a consequence, only bandwidths of income containing close-equals must be chosen.

Being more general, neither post-tax group means maintain the same order they had for the pre-tax income parade nor, within each group, the order of the incomes remains unchanged in the transition from the pre- to the post-tax incomes; in this case the residual of the *RE* decomposition is generally not equal to the APK index, whichcan be more generally defined as $R^{APK} = G_{y-T} - D_{y-T} = G_{y-T} - \left(D^B_{y-T} + D^W_{y-T}\right)$. D_{y-T} is the concentration index for the post-tax income parade when incomes are ranked according the pre-tax income non-decreasing ranking; D^B_{y-T} and D^W_{y-T} are, respectively, the between and the within group concentration indexes for post-tax income parade[5]. We can confirm these violations using a SHIW dataset, even if the magnitude of these unpleasant outcomes depends on the income range (bandwidth) chosen for each group. It is worth to stress that, according to empirical evidence[6], the income bandwidth acts in opposite directions towards group reranking and within-group reranking: the larger the bandwidth is, the less probable is the former and the more frequent happens to be the latter.

In addition, as the bandwidth increases, G^W_y can be no more close to zero, so that the redistributive effect can be no more evaluated as $RE = \left(G_y - G^B_{y-T}\right) - G^W_{y-T} - R^{APK} = \left(G^B_y - G^B_{y-T}\right) - G^W_{y-T} - R^{APK}$; it becomes more realistic to turn back to the more complete decomposition

$$RE = \left(G^B_y - G^B_{y-T}\right) - \left(G^W_{y-T} - G^W_y\right) - R^{AJL} = V^{VCL} - H^{VCL} - R^{AJL} \qquad (1)$$

having defined $V^{VCL} = \left(G^B_y - G^B_{y-T}\right)$ and $H^{VCL} = G^W_{y-T} - G^W_y$.

When using the above decomposition, one gives back the idea of constituting close-equals groups, and focuses on the eventual enlargement of the within-group inequality

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5 D^B_{y-T} is defined as the concentration index when all incomes inside each group are substituted by the group income average and, moreover, groups are ordered according to pre-tax group averages. $D^W_{y-T} = \sum_k a_{k,y-T} D_{k,y-T}$, where $D_{k,y-T}$ is the concentration index for the *k*-th group, when after tax incomes are ordered according to their pre-tax order, and $a_{k,y-T}$ is the product of the *k*-th group population share and post-tax income share.

6 Lambert and Urban (2005), Vernizzi and Pellegrino (2007).

$\left(G_{y-T}^{W}-G_{y}^{W}\right)=H^{VCL}$ term, together with the group overlapping term R^{AJL}, to measure the loss in potential vertical redistribution effect which is measured by $\left(G_{y}^{B}-G_{y-T}^{B}\right)=V^{VCL}$.

UL present other *RE* decompositions which holds also either when groups do not include just equals or between or within groups rerankings are introduced by taxation. Here we shall consider two of these decompositions, both of them apply the idea of smoothed taxation within group, which is introduced by UL in coherence with the principle of close equals groups: if groups contain close equals, their incomes should be taxed by a same tax rate, which can be properly estimated by the group average tax rate. After having applied a same tax rate to all incomes in group *k*, the Gini index for group *k* remains exactly equal to the pre-tax $G_{k,y}$; however the smoothed within group Gini index $G_{y-T}^{SW}=\sum_{k}a_{k,y-T}G_{k,y}$ is generally different from $G_{y}^{W}=\sum_{k}a_{k,y}G_{k,y}$, because in general $a_{k,y}\neq a_{k,y-T}$.

UL define $RE=V^{AJL}-H^{AJL}-R^{AJL}$, where $V^{AJL}=G_{y}-\left(G_{y-T}^{B}+G_{y-T}^{SW}\right)$ and $H^{AJL}=G_{y-T}^{W}-G_{y-T}^{SW}$, so that:

$$RE=V^{AJL}-H^{AJL}-R^{AJL}=\left(G_{y}-G_{y-T}^{B}-G_{y-T}^{SW}\right)-\left(G_{y-T}^{W}-G_{y-T}^{SW}\right)-R^{AJL} \qquad (2)$$

In expression (2) the potential vertical effect is measured by the difference between the pre-tax Gini index and the Gini index for an artificial post tax income parade, which, by constructions, excludes any group overlapping[7].

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[7] UL define V^{AJL} and H^{AJL} in an apparently different way. They define D_5 and D_6 as concentration indexes calculated on smoothed net incomes: the D_5 index ranks groups according to the same order they had before taxation, even if the taxation changed the income average order among groups; the D_6 index ranks groups that are ranked in a non decreasing order with respect

The "pure" horizontal inequity is measured by the enlargement of within group inequality, with respect to what would induce a smoothed taxation; group overlapping introduced by taxation, is measured by R^{AJL} as in equation (1).

Both expression (1) and (2) keep into account only a part of horizontal inequity, eventually introduced by a taxation system, in fact the two *RE* decompositions do not consider within group and between group eventual rerankings. Actually, the Atkinson-Plotnick-Kakwani index R^{APK} can be decomposed into three terms [8]: $R^{APK} = R^{AJL} + R^{EG} + R^{WG}$. Together with the overlapping term R^{AJL} which has been already described, there are two further terms: the former, R^{EG}, measures the horizontal inequity due to the reranking of the mean post-tax income among groups, whilst R^{WG} measures the reranking effect due to within groups reshuffling. More in detail [9] $R^{EG} = G^{B}_{y-T} - D^{B}_{y-T}$ and $R^{WG} = G^{W}_{y-T} - D^{W}_{y-T}$.

The latter UL decomposition we consider is $RE = V - H - R^{APK}$, where $V = G_y - \left(D^{B}_{y-T} + G^{SW}_{y-T}\right)$ and $H = D^{W}_{y-T} - G^{SW}_{y-T}$. Then:

$$RE = V - H - R^{APK} = \left(G_y - D^{B}_{y-T} - G^{SW}_{y-T}\right) - \left(D^{W}_{y-T} - G^{SW}_{y-T}\right) - R^{APK} \qquad (3)$$

UL notice that decomposition (3) has the advantage of synthesizing the whole information set into one equation[10]. Table 1 summarizes Gini and concentration indexes definitions.

TABLE 1 ABOUT HERE

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to their post-tax average incomes. D_3 is the concentration index for (non-smoothed) after tax incomes, when groups follow the same order as before taxation, whilst within group incomes are in non decreasing order; then $V^{AJL} = G_y - D_6$ and $H^{AJL} = D_4 - D_6 = D_3 - D_5$.

[8] Lambert and Urban (2005). See also Vernizzi (2006) for analytical details.

[9] UL define $R^{EG} = D_4 - D_3$ and $R^{WG} = D_3 - D_1$.

[10] UL define *V* and *H*, respectively, as $V = G_y - D_5$ and $H = D_1 - D_5$.

What decomposition is more suitable to analyze the redistributive effect and what bandwidth should be chosen is a problem not definitely solved: VCL suggest choosing a bandwidth where $\left(V^{VCL}/RE\right)$ is maximum. This ratio may have more than one relative maximum and presents a layout which may be irregular, and somewhere quite irregular; as a consequence, identifying univocally the maximum is not so obvious. We got through this problem.

3. A priori considerations on indexes behavior

On a priori considerations, we can easily state the values that the here considered indexes assume at bandwidth limits, that is either when bandwidth tends to zero or the maximum available range (Table 2).

TABLE 2 ABOUT HERE

When the bandwidth tends to zero, $V^{VCL} = V^{AJL} = RE$, $V = G_y - D_{y-T}$ (the Reynolds-Smolenky total redistribution index) and $H^{VCL} = H^{AJL} = H = 0$; it follows that at bandwidth zero $V \geq V^{VCL} = V^{AJL}$. Conversely, when the bandwidth is maximum, that is equal to the observed income range, $V^{VCL} = V^{AJL} = V = 0$, $H^{VCL} = H^{AJL} = -RE$ and $H = -\left(G_y - D_{y-T}\right)$, so that when the bandwidth coincides with the maximum range $H \leq H^{VCL} = H^{AJL}$. In what it concerns the reranking effects, we have that when the bandwidth is zero $R^{AJL} = R^{WG} = 0$ and $R^{EG} = R^{APK}$, whilst for maximum bandwidth $R^{AJL} = R^{EG} = 0$ and $R^{WG} = R^{APK}$.

The difference $V - V^{AJL}$ is equal to R^{EG}, which being non-negative, implies V to dominate V^{AJL}. Less evident is the relation between V^{VCL} and V^{AJL} and, especially, that between V and V^{VCL}: in fact $V^{VCL} - V^{AJL} = G^{SW}_{y-T} - G^{W}_{y}$ and $V - V^{VCL} = R^{EG} - \left(G^{SW}_{y-T} - G^{W}_{y}\right)$.

In order to throw some light on these relations, we recall how G^{W}_{y} and G^{SW}_{y-T} can be represented as weighted sums of average absolute differences, calculated within each group:

$$\begin{aligned} G^{W}_{y} &= \frac{1}{2n^2\mu}\sum_{i}^{K}\Delta_{i,y}n_i^2 \\ G^{SW}_{y-T} &= \frac{1}{2n^2\mu\left(1-\overline{t}\right)}\sum_{i}^{K}\Delta_{i,y}\left(1-t_i\right)n_i^2 \end{aligned} \tag{4}$$

where μ is the average income for the whole subjects considered in the sample, n is the number of equivalent subjects in the sample, n_i is the number of equivalent subjects [11] in group k, t_i is the i-th group average tax rate, $\overline{t}$ is average tax rate for the whole sample, $\Delta_{i,y} = \left(2/n_i^2\right)\sum_{s=1}^{k_i}\sum_{h>s}^{k_i}\left(y_{i,h} - y_{i,s}\right)n_{i,s}n_{i,h}$, having defined with k_i the number of cases registered in group i and with $n_{i,s}$ the weight associated to income y_i; within each group incomes are ranked in a non decreasing order. We can then write:

$$\begin{aligned} G^{SW}_{y-T} - G^{W}_{y} &= \frac{1}{n^2\mu\left(1-\overline{t}\right)}\sum_{i=1}^{K}G_{i,y}n_i^2\mu_i\left(\overline{t}-t_i\right) = \\ &= \frac{1}{2n^2\mu\left(1-\overline{t}\right)}\sum_{i=1}^{K}\Delta_{i,y}\left(\overline{t}-t_i\right)n_i^2 \end{aligned} \tag{5}$$

Due to the asymmetry of income distributions, which makes bandwidths in the left tail to be more crowded than those in the right tail, in (5) positive $\left(\overline{t}-t_i\right)$'s, are likely to

[11] A sum of equivalent subjects may be a non integer number.

receive a weight more than proportional than the negative $\left(\overline{t}-t_i\right)$'s; if this is the case, V^{VCL} is expected to be not lower than V^{AJL}[12].

Turning to V and V^{VCL}, the sign of the difference $V-V^{VCL}$ depends on the difference $R^{EG}-\left(G^{SW}_{y-T}-G^{W}_{y}\right)$, where $G^{SW}_{y-T}-G^{W}_{y}$ is likely non negative, due to the above considerations, and R^{EG} is surely non-negative. So, on a priori considerations, we can but conclude that for the bandwidth tending to zero $V-V^{VCL}$ has R^{APK} as its limit, and for the bandwidth tending to the maximum range, $V-V^{VCL}$ has zero as its limit.

Turning now to the horizontal loss measures, we observe that $H^{AJL}-H$ is equal to R^{WG}, the within group reranking index[13], which is non-negative and that the difference between $H^{VCL}-H$ is equal to the sum of R^{WG} and $\left(G^{SW}_{y-T}-G^{W}_{y}\right)$: being the former always non-negative and the latter likely non negative, we expect that $H^{AJL}\geq H$, moreover given that $\left(G^{SW}_{y-T}-G^{W}_{y}\right)$ is non-negative, H^{VCL} should be not lower than H^{AJL}, so that, summarizing, we expect that $H^{VCL}\geq H^{AJL}\geq H$, where the second inequality always holds.

4. Empirical analysis

In this section we investigate by an empirical analysis how the group bandwidth influences the components of the redistribution index V^{VCL}, H^{VCL} and R^{AJL} in equation

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[12] For instance when the income range is split into two groups, each having the same spread but not necessarily the same number of subjects, being $\overline{t}=\left(t_1\mu_1 n_1+t_2\mu_2 n_2\right)/\left(\mu_1 n_1+\mu_2 n_2\right)$ and assuming that $\Delta_{1,y}$=$\Delta_{2,y}$=$\Delta_{\cdot,y}$, we can write $G^{SW}_{y-T}-G^{W}_{y}=\left\{1/\left[n^2\mu\left(1-\overline{t}\right)\right]\right\}\cdot\left\{\Delta_{\cdot,y}\left[\left(\overline{t}-t_1\right)n_1^2+\left(\overline{t}-t_2\right)n_2^2\right]\right\}=\left\{1/\left[n^2\mu\left(1-\overline{t}\right)\right]\right\}\cdot\left(\mu_2 n_1-\mu_1 n_2\right)n_1 n_2$ which is greater than zero if $n_1>n_2$.

[13] $R^{WG}=G^{W}_{y-T}-D^{W}_{y-T}=\dfrac{1}{2n^2\mu\left(1-\overline{t}\right)}\sum_{i=1}^{K}\left(G_{i,y-T}-D_{i,y-T}\right)n_i^2\mu_i\left(1-t_i\right)=\dfrac{1}{n^2\mu\left(1-\overline{t}\right)}\sum_{i=1}^{K}R_i^{APK}n_i^2\mu_i\left(1-t_i\right)$, having defined with R_i^{APK} the Atkinson-Plotnick-Kakwani for the *i*-th group.

(1), V^{AJL} and H^{AJL} in equation (2), and V in equation (3), together with the components of the Atkinson-Plotnick-Kakwani index $R^{APK} = R^{AJL} + R^{EG} + R^{WG}$.

As stated before, our aim is either to contribute to the discussion about the choice of a proper bandwidth: a proper bandwidth should catch as much as possible of the potential redistributive effect and, in the meanwhile, should get as close as possible measures from the three indexes V^{VCL}, V^{AJL} and V.

Our experiment was conducted on the basis of the Bank of Italy survey on households incomes and wealth (SHIW). The 2004 Italian SHIW dataset provides demographic and post-tax income microdata for a representative cross-section of 12,713 taxpayers and 8,012 households (20,581 individuals). This data were used to obtain gross and net incomes according to the Italian Personal Income Tax (Pellegrino, 2007b). In order to deal in some way with two different data bases, the experiment was conducted with respect to both individual and family equivalent incomes. Equivalent incomes were obtained by dividing total family incomes by an equivalence scale; the scale here adopted is the Cutler scale which can be expressed as $CS_h = \left(NA_h + \alpha NC_h\right)^{\beta}$, having (arbitrarily) set $\alpha = 0.5$ and $\beta = 0.65$. Ebert and Moyes (2000) observe that, in applying equivalence scales, the choice of the weight may be arbitrary: we consequently decided to weigh equivalent incomes by the lower and the upper bound, the former being 1[14] and the latter being the component number associated to each family[15]. Once the 2004 gross income parade was obtained, the 2006 and 2007 distributions were estimated considering the impact of the inflation rate (Pellegrino, 2007c). We found that results are quite analogous either for weights equal to family components or for all

[14] $\alpha = \beta = 0$.

[15] $\alpha = \beta = 1$.

weights equal to 1; moreover, results are also very similar across years, so that, for the sake of simplification, here we only report results referred to year 2004, for individuals and for household equivalent incomes - weight 1[16]. Figures 1 and 1bis show the behavior of the three potential redistributive effects, V^{VCL}, V^{AJL} and V, are plotted together with the constant line of the Reynolds-Smolenky total redistributive effect.

The three indexes which measure the loss in horizontal equity, H^{VCL}, H^{AJL} and H, are reported in Figure 2 and 2bis, together with R^{AJL} and R^{APK}, the latter being constant; all the above measures are expressed as percentages of the redistributive effect RE.

The decomposition of R^{APK} is represented in Figure 3 and 3bis: R^{AJL}, R^{EG} and R^{WG} are there expressed as percentages of R^{APK}.

As we noticed in the previous section, in correspondence of a zero bandwidth, both V^{VCL} and V^{AJL} are equal to RE, whilst V is equal to $G_y - D_{y-T}$, the Reynolds-Smolensky redistribution index, which is greater than RE. For our minimum bandwidth, which is 10 euro large, in Figure 1 V^{VCL} and V^{AJL} show a 0.7 % increase (0.8 % when dealing with family equivalent incomes: Figure 1bis) with respect to the limit value for the bandwidth tending to zero, that is RE. Both Figure 1 the two lines are not distinguishable and show a steep ascent up to bandwidths around 300 euro large; then V^{AJL} leaves V^{VCL} and becomes undistinguishable from V for bandwidths larger than 400, when considering individuals, and larger than 550-600 when considering family equivalent incomes. V shows a decreasing trend; before becoming undistinguishable from V^{AJL} it dominates V^{VCL} and V^{AJL}, then the V line crosses V^{VCL} and continues descending together with V^{AJL}, leaving the V^{VCL} line above. When the

[16] Even if limited to V^{VCL}, H^{VCL}, R^{AJL}, R^{EG} and R^{WG}, Vernizzi and Pellegrino (2007) reports all graphs for the three tax systems (2004, 2004 and 2006), concerning both individuals and family equivalent incomes (weight 1 and weight equal to family components).

bandwidth is 3,000 euro large, for individuals the three lines are still greater than *RE*: V^{VCL} is almost 1.0% greater than *RE* (1.2% families), V^{AJL} and *V* are only 0.4% greater than *RE* (0.6% families).

Even if our analysis tries to focus on small bandwidths than LU do, our findings are substantially consistent with LU results; what appears to be different is that the lines presented by LU look much more regular the ones here represented. Our lines are the more irregular the more they depart from the axes origin: the irregularities are more similar to irregular waves than to completely random white noises.

More in detail, we observe that:

(*i*) $V^{VCL} \approx V^{AJL}$ as long as $G^{SW}_{y-T} \approx G^{W}_{y}$; V^{VCL} becomes greater than V^{AJL} when G^{SW}_{y-T} becomes sensibly greater than G^{W}_{y} ;

(*ii*) $V^{AJL} \approx V$ after R^{EG} becomes ≈ 0; as long as R^{EG} is not negligible $V^{AJL} < V$;

(*iii*) $V^{VCL} \approx V$ for bandwidths where $R^{EG} \approx \left(G^{SW}_{y-T} - G^{W}_{y}\right)$; $V^{VCL} < V$ as long as $R^{EG} > \left(G^{SW}_{y-T} - G^{W}_{y}\right)$ and, conversely, $V^{VCL} > V$ after R^{EG} becomes lower than $\left(G^{SW}_{y-T} - G^{W}_{y}\right)$.

FIGURES 1-2bis ABOUT HERE

Figure 2 and 2bis represent the behavior of the three indexes which measures the horizontal effect, together with R^{AJL}, the overlapping index, and R^{APK}, the global reranking index; as it is expected, the three indexes here considered, H^{VCL}, H^{AJL} and *H*, assume a value which is very close to zero when the bandwidth is 10 euro large. *H* presents few and insignificant positive values just for the tiniest bandwidth; then it starts a descending trend towards the limit value $-\left(G_{y} - D_{y-T}\right)$. Conversely in correspondence of bandwidths 10-3,000 large, here considered, H^{VCL} and H^{AJL} always present positive values. In particular when the bandwidth is 3,000 euro large, H^{VCL} looks to be still increasing, whilst H^{AJL} has already started the descending trend. Similarly to V^{VCL}, V^{AJL} and V, when bandwidths become large, H^{VCL}, H^{AJL} and H present relatively strong irregularities.

FIGURES 3-3bis ABOUT HERE

The decomposition of R^{APK} is represented in Figure 3 and 3bis: R^{AJL}, R^{EG} and R^{WG} are expressed as percentages of R^{APK}. R^{AJL}, which is zero both at bandwidth zero and at bandwidth maximum, shows a quite asymmetric line (as it could be noticed also from Figures 2-2bis, where it has just been rescaled by R^{APK}/RE): for individuals, at 10 euro bandwidth it has already jumped up to 67% of R^{APK} (58% for families) and it reaches its maximum value, 88%, at the 100 euro bandwidth (86% for families) then it begins to descend and at a 3,000 euro bandwidth it is roughly at a 25% of R^{APK}. R^{EG}, which coincides with is R^{APK} when the bandwidth is a point bandwidth, is 32,4% of R^{APK} at the 10 euro bandwidth (40% for families); it decreases quite soon and at a 300 euro bandwidth is already less that 1% of R^{APK}. R^{WG} appears to be a direct function of the bandwidth, even if at decreasing rates: at a 3,000 bandwidth it is nearly 80% of R^{APK} (70% for families). Similarly to what happens for the potential vertical indexes and the horizontal iniquity indexes, as bandwidths becomes large, R^{AJL} and R^{WG} present relatively strong irregularities: which does happen for R^{EG}, due to the fact that this index is quite low for large bandwidths.

To better investigate the behavior of H^{VCL}, H^{AJL} and H, we recall expression (4) where G_y^W and G_{y-T}^{SW} and G_y^W were represented as weighted sums of average absolute differences, calculated within each group: moreover we can similarly define G_{y-T}^W as

$$G_{y-T}^W = \frac{1}{2n^2\mu\left(1-\bar{t}\right)}\sum_i^K \Delta_{i,y-T} n_i^2 \tag{6}$$

From (4) and (6) it follows that we can represent H^{VCL} as:

$$\begin{aligned} H^{VCL} = G_{y-T}^W - G_y^W &= \sum_{i=1}^K \left(G_{i,y-T} \frac{n_i^2 \mu_i \left(1-t_i\right)}{n^2 \mu\left(1-\bar{t}\right)} - G_{i,y} \frac{n_i^2 \mu_i}{n^2 \mu} \right) = \\ &= \frac{1}{2n^2\mu\left(1-\bar{t}\right)} \sum_{i=1}^K \left[\Delta_{i,y-T} - \Delta_{i,y}\left(1-\bar{t}\right)\right] n_i^2 \end{aligned} \tag{7}$$

and H^{AJL} as:

$$\begin{aligned} H^{AJL} = G_{y-T}^W - G_{y-T}^{SW} &= \sum_{i=1}^K \left(G_{i,y-T} - G_{i,y}\right) \frac{n_i^2 \mu_i}{n^2 \mu} = \\ &= \frac{1}{2n^2\mu\left(1-\bar{t}\right)} \sum_{i=1}^K \left[\Delta_{i,y-T} - \Delta_{i,y}\left(1-t_i\right)\right] n_i^2 \end{aligned} \tag{8}$$

For lower income groups, where the tax rate for each subject may be much lower than $\bar{t}$, $\Delta_{i,y-T}$ may be greater than $\Delta_{i,y}\left(1-\bar{t}\right)$, even if $\Delta_{i,y-T} \le \Delta_{i,y}$ which may cause H^{VCL} to result positive: in fact due to the asymmetry of income distributions, it is likely that lower income intervals contain more subjects than higher income groups do, so that the weighed sum represented in (5) may result to be positive even if $\Delta_{i,y-T} \le \Delta_{i,y}, \forall k$. Then we can conclude that the H^{VCL} remains positive until bandwidths are large enough to make a sufficient number of $\Delta_{i,y-T}$'s –especially in the left hand side of the distribution- small enough to be less than their corresponding pre-tax $\Delta_{i,y}$, multiplied by $\left(1-\overline{t}\right)$[17].

When dealing with incomes in the right distribution queue, the contrary happens, but, due to the distribution asymmetry, in the left hand tail income groups generally present weights greater than those in the right hand tail.

Turning now to H^{AJL}, being for lower incomes $\left(1-t_i\right) \geq \left(1-\overline{t}\right)$, in the left distribution queue, the relation $\Delta_{i,y-T} \leq \Delta_{i,y}\left(1-t_i\right)$ is more likely to be verified than the relation $\Delta_{i,y-T} \leq \Delta_{i,y}\left(1-\overline{t}\right)$. this consideration should explain why H^{AJL} starts to decrease much before than H^{VCL}. In any case it is excluded that H^{AJL} is positive when all groups post-tax Gini indexes are lower than the corresponding pre-tax ones.

Let's now define $\Delta^{D}_{i,y-T} = \left(2/n_i^2\right)\sum_{s=1}^{k_i}\sum_{h>s}^{k_i}\left(\tilde{y}_{i,h}-\tilde{y}_{i,s}\right)n_{i,s}n_{i,h}$, where $\tilde{y}_{i,h}$ is net income for subject *h* in group *i*, being incomes here ordered according to pre-tax ranking; we can now express *H* as:

$$
\begin{aligned}
H = D^{W}_{y-T} - G^{SW}_{y-T} &= \sum_{i=1}^{K}\left(D_{i,y-T} - G_{i,y}\right)\frac{n_i^2 \mu_i \left(1-t_i\right)}{n^2 \mu \left(1-\overline{t}\right)} = \\
&= \frac{1}{2n^2 \mu\left(1-\overline{t}\right)}\sum_{i=1}^{K}\left[\Delta^{D}_{i,y-T} - \Delta_{i,y}\left(1-t_i\right)\right]n_i^2
\end{aligned}
\tag{9}
$$

Looking at Figure 2 and 2bis an we realize that $\Delta_{i,y}\left(1-t_i\right)$ becomes greater than $\Delta^{D}_{i,y-T}$ even when bandwidths are not so large[18], at least for the left distribution queue where

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[17] We observe also that H^{VCL} may be positive even when all groups post-tax Gini indexes are lower than the corresponding pre-tax ones, due two the different weight system: for lower incomes after tax weights should in fact be higher then the corresponding pre-tax ones and the reverse should hold for higher incomes.

[18] We observe that $\Delta^{D}_{i,y-T}$ is $\leq \Delta_{i,y-T}$, which helps to explain why *H* becomes negative much before than H^{AJL}.

groups are more crowded, and, consequently, receive a weight which is heavier than in the right one.

Table 3 reports the values for RE and R^{APK} decompositions, evaluated at bandwidths 100, 200, 300, 400, 500, 600, 700 and 2,000; together with their standard errors obtained by 2,000 bootstrap replications. From the figures reported in the tables it stems that the ratios between the indexes and their standard errors are generally quite high, but those which concern $\left(R^{EG}/R\right)$. The ratios $\left(R^{EG}/R\right)/SE\left\{R^{EG}/R\right\}$ range from 6.78 to 8.13 when the bandwidth is 100 euros, from 0.75 to 2.20 when the bandwidth is enlarged up to 700 euros, and they are not greater than 1.36 when the bandwidth is 2000. It is worth stressing that the 95% bootstrap percentiles are generally quite similar to those calculated assuming normality but those related to $\left(R^{EG}/R\right)$; this result is in line with UL findings: their simulations lead to the conclusion that the distribution for R^{EG} is asymmetric while the distributions for the other indexes they consider are symmetric and, moreover, that the bootstrap estimated standard error for R^{EG} is almost twice than that of the true distribution. Then we can conclude that the point estimates for RE components should be quite reliable. The same should hold for R^{WG}/R and R^{AJL}/R; $\left(R^{EG}/R\right)$ stays apart, perhaps due its relatively small magnitude, but not only for this reason: R^{WG}/R is small when the bandwidth is 100 euros, nevertheless it shows lower standard errors and bootstrap confidence intervals more similar to those obtained by the normal distribution. Some cautiousness should be adopted also for H when it assumes small absolute values.

Tables 3 and 4 ABOUT HERE

5. On determining an "optimal" bandwidth

VCL propose their decomposition with the idea that, in measuring the vertical effect by the pre-tax and post-tax distribution of group income averages, V^{VCL} should eliminate either of measurement errors or anomalous values, by averaging within group incomes: of course, on one side the larger the groups are the more efficacious the smoothing performed by averaging is, but, on another side, the larger the groups are the less equals incomes are within groups. VCL suggest to choose the bandwidth which maximizes the potential redistributive effect. AJL appears to be quite appealing for the horizontal effect measure adopted: as we stressed in the previous paragraph, H^{AJL} cannot result in being positive when all groups post-tax Gini indexes are lower than the corresponding pre-tax ones, which cannot be excluded at all for H^{VCL}.

H presents the undoubted advantage of been considered together R^{APK} and then not only with R^{AJL}: however its interpretation is not straight as LU notice.

Looking either at Figure 1 and 1bis in the present paper or to LU figures, we can see that, in correspondence of some bandwidths, the three potential redistributive effects are quite close, or even coincide. We would observe even closer values when the bandwidth tends to cover the whole income range: in this case, however, as already noticed, the three indexes would tend to zero and would not capture any potential vertical redistribution at all!

We would then suggest to identify as "optimal" the bandwidth where (*i*) the maximum distance among V, V^{VCL} and V^{AJL} is minimum, (*ii*) provided that there the highest of V, V^{VCL} and V^{AJL} is not lower then the lowest among the global maxima that the three indexes assume over the whole set of bandwidths ranging from zero to the maximum income spread.

Figures 4 and 4bis ABOUT HERE

If we consider the three distances $\left|V - V^{VCL}\right| = \left|R^{EG} - \left(G_{y-T}^{SW} - G_{y}^{W}\right)\right|$, $V - V^{AJL} = R^{EG}$ and $V^{VCL} - V^{AJL} = \left(G_{y-T}^{SW} - G_{y}^{W}\right)$, looking at Figures 4 and 4bis, the minimum for the maximum of the distances is reached when $R^{EG} = \left(G_{y-T}^{SW} - G_{y}^{W}\right)$, at a 300 euro bandwidth for individuals and at a 380 euro bandwidth for family equivalent incomes.

The behavior shown by R^{EG} in our empirical analysis is confirmed also by LU analysis: when bandwidths become large group average re-rankings annihilates, soon or later, depending on tax fairness: for individuals R^{EG} becomes zero sooner then for families. If we go back to expression (5) we can understand that $\left(G_{y-T}^{SW} - G_{y}^{W}\right)$ is totally negligible for small bandwidths, where $\left(n_i^2/n^2\right)$ results to be quite small and their sum is much less than 1 when bandwidths are tiny and then little crowded. As bandwidths increase, a more than proportional increase in $\left(n_i^2/n^2\right)$ is not compensated by a convergence of the t_i towards their average $\bar{t}$ and the difference between G_{y-T}^{SW} and G_{y}^{W} increases more than proportionally. Surely $\left(G_{y-T}^{SW} - G_{y}^{W}\right)$ is expected to decrease and to tend to zero as bandwidths enlarge, being $\lim_{b \to MAX} G_{y-T}^{SW} = \lim_{b \to MAX} G_{y}^{W} = G_{y}$, but, as already observed, when their difference reaches zero, *V*, V^{VCL} and V^{AJL} become much lower than *RE*, that is much lower than the global maximum of V^{AJL}.

Looking at individuals, for 280-300 euro bandwidths $R^{EG} = G_{y-T}^{SW} - G_{y}^{W}$ is 0.005% of *RE*, which means the 1.5% of the maximum value attained by R^{EG} (bandwidth 10 euro large) and the 1% of the value attained by $\left(G^{SW}_{y-T} - G^{W}_{y}\right)$ at 3000 euro bandwidth; this explains why at a 280 euro bandwidth the three potential vertical redistribution indexes look to be equal. Bit larger percentages hold for family equivalent incomes [19]: in Figure 1bis the distance between V^{AJL} and V=V^{VCL} appears to be a little greater than that observed for individuals, even if, in any case, quite limited.

Looking at Figures we can notice that the maximum for V^{AJL}, the lowest among the three indexes, lies quite close to the point where V^{VCL} crosses V, so we can confirm that according the suggested criteria, the bandwidth where R^{EG} crosses $G^{SW}_{y-T} - G^{W}_{y}$ can be identified as optimal. Observe that $V^{AJL} \le V^{VCL}$ holds together with, $V^{AJL} \le V$, V^{VCL} crosses V when the former is still increasing and the latter already decreasing, the global maximum for V^{AJL} should fall, as it actually falls, between the bandwidth where the separation between V^{VCL} and V^{AJL} becomes evident, and the bandwidth where V^{VCL} and V become no more distinguishible.

We add that in our empirical analysis, in the interval between 250-370 euro, V^{AJL} oscillates from 99.97% and 99.99% of the Reynolds-Smolensky index, for individual; for families the percentage ranges from 99.97% to 99.98% when the interval is 340-440 euro: in a neighbourhood of the optimal bandwidth the three indexes absorb most of the total redistributive effect. We conclude observing that at the optimal bandwidth the horizontal loss measured by H^{VCL} and H^{AJL} is much lower than the loss due to overlapping among groups, measured by R^{AJL}.

[19] R^{EG} and $\left(G^{SW}_{y-T} - G^{W}_{y}\right)$ represent the 0.012% of *RE* at the 400 euro bandwidth; which means the 2.3% of the maximum value attained by R^{EG} (bandwidth 10 euro large) and the 2.1% of the maximum attained by $\left(G^{SW}_{y-T} - G^{W}_{y}\right)$ (bandwidth 3000 euro large).

6. Conclusions

The original Aronson, Johnson and Lambert (1994) decomposition of the redistributive effect considers groups of exact equals in portioning the whole pre-tax income distribution and restrict the analysis to the special cases in which the group averages and the within group orderings maintain the same ranking as before taxation. This means that the AJL decomposition of the redistributive effect considers only overlapping effect among groups of exact equals.

As the following literature pointed out, exact equals are rare in the real world data, so that only groups with close pre-tax incomes can be considered. If this is the case, also the reranking of the mean post-tax income among groups and the reranking within groups must be considered. The intensity of the three possible rerankings here considered varies according to the bandwidth defining the close equals. Then a problem arises: an optimal bandwidth must be chosen in order to properly decompose the redistributive effect into vertical, horizontal and reranking effect.

The choice of the optimal bandwidth is not obvious. Van de Ven, Creedy and Lambert (2001) individuate the optimal bandwidth that should be used in decomposing the redistributive effect as the Aronson, Johnson and Lambert (1994) methodology suggests without considering the different contribution of the reranking of the mean post-tax income among groups and the reranking within groups. They suggest choosing as the optimal bandwidth the one which maximizes the ratio between the potential vertical effect and the actual redistributive effect. As the empirical analysis shows, this ratio may have more than one relative maximum and presents a layout which may be irregular, so that this condition is difficult to be applied in real data elaborations.

Lambert and Urban (2005) got though this problem by identifying a set of possible decompositions of the redistributive effect. They also notice that when close pre-tax equals groups instead of exact pre-tax ones are considered, the residual component in the original Aronson, Johnson and Lambert (1994) model is not the Atkinson-Plotnick-Kakwani index, but only one of its components, that is the one which measures group overlapping introduced by taxation.

In this paper we use this decomposition of the Atkinson-Plotnick-Kakwani index, and intensively look to the empirical analysis in order to individuate the relationships among the main three possible decompositions of the redistributive effect analyzed by Lambert and Urban (2005). We suggest that the optimal bandwidth should be chosen where the maximum distance among the considered possible definitions of potential vertical effect is minimum, provided that the greatest of the three indexes is not lower than the global maximum assumed by the lowest of the three indexes over the whole income distribution range: the optimal bandwidth can be individuated at the point where the between group reranking index crosses the difference between the within group Gini indexes calculated, respectively, for the post-tax smoothed income parade and the pre-tax one. We find empirical evidence that in this bandwidth neighborhood the three measures are also nearly converging and, moreover, absorb most of the Reynolds-Smolenky total redistribution measure.

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Table 1 Summary of index definitions

Groups	are constituted by subjects belonging to a same pre-tax income bracket; income brackets are created by splitting the pre-tax non decreasing incomes parade into contiguous intervals characterized by a same income spread. Groups contains the same subjects both before and after taxation, whatever ordering criterion is adopted. Before taxation no overlapping exists by construction; taxation may result in group overlapping.
G_y	Gini index for pre-tax income parade.
G_y^B	between groups Gini index for pre-tax income parade: it is defined as the Gini index when all incomes inside each group are substituted by the group income average.
G_y^W	within groups Gini index for pre-tax income parade: $G_y^W = \sum_k a_{k,y} G_{k,y}$, where $G_{k,y}$ is the Gini index for the *k*-th group and $a_{k,y}$ is the product of the *k*-th group population share and pre-tax income share.
G_{y-T}	Gini index for post-tax income parade.
G_{y-T}^B	it is analog to G_y^B for the post-tax income parade.
G_{y-T}^W	within groups Gini index for post-tax income parade: $G_{y-T}^W = \sum_k a_{k,y-T} G_{k,y-T}$, where $G_{k,y-T}$ is the post-tax Gini index for the *k*-th group and $a_{k,y-T}$ is the product of the *k*-th group population share and post-tax income share.
D_{y-T}	concentration index for post-tax income parade when ordered according to the pre-tax order.
D_{y-T}^B	between groups concentration index for post-tax income parade: it is defined as the concentration index when all incomes inside each group are substituted by the group income average, moreover groups are ordered according to pre-tax group averages.
D_{y-T}^W	within groups concentration index for post-tax income parade: $D_{y-T}^W = \sum_k a_{k,y-T} D_{k,y-T}$; $D_{k,y-T}$ is the concentration index for the *k*-th group, when the *k*-th group incomes are ordered according to the pretax within group order, and $a_{k,y-T}$ is the product of the *k*-th group population share and post-tax income share.
G_{y-T}^{SW}	within groups Gini index for post-tax smoothed income parade. Smoothed taxation consists in taxing all income in a group by the group average tax rate. $G_{y-T}^{SW} = \sum_k a_{k,y-T} G_{k,y-T}$, as the Gini index for the *k*-th group remains unchanged, when all group incomes are taxed by a same tax rate.

Table 2 Summary of equations and components

$RE = V^{VCL} - H^{VCL} - R^{AJL}$ $V^{VCL} = G_y^B - G_{y-T}^B$ $H^{VCL} = G_{y-T}^W - G_y^W$ $R^{AJL} = G_{y-T}^t = G_{y-T} - G_{y-T}^B - G_{y-T}^W$ $\lim_{b\to 0} V^{VCL} = RE$ $\quad \lim_{b\to 0} H^{VCL} = 0$ $\quad$ e $\quad \lim_{b\to 0} R^{AJL} = 0$ $\lim_{b\to MAX} V^{VCL} = 0$ $\quad \lim_{b\to MAX} H^{VCL} = -RE$ $\quad$ e $\quad \lim_{b\to MAX} R^{AJL} = 0$
$RE = V^{AJL} - H^{AJL} - R^{AJL}$ $V^{AJL} = G_y - G_{y-T}^B - G_{y-T}^{SW} =$ $= V^{VCL} - \left(G_{y-T}^{SW} - G_y^W\right)$ $H^{AJL} = G_{y-T}^W - G_{y-T}^{SW}$ $\lim_{b\to 0} V^{AJL} = RE$ $\quad \lim_{b\to 0} H^{AJL} = 0$ $\quad$ e $\quad \lim_{b\to 0} R^{AJL} = 0$ $\lim_{b\to MAX} V^{AJL} = 0$ $\quad \lim_{b\to MAX} H^{AJL} = -RE$ $\quad$ e $\quad \lim_{b\to MAX} R^{AJL} = 0$
$RE = V - H - R^{APK}$ $V = G_y - D_{y-T}^B - G_{y-T}^{SW} =$ $= V^{AJL} + \left(G_{y-T}^B - D_{y-T}^B\right) =$ $= V^{VCL} - \left(G_{y-T}^{SW} - G_y^W\right) + \left(G_{y-T}^B - D_{y-T}^B\right)$ $H = D_{y-T}^W - G_{y-T}^{SW}$ $\lim_{b\to 0} V = G_y - D_{y-T}$ $\quad$ e $\quad \lim_{b\to 0} H = 0$ $\lim_{b\to MAX} V = 0$ $\quad$ e $\quad \lim_{b\to MAX} H = D_{y-T} - G_y$
$R^{APK} = R^{AJL} + R^{EG} + R^{WG}$ $R^{EG} = \left(G_{y-T}^B - D_{y-T}^B\right)$ $R^{WG} = \left(G_{y-T}^W - D_{y-T}^W\right)$ $\lim_{b\to 0} R^{AJL} = 0$ $\quad \lim_{b\to 0} R^{EG} = R^{APK}$ $\quad$ e $\quad \lim_{b\to 0} R^{WG} = 0$ $\lim_{b\to MAX} R^{AJL} = 0$ $\quad \lim_{b\to MAX} R^{EG} = 0$ $\quad$ e $\quad \lim_{b\to MAX} R^{WG} = R^{APK}$

Figure 1: V, V^{VCL} and V^{AJL} (%RE) - Individuals

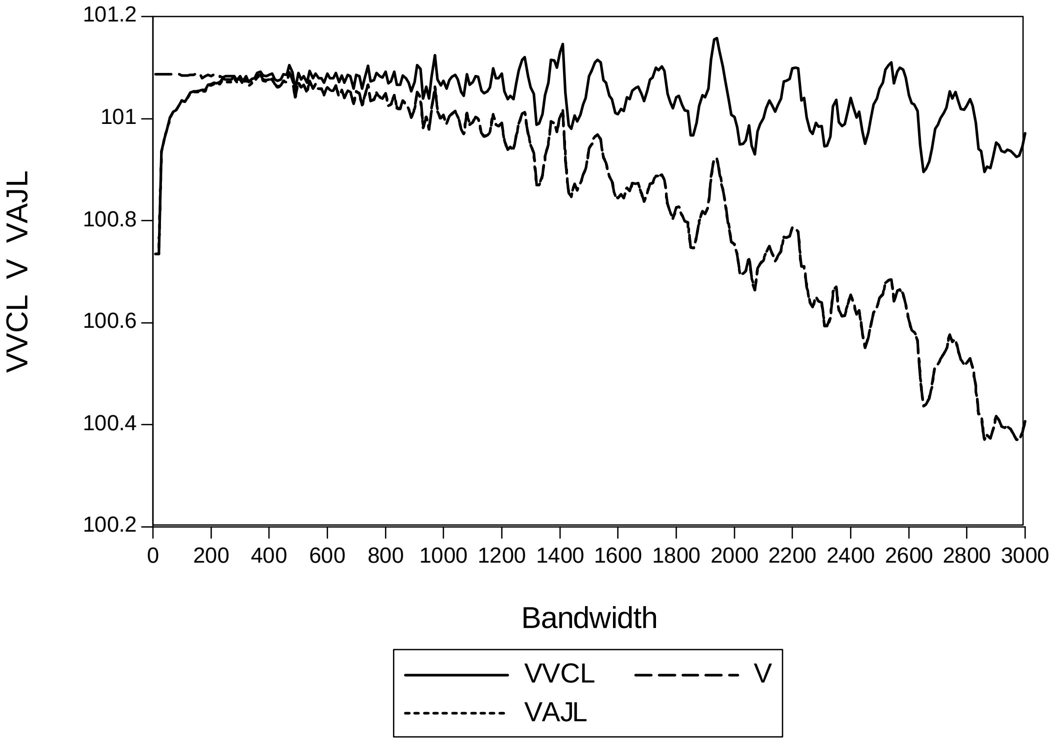

Figure 1: V, V^{VCL} and V^{AJL} (%RE) – Individuals (focus)

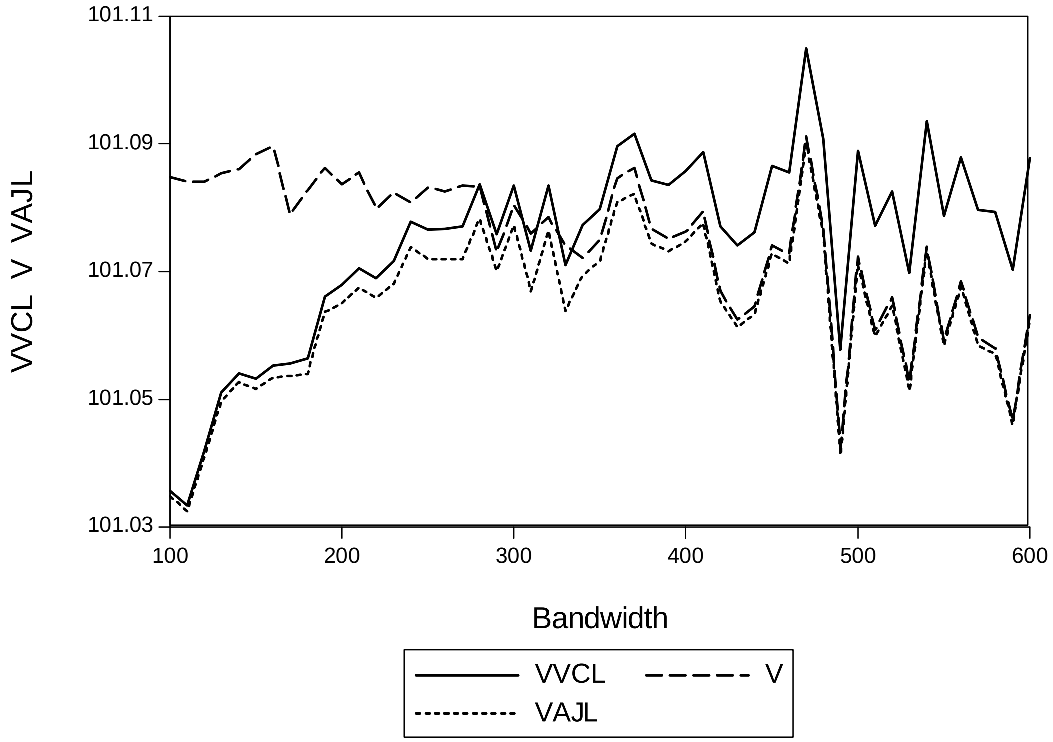

Figure 1 bis: V, V^{VCL} and V^{AJL} (%RE) – m(1) Households

m(1) means Cutler scale α=0.50 β=0.65 and family weight = 1

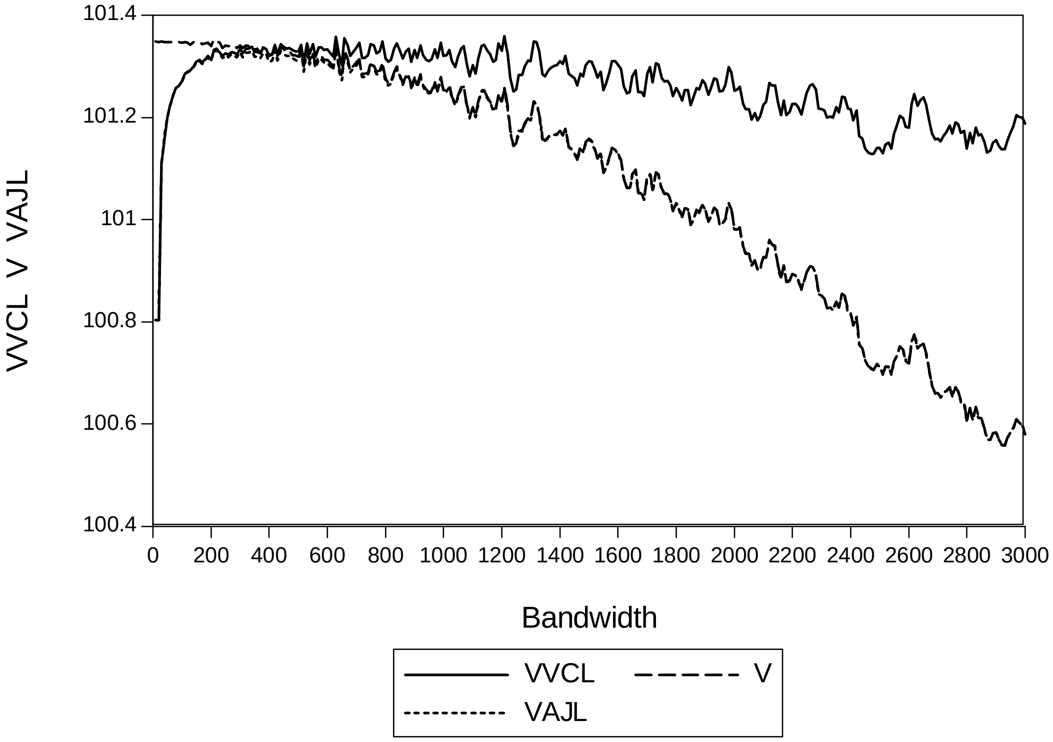

Figure 1 bis: *V*, V^{VCL} and V^{AJL} (%*RE*) – *m*(1) Households (focus)

m(1) means Cutler scale α=0.50 β=0.65 and family weight = 1

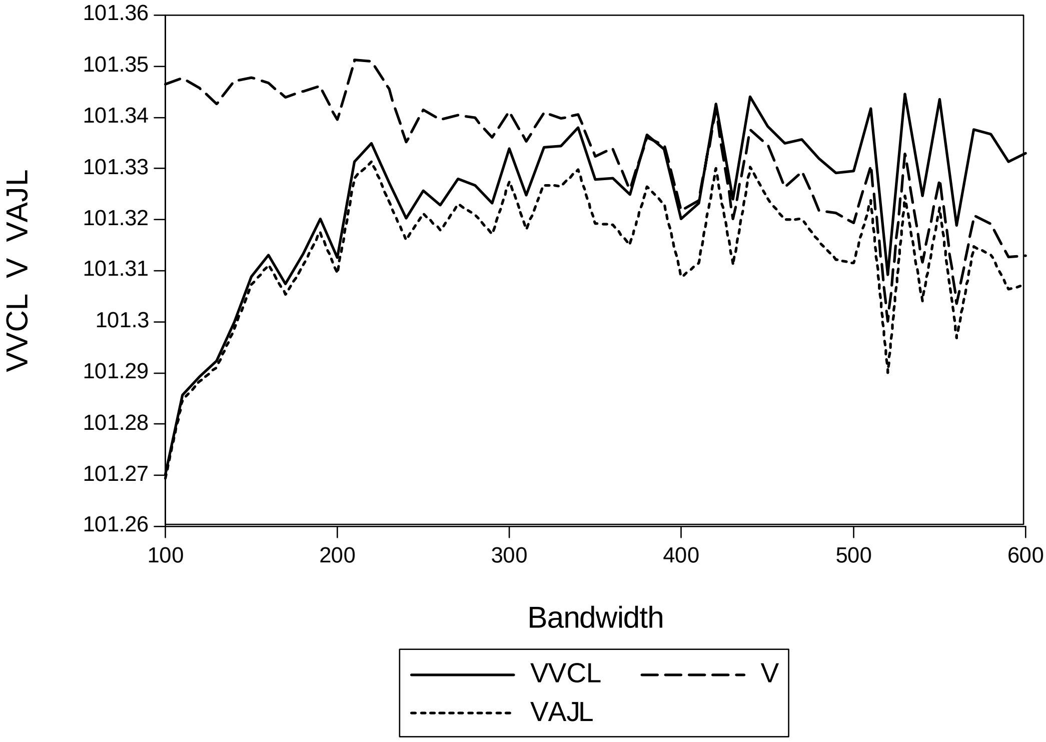

Figure 2: *H*, H^{VCL} and H^{AJL} with R^{AJL} and R^{APK} (%*RE*) - Individuals

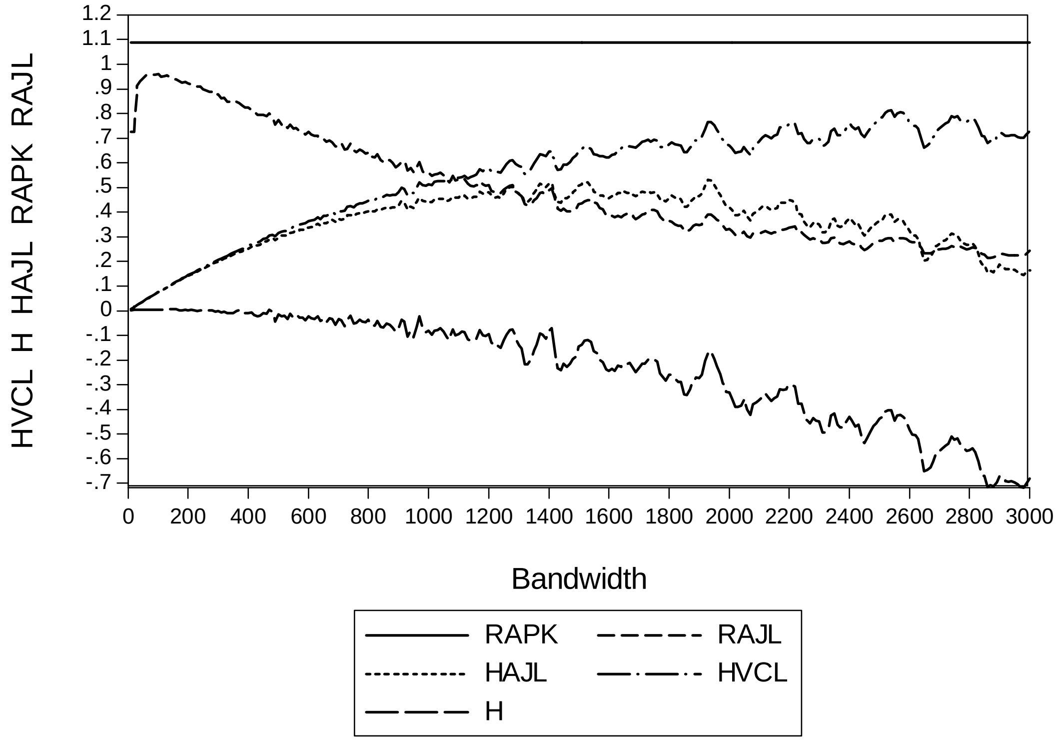

Figure 2 bis: *H*, H^{VCL} and H^{AJL} with R^{AJL} and R^{APK} (%*RE*) - *m*(1) Households

§ *m*(1) means Cutler scale α=0.50 β=0.65 and family weight = 1.

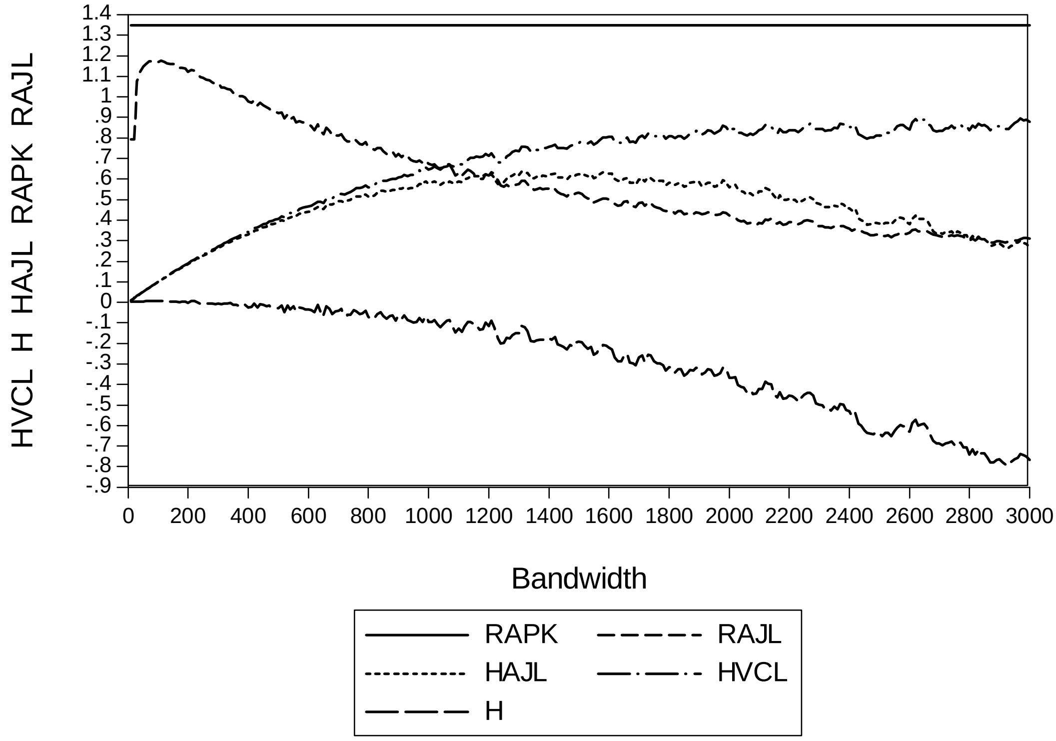

Figure 3: R^{APK} % decomposition - Individuals

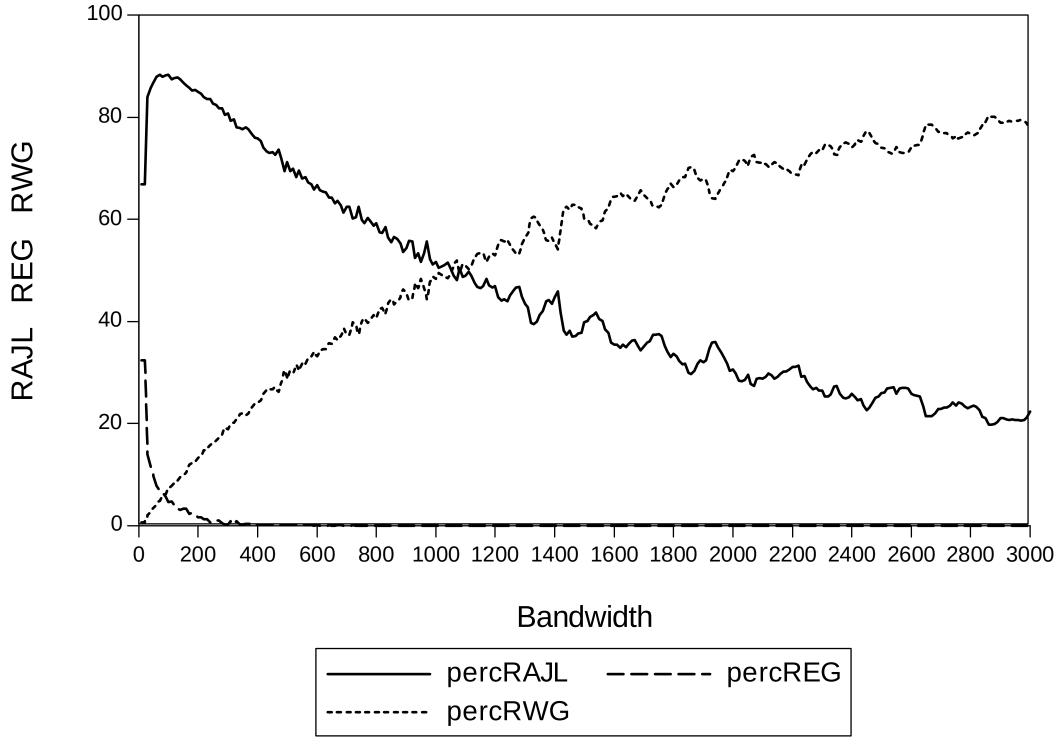

Figure 3 bis: R^{APK} % decomposition – *m*(1) Households

§ *m*(1) means Cutler scale α=0.50 β=0.65 and family weight = 1.

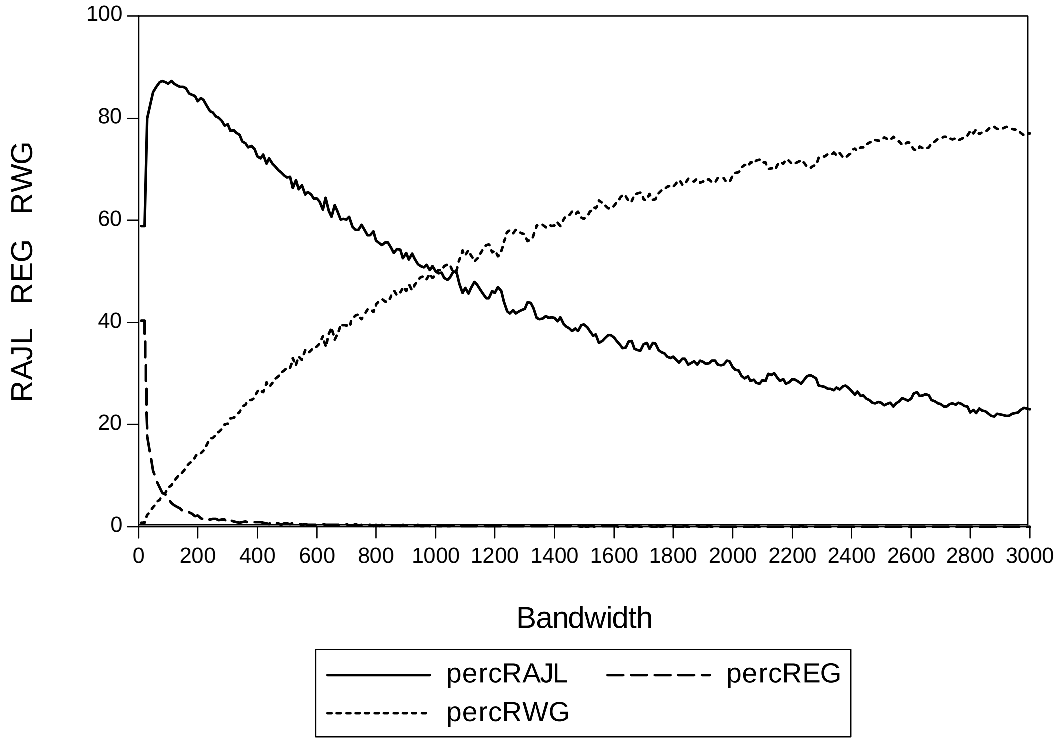

Table 3: *RE* decomposition – Individuals

(bootstrap estimated standard errors in parentheses-2,000 replications)

Component	*Bandwidths*						
	100	300	400	500	600	700	2000
% RE/G_y	14.3699	14.3699	14.3699	14.3699	14.3699	14.3699	14.3699
	(0.1266)	(0.1266)	(0.1266)	(0.1266)	(0.1266)	(0.1266)	(0.1266)
$\%\left(V^{VCL}/RE\right)$	101.0357	101.0835	101.0858	101.0890	101.0878	101.0855	101.0039
	(0.0381)	(0.0388)	(0.0390)	(0.0392)	(0.0397)	(0.0392)	(0.0380)
$\%\left(V/RE\right)$	101.0847	101.0806	101.0762	101.0723	101.0634	101.0537	100.7546
	(0.0395)	(0.0387)	(0.0392)	(0.0392)	(0.0398)	(0.0390)	(0.0375)
$\%\left(H^{VCL}/RE\right)$	0.0759	0.2057	0.2621	0.3157	0.3628	0.4065	0.6713
	(0.0025)	(0.0064)	(0.0083)	(0.0098)	(0.0115)	(0.0126)	(0.0232)
$\%\left(H/RE\right)$	0.0022	0.0063	0.0107	0.0146	0.0235	0.0332	0.3323
	(0.0012)	(0.0031)	(0.0042)	(0.0052)	(0.0061)	(0.0070)	(0.0222)
$\%\left(R^{AJL}/RE\right)$	0.9598	0.8778	0.8237	0.7733	0.7250	0.6789	0.3325
	(0.0349)	(0.0334)	(0.0316)	(0.0307)	(0.0283)	(0.0270)	(0.0170)
$\%\left(R^{APK}/RE\right)$	1.0871	1.0871	1.0871	1.0871	1.0871	1.0871	1.0871
	(0.0391)	(0.0391)	(0.0391)	(0.0391)	(0.0391)	(0.0391)	(0.0391)
$\%\left(R^{AJL}/R^{APK}\right)$	88.2865	80.7452	75.7697	71.1290	66.6890	62.4498	30.5890
	(0.5885)	(0.4274)	(0.4252)	(0.4923)	(0.5651)	(0.5982)	(0.8749)
$\%\left(R^{EG}/R^{APK}\right)$	4.5962	0.2900	0.1338	0.1116	0.0669	0.0669	0.0000
	(0.6003)	(0.3100)	(0.1310)	(0.1115)	(0.0549)	(0.0495)	(0.0146)
$\%\left(R^{WG}/R^{APK}\right)$	7.1174	18.9648	24.0964	28.7595	33.2441	37.4833	69.4110
	(0.1467)	(0.3454)	(0.4105)	(0.5073)	(0.5693)	(0.6197)	(0.9081)

Source: Own elaborations.

Table 4: *RE* decomposition – Households

(bootstrap estimated standard errors in parentheses-2,000 replications)

Component	*Bandwidths*						
	100	300	400	500	600	700	2000
% RE/G_y	13.9266	13.9266	13.9266	13.9266	13.9266	13.9266	13.9266
	(0.1910)	(0.1910)	(0.1910)	(0.1910)	(0.1910)	(0.1910)	(0.1910)
$\%\left(V^{VCL}/RE\right)$	101.2702	101.3349	101.3201	101.3300	101.3330	101.3351	101.2527
	(0.0543)	(0.0574)	(0.0577)	(0.0574)	(0.0575)	(0.0565)	(0.0565)
$\%\left(V/RE\right)$	101.3468	101.3412	101.3217	101.3193	101.3129	101.3079	100.9817
	(0.0606)	(0.0577)	(0.0574)	(0.0585)	(0.0592)	(0.0592)	(0.0574)
$\%\left(H^{VCL}/RE\right)$	0.1001	0.2718	0.3422	0.4078	0.4664	0.5245	0.8316
	(0.0040)	(0.0106)	(0.0136)	(0.0158)	(0.0188)	(0.0202)	(0.0335)
$\%\left(H/RE\right)$	0.0016	0.0072	0.0266	0.0290	0.0354	0.0405	0.3667
	(0.0017)	(0.0044)	(0.0061)	(0.0074)	(0.0091)	(0.0101)	(0.0313)
$\%\left(R^{AJL}/RE\right)$	1.1701	1.0622	0.9780	0.9217	0.8666	0.8106	0.4210
	(0.0504)	(0.0486)	(0.0453)	(0.0433)	(0.0420)	(0.0394)	(0.0246)
$\%\left(R^{APK}/RE\right)$	1.3486	1.3486	1.3486	1.3486	1.3486	1.3486	1.3486
	(0.0605)	(0.0605)	(0.0605)	(0.0605)	(0.0605)	(0.0605)	(0.0605)
$\%\left(R^{AJL}/R^{APK}\right)$	86.7615	78.7591	72.5153	68.3462	64.2561	60.1067	31.2191
	(0.7555)	(0.5072)	(0.5598)	(0.6584)	(0.6792)	(0.7271)	(1.0030)
$\%\left(R^{EG}/R^{APK}\right)$	5.7103	1.0275	0.9484	0.5928	0.4347	0.4742	0.0395
	(0.8019)	(0.3685)	(0.3194)	(0.2602)	(0.1983)	(0.2152)	(0.0384)
$\%\left(R^{WG}/R^{APK}\right)$	7.5282	20.2134	26.5363	31.0611	35.3092	39.4191	68.7414
	(0.1758)	(0.4143)	(0.5429)	(0.6269)	(0.6628)	(0.7425)	(0.9690)

Source: Own elaborations.

Figure 4: R^{EG} and $G_{y-T}^{SW} - G_{y}^{W}$ in percentage of RE – Individuals

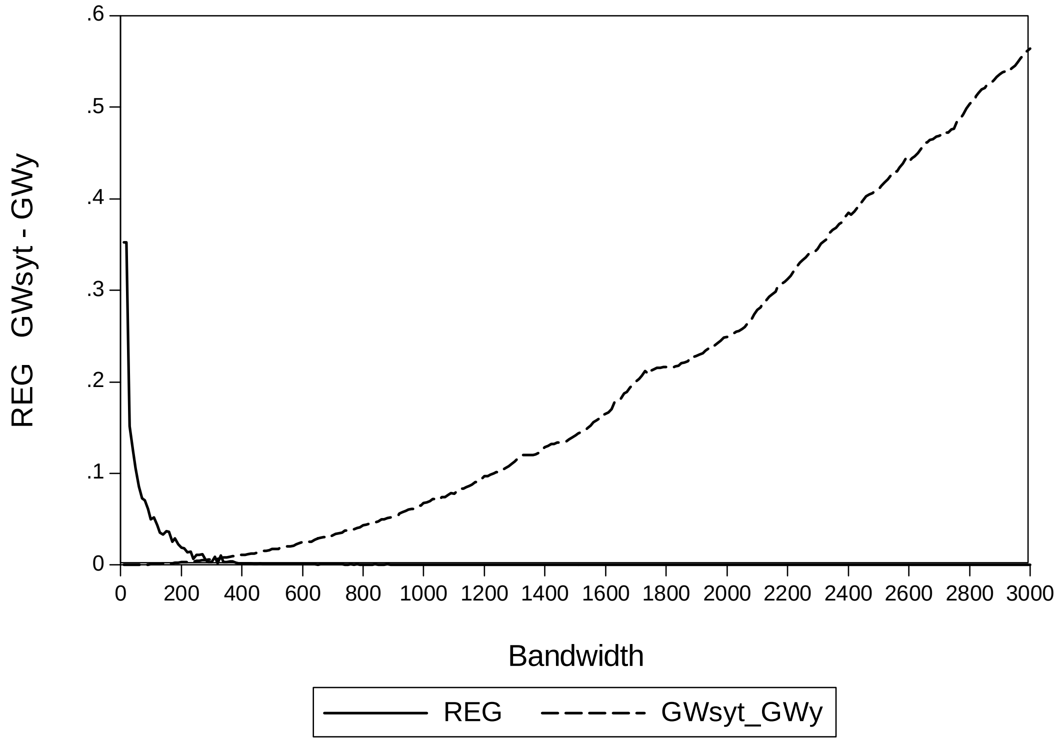

Figure 4 bis: R^{EG} and $G_{y-T}^{SW} - G_y^{W}$ in percentage of RE – m(1) Households

§ m(1) means Cutler scale α=0.50 β=0.65 and family weight = 1.