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CONTROL TOOLS OF ECONOMIC NON LINEAR SYSTEMS

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Control Tools of Economic Non linear Systems

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Abstract:

It is generally accepted that economy belongs to complex systems and both deterministic and stochastic descriptions are needed to define main features of its dynamics. This awareness and consequently the requirement of more realistic models have lead to powerful new concepts and tools to deal with apparently random phenomena that at deeper level could be complex and/or chaotic. Despite the difficult to manage chaotic systems many researchers have been pushed to find a control methods and tools of these systems. The control of nonlinear systems can actually be easier than the control of linear ones, because it might take only a small push to engender a big change in the system. In fact, controlled chaotic systems offer an advantage in flexibility: any one of a number of different orbits can be stabilized by the small control and the choice can be switched from one periodic orbit to an other by very small correction of its parameters, without drastically altering the systems configuration or interfering with their inherent properties. Therefore this richness of possible behaviours in chaotic systems may be exploited to enhance the performance of a dynamical system in a manner that would not be possible to have if the system's evolution is not chaotic. In this paper after a brief survey about the meaning and methods of chaos control we will indicate a new tools used to detect and control a chaotic behaviour and their application in economics.

Keywords: Non linear systems, Chaos Control, Floquet Theory, Recurrence Analysis

Introduction

In the last decades there has been an increasing interest in non-linear dynamic models, in all scientific fields (mathematics, chemistry, physics and son on). The discovery that simple nonlinear models can show complex and chaotic dynamics has pushed also some economists to be interested in this field [1]. In fact in literature there are a lot of examples of nonlinear economic models that exhibit chaotic dynamics [2]. So, it is generally accepted that economy belongs to complex systems and both deterministic and stochastic descriptions are needed to define main features of its dynamics [3]. It is, therefore, not surprising that this awareness and consequently the requirement of more realistic models have lead to powerful new concepts and tools to detect, analyze and control apparently random phenomena that at deeper level could be *complex economic dynamics* [4].

In the literature there is no standard definition [5] of chaos. In fact it is possible to define it outlining its typical features that are:

Nonlinearity. If the phenomenon is linear, it cannot be chaotic.

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[1] "Interest in nonlinear dynamics models in economics is not new, however, and dates back to the time before economists had learned about chaos. Kaldor (1940), Hicks (1950), and Goodwin (1951) have already tried to model economic fluctuations by nonlinear deterministic business cycle models. At that time, attention was focused on regular periodic dynamic behaviour rather than an irregularity and chaos" C. Hommes (1995).

[2] Some works about the application of complex approach are the following. Of course, this list to is not exhaustive but considers the main topics ranging from theories of choice, business cycle, time series analysis to growth theories, redistributive taxation, policy implication and so on: Alchian A., (1950), J. Benhabib, R. H. Day (1981,1982), R. H. Day (1982, 1983),J. M: Grandmont P81985), M. Boldrin, L. Montrucchio (1986), G. Gabisch, H.-W. Lorenz (1987); W.A Brock., W.D. Dechert, J. Scheinkman (1987), J. Scheinkman (1990), W.A. Brock, D.A. Hsieh, B. LeBaron (1991), D.A Hsieh. (1991), A. Medio (1992), Pesaran M. N., Potter M. (1992), J. Bullard, A. Butler (1993), A. Serletis (1993), M. Laaksonen (1994), C. J., Granger (1994), B. Lebaron (1994), K. Nishimura, M. Yano (1995), W. A Brock, Pedro J.F. de Lima (1996), S. N. Durlauf (1997), W.B Arthur., S.N. Durlauf and D.A. Lane (Eds), (1997), Barnett, S. N. Durlauf (1999), M. Scheicher (1999), W Barnett (2000), O. Moritz, (2000). Moreover for survey on chaos and economics see Baumol-Benhabib (1985), Brock et alt. (1992), Boldrin-Woodford (1990), Medio (1992), Lorenz (1993)

[3] J. A. Holyst, K. Urbanowicx 2000

[4] "[...] term 'complex economics dynamics' to designate deterministic economic models whose trajectories exhibit irregular (nonperiodic) fluctuations or endogenous phase switching. The first properties includes chaotic trajectories [...]the second [...] change in the systems states [...] according to intrinsic rules" R. H. Day (1992)

5 W. Ditto, Munakata T., (1995)

Determinism. A chaotic phenomenon has *deterministic* rather than probabilistic underlying rules every future state of the system must follow.

S*ensitivity to initial conditions*. Small changes in its initial state can lead to radically different behaviours in its final state. This property implies that two trajectories emerging from two different close position in the course of time separate exponentially. This critical dependence on the initial conditions, and the fact that experimental initial conditions are never known perfectly, make these systems intrinsically unpredictable.

Sustained irregularity. Hidden order including a large or infinite number of unstable periodic patterns characterises a chaotic phenomenon. This hidden order forms the infrastructures of systems: a chaotic attractor. The dynamics in the chaotic attractor is ergodic, which implies that during its temporal evolution the system ergodically visits small neighbourhood of every point in each one of the unstable periodic orbits embedded within the chaotic attractor.

Long-term prediction but not control is mostly impossible due to sensitivity to initial conditions, which can be known only to a finite degree of precision.

Despite the difficult to managing a chaotic systems many researchers have been pushed to find a control methods and tools of these systems.

The control of nonlinear systems can actually be easier than the control of linear ones, because it might take only a small push to engender a big change in the system (**sensitivity to initial conditions**). In fact, controlled chaotic systems offer an advantage in flexibility: any one of a number of different orbits can be stabilized by the small control and the choice can be switched from one periodic orbit to an other by very small correction of its parameters, without drastically altering the systems configuration or interfering with their inherent properties. Therefore this richness of possible behaviours (**infinite unstable orbits**) in chaotic systems may be exploited to enhance the performance of a dynamical system in a manner that would not be possible to have if the system's evolution is not chaotic.

This means, if we want to consider the economic application of chaos, that small, low-cost policy changes could have a large impact on overall social welfare.

Therefore, also in economics the control of a time periodic system is a challenging task due to the time varying nature of the coefficients and in particular the control of dynamical systems from chaotic and unpredictable to periodic and predictable behaviour has become an intense field of research in the last years, and therefore renewed interest in different control methods has been stimulated.

In this paper after a brief survey about the meaning and methods of chaos control we will indicate new tools to detect and to control a chaotic behaviour; suggests the meaning of chaos control for economics policy. Finally we will propose an application.

Control of chaos

The term "controlling chaos" was coined by E. Ott, C. Grebogi and J. Yorke when they published in Physical Review Letters in 1990 the paper "Controlling Chaos". The key element in this paper was the demonstration of the fact that a significant change in the behaviour of a chaotic system can be made by a very small, tiny correction of its parameters and in particular, this correction can be made without interfering with inherent properties of the system[6]. After this paper the chaotic systems control has attracted increasing attention of researchers from different field.

In general the methods[7] for the control of chaos can be classified into two main classes: 1) **closed loop or feedback methods** which apply perturbations independent of the state of the system; 2) **open loop or non feedback methods** where perturbations are derived from state information. The idea of this method is to change the behaviour of non linear systems by applying a properly chosen input function.

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[6] FradkovA. L., EvansJ. R., Control of Chaos: Survey 1997-2000, 2002

[7] An interesting and a deepening survey can be found in Fradkov A. L., Evans J. R., Control of Chaos: Survey 1997-2000, 2002

One can further distinguish between time-discrete and time-continuous methods, and between methods in which perturbations are applied to parameters and to a dynamical variable, respectively [8].

Closed loop or feedback methods

This class includes those methods which select the perturbation based upon a knowledge of the state of the system, and oriented to control a prescribed dynamics. Besides OGY, among them, we can consider the so called occasional proportional feedback (OPF) simultaneously introduced by Hunt[9] and Showalter[10] , the method of Huebler[11], and the method introduced by Pyragas[12], which apply a delayed feedback on one of the system variables. All these methods are model independent, in the sense that the knowledge on the system necessary to select the perturbation can be done by simply observing the system for a suitable learning time.

The OGY method is based on identifying a periodic orbit and applying small perturbations to a system parameters to stabilize unstable steady states or unstable periodic orbits. Although this perturbations is applied only when the system is close to the desired UPO and when the only time series is available, using it for stabilization of general UPOs requires precise information about the target UPO.

Therefore this method is inadequate for non stationary systems or the targeting problem. This method requires, however, initially large parameter perturbations and is limited to the stabilization of flip-saddle unstable periodic fixed points. Although the OGY-method is well understood from the theoretical point of view its experimental implementations are seriously limited by the fact that all quantities needed to calculate values of a system control parameter are not directly given in an experimental data chain and to perform the control one needs to apply a complex data analysis [13].

Some further extension of this method has lately been proposed and they are quite popular in the fields of applied physics and nonlinear dynamics today.

In contrast to the OGY method the method of chaos control introduced by Pyragas can be easily applied to experimental systems where the equations of motion are not know. The basic idea of Pyragas' method is to simply use a delayed state as feedback. The advantage of this method is that it does not require full information about the target UPO; but rather, it only uses a constant delayed time in the feedback controller[14].

Open loop or non feedback methods

This class includes those strategies which consider the effect of external perturbations (independent ofthe knowledge of the actual dynamical state) on the evolution of the system. Periodic or stochastic perturbations have been seen to produce drastic changes in the dynamics of chaotic systems, leading eventually to the stabilization of some periodic behaviours. These approaches, however, are in general limited by the fact that their action is not goal oriented, i.e. the final periodic state cannot be decided by the operator.

The crucial points of all these methods to perform a chaos control are: a) the consideration that a chaos, while signifying sensitive dependence on small changes to the current state and henceforth rendering unpredictable the system state in the long run, also implies that the system's behaviour can be altered by using small perturbations; b) the observation that a chaotic set, on which the trajectory of the chaotic process lives, has embedded within it a large number of unstable periodic orbits (UPOs), so, unlike a linear system in which a given parameter renders only one type of motion, many different time evolutions are simultaneously possible; c) in addition, because of ergodicity, the trajectory visits the neighbourhood of each one of these periodic orbits.

[8] S. Boccaletti et al. 2000
[9] E.R. Hunt, (1991)
[10] V. Petrov, V. GaH spaH r, J. Masere, K. Showalter, (1993)
[11] B.B. Plapp, A.W. Huebler, (1990)
[12] K. Pyragas, (1992)
[13] Janusz A. Ho lyst_, Krzysztof Urbanowicz, 2000
[14] Y. Tian, C. Chen 2001

To control a chaotic systems means stabilizing unstable periodic orbits[15]. The main idea consisted in waiting for a natural passage of the chaotic orbit close to the desired periodic behaviour and when a trajectory approaches ergodically this desired periodic orbit embedded in the attractor, one applies small perturbations in order to stabilize such an orbit. This fact has suggested the idea that the critical sensitivity of a chaotic system to changes (perturbations) in its initial conditions may be, in fact, very desirable in practical experimental situations.

Economic Applications

Historically, economists have used linear equations to model economic phenomena, because they are easy to manipulate and usually yield unique solutions. However, as the mathematical and statistical tools available to economists have become more sophisticated, it has become impossible to ignore the fact that many important and interesting phenomena are not amenable to such treatment. Therefore controlling of at least some economical processes seems to be one of the most important and challenging tasks facing the economists and politicians responsible for economical policy.

Important phenomena for which linear models are not appropriate include ``depressions and recessionary periods, stock market price bubbles and corresponding crashes, persistent exchange rate movements . . . and the occurrence of regular and irregular business cycles''[16]. Therefore, economic theorists are turning to the study of non-linear dynamics and chaos theory as possible tools to model these and other phenomena

In fact recently, there have been some applications of chaos control methods in economic contexts considering that recognition and control of cyclical patterns and estimation of complex dynamics may lead to many applications, for instance, business cycle detection, seasonal changes in meteorology and population variations in ecology.

Examples of these application are: Holyst et al. (1996) applied the Ott-Grebogi-Yorke method to a model of two competing firms; Kopel (1997) showed using a simple model of evolutionary market dynamics how chaotic behaviour can be controlled by making small changes in a parameter that is accessible to the decision makers and how firms can improve their performance measures by use of the targeting method[17]. Xu et al. (2001) have introduced an approach to detect UPOs pattern from chaotic time series from Kaldor business cycle model. Kaas (1998) has proved that within a macroeconomic disequilibrium model that stationary and simple adaptive policies are not capable of stabilizing efficient steady states and lead to periodic or irregular fluctuations for large sets of policy parameters. The application of control methods for chaotic dynamical systems shows that the government can, in principle, stabilize an unstable Walrasian equilibrium in a short time by varying income tax rates or government expenditures.

Tools to detect and stabilize UPO’s

The reason why the periodic orbits of a dynamical system are not easily detectable is their instability: trajectories neighbouring an UPO are repelled from it. As periodic orbits open a door to the understanding of the chaotic dynamics, many efforts have been made to develop methods to detect these orbits despite their instability from both time series and knowledge of the system under examination.

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[15] “The possibility of transformation of periodic motion into chaotic motion and viceversa was demonstrated by Alexeev and Loskutov (1987)” see Alexander L., F., Radkov R. J.Evans 2002

[16] Creedy, John and Vance L. Martin. Chaos and Non-Linear Models in Economics. Edward Elgar Publishing Limited: Aldershot, 1994.

[17] “The targeting procedure may be seen as a preliminary task for chaos control, because, as we have already pointed out, the control algorithms (see, e.g. [1}3]) use linearization of the dynamics that are valid only in a rather small neighborhood of the desired saddle point, and therefore need the system to target such a small neighborhood before the switch on. The first targeting method was introduced by Shinbrot et al. (1990) who have suggested to use the exponential sensitivity of a chaotic process to tiny perturbations in some accessible control parameter. S. Boccaletti et al 2000

Detecting UPO's: Recurrence analysis

In economics there have been numerous works both theoretical and empirical concerning the detection of complex and /or chaotic behaviours.

Many problems there have been in particularly for empirical works whose results have tended to be inconclusive, due to lack of appropriate testing methods (Gilmore 1993). Taking into account that standard techniques, such as spectral analysis or the autocorrelation function, cannot distinguish whether a time series was generated by a deterministic or a stochastic mechanism, also the *complex tools* are lacking to perform reliable outcomes. In fact, the correlation dimension test, a metric approach[18] developed by Grassberger and Procaccia (1983), has been widely used in the natural sciences, and generally in conjunction with related procedures such as the calculation of Lyapunov exponent, but its application to economic data has been problematic[19]. In fact the implementation of these algorithms is connected with specific requirements as extensive amounts of data, which are not always available in the experimental settings, and stationarity of data under investigation, and distributional behaviours while many time series variables are nonlinear or do not behave as Gaussians.

So, the application of metric approaches to relatively small, noisy data sets, which are common in economics, is of dubious validity[20]. To avoid these difficulties in metric approach a new method called topological approach[21] to detecting deterministic chaos has been developed: [Mindlin et al. (1990), Tufillaro et al. (1990), Mindlin et al. (1991)].

The topological method has several important advantages over the metric approach:

1. It is applicable to relatively small data sets, such as are typical in economics and finance;
2. It is robust against noise;
3. Since the topological analysis maintains the time-ordering of the data, it is able to provide additional information about the underlying system generating chaotic behaviour;
4. Falsificability is possible, as verification can be made of the reconstruction of the strange attractor [Mindlin et al. (1991)].

Moreover the discovery of invariants by topological approach allows identifying models to explain the data and the consequent topological classification of chaotic sets is a promising step to develop predictive models in non-linear systems[22]

The Recurrence Analysis is an example of topological method and can represent a useful methodology to detect non-stationarity[23], chaotic behaviours and bifurcations in time series (Zbilut, Webber, Giuliani, Trulla 2000; J. S. Iwanski 1998).

In the first time this approach has been used to show recurring patterns and non-stationarity[24] in time series (J. Cao, H. Cai 2000); then the Recurrence Analysis has been applied to study of chaotic systems because recurring patterns are among the most important features of chaotic systems (J. Cao, H. Cai 2000). This methodology by Recurrence Plot allows revealing correlation in the data that is not possible to detect in the original time series. It not

[18] The metric approach is characterized by the study of distances between points on a strange attractor. C. G. Gilmore 1993

[19] "Though in some cases it appears that dimensions can be reliably extracted with as few as 500 data points, the minimum sufficient number of data points, and optimum data sampling rate and embedding delay, all depend critically on the uniformity of the strange attractor and its dimension. [...] (Note that Smith (Smith 1988) has a proof that the lower bound of the number of points to avoid spurios results is $N = 42^d$". Abrham N. B., Albano A. M., Tufillaro N. B. 1989

[20] for the problem associated with metric approaches implementation see C. G. Gilmore 1993

[21] This method includes a `close returns' test for detecting chaos which is of particular interest for researchers in fields such as finance and economics, since it works well on relatively small, noisy data sets. Further, the topological approach is potentially far more useful than metric methods since it is capable of promoting additional information about the underlying system generating chaotic behavior, once evidence of chaos is detected. C. G. Gilmore 1993

[22] N Tufillaro. 1994

[23] "system properties that cannot be observed using other linear and non linear approaches and is specially useful for analysis of non stationarity systems with high dimensional and /or noisy dynamics", see Holyst J. A. et alt. 2000 pp. 1

[24] The recurrence periods obtained by RQA are only partially linked to the ones derived from FFT analysis. Whereas the FFT is constrained by data stationarity and imposes a filter (sinusoid model) to the observed periodicities, RQA, being based on the simple recurrence in time of very short patches of the studied signal, is independent from stationarity assumptions and does not impose any "filter function" on the data

requires any assumptions on the stationarity of time series, any assumptions regarding the underlying equations of motions[25] and distributional behaviours.

It is quite robust in the face of the noise and RP of dynamical system preserves the invariant of the dynamics (Bradley and Mantilla 2001).

It seems especially useful for cases in which there is modest data availability and can be compared to classical approaches for analysing chaotic data, especially in its ability to detect bifurcation (J.P. Ekmann S. O. Kamphorst, D. Ruelle, 1987; L., Giuliani A., Zbilut J. P., Weber C. L. Jr 2000, E. Kononov). Recurrence Analysis is particularly suitable to investigate the economic time series that are characterised by noise, lack of data and being output of high dimensional systems[26].

The RPs approach is not gained much popularity because its graphical output is not easy to interpret. As consequence Zbilut et alt. (1998) proposed statistical quantification of RPs, well-know as Recurrence Quantification analysis (RQA). RQA defines measures for diagonal segments in a recurrence plots. These measures are recurrence rate, determinism, averaged length of diagonal structures, entropy and trend (Zbilut J.P 2000).

The RP is a two dimensional representation of single trajectory. It is formed by a 2-dimensional M x M (matrix) where M is the number of embedding vectors Y(i) obtained from the delay co-ordinates of the input signal. In the matrix the point value of coordinates (i,j), is the Euclidean distances between vectors Y(i) and Y(j). In this matrix horizontal axis represents the time index Y(i) while the vertical one represents the time shift Y(j). A point is placed in the array (i,j) if Y(i) is sufficiently close to Y(j). the closeness between Y(i) and Y(j) is simply expressed by $\left\| Y_i - Y_j \right\| \leq d$ where d is a prescribed number.

There are two type of RP thresholded (also known as recurrence matrix) and unthresholded[27]. The thresholded RPs are symmetric[28] around the main diagonal (45° axis).

The points in this array are coloured according to the $i - j$ vectors distance. Usually the dark colour shows the long distances and light colour short one. If the texture of the pattern within such a block is homogeneous, stationarity can be assumed for the given signal within the corresponding period of time; non-stationary systems causes changes in the distribution of recurrence points in the plot which is visible by brightened areas

Recurrence Analysis is used also to detect unstable periodic orbits in chaotic time series. From Bradley and Mantilla (2001) we could derive an example of application of RP to chaotic time series analysis. This is indicated in the Fig.2 in the Appendix. Here, the repeated patterns are building blocks in RP. These blocks reflect time intervals when the trajectory is travelling on or near the corresponding UPO.

To find an UPO, we have to construct the RP of trajectory on a chaotic attractor, analyse the repeated structures using also the quantification of RP, that is the RQA and use the information extracted from Recurrence to index into trajectory and find the associated state variable values.

Moreover the RP represents an useful way to compare two chaotic system; for example if RP of two trajectory have different building blocks they can be not from the same system, instead identical block RP structure identify identical dynamics. The recurrence analysis is an useful tool for locating unstable periodic orbits in chaotic time-series data and bifurcation behaviour[29] and for identifying a dynamical system[30].

[25] [...]Such an approach to dynamical system may be critically significant for analysing [...] system whose complex mathematics are unknown, but whose state dependent fluctuations are essential for a complete systemic characterization .Trulla 1996

[26] Trulla L. et al. 1996 pp. 255, 1996

[27] "[...] In an unthresholded PR the pixel lying at (i,j) is colored-coded according to the distance, while in a thresholded RP the pixel lying at (i,j) is black if the distance falls within a specified threshold corridor and white otherwise." See Iwanski J. S. and Bradley E. 1988

[28] The recurrence matrix is symmetric across its diagonal if ||Y(i)-Y(j)||=||Y(j)-Y(i)|| (McGuire et alt.1997)

[29] [...] RQA can localize bifurcation behavior in this system without making any a priori assumptions regarding the underlying equations of motion. Trulla et al. 1996

[30] Bradley E., R. Mantilla 2001

Stabilizing UPO's: Floquet theory

The control of a time periodic system is a challenging task due to the time varying nature of the coefficients. The main problem is that the time varying eigenvalues of the periodic matrix do not determine the stability of the system and the standard methods of control theory cannot be applied directly[31].

Therefore, one possible approach to handle such problems would be to construct equivalent time invariant systems suitable for the application of conventional techniques. A time invariant system can be obtained using the Lyapunov-Floquet (L-F) transformation. The Floquet theory is at the core of what is now known as Floquet-Lyapunov Theory which transforms the linear part of a periodic quasi linear equation into a time invariant form preserving the original dynamic characteristic of the system.

While other methods[32] can be used for those systems where the periodic coefficients can be expressed in terms of a small parameter the Lyapunov-Floquet transformation technique does not have such limitations and hence it can be applied to general periodic systems[33].

Stability of the system is determined by eigenvalues of transition matrix, so if the real part of all Floquet exponents is negative the solution is stable, while the positive exponents indicate instability.

The proposed techniques would provide useful tools in the simplification of linear and nonlinear time-periodic systems. Since the analysis and control techniques for time-invariant systems are well-developed, it would now be possible to use these methods for time-periodic systems.

Nevertheless, this approach has been widely used for the assessment of stability of small dimensional systems with periodic coefficients[34]. When the system is characterized by large number of degrees of freedom a novel approach has been proposed, the *implicit Floquet analysis*[35], which evaluates the dominant eigenvalues of the transition matrix using the Arnoldi algorithm, without the explicit computation of this matrix. This method is far more computationally efficient than the classical approach and is ideally suited for systems involving a large number of degrees of freedom.

Over the last thirty years, Floquet-Lyapunov theory has been mainly used for stability analysis alone. Attempts to use it with a view to control a system are referred to few work [36].

The Floquet theory can be used to analyse a bifurcation behavior that provides a means for studying dynamic mechanisms which may change structural stability of the system as some parameter slowly varies with time.

The types of bifurcation are determined from the manner in which Floquet multipliers leave the unit circle. There are three possibilities: (a) if the Floquet multiplier leaves the unit circle through +1, we have either a transcritical, symmetry-breaking, or cyclic-fold bifurcation; (b) if the Floquet multiplier leaves through -1, we have a period-doubling bifurcation (flip bifurcation); and (c) if complex conjugate Floquet multipliers leave the unit circle from the imaginary axis, we have a secondary Hopf bifurcation[37].

Consider a system of linear, homogeneous differential equations with periodic coefficients:

$$\dot{x} = G(t)x \qquad \textbf{(1)}$$

where **G**(t), with t$\in$ **R** is a real m x m matrix function. The vector x is a column vector of dimension m. Let G(t) be periodic with minimum period of T.

Let $x_1(t), x_2(t)........, x_m(t)$ be any set of m solutions to the system (1), linearly independent for any independent t$\in$ **R**. The matrix **X**(t) with columns $x_1(t), x_2(t)........, x_m(t)$

[31] D. Boghiu et al. 1998
[32] The averaging method Anderson, G. L., and Tadjbakhsh, I. G, 1989
[33] Sihna and Henrichs 1997
[34] Gaonkar G.H., Peters D. A., 1987
[35] Bauchau O.A., Nikishkov Y.G., 2000a and 2000b
[36] R. A. Calico and W.E. Wiesel 1984, H.M. Al-Rahmani and G.F. Franklin 1989, H.M. Al-Rahmani and G.F. Franklin 1990
[37] David and Sihna 1999

is called a fundamental matrix. If **X**(0) = **I** where **I** is the m x m identity matrix, **X**(t) is called a principal fundamental matrix, the matrix given by **F** = **X**(T) is called the Floquet transition matrix or the monodromy matrix. The computation of the Floquet transition matrix is related all the states of the system at a given instant to the same states one period later. The size of this transition matrix is equal to the total number of states of the system. So, the computation of the transition matrix of a system with N states requires N integrations of the system response over one period, for a set of N linearly independent initial conditions.

The eigenvalues of **F** are called the characteristic multipliers for the system (1). The analysis of characteristic multipliers (eigenvalues of monodromy matrix) allows determining the stability of the solutions of the system represented by (1).

The eigenvalues closest to the imaginary axis at either side play an important role, and they are called the leading eigenvalues[38].

In fact, if all the eigenvalues (characteristic multipliers) are situated inside the unit circle in the complex plane, all the solutions turn to zero as $t \rightarrow +\infty$. If any of the characteristic multipliers are outside the unit circle, unbounded solutions exist. If all multipliers are inside or on the unit circle, the stability conditions are determined by the difference between the algebraic and the geometric multiplicity of the multipliers situated on the unit circle[39].

Recurrence Analysis and Floquet theory.

After described Recurrence Analysis and Floquet theory and the particular application they could have in economics, we propose an idea to combine the two tools in the field of time series analysis.

Here we will start from consideration of a basic idea from Auerbach et al. 1987. The goal of this work was: a) to extract all the periodic orbits from an experimental chaotic time series and calculate their stabilities by Lyapunov exponent; b) This information can be used to describe important properties of general chaotic sets. He considered a time series sufficiently large outlining that the unstable periodic orbits can be extracted from chaotic signal for an order n that depends on the amount of data available[40]. After localizing the periodic orbits by methods that look like recurrence plot[41], to calculate the eigenvalues and eigenvector for each point of a periodic cycle he used a Jacobian matrix.

The combining[42] the Recurrence Analysis and Floquet Theory allows to overcome some pitfall of that approach.

In fact, given a time series we could use Recurrence analysis to detect chaotic behaviour, in particularly to localize unstable orbits and bifurcation. As highlighted above the detection of periodic orbits in experimental data is a central issue in the field of chaotic control[43]. Moreover unstable periodic orbits embedded in chaotic attractors are fundamental to an understanding of chaotic dynamics.

The instability that characterises these orbits makes hard to find them. The tools[44] used to recognize the UPOs in time series don't work well[45]

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[38] Zheng Z., Lust K., Roose D., 2001

[39] Seyranian A.P., et al. 2000

[40] Auerbach et al.1986

[41] [...] to locate the periodic orbits by scanning of the time series for pairs of points separated by n time steps that are within a small preassigned spatial difference (r_1) of one another. Auerbach et al. 1987

[42] We could apply the combination of these two tools also for studying bifurcation behavior and chaotic itinerancy; these are typical behavior of systems with high freedom degree. For analysing the stability and what kind of bifurcation occurs, we apply the Floquet theory, while for evaluating the weak instability that characterises the CI's attractors we could use the implicit Floquet theory.

[43] So P., et al. 1996

[44] [...] the method utilizes the linear dynamics around an unstable periodic point to produce a statistical measure which is singular at periodic point. By construction, all points that lie within the linear regions of periodic orbits are utilizes. There is no need to search for an optimal neighborhood size as in other recurrence methods. Using this method, unstable fixed point were reliably identified in noisy numerically generated data. Do P. et alt. 1996

[45] [...]one watches for close returns one plane of section, then bins and averages several occurrences in order to reduce noise. This procedure is quite time consuming for two reason: it non involves an ensemble of nearest-neighbor searches, but also relies on the ergodicity of the orbit in order to visit each UPOs. One can accelerate

Using the RP we have extract the periodic orbits from time series data and now we have to calculate the their stability. This an important step because UPO's stability properties dictate how the trajectories travel upon and around it. This question of stability can be resolved using the Floquet theory. In fact calculating the eigenvalues and eigenvectors of monodromy matrix we can know the stability of periodic orbit.

To calculate periodic orbit stability we could apply Floquet theory rather then Lyapunov exponents, as Auerbach et alt (1987) did. This because the economic time series are very short and the reliability of Lyapunov exponents is based on large amount of data. Furthermore, we use the monodromy and not the Jacobian matrix to calculate the eigenvalues because for fixed-point solutions, local stability of the system is determined from the eigenvalues of the Jacobian matrix of the linearized system. On the other hand, for periodic solutions, stability of the system depends on eigenvalues of the monodromy matrix, i.e., Floquet multipliers.

Moreover, in this context the suggested method of control could be the Pyragas method. This because, while the OGY method is well understood from theoretical point of view its practical implementation is limited by fact that all quantities needed to calculate values of system control parameters are not directly given in experimental data, the Pyragas method[46] can be easily applied to experimental systems where the equations of motion are unknown[47].

Conclusion

The controlling of at least some economical processes seems to be one of the most important and challenging tasks facing the economists and politicians responsible for economical policy.

In the traditional approach the economic modelling on which to base the advices of policy, has been dominated by linearity assumption of dynamic economic systems. The main a priori argument in favour of linearity and the reason for its original adoption is its simplicity[48]. Nevertheless linear models have been proved to be fundamentally wrong or misleading, skewing the understanding of the economy and sometimes corrupting the associated policy advice[49].

In this context chaos represents a radical change of perspective. This because it is not only capable of explaining irregular dynamic behaviors, that characterize the economic phenomena but also providing a useful tool for the stabilization of nonlinear dynamical systems. In fact many nonlinear dynamical systems, even though they exhibit very irregular behavior, are in fact stabilizable, quite in contrast to systems with irregularities depending solely on stochastic shocks. As illustrated above chaotic systems display continuous dependence on parameters and their control is based on small variations in these parameters which lead to changes in the dynamic properties of the model. Some of these parameters represent policy rules such as the rate of taxation, the rate of money growth or government expenditure and are set by the policy authorities. Therefore, the authorities have considerable control over the dynamic outcome.

So, exploiting the fundamental features of chaotic systems as the sensitivity to initial conditions and the presence of unstable orbits the policy authorities can perform their outcome by small interventions.

Therefore the policy authorities that want to perform a best outcome in term of increasing employment, growth and welfare cannot use economic models based on the linearity and simplicity assumption of traditional economic models. The interventions of policy instead must be based on the considerations that economics is a complex system. Of course, this implies the use of typical tools of complexity.

From this point of view the Recurrence Analysis and Floquet Theory represent useful tools to analyze and control complex system. Moreover, in the time series analysis the suggested methodology, (that is the combined use of these tools) allows to overcome application's

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matters somewhat by using estimates of the local dynamics, but the computational complexity is largely inescapable. Bradley and Mantilla 2001

[46] for deepening Pyragas method as chaos control see H. Nakajima 1997

[47] Holyst and Urabanowicz 2000

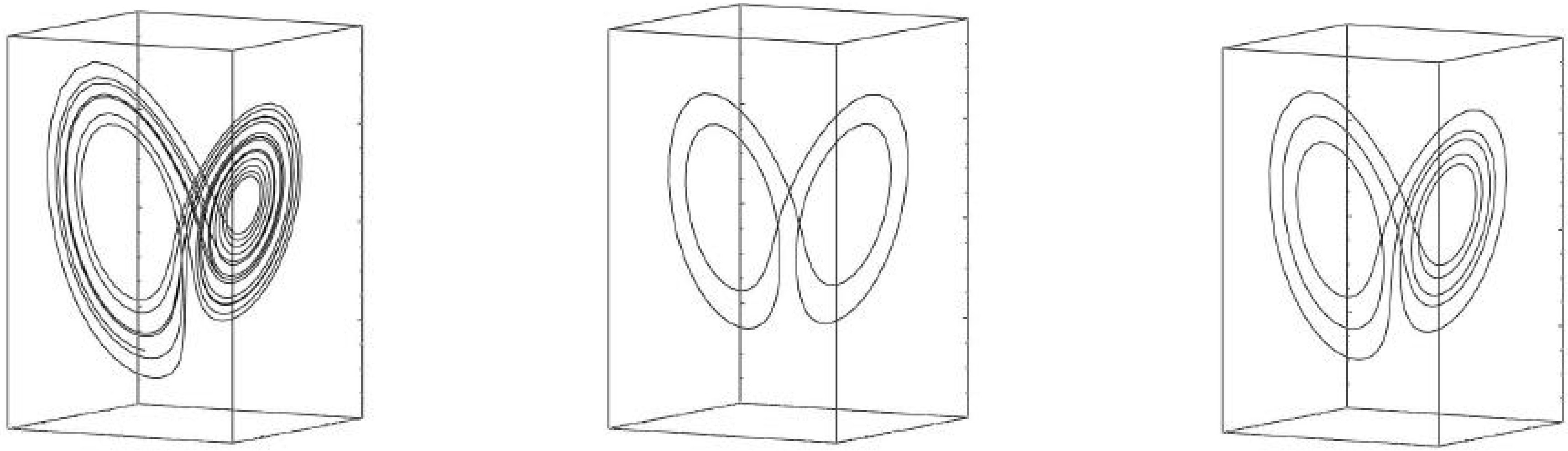
[48] Pesaran M. H., Potter S. M. (1992)

[49] J. Bullard, A. Butler (1993)

difficulties of traditional tools and also of some more known complex tools, as Lyapunov exponent.

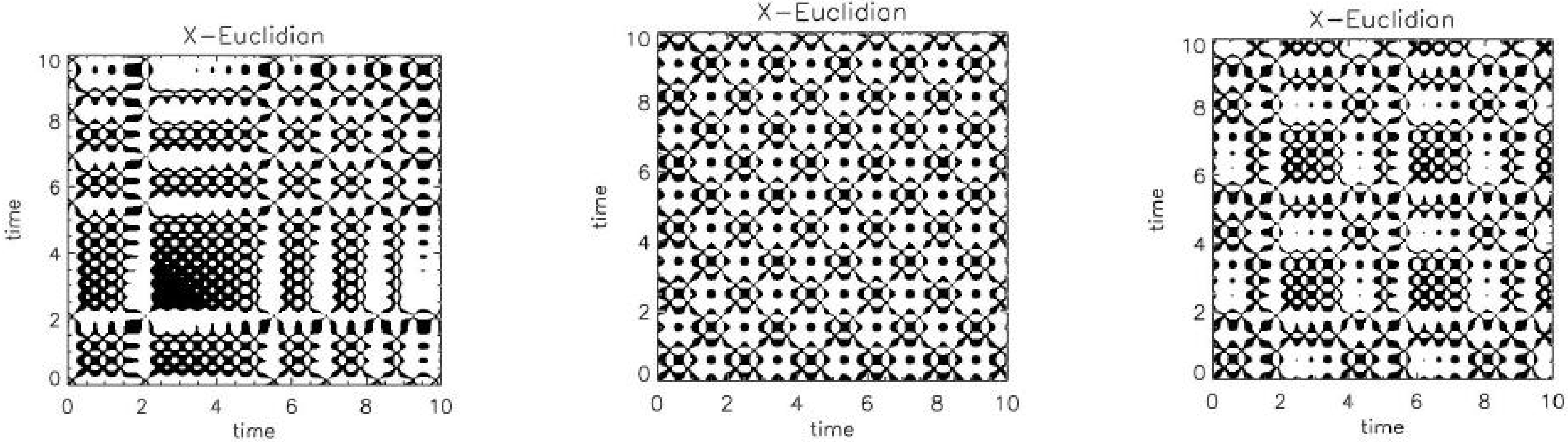
In fact for example, Recurrence analysis seems especially useful for cases in which there is modest data availability and to detect unstable periodic orbits because preserves the invariant of the dynamics. Floquet theory provides a means for studying dynamic mechanisms which may change structural stability of the system as some parameter slowly varies with time. While other methods can be used for those systems where the periodic coefficients can be expressed in terms of a small parameter the Lyapunov-Floquet transformation technique does not have such limitations and hence it can be applied to general periodic system.

Appendix



(a) (b) (c)

Fig. 2: source Bradley Mansilla 2001. Unstable periodic orbits embedded within the Lorenz attractor: a) full attractor, b) an unstable two cycle, c) five-cycle



Rp (a) Rp (b) Rp (c)

Fig. 3. Recurrence plot of the x component of the trajectories of a, b, c constructed with a threshold corridor [0,2] and the Euclidean 2-norm

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