Società italiana di economia pubblica

XIV CONFERENZA

IL FUTURO DEI SISTEMI DI
WELFARE NAZIONALI TRA INTEGRAZIONE EUROPEA E DECENTRAMENTO REGIONALE
coordinamento, competizione, mobilità
Pavia, Università, 4 - 5 ottobre 2002

PRODUCT DIFFERENTIATION AND CARTEL STABILITY WITH STOCHASTIC MARKET DEMAND

LUCA COLOMBO

pubblicazione internet realizzata con contributo della COMPAGNIA di San Paolo

Product Differentiation and Cartel Stability with Stochastic Market Demand [1]

Luca Colombo

Department of Economics

University of Bologna

Strada Maggiore 45, 40125 Bologna, Italy

email:colombo@spbo.unibo.it

tel.0039-051-2092629

fax:0039-051-2092664

September 3, 2002

[1] I would like to thank Luca Lambertini for his guidance throughout the paper's evolution. The usual disclaimer applies.

Abstract:

I adopt a supergame theoretic model to study: (i) how product differentiation affects the stability of cartels, either in prices or quantities, when market demands are subject to observable shocks; (ii) whether collusive agreements are less or more sustainable during booms as opposed to busts. I prove that, albeit firms selling stronger substitutes tend to find it easier to collude, there exists a non-monotone relationship betIen the degree of product differentiation and the critical discount factor. Moreover, I prove that, under Bertrand competition, collusion is always more stable in periods of low demand, while, under Cournot competition, quantity wars occur in booms only if products are sufficiently homogeneous. Finally, from a comparison betIen price and quantity competition, I prove that whether cartels in quantities are more stable than in prices crucially depends on the sign and the amplitude of the shock: product differentiation matters only in the case in which the demand turns out to be quite stable.

Keywords: cartel stability, product differentiation, uncertainty, price and quantity wars.

JEL Classification: D43, L41

1. Introduction

The effects of product differentiation on the stability of implicit collusion have been investigated in several papers, either under Cournot or Bertrand competition (Deneckere, 1983; Chang, 1991; Ross, 1992; Lambertini, Poddar and Sasaki, 1997, 1998, among others). In the light of the existing literature, however, almost nothing can be said about when uncertainty is intended to be explicitely accounted for. With very few exceptions (see Raith, 1996), uncertainty has been dealt with in repeated games without product differentiation. Yet, most of the actual oligopolistic firms competes or colludes on segmented markets facing some kind of uncertainty.

In this paper, I aim at moving the analysis along the intersection between two streams of research almost completely disjointed up to now: the one focussing on product differentiation in a deterministic setting and the one focussing on uncertainty in a setting with homogeneous products.

The existing deterministic literature has arrived to consider product heterogeneity as a facilitating factor w.r.t.collusion. The empirical evidence (see Hay and Kelley, 1974; Symeonidis, 1998), in line with the informal conventional wisdom (see Scherer and Ross, 1990), has contrasted such a theoretical prediction, showing how oligopolistic firms tend to find it tougher to collude the higher the degree of product differentiation.

According to Symeonidis (1999), one possible reason of the discrepancy between game theoretic results and the empirical evidence (and also the conventional wisdom) could have been an excessive emphasis on horizontal rather than vertical differentiation. Indeed, much less attention has been paid to understand whether quality differences may enhance cartel stability. The state of the art does not authorize one to draw general conclusions: Hackner (1994) found that when quality affects fixed costs, as the quality differential increases, it becomes more difficult to sustain price collusion[2]; Ecchia and Lambertini (1997) found the opposite, the crucial difference consisting in the type of cost function adopted[3]. Another possible explanation of the aforementioned discrepancy, more relevant to my paper, has been formulated by Raith (1996). In his paper, the author proposed to extend the framework of spatial differentiation à la Hotelling to a setting with stochastic demand. As a result of the introduction of uncertainty into the

[2] Symeonidis (1999) extends Hackner's results.
[3] Quality affects marginal rather than fixed cost.

analysis, horizontal product differentiation has been shown to hinder cartel stability.

Turning to the effects of uncertainty on the stability of implicit collusion, whether a price or quantity agreement is more sustainable during booms as opposed to busts is far from trivial, even dealing with homogenous products. The two key papers in this respect are that by Rotemberg and Saloner (1986) and that by Green and Porter (1984). In the latter, when firms realize that their profits are low they do not know if this is due to a bad shock or to a rivals price reduction (quantity increase). Therefore, given that the shock is never observable, price wars occur when demand is unexpectedly low. Conversely, in the former, at each period firms take their decisions once the current shock has become common knowledge; nevertheless, the level of demand from the successive period onwards, is a priori undetermined. Hence, in choosing about collusion, each firm compares the deterministic gain from deviating with the discounted expected value of the consequent losses due to the punishment phase[4]. Since such a value is independent of the shock realization, the higher the observed demand level, the higher the incentive to deviate, i.e., the lower the stability of collusion. The empirical evidence suggests that cartels tend to breakdown in periods of high demand (see Tirole, ch.6, 1988).

In order to gain further insight into how product differentiation affects the analysis of collusive behavior, I propose a simple duopoly model with a representative consumer whose preferences are described by means of a quadratic utility function. I let the resulting linear market demands for differentiated products be subject to a macro observable[5] shock on their intercept term and firms playing an infinitely repeated game in marketing, either in prices or quantities. The present analysis reveals that some implications drawn from previous deterministic models, in particular those that heterogeneity tends to facilitate collusions, turn out to be not robust to a stochastic setting. Furthermore, it reveals that, under Bertrand competition, collusion is always more stable in periods of low demand, while, under Cournot competition, whether price wars really occur in booms depends on the degree of product differentiation. Finally, the analysis deals with a comparison between the two kinds of competition, showing

[4] The perfect observability of the shock allows each firm to recognize any kind of deviation. If the shock were not perfectly observable, marginal deviations could not be detected.

[5] A further important difference between Raith's model and mine is that I assume perfect observability of the shock while he introduces imperfect monitoring, so, in his model, firms are not able to discriminate between random demand shocks and marginal deviations. In my model, instead, any deviation is detected with probability 1. Therefore, Raith's model follows Green and Porter (1984) while my model is in Rotemberg and Saloner's vein.

that whether cartels in quantities are more stable than in prices crucially depends on the sign and the amplitude of the shock.

The remainder of this paper is structured as follows: the model is laid out in section 2; section 3 studies the effects of demand uncertainty on the stability of collusion; section 4 compares the two kinds of competition; section 5 investigates upon product differentiation and section 6 provides concluding remarks.

2. The model

I employ a quadratic utility function for a representative consumer as in Bowley (1924), Dixit (1979) and Singh and Vives (1984):

$$U = aq_1 + aq_2 - \frac{1}{2}(bq_1^2 + 2\gamma q_1 q_2 + bq_2^2) + Y \tag{0.1}$$

where Y is the *numeraire* good. Without any loss of generality, let parameters a and b be unitary. Given such restrictions, (1) yields the following inverse demand function ($p_Y = 1$):

$$p_i = 1 - q_i - \gamma q_j \qquad i = 1, 2 \tag{0.2}$$

where $\gamma \in [0, 0.91667]$ represents the degree of substitutability (or standardization) between the product offered by firm i and the product offered by firm j, as perceived by the consumer[6]. Since I am interested in exploring the link betIen product differentiation and the stability of collusion in a setting with market demand uncertainty, I introduce a random shock to the intercept term of (2), as follows[7]:

$$p_i = 1 + \widetilde{s} - q_i - \gamma q_j \qquad i = 1, 2 \tag{0.3}$$

[6] $\gamma \in [0, 0.91667]$ because for $\gamma > 0.91667$ the non negative constraint on cartel quantities becomes binding.

[7] Such a perturbated market demand dates back to Klemperer and Meyer (1986). Another interesting specification could have been: $p_i = \widetilde{s}(1 - q_i - \gamma q_j)$ with $\widetilde{s} \neq 0$.

where $\widetilde{s}$ is the observable shock. By inverting (3), the direct demand function obtains:

$$q_i = \frac{1}{1+\gamma}(1+\widetilde{s}) - \frac{1}{1-\gamma^2}p_i + \frac{\gamma}{1-\gamma^2}(p_j - \widetilde{s}) \qquad i = 1, 2 \tag{0.4}$$

I assume that $\widetilde{s}$ has domain $[-\theta, \theta]$ and a uniform distribution function[8] $dF(\widetilde{s})$, with $\theta \in [0, \frac{1}{10}]$. Moreover, I assume that $\widetilde{s}$ be independently and identically distributed.

Without any loss of generality, I also assume that production entails constant marginal cost[9], normalized to zero for the sake of simplicity.

There are neither financial constraints nor entry threats into the model. This amounts to saying that, during recessions, firms face any kind of bankruptcy risk, while, during booms, they are allowed to act regardless of the behaviors of their potential competitors.

I model firms interactions on the marketplace as a repeated game over an infinite horizon, either in prices or quantities. The time structure is as follows. At the beginning of each period, each firm learns the realization of $\widetilde{s}$ (more precisely s_t becomes common knowledge). After having observed s_t, firms simultaneously choose their prices (quantities) from the strategy space . The choices of both firms then become common knowledge and the stage game is repeated.

There are really just three prices (quantities) which matter in this model. Denote these with $\alpha, \beta, \phi \in$. When α is chosen by both firms, the expected value of joint profits results maximized. Supposed α be the price (quantity) to which firms would like to commit in the long run. Depending on the value attached to the future, expressed by $\delta \in [0, 1]$, firm $i(j)$ could find it more profitable to breakdown the implicit agreement, playing its deviation strategy, β, given the rival plays the cartel strategy. Once cheated, both firms are aware of the fact that a kind of punishment phase will take place for a certain horizon: let $\phi \in \Phi \subset$ be such a punishment strategy, with Φ being the punishments set. Of course, cooperation will be sustained in the repeated setting for sufficiently severe punishments. As in Rotemberg and Saloner (1986), the losses from being punished depend on the expected value of $\widetilde{s}$, as well as for the cartel profits, while the gains from cheating are a function of its current realization, s_t.

---

[8] $dF(\widetilde{s}) = \frac{1}{2\theta}$ with $E[\widetilde{s}] = \mu = 0$; $Var[\widetilde{s}] = \sigma^2 = \frac{\theta^2}{6}$.

[9] In particular, I assume that there are no capacity constraints, so each firm is allowed to face the entire demand function also during booms.

Let us consider grim strategies (Friedman, 1971): each firm goes ahead in playing α until β is detected, say at time t. In this case, the firm being cheated reverts to the one shot Nash equilibrium strategy from time $t+1$ to infinity. According to this, the critical threshold of the discount factor under either Cournot or Bertrand competition turns out to be: $\delta^* = \dfrac{\Pi_K^D - E[\Pi_K^M]}{\Pi_K^D - E[\Pi_K^N]}$, where superscripts D, M, N stand for deviation, monopoly (cartel) and one-shot Nash equilibrium profits and $K = \{B, C\}$ indicates the kind of competition. By denoting the minimum value of δ that supports the cartel profit as a subgame perfect equilibrium in the infinitely repeated game, δ^* measures the scope of collusion. The higher such a value, the lower the likelihood of collusive agreements. In order to compute it, the preliminary task is to find out the level of the deterministic profits under the different regimes. Straightforward calculations lead to:

$$\Pi_C^N = \left(\frac{1+s_t}{2+\gamma}\right)^2 \tag{0.5}$$

$$\Pi_B^N = \frac{(\gamma + 2s_t\gamma - 1 - s_t)^2}{(\gamma-2)^2(1-\gamma^2)} \tag{0.6}$$

$$\Pi_{C,B}^M = \frac{(\gamma + 2s_t\gamma - 1 - s_t)^2}{4(\gamma-1)^2(1+\gamma)} \tag{0.7}$$

$$\Pi_C^D = \frac{(\gamma^2 - 2 - 2s_t + \gamma + s_t\gamma)^2}{16(\gamma-1)^2(1+\gamma)^2} \tag{0.8}$$

$$\Pi_B^D = \begin{cases} \dfrac{(\gamma + 2s_t\gamma - 2 - 2s_t)^2}{16(1+\gamma)(1-\gamma)} & \forall \gamma \in (0, \widehat{\gamma}] \\ (1+2s_t)\dfrac{2\gamma + 2\gamma s_t - 1 - 2s_t}{4\gamma^2} & \forall \gamma \in (\widehat{\gamma}, 0.91667] \end{cases} \tag{0.9}$$

where $\widehat{\gamma} = 0.6821$ in case of negative shock, and $\widehat{\gamma} = 0.76865$ in case of positive shock. Parameter $\widehat{\gamma}$ denotes the threshold above which the non negative constraint on the quantity of the firm being cheated becomes binding. In the deterministic case, $\widehat{\gamma} = \sqrt{3} - 1$ (see Deneckere, 1983). Notice that the region where the deviating firm remains monopolist is greater the lower the realization of the shock. The meaning of superscripts N, M, D and subscripts C, B, has been previously specified and still holds. Looking at the deviation profits, it is immediate to verify that when $s_t = 0$, then $\Pi_C^D = \dfrac{(\gamma+2)^2}{16(1+\gamma)^2}$, $\Pi_B^D = \dfrac{(\gamma-2)^2}{16(1+\gamma)(1-\gamma)}$ $\forall \gamma \in (0, \sqrt{3}-1]$ and $\Pi_B^D = \dfrac{2\gamma - 1}{4\gamma^2}$ $\forall \gamma \in (\sqrt{3}-1, 1]$, which correspond to the deterministic case (see Deneckere, 1983).

Now, the second step is to compute the level of the expected profits under both the monopoly and the competitive regime[10]. This is done by applying the usual formula for the expectation of a generic function $g(\widetilde{x})$, $\widetilde{x} \in R$ being a random variable:

$$E_{(\widetilde{s})}[\Pi_K^R] = \int_{-\theta}^{\theta} \Pi_K^R(\widetilde{s}) dF(\widetilde{s}) d\widetilde{s} \tag{0.10}$$

where superscript $R = \{M, N\}$ denotes the regime. Simple calculations lead to the following expressions, which hold $\forall(\theta, s_t)|\, s_t \leq \theta \in [0, \frac{1}{10}]$:

$$E_{\widetilde{s}}[\Pi_C^N] = \frac{3+\theta^2}{3\,(\gamma+2)^2} \tag{0.11}$$

$$E_{\widetilde{s}}[\Pi_B^N] = \frac{3\gamma^2 + 4\theta^2\gamma^2 - 6\gamma - 4\theta^2\gamma + \theta^2 + 3}{3\,(1-\gamma^2)\,(\gamma-2)^2} \tag{0.12}$$

$$E_{\widetilde{s}}[\Pi_{C,B}^M] = \frac{4\theta^2\gamma^2 + 3\gamma^2 - 6\gamma - 4\theta^2\gamma + 3 + \theta^2}{12\,(\gamma-1)^2\,(1+\gamma)} \tag{0.13}$$

As before, it is immediate to verify that when $\theta = 0$, then $E_{\widetilde{s}}[\Pi_C^N] = \frac{1}{(2+\gamma)^2}$, $E_{\widetilde{s}}[\Pi_B^N] = \frac{1-\gamma}{(\gamma-2)^2\,(\gamma+1)}$, $E_{\widetilde{s}}[\Pi^M] = \frac{1}{4(1+\gamma)}$ which correspond to the deterministic case (see Singh and Vives, 1984). Of course, given the constraint $s_t \leq \theta$, if θ vanishes s_t does likewise, and the same expressions can be obtained by canceling out s_t in (5), (6), (7).

Without imposing any restriction on the range of parameters, the analytical expression for δ_K^* appears quite cumbersome[11]. Hence, let us consider the case $\theta = \frac{1}{10}$, meaning that the demand level is allowed to fluctuate, either upwards or downwards, up to 10%. Notice that the fixing of θ does not affect the qualitative properties of the model, albeit it makes life much easier. In what follows, I plot δ_K^* against γ (the independent variable), for different values[12]

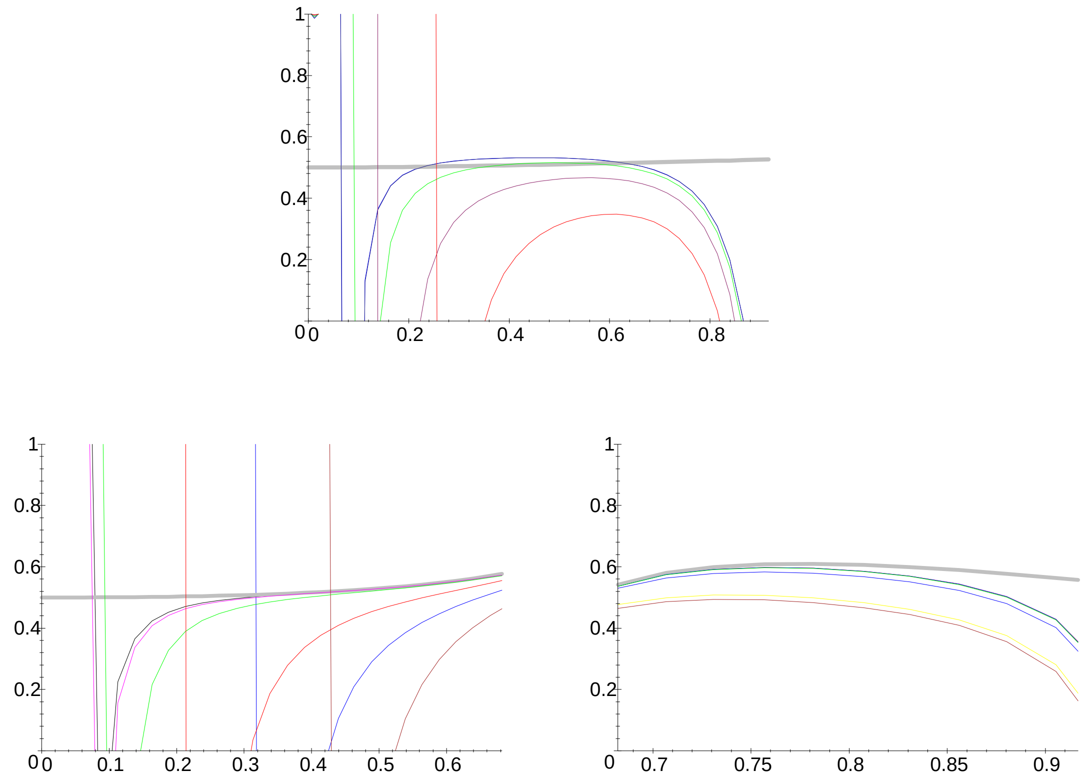
[10] Notice that since deviation occurs after having observed the realization of the shock, the corresponding level of the expected profit is a degenerate stochastic function.

[11] I report only the expression for δ_C^*; the one for δ_B^* is available upon request. $\delta_C^* = \frac{\left(24s+3\gamma^2-6\gamma^3-24s\gamma+3\gamma^4-6\gamma^2 s+6\gamma^3 s+12s^2-12s^2\gamma+3s^2\gamma^2-16\theta^2\gamma^3+12\theta^2\gamma-4\theta^2\right)(\gamma+2)^2}{s(96+48s)-16\theta^2+\gamma^2(24(1-\gamma)-3\gamma^2(7-\gamma^2-6\gamma)-96s-24s(\gamma+s)+3\gamma^2 s(6+2\gamma+s)+16\theta^2(2-\gamma^2))}$

Notice that when $\theta = s = 0$, $\delta_C^* = \frac{(\gamma+2)^2}{8+8\gamma+\gamma^2}$ which corresponds to the deterministic case.

[12] The curves closer to the thick line corresponds to the lower values of s_t. The rationale is that the higher the

of negative shocks. The highest figure is for Cournot competition, while the row below is for Bertrand competition:



where the thick line represents the deterministic case. All the information I need is contained into this graphical analysis[13]. Next three sections will deal with such an information from three different perspectives, aiming at answering three distinct questions.

3. Is competition more pervasive during booms?

Rotemberg and Saloner (1986) concludes that with linear market demand and linear marginal cost, competition is always more pervasive in booms, independently of whether firms set prices or quantities.

In this section, I test the robustness of their result in case of differentiated products. I show that, under Cournot competition, when products are almost independent, the result is

amplitude of the shocks, the greater the differences w.r.t. the deterministic case.

[13]The figures referred to positive shocks are specular to those referred to negative shocks, therefore omitted.

reverted, while under Bertrand competition, it is confirmed regardless of the degree of product differentiation.

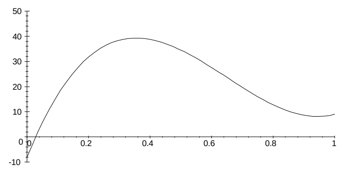
Proposition 1 *At time t, for all $\gamma \in (0, 0.91667]$ and for all $s_t \in [-\frac{1}{10}, \frac{1}{10}]$, consider the size of the market grows: under Cournot competition, whenever $\gamma \in (0, 0.02\,769\,7]$ the likelihood of collusion increases, while it decreases whenever $\gamma \in (0.02\,769\,7, 0.91667]$; under Bertrand competition, the likelihood of collusion always decreases.*

P roof. Competition is more pervasive during booms as long as the critical discount factor is increasing in s_t: $\dfrac{\partial \delta_K^*}{\partial s_t} > 0$. Let me move from Cournot competition. It is easy to find the following:

$sign(\dfrac{\partial \delta_C^*}{\partial s_t}) = sign(304\gamma^3 - 592\gamma^2 + 305\gamma - 8) = sign(\Xi)$

$(304\gamma^3 - 592\gamma^2 + 305\gamma - 8) = 0 \rightarrow \gamma = 0.02\,769\,7$

Plot of Ξ :



$\dfrac{\partial \delta_C^*}{\partial s_t} \leq 0 \ \forall \gamma \in (0, 0.02\,769\,7]$; $\dfrac{\partial \delta_C^*}{\partial s_t} > 0 \ \forall \gamma \in (0.02\,769\,7, 0.91667]$.

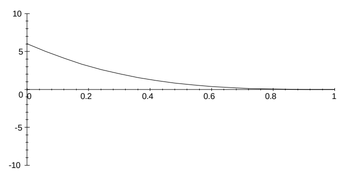
(i) unconstrained region:

Turning to Bertrand competition, consider first the range $\gamma \in (0, \widehat{\gamma}]$

$sign(\dfrac{\partial \delta_B^*}{\partial s_t}) = -sign(\frac{76}{25}\gamma^3 + \frac{152}{25}\gamma^3 s_t - \frac{303}{25}\gamma^2 - \frac{454}{25}\gamma^2 s_t + \frac{1509}{100}\gamma + \frac{181}{10} s_t \gamma - \frac{301}{50} - \frac{301}{50} s_t) = sign(\Sigma)$

$\dfrac{\partial \Sigma}{\partial s} > 0 \ \forall \gamma \in [0, \widehat{\gamma}]$; $\dfrac{\partial \Sigma}{\partial s_t} = 0$ if $\gamma = 1 \notin [0, \widehat{\gamma}]$

Plot of $\dfrac{\partial \Sigma}{\partial s_t}$:



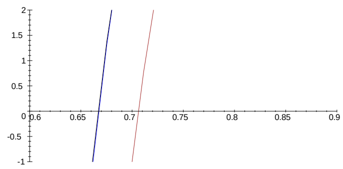
(ii) constrained region:

Now, consider the range $\gamma \in (\widehat{\gamma}, 0.91667]$

$sign(\dfrac{\partial\delta_B^*}{\partial s_t}) = sign(9.12\gamma^4+12.16\gamma^4 s_t-24.16\gamma^3 s_t-15.08\gamma^3-3.09\gamma^2-1.2s_t^2\gamma^2+24.0s_t\gamma+15.09\gamma+1.6\gamma s_t^2-12.0s_t-6.02-.4s_t^2) = sign(\Psi)$

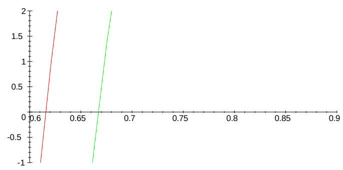
$\dfrac{\partial\Psi}{\partial s_t} > 0\ \forall\gamma\in[\widehat{\gamma},0.91667]$; $\dfrac{\partial\Psi}{\partial s_t} < 0\ \forall\gamma\in[0,\gamma^x]$ with $\gamma^x<\widehat{\gamma}$; $\dfrac{\partial\Psi}{\partial s_t} = 0$ if $\gamma = 1 \notin [\widehat{\gamma},0.91667]$

Plot of Ψ:



$$s_t = \{10^{-8}, 10^{-1}\}$$

In case of positive shocks, $\Psi(0.76865) = 0.088449 - 0.26753s_t$; since $s_t \in [0,.1]$, $\Psi(0.76865) = 0.088449 - 0.26753(.1) = 0.061696$ is the minimum value, still positive.



$$s_t = \{-10^{-1}, -10^{-8}\}$$

In case of negative shocks, $\Psi(\gamma = 0.6821) = 0.023715 - .65133s_t$; since $s_t \in [-0.1, 0]$, $\Psi(\gamma = 0.6821) = 0.023715$ is the minimum value, still positive. This completes the proof. ■

Therefore, when competition takes place à la Cournot and products are highly differentiated, the common antitrust practice is no more appropriate, since it fails to be always true that competition is more pervasive in periods of high demands. In fact, it has been shown that attention should be paid also to those markets in which firms compete in quantities and, indeed, the degree of product standardization is very low. The scope of this result becomes even stronger as one considers a greater uncertainty.

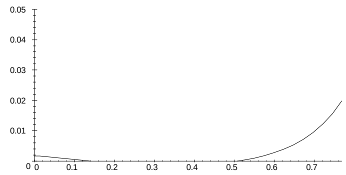
4. Is collusion in quantities more stable?

Deneckere (1983) proves that collusion in quantities is more stable than in prices for $\gamma \in (0, 0.962]$, while the opposite holds for $\gamma \in [0.962, 1]$. In this section, I address the same issue in a stochastic setting, showing that above a certain amplitude of the positive shocks, collusion in prices is more stable than in quantities for $\gamma \in (0, 0.91667]$, while above a certain amplitude of the negative shocks, the opposite holds. Between the negative and the positive threshold, there exists a region where one has to take into account both the amplitude of the shocks and the degree of product differentiation.

Proposition 2 *For all* $\gamma \in (0,\ 0.91667]$, *with* $s_t \in [0.0508, 0.1]$ *collusion in prices is more stable, with* $s_t \in [-0.1, -0.0083]$ *collusion in quantities is more stable. With* $s_t \in [-0.0083, 0.0508]$ *, see corollaries 3-6.*

Corollary 3 *The relationship betIen the critical threshold of* $s_t\ > 0$ *and* $\gamma \in (0,\ 0.76865]$ *is non monotone and convex. With* $s_t > 0.02$ *collusion in prices is more stable* $\forall \gamma \in (0,\ 0.76865]$*. With* $s_t > 0.002$ *collusion in prices is more stable* $\forall \gamma \in (0,\ 0.5823]$, *while* $\forall \gamma \in (0.5823,\ 0.76865]$ *it depends on* $s_t \in (0, 0.2]$

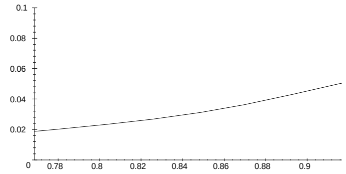
P roof. In all the following proofs, I rely on graphical representation, being the involved analytical expressions really cumbersome. I compute the difference betIen δ^*_C and δ^*_B, then I solve for s_t and make the corresponding graph. The curve in the space γ, s_t is where $\delta^*_C = \delta^*_B$.



Its upper counter set is where price collusion is more stable. ■

Corollary 4 *The relationship betIen the critical threshold of* $s_t\ > 0$ *and* $\gamma \in (0.76865,\ 0.91667]$ *is increasing and convex. With* $s_t\ > 0.0508$ *collusion in prices is more stable* $\forall \gamma \in (0.76865,\ 0.91667]$*. With* $s_t < 0.0182$ *collusion in quantities is more stable* $\forall \gamma \in (0.76865,\ 0.91667]$*.*

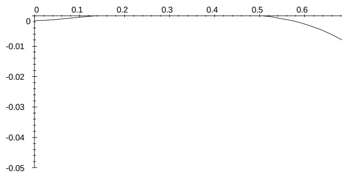
P roof. I compute the difference between δ^*_C and δ^*_B, then I solve for s_t and make the corresponding graph. The curve in the space γ, s_t is where $\delta^*_C = \delta^*_B$.



Its upper counter set is where price collusion is more stable. ■

Corollary 5 *The relationship between the critical threshold of* $s_t < 0$ *and* $\gamma \in (0,\ 0.6821]$ *is non monotone and concave.*$\forall \gamma \in (0,\ 0.76865]$ *with* $s_t < -0.0083$ *collusion in quantities always occur.* $\forall \gamma \in (0.123,\ 0.5167]$ *collusion in quantities is always more stable, otherwise it depends on* $s_t \in (0, 0.0083]$.

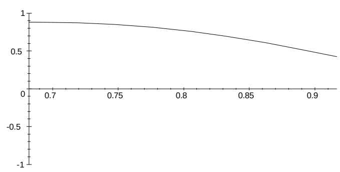
P roof. I compute the difference between δ_C^* and δ_B^*, then I solve for s_t and make the corresponding graph. The curve in the space γ, s_t is where $\delta_C^* = \delta_B^*$.



Its lower counter set is where quantity collusion is more stable. ■

Corollary 6 *With* $s_t < 0, \forall \gamma \in (0.76865, 0.91667]$ *collusion in quantities is always more stable.*

P roof. I compute the difference between δ_C^* and δ_B^*, then I solve for s_t and make the corresponding graph. The curve in the space γ, s_t is where $\delta_C^* = \delta_B^*$.



Its lower counter set is where quantity collusion is more stable. ■

Hence, during booms, firms would like to collude in prices while, during recessions, they would like to collude in quantities. The rationale behind this result is quite intuitive: it is always true that the possibility to cheat is lower under Cournot competition than otherwise; however, during booms, firms would like to collude in prices to decrease the likelihood of the punishment phase.

5. Does product differentiation enhance cartel stability?

As said in the introduction, many authors have tried to answer this question, arriving at demonstrating that, at least in the unconstrained region, product differentiation is a facilitating factor towards collusion. This believe is challenged by this section, which shows that, also in the unconstrained region, the less products are differentiated, the higher may be the stability of the cartel. With regard to Cournot competition, where the constraint on the quantity of the firm being cheated is never binding, it is shown that, again, product differentiation may be an hindering factor. This finding is in contrast with the existing deterministic analysis.

Corollary 7 *Under Cournot competition, the relationship betIen γ and δ_K^* is non monotone for $s_t < \widehat{s}_t \simeq 0.002$, while, except in the case where products are almost totally independent, the link is negative.*

Corollary 8 *Under Bertrand competition, $\forall \gamma \in (0, 0.76865]$ and $\forall s_t \in [0, 0.1]$ the relationship betIen γ and δ_K^* is negative for $s_t > \widehat{s}_t \simeq 0.09$, and non monotone and convex for $s_t < \widehat{s}_t$; $\forall \gamma \in (0, 0.6821]$ and $\forall s_t \in [-.1, 0]$ the relationship betIen γ and δ_K^* is positive*

Corollary 9 *Under Bertrand competition; $\forall \gamma \in (0.76865, 0.91667]$ and $\forall s_t \in [0, 0.1]$ the relationship betIen γ and δ_K^* is negative. $\forall \gamma \in (0.6821, 0.91667]$ and $\forall s_t \in [-.1, 0]$ the relationship betIen γ and δ_K^* is non monotone and concave.*

The above discussion can be summarized in the following:

Proposition 10 *Under both Cournot and Bertrand competion, there exists a range of parameters (s_t, γ) in which the relationship betIen γ and δ_K^* can be decreasing, either monotonically or not. Such a relationship can never be monotonically increasing.*

Notice that, in the deterministic case, the relationship betIen γ and δ_K^* can be monotonically increasing. This is true under Cournot competition and, at least in the unconstrained region, under Bertrand competition. As said in the introduction, such a prediction strongly contrasts with both the conventional wisdom and the prevailing empirical evidence. The introduction of a stochastic variable into the demand function has resulted sufficient to partially overcome that undesirable feature.

6. Concluding remarks

I have proposed a simple supergame duopoly model where each firm selling differentiated products faces a linear market demand subject to observable shocks. I have investigated upon the joint effects of product differentiation and uncertainty on the stability of implicit collusion, unveiling interesting results. First, I have challenged the believe that in a setting with linear market demand, linear marginal cost and observable shock of the kind adopted in this paper, competition is always more pervasive during booms as opposed to busts (Rotemberg and Saloner, 1986), having proved that, when competition takes place à la Cournot and products are strongly differentiated, cartels may be more stable in periods of high demand. Second, I have compared price with quantity competition in terms of cartel stability, and I have shown that, regardless of the degree of product differentiation, collusion in prices is more stable than in quantities for sufficiently relevant positive shocks, while the opposite holds for sufficiently relevant negative shocks, where the term relevant is referred to their amplitude. In contrast with Deneckere (1983), for whom collusion in quantities is always more stable than in prices except when products are almost completely standardized, I have shown that when the market is expanding, firms competing in prices will find it easier to collude, while when the market is affected by a recession, firms competing in prices will find it tougher to collude. Hence, as long as the market is expanding, I should expect firms to compete in prices. Finally, focussing on product differentiation, I have proved that its link with the critical discount factor is never increasing along the entire parameter range.

References

Albaek, S., and L. Lambertini, 1998. Collusion in differentiated duopolies revisited. *Economics Letters*, **59**, 305-8.

Bowley, A.L., 1924. The mathematical groundwork of economics. Oxford, *Oxford University Press.*

Chang, M.-H., 1991. The effects of product differentiation on collusive pricing. *International Journal of Industrial Organization,* **9**, 453-469.

Clarke, R., 1985. Industrial Economics. *Basil BlackIll.*

Deneckere, R., 1983. Duopoly supergames with product differentiation. *Economics Letters,* **11**, 72

Dixit, A., 1979. A model of duopoly suggesting a theory of entry barriers. *Bell Journal of Economics*, **10**, 20-32.

Ecchia, G. and L. Lambertini, 1997. Minimum quality standards and collusion. Journal of Industrial Economics, **45**, 103-18.

Friedman, J.W., 1971. A non cooperative equilibrium for supergames. *Review of Economic Studies*, **28**, 1-12.

Green, E. and R. Porter, 1984. Non-cooperative collusion under imperfect price information, *Econometrica,* **52**, 87-100.

Klemperer, P.D. and M.A. Meyer, 1986. Price competition vs quantity competition: the role of uncertainty. *Rand Journal of Economics*, **17**, 618-638.

Klemperer, P.D. and M.A. Meyer, 1989. Supply function equilibria in oligopoly under uncertainty. *Econometrica*, **57**, 1243-1277.

Hackner, J., 1994. Collusive pricing in markets for vertically differentiated products. *International Journal of Industrial Organization*, **12**, 155-177.

Hay, G. and D. Kelley, 1974. An empirical survey of price-fixing conspiracies. Journal of Law and Economics, **17**, 13-38.

Lambertini, L., Poddar, S. and D. Sasaki, 1998. Standardization and the stability of collusion. *Economics Letters*, **58**, 303-310.

Majerus, D.W., 1988. Price vs Quantity competition in oligopoly supegames. *Economics Letters,* **27**, 293-297.

Martin, S., 1989. Product differntiation and the stability of non-cooperative collusion. *Mimeo*, European University Institute.

Penard, T., 1998. Some implications of repeated games for antitrust policy. *Revue Economique*, **49**, 743-753.

Posner, R., 1976. Antitrust law - An economic perspective. *Univ. of Chicago Press,* Chicago and London.

Raith, M., 1996. Product differentiation, uncertainty and the stability of collusion. *Mimeo*, STICERD Working Paper no. EI/16 London School of Economics.

Ross, T.W., 1992. Cartel stability and product differentiation. *International Journal of Industrial Organization,* **10**, 1-13.

Rotemberg, J. and G. Saloner, 1986. A supergame-theoretic model of price wars during booms. *American Economic Review,* **76**, 390-407.

Rothschild, R., 1992. On the sustainability of collusion in differentiated duopolies. *Economics Letters,* **40**, 33-37.

Scherer, F.M. and D. Ross, 1990. Industrial market structure and economic performance. *Houghton Mi- in.*

Singh, N. and X. Vives, 1984. Price and quantity competition in a differentiated duopoly. *Rand Journal of Economics,* **15**, winter, 546-554.

Stigler, G., 1964. A theory of oligopoly. *Journal of Political Economy*, **72**, 44-61.

Symeonidis, G., 1999. Cartel stability in advertising-intensive and R&D-intensive industries. *Economics Letters*, **62**, 121-129.

Tirole, J., 1988. The theory of industrial organization. *MIT Press*, Cambridge, MA.

Irnefelt, B., 1989. Tacit collusion in differentiated Cournot games. *Economics Letters,* **29**, 303-306.