

FIRM INVESTMENT, CORPORATE FINANCE, AND TAXATION

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Abstract

This paper examines the effect of corporate income taxation on the investment behaviour of a firm that faces imperfect capital markets and optimally chooses its financial policy. It shows that when capital markets are imperfect, the optimizing firm goes through different phases of growth. Given the starting stock of capital, the firm initially enters a phase of intensive growth during which it finances new investment both by retaining earnings and by issuing new debt. While accumulating, it moves to a consolidation phase during which it fully finances investment by debt. Along this path, the firm approaches a steady state stock of capital. In this dynamic setting, the standard cost-of-capital formulae do not always apply. Corporate taxation of profits does not necessarily reduce firm investment, and its effect varies over time. An increase in the corporate profit tax rate initially reduces investment, but the effect is reversed over time as the firm adjusts its financing policy. Indeed, the overall effect depends on the temporal horizon.

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Firm Investment, Corporate Finance, and Taxation

1.1 Introduction

Corporate taxes play a central role in the tax policy debate. Reductions in corporate taxation are envisaged in many countries as a means of affecting the investment decisions of firms and fostering capital accumulation. In the United States, for example, there is considerable debate on the need to cut corporate taxation in order to support accumulation. In the European Union, many countries have already reduced corporate taxes. However, despite the apparent link between corporate taxation and capital accumulation over time, very little of the theoretical debate has been interested in the intertemporal effects of corporate taxes. In fact, little of the theoretical literature on corporate taxation is based on any very satisfactory analysis of the factors underlying the investment and financing behaviour of firms over time. The analysis usually relies on static cost-of-capital formulae and the firm's financial decisions are arbitrarily postulated (Sinn, 1991b).

The aim of this paper is to provide a theoretical model to examine the effect of corporate taxes over time and the role of financial decisions. To this purpose, we need a dynamic theory of firm investment and a modelling of the imperfections in capital markets, particularly those affecting the financing decisions of firms. In fact, the impact of taxes may be very sensitive to the firm's financing choices (Atkinson and Stiglitz, 1980). In addition, we have to consider that

investment is an intertemporal phenomenon, and we need, therefore, to examine the dynamics of tax effects. A tax cut may increase investment, but the effect varies over time as firms adjust to the tax change; indeed, the underlying timescale of the effects may be critical for governments. To provide a rigorous discussion of these issues, I set up a dynamic model of firm investment where financial and real investment policies are simultaneously derived from the firm's decision problem. While this model involves a number of strong assumptions, it does capture some important features of firm behaviour that would otherwise be lost.

The general framework underlying the model of the firm is described in section 2; this is formally set up so as to permit use of the standard methods of optimal control. The model is of the "adjustment costs" type that has been discussed a great deal in the literature, but incorporates in addition the firm's financial decisions, and imperfections in capital markets. The role of financial imperfections has in fact been examined, amongst others, by Sinn (1991) and Steigum (1983); however, these authors ignore the costs of installing capital, and consider different financial constraints. In these respects, the model described here is closer to a number of recent empirical contributions such as Bond *et al.* (1997), Bond and Meghir (1994), and Fazzari *et al.* (1988, 1996). However, these authors do not develop the real and financial choices of the firm as a simultaneous decisions-making process. Since this is clearly central to the problem discussed in this paper, I have extended the analysis in this direction. This extended model of investment is used in section 3 to determine the financial and real investment decisions of the firm over time. It is shown that the firm goes through different phases of growth. Given the starting stock of capital, the firm initially enters a phase of "intensive" growth. During this phase, it finances new investment both by retaining earnings and by issuing new debt. While accumulating, it moves to a "consolidation" phase during which it fully finances investment by debt. Along this path, the firm approaches a steady state stock of capital.

This model of investment is meant to prepare the ground for a discussion of the effects of corporate taxation and, in section 4, it is re-stated to incorporate a corporate tax on profits. This extended model is used in section 5 to examine the intertemporal effects of profit taxation. In discussing these effects, the standard method of comparative-dynamics is initially presented and criticised for it does not explicitly consider time. Then, an alternative approach of dynamic analysis is developed. This consists in examining the variations in the temporal path of invest-

ment as they derive from changes in the tax policy. A similar method has in fact been used by Atkinson (1971), to whose work I owe a considerable debt; however, the method is extended to a several variables problem and to include no endpoint conditions, which is the central framework of the problem discussed in this paper. The dynamic analysis shows that the straightforward application of the standard cost-of-capital formulae may be misleading. One can only apply the standard results to a dynamic model under certain conditions. In particular, taxation does not necessarily reduce investment, and its effect varies over time. An increase in the corporate profit tax rate initially reduces investment, but the effect is reversed over time as the firm adjusts its financing policy; thus, the overall effect depends on the temporal horizon. The intertemporal perspective makes a great deal of difference to the way we think about the effect of corporate taxes and poses a number of problems. Section 6 deals with some of the problems of examining taxes in a dynamic model, problems that have received very little attention in the literature.

The theoretical results of this paper are of great importance for tax policy purposes. They suggest that reductions in corporate profit taxes increase firm investment, but this effect is only temporary and it is reversed over time. Thus, a policy of reducing tax rates, following for example fierce tax competition, is expected to have only limited effect on capital accumulation of domestic firms. The relevance of these intertemporal implications depends, of course, on the timescale of the effects; however, it should be clear that to confine the attention to the immediate impact of taxes, as in the standard literature, may be misleading. Indeed, the intertemporal aspects can make a great deal of difference to the way we think about corporate taxes.

1.2 A model of the firm with imperfect capital markets

To examine the effect of corporate taxation, we need a theory of firm investment, and a modelling of the imperfections in capital markets. In addition, we need to envisage the mechanisms through which these imperfections affect the firm's financing decisions. In fact, the literature on taxation has demonstrated how the impact of taxes may be very sensitive to the firm's financing decisions (Atkinson and Stiglitz, 1980). This section sets forth a dynamic model of investment where capital markets are imperfect and financial and real investment policies are

simultaneously derived from the firm maximization problem.

1.2.1 Firm investment and financing constraints

As noted at the outset, we consider the case of the single firm that plans its investment over time and is concerned with how to finance the accumulation process. For simplicity, it is assumed to face market conditions that do not change over time. As a result of these assumptions, the firm takes real and financial decisions simultaneously, and has static expectations.

One of the basic assumptions of the model concerns the characterization of the firm investment behaviour. In the spirit of most of the theory of investment, let us assume that when the firm invests in real assets it encounters some costs of installing new capital. In particular, the firm that at time t undertakes gross investment I_t must set aside I_t units as capital, together with $c(I_t)$ units which are used during installation. The amount $c(I_t)$ is used up in transforming goods into capital, and represents the cost of buying an increment in the stock of capital, namely the cost of adjusting capital upwards. For convenience, these costs are assumed to be non-negative (with $c(0) = 0$), increasing (with $c'(0) = 0$), and strictly convex, so that at each time the firm finds it convenient to accumulate only a finite amount of new capital. Models with strictly convex adjustment costs are widespread in economics, and are often used in the literature of investment behaviour to describe the accumulation decisions of firms (see for example Eisner and Strotz, 1963; Lucas, 1967; Uzawa, 1969; Abel and Blanchard, 1983). However, one of the basic premises of the present model is that financing decisions are important in examining the firm investment behaviour, and they have to be explicitly considered into the analysis.

Financial decisions are introduced into the model by assuming that the firm faces specific imperfections in capital markets. The first imperfection is a limitation on the firm's ability to issue new shares. This constraint is not new to economists, and the justification has been well developed in the microeconomic literatures on adverse selection and moral hazard (for example in Greenwald *et al.*, 1984). For simplicity, let us assume this constraint in the extreme form of no equity selling on the part of the firm. As a result of this assumption, the firm cannot issue new shares and has to finance new investment exclusively by means of retained profits and by

borrowing in credit markets.¹

At this point, we can introduce the second important assumption of the model: the firm can borrow external funds, but has access only to imperfect credit markets. These are seen as decentralised markets characterised by search on the part of firms and lenders. On the one hand, firms look for lenders who can match their demand for funds. On the other hand, lenders try to select better borrowers or less risky debtors in a context where information is imperfect. In this setting, imperfections arise as a problem of coordination, one that requires an explicit and costly searching process for new funds. More specifically, to expand external finance the firm has to carry out costly search activities, and these searching costs are rising with the amount of new debt requirements.

To make this more concrete, let us denote by b_t the stock of debt at time t , and by $\dot{b}_t \equiv db_t/dt$ its derivative with respect to time, that is the issues of new debt. At this point, we can define a searching or matching cost function $\sigma(\dot{b}_t)$. This is positive for any positive issue of new debt \dot{b}_t , and zero otherwise. Moreover, it is increasing in the new debt issues so that its first derivative $\sigma'(\dot{b}_t)$ is positive as well. As a result, at each time, larger issues of new debt imply greater search costs to the firm.

To complete the building blocks of our model of accumulation, let us assume that the searching cost function $\sigma(\dot{b}_t)$ is convex; thus, each further increase in debt requires a successively larger increase in search expenses. This assumption captures the idea of a quality sequential search approach adopted by the firm, the first effort of which is supposed to be the most productive. For example, let us consider the firm that promotes new bonds issues by a sequence of advertisements. We would expect that the number of answers to successive announcements decreases, so that decreasing returns characterise these expenses. Put the other way around, the firm is facing uprisng costs of issuing new bonds.²

Armed with this information, we can draw the matching cost function shown in the following

¹Empirical evidence suggests that firms tend not to issue shares, indeed, and to primary rely on internal flows and external debt. For instance, in the period from 1980 to 1985 in the United States, only 1,2% of gross investment by non-financial corporations was financed by share issues, while 67,8% was internally financed, and 31% was financed by debt (see Sinn, 1991).

²In practice, when debt issues exceed a “safety” level, lenders’ requirements (in terms of information, collateral and other covenants) do increase. In the extreme case, they can prevent some borrowers from accessing credit arrangements. Thereby, credit costs may well be assumed to rise at an increasing rate with the amount of new funds.

figure:

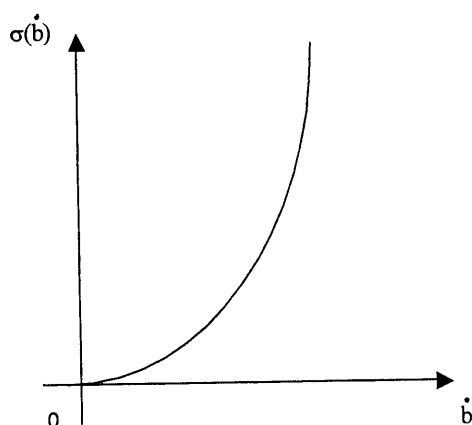


Figure 1: The searching cost function

The matching or searching costs are non-negative and rise at an increasing rate as b_t gets greater. We also assume, for convenience, that $\sigma'(0) = A = 0$, where A is a constant.

As the reader may have already detected, there is an analogy, in this model, between the treatment of search expenditures for new debt and the installing costs for investment. Search expenditures may be seen as buying an increment in the stock of “trust” or “informational capital” of the firm, for example, by providing information about the company. In that sense, Fig. 1 can be regarded as a deterministic description of an uncertain world where the increase in the cost for external funds results from lack of information and fear of bankruptcy.

Ideally, the search process for external funds should be derived from first principles rather than being simply postulated. The matching cost function $\sigma(b_t)$ ought to be achieved from a specification of the search technology used by firms and lenders; and, uncertainty should be treated more explicitly. However, to define the search process is beyond the scope of this analysis. In this sense, our representation is admittedly extreme in some respects. The form and the position of function $\sigma(\cdot)$ may well reflect a number of factors different from the issues of new debt, including the development of financial markets, the nature of the firm’s business, its outstanding debt liabilities and stock of equities. Indeed, in large part of the theory of firm investment, the yields demanded by lenders are assumed to increase with the debt-equity ratio of the firm (Modigliani and Miller, 1958; Steigum, 1983). Here, I have preferred to confine the attention to the effects of current financing decisions because this is an important aspect which has implications that are not always appreciated.

1.2.2 The firm's problem

To characterise the decision process of the firm, we assume that it acts optimally so as to maximise an objective function. Specifically, in the spirit of much of the neoclassical theory of investment, let us suppose that the firm chooses its real investment and financial strategies so as to maximise the present value of current and future cash flows discounted at an appropriate rate. This objective function is open to question, and different formulations could be proposed. What is important for our purposes, however, is not what the firm exactly maximises, but how it responds to changes in data like tax rates. In this respect, it is possible that the qualitative response to this practical question is more or less insensitive, within limits, to the precise nature of the firm's objective (see for example Solow, 1971). Therefore, we ignore the problem of what the firm exactly maximises, and simply look at its cash flow.³

To define the objective function of the firm, we need to determine its cash flow at each time. This is the difference between the firm's current receipts and disbursements. At each time t , the firm receives revenue from its productive activity and from new debt issues \dot{b}_t . For convenience, the firm is assumed to produce its output only with capital k_t , so that its revenue, profit and output can all be described by a function $R(k_t)$. This is increasing and strictly concave, thus $R' > 0$ and $R'' < 0$, and for technical reasons, I assume that $R''' > 0$ (which is a sufficient condition for the diagrammatic analysis).⁴ On the expenditure side, the firm faces expenses for investment I_t , and for installing costs $c(I_t)$, as well as search costs for external financing $\sigma(\dot{b}_t)$. Moreover, the firm borrows at an interest rate ρ that is supposed, for simplicity, to be constant; thus, at each time it has to pay interests ρ on debt outstanding at the beginning of the period.

As a result of these assumptions, and taking the price of capital goods as numeraire, the cash flow of the firm at any time t is:

$$\underbrace{R(k_t) + \dot{b}_t}_{\text{Revenue}} - \underbrace{I_t - c(I_t) - \sigma(\dot{b}_t) - \rho b_t}_{\text{Disbursements}}$$

³For alternative formulations of the firm's objective, and references to the literature, see Solow (1971). A recent discussion of the cash flow objective with an explicit treatment of uncertainty is in Dutta and Radner (1999).

⁴It is also assumed that the boundary Inada conditions hold, so that $R(0) = 0$, $R'(0) = \infty$, and $R'(\infty) = 0$.

At this point, the firm's objective function can be defined as the discounted sum of each single period cash flow

$$v(0) = \int_0^{\infty} \left(R(k_t) - \rho b_t - I_t - c(I_t) + \dot{b}_t - \sigma(\dot{b}_t) \right) e^{-\theta t} dt \quad (1.1)$$

where future cash flows are discounted at whatever rate θ is appropriate, given the rate of interest ρ , the riskiness of the business, and whatever else may matter. In particular, we would expect the discount rate θ to be larger than the market interest rate, so as to justify the existence of productive activities even when capital markets are imperfect (see for example Kalecki, 1937; Hahn and Matthews, 1964; Uzawa, 1969).

In choosing its policies, the firm maximises its objective function, but has to satisfy a number of constraints. It has to consider the law of motion of capital and that for debt. If we assume, for simplicity, that there is no physical depreciation, then net investment $\dot{k}_t \equiv dk_t/dt$ equals gross investment I_t , and

$$\dot{k}_t = I_t \quad (1.2)$$

hence, whatever is invested augments the existing stock of capital. The law of motion for debt can be now defined by stating how investment is financed. In particular, the firm finances new investment by retaining profits and by borrowing. If s_t denotes the profits retained in the firm, that is its saving, at time t , the law of motion for debt is derived from the following financing equation

$$I_t = s_t + \dot{b}_t \quad (1.3)$$

From this condition, the issues of new debt are the difference between current investment and internal saving. In what follows, let us assume that the firm can neither run down capital nor reduce its outstanding debt (or lend money), so that $I_t \geq 0$ and $\dot{b}_t \geq 0$. Moreover, let us suppose that the stock of own capital cannot decrease either so that $s_t \geq 0$. These assumptions are admittedly restrictive, but they are motivated by the attempt to treat one difficulty at a time. In fact, they allow us to focus on real capital accumulation and to exclude financial capital issues; indeed, real capital accumulation is what we are interested in. This greatly

simplifies the analysis.⁵

As a result of these constraints, the firm may be seen as designing its investment and financial policy so as to maximise the objective function (1.1) subject to the capital accumulation constraint (1.2), the financing condition (1.3), and the non-negativity constraints on I_t , \dot{b}_t , and s_t . In formalising this decision process, we need, at this point, to choose the appropriate decision or control variables of the problem. In particular, the financing constraint (1.3) allows us to reduce the number of controls, and to only use investment I_t and the new issues of debt \dot{b}_t . Retained earnings are in fact determined as a residual, the difference between total investment and new debt issues ($s_t = I_t - \dot{b}_t$). Hence, the firm has only two margins along which to maximise its objective.

Armed with this information, the firm's maximisation problem can be stated as follows:

$$\max_{I_t, \dot{b}_t} v(0) = \max_{I_t, \dot{b}_t} \int_0^{\infty} \left(R(k_t) - \rho b_t - I_t - c(I_t) + \dot{b}_t - \sigma(\dot{b}_t) \right) e^{-\theta t} dt$$

subject to

$$\dot{k}_t = I_t$$

$$I_t = s_t + \dot{b}_t$$

and to the non-negativity constraints

$$I_t \geq 0 \quad \dot{b}_t \geq 0 \quad s_t \geq 0$$

where the initial values of the state variables at time $t = 0$ are given, with $b(0) = b_0 \geq 0$, and $k(0) = k_0 > 0$, so that the firm can get production off the ground.

It may be noticed that this problem differs from the standard cost of adjustment model in the search cost term $\sigma(\dot{b}_t)$, and in the presence of financial imperfections. Indeed, in the absence of an equity finance constraint and with no search cost for external debt, investment could be indifferently financed by all means; the firm's problem would then reduce to the usual cost of adjustment formulation (as for example in Abel and Blanchard, 1983).

⁵For an extended version of this model, including accumulation of financial capital (lending money), and reduction in the stock of real capital, see Palomba (2001).

1.2.3 Optimality conditions

The firm's intertemporal maximisation problem is now set up as a typical problem of optimal control. We can then apply the standard techniques of the optimal control theory, and solve for the path of the controls I_t and \dot{b}_t . To this end, we can restrict the attention to the case where $\theta > \rho$, that is the discounting rate exceeds the market interest rate; indeed, this is the only possible case. If we assumed, for example, that $\theta \leq \rho$, then the optimal plan would involve continuous accumulation of financial assets (with a negative \dot{b}_t representing financial accumulation); but, this possibility has been ruled out by imposing the constraint $\dot{b}_t \geq 0$.

At this point, we can apply the Pontryagin's Maximum Principle. Introducing the multipliers q_{1t} and q_{2t} for the constraints (1.2) and (1.3), the current value Hamiltonian function of the problem can be written as:

$$H = R(k_t) - \rho b_t - I_t - c(I_t) + \dot{b}_t - \sigma(\dot{b}_t) + q_{1t}I_t - q_{2t}\dot{b}_t$$

where the q_{1t} and q_{2t} functions are the co-state variables of the problem. These can be interpreted as giving the marginal value of the associated state variables at time t in terms of values at t .

This Hamiltonian is to be maximised subject to the non-negativity constraints on the control variables I_t and \dot{b}_t , and on savings s_t ; therefore, we need to apply the Lagrangian approach. Associating the non-negative Lagrange multipliers μ_{I_t} , μ_{b_t} and μ_{s_t} with, respectively, the constraints on the controls I_t , \dot{b}_t and on s_t , the Lagrangian function of the problem is:

$$L = R(k_t) - \rho b_t - I_t - c(I_t) + \dot{b}_t - \sigma(\dot{b}_t) + q_{1t}I_t - q_{2t}\dot{b}_t + \mu_{I_t}I_t + \mu_{b_t}\dot{b}_t + \mu_{s_t}(I_t - \dot{b}_t)$$

where the condition (1.3) has been used. The optimal investment and financing policy of the firm can be now calculated as a maximum of L .

The necessary first-order conditions for optimality can be set out by differentiating the Lagrangian with respect to the control variables I_t and \dot{b}_t

$$q_{1t} = 1 + c'(I_t) - \mu_{I_t} - \mu_{s_t}$$

$$q_{2t} = 1 - \sigma'(\dot{b}_t) + \mu_{bt} - \mu_{st}$$

where c' and σ' denote the derivatives of functions $c(\cdot)$ and $\sigma(\cdot)$ with respect to their arguments. Moreover, at the optimum the following complementary slackness conditions must hold:

$$\mu_{I_t} \geq 0 \quad I_t \geq 0 \quad \mu_{I_t} I_t = 0$$

$$\mu_{b_t} \geq 0 \quad \dot{b}_t \geq 0 \quad \mu_{b_t} \dot{b}_t = 0$$

$$\mu_{s_t} \geq 0 \quad s_t \geq 0 \quad \mu_{s_t} s_t = 0$$

In addition, differentiating the Lagrangian with respect to the states k_t and b_t , it gives us the multiplier equations

$$\dot{q}_{1t} = \theta q_{1t} - R'(k_t)$$

$$\dot{q}_{2t} = \theta q_{2t} - \rho$$

where $\dot{q}_{xt} \equiv \frac{dq_{xt}}{dt}$, and from the dynamic constraints of the problem, we have the laws of motion for the state variables

$$\dot{k}_t = I_t$$

$$I_t = \dot{b}_t + s_t$$

Finally, on the boundary, optimality also requires the transversality conditions

$$\lim_{t \rightarrow \infty} q_{1t} e^{-\theta t} k_t = 0$$

$$\lim_{t \rightarrow \infty} q_{2t} e^{-\theta t} b_t = 0$$

These conditions may be simplified. We can solve the differential equation in q_{1t} subject to the transversality condition for capital to give

$$q_{1t} = e^{\theta t} \int_t^{\infty} R'(k_v) e^{-\theta v} dv$$

Thus, at the optimum, the shadow price of capital is equal to the present discounted value

of future marginal products. Like in the standard cost of adjustment model, it does not directly depend on financial aspects.

In the same way, we can integrate over time the differential equation in q_{2t} , and using the transversality condition for debt, we obtain

$$q_{2t} = \frac{\rho}{\theta}$$

This tells us that along the optimal path, the shadow cost or marginal valuation of debt is constant, and depends both on the market interest rate, ρ , and on the expected rate of return θ . In particular, since $\theta > \rho$, the marginal cost of debt is lower than one. This suggests that, at the margin, debt costs less than what it can buy. In this sense, debt is likely to be a “bargain” for the firm, and it may be the preferred source of funds. However, in choosing its optimal borrowing policy, the firm shall trade off this advantage against the search costs for obtaining new funds, and whether debt is the preferred source of funds will depend on the level of search costs, an issue we discuss later.

Looking for the optimal growth path of the firm means finding a temporal paths of the controls I_t and \dot{b}_t that solve the optimality conditions given above, and that at the optimum satisfies the dynamics and the constraints of the system.

Along the optimal path the firm may adopt different financial policies and be in different regimes or activity phases. In particular, because of condition (1.3), if the firm invests in new capital ($I_t > 0$), then it has to either issue new debt ($\dot{b}_t > 0$) or retain earnings ($s_t > 0$) or both ($\dot{b}_t > 0, s_t > 0$); but, if it does not invest, then it neither retains profit nor collects external funds ($I_t = \dot{b}_t = s_t = 0$). This leaves us with four feasible regimes or activity phases:

- | | | |
|-----------|-----------------------------------|--|
| Regime 1. | $I_t = \dot{b}_t = s_t = 0$ | <i>The stationary state</i> |
| Regime 2. | $I_t > 0, \dot{b}_t > 0, s_t > 0$ | <i>The singular solution</i> |
| Regime 3. | $I_t > 0, \dot{b}_t > 0, s_t = 0$ | <i>The pure debt-financing policy</i> |
| Regime 4. | $I_t > 0, \dot{b}_t = 0, s_t > 0$ | <i>The pure Internal growth policy</i> |

The names of these regimes reflect different strategies of investment financing.

1.3 The optimal accumulation path of the firm

To examine the effect of taxes on firm behaviour, we need to determine the optimal accumulation path of the firm. This is a combination of different financial regimes that satisfies the Maximum Principle's optimality requirements. In turn, each regime is characterised by its own specific first-order conditions and dynamics, and to find the overall path we need to investigate the different regimes separately. In this section, I begin to examine the stationary states in regime 1 and the dynamics of the model. I then consider the investment and financial decision rules of the firm in the other regimes. Finally, these results are combined together to determine the firm's policy along the optimal accumulation path.

1.3.1 Stationary states and dynamics

Let us start by examining the stationary states in regime 1, and by describing the general dynamics of the model. While these do not tell us about the optimal policies of the firm, they are useful to illustrate the behaviour of the model.

Regime 1 and stationary states

In the financial regime 1, the firm is in a stationary state; it does not invest in new capital and does not take any financing decision either, so that $I_t = s_t = \dot{b}_t = 0$. As a result of this behaviour, both the stock of capital and the level of outstanding debt are constant, and we can denote these steady-state values by k_s and b_s , respectively.

In this regime, the optimal behaviour of the firm is described by the first-order conditions and dynamics discussed in the previous section; but, these greatly simplify. Provided that $c'(0) = 0$ and $\sigma'(0) = A = 0$, they can be reformulated as

$$q_{1t} = 1 - \mu_{It} - \mu_{st}$$

$$q_{2t} = 1 - \mu_{st} + \mu_{bt}$$

$$q_{1t} = e^{\theta t} \int_t^{\infty} R'(k_v) e^{-\theta v} dv$$

$$q_{2t} = \frac{\rho}{\theta}$$

Since the stock of capital is constant, the integral for q_{1t} can be explicitly solved so as to

have

$$q_{1t} = \frac{R'(k_s)}{\theta} \quad (1.4)$$

From this equation and the condition that $q_{2t} = \frac{\rho}{\theta}$, it can be deduced that both the co-state variables q_{1t} and q_{2t} are constant, so that

$$\dot{q}_{1t} = \dot{q}_{2t} = 0 \quad (1.5)$$

where $\dot{q}_{xt} = \frac{dq_{xt}}{dt}$. From these results, it follows that in the financial regime 1 there are no dynamics, and once the firm is in this activity phase it does not find desirable to move away.

At this point, we can explicitly characterise the possible stationary states of the model. Let us consider the first two optimality conditions; if we use the non-negativity property of Kuhn-Tucker multipliers, we obtain that at the optimum

$$q_{1t} \leq q_{2t} \quad (1.6)$$

Substituting for the value of the co-state variable q_{2t} in this condition, it follows that

$$q_{1t} \leq \frac{\rho}{\theta} \quad (1.7)$$

In the steady state, the shadow cost of capital q_{1t} is, therefore, non-greater than its replacement cost in terms of external funds $q_{2t} = \frac{\rho}{\theta}$. That is, its value does not compensate for the cost that a marginal addition of capital would generate when financed by borrowing, even if the cost of borrowing, $\frac{\rho}{\theta}$, is smaller than one.

If we now substitute equation (1.4) in previous condition (1.7), we obtain the optimal investment rule in this regime:

$$R'(k_s) \leq \rho \quad (1.8)$$

The equation (1.8) determines the firm's optimal steady state level of capital as a function of the market rate of interest. It may be noticed that, in this model, the steady state stock of capital is determined like in a model with perfect capital markets, in which ρ is the market interest rate. However, condition (1.8) does not identify a unique steady state stock of capital

as in a standard model with adjustment costs; instead, it defines a wide range of possible values, so that different stationary states are possible.⁶

Armed with this information, we can illustrate the set of possible stationary states in the (k, q_1) phase diagram shown in Fig. 2. All points in the positive orthant of this figure are feasible, except for points on the vertical axis above the origin; under the previous assumptions, k_0 is in fact positive, and capital cannot be reduced (as $I_t \geq 0$), so that the stock of capital can never be zero. The points along the horizontal axis also are excluded by the Inada conditions on $R(k_t)$ which do not allow q_1 to be zero (recall the equation (1.4)).

The set of possible stationary states can be depicted by considering the conditions (1.5) and (1.7). The locus $\dot{q}_1 = 0$ in the figure corresponds to condition (1.5) (recall that, from the optimality conditions, $\dot{q}_{1t} = \theta q_{1t} - R'(k_t)$, therefore $\dot{q}_1 = 0$ implies that $q_{1t} = \frac{R'(k_t)}{\theta}$); this is a downward sloping curve, and its concavity properties derive from the assumptions made on the marginal product function $R'(k)$. The equation (1.7) requires that $q_1 \leq \frac{\rho}{\theta}$; this condition identifies an area below the $q_{1t} = \frac{\rho}{\theta}$ locus in the figure where $I = 0$. The points on the $\dot{q}_1 = 0$ locus and below the $\frac{\rho}{\theta}$ line (the line AB) satisfy both the optimality conditions (1.5) and (1.7), and identify the set of stationary-state levels of capital $\{k_s\}$ compatible with the optimality condition (1.8). As noted above, there is a wide set of possible steady states. Once the firm is at one of these points, it can stay there for ever, since there are no dynamics, and all transversality conditions are satisfied.

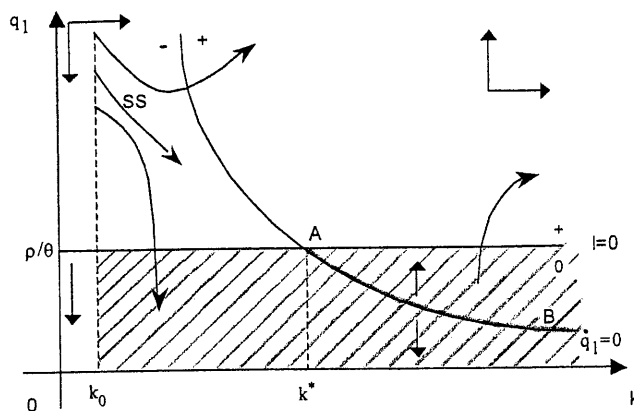


Figure 2: Steady states and dynamics

⁶For a discussion of cost of adjustment models with perfect financial markets, and references to the literature, see for example Abel and Blanchard (1983); and for their use in models of growth, see Atkinson (1999).

Dynamics

At this stage, we can characterise neither the exact accumulation path nor the optimal policy of the firm; but, we may use the phase diagram of previous Fig. 2 to sketch the directions of movement compatible with the optimality conditions, and to conjecture a possible solution to the firm's problem.

From Fig. 2, it can be deduced that anywhere above the $I_t = 0$ locus capital k_t is increasing; in fact, the marginal valuation of new capital, q_{1t} , exceeds the cost of borrowing money, $\frac{\rho}{\theta}$, and the necessary condition (1.7) for a stationary state is violated. In this case, the firm finds it desirable to invest against borrowing external funds, and $I_t > 0$. Similarly, the stock of capital k_t is constant at points below the $I_t = 0$ locus where $q_1 \leq \frac{\rho}{\theta}$, and it is not optimal to invest in new capital. In the case of the $\dot{q}_1 = 0$ locus, the marginal value of capital q_{1t} is increasing at points above the locus, where $q_{1t} > \frac{R'(k_t)}{\theta}$ and, from dynamics, $\dot{q}_1 > 0$; but, it is decreasing at points below the locus, where $\dot{q}_1 < 0$. The arrows in Fig. 2 demonstrate these possible directions of motions, and the trajectories illustrate the dynamics of the model consistent with the optimality conditions.

As it is clear from the figure, there are three possible sets of equilibria: the set of stationary states along the $\dot{q}_1 = 0$ locus and below the $\frac{\rho}{\theta}$ locus, namely the line AB ; the points along the horizontal axis; finally, the points at the top-right hand side of the phase diagram where both capital and the co-state variable q_1 take infinite values. However, only the trajectory SS that converges to A and the set of stationary states on the AB line satisfy the optimality conditions of the problem. All other paths, either violate the transversality conditions or do not satisfy some of the optimality requirements. The trajectory going to infinity, for example, violates the transversality condition for capital, and cannot, therefore, be optimal. The paths leading to a zero marginal value of capital (the horizontal axis) are not possible either, as they violate the Inada conditions and the fact that the equilibrium cannot be on the horizontal axis. It follows that the only possible optimal paths are those leading towards one of the stationary state levels of capital, as these are the only long-run equilibria of the model.

It is of course unfortunate that there is an infinite set of long-run equilibria; indeed, the nature of the optimal solution crucially depends on the initial conditions of the problem. For example, let us assume that the starting stock of capital k_0 is greater than the smallest possible

steady state level of capital k^* (see Fig. 2), then there is no optimal path, and any trajectory would violate some of the conditions of the problem. In this case, the only long-run equilibrium has to be the initial stock of capital itself, if this is a stationary state of the model (that is, k_0 has to be on the line AB in Fig. 2). If $k_0 < k^*$, there may exist an optimal path such as the trajectory SS which approaches point A and leads to the steady state level of capital k^* . Along this path the firm accumulates capital, and has to finance the growth process adequately. This is the only optimal accumulation path of the model, and this is the case on which I will focus my attention.

1.3.2 The investment and financial decision rules of the firm

How does the firm choose its investment and financial policy along the accumulation path? Along this path, there are different financial regimes through which the firm may grow, and each regime corresponds to different investment and financial policy choices. To determine the firm's optimal policy along the accumulation path, we then need to examine its decisions in each regime separately, even though this means that the analysis becomes regrettably taxonomic. The different regimes can be then joined together to characterise the firm's optimal policy along the accumulation path.

Regime 2 and the singular solution

Let us start considering the financial regime 2. In this regime the firm invests in new capital, and finances investment both by issuing new debt and by retaining earnings. The decision variables I_t , \dot{b}_t , and s_t are, therefore, positive, and the regime identifies a singular solution of the firm's problem.

In this regime, the complementary slackness conditions require the Kuhn-Tucker multipliers μ_{I_t} , μ_{b_t} and μ_{s_t} to be zero, and the first of the optimality conditions becomes:

$$q_{1t} = 1 + c'(I_t)$$

This condition contains a straightforward result; namely, that the rate of investment (relative to the stock of capital) is a function only of the marginal valuation of installed capital q_{1t} . More

specifically, investment takes place until the marginal valuation of capital equates the marginal cost of investing, which rises with the rate of investment because of adjustment costs. These costs are strictly convex ($c''(\cdot) > 0$), and we can then define an inverse function $\varphi(\cdot)$ to give the investment rule of the firm,

$$I_t = \varphi(q_{1t} - 1) \quad (1.9)$$

where from the properties of $c(\cdot)$, it follows that $\varphi'(\cdot) > 0$ and $\varphi(0) = 0$, so that at $q_{1t} = 1$ investment is zero.

The equation (1.9) describes the firm's optimal investment rule as an increasing function of q_{1t} . It may be noticed that in this regime investment is always positive, therefore q_{1t} has to be greater than one; that is, this regime is characterised by $q_{1t} > 1$. It is also interesting to observe that investment decisions depend only on the shadow value of capital q_{1t} , and are independent of the firm's financial decisions.

Even if, in this regime, finance does not explicitly affect investment decisions, the different sources of funds are not equivalent. The second of the optimality conditions in section 2 shows, in particular, the optimal borrowing rule of the firm,

$$\sigma'(b_t) = 1 - \frac{\rho}{\theta}$$

where we have substituted the shadow cost of debt $q_{2t} = \frac{\rho}{\theta}$.

The term on the left-hand side of this equation gives the marginal search costs of issuing new debt, whilst the term on the right-hand side gives the advantage that results from financing one unit of new investment by external debt. In deciding its optimal borrowing policy, the firm trades off these advantages against the marginal search costs, and borrows until the advantages have been fully exploited.

To make this more concrete, we can invert the cost function $\sigma'(\cdot)$ so as to obtain the firm's optimal borrowing policy:

$$b_t = \omega \left(1 - \frac{\rho}{\theta}\right) \equiv \bar{w} \quad (1.10)$$

where \bar{w} is constant and positive as $\frac{\rho}{\theta} < 1$. From this, we deduce that in the financial regime 2 the firm issues a constant amount of new debt \bar{w} , and this is independent of its investment

decisions.

If we now substitute equations (1.9) and (1.10) in the financing constraint $I_t = \dot{b}_t + s_t$, and re-arrange, we can determine the firm's retaining profit rule,

$$s_t = I_t - \dot{b}_t = \varphi(q_{1t} - 1) - \bar{\varpi} = s(q_{1t}) > 0 \quad (1.11)$$

where \dot{b}_t is constant as in equation (1.10), and investment varies with the marginal value of capital q_{1t} , so that retained profits also vary with q_{1t} . The equations (1.9), (1.10) and (1.11) describe the investment and financing policy of the firm in regime 2.

In this regime, the exact level of investment, and the combination of the different sources of funds may vary over time, and we need to investigate the dynamics of the firm behaviour. In particular, we need to examine how the co-state variables q_{1t} and q_{2t} vary over time because, through the equations (1.9) and (1.10), these determine the evolution of the firm's investment and financial policy. We have already seen that q_{2t} does not change over time, and that the new debt issues are constant. Therefore, it only remains to examine the dynamics of q_{1t} and the path of investment. In this regime, the marginal valuation of capital is steadily decreasing so that $\dot{q}_{1t} < 0$ (see appendix A). From this result and equation (1.9), it follows that investment is monotonically decreasing over time together with the marginal valuation q_{1t} .

Armed with this information, we conclude that, in the financial regime 2, the firm invests in new capital at a decreasing rate, while issuing a constant amount of new debt. As a result, investment tends to be financed by retained earnings at a decreasing rate (and by debt at an increasing rate). These conclusions are illustrated in the following figure.

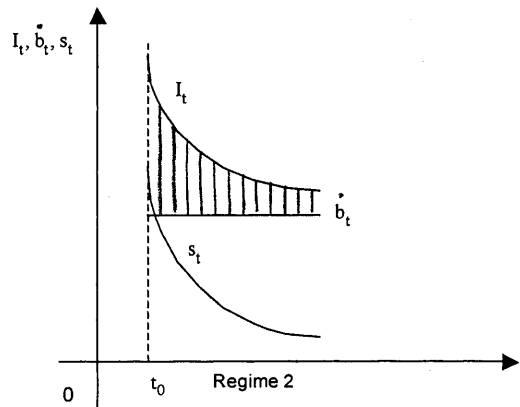
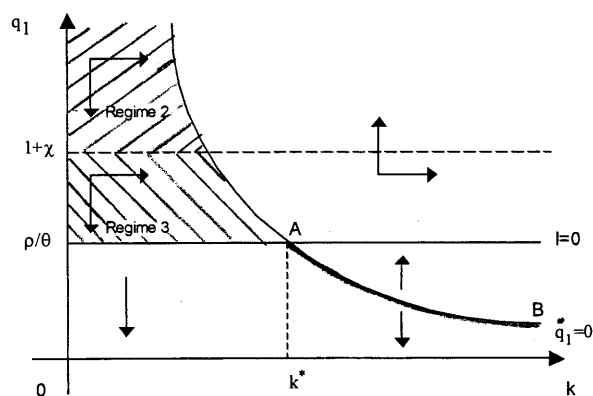


Figure 3: Financial and real investment decisions in regime 2

The firm invests I_t and finances part of this investment according to the optimal debt policy (1.10), the straight line. Given this optimal use of debt, there are sufficiently attractive investment opportunities remaining (the shadowed area), and the firm finances this extra investment by retaining earnings (the equation (1.11)). The retained-profit curve s_t is, thus, determined as a residual, namely the difference between investment and new debt issues. In this regime, retained earnings are, therefore, the marginal source of financing.

At times, it is convenient to work with the standard phase diagram shown in Fig. 4. The top shadowed area in this figure denotes the possible values for the co-state variable q_{1t} and the stock of capital k_t during the financial regime 2. This area lies below the $\dot{q}_1 = 0$ locus since during this regime $\dot{q}_{1t} < 0$ (see appendix A). The marginal value of capital q_{1t} has to also be greater than one; thus, there exists a regime 2-lower border at some $q_{1t} > 1$, say $q_{1t} = 1 + \chi$, where $\chi > 0$. At $q_{1t} = 1 + \chi$, the firm leaves regime 2, and this constitutes the lower limit for this activity phase as shown in the figure. The arrows indicate the optimal directions of motion described above, with capital be increasing as q_{1t} decreases.



Note: The AB line denotes the steady state levels of capital in regime 1.

Figure 4: Steady states, dynamics and financial regimes

Regime 3 and pure debt-financed growth

To finance its accumulation process the firm can follow alternative strategies; in the case of the financial regime 3, for example, it entirely finances investment by borrowing funds and is, therefore, in a pure debt-financing regime where $I_t = \dot{b}_t$.

Under this assumption, the complementary slackness conditions imply that the Kuhn-Tucker

multipliers μ_{I_t} and μ_{b_t} are zero; we can then reformulate the optimality conditions for this regime as

$$\begin{aligned} q_{1t} &= 1 + c'(I_t) - \mu_{st} \\ \frac{\rho}{\theta} &= 1 - \sigma'(\dot{b}_t) - \mu_{st} \end{aligned}$$

where the shadow cost of debt $q_{2t} = \frac{\rho}{\theta}$ has been substituted in the second of these conditions, and the integral for q_{1t} has been omitted.

If we substitute the first condition into the second, we obtain

$$c'(I_t) + \sigma'(\dot{b}_t) = q_{1t} - \frac{\rho}{\theta} \quad (1.12)$$

The term on the left-hand side of this equation gives the marginal cost of investing by issuing new debt, whilst the term on the right-hand side represents the net marginal advantage that result from financing investment by debt (this is the difference between the marginal valuation of capital, q_{1t} , and the shadow cost of debt, $\frac{\rho}{\theta}$). At the optimum, the firm trades off these advantages against the marginal costs of investing, and will invest till the advantages have been fully exploited.

To examine the firm behaviour, it may be more straightforward, however, to reformulate previous investment rule (1.12) in terms of the relation $\Phi(\cdot)$,

$$\Phi(I_t) = q_{1t} - \frac{\rho}{\theta}$$

where $\Phi(\cdot)$ is a monotonically increasing function, with $\Phi'(\cdot) > 0$, and $\Phi(0) = 0$ so that at $q_{1t} = \frac{\rho}{\theta}$ investment is zero. At this point, we can define the inverse function $\Psi(\cdot)$, and give the firm's optimal investment-borrowing policy as a function of q_{1t} only

$$I_t = \Psi\left(q_{1t} - \frac{\rho}{\theta}\right) = \dot{b}_t \quad (1.13)$$

where from the properties of $\Phi(\cdot)$, it follows that $\Psi'(\cdot) > 0$ with $\Psi(0) = 0$, namely at $q_{1t} = \frac{\rho}{\theta}$.

The equation (1.13) describes the investment policy of the firm in regime 3. From this, it turns out that investment takes place until the marginal value of installed capital, q_{1t} , equals the marginal cost of debt. Since in this regime investment is positive, it follows that q_{1t} must always

be greater than $\frac{\rho}{\theta}$; but, it can be greater or lower than one, without violating the necessary conditions for an optimum. The shadow value of capital q_{1t} can be smaller than one because in this regime, at the margin, the firm finances new investment by borrowing external funds; in turn, the cost of external funds can be as low as $\frac{\rho}{\theta}$ and, under previous assumptions, this is smaller than one.

The reader may probably find it interesting, at this point, to compare the firm behaviour in this regime with that in regime 2. From equation (1.13), it may be noticed that the issues of new debt are not constant, as it was the case in regime 2. More interestingly, if we re-arrange the second of the first-order condition, and consider the inverse function of $\sigma'(\cdot)$, we obtain the firm's optimal debt rule

$$\dot{b}_t = \omega \left(1 - \frac{\rho}{\theta} - \mu_{st} \right) \quad (1.14)$$

where $\omega'(\cdot) > 0$. This equation characterises the firm's optimal debt policy during regime 3, and also determines the investment policy as a function of the Kuhn-Tucker multiplier μ_{st} .

By comparing equation (1.14) with the corresponding condition (1.10) for regime 2, we see that the issues of new debt are now non-greater than in regime 2, as $\mu_{st} \geq 0$. Moreover, if we recall that in the previous regime debt issues were smaller than investment, it follows that the firm now issues a smaller amount of new debt and invests less than in regime 2.

At this point, the dynamics characterising the firm's optimal policy in this regime should also be discussed. As shown in appendix A, the marginal valuation of capital q_{1t} in regime 3 is steadily decreasing over time, so that $\dot{q}_{1t} < 0$. From the equation (1.13), it then follows that both investment and new debt issues are decreasing (as a corollary, it also follows that $\dot{\mu}_{st} > 0$; see appendix A).

Previous conclusions can be illustrated in the phase diagram of previous Fig. 4. The lowest shadowed area in this figure denotes the possible values for the co-state variable q_{1t} and the stock of capital during the financial regime 3. This area lies below the $\dot{q}_1 = 0$ locus since during regime 3 the marginal valuation of capital is decreasing, and $\dot{q}_{1t} < 0$ (see appendix A). In this regime, the shadow value of capital q_{1t} has to also be greater than $\frac{\rho}{\theta}$, as investment is positive (recall (1.13)); it will then exist a regime 3-lower limit at the $q_{1t} = \frac{\rho}{\theta}$ level. Finally, an upper constraint will also exist; indeed, above the $q_{1t} = 1 + \chi$ locus the level of investment differs from the new debt issues and the firm can no longer be in regime 3. The arrows demonstrate the

optimal directions of motion described above, with capital steadily increasing as q_{1t} decreases.

Regime 4 and pure internal growth

Finally, it only remains to consider the alternative strategy offered by the financial regime 4. In this regime, the firm invests in new capital and entirely finances investment by retaining earnings. As a result, the firm is in a pure internal growth phase where $I_t = s_t$, and external funds play no role.

In this regime, the complementary slackness conditions require both the Kuhn-Tucker multipliers μ_{I_t} and μ_{s_t} to be zero; thus, provided that $\sigma'(0) = A = 0$, we can reformulate the first-order conditions and dynamics as

$$q_{1t} = 1 + c'(I_t)$$

$$q_{2t} = 1 + \mu_{bt}$$

$$q_{1t} = e^{\theta t} \int_t^\infty R'(k_v) e^{-\theta v} dv$$

$$q_{2t} = \frac{\rho}{\theta}$$

These optimality conditions identify a path that is not, in general, optimal. On the one hand, the second of the optimality conditions implies that the shadow cost of debt q_{2t} is non-smaller than one; in fact, from the Kuhn-Tucker conditions $\mu_{bt} \geq 0$, and $q_{2t} = 1 + \mu_{bt} \geq 1$. On the other hand, the marginal valuation of debt q_{2t} is such that $q_{2t} = \frac{\rho}{\theta}$, but under our assumptions this is smaller than one, that is $q_{2t} < 1$. This is clearly a contradiction; thus, the financial regime 4 cannot be part of the firm's optimal growth path. This follows because, in this model, borrowing is an attractive option for the firm ($\rho < \theta$) as the marginal cost for searching external funds, $\sigma'(\cdot)$, becomes close to zero for sufficiently small amounts.⁷

Armed with these results, we conclude that given the set of stationary states in regime 1, the financial regimes 2 and 3 are the only feasible activity phases. Therefore, the optimal growth path of the firm can only go through these two regimes.

⁷For a discussion of the conditions under which debt may not be an optimal source of funds, see Palomba (2001).

1.3.3 The optimal growth path of the firm

What can we conclude from this about the optimal investment and financial decisions of the firm? To consider this, we need to determine the policy of the firm along the optimal accumulation path. This may be seen as a combination of the different financial regimes described above. But, the combination is not, of course, arbitrary. It has to satisfy the transversality conditions and the Maximum Principle's general requirement that there be no jumps in the state and co-state variables. We have already seen, however, that the discounting and the market interest rates are constant, so that the co-state variable $q_{2t} = \frac{p}{\theta}$ is necessarily continuous. The growth path can then be uniquely determined in a (k, q_1) phase diagram used above, where k_t and q_{1t} are the only state and co-state variables that we require to be continuous.

Crucial to the characterisation of the optimal growth path is the choice of the starting point or initial stock of capital. The earlier analysis has demonstrated how different starting stocks of capital may lead to different optimal patterns and to alternative steady states. For convenience, let us confine the attention to the case of a small level of initial capital; in particular, let us assume that the starting capital, k_0 , is smaller than the smallest steady state level k^* , as illustrated in Fig. 5. This assumption is clearly restrictive, but the alternative condition that $k_0 \geq k^*$ would imply no accumulation at all, and this is not a case of much interest to the present purposes. As a result of this assumption, there is only one direction of motion compatible with optimality, and this leads the firm to approach the steady state stock of capital k^* . This is shown in Fig. 5 by the continuous line SS .

The question at issue here is then to find, if it exists, a succession of financial regimes such that the firm starts off from the initial condition $k_0 \leq k^*$ and, following a continuous path, approaches the steady state and long-run equilibrium level of capital k^* in the financial regime 1. To determine such an accumulation path, it is convenient to adopt a backward solution strategy. Assume then to be at the long-run equilibrium k^* , and according to the optimality conditions let us trace out a complete reverse accumulation path, till to reach the initial stock k_0 .

Following the backward strategy, the firm is initially in the stationary state k^* where it acts according to the financial regime 1 (see Fig. 5). In this regime, the firm follows the *laissez faire* investment rule (1.8), and the level of capital k^* is such that $R'(k^*) = \rho$. In this steady state,

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optimality also requires that

$$q_{1t} = q_{2t} = \frac{\rho}{\theta}$$

thus, the shadow price of capital q_{1t} is equal to its replacement cost in terms of external funds q_{2t} . That is, the value of capital just compensates for the cost that an additional (infinitesimal) unit of capital would generate when financed by borrowing.⁸

In this steady state, all transversality conditions are satisfied, and $\dot{q}_{1t} = \dot{q}_{2t} = 0$; it follows that the firm can stay at this stage for ever as there is no incentive to invest and move away.

As a result of this behaviour, the financial regime 1 cannot be a moving phase. It cannot be a starting phase either, as $k_0 < k^*$ and the firm has to raise enough capital to get there. Thus, regime 1 can only be a final phase, and k^* is the long-run equilibrium of the model.

With the financial regime 1 as the final phase, it remains to explain how we can approach the long-run equilibrium k^* , and the potential candidates are the financial regimes 2 and 3.

The financial regime 2 cannot lead to the long-run equilibrium k^* , as this would imply a jump in the optimal path of the co-state variable q_{1t} (see Fig. 5). In this regime, the marginal valuation of capital is, in fact, greater than one, that is $q_{1t} > 1$. But, in the stationary state, $q_{1t} = \frac{\rho}{\theta}$ so that, under our assumptions, $q_{1t} < 1$. It follows that any move from the financial regime 2 to the steady state k^* requires a forbidden jump in the co-state variable q_{1t} , and the Maximum Principle's requirement of continuity would be violated.

It should be clear, at this point, that the only regime potentially leading to the stationary state is the financial regime 3. Indeed, in this regime the first-order optimality conditions define a set of paths for the co-state variable q_{1t} in the (k, q_1) space, one of which can eventually approach the stationary state k^* . In particular, the slope of these paths is given by

$$\frac{dq_t}{dk_t} = \frac{\dot{q}_{1t}}{\dot{k}_t} = \frac{\theta q_{1t} - R'(k_t)}{\dot{k}_t}$$

Since in regime 3 the marginal valuation of capital is decreasing over time, that is $\dot{q}_{1t} < 0$ (see appendix A), and capital steadily rises, so that $\dot{k}_t > 0$, it follows that the slope of these paths is negative. As a result, the co-state variable q_{1t} is monotonically decreasing as capital rises, and

⁸As far as the steady state Kuhn-Tucker multipliers are concerned, from the optimality conditions, we have that at $k_t = k^*$, $\mu_{1t} = \mu_{2t} = 0$ and $\mu_{3t} > 0$ with $\mu_{3t} = 1 - \frac{\rho}{\theta}$.

the arrows in Fig. 5 illustrate these directions of motion. From the equation (1.13), it can be then deduced that in regime 3 the rate of investment is monotonically decreasing, and the firm will eventually approach a point where $I_t = 0$. This is the case when $q_{1t} = \frac{\rho}{\theta} = 1 - \mu_{st}$ (recall (1.13) and (1.14)), and the convergence to such a point is ensured by the fact that in this regime $\dot{q}_{1t} < 0$, while at the same time $\dot{\mu}_{st} > 0$ (see appendix A). Once at this point, $q_{1t} = q_{2t} = \frac{\rho}{\theta}$, and capital is such that $R'(k_t) = \rho$, thereby the firm is at the long-run equilibrium in regime 1 where $k_t = k^*$, and there is no incentive to move away.

We then conclude that from the financial regime 3, an optimal growth path exists such that the firm converges towards the long-run equilibrium level of capital k^* in the financial regime 1. This conclusion is most obvious from the phase diagram of Fig. 5, where the continuous optimal path is drawn such that the firm in the financial regime 3 approaches the level of capital k^* in regime 1 with no jumps.

With regime 3 as the only phase approaching regime 1, and regime 1 as the necessary final phase, there only remains the question of the role of the financial regime 2; is this regime needed at all?

It can be shown that optimality does require the firm to go from the financial regime 2 to regime 3, so that regime 2 is the necessary link between the initial condition k_0 and the optimal growth path. To illustrate this point, let us examine the firm's optimal policy in regime 2. In this regime, the firm invests in new capital and accumulates at a decreasing rate, as q_{1t} decreases over time (recall the equation (1.9)). At the same time, it issues a constant amount of new debt, as described by the optimal debt rule (1.10); thus, the firm increasingly finances investment by debt. As a result of this behaviour, there will be a level of q_{1t} , e.g. $q_{1t} = 1 + \chi > 1$, at which investment eventually equals the new debt issues \dot{b}_t , and $I_t = \dot{b}_t$. At this point, the firm would finance new investment entirely by debt, thereby shifting to the financial rule of regime 3. We conclude that regime 2 is the necessary link between the sufficiently small initial condition k_0 and the activity phase 3.

The switch from regime 2 to regime 3 is shown in the phase diagram of Fig. 5, and can be explicitly calculated. Let t_1 be the point in time at which the firm changes regime, and the financial regime 3 starts. At this point, investment I_{t_1} has to equate the new issues of debt \dot{b}_{t_1} ,

and from equations (1.9) and (1.10)

$$\varphi(q_{1t_1} - 1) = \omega\left(1 - \frac{\rho}{\theta}\right)$$

The function $\varphi(\cdot)$ is monotonically increasing and can be inverted to give q_{1t_1} as

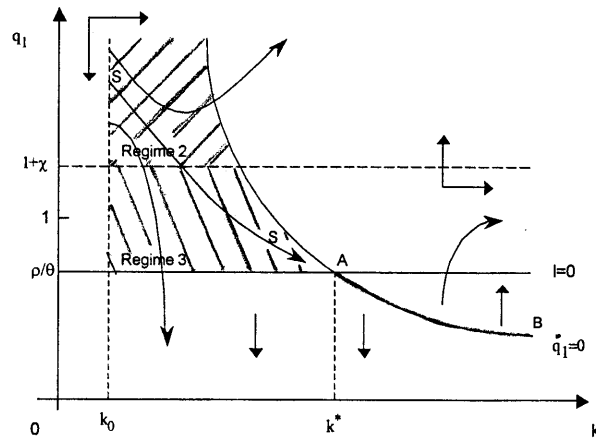
$$q_{1t_1} = 1 + \varphi^{-1}\left[\omega\left(1 - \frac{\rho}{\theta}\right)\right] = 1 + c'(\bar{\omega}) = 1 + \chi \quad (1.15)$$

where from the properties of $\varphi(\cdot)$, it turns out that $\varphi^{-1}(\cdot) = c'(\cdot)$; in addition, let us assume that $c'(\bar{\omega}) = \chi$, with χ being a positive constant.⁹

Unfortunately, we cannot find the value of the constant χ (and the switching point q_{1t_1}), without specification of the φ and ω functions. It may, however, be noticed that the switching value q_{1t_1} crucially depends on the shadow cost of debt $\frac{\rho}{\theta}$. More specifically, equation (1.15) brings out that the lower the ratio $\frac{\rho}{\theta}$, the greater is the χ value, and therefore the higher is the switching point for the q_{1t} variable. This conclusion can be seen in Fig. 5 where the $q_1 = 1 + \chi$ locus would move up as the $\frac{\rho}{\theta}$ ratio decreases, thereby leading the financial regime areas to change.

At this point, we have demonstrated that regime 2 is the necessary link between the initial condition k_0 and regime 3; in turn, regime 3 is the only phase approaching regime 1, and this represents the necessary final state. Armed with this information, we can summarise our conclusions by drawing the overall optimal growth path in the phase diagram of the following figure:

⁹It may be interesting to observe that along the optimal path, $\dot{q}_{1t} < 0$ and, q_{1t} is converging to a stationary level, i.e. $\lim_{t \rightarrow \infty} q_{1t} = \frac{\rho}{\theta} < 1$. It follows that a finite time t_1 necessarily exists such that $q_{1t_1} = 1 + \chi > 1$, where $\chi > 0$; hence, the financial regime 3 is always reached.



Note: The AB line denotes the steady state levels of capital in regime 1.

Figure 5: The optimal growth path of the firm

This figure shows that there is a unique accumulation path converging to a stationary state, the downward-sloping path SS . This starts from the initial condition $k_0 < k^*$ and approaches the stationary state and long-run equilibrium level of capital k^* . The transversality conditions require the system to be on this unique stable path. The stationary state k^* is approached monotonically with k_t growing and q_{1t} falling.

Along this optimal growth path, the firm goes through different financial regimes. It starts off from regime 2 where the marginal valuation of capital q_{1t} is greater than one and investment is positive. While accumulating, the firm will find it optimal to shift to the financial regime 3. In this regime, q_{1t} is initially greater than one but, as capital accumulates, it decreases to a value lower than unity till to approach the steady state in which there is no investment.

Each of these regimes corresponds to different policy choices on the part of the firm; it may then be useful to summarise our findings in terms of the firm's financial and investment decisions.

Proposition 1 *Given an initial condition k_0 , a phase of growth initially arises during which the firm finances new investment both by retaining earnings and by issuing new debt, the regime 2. During this regime, retained earnings are the marginal source of funds. While accumulating capital, the firm moves to a phase of pure debt-financed growth, the regime 3, during which capital is accumulated at a decreasing rate and debt is the marginal source of funds. Along this path, the firm approaches a level of capital that equates its marginal product to the market*

interest rate (regime 1).

From Fig. 5 it may be noticed that, with a different initial stock of capital, the firm might start off from the financial regime 3. Alternatively, when the initial condition is such that $k_0 \geq k^*$, then there may be no dynamics at all; in this case the only optimal policy is to stay at the initial situation.¹⁰

As the reader will have already detected, previous conclusions refer to the pattern of capital accumulation, but they are independent of time. The investigation of the financial and investment policy over time may, however, provide a useful insight into the decision-making process of the firm. How much does the firm invest in each period? How do its decisions change over time?

1.3.4 The life cycle of the firm: investment and financial policies over time

One of the merits of the model described here is to provide a life-cycle story for the firm, something which is usually missing, even from dynamic models of the firm (which tend to assume steady state). In particular, armed with previous information, we can draw the optimal temporal paths of the variables I_t , \dot{b}_t and s_t shown in Fig. 6. This figure also illustrates the patterns for the co-state variables q_{1t} and q_{2t} , and the multiplier μ_{st} (Fig. 6b).¹¹

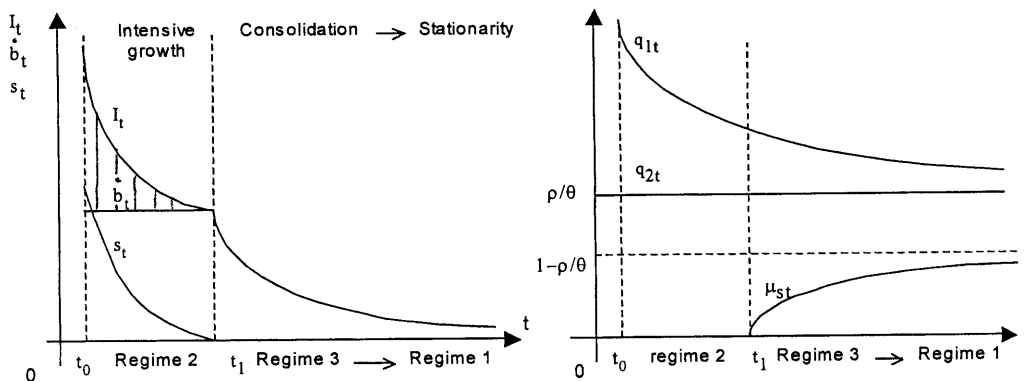


Figure 6a
Figure 6b
Figure 6: The life cycle of the firm: investment and financial policy over time

¹⁰For a model of firm investment with distinctive growth phases, see also Sinn (1991). In the Sinn's model, however, the phases of growth depend on the existence of a positive dividend tax. Indeed, in the absence of taxation, there would be no accumulation path, and the firm would jump to the steady state. In the present model, the accumulation path is independent of the existence of taxation.

¹¹The concavity properties shown in figure 6 follow from specific assumptions on the third grade derivatives of functions $c(\cdot)$ and $\sigma(\cdot)$.

The optimal path for investment I_t is obtained by combining the equations (1.9) and (1.13) with the temporal path of the co-state variable q_{1t} (see Fig. 6b). In particular, in each regime investment I_t is an increasing function of q_{1t} , and this is steadily decreasing over time (see appendix A). As a result, the investment rate is also decreasing monotonically. This is shown in Fig. 6a where the path of investment starts at t_0 , and approaches a steady state value of nil investment. At the early stages of development the firm is in an “intensive growth” phase where investment expenses I_t are particularly large. This initial policy of high investment is followed by a reduced growth, or “consolidation” phase where investment expenses are lower and capital accumulates at a lower and decreasing rate. The firm then approaches a phase of “stationarity” where no further investment is desirable.

Along this accumulation path, the firm’s optimal financing policy changes between the different phases of growth, but the firm acts according to a clear financial hierarchy. In the “intensive growth” phase, the firm follows the financial rule of regime 2 (see Fig. 6a). It partially finances investment by borrowing a constant amount of new debt up to its maximum debt capacity. As shown in the figure, given this optimal debt policy, the firm has, however, sufficiently attractive investment opportunities remaining (the shadowed area), and it finances the higher level of investment by retaining earnings. From this, we deduce that the firm acts according to a financial hierarchy: it prefers debt to retained earnings, and these are considered a residual or marginal source of funds.

As the firm grows, its investment opportunities reduce, and it is eventually able to finance investment expenses by borrowing only. At this point, the firm shifts to the financial regime 3, the pure debt-financing regime. During this regime, borrowing is the marginal source of funds. Following this path, the firm approaches a steady state level of capital, and this is a long-run equilibrium of the model.

From Fig. 6a, it may be noted that the optimal path of investment has different slopes at the different phases, so that the speed of accumulation changes with the financial regime. We conclude that, in this model, financial constraints do affect the firm’s real investment decisions and the optimal accumulation path.¹²

¹²The different speed in capital accumulation in regimes 2 and 3 derives from the optimal investment rules in each regime. In regime 2, optimality requires that $c'(I_t) = q_{1t} - 1$, and the slope of the function I_t with respect

1.3.5 Contrasting theoretical predictions and empirical evidence

The theoretical conclusions described above may appear remote from reality, but they echo the concerns of many practical commentators. The importance of financial constraints for corporate investment is an issue of much current interest and some recent empirical studies are brought together in Table 1. These range over different countries, and provide evidence to be contrasted with the predictions of our model. In particular, the model suggests that the firm's investment and financial decisions are closely related, and that the nature of this relation depends on the stage of development of the firm.

The conclusion of most empirical studies is that investment by firms is highly sensitive to financial factors, as may be seen by looking at the last column of Table 1. This shows the significance of different financial indicators in explaining investment. For example, Fazzari, Hubbard, and Petersen (1988) have examined a sample of U.S. firms, and conclude:

“Our results show that financial factors affect investment. Our approach emphasizes that the link between financing constraints and investment varies by type of firm. ... the results consistently indicate a substantially greater sensitivity of investment to cash flow and liquidity in firms that retain nearly all their income.” (Fazzari *et al.*, 1988, p. 183-184)

This conclusion is based on a special method of analysis. This method consists in estimating investment functions across groups of firms with different retention practices; this allows us to separately investigate the behaviour of firms that are likely to face binding financial constraints and retain a large part of their profits (e.g. those at the first stages of development) *versus* those which are “unconstrained”. In particular, the authors observe that “constrained firms borrow up to their debt capacity.” (p. 161); a conclusion which would be expected under the

to time is:

$$\dot{I}_t = \dot{q}_{1t} \frac{1}{c''(\cdot)} < 0$$

where dots denote the derivatives with respect to time. In the financial regime 3, instead, the optimality conditions imply that:

$$\dot{I}_t = \dot{q}_{1t} \frac{1}{c''(\cdot) + \sigma''(\cdot)} < 0$$

From these equations, it turns out that the slope of the investment curve in the two regimes is different, and at the switching point t_1 , given \dot{q}_{1t} , there must be a kink as illustrated in Fig. 6a.

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predictions of the model described here. Results do not change if alternative approaches are used, and if different countries are considered. In their investigation of U.K. manufacturing firms, for example, Bond and Meghir (1994) use a different sample separation device, and observe:

“Our results suggest that there are significant differences in the investment behaviour of sub-samples of firms allocated according to their financial policies.” (Bond and Meghir, 1994, p. 215)

Clearly, the empirical evidence does confirm the predictions of our model that investment and financing decisions are closely related, and that there is a limit on the borrowing capacity of firms.

The other merit of the model described here is to provide a life-cycle story of the firm. It suggests that along the growth path the firm has different investment opportunities, and is subject to different financial constraints. Can these predictions be empirically verified? Unfortunately, one of the assumptions common to empirical studies is that the sub-samples of “constrained” and “unconstrained” firms do not vary over time; as a result, intertemporal aspects cannot be fully incorporated in empirical investigations. Therefore, while evidence confirms the existence of a link between investment and financing decisions, the dynamic implications of our model remain to be verified. Certainly, more analysis is required in this respect.¹³

1.4 Introducing corporate income taxation

The model of accumulation and financing decisions described in previous sections is meant to provide the ground for a discussion of the effect of corporate taxation on the firm investment behaviour. In fact, the literature on taxation has demonstrated how the impact of taxes on firm investment may be sensitive to the firm’s financing decisions. Ideally, the firm financial behaviour should then be determined simultaneously with its investment decisions, and the cost of tax and financial distortions ought to be considered together. However, very little of the literature on corporate taxation relies on any satisfactory analysis of the factors underlying the

¹³For discussion of empirical work on financial constraints and firm investment, see for example Hubbard (1998).

financing behaviour of firms. The analysis usually relies on cost-of-capital formulae where the firm's financial decisions are arbitrary postulated. The model of accumulation described above provides a theory of firm investment and financing behaviour and, in this respect, represents a good starting point to examining the effect on taxes on firm behaviour.

In order to examine the effects of corporate taxation, let us to explicitly introduce taxes in the model of the firm. For simplicity, let us consider only one kind of taxation:

- Corporate income tax - a corporate tax on the firm's profits.

This tax is assumed to be proportional and constant, but the results can in general be extended to cover different tax schedules. For convenience, let us also assume that the proceeds from the tax are used outside the corporate sector, so as to ignore the effects of public spending.

The consideration of profit taxation confronts us with one of the thorniest issues in the analysis of tax systems: the choice of the profit tax rate. In each country, firms could face alternative tax rates. For example, national authorities may adopt a dual system of corporate taxation, introduce preferential tax regimes or consider temporary tax incentive. In addition, firms may operate in more than one country, and a domestic firm may generate some of its profit abroad. In that case, part of the profits may be taxed at the foreign rate, and the effect of domestic taxes on firm behaviour become difficult to investigate.¹⁴ However, the aim here is to examine the effect of domestic taxation on firm investment; to this purpose, we abstract from these complications and assume that the firm faces a unique tax rate on profits, the rate τ_C . While we shall concentrate on this simplified case, previous qualifications should clearly borne in mind when applying results to empirical evidence.

In order to introduce the profit tax in the model of the firm, we assume that in calculating taxable profits the firm is not allowed to deduct all costs. It can deduct expenses for installing new capital, and interest payments to bond-holders, but it cannot fully deduct search costs for external funds. As a matter of fact, the search process may involve costs that do not correspond to actual expenses for the firm or costs that are not recognised as such by tax provisions, for example, the cost of providing collateral and guarantee funds. These costs are

¹⁴For discussion of corporate taxation in an international context, and of taxation of multinational enterprises, see for example Alworth (1988), Feldstein *et al.* (1995) Giovannini *et al.* (1993), Mintz and Tulkens (1996), and Hines (1997).

not tax deductible. Indeed, debt expenses are not, in general, fully deductible.

To make this more concrete, let us assume that only a fraction a of the total searching costs $\sigma(\dot{b}_t)$ is permitted as a deduction, with $0 \leq a \leq 1$; where if $a < 1$, then some costs upon search are not tax deductible. Using the notation of previous sections, we can define the profit tax liability of the firm at each time t as:

$$\left(R(k_t) - \rho b_t - c(I_t) - a\sigma(\dot{b}_t) \right) \tau_C$$

and the net-of-tax cash flow

$$\left(R(k_t) - \rho b_t - c(I_t) - a\sigma(\dot{b}_t) \right) (1 - \tau_C) - I_t + \dot{b}_t - (1 - a)\sigma(\dot{b}_t)$$

This is the after-tax profits net of earnings re-invested in the firm $(I_t - \dot{b}_t)$, and net of non-deductible search costs.

At this point, we can reformulate the firm's maximisation problem, now allowing for the profit tax. More specifically, the firm may be seen as maximising the present discounted sum of current and future net-of-tax cash flows, and its decision problem can be re-stated as follows:

$$\max_{I_t, \dot{b}_t} \int_0^{\infty} \left\{ \left(R(k_t) - \rho b_t - c(I_t) - a\sigma(\dot{b}_t) \right) \Upsilon_C - I_t + \dot{b}_t - (1 - a)\sigma(\dot{b}_t) \right\} e^{-\theta t} dt$$

subject to

$$\dot{k}_t = I_t$$

$$I_t = \dot{b}_t + s_t$$

and to the non-negativity constraints

$$I_t \geq 0 \quad \dot{b}_t \geq 0 \quad s_t \geq 0$$

with initial conditions $b(0) = b_0 \geq 0$, $k(0) = k_0 > 0$, and where $\Upsilon_C \equiv 1 - \tau_C$.¹⁵ It may be noticed that what distinguishes this problem from that of the previous sections is the term in

¹⁵In what follows, I focus on the basic version of the model where $\sigma'(0) = 0$. For comparison, I also consider the possibility that $\sigma'(0) = A > 0$; however, in this case, the treatment of tax effects is confined to footnotes.

the tax rate Υ_C . This incorporates a proportional tax on the firm's profits, but other forms of corporate and personal income taxation could, of course, be considered. There may be capital gains taxes and property taxes. In addition, corporate profits can be taxed at firm level, and again at personal level for the part paid out as dividends. For simplicity, however, let us only consider a corporate profits tax.

1.4.1 Optimality conditions

The firm's maximisation problem may be reformulated in terms of the Lagrangian function (using \bar{L} in place of L):

$$\begin{aligned} \bar{L} = & \left(R(k_t) - \rho b_t - c(I_t) - a\sigma(\dot{b}_t) \right) \Upsilon_C - I_t + \dot{b}_t - (1-a)\sigma(\dot{b}_t) + \\ & q_{1t}I_t - q_{2t}\dot{b}_t + \mu_{It}I_t + \mu_{bt}\dot{b}_t + \mu_{st}(I_t - \dot{b}_t) \end{aligned}$$

where q_{1t} and q_{2t} are the co-state variables, and the functions μ_{mt} , with $m = I_t, \dot{b}_t, s_t$, are the Kuhn-Tucker multipliers for the non-negativity constraints of the problem. The optimal investment and financing policy of the firm can then be calculated as a maximum of \bar{L} .¹⁶

The necessary first-order conditions for optimality now become:

$$\begin{aligned} q_{1t} &= 1 + c'(I_t) \Upsilon_C - \mu_{It} - \mu_{st} \\ q_{2t} &= 1 - a\sigma'(\dot{b}_t) \Upsilon_C - (1-a)\sigma'(\dot{b}_t) - \mu_{st} + \mu_{bt} \end{aligned}$$

At the optimum, the following complementary slackness conditions must also hold:

$$m_t \geq 0 \quad \mu_{mt} \geq 0 \quad \mu_{mt}m_t = 0 \quad \text{with } m_t = I_t, \dot{b}_t, s_t$$

In addition, along the accumulation path, we have the multiplier equations

$$\dot{q}_{1t} = \theta q_{1t} - R'(k_t) \Upsilon_C$$

¹⁶It should be noted that using the same co-state and Lagrangian multipliers as in previous sections, we are in fact abusing notation.

$$\dot{q}_{2t} = \theta q_{2t} - \rho \Upsilon_C$$

and the laws of motion for the state variables

$$\dot{k}_t = I_t$$

$$I_t = \dot{b}_t + s_t$$

Finally, on the boundary, optimality also requires the transversality conditions

$$\lim_{t \rightarrow \infty} q_{1t} e^{-\theta t} k_t = 0$$

$$\lim_{t \rightarrow \infty} q_{2t} e^{-\theta t} b_t = 0$$

These conditions may be simplified. We can integrate over time the equations of motion for q_{1t} and q_{2t} , and using the transversality conditions, we obtain:

$$q_{1t} = e^{\theta t} \Upsilon_C \int_t^{\infty} R'(k_v) e^{-\theta v} dv$$

$$q_{2t} = \frac{\rho}{\theta} \Upsilon_C$$

In what follows, we turn to the interpretation of these results. We characterise the investment and debt policy rules of the firm as a function of the tax rate, and examine how the firm reacts to unanticipated changes in tax provisions.

1.4.2 Corporate taxation and firm behaviour

How does corporate taxation of profits affect the firm behaviour? The different channels may be seen from previous optimality conditions. To this purpose, it is convenient to work with a simplified version of these conditions with

$$\tilde{q}_{1t} = \frac{1}{\Upsilon_C} + c'(I_t) - \tilde{\mu}_{It} - \tilde{\mu}_{st}$$

$$\tilde{q}_{2t} = \frac{1}{\Upsilon_C} - \frac{(1 - a\tau_C)}{(1 - \tau_C)} \sigma'(\dot{b}_t) - \tilde{\mu}_{st} + \tilde{\mu}_{bt}$$

$$\dot{\tilde{q}}_{1t} = \theta \tilde{q}_{1t} - R'(k_t)$$

$$\tilde{q}_{2t} = \frac{\rho}{\theta}$$

where, for convenience, I have re-scaled up the multipliers of the problem by the term Υ_C , so that

$$\tilde{q}_{1t} = \frac{q_{1t}}{\Upsilon_C} \quad \tilde{q}_{2t} = \frac{q_{2t}}{\Upsilon_C} \quad \tilde{\mu}_{It} = \frac{\mu_{It}}{\Upsilon_C} \quad \tilde{\mu}_{bt} = \frac{\mu_{bt}}{\Upsilon_C} \quad \tilde{\mu}_{st} = \frac{\mu_{st}}{\Upsilon_C}$$

As the reader will have already detected, with this reformulation the optimality conditions are analogous to those obtained in the previous sections for the no-tax case. In particular, they only differ in the constant terms in τ_C in the first two equations. From this similarity, we deduce that the firm qualitatively behaves as described in previous sections. Therefore, it accumulates capital through distinctive growth stages. It finds initially desirable to finance new investment both by issuing new debt and by retaining earnings, what we defined the financial regime 2. As it grows, it moves to a regime of debt-financed investment, the regime 3; finally, the firm approaches a steady state and long-run equilibrium level of capital in regime 1.

As in the previous sections, the firm's accumulation path can be illustrated in a phase diagram as shown in Fig. 7.

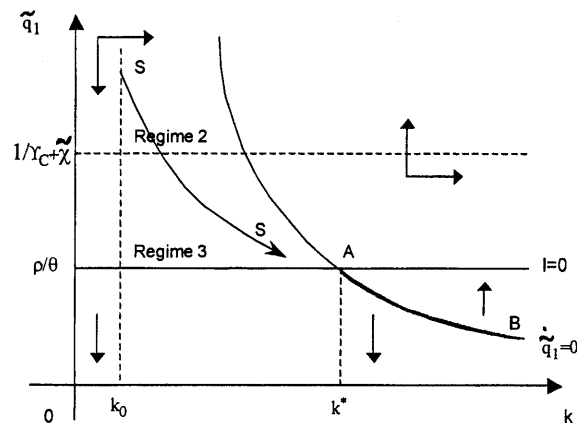


Figure 7: The accumulation path of the firm with corporate profits taxation

This figure is similar to those used before, the only difference being in the co-state variable, which is now re-scaled by Υ_C . Again, there is a unique converging path of accumulation, the downward-sloping path SS . This starts off from the initial condition $k_0 < k^*$, goes through the different financial regimes, and approaches the stationary state, and long-run equilibrium level of capital k^* . This is not, of course, the only possible long-run equilibrium level of capital. The

points on the $\tilde{q}_{1t} = 0$ locus and below the $\frac{\rho}{\theta}$ line (the line AB) also are possible equilibria for the firm.

Along this accumulation path, the profit tax influences the firm's financial and investment decision rules in each regime.

REGIME 2. - In the initial regime 2, the firm finances investment both by issuing new debt and by retaining earnings; thus, the multipliers $\tilde{\mu}_{I_t}$, $\tilde{\mu}_{s_t}$ and $\tilde{\mu}_{b_t}$ are zero. In this case, the first of the optimality conditions gives us the new investment rule of the firm, now incorporating profit taxation,

$$c'(I_t) = \tilde{q}_{1t} - \frac{1}{\Upsilon_C} \quad (1.16)$$

This equation describes implicitly the firm's investment policy as a decreasing function in the tax factor $\frac{1}{\Upsilon_C}$, and increasing in the re-scaled marginal valuation of capital, the term \tilde{q}_{1t} . But, for each given stream of capital, \tilde{q}_{1t} is independent of the tax rate as $\tilde{q}_{1t} = e^{\theta t} \int_t^{\infty} R'(k_v) e^{-\theta v} dv$. It then follows that the introduction of the profit tax reduces the investment of the firm for any given \tilde{q}_{1t} . This caveat notwithstanding, it should be noticed that previous equation (1.16) is not a closed form solution for investment because I_t affects k_t and thereby \tilde{q}_{1t} . As a result, we cannot conclude from (1.16) that the tax has a negative effect on the "temporal" path of investment, a question I discuss further below. A second important aspect to observe is that investment is independent of the firm's financial decisions, and of how these respond to the tax.

From the second of the optimality conditions, we can derive the new borrowing policy of the firm, now incorporating the profit tax,

$$\dot{b}_t = \omega \left(\frac{1 - \tau_C}{1 - a\tau_C} \left(\frac{1}{\Upsilon_C} - \frac{\rho}{\theta} \right) \right) \equiv \tilde{\omega} \quad (1.17)$$

where $\omega(\cdot)$ is the inverse of the search cost function $\sigma'(\cdot)$ whose properties imply that $\omega'(\cdot) > 0$. From this equation, we deduce that the firm issues a constant and positive amount of new debt, and this amount depends on the tax rate τ_C . In particular, a higher tax rate increases the issues of debt, as $\frac{db_t}{d\tau_C} > 0$, and the effect is larger the higher is the tax deductibility parameter a . In fact, with profit taxation, interest and debt costs deductibility reduce the tax burden, thereby encouraging the firm to choose debt financing.

If we now substitute previous equation (1.17) and the inverse of (1.16) in the financing

constraint $s_t = I_t - \dot{b}_t$, we can determine the firm's retaining profit rule in regime 2; in this rule, both investment and debt issues depend on the profit tax rate τ_C .

REGIME 3. - Along the accumulation path, the firm shifts from the financial regime 2 to regime 3 (see Fig. 7). In this regime, it entirely finances new investment by borrowing funds so that $I_t = \dot{b}_t > 0$, and the multipliers $\tilde{\mu}_{I_t}$ and $\tilde{\mu}_{b_t}$ are zero. From the first two optimality conditions, we then obtain a simple expression for the firm's investment-borrowing rule

$$c'(I_t) + \sigma'(\dot{b}_t) \frac{(1 - a\tau_C)}{(1 - \tau_C)} = \tilde{q}_{1t} - \frac{\rho}{\theta} \quad (1.18)$$

This equation describes implicitly the new investment rule of the firm in regime 3 after introducing profit taxation. It may be noticed that in this regime if $a = 1$, then the introduction of the profit tax has no effect on the firm's investment function. But, if only a fraction of search costs is permitted as a deduction so that $a < 1$, then the tax does change the investment rule of the firm and, for each value of the multiplier \tilde{q}_{1t} , it results in lower investment.

At times, it may be more straightforward to work with a reformulation of the previous rule; in particular, we can write the firm's investment as a direct function of the shadow price \tilde{q}_{1t} , and of the profit tax rate τ_C :

$$I_t = \tilde{\Psi}\left(\tilde{q}_{1t} - \frac{\rho}{\theta}, \tau_C\right) \quad (1.19)$$

where from the properties of functions $c(\cdot)$ and $\sigma(\cdot)$, it follows that $\tilde{\Psi}'_{\tilde{q}_1}(\cdot) > 0$, and $\tilde{\Psi}'_{\tau_C}(\cdot) < 0$, but $\tilde{\Psi}'_{\tau_C}(\cdot) = 0$ when $a = 1$.¹⁷

REGIME 1. - Along this path, the firm approaches a steady state or long-run equilibrium level of capital in regime 1 (see Fig. 7). In this regime, the firm does not invest in new capital and does not borrow so that the stocks of capital and debt are constant and $\dot{k}_t = \dot{b}_t = 0$. From previous optimality conditions, and provided that $c'(0) = \sigma'(0) = 0$, it follows that the stationary state stock of capital k_s is such that

$$R'(k_s) \leq \rho$$

¹⁷As in the previous sections, we can also determine the exact value of \tilde{q}_{1t} at the switching point between regimes 2 and 3, namely the $\frac{1}{r_C} + \tilde{\chi}$ locus in Fig. 7. At the switching point, $I_t = \dot{b}_t$, and for the continuity of the co-state variables, we can use the investment and debt functions in regime 2 to obtain that $\tilde{q}_{1t} = \frac{1}{r_C} + c'(\omega(\tau_C)) = \frac{1}{r_C} + \tilde{\chi}(\tau_C)$, where $\tilde{\chi}'_{\tau_C} > 0$.

where, for convenience, I focus on the case in which the condition holds with equality, and $k_s = k^*$. This condition determines the steady state level of capital as a function only of the interest rate ρ ; therefore, the corporate income tax has no effect on the long-run equilibrium of the firm, k^* . An exception arises when $\sigma'(0) = A > 0$ and $a < 1$. In this special case, the steady-state condition becomes $R'(k_s) \leq \rho + \frac{1-a\tau_C}{1-\tau_C} A\theta$, and, since $R'' < 0$, the introduction of the tax reduces the long-run equilibrium level of capital k^* .¹⁸

SUMMING UP. - At this point, we can summarise the different channels through which the corporate profits tax influences the firm behaviour. In the case of $a < 1$, the profit tax affects the investment and debt functions of the firm both during the initial regime 2 (recall equations (1.16) and (1.17)) and during the financial regime 3 (the equation (1.18)). However, this result depends on the search costs deductibility parameter a , and if $a = 1$, then the tax has no effect on the investment-borrowing policy of the firm in regime 3. Nonetheless, the tax is in general neutral in the steady state.

1.4.3 An example with quadratic cost functions

To make previous results more concrete, it may be useful to work out a specific example. In particular, let us consider the more restrictive class of quadratic cost functions and constant returns to scale

$$c(I_t) = \alpha_0 + \frac{\alpha}{2} I_t^2 \quad \sigma(\dot{b}_t) = \beta_0 + \frac{\beta}{2} \dot{b}_t^2 \quad R(k_t) = \left(\delta_0 - \frac{\delta}{2} k_t \right) k_t$$

where the constants α_0 and β_0 are non-negative, $\alpha, \beta, \delta_0, \delta$ are positive, and $\delta_0 > \theta > \rho$ (which is a sufficient condition for the steady state level of capital to be positive).¹⁹ This example is difficult to solve for the entire accumulation path, but is specially informative if we confine the attention to the firm which starts off from the financial regime 3.

Let us solve our problem for this special case. In particular, let us suppose to initially be in the steady state and solve the firm's problem by working backward in time. From previous

¹⁸For a detailed discussion of this case, see Palomba (2001).

¹⁹It may be noticed that in this example $R'''(\cdot) = 0$ and this differs from the assumptions used in previous sections where $R'''(\cdot) > 0$. However, the assumption of $R'''(\cdot) > 0$, it should be recalled, was introduced only to simplify the diagrammatic discussion; thus, it may be omitted, without jeopardising the analysis.

optimality conditions, it follows that the long-run equilibrium level of capital k^* is:

$$k^* = \frac{\delta_0 - \rho}{\delta}$$

where $\delta_0 > \rho$, so that $k^* > 0$. As noted above, this is independent of the profit tax.

Approaching this steady state, the firm is in the financial regime 3 (Fig. 7). From the optimality condition (1.18), and the dynamic equations for \tilde{q}_{1t} , it follows that at the optimum,

$$\begin{aligned} \left(\alpha + \beta \frac{1-a\tau_C}{1-\tau_C} \right) \dot{k}_t &= \tilde{q}_{1t} - \frac{\rho}{\theta} \\ \dot{\tilde{q}}_{1t} &= \theta \tilde{q}_{1t} + \delta k_t - \delta_0 \end{aligned}$$

This is a non-homogeneous and autonomous system of two linear first order differential equations in k_t and \tilde{q}_{1t} , and its general converging solution can be written as:

$$k_t = \tilde{c}_2 e^{\tilde{r}_2 t} + \frac{1}{\delta} (\delta_0 - \rho) \quad (1.20)$$

$$\tilde{q}_{1t} = \left(\alpha + \beta \frac{1-a\tau_C}{1-\tau_C} \right) \tilde{c}_2 \tilde{r}_2 e^{\tilde{r}_2 t} + \frac{\rho}{\theta}$$

where \tilde{r}_2 is the negative root of the characteristic equation associated with the system and it depends on the tax rate τ_C (as $\tilde{r}_2 = \frac{1}{2} \left(\theta - \sqrt{\theta^2 + \frac{4\delta}{\alpha + \beta \frac{1-a\tau_C}{1-\tau_C}}} \right)$), and \tilde{c}_2 is an arbitrary constant. To determine this constant, we can use the value of the function k_t at the initial time $t = 0$; this gives us

$$\tilde{c}_2 = k_0 - \frac{1}{\delta} (\delta_0 - \rho) < 0$$

which is negative and independent of the tax rate.

These equations provide an explicit solution to the model of accumulation described above. From this, it can be noticed that the long-run equilibrium is approached monotonically with capital k_t growing steadily ($\dot{k}_t > 0$) and \tilde{q}_{1t} falling ($\dot{\tilde{q}}_{1t} < 0$), and both paths depend on the tax rate. This information may be also used to determine the optimal pattern of investment shown in the following Fig. 8.

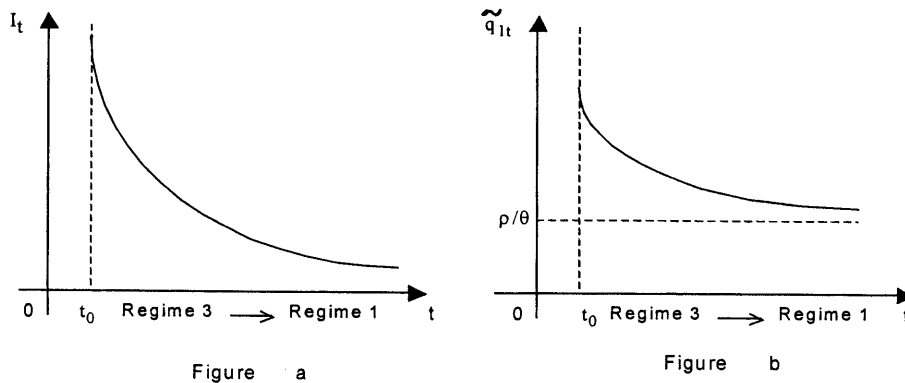


Figure 8: The optimal investment path with quadratic cost functions

This path is obtained by combining the solution for the co-state variable \tilde{q}_{1t} (shown in Fig. 8b), with the optimality condition for investment (1.19). Since I_t is an increasing function of \tilde{q}_{1t} , and this is decreasing, it follows that the investment rate is also decreasing monotonically, and the path depends on the tax rate τ_C .²⁰

1.5 Dynamic effects of corporate income taxation

What can we deduce from this about the effect of the profit tax? Would profit taxation reduce corporate investment? In considering these questions, I am concerned with the immediate impact of the tax, but I am also interested in its intertemporal effects. While the standard analysis of tax incidence has, in fact, concentrated on the single period impact, taxes have important intertemporal implications which are not always fully appreciated.

In order to examine these intertemporal effects, we need in principle to characterise the full dynamic path and the dependence of all variables with respect to the tax. Unfortunately, we do not know the optimal path explicitly, and we cannot calculate the impact of the tax over time. Nevertheless, we can attempt to qualitatively characterise these effects by means of the well-known comparative-dynamics analysis (see for example Kamien and Schwartz, 1991). As I shall argue below, this approach is far from fully satisfactory, but it provides a convenient starting point for our discussion.

²⁰As an alternative, the reader may derive Fig. 8a by differentiating the first of equations (1.20) with respect to time.

1.5.1 The method of comparative dynamics

The method of comparative dynamics consists in comparing the pattern of accumulation described in a conventional phase diagram (as in Fig. 7), with that deriving from a parametric change in tax provisions. To investigate the effects of profit taxation, let us then examine how the optimal accumulation path of the firm changes with variations in the corporate profits tax rate. In particular, let us assume that at the initial time $t = 0$ an unexpected marginal increase in the tax rate τ_C occurs. Let us also assume that the change persists for the entire future beyond period 0, so as to consider the effect of a single change on the entire accumulation path. Moreover, suppose for simplicity that the firm is in regime 2 when the tax rate rises, and confine the attention to the general case of $a < 1$, that is only a fraction of search costs is permitted as a deduction.

At this point, we can use the diagrammatic analysis, and illustrate the effects of the higher profit tax rate in the (k, \tilde{q}_1) phase diagram of Fig. 9.

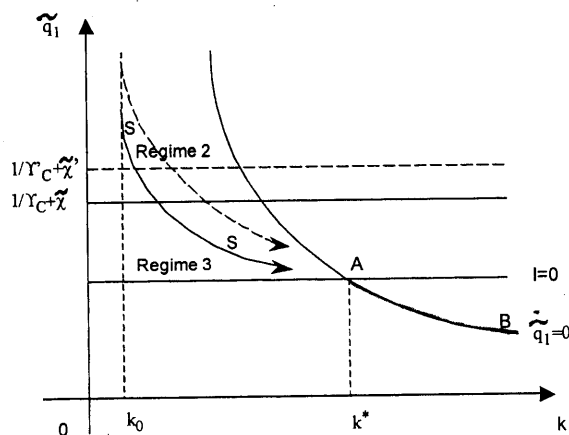


Figure 9: The comparative dynamics analysis of profit taxation

As shown in this figure, the increase in the tax rate shifts the $\tilde{q}_{1t} = \frac{1}{Y_C} + \tilde{\chi}(\tau_C)$ line upwards (the dotted line), while leaving the loci $I = 0$ and $\tilde{q}_{1t} = 0$ unaffected. As noted above, the tax has no effect on the steady state or long-run equilibrium level of capital.²¹

This result does not tell us anything about the accumulation decisions, and we need to examine the equilibrium path. Along this path, the higher profit tax discriminates in favour

²¹These comparative statics results can be found by differentiating the switching value $\tilde{q}_{1t} = \frac{1}{Y_C} + \tilde{\chi}(\tau_C)$ with respect to τ_C (see previous note 17 where $\tilde{\chi}'_{\tau_c} > 0$), and by noting that the equations of the loci $I_t = 0$ and \tilde{q}_{1t} are independent of the tax rate.

of debt and against retained earnings. Since in the initial regime 2 retained earnings are the marginal source of finance, it follows that investment has to initially decrease as a result of the tax change. At the same time, a greater debt reduces the burden of taxation, and thus increases the marginal value or shadow price of capital \tilde{q}_{1t} ; hence, \tilde{q}_{1t} is initially higher. If we go back to Fig. 9, the higher \tilde{q}_{1t} requires the optimal path to initially shift upwards as shown by the dashed line; in addition, we know that investment is initially reduced. However, the steady state stock of capital remains unaffected, thus the investment corresponding to each level of capital has to increase above its original path at some point. Given the investment equations (1.16) and (1.19), this necessarily requires a higher value of \tilde{q}_{1t} . As shown in Fig. 9, the new path then steadily lies above the original one; along this path, investment is initially lower than on the original pattern, but there must be some level of capital for which it is higher. A formal proof of these results is given in appendix B.²²

As the reader will have already detected, time does not explicitly enter in the analysis of comparative-dynamics, and this gives rise to a number of problems. The fundamental problem with this method is that it rests on the assumption that tax effects are examined with respect to a given level of capital. This “equal capital repercussion” assumption simplifies the analysis, but it prevents the method from providing information on the effect of the tax on the “temporal” path of investment. For example, we cannot know the speed of capital accumulation as this depends on the stock of capital at each instant which, in turn, is affected by the tax. As a result, at any time investment may rise above, or fall below, its original path, depending on the effect of the tax on the stock of capital. But, this is like to redefine the question at issue. We want to know whether investment will be higher after a tax change; we cannot say that it will be higher if \tilde{q}_{1t} is higher given a stock of capital, since that simply shifts the question about whether \tilde{q}_{1t} is higher. Therefore, the problem with comparative-dynamics analysis is that, while it may tell us about the relation between the investment I_t and the co-state variable \tilde{q}_{1t} for each given level of capital, it does not tell us about the impact on I_t at all instants t , since \tilde{q}_{1t} depends on

²²It is important to realise that we examine infinitesimal changes in the tax rate under the assumption that the initial rate is positive. Because of continuity properties of the model, it is therefore legitimate to assume that the tax change does not affect the initial standing regime of the firm. However, when the initial tax rate is zero, it may well be the case that the introduction of the tax encourages the firm to transfer to a different regime. In this special case, previous results hold under the condition that the firm does not change regime as a result of the tax.

capital at each time, a variable that is, in turn, affected by the policy change.

These theoretical considerations may be thought to be of little practical relevance, but they have implications of great importance for policy purposes. Governments are likely to be more concerned with the impact of taxes over time than with the effect for a hypothetical stock of capital. Certainly, in many countries, the discussions regarding tax reforms have centred round the timing of tax effects. Governments want to know, for example, whether investment in 3 years time will be higher as a result of a proposed reform. However, if we rely on the method of comparative-dynamics, then the intertemporal effect of taxation is a question to which no definite answer can be given. In this respect, the method does not seem particularly appealing as a tool of analysis. This brings out the need for a different approach to the dynamic analysis of taxation, an issue which has received very little attention in the literature.

1.5.2 The method of direct variations

To consider the effect of taxes over time, we develop an alternative method of dynamic analysis, what we can call “the method of direct variations”. This consists in directly examining the variations in the temporal path of investment as they derive from a change in the tax rate. While this approach involves the examination of systems of differential equations which is not always straightforward, it results in a very simple diagrammatic analysis, and does permit a rigorous discussion of the intertemporal effects of taxes.

In order to simplify the analysis, let us examine first the firm which is in the initial regime 2 when the tax rate rises, and then the firm which is in regime 3 when the tax increase occurs under the common assumption that search costs are fully deductible; finally, let us consider the implications of introducing partial search costs deductibility.

The mixed-financed investment regime

Let us start assuming that the firm is in the mixed financial regime 2 when the statutory tax rate τ_C rises. Let us also consider for convenience the restrictive case of $a = 1$; that is, costs upon search are fully tax deductible. This assumption is clearly strong, and it will be relaxed later; nevertheless, it provides a useful benchmark for the examination of more complicated cases.

The effect of the profit tax on investment may be seen by differentiating the optimality conditions for I_t , and the equation of motion for \tilde{q}_{1t} along the accumulation path with respect to τ_C . In particular, in the starting regime 2, we can differentiate the optimality condition (1.16) for I_t , and the dynamic equations for \tilde{q}_{1t} so as to have:²³

$$c''(I_t) \frac{d}{d\tau_C} \left(\frac{dk_t}{dt} \right) = c''(I_t) \frac{d}{dt} \left(\frac{dk_t}{d\tau_C} \right) = \frac{d\tilde{q}_{1t}}{d\tau_C} - \frac{1}{\Upsilon_C^2} \quad (1.21)$$

$$\frac{d}{d\tau_C} \left(\frac{d\tilde{q}_{1t}}{dt} \right) = \frac{d}{dt} \left(\frac{d\tilde{q}_{1t}}{d\tau_C} \right) = \theta \frac{d\tilde{q}_{1t}}{d\tau_C} - R''(k_t) \frac{dk_t}{d\tau_C}$$

These equations give us a non-homogeneous system of two linear differential equations in $\frac{dk_t}{d\tau_C}$ and $\frac{d\tilde{q}_{1t}}{d\tau_C}$, from which we can draw the phase diagram shown in Fig. 10. This diagram illustrates the loci $\frac{d}{dt} \left(\frac{d\tilde{q}_{1t}}{d\tau_C} \right) = 0$ and $\frac{d}{dt} \left(\frac{dk_t}{d\tau_C} \right) = 0$ derived from previous equations, and the directions of motion associated with the system. In drawing this figure, it must be realised that $R''(k_t)$ varies over time, and the $\frac{d}{dt} \left(\frac{d\tilde{q}_{1t}}{d\tau_C} \right) = 0$ line has, therefore, a changing slope around the origin. Nonetheless, under the assumptions of the previous chapter $R''(k_t) < 0$ and $R'''(k_t) > 0$ for all k_t , so that the slope of this line is always negative, and increasing with the stock of capital. In particular, as capital accumulates over time, the slope of the $\frac{d}{dt} \left(\frac{d\tilde{q}_{1t}}{d\tau_C} \right) = 0$ line steadily rises, with the locus being flatter. This is illustrated in the following figure by the double arrow demonstrating the direction of this change.²⁴

²³The reversal of the order of differentiation is legitimate since the variables \tilde{q}_{1t} and k_t are continuous in τ_C .

²⁴It should be noted that if $R''' = 0$, as in the previous example, then the slope of the locus $\frac{d}{dt} \left(\frac{d\tilde{q}_{1t}}{d\tau_C} \right) = 0$ would not change.

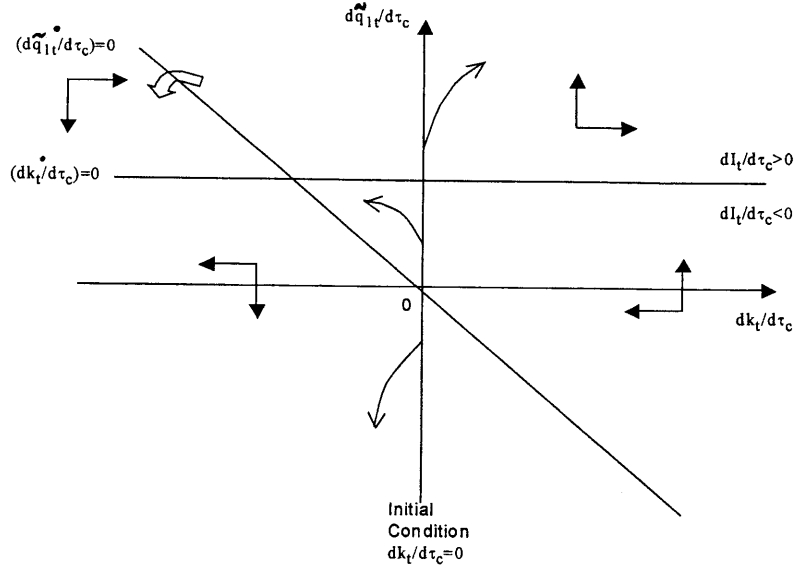


Figure 10: The basic diagram

As an initial condition we have $\frac{dk_0}{d\tau_C} = 0$ because the stock of installed capital is predetermined in the short-run and invariant to taxation; therefore, we have to start on the vertical initial condition line of the diagram. According to the directions of motion, this initial condition defines the three possible directions of adjustment illustrated in the figure. To determine which of these is optimal, we clearly need a terminal condition. However, this is *a priori* undetermined because along the optimal path the firm moves from the financial regime 2 to regime 3. The terminal condition for regime 2 has to be then obtained as initial condition of the adjustment process in regime 3. At this point, the adjustment in this regime has to be clearly discussed.

During regime 3, the effect of a higher profit tax can be seen from the equation of motion for \tilde{q}_{1t} , and the optimality condition (1.18) characterising I_t in this regime. In particular, differentiating these equation with respect to τ_C , we have:

$$\frac{d}{d\tau_C} \left(\frac{d\tilde{q}_{1t}}{dt} \right) = \frac{d}{dt} \left(\frac{d\tilde{q}_{1t}}{d\tau_C} \right) = \theta \frac{d\tilde{q}_{1t}}{d\tau_C} - R''(k_t) \frac{dk_t}{d\tau_C} \quad (1.22)$$

$$\left(c''(I_t) + \sigma''(b_t) \frac{1 - a\tau_C}{1 - \tau_C} \right) \frac{d}{dt} \left(\frac{dk_t}{d\tau_C} \right) = \frac{d\tilde{q}_{1t}}{d\tau_C} - \sigma'(b_t) \frac{1 - a}{(1 - \tau_C)^2}$$

where since $a = 1$, the term multiplying $\sigma''(b_t)$ on the left-hand side of the second equation is equal to one, while the last term on the right-hand side disappears.

These conditions define a homogeneous system of two linear differential equations. As in the case of regime 2, we can draw a phase diagram in $\left(\frac{dk_t}{d\tau_C}, \frac{d\tilde{q}_{1t}}{d\tau_C}\right)$ space where the line $\frac{d}{dt}\left(\frac{dk_t}{d\tau_C}\right) = 0$ overlaps the horizontal axis, and the locus $\frac{d}{dt}\left(\frac{d\tilde{q}_{1t}}{d\tau_C}\right) = 0$ shows a varying, and increasing slope. This is illustrated in Fig. 11 where for, convenience, I also draw the loci for regime 2, so that moving from regime 2 to regime 3, the $\frac{d}{dt}\left(\frac{dk_t}{d\tau_C}\right) = 0$ line jumps from the initial value $\frac{d\tilde{q}_{1t}}{d\tau_C} = \frac{1}{Y_C^2}$ down to $\frac{d\tilde{q}_{1t}}{d\tau_C} = 0$.

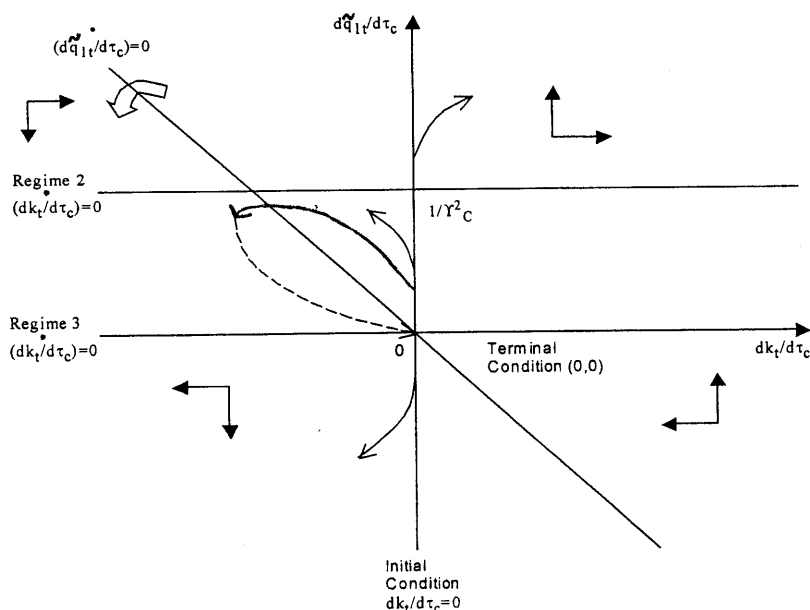


Figure 11: The effect over time of an increase in the corporate profits tax during regime 2 (case $a = 1$; full search costs deductibility)

Since along the optimal path the firm approaches the stationary state in which the profit tax is neutral (recall the long-run equilibrium condition), we must approach the terminal point $\frac{dk_t}{d\tau_C} = \frac{d\tilde{q}_{1t}}{d\tau_C} = 0$. Armed with this information, we can deduce that the adjustment path in regime 3 can start off from (and regime 2 can therefore terminate at) neither a point in the bottom left-hand quadrant, nor in the top right-hand one. For if we were to begin in one of these areas, we would never approach the terminal condition. Using the analysis for regime 2, the adjustment path has to begin between the point $\frac{1}{Y_C^2}$ and the origin of the axes on the initial condition line. The only path converging to the terminal point is, therefore, that illustrated in Fig. 11, where the continuous line denotes the adjustment process in regime 2, and the dashed part the effect during regime 3.

From the optimal adjustment path of Fig. 11, it can be deduced that the immediate effect of the profit tax is to reduce the investment of the firm; in fact, during regime 2, $\frac{dI_t}{d\tau_C}$ is negative (the path is below the $\frac{d}{dt}\left(\frac{dk_t}{d\tau_C}\right) = 0$ line), but this effect reverses over time. After this initial effect, the firm moves to regime 3; the $\frac{d}{dt}\left(\frac{dk_t}{d\tau_C}\right) = 0$ locus jumps down, and the higher tax rate leads to a greater investment which eventually offsets the initial negative impact. We then conclude that the effect of the tax on investment varies over time.

Indeed, there is an important distinction, in this model, between the short-run impact of the tax, which is negative, and its intertemporal and long-run implications, which compensate the initial impact; as a result, the profit tax does not necessarily reduce investment along the entire accumulation path. However, $\frac{dk_t}{d\tau_C}$ is everywhere negative, so that the profit tax does reduce the stock of capital at every point along the adjustment path.²⁵

The second important effect of the profit tax is on the firm's financing policy. From the equation (1.17), it may be seen that in the initial regime 2 the tax induces the firm to increase the issues of new debt. We can then conclude that an increase in the profit tax has initially two effects, it: (1) reduces investment spending, and (2) increases debt issues. As a result, the leverage ratio of the firm is at each time greater. In this respect, the profit tax can be seen as an instrument that temporarily reduces investment, but permanently affect the firm's financial structure.

These conclusions may appear remote from reality, and the possibility of a positive effect of the tax on investment is particularly striking. These results have, however, a straightforward economic interpretation. Intuitively, the increase in the profit tax rate lowers the return to capital, and if at the margin the firm finances investment by retaining earnings then it finds desirable to reduce investment. But, the tax also discriminates against retained profits and in favour of debt finance. Since in regime 3 debt is the marginal source of finance, it follows that once in this regime the firm will take advantage of the lower cost of external funds and will invest more, despite the full deductibility of debt-related costs. These conclusions are not totally new to empirical economists either; in fact, a number of studies suggest that profit taxes

²⁵It may be interesting to notice that when $\sigma'(0) = A > 0$ the tax reduces the steady state level of capital, and the terminal condition is in the top left-hand quadrant of Fig. 11. As a result, the adjustment path need not to be of the form illustrated, and may involve a steady fall in investment.

do work as an incentive to borrow funds.²⁶ That profit taxation reduces investment also is a common result. In their recent study on ‘tax policy and investment’, for example, Hassett and Hubbard observe that:

“Recent studies (*based on firm-level data*) appear to have reached a consensus that the elasticity of investment with respect to the user cost of capital (*including taxes*) is between -0.5 and -1.0 .” (Hassett and Hubbard, 1997, p. 365)

However, what our analysis suggests is that the negative impact of the tax on investment is likely to be only a temporary effect. This consideration is important for policy purposes, and clearly brings out the relevance of carrying out an explicit dynamic analysis of taxation.

Debt-financed investment

The discussion above applies to the firm which is in regime 2 when the tax rate rises; but, the analysis may be extended to the firm which is in the debt-financed investment regime 3 at the time of the tax change. This discussion is particularly important because firms with different financing policies may react differently to taxation (Atkinson and Stiglitz, 1980).

In this case, the profit tax becomes a cash-flow tax and it affects neither the investment nor the financing policy of the firm. With debt-financed investment and full cost deductibility ($a = 1$), the tax is, in fact, equivalent to multiply the firm’s objective function in section 4 times the constant Υ_C . As a result, there are no effects on the optimality conditions at the margin, and the profit tax does not influence the behaviour of the firm. It thus follows that when costs for external funds (in this model, interest payments and search expenses) are fully deductible, and investment is financed at margin by debt, then the profit tax invariably burdens on the infra-marginal returns to investment, leaving the marginal conditions for optimality unaffected. In this sense, the tax may be said to be “neutral”.

Formally, the neutrality of the profit tax can be seen by differentiating the dynamic equations for \tilde{q}_{1t} and k_t characterising regime 3 with respect to τ_C . This gives us the previous system of differential equations (1.22) where the term $\sigma' \left(b_t \right) \frac{1-a}{(1-\tau_D)^2}$ is zero as $a = 1$. Similarly, we can

²⁶For discussion of the effects of taxation on firm’s debt policy, and references to the literature, see for example Graham (1996).

draw a phase diagram in $\left(\frac{dk_t}{d\tau_C}, \frac{d\tilde{q}_{1t}}{d\tau_C}\right)$ space as in illustrated in the previous Fig.11 where the $\frac{d}{dt}\left(\frac{dk_t}{d\tau_C}\right) = 0$ locus overlaps the horizontal axis. We must begin along the initial condition line $\frac{dk_0}{d\tau_C} = 0$, and approach the point $\frac{dk_t}{d\tau_C} = \frac{d\tilde{q}_{1t}}{d\tau_C} = 0$. It follows that the only possible adjustment path is that which does not move from the origin of axes. For, if we were to begin above or below the origin, we should never approach the terminal point, and optimality conditions would be violated. As a result, the profit tax does not affect the optimal path of investment.

It may be noticed that in this special case the immediate impact of the tax coincides with the long-run effect, and we have a unique conclusion about the effect of taxation over time. Not surprisingly, this conclusion coincides with the predictions from the standard theory of corporate taxation which concentrates on single period effects (see for example Atkinson and Stiglitz, 1980, lecture 5).

Implications of partial search costs deductibility

So far we have discussed the effect of the profit tax for the special case where $a = 1$; it may be now interesting to relax this assumption, and look at the general case of $a < 1$. In this event, costs upon search are not entirely deductible; indeed, this is the most likely case in practice because search expenses and, in general, debt-related costs are unlikely to be fully deductible.

For convenience, let us assume that the firm is in regime 2 when the profit tax rate τ_C rises. Making use of the method of direct variations, the effect of the tax on investment can be seen by differentiating the optimality condition (1.16) for I_t , and the equation of motion for \tilde{q}_{1t} with respect to τ_C . This gives us a pair of differential equations like those in the previous system (1.21). Again, we can draw a phase diagram in $\left(\frac{dk_t}{d\tau_C}, \frac{d\tilde{q}_{1t}}{d\tau_C}\right)$ space as illustrated in Fig. 12, where we show the loci $\frac{d}{dt}\left(\frac{d\tilde{q}_{1t}}{d\tau_C}\right) = 0$ and $\frac{d}{dt}\left(\frac{dk_t}{d\tau_C}\right) = 0$, and the $\frac{d}{dt}\left(\frac{d\tilde{q}_{1t}}{d\tau_C}\right) = 0$ line has a varying, and increasing slope over time.

As initial condition we have $\frac{dk_0}{d\tau_C} = 0$ since the initial stock of capital is unchanged by the tax; to determine the terminal condition of this regime, we need to examine the adjustment process in regime 3.

During regime 3, the effect of the tax may be seen from the optimality condition (1.18) for I_t and the equation of motion for \tilde{q}_{1t} associated with this regime. Differentiating these equations

with respect to τ_C , it gives us the system of differential equations (1.22), where now $a < 1$. As in the case of regime 2, we can draw a phase diagram in $\left(\frac{dk_t}{d\tau_C}, \frac{d\tilde{q}_{1t}}{d\tau_C}\right)$ space. However, the problem is more complicated than before, since the second equation is now non-homogeneous, and both equations are non-autonomous, so that the lines $\frac{d}{dt}\left(\frac{dk_t}{d\tau_C}\right) = 0$ and $\frac{d}{dt}\left(\frac{d\tilde{q}_{1t}}{d\tau_C}\right) = 0$ are shifting over time. Nonetheless, from (1.22) we can deduce that the locus $\frac{d}{dt}\left(\frac{dk_t}{d\tau_C}\right) = 0$ always lies above the origin as the term $\sigma'(\dot{b}_t) \frac{1-a}{(1-\tau_C)^2}$ is positive ($a < 1$), and it shifts downwards because the issues of new debt \dot{b}_t are decreasing over time. Moreover, as discussed above, the line $\frac{d}{dt}\left(\frac{d\tilde{q}_{1t}}{d\tau_C}\right) = 0$ shows a varying, and increasing slope around the origin as capital accumulates. This is shown in the following Fig. 12 where the double arrows indicate the movement over time of these two lines, and where the loci from regime 2 are also shown. In drawing this figure, we have to observe that optimality requires $\frac{1}{\Upsilon_C^2} > \sigma'(\dot{b}_t) \frac{1-a}{(1-\tau_C)^2}$, so that switching from the financial regime 2 to regime 3 the locus $\frac{d}{dt}\left(\frac{dk_t}{d\tau_C}\right) = 0$ associated with this regime must lie below the line $\frac{d\tilde{q}_{1t}}{d\tau_C} = \frac{1}{\Upsilon_C^2}$.²⁷

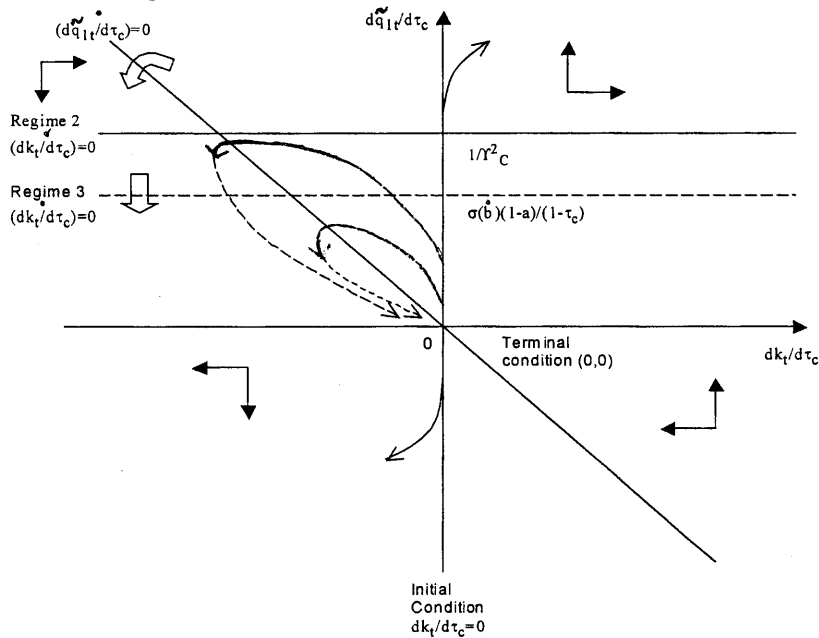


Figure 12: The effects over time of an increase in the corporate profits tax during regime 2 (case $a < 1$; partial search costs deductibility)

²⁷To demonstrate this result, we can observe that at the switching point between regimes 2 and 3, condition (1.17) implies that $\sigma'(\dot{b}_t) = \frac{1-\tau_C}{1-a\tau_C} \left(\frac{1}{\theta_C} - \frac{\rho}{\theta}\right)$. Substituting this term in the inequality in the text, the result follows.

Thus, we must begin with $\frac{dk_0}{d\tau_C} = 0$ and approach the steady state where the tax is neutral (recall the long-run equilibrium condition), that is the terminal point $\frac{dk_t}{d\tau_D} = \frac{d\tilde{q}_{1,t}}{d\tau_D} = 0$. From Fig. 12, we then deduce that the adjustment path in regime 2 can terminate (and the path in regime 3 can begin) neither in the bottom left-hand quadrant nor in the top right-hand area of the figure. This means that we must follow a path beginning between the origin of the axes and the point $\frac{1}{\tau_C}$ on the initial condition line. The possible converging paths are then those illustrated in the figure, where the continuous line denotes the adjustment during regime 2, and the dashed part the effect during regime 3.

From the optimal patterns of Fig. 12, it can be deduced that the effect of the tax on firm investment varies over time, and the short-run impact differs from the long-run implications. In the short-run, the profit tax reduces the investment by the firm; in fact, $\frac{d}{dt} \left(\frac{dk_t}{d\tau_C} \right)$ is necessarily negative at the outset. However, approaching the terminal condition investment will rise again till to offset the initial reduction.²⁸

This adjustment path is similar to that obtained in the case with $a = 1$, but how enduring is the initial impact of the tax is now difficult to determine; in particular, it does not necessarily depend on the marginal source of funds. For example, the initial negative impact may affect the firm both in the initial regime 2 and partially in regime 3, as shown by the lowest path in Fig. 12. In this case, the effect of the tax on investment does not change immediately, with changes in the marginal source of funds. On the other hand, the short-run negative effect may be limited to the initial regime 2 only, and terminate when the pure-debt financed regime starts, as demonstrated by the upper path.

The profit tax influences also the firm's financial policy. The deductibility of debt costs discriminates against internal finance and in favour of debt as this reduces the firm's tax burden (see equation (1.17)). Again, profit taxation leads to high leverage along the accumulation path, but the incentive to increase debt is now lower than in the previous case where $a = 1$. Indeed, the difference with the case of full cost deductibility is in the extent to which profit taxation favours debt financing, and in how long the initial negative effect on investment lasts; these issues are clearly of great importance for policy purposes.

²⁸It may be noticed that where $\sigma'(0) = A > 0$, the optimal path need not to be of this form. In this case, the terminal condition is in the top left-hand quadrant, and the adjustment path could involve a steady fall in investment.

This analysis applies to the firm which is in the mixed-financial regime 2 when the tax rate raises; what about the mature and debt-financed company which is in regime 3 at the time of the tax change?

Formally, the effect of the profit tax in regime 3 may be seen by differentiating the dynamic equation for \tilde{q}_{1t} , and the optimality condition (1.18) characterising I_t in this regime with respect to τ_C . From this we obtain a pair of differential equations equal to the previous system (1.22). Again, as initial condition we have $\frac{dk_0}{d\tau_C} = 0$; moreover, in the steady state the tax is neutral, and we must therefore approach the terminal point $\frac{dk_t}{d\tau_C} = \frac{d\tilde{q}_{1t}}{d\tau_C} = 0$.

With these information, we can draw the phase diagram shown in the following Fig. 13; this illustrates the loci $\frac{d}{dt} \left(\frac{d\tilde{q}_{1t}}{d\tau_C} \right) = 0$ and $\frac{d}{dt} \left(\frac{dk_t}{d\tau_C} \right) = 0$ from (1.22), and describes the optimal directions of motion.

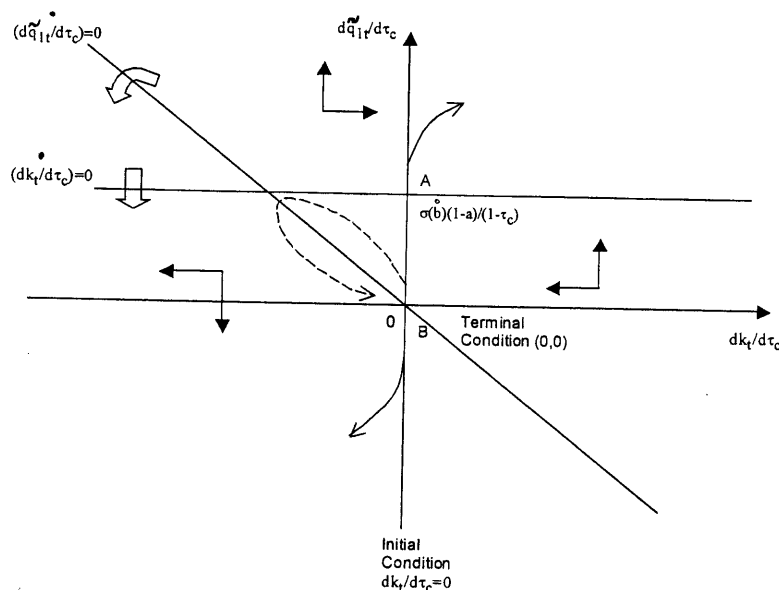


Figure 13: The effects over time of an increase in the profit tax when investment is financed by debt (case $a < 1$; partial search costs deductibility)

If we were to begin above the line $\frac{d}{dt} \left(\frac{dk_t}{d\tau_C} \right) = 0$, or below the origin, we should obviously never approach the terminal condition. This means that we must follow a path beginning between the points A and B on the initial condition line. The only adjustment path is therefore that illustrated in the figure.

As shown in the figure, along the adjustment path, investment initially decreases as a result of the higher tax rate, then it rises as the firm approaches the stationary point; but, the marginal

source of funds does not change. Thus, the effect of the tax on investment is independent of the marginal source of funds. Whether the initial impact of the tax has any enduring effects depends, of course, on the form of functions $c(I_t)$, $\sigma(\dot{b}_t)$ and $R(k_t)$, and on the term a , thus the timescale of the model may change with the exact specification of these aspects.

If we confine the attention to the case of quadratic cost functions, we can calculate the precise effect of the tax on the pattern of investment, namely the function $\frac{dI_t}{d\tau_C}$. In particular, differentiating the optimal path for capital k_t from equation (1.20) with respect to time, and then with respect to the tax rate τ_C , we obtain:

$$\frac{dI_t}{d\tau_C} = \frac{d\tilde{r}_2}{d\tau_C} \tilde{c}_2 e^{\tilde{r}_2 t} (1 + \tilde{r}_2 t)$$

where \tilde{c}_2 is negative and independent of the tax rate τ_C , and $\frac{d\tilde{r}_2}{d\tau_C}$ is positive.²⁹

The function $\frac{dI_t}{d\tau_C}$ can be illustrated in the following figure:

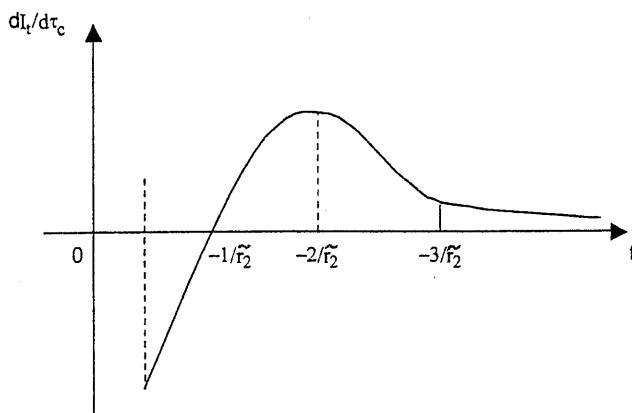


Figure 14: Example with quadratic cost functions: the effects of an increase in the profit tax rate when investment is financed by debt (case $a < 1$)

It may be noticed from this figure that the function $\frac{dI_t}{d\tau_C}$ is the analogue of the adjustment process described for the general case, with the effect of the tax varying over time. Investment initially decreases as a result of the increase in the profit tax rate; then, it rises till to compensate the initial effect with the tax being neutral in the long-run.

²⁹This result follows by differentiating \tilde{r}_2 (see the example in previous section 4) with respect to τ_C , so as to have:

$$\frac{d\tilde{r}_2}{d\tau_C} = \frac{\delta\beta(1-\tau_C)^2}{\left[(\alpha + \beta(1-a\tau_C))^3 (\theta^2(\alpha + \beta(1-a\tau_C)) + 4\delta(1-\tau_C))^{\frac{1}{2}}\right]} > 0$$

This simple example also allows us to consider an important question underlying the dynamic analysis of taxation, that of the timescale of the tax effect. Indeed, the speed of the adjustment may make a great deal of difference to the way we think about the effect of taxes. In this example with quadratic cost functions, we can calculate explicitly the timescale of the tax effect. For instance, suppose for convenience that the parameters α , β , and δ are similar and equal to 0.5, that the discounting rate θ is 0.3, and that the government levies a profit tax rate of 40 percent per year. Then, it can be seen that the switching point $-\frac{1}{r_2}$ in Fig. 14 is 1.8. In this case, the initial negative impact of the tax is clearly limited over time as it takes only 1.8 years for the initial impact to be reversed. It may also be interesting to observe that the timescale of the tax effect depends on the initial tax rate. A lower tax rate, for example 10 percent, increases the switching period to 2.7 years. Put the other way around, the highest the tax rate, the fastest is the adjustment process to a tax change. This is important for policy purposes; it means that governments with high tax rates can only hope to have temporary positive effects on the investment of domestic firms by reducing taxation. This is of course only an example, and the parameterisation is admittedly arbitrary, but this simple “back to an envelope” calculation clearly brings out the importance of considering the intertemporal effects in the analysis of tax policy.

Summing up

The intertemporal analysis of profit taxation has often been thought to be of little practical importance, but this discussion clearly suggests that it does deserve attention. Previous results show that an important distinction exists between the immediate impact of the tax and its implications over time. It is true, for example, that an increase in the profit tax rate reduces investment by firms. This conclusion is often suggested in the literature on corporate taxation which draws on single period analysis. But, the negative impact of the tax is only temporary and it is shown to be reversed, and offset over time independently of the marginal source of funds. This is important for policy purposes; if the benefits of a tax cut are only temporary, then lower taxes, following for example a tax reform, may have only limited effect on capital accumulation of domestic firms. The relevance of these intertemporal implications depends, of course, on the timescale of the effects. Unfortunately, whether the initial effect is enduring

is influenced by the specific parameterisation of the model. Nonetheless, it should be clear that to confine the attention to the immediate impact of taxes may be misleading; indeed, the intertemporal aspects can make a great deal of difference to the way we think about corporate taxation of profits.

1.6 The effects of corporate taxation—comparing with the literature

In previous sections, I have discussed the effect of a corporate profits tax; I now consider how the results given above can be used to shed light on some of the most debated issues in the literature on corporate taxation. In particular, I examine two questions: the concept of tax effect in a dynamic model, and the role of financial optimisation in determining the effects of taxes. The discussion of these issues is of great importance for understanding the advantage of our dynamic approach with endogenous financial decisions compared to the conventional cost-of-capital formulae (Sinn, 1991b). However, it involves a number of problems of considerable complexity. As a result, the conclusions reached in this section are only very limited in scope. Indeed, the primary aim of the section is not to present results, but to bring out the advantages and the problems of examining taxes in a dynamic model with financial imperfections, aspects which have received very little attention in the literature.

1.6.1 The concept of tax effect in a dynamic model

The principal argument for reforming corporate taxation is that of affecting the investment behaviour of firms, and this is clearly the first issue we should discuss. However, the concept of tax effect on investment is not as straightforward as may appear, and we need to exactly define what is meant by “affecting investment”.

The underlying concepts of tax effect have received very little attention in the literature on corporate taxation. The standard cost-of-capital analysis is usually carried out in highly stylised models of accumulation in which there are no stock adjustments, and no dynamics of growth. In this context, the initial impact of the tax coincides with its effect on the long-run level of capital, and there are no intertemporal implications; in fact, this is a single-period

analysis.³⁰ However, in terms of the dynamic model described here, this approach means that if a tax does not affect the long-run level of capital, then it may be said to be neutral, even if it changed the level of investment at each single instant along the accumulation path. There would be neutrality because the effects at different instants offset each other, eventually leading to an unchanged long-run stock of capital. This brings out the difficulties implicit in using models of accumulation with no explicit dynamics for the analysis of the effects of taxes on investment; indeed, investment is typically a flow variable.

For this reason, the standard method of single-period investigation is neither particularly appealing nor appropriate for the study of corporate taxation; instead, the analysis ought to be based on explicit dynamic models. The effects of taxes should be examined in terms of the impact on investment at each instant, and not just on the long-run level of capital. In this sense, we should regard as non-neutral a tax which affects the investment decisions at each time, even though this implies an unchanged long-run level of capital. In fact, governments are more concerned with the timing of policy repercussions than with the long-run impact. On these grounds, I have considered the effect of a corporate profits tax in terms of its immediate, and dynamic impact on firm investment, as well as in terms of the long-run effect on the stock of capital.

It should be noticed that this intertemporal approach repeats some of the results of the conventional single-period analysis, for example, about the long-run effect of taxes; but, it gives rather different results if we consider the overall effect over time. We may have a situation in which a particular tax is neutral with respect to the long-run equilibrium level of capital, but by changing the intertemporal path of investment it leads to different levels of investment at each instant. Moreover, we should consider that in adopting this intertemporal approach the speed of the tax adjustment can make a great deal of difference to the way in which we think about the effect of tax policies. For example, even if tax changes were to be brought about by an initial temporary reduction in investment, this would not cause the same concern as if the effects were enduring. This is important for the analysis of tax reforms. If the path of tax effects varies over time, then the choice of a tax policy becomes complicated. Suppose that a country has to

³⁰For discussion on taxation of the firm, and references to the literature, see for example Atkinson and Stiglitz (1980), Auerbach (1983, 1984), Sinn (1987, 1991b), and more recently Sorensen (1994), Mintz (1995), and Head (1997).

decide whether to cut taxes or not. It will have to compare the different paths of investment corresponding to alternative levels of domestic tax rates. In this setting, the choice will depend on the temporal horizon of the government: a strategy with appealing initial effects may have undesired implications in the long-run. It should be clear that obtaining analytical results on these bases is going to be extremely difficult, even with strong simplifying assumptions. While for simplicity I have ignored the timescale of the tax effects and strategic considerations, these qualifications should be borne in mind in interpreting the results.

1.6.2 Effects of corporate taxation and imperfections in capital markets

The second objection to the conventional or cost-of-capital analysis of corporate taxation is that it does not explicitly consider imperfections in capital markets. The analysis derives predictions on how the effect of taxes depend on the firm's financing decisions (see Atkinson and Stiglitz, 1980), but no satisfactory analysis is provided of the factors underlying the functioning of capital markets and the financing decision-making process of the firm. Ideally, the financial behaviour of the firm should be determined simultaneously with its investment decisions, and the cost of tax and financial distortions ought to be considered together. Unfortunately, in the absence of sound theories of financial markets, this approach has rarely been taken in the literature. But, these considerations may have important implications for the analysis of taxation. In the model described here, for example, the effect of taxes varies even if the marginal source of finance remains unchanged. The effect varies with the stage of development of the firm and with the specific constraints on financial markets. More interestingly, in any given country, the same tax policy, for example lower corporate taxation, may have different effects depending on the "average" firm. Clearly, results could have been different with alternative assumptions about capital market imperfections and if firms could rely, to some extent, on equity finance.

It is not, of course, suggested that the standard cost-of-capital approach is unimportant; however, it is interesting that even when the marginal form of finance is given, we may still get different conclusions depending on the functioning of capital markets. In this respect, the conventional approach to corporate taxation is not fully satisfactory, and a more complex view of the problem should be taken. Models of accumulation with explicit dynamics and financial imperfections have to be used. This may give rise to a *plethora* of different models, but it would

eventually lead to a better understanding of the "black box" of the firm's decision process, and of the effects of corporate taxation.

1.7 Conclusions

The primary aim of this paper has been to set forth a simple intertemporal model to discuss the determinants of firm investment and examine the effect of taxes over time. In particular, the paper brings into the analysis imperfections in capital markets and financial constraints as these aspects are clearly necessary if we are to examine the impact of taxes on firm's investment decisions.

The model of investment described in this paper shows that financial and real investment decisions are closely related; indeed, the firm behaviour is more complex than it is often supposed. Financing constraints influence the firm's accumulation process, and its investment strategy over time. The impact of financial constraints on the firm's financing decisions also are less straightforward than generally thought. It is true, for example, that the cost of borrowing funds discriminates debt in favour of internal saving. But, this does not necessarily imply a financial hierarchy with firms preferring internal funds to external finance; the result depends on each firm-specific characteristics. Indeed, firms may behave in a number of different ways.

This intertemporal model is then used to examine how taxes alter the firms' investment decisions over time. Setting the discussion out in this dynamic setting shows that the effect of taxation is more subtle than often supposed. In particular, the standard cost-of-capital analysis does not always apply to a dynamic setting, and needs to be qualified in a number of respects. We need to distinguish between the short-term impact of taxes and the effects along the temporal path of accumulation. It is true, for example, that an increase in the profit tax rate reduces investment by firms, but the negative impact of the tax is only temporary. We also need to consider the stages of development of the firm and the connections with its financing strategy.

The conclusions reached in this paper are only limited in scope, and rely on a very special view of financial markets. They are sufficient, however, to cast light on a substantive issue: the effect of profit taxes on firm investment. It is widely assumed that a reduction in profit taxation

is beneficial to corporate investment, but if we look at the effects over time, then lower profit taxes do not necessarily increase investment along the entire accumulation path. Moreover, the impact of taxes may change with the firm, and we have seen that enterprises with different financing constraints and at different stages of development respond in a variety of ways to the same tax change. These results are important for policy purposes. If the benefits of a tax cut are only temporary, then lower taxes, following for example a tax reform or fierce tax competition, may have only limited effect on capital accumulation of domestic firms. Clearly, the choice of a tax reform strategy depends on the optimising time horizons of governments.

This discussion is served, I hope, to demonstrate the importance of a thorough analysis of the firm decision-making process for the investigation of taxation. Indeed, more than fifteen years after Auerbach wrote his survey on ‘the effects of taxation on the incentive to invest’, we can still conclude that

“There are many problems in this area awaiting satisfactory solution.” (Auerbach, 1983, p. 906).

I hope this analysis has partially moved in the direction of filling this gap.

1.8 Appendix A: Optimal paths

Appendix A.1: Derivation of the optimal path of the co-state variable q_{1t}

In this appendix, we show that during the financial regimes 2 and 3 optimality conditions require the co-state variable q_{1t} to steadily decrease over time, so that

$$\dot{q}_{1t} < 0$$

Proof. From the multiplier equation for q_{1t} , at the optimum,

$$\dot{q}_{1t} = \theta q_{1t} - R'(k_t)$$

and, using the transversality condition for capital, we have

$$q_{1t} = e^{\theta t} \int_t^{\infty} R'(k_v) e^{-\theta v} dv$$

Integrating by parts this integral, we can also write

$$q_{1t} = \frac{R'(k_t)}{\theta} + \frac{e^{\theta t}}{\theta} \int_t^{\infty} R''(k_v) \dot{k}_v e^{-\theta v} dv$$

At this point, from the previous multiplier equation $\dot{q}_{1t} < 0$ implies that $q_{1t} < \frac{R'(k_t)}{\theta}$, therefore,

$$\dot{q}_{1t} \leq 0 \iff \frac{R'(k_t)}{\theta} + \frac{e^{\theta t}}{\theta} \int_t^{\infty} R''(k_v) \dot{k}_v e^{-\theta v} dv \leq \frac{R'(k_t)}{\theta}$$

that, in turn, requires

$$\dot{q}_{1t} < 0 \iff \frac{e^{\theta t}}{\theta} \int_t^{\infty} R''(k_v) \dot{k}_v e^{-\theta v} dv < 0$$

Since both in the financial regime 2 and in regime 3, $R''(k_t)$ is negative for assumption, while $\dot{k}_t > 0$, it follows that \dot{q}_{1t} is always negative, as it was to be shown. ■

Appendix A.2: Derivation of the optimal path of the multiplier μ_{st}

This appendix shows that during the financial regime 3 the Kuhn-Tucker multiplier μ_{st} has a positive slope, so that

$$\dot{\mu}_{st} > 0$$

Proof. From the first order conditions characterising regime 3

$$\mu_{st} = 1 - \frac{\rho}{\theta} - \sigma'(\dot{b}_t)$$

and at the optimum $\dot{b}_t = I_t = \Psi(q_{1t} - \frac{\rho}{\theta})$. (see section 3)

Differentiating these expressions with respect to time, we have that

$$\dot{\mu}_{st} = -\sigma''(\cdot) \ddot{b}_t$$

$$\ddot{b}_t = \Psi'(\cdot) \dot{q}_{1t}$$

where $\dot{q}_{1t} < 0$ (see Appendix A.1), and $\Psi'(\cdot) > 0$, so that $\ddot{b}_t < 0$.

At this point, from the expression for $\dot{\mu}_{st}$, and since $\sigma''(\cdot) > 0$, it follows that,

$$\dot{\mu}_{st} > 0$$

as it was to be shown. ■

1.9 Appendix B: The comparative-dynamics analysis of profit taxation

In this appendix we use the method of comparative dynamics to examine the effects of an increase in the profit tax rate on the firm's accumulation path (section 5). To determine these effects, let us denote the new tax rate τ_{C_2} , and let $\tau_{C_2} > \tau_{C_1}$ where τ_{C_1} is the original rate of taxation. The implications of this tax change are illustrated in the phase diagram of Fig. 9 in section 5 (using dashed lines). The neutrality of the corporate profit tax in the steady state has been already discussed in section 4. Furthermore, we have also showed that the higher profit tax leads to a higher $\tilde{q}_{1t} = \frac{1}{\tau_C} + \tilde{\chi}$ locus (the switching point between regimes 2 and 3). We now show that the optimal path corresponding to τ_{C_2} lies initially above the original path as in Fig. 9, by demonstrating that the new path can neither be uniformly lower than nor cross from below the old path. From Fig. 9, we can deduce that a steadily lower accumulation path would imply a smaller \tilde{q}_{1t} corresponding to each level of capital; in turn, from the investment rules (1.16) and (1.19) this also means a lower level of investment. But, the steady-state stock of capital is unaffected by the tax, and the investment corresponding to each level of capital must, therefore, rise at some point. As a result, the new pattern of accumulation cannot steadily lie below the original path, and it has to necessarily cross the original pattern of accumulation. However, the new path cannot cross the original one from below. Using Fig. 9, we can notice that the new path crosses the original pattern from below if at the intersection point its slope

is relatively greater (the path is flatter). The slope of the optimal path is

$$\frac{d\tilde{q}_{1t}}{dk} = \frac{\dot{\tilde{q}}_{1t}}{\dot{k}_t} = \frac{\theta\tilde{q}_{1t} - R'(k_t)}{I_t}$$

Moving from the initial conditions toward the steady state, it is easily verified that at any possible intersection point the required inequality implies that $\tau_{C_2} \leq \tau_{C_1}$, which is a contradiction; therefore, the new path cannot cross the old one from below. As a result, a higher profit tax rate has to result in a new path initially lying above the original one. At this point, the new path may steadily lie above or cross the original path. However, if it crosses the original path, then at any point below this path, the new pattern should be flatter (it has a greater slope) and, given the level of capital, this requires that $\tau_{C_2} \leq \tau_{C_1}$, which is again a contradiction. Therefore, the new pattern of accumulation must steadily lie above the original path as illustrated by the dashed line in Fig. 9, and discussed in section 5.

Table 6.1: Empirical studies on firm investment and financing constraints

Study	Coverage	Period	Model	Financial Variables	Results
Fazzari, Hubbard, and Peterson (1988)	Firm-level panel data USA	1969-1984	Tax-adjusted q model	Cash flow, liquidity	Cash flow significant, and improves the fit; specially relevant for low dividend firms
Bond and Meghir (1994)	Firm-level panel data UK	1971-1986	Euler equation with a hierarchy of sources of finance	Cash, debt	Cash flow relevant for financing constrained firms
Blundell, Bond, Devereux, and Schiantarelli (1992)	Firm-level panel data UK	1975-1986	Tax-adjusted q model	Cash flow	Cash flow significant
Galeotti, Schiantarelli, and Jaramillo (1994)	Firm-level panel data ITALY	1976-1987	Tax-adjusted q model Euler equation Free estimations	Cash flow, debt, liquid assets	Cash flow significant for small firms; financial indexes irrelevant to large firms
Cummins, Hassett, and Hubbard (1994)	Firm-level panel data USA	1962-1988	Tax-adjusted q model User cost of capital	Cash flow	Cash flow generally significant, and improves the fit
Cummins, Hassett, and Hubbard (1995)	Firm-level panel data 14 OECD	1982-1992	Tax-adjusted q model	Cash flow	Cash flow generally significant and improves the fit in 6 countries
Hubbard, Kashyap, and Whited (1995)	Firm-level panel data USA	1976-1987	Euler equation	Cash flow, spread between risky and risk-free interest rate	Both financial variables are significant, and improve the fit

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