A LOGISTIC GROWTH THEORY FOR GOVERNMENT EXPENDITURES: A STUDY OF FIVE COUNTRIES OVER 100 YEARS

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Abstract

This paper offers a new theory and empirical testing of long-term trends of public expenditures. Wagner's Law would imply an exponential growth process of the ratio between public expenditures and national income (G/Y). The law may be rejected both on theoretical and empirical reasons, because it disregards the role of ever increasing distortionary taxation. But, under some conditions, the combination of Wagner's Law and the Pigou's conjecture that the excess burden of taxation constraints the growth of public expenditures, may be captured by a non-linear first order differential equation. The equation is the Verhulst's logistic, originally invented to model Malthusian predictions on population growth. The integration of a Verhulst equation generates a S-shaped curve and preliminary econometric estimates on very long run trends (around 1870-1990) of G/Y in US, UK, France, Germany, Italy confirm a pattern of similar logistic trajectories, in spite of different individual parameters.

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Introduction

This paper offers a new theory of the secular growth of public expenditures and empirical testing for five countries.

The growth of government in the last 150 years has been huge. A rough measure of this process is the increase in the ratio between public expenditures and national income (G/Y). This ratio was between 5% and 10% in the second half of XIX century in US, UK, France, Germany, Italy, Sweden and for many other countries involved in modern economic growth (Tanzi, Schuknecht 2000). At the end of XX century the ratio is typically in a range between 35-65%. Quite often public finance students and policy-makers have been interested in the *level* of G or G/Y. However the long-run dynamics of the process may be also very interesting. For example, and contrary to popular perceptions, since the beginning of this century the increase of G/Y has been greater in the US and the UK, while it was slower in some "big-spender" countries, as France and Italy, where the ratio was already high at the beginning of the century. Moreover in most countries the growth of G/Y started to *decrease* after World War II, in the decades usually perceived as the booming of the welfare state (See Table 1). Looking at levels may be very misleading in a long run process.

Several theories have been proposed in order to explain the level of G or G/Y or public expenditures per capita, however the long term dynamic growth mechanism is elusive. However, when we look carefully to available data (scant and imprecise as they are), this pattern can be depicted as a sigmoid curve. The rate of growth of G/Y increases continuously over time until a certain point then the process is reversed: while G/Y still increases, its growth rate declines. At the end of XX century the latter is near to zero, indicating perhaps the convergence to a steady state.

We explore this pattern for several countries and offer a simple theory that may explain it. The theory we propose assumes (Section 1) that virtually the growth of the demand for G/Y is an exponential process, as was first suggested by Wagner (1893). It well may be true that most goods and services (including the administration of transfers) efficiently offered by the state show demand elasticity to income greater than 1. However if this were unconditionally true, the trajectory of G/Y over time we should observe would be an exponential curve (in fact this was approximately true at Wagner's time and for subsequent decades). We suggest that Wagner's law disregards the social cost of distortionary taxation (Section 2). Following a conjecture by Pigou (1947), who in fact wrote at a time of changing trends, we assume the excess burden of taxation acts as a brake to the supply of public goods (and transfers). The increase over time of the excess burden of taxation is a function of the square of T/Y, where T are distortionary tax revenues. If G = T, or the budget is balanced in the long run, the combination of Wagner's law and Pigou's conjecture may be captured by a non linear differential equation in G/Y. The equation is Verhulst's logistic (1847), firstly proposed to model Malthus' population law, and than widely used by demographers (Peerl and Reed, 1920), mathematical biologists (e.g. Lotka or Volterra, 1931) and more recently in deterministic chaos theory (Section 3). The solution of the equation gives a S-shaped curve over time, with some interesting properties. We then study long term data on G/Y for five countries: US, UK, Germany, France, Italy (Section 4). We selected a limited number of sources out of a longer list of possible candidates (see bibliography).

The selection criteria was comparability of definitions and not the lenght of the time series. In some cases we used series comprising a very limited number of data, but we tested our model on a much longer set of sources with similar results. Simple estimation procedure is used to test an exponential process against a logistic one (Section 5).

Table 1. The ratio between public expenditures and national income around selected years.

Sources	1870	1880	1890	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990
France													
Delorme	11.0	14.6	14.3	14.4	12.6	32.8	18.8	26.5	41.1	46.4	50.8	n.a.	n.a.
Flora	11.6	15.4	15.0	15.2	12.0	34.2	22.1	29.2	37.2	39.2	38.7	n.a.	n.a.
Maddison	n.a.	11.2	n.a.	n.a.	8.9	n.a.	12.4	23.2	27.6	33.9	38.8	48.7	51.0
Germany													
Delorme	n.a.	7.8	9.2	11.7	11.3	23.1	34.2	n.a.	35.7	38.8	35.9	n.a.	n.a.
Flora	n.a.	9.9	12.9	14.2	17.0	22.4	29.4	36.9	n.a.	32.5	38.0	n.a.	n.a.
Maddison	n.a.	10.0	n.a.	n.a.	17.7	n.a.	30.6	42.4	30.4	33.9	42.0	48.7	47.8
Italy													
Brosio	14.4	13.7	18.4	16.2	17.2	30.1	22.0	41.4	30.2	37.3	48.3	n.a.	n.a.
Tanzi	11.9	n.a.	n.a.	n.a.	11.1	22.5	n.a.	24.5	n.a.	30.1	n.a.	41.9	53.2
United Kingdom													
Maddison	n.a.	9.9	n.a.	n.a.	13.3	n.a.	23.8	28.8	34.2	32.9	41.5	46.4	51.2
Middleton	9.0	10.0	8.0	13.3	11.9	20.5	25.6	28.1	37.5	37.1	42.9	45.9	n.a.
United States													
Delorme	n.a.	n.a.	n.a.	6.8	8.0	11.1	18.5	17.8	22.0	27.8	32.2	33.2	n.a.
Maddison	n.a.	n.a.	n.a.	n.a.	8.0	n.a.	10.0	18.5	21.4	27.9	31.1	34.4	38.5
Musgrave	n.a.	n.a.	7.1	7.9	8.5	12.6	21.3	22.2	24.6	28.2	34.2	36.5	n.a.

Notes:

France: Delorme. Public expenditure as percentage of GDP; 1870 data refers to 1872, 1910 to 1912, 1930 to 1929, 1940 to 1938.
Flora. Public expenditure as percentage of GDP; 1870 to 1872, 1910 to 1912, 1930 to 1929, 1940 to 1938, 1950 to 1947, 1960 to 1959, 1970 to 1971.
Maddison. Public expenditure as percentage of GDP; 1910 data refers to 1913, 1930 to 1929, 1940 to 1938, 1970 to 1973.

Germany: Delorme. Public expenditure as percentage of GNP; 1880 data refers to 1881, 1890 to 1891, 1900 to 1901, 1910 to 1907.
Flora. Public expenditure as percentage of GDP; 1880 data refers to 1881, 1890 to 1891, 1900 to 1901, 1910 to 1913, 1940 to 1938.
Maddison. Public expenditure as percentage of GDP; 1880 data refers to 1881, 1910 to 1913, 1930 to 1929, 1940 to 1938, 1970 to 1973, 1980 to 1981 and 1990 to 1985.

Italy: Brosio. Public expenditure as percentage of GDP at factors cost.
Tanzi. Public expenditure as percentage of GDP; 1910 data refers to 1913, 1940 to 1937.

United Kingdom: Maddison. Public expenditure as percentage of GDP; 1910 data refers to 1913, 1930 to 1929, 1940 to 1938, 1970 to 1973, 1980 to 1981 and 1990 to 1992.
Middleton. Public expenditure as percentage of GDP; 1910 data refers to 1913, 1940 to 1938, 1950 to 1951, 1970 to 1973, 1980 to 1979.

United States: Delorme. Public expenditure as percentage of GNP; 1900 data refers to 1902, 1910 to 1913, 1920 to 1922, 1930 to 1932, 1940 to 1938.
Maddison. Maddison. Public expenditure as percentage of GDP; 1910 figure refers to 1913, 1930 to 1929, 1940 to 1939, 1970 to 1973, 1980 to 1981, 1990 to 1992.
Musgrave. Public expenditure as percentage of GNP; 1900 data refers to 1902, 1910 to 1913, 1920 to 1922, 1930 to 1932

Sources:

Brosio, Marchese, 1986. Delorme , André 1979. Flora Peter (et al.), 1983. Maddison A., *1984, 1989,1991,1995*. Middleton R., 1996. Musgrave R.A., 1953, 1969, 1995. Tanzi, Schuknecht, 1995

The test, while a preliminary one, confirms that for all the five countries a logistic equation in G/Y offers a better fit than an exponential one. Thus data do not reject our sample growth model. The paper is concluded by some remarks and indications for future research.

1. The demand of public expenditures: Wagner's Law restated

According to a straightforward interpretation of Wagner's work (1893)[1], the ratio G/Y should increase over time because the goods and services offered by the State are superior goods. The intuition is that

a) some goods can be supplied efficiently by the state

b) the demand for these goods is such that as Y, or Y per capita, increases over time (rather than across countries), their demand increases even more.

While many economists or politicians may disagree on both propositions it is a matter of fact that in the last 100-150 years, and in many countries such goods as education, health and social insurance have been mostly delegated to the state, indicating in some way a social preference for this public provision; it may also be true, albeit less easy to test empirically, that many of these goods show income elasticity greater than 1, while many goods typically offered by the private sector, typically food, are inferior goods in respect to income (food is a typical example).

The above propositions do not need to be true for all the items of the public expenditures, it is sufficient that they are true for the average or typical item comprised in G.

Having said this, the intuition behind Wagner's Law may be modeled in different ways. According to Henrekson (1990, 1993, Fölster S., Henrekson 2001) in the literature on this subject at least five different testable models can be found (we report also the elasticity values η_n of the dependent variable to the independent variable which may be seen as confirming Wagner Law):

W.1) $G/Y = f(Y^*/N)$ $\quad \eta_1>0$

W.2) $G= f(Y)$ $\quad \eta_2>1$

W.3) $G/N = f(Y^*/N)$ $\quad \eta_3>1$

W.4) $G = f(Y^*/N)$ $\quad \eta_4>1$

W.5) $G/Y = f(Y)$ $\quad \eta_5>0$.

[1] The interpretation of Wagner's Law is rather controversial ans has a long history, see Peacok, Scott (2000).

G should typically be understood as total public expenditures, excluding transfers, of general government (both central and local), Y and Y* as respectively nominal and real national income (probably GDP may be the appropriate variable), N is population. Henrekson shows that 1) and 3) are equivalent for a monotonic transformation; so are 2) and 5), while 4) is conceptually different (and the interpretation of the elasticity is also more loosely related to Wagner's Law). Moreover it can be shown that the elasticity for 1) and 2) is the same if N is constant and Y/N is always increasing.

We do not want at this stage to discuss the most appropriate empirical definitions of variables for G, Y, N, an issue by far much more complex that it may look at first sight (for instance the issue of transfer expenditures[2], or the relevance of constant prices variables in this framework).

We fully agree with Henrekson (1993), Fölster, Henrekson (2001) that there is not much insight to gain from cross country analysis, and that long time series may be more revealing. However in this case we need to model explicitly the dynamic process, while none of the above equations is dynamic.

To restate Wagner's Law over time, let us first think to a progressive economy with stable population. Thus:

Y = Y(t); N = N(t); G = G(t) and dY/dt >0, dN/dt =0

Here and elsewhere derivatives are over time. In this setting we want to test W.1 or equivalently W.2, implicitly then we test also W.3 and W.5 (W.4 is excluded frtom the scope of our study, because we think it is poorly related to Wagner's intuition).

Our interest is to study the trajectory of G/Y.

It is convenient to use the following notation:

$$g = \frac{d \log G}{dt} = \frac{1}{G}\frac{dG}{dt} \qquad [1]$$

$$y = \frac{d \log Y}{dt} = \frac{1}{Y}\frac{dY}{dt} \qquad [2]$$

$$R = G/Y$$

It easy to see that

$$dR/dt = hR$$

where

$h = g\text{-}y$

In fact it will then be

$$h(G/Y) = 1/Y(dG/dt) - G/Y\,[1/Y(dY/dt)]$$

The right side of the above is exactly the same thing as the derivative of G/Y

$$dR/dt = [Y(dG/dt) - G(dY/dt)]/Y^2 \qquad [3]$$

[2] We can think that transfers are a way to provide redistribution and that this may also be a superior good.

Then h > 0 implies dR/dt >0, for R_0>0, which we can easily assume. There is also a simple relationship between η and h:

$h = y\ (\eta-1)$. [4]

Thus for y >0 and constant by assumption, also any value of η > 1 and constant over time implies a simple exponential process integrated by

$R = R_0 e^{ht}$.

If y changes over time, the same exponential process to be generated needs an offsetting change in the opposite direction of the income elasticity of G. In the more general case h may vary over time and the integration of the differential equation above may generate many different type of trajectories.

However what follows we are going to confine ourselves to the case of constant h, but it is important to remember that this assumption is compatible either with loglinear trend of GDP and fixed elasticity of public expenditures to income, or with appropriate offsetting trends in these parameters.

Having said this, it is clear why the Wagner Law in the long run is both theoretically and empirically implausible. An exponential process implies that G/Y increases over time without limits, and this in turn implies either ever increasing debt or a very costly process of transfer and tax. The latter is true for G=T, where T is tax revenue, when G/Y >1, because in this case there is no other way to levy taxes than to tax public expenditures themselves (e.g. public pensions). The alternative for G> T is a debt rising without limits. And this also seems implausible.

However it may well be that Wagner's Law is only a part of the long-run dynamics but that other forces put a brake on it. To this inquiry we turn now.

2. The excess burden of taxation: Pigou' s Conjecture

Wagner's Law ignores the social cost of financing government expenditures. Pigou (1947), writing one hundred year later, was more aware of the issue.

Public expenditures may be financed by incomes directly accruing to government (e.g. statutory monopolies), by seignorage, by debt, and by taxation. In the last century most of public expenditures have been financed by taxation, hence we shall simply ignore the other sources of government revenue. Thus we shall suppose that normally G=T or G/Y = T/Y. In order to study the supply of good offered by the state we should think on their production and cost functions.

We introduce here two assumptions:

a) the production function of the average good produced by the state shows constant returns to production factors;

b) there is an extra-cost of taxation, its excess burden, and this is quadratic in T/Y.

As for assumption a), only a detailed analysis of the production function of such goods as education, health, defence, transport, social insurance, etc. may establish whether these goods show increasing or decreasing or constant returns. Baumol (1967, 1985) tried to show that under certain conditions if one sector shows stagnating productivity while the other one is progressive, the first one will absorb most of the expenditures of the economy. He thought that this proposition was relevant in order to explain the growth of public expenditures. However, the pattern that would emerge if Baumol theory holds is again exponential growth of G/Y and this is not plausible. Moreover, there is no evidence that on average and over the long run government activity is based on technologies that do not allow for (absolute or relative) rise in productivity. One may think many counter-examples.

Thus it seems more prudent to assume constant returns as a benchmark case. Under fixed production costs on average one extra euro of inputs provides one extra euro of government output in real terms. We may suppose that the gross marginal social benefit of one euro of G is not less than its marginal production cost. This is approximately true for pure transfers (when we disregard administration costs), while it may be an understatement for such goods as education or health (standard national income accounting conventions make the assumption that one more euro of public sector costs adds one more euro to GDP, hence to national welfare, if we are not interested in income distribution: a particularly crude, but practical, way of measuring economic welfare).

However, consumers of G do not pay typically an individual price for the goods they consume, they pay taxes that cover the aggregate cost of G. Because taxation is distortionary (quasi lump-sum taxes may be levied, but they play a minor role) a positive T/Y implies a cost in excess to the social benefit of G/Y, the excess burden of taxation.

How large is this cost? There are several ways to compute it, all of them based on specific assumptions on the measurement of consumer welfare: again this is not the place to study this in detail or to review a well developed literature. However it is easy to show that the excess burden for small increases of taxation should be a function of the square of T/Y.

Starting with the simplest case, suppose there is an economy with one consumer, one private good and one publicly provided good. Quantity of the private good is x, while its fixed unit production cost is p. If the only taxable good is the private one (leisure is untaxed, or there is no lump sum taxation), tax revenues for the state are

$T=\zeta px$

where ζ is the tax rate. If the public good is provided free, national income at consumers prices is

$Y=qx$

where q is the consumer price. In this simple context suppose we start at t=0 with p=q. Thus for a small tax

$dp=\zeta p$.

The Marshallian demand price elasticity of the private good is

$\varepsilon = - (dp/dx)\,(p/x)$

and

$dx= \varepsilon x\zeta$.

Thus the simple standard definition of the excess burden of taxation implies:

$$E = dxdq/2 = -Y\varepsilon\zeta^2/2 \qquad [5]$$

But because G=T, the ratio of the aggregate excess burden to national income is then quadratic in the ratio of public expenditures to national income itself:

$$E/Y = -(\varepsilon/2)(G/Y)^2 . \qquad [6]$$

We can think that the general form of a differential equation for the growth of public expenditures isthus:

$d(G/Y)/dt = f(G/Y, E/Y)$

where the expected sign for E/Y coefficient is negative.

The simplest specification of this equation is a linear combination of Wagner's law (as restated above) and of Pigou's conjecture of increasing excess burden of taxation as a brake to public expenditures:

$$d(G/Y)/dt = \alpha y(\eta - 1)(G/Y) - \beta(\varepsilon/2)(G/Y)^2 \qquad [7]$$

where α, β are parameters that may reflects institutional factors, e.g. the time needed to the political system to react to welfare changes of voters.

We are not going to further discuss on the political economy of public expenditure process, but we wish to stress that the model itself can be interpreted in a way consistent with the view that in the long run the consumer-tax payer-voter is able to influence policy makers.

The above result needs a number of qualifications:

a) consumer surplus measured on the Marshallian curve is generally an insatisfactory basis for the evaluation of the excess burden of taxation, for well known reasons. A better measure is a metric based on the equivalent variation on compensated or Hicksian demand curve. In this case, differently from the compensating variation measure, the tax revenue is the effective one and this is particularly useful in our context.

b) moreover, we are interested to a dynamic process where taxes are already existing in previous periods (in fact we use a continuous process). It is known that in such a case first order welfare effects will appear (Harberger 1964, Auerbach 1990, Feldstein 1997). The "Harberger triangle" becomes a trapezoid.

A second order Taylor expansion formula in such case is

$$E = (dE/dq)s + 1/2\left(d^2E/dq^2\right)s + \cdots$$

where s is the tax per unit of good, q= p + s.

For linear compensated demand x we have then the approximate value

$$E = -sdx + 1/2(dsdx) \qquad [8]$$

where ds is a small change of the tax over time. Thus we can write:

$$dx = \varepsilon(x/p)dp = \varepsilon x\zeta$$

Thus the first addendum is $-\zeta^2 Y\varepsilon$ and the second addendum is $1/2\,(Ydx)$.

Thus the approximate value of the excess burden relative to income, with pre-existing taxes, remembering T=G is circa:

$$E/Y = -\left(\varepsilon(G/Y)^2 + 1/2\,d(G/Y)\right) \qquad [9]$$

Thus again we have a quadratic term in the tax rate, plus an additional term.

Even if ignore the latter it is clear that the excess burden coefficient in the equation above is now greater than in the benchmark case. For example, a country where G/Y is around 0.50, as e.g. Italy or France, and where the aggregate compensated elasticity to prices is, say, 0.20, would have an excess burden of 5% of GDP plus half the increase of G/Y.

c) The consideration of flexible production prices and of many different consumers adds complexity to the evaluation of the excess burden of taxation. The literature on the subject offers different formulas to deal with this reacher framework of analysis. Unfortunately generally it is showed that E depends upon the initial income distribution among agents and we need then a social welfare function to consider this. We cannot dwell here on these additional difficulties, important as they are: we must confine ourselves to a much simpler framework

d) Other difficulties that we shall ignore at this stage arise from the distinction between different types of taxes (e.g. indirect versus direct ones) and between types of public expenditures (public consumption, investment and transfers). Both income taxes and income (or goods) subsidies have distortionary effects and in principle generate an excess burden, but the welfare impact may be different. It seems worthwhile to avoid disaggregation of taxes and expenditures by categories at this very preliminary stage of analysis. Readers should be aware of this point.

Having said all this, it seems that there is a theoretical case for a differential equation of public expenditure growth of the general form:

$$d(G/Y)/dt = \alpha y(\eta - 1)(G/Y) - \beta\varepsilon(G/Y)^2 \qquad [10]$$

where the key parameters are two elasticities: respectively the income elasticity of public goods and the compensated price elasticity of private goods; plus the growth rate of national income, and institutional factors affecting the reactivity of the political system to the change in consumer and taxpayer welfare.

The above equation is a nonlinear first-order ordinary differential equation, that can be written as

$$d(G/Y)/dt = (G/Y)(h - k(G/Y)). \qquad [11]$$

where $h = \alpha y(\eta - 1)$ and $k = \beta\varepsilon$

In this form the equation is the logistic, firstly introduced by Verhulst (1838), following a suggestion by Quetelet, to study Malthusian population growth. This equation has some interesting properties that we shall discuss in the next section. Looking at the history of government through it may give us a new perception of long term trends of public expenditures.

3. Logistic growth

Let us resume the above assumptions. Consumers-taxpayers want more and more G as their income increases, but they suffer an extra loss of welfare that increases with the square of T/Y. Because T/Y = G/Y, their excess burden will be a function of the square of G/Y itself. At the beginning of the process, for reasonable values of the key parameters, the welfare loss will be small enough and the growth of G/Y may look similar to an exponential process. However, after some time the process will be hindered and the logistic trajectory will become apparent. As said the Verhulst logistic equation, was originally introduced to study the Malthusian hypothesis on population growth and in subsequent years was vastly used by demographers (Pearl, Reed) and mathematical biologists (Volterra) and more recently in the study of innovation, learning processes and in deterministic chaos theory.

The curve has some well known properties:

- let us denote R = G/Y; dR/dt >0 only if h/k >R; with h>0, k>0 and fixed parameters, their ratio is then the upper limit of R. When R = h/k, dR/dt=0 and there is no more increase of G/Y ratio, i.e. the two variables increase at the same speed. At the beginning of the process, near t=0 and R=0, then dR/dt = hR, as in the exponential growth;
- the integral value of the function is

$$S_t = K/1 + Ce^{-ht} \qquad [12]$$

where K=h/k, C is a constant term;

- the proportional derivative is a simple linear function:

$$d\log S/dt = 1/S\,(h - kS)S = h - kS\,; \qquad [13]$$

- flexpoint occurs at point $t = (1/h)\log C$ and there the G/Y ratio is at half of its maximum value: i.e. K/2. At that point dS/dt reaches its maximum value or hK/4.

The logistic process over time generates a S-shaped curve that at the beginning is similar to an exponential: the process may be seen as the prevalence of expanding forces, while at a certain point the brake given by countervailing forces reverses the process. Thus the flex-point divides the trajectory of government expenditures in two histories: the first part is convex to the horizontal axis, the later part is concave. In the first part of the history the growth is accelerating, in the second part it is still positive, but decelerating. At a certain point the process converges to a steady state.

If the actual process governing G/Y is a logistic one, we should observe something similar to an S-curve through actual data. In fact, the empirical trajectories we observe are compatible with logistic growth, as we shall show below. Other sigmoid curves may fit well with data, but at this stage we are not interested mainly in empirical analysis, but simply to make a preliminary test of the theory.

4. Data

Long-run data on public expenditures and GDP are difficult to find and we need to rely on work by economic historians, more than on official sources.

The sources of data we have selected are the following one:

- France: André, Delorme (1984), Flora (1983), Maddison (1989-1995).
- Germany: André, Delorme (1979), Flora (1983), Maddison (1989-1995).
- Italy: Brosio, Marchese (1981), Luzzati, Portesi (1984), Tanzi, Schucknet (1995).
- United Kingdom: Maddison (1989-1995), Middleton (1996).
- United States: André, Delorme (1979), Maddison (1989-1995), Musgrave (1995).

The available time series are incomplete and revealing wide variance among different sources for the same country and the same time period. However, in what follows we rely on the evidence already available by the cited established experts.

Because data differ among authors, and because there are important data issues that should be solved, e.g. whether to exclude war years from time series, and how to control for volatility of Y, we decided that at this stage the empirical analysis should be as simple as possible. Thus, what follows should be regarded just as a preliminary test. We repeat the test on different sources for each individual country.

5.Estimation procedure.

The empirical curve to be estimated can be linearized in the following way:

We consider the function $S_t = S_0 + K/\left(1 + Ce^{-ht}\right)$ [14]

where R_0 is the low asymptote and R, K, C >0.

It is convenient to consider the following transformations:

$$\left(K + S_0 - S_t\right)/\left(S_t - S_0\right) = Ce^{-ht}$$

$$\log_e\left(K + S_0 - S_t\right)/\left(S_t - S_0\right) = \log_e Ce^{-ht}$$

$$\log_e\left(K + S_0 - S_t\right)/\left(S_t - S_0\right) = \log_e C + ht$$

Let us now define $\log_e\left(K + S_0 - S_t\right)/\left(S_t - S_0\right) = Z_t$. This new variable, a transformation of S_t is linearly related to *t* through the parameters C ed *h*:

$Z_t = a + bt$ with $a = \log_e C$ and *h*=*b*.

We need now to determine two values R' e K' that constitute the extremes of the above mentioned Z variable and compatible with the linear relationship between the N observation pairs Z_t e *t*. Thus, fixed S e K we can estimate the other two parameters (C e *h*) with an OLS estimate of a and b.

We can also use non linear regression to estimate the parameters of the logistic function. In order to start the non linear estimation algorithm, we must have initial values for the parameter. Unfortunately the results of non linear estimation often depend on having good starting value for the parameters. Poor initial values can result in non convergence, a local rather than global solution or a physically impossible solution. There are several methods for obtaining starting values. Sometimes a linear model can be derived and linear regression can then be used to obtain initial values. This is the case: we specify the results of the linear regression described above as starting value in non linear regression. In non linear regression, just as in linear regression, we choose values for the parameters so that the sum of squared residuals is a minimum. There is not, however, a closed solution. We must solve for the values iteratively. There are several algorithms for the estimation of non linear models. With SPSS we can use a Levenberg-Marquardt algorithm by default or we can try the sequential quadratic programming algorithm. For a particular problem, one algorithm may perform better than the other. Additional optional are available in SPSS, when the sequential quadratic programming algorithm is used. We can supply linear and non linear constraints for the values of the parameter estimates and we can specify our own loss function (By default, the loss function that is minimized is the sum of the squared residuals). In our estimates we used linear constraint only when the upper asymptote estimated by the Levenberg-Marquardt algorithm was upper than 100 or the low asymptote was < 0 or upper than the value minimum of the series. For a non linear model, the tests used for linear models are not appropriate. In this situation, the residual mean square is not an unbiased estimate of the error variance, even if the model is correct. For practical purpose we can still compare the residual variance with an estimate of the total variance, but the usual *F statistics* cannot be used for testing hypotheses. The R^2, however, can be applied in its conventional sense to a non linear regression. It may be interpreted as the proportion of the total variation of the dependent variables around its mean that is explained by the fitted model. For non linear models, its value can be negative if the selected model fits worse than the mean. In the case of non linear regression it is not possible to obtain exact confidence interval for each of the parameter. Instead, we must rely on asymptotic approximations.

For a wider discussion of estimating techniques of the logistic curve see Oliver (1966, 1969, 1982), Nelder (1961), Pearl, Reed (1977), Ratkowsky (1986, 1993).

6. Main results

Our main results for the countries we considered are the following ones:

a) US

Fig.1 and Table A1, show data and regression results for the data by Delorme (1979), Musgrave (1995), and Maddison (1984-1995). The number of observations is limited, respectively 19, 14, 11. A larger number of observation (33) is available for US Stat (US Bureau of Census, 1975), however that set of data stops at 1970 and includes some strong outliers for Second World War years, with a bad fit for either an exponential or a logistic process (however in table 1b we report US STAT data for the years included in the other sources). The Delorme, Maddison, Musgrave three sets of data are generally similar in levels: for example for year 1902 the range of G/Y is between 6.8% and 7.9%; for year 1913 it is 8-8.5%, 17.8-19.8% in 1938, 32.2-34.2 in year 1970. The latest year we have is 1992, at 38.5% by Maddison (obviously we have recent data from other official sources, but - as said - we do not want here to integrate various sources). The first observation we have is for year 1890, by Musgrave, where G/Y was 7.1%. The overall picture seems clear enough: G/Y increased by 3 times between the end of XIX century and the '30s, then in the following 50-60 years the increase was much less, around 2 times. The overall fit (adjusted R-squared) of a logistic curve is only marginally better than an exponential one, however the extrapolation by an exponential gives a completely wrong trend for the most recent period, while the logistic captures well the history of G/Y at the end of XX century. Most interesting, the flex-point year in our estimates is 1944 for both Delorme and Musgrave, and 1954 for Maddison. The change in the rate of increase of G/Y greatly anticipates the perception of a change of attitudes towards public spending in the '80s. The value of the time coefficient h (linear regression), is very similar for Musgrave and Maddison: around 0.074, and 0.066 for Delorme series.

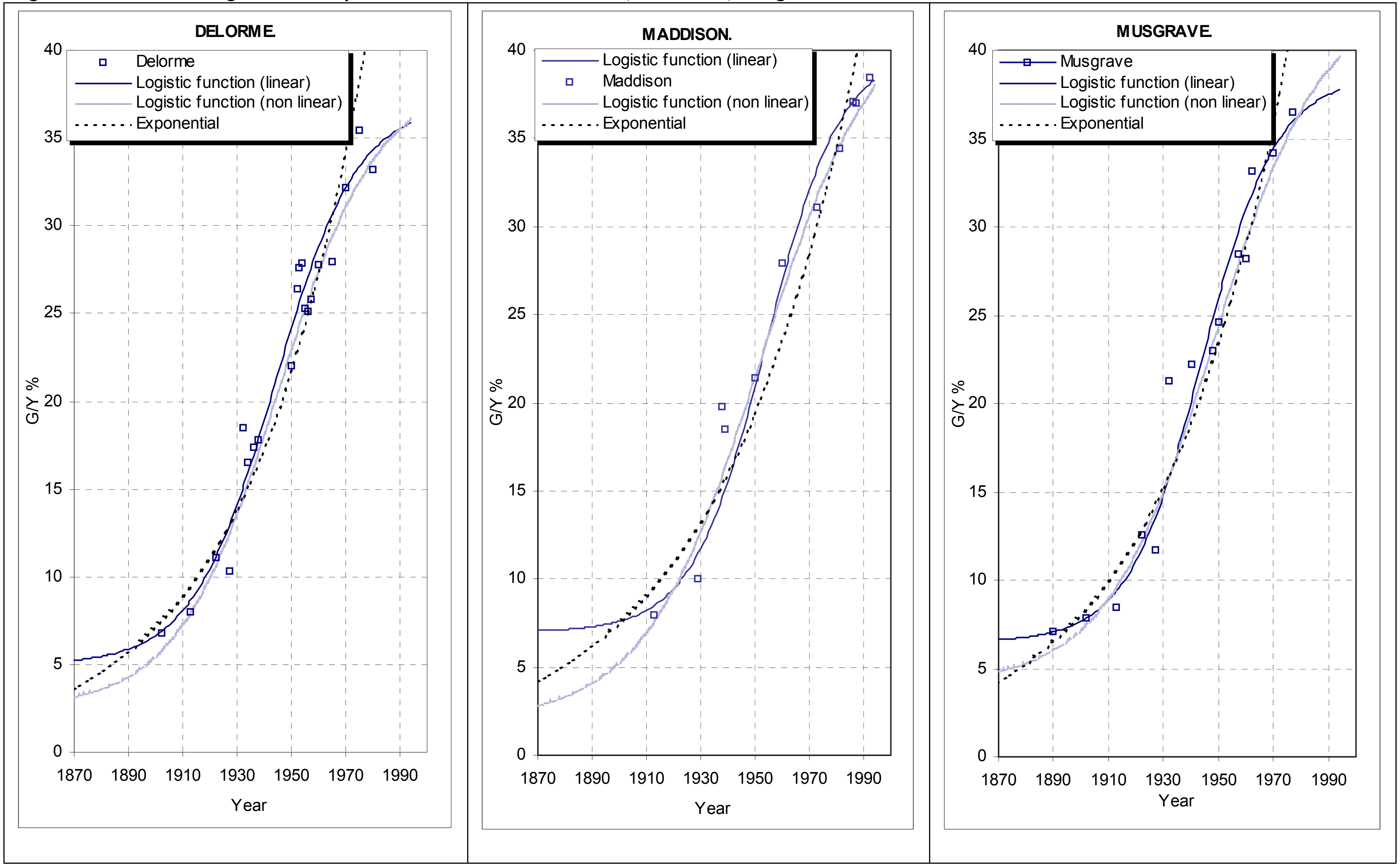
Tab 1a. United States. Output regression summary - Delorme, Maddison, Musgrave data.

		Logistic linear estimate		
		DELORME	MADDISON	MUSGRAVE
R_0		5.000	7.000	6.500
K		32.000	33.000	32.000
R Square		0.947	0.961	0.972
Adjusted R Square		0.944	0.957	0.969
Standard Error		0.332	0.418	0.345
Observations		19	11	14
Constant:	Coefficients (a)	4.89404	3.03720	5.50586
	Standard Error	0.30388	0.25832	0.27048
	Stat t	16.10506	11.75738	20.35616
	Signif. T	0.00000	0.00000	0.00000
	Lower 95%	4.25291	2.45283	4.91655
	Upper 95%	5.53518	3.62156	6.09518
Variable t:	Coefficients (b)	-0.06618	-0.07351	-0.07434
	Standard Error	0.00379	0.00491	0.00367
	Stat t	-17.45117	-14.96011	-20.25433
	Signif. T	0.00000	0.00000	0.00000
	Lower 95%	-0.07418	-0.08462	-0.08233
	Upper 95%	-0.05818	-0.06239	-0.06634
C		133.49197	20.84671	246.13070
Flex point		73.95231	41.31921	74.06678
Flex Year		1944	1954	1944
		Logistic non linear estimate		
		DELORME	MADDISON	MUSGRAVE
R_0		2.5800	2.0000	4.1968
K		35.7500	40.8198	38.7136
C		9.6918	5.9638	19.0034
H		0.0544	0.0472	0.0515
R Square		0.9623	0.9791	0.9653
Flex Year		1943	1950	1946
		Exponential estimate		
		DELORME	MADDISON	MUSGRAVE
R^2		0.9251	0.914	0.943
C		2E-18	1E-15	1E-17
B		0.066	0.0191	0.0215

Note: Exponential Equation: $Y=ce^{bx}$; c,b are constant.

The range of the same coefficients estimated with the non linear regression is between 0.047 and 0.054. As we shall see this is a rather high value, that exceeds the corresponding value for some other countries in other samples: this implies that, contrary to popular views, the US have a history of sustained increase in public expenditures, and if their current level of G/Y is much lower than in Europe, this is due to the low starting level and not to a different trend.

Fig.1 United States. Logistic and exponential functions - Delorme, Maddison, Musgrave G/Y% data.



b) France

We considered three data sources: Delorme (1984), Flora (1983), Maddison (1984-95). We have also data from Brosio (1993) that are usually consistent with Delorme (we report them in table A2). Figure 2 and tab 2a show estimation results. Around 1870, G/Y in France was around 11%, and by the end of the XIX Century it exceeded 14%: two times the corresponding level of the US. The increase was around 2 times between 1870 and the early '30s (while as said it was more than 3 times in the US), then it more than doubled again until it nearly stabilized around 50% since the '60s. As in the US, the stabilization greatly precedes the hot debates about budgetary issues in the '80s. Unfortunately the three series of data we use are very different as for the number of observations: 55 for Delorme, 23 for Flora, just 10 for Maddison. This makes the estimations difficult to be compared. Moreover there are quite different values for individual years, probably due to different definitions of G or Y considered by the authors (but for other years the G/Y ratios are very similar).

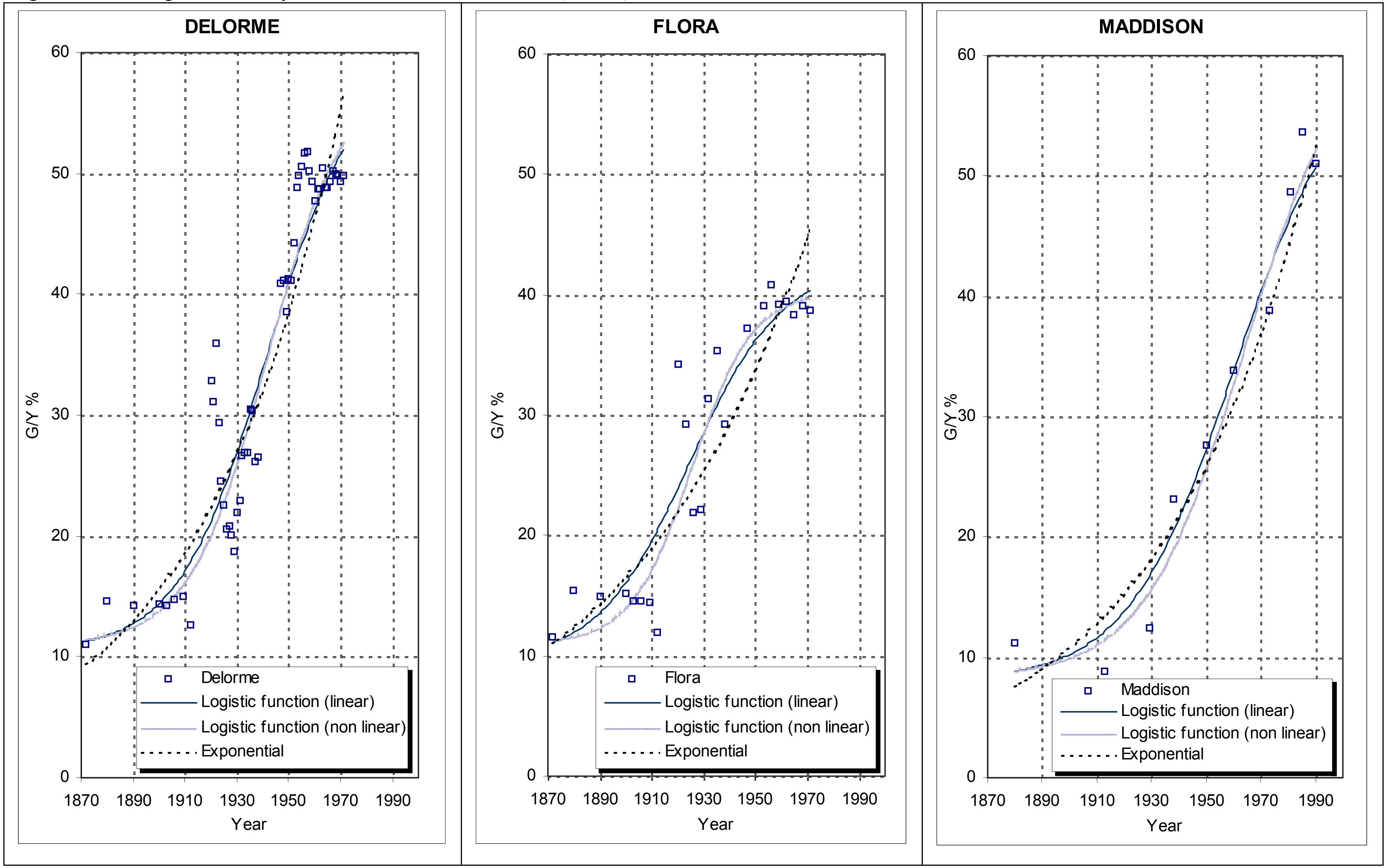
The fit for a logistic (linear and non linear) is slightly better than with an exponential with Delorme and Flora data, but not with Maddison (only the non linear estimation gives a R square higher than the exponential function). Flex points in the estimated logistic are very different, from 1924 with Flora data (that stop much earlier than the other two series), to 1960 with Maddison data, with Delorme in between, 1941. As for the Us, the superiority of the logistic as compared with the exponential is best seen looking at most recent years. When extrapolated to year 2000 the exponential widely overstates G/Y, while the logistic is very near to current data for France from official sources. The estimates for *h* coefficient are in the region of 0.05-0.08.

Tab 2a. France. Output regression summary - Delorme, Flora, Maddison data

		Logistic estimate		
		DELORME	FLORA	MADDISON
R_0		10.500	9.500	8.000
K		48.700	33.000	52.000
R Square		0.891	0.833	0.837
Adjusted R Square		0.889	0.825	0.816
Standard Error		0.528	0.752	0.854
Observations		55	23	10
Constant:	Coefficients (a)	4.12269	2.98610	4.13198
	Standard Error	0.21077	0.35062	0.62346
	Stat t	19.55987	8.51662	6.62753
	Signif. T	0.00000	0.00000	0.00016
	Lower 95%	3.69993	2.25695	2.69428
	Upper 95%	4.54544	3.71526	5.56967
Variable t:	Coefficients (b)	-0.06033	-0.05680	-0.05145
	Standard Error	0.00290	0.00555	0.00804
	Stat t	-20.77407	-10.23626	-6.39900
	Signif. T	0.00000	0.00000	0.00021
	Lower 95%	-0.06616	-0.06834	-0.06999
	Upper 95%	-0.05451	-0.04526	-0.03291
C		59.81384	19.80833	62.30089
Flex point		69.45355	52.57199	80.31217
Flex Year		1941	1924	1960
		Logistic non linear estimate		
		DELORME	FLORA	MADDISON
R_0		11,0000	11,0000	8,3359
K		47,1319	29,2843	53,7877
C		103,5061	98,8034	99,8623
H		0,0669	0,0860	0,0553
R Square		0,9009	0,8818	0,9726
Flex Year		1941	1925	1955
		Exponential estimate		
		DELORME	FLORA	MADDISON
R^2		0.8629	0.8056	0.8741
C		2E-14	2E-11	3E-14
B		0.0181	0.0143	0.0177

Note: Exponential Equation: $Y=ce^{bx}$; c,b are constant

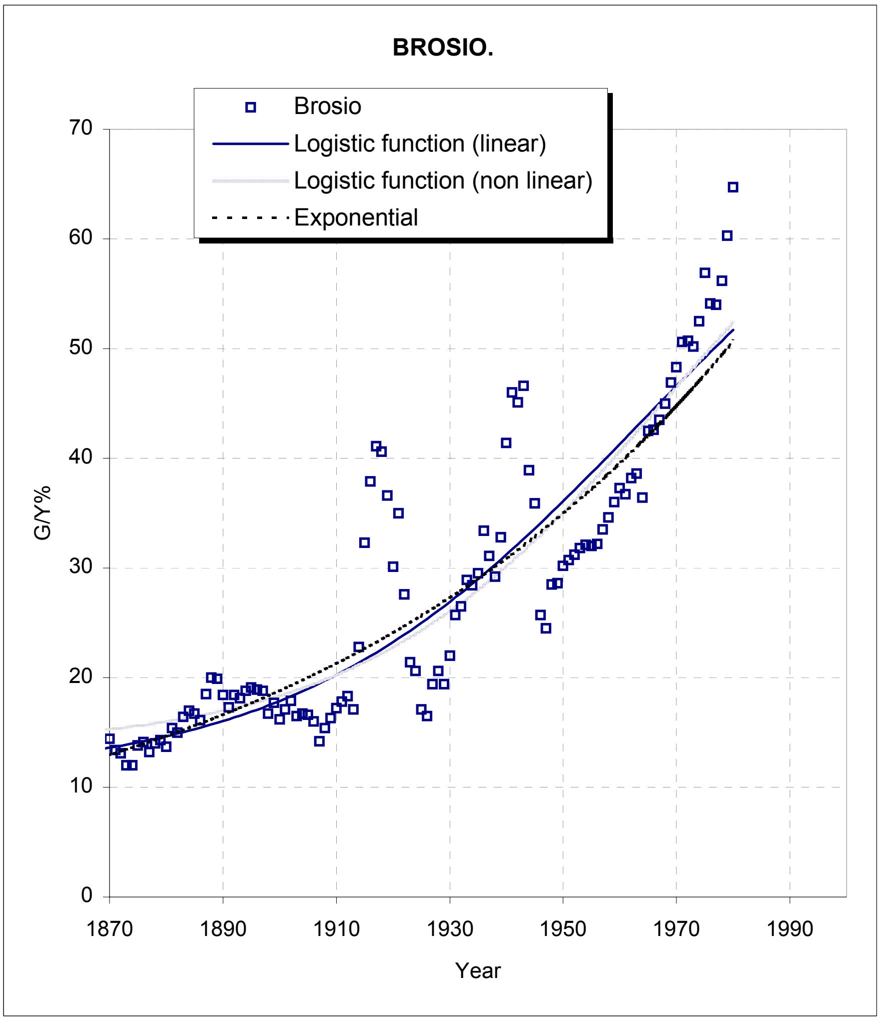
Fig.2 France. Logistic and exponential functions - Delorme, Flora, Maddison G/Y% data.



c) Italy

We considered here a number of sources, but we report results for just one author: Brosio (1986). According to this author, G/Y was around 14% in 1870, thus a starting point higher than France. The doubling time was around 60 years (in the early '30s), than G/Y doubled again at the end of the '70s. The fit of the logistic is marginally better than the exponential, and the "extrapolation test" is less clear cut than in the previous cases. In the case of Italy data may support either an exponential process or a logistic one: if we accept the second one, it is interesting to remark that the flex point year is in the '60s, around 20 years later than France or US. However, the *h* coefficient is just 0.03, half than the US and much less than France; Italy seems to be more a late comer, starting from very high level of government expenditures at its birth as a state, than a big spender.

Fig.3 Italy. Logistic and exponential functions - Brosio G/Y% data.



Tab 3a. Italy. Output regression summary – Brosio data

	Logistic linear estimate
	BROSIO
R_0	10.800
K	64.000
R Square	0.819
Adjusted R Square	0.817
Standard Error	0.523
Observations	115
Constant:	
Coefficients (a)	3.21569
Standard Error	0.09688
Stat t	33.19155
Signif. T	0.00000
Lower 95%	3.02374
Upper 95%	3.40763
Variable t:	
Coefficients (b)	-0.03321
Standard Error	0.00147
Stat t	-22.61028
Signif. T	0.00000
Lower 95%	-0.03612
Upper 95%	-0.03030
C	24.92037
Flex point	96.83138
Flex Year	1962/63
	Logistic non linear estimate
	BROSIO
R_0	13.6727
K	64.0000
C	44.7607
h	0.0370
R Square	0.8038
Flex Year	1969
	Exponential function
	BROSIO
R^2	0.8175
c	1E-09
b	0.0124

Note: Exponential Equation: $Y=ce^{bx}$; c,b are constant

d) Germany

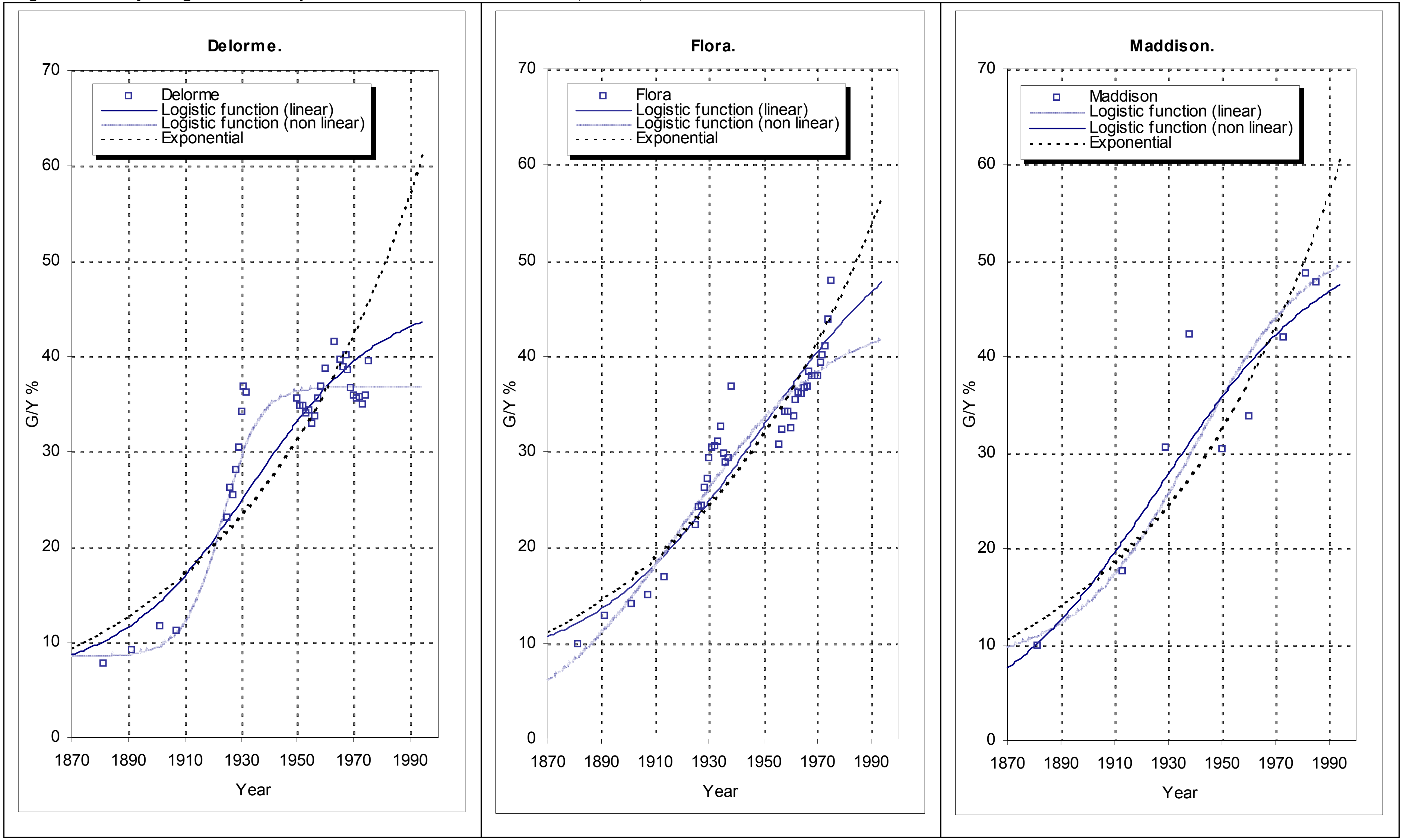
The sources we have selected are Maddison, Flora, Delorme. The number of observations for the first author are just 9, but they are consistent more or less with the other authors. Flora has 39 years, Delorme 34, but these two authors stop their series in 1974, while the last year for Maddison is 1985.

Tab 4a. Germany. Output regression summary - Delorme, Flora, Maddison data.

		Logistic estimate		
		DELORME	**FLORA**	**MADDISON**
R_0		6.300	7.000	8.000
K		40.000	50.000	45.000
R Square		0.791	0.875	0.871
Adjusted R Square		0.785	0.871	0.853
Standard Error		0.568	0.306	0.641
Observations		34	39	9
Constant:	Coefficients (a)	2.74862	2.52494	3.16292
	Standard Error	0.31697	0.15796	0.54470
	Stat t	8.67160	15.98516	5.80671
	Signif. T	0.00000	0.00000	0.00066
	Lower 95%	2.10298	2.20489	1.87491
	Upper 95%	3.39427	2.84498	4.45094
Variable t:	Coefficients (b)	-0.04331	-0.03227	-0.04559
	Standard Error	0.00393	0.00201	0.00663
	Stat t	-11.00917	-16.05810	-6.87493
	Signif. T	0.00000	0.00000	0.00024
	Lower 95%	-0.05132	-0.03634	-0.06127
	Upper 95%	-0.03530	-0.02820	-0.02991
C		15.62112	12.49010	23.63958
Flex point		63.46598	78.25103	69.38127
Flex Year		1933	1948	1939
		Logistic non linear estimate		
		DELORME	**FLORA**	**MADDISON**
R_0		8.5810	0.0000	0.0000
K		28.3312	44.5866	53.3418
C		449.6581	4.0164	4.1667
h		0.1489	0.0365	0.0314
R Square		0.9289	0.8934	0.8434
Flex Year		1921	1918	1925
		Exponential estimate		
		DELORME	**FLORA**	**MADDISON**
R^2		0.7657	0.8658	0.8506
c		5E-12	2E-10	4E-10
b		0.0151	0.0131	0.0141

Note: Exponential Equation: $Y=ce^{bx}$; c,b are constant

Fig.4 Germany. Logistic and exponential functions – Delorme, Flora, Maddison G/Y% data.



There are also some surprising differences for individual years. The history these authors tell is the following one: G/Y in Germany was between 8-10% in 1881 (thus less than Italy or France, but more than in the US). In the early '20s the ratio doubled (in 50 years), then doubled again in the early '70s (again in 50 years). In the following decades the increase was limited. A logistic interpretation of this story is marginally better than an exponential one in all three cases in terms of overall fit, however the extrapolation test to year 2000 with an exponential implies that G/Y would exceed 60%, while the logistic is closer to observation (around 50%) for Maddison and Flora, while it underscores the target with Delorme data. The range of estimates for *h* is between 0.03 and 0.04. Again this is less than for the US, and in between the corresponding values for Italy and France.

e) United Kingdom

The sources we have selected for the United Kingdom are Maddison (11 observations) and Middleton (23 observations). We have considered also Peacock, Galloway, Flora. Peacock data (45 years) stop in year 1955, and contain a high number of war years. As a result that series is of limited use. The history of G/Y in the UK that emerges from our sources is the following one: between 1870-1895 the ratio was between 9-10%, half way the case of the US and the case of Germany. The increase was modest until the first world war (in 1913 G/Y was still around 12%). However in the '20s the ratio doubled (offering some superficial justification to the well known Peacock-Wiseman argument), and it doubled again in the following 50 years, through the '70s. Then the increase was limited.

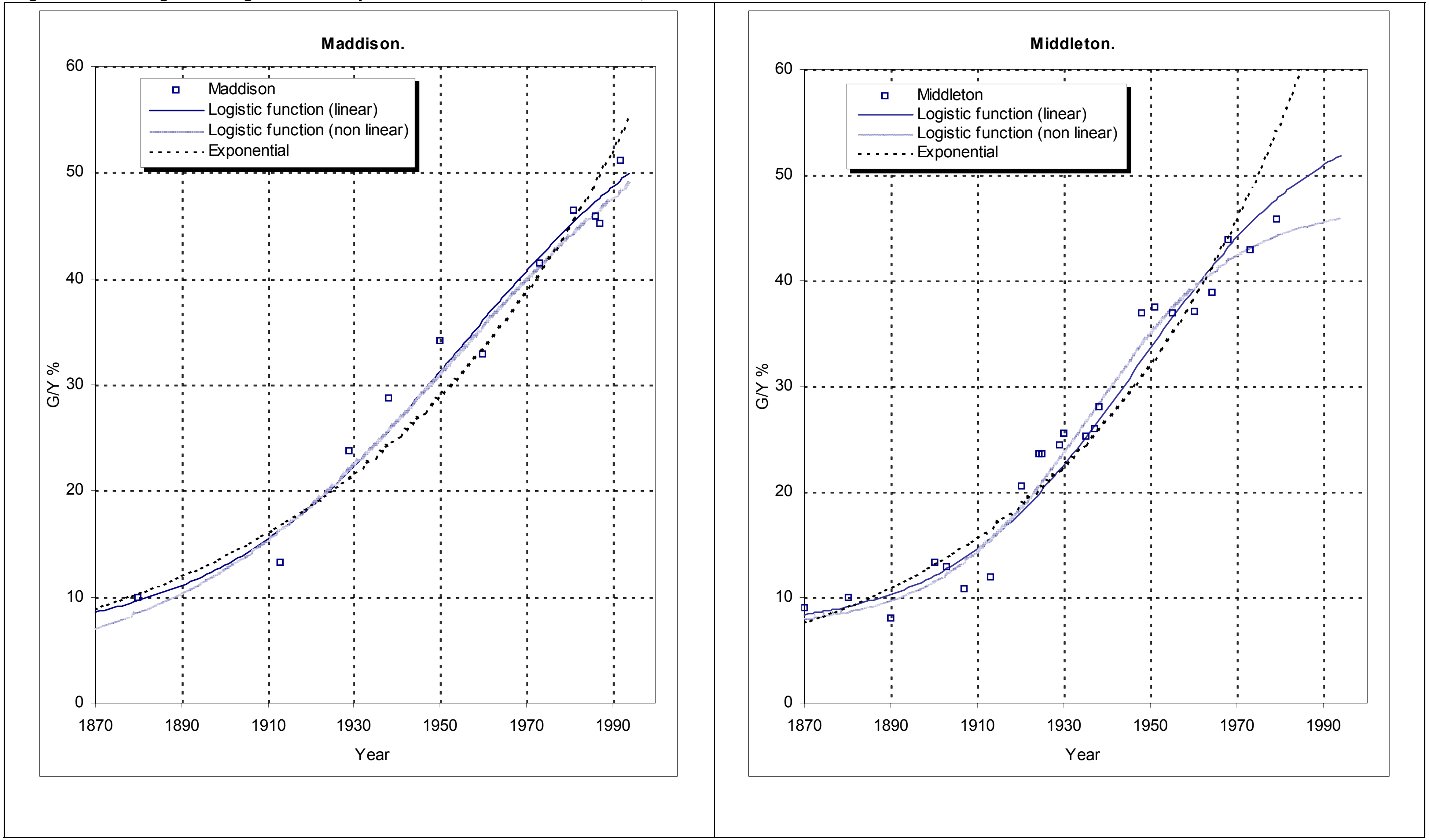
The overall fit of a logistic is better than an exponential with Maddison data (both linear and non linear estimation), but equal with the Middleton data in the case of linear regression. However, the extrapolation with an exponential to year 2000 largely overshoots the target, while the logistic is much nearer to it. The flex point for a logistic process is 1954 for Maddison and 1947 for Middleton, and the *h* coefficients are between 0.03 and 0.058.

60Tab 5a. United Kingdom. Output regression summary –Maddison, Middleton data.

	Logistic estimate	
	MADDISON	**MIDDLETON**
R_0	5.900	7.000
K	55.000	50.000
R Square	0.970	0.922
Adjusted R Square	0.967	0.919
Standard Error	0.231	0.410
Observations	11	23
Constant:		
Coefficients (a)	2.96807	3.57447
Standard Error	0.18481	0.19665
Stat t	16.06011	18.17667
Signif. T	0.00000	0.00000
Lower 95%	2.55000	3.16551
Upper 95%	3.38614	3.98343
Variable t:		
Coefficients (b)	-0.03512	-0.04631
Standard Error	0.00205	0.00293
Stat t	-17.14027	-15.79550
Signif. T	0.00000	0.00000
Lower 95%	-0.03975	-0.05241
Upper 95%	-0.03048	-0.04021
C	19.45440	35.67557
Flex point	84.52350	77.18234
Flex Year	1954/55	1947
	Logistic non linear estimate	
	MADDISON	**MIDDLETON**
R_0	2.6283	7.1964
K	60.1821	40.0272
C	8.9246	47.6568
h	0.0299	0.0585
R Square	0.9738	0.9748
Flex Year	1952	1931
	Exponential function	
	MADDISON	**MIDDLETON**
R^2	0.9251	0.9229
c	2E-18	2E-14
b	0.0224	0.018

Note: Exponential Equation: $Y=ce^{bx}$; c,b are constant

Fig.5 United Kingdom. Logistic and exponential functions - Maddison, Middleton G/Y% data.



7. Conclusions

This paper has offered a theory of the growth of government consistent with observed facts. We suggested that a very simple dynamic process arises by the combination of Wagner Law and the aggregate excess burden of taxation. The theory predicts a S-shaped dynamics, and this is what we observe in several countries in last 100-150 years.

In the table 6 we present some summary results of our estimations. The logistic curve fits well with data in all the five cases here reported. The range of value of the time coefficient H for the five countries is between 0.036 and 0.067 and also the US show a sustained increase of the public expenditure in spite of the lower level of G/Y.

Tab.6 Selected Results summary

Country	Author	Parameter				Flex year
		R_0	K	C	H	
United States	Musgrave	4.1968	38.7136	19.0034	0.0515	1946
France	Delorme	11.0000	47.1319	103.5061	0.0669	1941
Italy	Brosio	13.6727	64.0000	44.7961	0.0370	1969
Germany	Flora	0.0000	44.5866	4.0164	0.0365	1918
United Kingdom	Middleton	7.1964	40.0272	47.6568	0.0585	1931

The only country for which the growth process of public expenditures is more similar to an exponential process than to a logistic one is Italy: however this can be seen rather as a confirmation of our theory, because Italy recurred much more to public debt to finance public expenditures than the other four countries.

A possible first extension of our research framework may include a more complex dynamic model where GNP, public spending and population growth are integrated.

A second opportunity for further research is a more detailed analysis of public spending trends, possibly by functions. The framework of analysis we have proposed seems to be a convenient one to look at public spending trends as dynamic trajectories generated by a small number of explanatory variables.

Bibliography

André C., Delorme R., "L'Évolution de longue Période des Dépenses Publiques en France: 1872-1971" in Recktenwald H.C. (ed.), *Tendances à long terme du secteur public*, XXXIIe Session, Congrès d'Edimbourg 1976, Éditions Cujas, Paris 1978.

André C., Delorme R., *Materiaux pour une comparaison internationale des dépenses publiques en longue période: le cas de six pays industrialisés*, in Statistiques & Etudes Financières, Ministère de l'Economie, des Finances et du Budget, Paris 1984.

Auerbach , Feldstein (Eds), *Handbook of Public Economy,* vol I, NHPC Amsterdam 1990

Baumol W., "The Macroeconomics of Unbalanced Growth" in *American Economic Review*, june 1967, pp. 415-26.

Baumol W., Baty Blackman S.E., Wolf E., "Unbalanced Growth Revisited. Asymptotic Stagnancy and New Evidence" in *American Economic Review*, September 1985, pp.806-17.

Beck M., *Government Spending. Trends and Issues*, Praeger Publishers, New York, 1981.

Borcherding Thomas E., "One Hundred Years of Public Spending, 1870-1970" in Borcherding (eds), *Budgets and Bureaucrats: The Sources of Government Growth*, Duke University Press, Durham North Carolina, pp.19-44, 1977.

Borcherding Thomas E., "The Sources of Growth of Public Expenditures in the United States, 1902-1970", in Borcherding (eds), *Budgets and Bureaucrats: The Sources of Government Growth*, Duke University Press, Durham North Carolina, pp. 45-70, 1977.

Brosio G., *Economia e finanza pubblica*, NIS, Roma, 1993.

Brosio G., Marchese C., *Il potere di spendere. Storia ed economia della spesa pubblica dall'unificazione ad oggi*, Il Mulino, Bologna, 1986.

Delorme R, André Ch., *Étude comparative de l'évolution des dépenses publiques en longue période dans six pays industrialisés occidentaux*, CEPREMAP, Paris, 1979.

Delorme R, André Ch., *L'État et l'économie. Un essai d'explication de l'évolution des dépenses publiques en France (1870-1980)*, Édition du Seuil, Paris, 1983.

Ercolani Paolo, "Documentazione statistica di base" in Fuà (a cura di) *Lo sviluppo economico in Italia*, Vol. III-Studi di settore e documentazione di base, Franco Angeli Editore, Milano, 1969.

Feinstein C.H.*, National Income, Expenditure and Output of the United Kingdom, 1855-1965*,Cambridge University Press, 1972.

Feldstein, "How big should government be" in *National Tax Journal,* Vol. XLX n°1, 1997.

Flora Peter (et al.), *State Economy and Society in Western Europe 1815-1975, Volume I: The Growth of Mass Democracies and Welfare States*, Campus Verlag, Frankfurt, 1983.

Florio M., "Stime della domanda di spesa pubblica con variabili di struttura sociale: applicazione all'Italia dell'approccio del Fmi" in Bernardi L. (a cura di) *Analisi e modelli per la gestione della finanza pubblica*, Il Mulino, Bologna, 199.

Fölster S., Henrekson M., "Growth Effects of Government Expenditure and Taxation in Rich Countries" in *European Economic Review*, vol.45, no.8, 2001, pp.1501-1520

Fratianni M., Spinelli F. "The growth of government in Italy: Evidence from 1861 to 1979" in *Public Choice*, pp.221-243

Gupta S., "Public Expenditure and Economic Growth. A Time-series Analysis" in *Public Finance/Finances Publiques*, Vol. XXII, No 4 1967.

Henrekson M., *An Economic Analysis of Swedish Government Expenditure*, FIEF Trade Union Institute for Economic Research, Gothenburg, 1990.

Henrekson M., "Wagner's Law – A Spurious Relationship?" In *Public Finance*, vol.48, no.3, 1993, pp.406-415

Hicks U.K., *British Public Finances. Their structure and development, 1880-1952*, Geoffrey Cumberlege, Oxford University Press, London 1954.

Hoffmann W.G., *Das Wachstum der deutschen Wirtschaft seit der Mitte des 19. Jahrhunderts,* Springer Verlag, Berlin, 1965.

Liesner T., *One Hundred Years of Economic statistics*, The Economist Publications, London, 1989.

Luzzati E., Portesi R., "La spesa pubblica" in AAVV, *Storia della società italiana dall'unità ad oggi*, Vol. 9: Cassese S. *L'amministrazione centrale*, UTET, Torino, 1984.

Lybeck J.A., *The Growth of Government in Developed Economies*, Gower Published Company Limited , Aldershot England 1986.

Maddison A., *Origins and impact of the welfare state 1883-1983*, Banca Nazionale del lavoro, Quarterly Review, March 1984.

Maddison A., *The World Economy in the 20th Century*, OECD, Paris, 1989.

Maddison A., *Dynamic Forces in Capitalist Development*, Oxford University Press, 1991.

Maddison A., *Monitoring the World Economy, 1820-1992*, OECD, Paris, 1995.

Matthews R.C.O., Feinstein C.H. and Odling-Smee J.C.*, British Economic Growth 1856-1973*, Clarendon Press, Oxford 1982.

Middleton R., *Government versus the Market. The growth of the public sector, economic management and British economic performance, c. 1890-1979*, Edward Elgar Published Limited, Cheltenham UK, 1996.

Mitchell B. R., *International Historical Statistics, Europe 1750-1988*, Abridged Edition, Sijthoff & Noordhoff, 1992.

Mitchell B. R., *International Historical Statistics, The Americas 1750-1988*, 2nd ed. New York Stockton Press, 1993.

Musgrave R. A., *Fiscal Systems*, New Haven and London Yale University Press, 1969.

Musgrave R. A., "Teoria delle crisi fiscali. Un saggio di sociologia fiscale" in Musgrave R., *Finanza pubblica, equità, democrazia*, Bologna, Il Mulino, 1995.

Musgrave R.A., Culbertson J.M., "The growth of public expenditures in the United States, 1890-1948" in *National Tax Journal* Vol. VI, n°2 June 1953.

Nelder J.A., "The fitting of a generalisation of the logistic curve" in *Biometrics*, March 1961, pp.89-110

Oliver F.R., "Methods of Estimating the Logistic Growth Function", *Applied Statistics,* Vol. 13, Issue 2 (1964), pp.57-66.

Oliver F.R., "Aspects of maximum likelihood estimation of the logistic growth function" in *American Statistical Association Journal*, September 1966, pp.697-705.

Oliver F.R., "Another generalisation of the logistic growth function" in *Econometrica*, Vol. 37, N°1, January 1969, pp144-47.

Oliver F.R., "Notes on the logistic curve for human populations" in *Journal of Royal Statistical society A 145*, Part 3, pp. 359-363, 1982.

Peacock A., Scott A., "The curious attraction of Wagner's Law", *Public choice*, vol 102 (1-2), p. 117, January 2000.

Peacock A., Wiseman J., *The Growth of Public Expenditure in the United Kingdom*, Princeton University Press, Princeton 1961.

Pearl R., Reed L.J., "On the rate of growth of the population of the United States since 1790 and its mathematical representation", 1920 in Smith D., Keyfitz N., *Mathematical Demography*. Selected Papers, Springer Verlag, 1977.0

Pigou A.C., *A Study in Public Finance*, MacMillan, London, 1947.

Ratkowsky D.A., *Handbook of non-linear regression models*, Marcel Dekker Inc., New York and Basel, 1993.

Ratkowsky D.A., Reedy T.J., "Choosing near-linear parameters in the four-parameter logistic model for radioligand and related Assays" in *Biometrics*, Vol. 42, September 1986, pp.575-582.

Recktenwald H.C. (ed.), "The public sector in transition", in Recktenwald H.C. (ed.), *Tendances à long terme du secteur public (Secular Trends of the Public Sector)*, XXXIIe Session, Congrès d'Edimbourg 1976, Éditions Cujas, Paris 1978.

Repaci F. A., *La finanza pubblica italiana nel secolo 1861-1960*, Zanichelli Editore, Bologna, 1962.

Tanzi V., Schuknecht L., *Public Spending in the 20th Century. A global Perspective,* Cambridge University Press, 2000.

US Bureau of Census, *Historical Statistics of the United States, Colonial Times to 1970*, Washington DC, US Government Printing Office, 1975.

Verhulst P., "A note on the law of population growth", 1838 in Smith D., Keyfitz N., *Mathematical Demography. Selected Papers*, Springer Verlag, 1977

Veverka J., "The Growth of Government Expenditure in the United Kingdom since 1790", in *Scottish Journal of Political Economy*, Vol. 10 (1), 1963.

Volterra V., *Theory of Functionals and of Integro-differential Equations*, Blackie & Son Limited, London and Glasgow, 1931.

Wagner A., *Grundlegung der Politischen Ökonomie,*Leipzig, 1893

Annex

Tab. A1 United States. Total Government Expenditure as a Percentage of GDP

Year	Delorme	Maddison	Musgrave	US Stat	Year	Delorme	Maddison	Musgrave	US Stat
1890			7.1		*1960*	27.8	27.9	28.2	30.0
1902	6.8		7.9	7.7	*1961*				31.7
1913	8	8	8.5	8.1	*1962*			33.2	31.5
1922	11.1		12.6	12.5	*1963*				31.3
1927	10.3		11.7	11.8	*1964*				31.1
1928					*1965*	28			30.0
1929		10			*1966*				30.0
1930					*1967*				32.5
1931					*1968*				32.7
1932	18.5		21.3	21.4	*1969*				33.1
1933					*1970*	32.2		34.2	34.1
1934	16.5			19.7	*1971*				
1935					*1972*				
1936	17.4			20.3	*1973*		31.1		
1937					*1974*				
1938	17.8	19.8		20.9	*1975*	35.4			
1939		18.5			*1976*				
1940			22.2	20.5	*1977*			36.5	
1942				28.9	*1978*				
1944				52.3	*1979*				
1946				38.2	*1980*	33.2			
1948			23	21.4	*1981*		34.4		
1949					*1982*				
1950	22	21.4	24.6	24.7	*1983*				
1951					*1984*				
1952	26.4			28.9	*1985*				
1953	27.6			30.2	*1986*		37.1		
1954	27.9			30.5	*1987*		37.0		
1955	25.3			27.8	*1988*				
1956	25.1			27.6	*1989*				
1957	25.8		28.5	28.4	*1990*				
1958				30.2	*1991*				
1959				30.1	*1992*		38.5		

Sources

Delorme , André, 1979. Maddison A*.,* 1984, 1989, 1991, 1995. Musgrave R.A., 1953, 1969, 1995.

Notes:

(1) Public expenditure as percentage of GNP

(2) Public expenditure as percentage of GDP

(3) Public expenditure as percentage of GNP

Tab. A2 France. Total Government Expenditure as a Percentage of GDP

Year	**Delorme**[1]	**Flora**[2]	**Maddison**[3]	Year	**Delorme**	**Flora**	**Maddison**
1870				1947	40.9	37.2	
1872	11.0	11.6		1948	41.1		
1880	14.6	15.4	11.2	1949	38.6		
1890	14.3	15.0		1950	41.3		27.6
1900	14.4	15.2		1951	41.1		
1903	14.2	14.6		1952	44.3		
1906	14.8	14.6		1953	48.8	39.1	
1909	15.0	14.4		1954	49.8		
1912	12.6	12.0		1955	50.6		
1913			8.9	1956	51.7	40.8	
1920	32.8	34.2		1957	51.8		
1921	31.1			1958	50.2		
1922	35.9			1959	49.4	39.2	
1923	29.4	29.3		1960	47.7		33.9
1924	24.6			1961	48.7		
1925	22.6			1962	48.7	39.4	
1926	20.6	21.9		1963	50.5		
1927	20.8			1964	48.8		
1928	20.1			1965	48.9	38.3	
1929	18.7	22.1	12.4	1966	49.4		
1930	21.9			1967	50.2		
1931	22.9			1968	50.0	39.1	
1932	26.6	31.4		1969	49.8		
1933	26.9			1970	49.3		
1934	26.9			1971	49.8	38.7	
1935	30.5	35.4		1973			38.8
1936	30.4			1980			
1937	26.2			1981			48.7
1938	26.5	29.2	23.2	1985			53.6
				1988			
				1990			51.0
				1994			

Sources:

Delorme R., André C., 1979. Flora Peter (et al.), 1983. Maddison A., 1984, 1991, 1995.

Notes

(1) Constant prices 1938.

(2) From 1872 to 1912 the figure are in percentage of NDP.

(3) We linked the series at current prices quoted in Maddison (1984, 1989, 1991, 1995), taking the most up-to-date figure for the years with various estimates. The sources quoted by Maddison are: for numerator L. Fontvieille, *Evolution et croissance de l'Etat Français 1815-1969*, ISMEA, Paris, 1976; for denominator J.C. Toutain, *Le Produit intérieur brut de la France de 1789 à 1962,* ISMEA, Paris, 1987. From 1950, figures are from OECD, *National Account*, various issues.

Tab.A3 Italy. Total Government Expenditure as a Percentage of GDP

Year	Brosio[1]	Year	Brosio[1]	Year	Brosio[1]
1866	16.7	*1909*	16.3	*1952*	31.2
1867	13.6	*1910*	17.2	*1953*	31.8
1868	13.0	*1911*	17.8	*1954*	32.1
1869	13.5	*1912*	18.3	*1955*	32.0
1870	14.4	*1913*	17.1	*1956*	32.2
1871	13.4	*1914*	22.8	*1957*	33.5
1872	13.1	*1915*	32.3	*1958*	34.6
1873	12.0	*1916*	37.9	*1959*	36.0
1874	12.0	*1917*	41.1	*1960*	37.3
1875	13.8	*1918*	40.6	*1961*	36.7
1876	14.1	*1919*	36.6	*1962*	38.2
1877	13.2	*1920*	30.1	*1963*	38.6
1878	14.0	*1921*	35.0	*1964*	36.4
1879	14.3	*1922*	27.6	*1965*	42.5
1880	13.7	*1923*	21.4	*1966*	42.6
1881	15.4	*1924*	20.6	*1967*	43.5
1882	15.0	*1925*	17.1	*1968*	45.0
1883	16.4	*1926*	16.5	*1969*	46.9
1884	17.0	*1927*	19.4	*1970*	48.3
1885	16.7	*1928*	20.6	*1971*	50.6
1886	16.1	*1929*	19.4	*1972*	50.7
1887	18.5	*1930*	22.0	*1973*	50.2
1888	20.0	*1931*	25.7	*1974*	52.5
1889	19.9	*1932*	26.5	*1975*	56.9
1890	18.4	*1933*	28.9	*1976*	54.1
1891	17.3	*1934*	28.4	*1977*	54.0
1892	18.4	*1935*	29.5	*1978*	56.2
1893	18.1	*1936*	33.4	*1979*	60.3
1894	18.8	*1937*	31.1	*1980*	64.7
1895	19.1	*1938*	29.2	*1981*	
1896	18.9	*1939*	32.8	*1982*	
1897	18.8	*1940*	41.4	*1983*	
1898	16.7	*1941*	46.0	*1984*	
1899	17.7	*1942*	45.1	*1985*	
1900	16.2	*1943*	46.6	*1986*	
1901	17.1	*1944*	38.9	*1987*	
1902	17.9	*1945*	35.9	*1988*	
1903	16.5	*1946*	25.7	*1989*	
1904	16.7	*1947*	24.5	*1990*	
1905	16.6	*1948*	28.5	*1991*	
1906	16.0	*1949*	28.6	*1992*	
1907	14.2	*1950*	30.2	*1993*	
1908	15.4	*1951*	30.7	*1994*	

Source:

Brosio G., Marchese C., 1986.

Notes:

(1) Public expenditure as percentage of GDP at factor cost

Tab. A4 Germany. Total Government Expenditure as a Percentage of GDP

Year	Delorme[1]	Flora[2]	Maddison[3]	Year	Delorme	Flora	Maddison
1880				1930		29.4	34.2
1881	7.8	9.9	10.0	1931		30.5	36.8
1882				1932		30.7	36.2
1883				1933		31.1	
1884				1934		32.7	
1885				1935		29.8	
1886				1936		28.9	
1887				1937		29.4	
1888				1938	42.4	36.9	
1889				1950	35.7		30.4
1890				1951	34.8		
1891	9.2	12.9		1952	34.9		
1892				1953	34.1		
1893				1954	34.4		
1894				1955	32.9		
1895				1956	33.8	30.8	
1896				1957	35.6	32.4	
1897				1958	36.9	34.3	
1898				1959		34.3	
1899				1960	38.8	32.5	33.9
1900				1961		33.7	
1901	11.7	14.2		1962		35.5	
1902				1963	41.6	36.2	
1903				1964		36.1	
1904				1965	39.7	36.7	
1905				1966	38.9	36.8	
1906				1967	40.2	38.5	
1907	11.3	15.1		1968	38.6	37.9	
1908				1969	36.7	37.9	
1909				1970	35.9	38.0	
1910				1971	35.6	39.3	
1911				1972	35.8	40.2	
1912				1973	35.0	41.0	42.0
1913		17.0	17.7	1974	35.9	43.9	
1924				1975	39.6	47.9	
1925		22.4	23.1	1981			48.7
1926		24.3	26.3	1982			
1927		24.5	25.5	1983			
1928		26.3	28.2	1984			
1929	30.6	27.3	30.5	1985			47.8

Sources: Delorme R., André C., 1979. Flora Peter (et al.), 1983. Maddison A., 1984, 1989, 1995.

Notes: (1) The figures are in percentage of NNP (2) The figures are in percentage of GDP.

(3) We linked the series at current prices quoted in Maddison (1984, 1989, 1991, 1995), taking the most up-to-date figure for the years with various estimates.

Tab. A5 United Kingdom. Total Government Expenditure as a Percentage of GDP

Year	Maddison	Middleton	Year	Maddison	Middleton	Year	Maddison	Middleton
1870		9.0	1911			1952		
1871			1912			1953		
1872			1913	13.3	11.9	1954		
1873			1914			1955		37.0
1874			1915			1956		
1875			1916			1957		
1876			1917			1958		
1877			1918			1959		
1878			1919			1960	32.9	37.1
1879			1920		20.5	1961		
1880	9.9	10.0	1921			1962		
1881			1922			1963		
1882			1923			1964		38.9
1883			1924		23.6	1965		
1884			1925		23.6	1966		
1885			1926			1967		
1886			1927			1968		43.9
1887			1928			1969		
1888			1929	23.8	24.5	1970		
1889			1930		25.6	1971		
1890		8.0	1931			1972		
1891			1932			1973	41.5	42.9
1892			1933			1974		
1893			1934			1975		
1894			1935		25.3	1976		
1895			1936			1977		
1896			1937		26.0	1978		
1897			1938	28.8	28.1	1979		45.9
1898			1939			1980		
1899			1940			1981	46.4	
1900		13.3	1941			1982		
1901			1942			1983		
1902			1943			1984		
1903		12.9	1944			1985		
1904			1945			1986	45.9	
1905			1946			1987	45.2	
1906			1947			1988		
1907		10.9	1948		37.0	1989		
1908			1949			1990		
1909			1950	34.2		1991		
1910			1951		37.5	1992	51.2	

Sources:

Maddison A, 1989, 1991, 1995. Middleton R., 1996.