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FEDERALISM, EDUCATION-RELATED PUBLIC GOOD AND GROWTH WHEN AGENTS ARE HETEROGENEOUS

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Federalism, Education-Related Public Good and Growth when Agents are Heterogeneous*

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Abstract

In this paper we use an endogeneous-growth model with human capital and heterogeneous agents to analyse the relationship between fiscal federalism and economic growth. Results show that federalism, which allows education-related public good levels to be tailored on the human capital of heterogeneous agents, increases human capital accumulation. This in turn leads to higher rates of growth. The benefits of federalism are stronger the larger the intra-jurisdiction variance of agents' human capital.

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1 Introduction

More than forty years have passed since economists formally addressed the theory of fiscal federalism. Richard Musgrave, in 1959, gave a definition of fiscal federalism as a system whose purpose "is to permit different groups living in various states to express different preferences for public services; and this inevitably leads to differences in the levels of taxation and public services" (Musgrave, 1959, p. 179). So, the more heterogeneous the federal population, the higher the necessity of decentralization. This argument was underpinned in the seminal works of Tiebout (1956), Olson (1969) and Oates (1972). Following these seminal studies, in the theoretical literature we have seen a huge amount of papers (for a survey see Oates (1999) and Ahmad and Brosio (2006)) which confirm this first insight, that is: with varying tastes and incomes of all citizens in a country, central and uniform provision - as opposed to decentralized provision - may result in a loss of welfare as the distance between the deciders and beneficiaries of the public goods increases. In this paper we intend to explore this strong argument in favour of decentralization in a dynamic setting of economic growth and to this aim we develop an endogenous-growth model with complete heterogeneity across agents in the economy. Only recently economists are trying to identify some potential transmission channels for an impact of fiscal federalism on economic growth[1]. To the best of our knowledge these papers are by Brueckner (2006); Hatfield (2006); Lejour and Verbon (1997); Koethenbuerger and Lockwood (2010). In these last three papers the impact of federalism on economic growth is established via a tax competition mechanism. Instead in Brueckner (2006) the source of economic growth derives from targeting public spending to the preferences of two different groups of individuals who live into two different jurisdictions. As it will be clear shortly our paper is related to Brueckner (2006). It is also worth saying that as far as the decentralization-growth nexus is concerned, the empirical research has been more intense than the theoretical one. However, even in the empirical literature results are mixed, some papers (for instance Davoodi and Zou (1998)) do not find robust evidence of a relationship between decentralization and growth; on the other hand other recent papers (see Thiessen (2003)) have found more robust evidence that fiscal federalism affects growth[2]. In any case what the review of the empirical literature shows is that these conflicting results may be due to the different measures of fiscal decentralization that can be used, as well as samples, estimation methods etc. but, most importantly, it seems that the empirical works are weak in properly testing the decentralization-growth nexus mainly because of a lack in theoretical works that explain clearly why we should expect this relationship to exist. So, in this paper we intend to make a contribution on the theoretical linkage between fiscal decentralization and growth

[1] In a recent survey Feld, Zimmermann and Doring (2007) identify four plausible transmission channels named as 1) the Tiebout thesis; 2) the Market Preservation Thesis; 3) the Structural Change Thesis; 4) the Political Innovation Thesis.

[2] For a survey on the empirical works see Martinez-Vazquez and Mcnab (2003) and Feld, Zimmermann and Doring (2007).

and we build on an endogenous-growth model with overlapping generations and heterogenous agents as far as human capital is concerned within each jurisdiction and across the jurisdictions. Furthermore in our set-up young agents get educated through the provision of a public good, and this enables them to enhance their earning power when they are old. As already said, our paper is related to a model recently published by Brueckner (2006) who points out that the "beneficial outcome (of federalism)...is achieved via sorting individuals into demand-homogeneous jurisdictions, each of which provides a different amount of the public good" (p. 2107). In the paper he shows that federalism, by allowing public good levels to be tailored to suit the different demands of young and old individuals, who live in different jurisdictions, increases the incentive to save. This strong incentive in turn leads to an increase in investment in human capital and as a result to a faster economic growth. But, notice that, in Brueckner's paper, heterogeneity across individuals - and therefore across jurisdictions - is only age-related. In other words heterogeneity is modelled in an *ad hoc* way given that the young individuals and the old individuals live in separate jurisdictions and therefore there are only two jurisdictions. This initial sorting assumption is crucial for the results in Brueckner's paper.

We modify Brueckner's analysis firstly by considering a publicly provided public good which is related to the educational process or accumulation of human capital. For instance let us think of such public good as training programmes which may be chosen by governments to improve the educational process in the economy or stated differently the human capital of a country. Secondly, more realistically, in our model agents are completely heterogeneous to one another according to the different endowment of human capital they may have. At this stage it is also worth recalling that Oates (1993) firstly informally conjectured about the link between decentralization and growth, suggesting that potentially a better targeting of human capital under a decentralized scenario may be growth-enhancing for the economy. In the paper in order to explore this potential link we start firstly analyzing an idealized economy, which is like an extreme Tiebout world (one individual in each jurisdiction). In such economy heterogeneity across individuals coincides with heterogeneity across jurisdictions and the provision of the public good is tailored to suite individual preferences. It follows a first-best provision of an education-related public good in each jurisdiction.

As second step in the analysis, we consider a centralized economy or a unitary system where a common public good level is provided regardless the heterogeneity across individuals. This allows us to compare the performance of the two benchmark cases, an extreme Tiebout world vs a centralized one. Finally, we consider what we call a decentralized economy or a federalist system which may be seen as a more realistic scenario of a world *à la* Tiebout. Stated differently by looking at the federalist system we want to consider a partitioning of the heterogeneous population into a finite number of jurisdictions.

Results show that in the idealized Tiebout world, where education-related public good levels are tailored on the human capital of heterogeneous agents, human capital accumulation is higher and this leads to higher growth, with respect to the centralized economy. Whereas, as for the federalist system, growth rates of each jurisdiction are higher the larger the intra-jurisdiction variance of agents' human capital. Furthermore we demonstrate that the federalist economy grows faster than the centralized economy.

The paper is organized as follows. Section 2 introduces the model. Section 3 describes the Tiebout world. Sections 4 presents the centralized economy. Section 5 analyzes the decentralized world, and Section 7 concludes.

2 The set-up

We set up a model that relies on Brueckner (2006), modified in order to explicitly consider an education-related public good, complete heterogeneity across agents in a small open economy with perfect mobility of capital[3]. The economy consists of overlapping generations (OLG) of a continuum of two-period lived agents with mass equal to 1. There is no population growth.

In the economy there exists a unique non-perishable good, the output good (Y). At any time the output good can be consumed (X), or purchased by the Government. The Government then uses these purchases to provide public good (or services) (Z) to individuals.

At any time t, the following equation holds:

$$Y_t = X_t + Z_t \tag{1}$$

In this economy there exists a unique production factor, human capital (H), which is heterogeneously distributed across individuals. H_t indicates aggregate human capital at time t. Individual variables (lowercase letters) are indexed both by a time subscript (like the aggregate ones), and by an individual (i) subscript and by a superscript, that indicates the agent's generation, or date of birth. Therefore h_{it}^t indicates the human capital of agent i, of generation t, at time t. Human capital heterogeneity implies $h_{it}^t \neq h_{jt}^t \; \forall i \neq j$. Notice that this hypothesis is different from Brueckner (2006) who assumes a young representative agent and an old representative agent. In other words, in Brueckner the young agents are identical to one another, and the old agents also are identical to one another. Therefore heterogeneity across individuals - and across jurisdictions - is only age-related: young agents and old agents live in separate jurisdictions and as a consequence there are only two jurisdictions. In our model instead agents are completely heterogeneous to one another according to the different endowment of human capital they may have. Notice that this set-up, as it will be more clear in what follows, allows us to consider heterogeneity along two dimensions: heterogeneity across individuals and heterogeneity across jurisdictions.

In what follows we focus on the dynamics from period t to period $t+1$. As it will be clearer later, human capital accumulation is endogenous, therefore

[3]The hypothesis of a small open economy allows us to consider the interest rate as exogenous.

complete human capital heterogeneity is an assumption only as far as time t is concerned, whereas agents endogenously differ to one another in human capital from time $t+1$ onwards.

We assume a production function with constant marginal product that is equal to the wage rate (w) for the profit maximization and the zero-profit condition, namely

$$Y_t = wH_t \tag{2}$$

Following Brueckner (2006), we also assume that young individuals fully inherit the human capital of their "old" parents, therefore the intergenerational transmission mechanism of human capital is the following:

$$h_{it+1}^{t+1} = h_{it+1}^{t} \tag{3}$$

This hypothesis is unusual. Actually the literature that studies the intergenerational accumulation/transmission of human capital either assumes a genetic mechanism, that is the young's human capital is equal to his father's one when young (we could define this as a Darwinian transmission mechanism), or assumes a stochastic mechanism, that is Nature extracts each generation's human capital from a time invariant distribution. Our hypothesis may seem Lamarckian at first sight, but captures a cultural transmission mechanism, more precisely the fact that parents' human capital affects the environment where the new born lives and, by this token, affect his human capital.

The public good (z) is education-related and enhances human capital of young individuals. In other words individual human capital technology is human capital intensive and positively depends on the education-related public good:

$$h_{it+1}^{t} = \phi(z_t)\, h_{it}^{t} \tag{4}$$

where $\phi(0) = 1$, indicating that human capital remains constant over the life cycle if no education is undertaken, whereas on the contrary $\phi(z_t) > 1\ \forall z_t > 0$. Moreover we assume diminishing returns from education, that is $\phi'(z_t) > 0$ and $\phi''(z_t) < 0$. This is identical to Brueckner's eq. (1), but for the public good that is education-related. Brueckner himself suggested an extension of his model in order to explicitly consider an education-related public good: "... note that if z were an education related public good, then it would be an argument of ϕ ...Treatment of this case could be a subject for future research." (Brueckner 2006, p. 2109).

Hereafter, in order to study explicit functional forms and have a clear-cut result, we assume

$$h_{it+1}^{t} = \left(1 + (z_t)^{1/2}\right) h_{it}^{t} \tag{5}$$

We assume preferences are captured by the following lifetime utility function[4]:

[4] This function is identical to Yakita (2003).

$$U_i = \ln x_{it}^t + \frac{1}{R}\ln x_{it+1}^t \tag{6}$$

where R is the exogenous[5] gross nominal interest rate, $\frac{1}{R}$ is the discount factor, x_{it}^t is consumption while young and x_{it+1}^t is consumption while old of the i-th agent of generation t.

When young, agents consume, pay the cost of the public good in order to get educated and work. Since preferences do not depend on leisure, agents supply their entire endowment of human capital. Human capital heterogeneity across agents implies income heterogeneity across agents, since individual income is the product of the wage rate times individual human capital (wh_{it}^t).

As in Brueckner (2006) we assume that the cost per-capita per unit of public good is equal to c, with the cost recovered via a head tax (cz_t). Young agent's disposable income is therefore $wh_{it}^t - cz_t$. Furthermore, given the constant exogenous interest rate, each i-th agent at time t has unlimited access to credit (l_{it}). The i-th young agent's budget constraint therefore becomes:

$$x_{it}^t = wh_{it}^t + l_{it} - cz_t \tag{7}$$

When old, agents get return on human capital, work, refund R units of output for any unit of loan and consume, therefore the old agent's budget constraint is:

$$x_{it+1}^t = wh_{it+1}^t - Rl_{it} \tag{8}$$

Substituting eq. (5) into eq. (8) we get:

$$x_{it+1}^t = w\left(1 + (z_t)^{1/2}\right)h_{it}^t - Rl_{it} \tag{9}$$

Solving eq. (7) for l_{it} and substituting it into eq. (9) we get the intertemporal budget constraint (IBC), that is

$$\frac{1}{R}x_{it+1}^t + x_{it}^t = \frac{1}{R}w\left(1 + (z_t)^{1/2}\right)h_{it}^t + \left(wh_{it}^t - cz_t\right) \tag{10}$$

The interpretation of the IBC is straightforward: the present value of lifetime consumption equals the present value of lifetime disposable income.

Therefore the maximization problem becomes:

$$\begin{array}{l} \underset{x_{it}^t, x_{it+1}^t}{Max}\ U_i \\ \text{subject to the IBC:} \\ x_{it+1}^t + R\left(x_{it}^t + cz_t\right) = w\left(1 + (z_t)^{1/2}\right)h_{it}^t + Rwh_{it}^t \end{array} \tag{11}$$

that gets the following solution:

$$x_{it}^{t*} = x_{it+1}^{t*} = \frac{1}{1+R}\left[w\left(1 + (z_t)^{1/2}\right)h_{it}^t + Rwh_{it}^t - Rcz_t\right] \tag{12}$$

[5] The exogeneity of the interest rate relies on the assumption of a small open economy.

In this setting the credit market allows individuals to smooth consumption along lifetime, therefore $x_{it}^{t*} = x_{it+1}^{t*}$. Notice that consumption is a non-monotonic function of z_t. Actually, in order to get educated, individuals need to pay marginal cost equal to c at time t, and once got educated they receive a marginal benefit which is equal to $\frac{\partial wh_{it+1}^t}{\partial z_t} = \frac{1}{2}\frac{wh_{it}^t}{z_t^{1/2}}$ at time $t+1$. Therefore, as long as $z_t < \left(\frac{1}{2}\frac{wh_{it}^t}{Rc}\right)^2$, the discounted value of the marginal benefit exceeds the marginal cost of getting education, as a consequence a higher level of education would increase consumption.

3 The Tiebout world

We now start analyzing our first benchmark case, that is an extreme Tiebout world (one individual in each jurisdiction), which obviously means a sorting of individuals into different jurisdictions, each of which provides a different amount of the education-related public good, according to individuals' different demands[6].

We firstly study the performance of each atomistic jurisdiction, more precisely we study the provision of the education-related public good and the accumulation of human capital and the growth in each jurisdiction. We then pass analyzing the performance of the aggregation of all the atomistic jurisdictions. We refer to this second case as the atomistic economy.

3.1 The atomistic jurisdiction

Let us assume now that the economy is like an extreme Tiebout world (one individual in each jurisdiction). This implies $z_{it} \neq z_{jt}$ for $i \neq j$. Obviously in such economy heterogeneity across individuals coincides with heterogeneity across jurisdictions. We index the public good with the subscript i, since in each i-th jurisdiction, the public good is tailored to solve the i-th agent's optimization problem.

Therefore, as far as each i-th jurisdiction is concerned, the atomistic solutions of the utility maximization problem are eq. (12) rewritten as follows, where the superscript A stands for "atomistic":

$$x_{it}^{tA} = x_{it+1}^{tA} = \frac{1}{1+R}\left[w\left(1+(z_{it})^{1/2}\right)h_{it}^t + Rwh_{it}^t - Rcz_{it}\right] \tag{13}$$

The Government/agent of the atomistic jurisdiction therefore chooses to provide the level of public good that maximizes individual consumption (eq. 13)

The first order condition for a maximum gives the optimal level of public

[6] Recall that in Brueckner the sorting of individuals is instead into two jurisdictions.

good provided in each atomistic jurisdiction:

$$z_{it}^{A}=\left(\frac{1}{2}\frac{wh_{it}^{t}}{Rc}\right)^{2} \tag{14}$$

Inspection of eq. (14) reveals that the provision of public good is optimal when the per-capita marginal cost (c) of education equals the marginal return from education $\left(\frac{1}{R}\frac{wz_{it}^{-1/2}}{2}h_{it}^{t}\right)$ discounted at time t. Moreover, since the marginal return on education increases as the young agents' human capital (h_{it}^{t}) increases, the optimal level of public good is increasing and convex in h_{it}^{t}. This is summarized in the proposition that follows.

Proposition 1 *The higher the level of human capital in a jurisdiction, the higher the provision of the education-related public good tailored to suite the preferences of i-th individual's, who constitutes the jurisdiction itself.*

This is basically the engine of a virtuous circle that, as shown in the following section and summarized in propositions 2 and 3, makes a jurisdiction growing at an increasing rate and, by the same token, allows a jurisdiction with a higher level of human capital to grow faster than another jurisdiction with less human capital.

3.1.1 Dynamics

In order to study the dynamics in the atomistic jurisdiction, let us first focus on figure 1. Panel (a) describes equation (14), that is the atomistic optimal level of public good, increasing and convex in h_{it}^{t}. Panel (b) only transfers z_{it}^{A} from the vertical to the horizontal axes. Both panel (c) and the straight lines in panel (d) sketches eq. (5) as far as the atomistic economy is concerned, that is

$$h_{it+1}^{tA}=\left(1+\left(z_{it}^{A}\right)^{1/2}\right)h_{it}^{t} \tag{15}$$

Therefore h_{it+1}^{t} is linearly increasing in h_{it}^{t} (given z_{it}^{A}) (see the increasing straight lines in panels (d)), and is increasing and concave in z_{it}^{A} (given h_{it}^{t}). This means that human capital technology enhances the young's human capital and there are diminishing returns in education (remind eq. (5)) However, once the public good is optimally chosen, the resulting relation between h_{it+1}^{t} and h_{it}^{t} becomes increasing and convex: the higher h_{it}^{t}, the higher the public good, the higher h_{it+1}^{t}. In other words, thanks to the first-best provision of the public good, increasing returns on human capital arise and agents can escape the trap of diminishing returns.

Formally, from eqq. 5 and 14, we get the individual human capital law of motion:

$$h_{it+1}^{tA}=h_{it}^{t}+\frac{1}{2Rc}w\left(h_{it}^{t}\right)^{2} \tag{16}$$

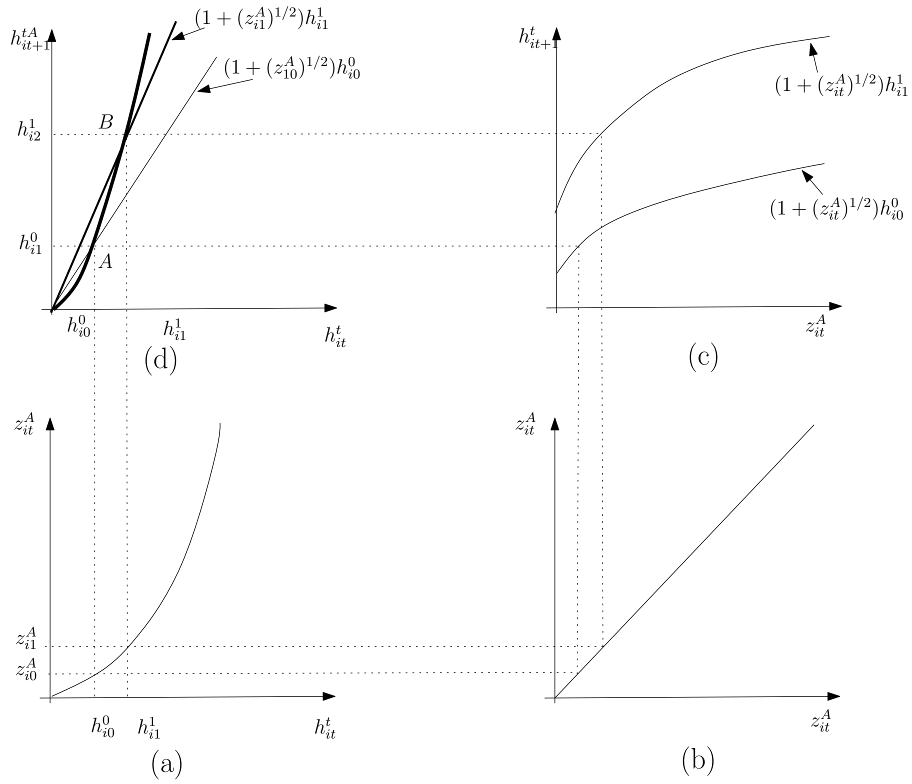

Figure 1: The increasing returns on human capital thanks to the first-best provision of the education-related public good

Note that this is both the individual lifetime accumulation of human capital and the intergenerational accumulation of human capital (remind the intergenerational transmission mechanism $h_{it+1}^{t+1} = h_{it+1}^{t}$). Therefore eq. (16) describes also the dynamics of the human capital of the i-th jurisdiction.

Proposition 2 *An atomistic jurisdiction grows at an increasing rate.*

Proof. From eq. (16), we derive that the rate of growth for a jurisdiction i is increasing in h_{it}^{t}:

$$\gamma_{it+1}^{A} \equiv \frac{h_{it+1}^{t} - h_{it}^{t}}{h_{it}^{t}} = \frac{wh_{it}^{t}}{2Rc} \tag{17}$$

Therefore during the human capital accumulation process, the rate of growth increases. The rationale for this result comes from proposition 1. ■

Proposition 3 *The higher the level of human capital in a jurisdiction, the higher the rate of growth performed by the jurisdiction itself.*

Proof. The proof follows from eq. (17) referred to two different jurisdictions: $\frac{h_{jt+1}^{t}-h_{jt}^{t}}{h_{jt}^{t}} > \frac{h_{it+1}^{t}-h_{it}^{t}}{h_{it}^{t}}$ if and only if $h_{jt}^{t} > h_{it}^{t}$ ■

Summarizing, each jurisdiction would provide an education-related public good in order to suite individual preferences. In other words heterogeneous demands for the education-related public good are completely fulfilled. This efficient provision of the education-related public good allows a jurisdiction to grow at an increasing rate and, by the same token, allows a jurisdiction with a higher level of human capital to grow faster than another jurisdiction with less human capital.

3.2 The atomistic economy

Let us pass now to analyze the performance of the entire economy that we call the atomistic economy, which consists of the aggregation of all the atomistic jurisdictions. Let us denote by $\overline{h}_{t+1}^{t} \left(\equiv \overline{h}_{t+1}^{t+1}\right)$ the average human capital of generation t while old (or equivalently of generation $t+1$ while young), and by $\overline{h}_{t}^{t}$ the average human capital of generation t while young. Taking the mean of eq. (16), we get:

$$\overline{h}_{t+1}^{tA} = \overline{h}_{t}^{t} + \frac{w\left(Var\left(h_{it}^{t}\right) + \left(\overline{h}_{t}^{t}\right)^{2}\right)}{2Rc} \tag{18}$$

Proposition 4 *The aggregate dynamics of an atomistic economy is positively affected both by the average human capital and by the variance of the distribution of human capital.*

This is a standard result in the literature on heterogeneous agents. Differently from the representative agent models, not only the first moment of the distribution of agents matters, but also and most importantly higher moments, namely the variance.

Proposition 5 *The rate of growth of an atomistic economy is higher the higher the variance of the distribution of human capital with respect to its mean.*

Proof. From eq. 18 it follows:

$$\gamma_{t+1}^{A} \equiv \frac{\overline{h}_{t+1}^{tA} - \overline{h}_{t}^{t}}{\overline{h}_{t}^{t}} = \frac{w\left(\frac{Var\left(h_{it}^{t}\right)}{\overline{h}_{t}^{t}} + \overline{h}_{t}^{t}\right)}{2Rc} \tag{19}$$

■

Proposition 6 *An atomistic economy grows at an increasing rate.*

Proof. The proof for this proposition comes straightforward from prop. 2. The aggregation of atomistic jurisdictions, each of which grows at an increasing rate, exhibits an increasing rate of growth itself. ■

Anyway, in order to formally evaluate the dynamics of the rate of growth of the atomistic economy, we need also to investigate the dynamics of the distribution of human capital in the whole economy. Actually the dynamics of the distribution of human capital, and by the same token the dynamics of the distribution of income, is beyond the growth process in our model, since the accumulation of human capital is the engine of growth. In particular the dynamics of the first two moments of the distribution is crucial. This is computed in details in Appendix A where we assume an exogenous uniform distribution of human capital at time t. The results are summarized in Table 1, which confirms unambiguously that, from time t onwards, the atomistic economy exhibits increasing mean and variance. The latter increases faster than the former and as a consequence the economy grows at an increasing rate.

Time	Mean	Variance	Rate of growth
t	0.5	0.08333	
$t+1$	0.8333	0.338	0.666
$t+2$	1.8667	2.8695	1.240
$t+3$			3.4039

Table 1

To sum-up, thanks to the increasing returns on human capital derived on the efficient provision of the education-related public good in each atomistic jurisdiction, the aggregation of atomistic jurisdictions accumulates human capital at an increasing rate.

4 The centralized world

We now pass to analyze another benchmark case, that is a centralized economy. Following much of the literature on fiscal federalism, we assume that in this scenario at any time t a common public good level z_t is provided regardless the heterogeneity across agents, therefore $z_{it} = z_t$ for any i. The level of z_t is chosen according to the average of the demands for the public good across agents, therefore taking the mean of all z_{it}^A (eq. 14), we get

$$z_t = E\left[\left(\frac{wh_{it}^t}{2Rc}\right)^2\right] \tag{20}$$

that is

$$z_t^C = \left(\frac{w}{2Rc}\right)^2 \left(Var\left(h_{it}^t\right) + \left(\overline{h}_t^t\right)^2\right) \tag{21}$$

where the superscript C stands for "centralized"

Proposition 7 *The Government of a centralized economy would supply a level of public good that increases both in the first and in the second moment of the distribution of human capital.*

Proposition 8 *In a representative agent economy with absence of heterogeneity, that is $Var\left(h_{it}^t\right) = 0$, then $z_{it}^A = z_t^C$.*

Proposition 9 *In a centralized economy the provision of the education-related public good is inefficient at the individual level, that is if $\left(h_{it}^t\right)^2 > \left(Var\left(h_{it}^t\right) + \left(\overline{h}_t^t\right)^2\right)$ then $z_{it}^A > z_t^C$ viceversa if $\left(h_{it}^t\right)^2 < \left(Var\left(h_{it}^t\right) + \left(\overline{h}_t^t\right)^2\right)$ then $z_{it}^A < z_t^C$*

Proof. *The proof follows by comparing eq. 14 with eq. 21.* ■

Summarizing, in a centralized economy, which supplies a common education-related public good (averaging individual demands) regardless the heterogeneity of population, the provision of the education-related public good departs from the individual first best and inefficiencies arise.

4.1 Dynamics

Substituting eq. 21 into eq. 5, we get the individual law of motion of human capital in a centralized economy:

$$h_{it+1}^{tC} = \left(1 + \left(\frac{w}{2Rc}\right)\sqrt{Var\left(h_{it}^t\right) + \left(\overline{h}_t^t\right)^2}\right) h_{it}^t \tag{22}$$

Proposition 10 *In a centralized economy the accumulation of individual human capital is inefficient, that is if* $\left(h_{it}^{t}\right)^{2} > \left(Var\left(h_{it}^{t}\right) + \left(\overline{h}_{t}^{t}\right)^{2}\right)$ *then* $h_{it+1}^{tA} > h_{it+1}^{tC}$ *viceversa if* $\left(h_{it}^{t}\right)^{2} < \left(Var\left(h_{it}^{t}\right) + \left(\overline{h}_{t}^{t}\right)^{2}\right)$ *then* $h_{it+1}^{tA} < h_{it+1}^{tC}$
Proof. *The proof follows from proposition 9.* ■

Aggregating, we get the centralized economy accumulation of human capital:

$$\overline{h}_{t+1}^{tC} = \overline{h}_{t}^{t} + \left(\frac{w}{2Rc}\right)\sqrt{Var\left(h_{it}^{t}\right) + \left(\overline{h}_{t}^{t}\right)^{2}}\,\overline{h}_{t}^{t} \tag{23}$$

From eq. 23, we get the centralized economy rate of growth:

$$\gamma_{t+1}^{C} \equiv \frac{\overline{h}_{t+1}^{tC} - \overline{h}_{t}^{t}}{\overline{h}_{t}^{t}} = \left(\frac{w}{2Rc}\right)\sqrt{Var\left(h_{it}^{t}\right) + \left(\overline{h}_{t}^{t}\right)^{2}} \tag{24}$$

First of all notice that the increasing marginal returns on human capital disappears, since the provision of public good is not tailored on individual human capital. In other words, heterogeneous demands for the education related public good are not fulfilled under this scenario.

Proposition 11 *Assuming that at time* t *the centralized and the atomistic economies are perfectly identical to each other in terms of distribution of human capital, the centralized economy accumulates less human capital at time* $t+1$ *and grows at a lower rate than the atomistic economy.*
Proof. *Comparing eq. 18 with eq. 23 and eq. 19 with eq. 24, and assuming that in time* t *the two economies are perfectly identical to each other, since* $\left(Var\left(h_{it}^{t}\right)\right)^{2} + Var\left(h_{it}^{t}\right)\left(\overline{h}_{t}^{t}\right)^{2} > 0$*, it results* $\overline{h}_{t+1}^{tA} > \overline{h}_{t+1}^{tC}$ *and* $\gamma_{t+1}^{A} > \gamma_{t+1}^{C}$*.* ■

Proposition 12 *Proposition 11 holds also from time* $t+1$ *onwards.*

As previously showed (eq. 19), also in this case (eq. 24), it follows that the dynamics of the rate of growth depends on the dynamics of the variance and of the mean of the distribution of human capital As we did for the atomistic economy, we verify proposition 12 assuming an exogenous uniform distribution of human capital at time t. The detailed dynamics is in Appendix B and Table 2 summarizes the results of the numerical example.

Time	Mean	Variance	Rate of growth
t	0.5	0.08333	
$t+1$	0.78863	0.20732	0.57735
$t+2$	1.5067	0.75677	0.91062
$t+3$			1.7398

Table 2

The comparison between Table 1 and Table 2 clarifies that the atomistic economy exhibits a higher growth than the centralized economy. Beyond this result there is the faster dynamics of the first two moments of the distribution of human capital in the atomistic economy.

Summarizing, the centralized economy accumulates less human capital and exhibits a lower rate of growth than an atomistic economy. As already discussed, this is due to the fact that in the atomistic economy the tailor-made provision of education-related public good generates increasing returns on education.

5 The decentralized world

Let us now consider the decentralized economy (federalist system).

What we mean by decentralized economy is a partitioning of the population into a finite number of jurisdictions (J). The reason why we consider this third environment is twofold: firstly this is a more realistic scenario of a world *à la* Tiebout, in other words the Tiebout world is the limit case of the decentralized world. Secondly under this scenario we are able to analyze the effects of heterogeneity which is now both intra-jurisdiction (across agents of the same jurisdiction) and inter-jurisdictions[7]. Actually the Tiebout world allowed us to analyze only inter-jurisdiction heterogeneity whereas the centralized world allowed us to analyze only intra-jurisdiction heterogeneity.

To perform such analysis we first study the provision of the public good and the dynamics of a decentralized jurisdiction. We then pass to consider two jurisdictions which differ to each other in terms of mean and variance. Finally by aggregating the two jurisdictions we can consider the performance of the whole decentralized economy.

5.1 The decentralized jurisdiction

In each jurisdiction at any time t a common public good level z_t is provided. Therefore $z_{it} = z_t^J$ for any $i \in J$. The level of z_t is chosen according to the average of the preferences across agents, therefore taking the mean of eq. 14 as far as each jurisdiction is concerned, we get

$$z_t^J = \left(\frac{w}{2Rc}\right)^2 \left(Var^J\left(h_{it}^t\right) + \left(\overline{h}_t^{tJ}\right)^2\right) \tag{25}$$

Proposition 13 *The Government of a decentralized jurisdiction would supply a level of public good that increases both in the first and in the second moment of the distribution of human capital in each jurisdiction.*

Since in our model there exists complete heterogeneity across agents, this implies that exists heterogeneity even inside each jurisdiction (J), that we call intra-jurisdiction heterogeneity. This is proxied by $Var^J\left(h_{it}^t\right)$.

[7] Notice that the literature on fiscal federalism typically analyzes heterogeneity only along one dimension - that is inter-jurisdictions heterogeneity - for instance a rich versus a poor jurisdiction.

5.1.1 Dynamics

For each jurisdiction, we get the following laws of human capital accumulation:

$$h_{it+1}^{tJ} = \left(1 + \left(\frac{w}{2Rc}\right)\sqrt{Var^J\left(h_{it}^t\right) + \left(\overline{h}_t^{tJ}\right)^2}\right) h_{it}^t \tag{26}$$

Averaging in each jurisdiction:

$$\overline{h}_{t+1}^{tJ} = \left(1 + \left(\frac{w}{2Rc}\right)\sqrt{Var^J\left(h_{it}^t\right) + \left(\overline{h}_t^{tJ}\right)^2}\right) \overline{h}_t^t \tag{27}$$

Therefore each jurisdiction grows at the following rates:

$$\gamma_{t+1}^J \equiv \frac{\overline{h}_{t+1}^{tJ} - \overline{h}_t^t}{\overline{h}_t^t} = \left(\frac{w}{2Rc}\right)\sqrt{Var^J\left(h_{it}^t\right) + \left(\overline{h}_t^{tJ}\right)^2} \tag{28}$$

Proposition 14 *Each jurisdiction J benefits from being in the decentralized economy rather than being in the centralized economy as long as*

$$Var^J\left(h_{it}^t\right) + \left(\overline{h}_t^{tJ}\right)^2 > Var\left(h_{it}^t\right) + \left(\overline{h}_t^t\right)^2 \tag{29}$$

Proof. *The proof comes by comparing eq. 28 with eq. 24.* ■

Summing up, each jurisdiction benefits from being in a decentralized economy, that is it accumulates more human capital and grows at a faster rate than it would have done belonging to a centralized economy, the larger the variance in agents' endowment of human capital.

- **Two jurisdictions: a numerical example**

We now consider a decentralized world consisting of two jurisdictions that we call Γ and Ω. The two jurisdictions differ to each other in terms of the mean of the distribution of human capital. This means that one jurisdiction is richer than the other one [8]. In our example the rich jurisdiction is Ω.

Let us firstly assume that the two jurisdictions (Γ and Ω) have the same variance. We refer to this scenario as Case 1. Secondly we assume that the two jurisdictions (Γ and Ω) have different variances and the poor jurisdiction exhibits higher variance. We refer to this scenario as Case 2.

As we did for the previous scenarios (i.e. the Tiebout world and the centralized world) all the dynamics is computed in Appendix C and the results are summarized in Table 3 and in Table 4.

[8] Notice that, since we aim to make a comparison with the centralized economy, we assume that the population of the of the aggregation of the two jursdictions is exogenously uniformly distributed at time t likewise we did for the centralized economy.

Case 1: (diff. mean, same var.)		Γ (poor)			Ω (rich)	
Time	Mean	Variance	Rate of growth	Mean	Variance	Rate of growth
t	0.25	0.02083		0.75	0.02083 3	
$t+1$	0.32217	0.03459	0.28867	1.3229	0.064818	0.76376
$t+2$	0.44201	0.065126	0.37201	3.1052	0.35713	1.3472
$t+3$			0.51039			3.1622

Table 3

Case 2: (diff. mean, diff. var.)		Γ (poor)			Ω (rich)	
Time	Mean	Variance	Rate of growth	Mean	Variance	Rate of growth
t	0.109 52	0.109 52		0.6	00008333	
$t+1$	0.777 53	0.230 85	0.590 37	0.960	0.002136	0.600 69
$t+2$	1. 488 2	0.845 68	0.914	1. 884	0.008216	0.961 55
$t+3$			1. 749 4			1. 886 2

Table 4

A number of comparison are in order here.

Firstly, comparing Table 2 (see section 4) and Table 3, we can conclude that, if the two jurisdictions were equally heterogenous, individuals belonging to the poor jurisdiction Γ would be better off in a unitary system, whereas individuals belonging to the rich jurisdiction Ω would be better off with decentralization. Actually it easy to verify that eq. (29) holds for jurisdiction Ω but not for jurisdiction Γ.

Secondly, comparing Table 2 (see section 4) and Table 4, we conclude that if the two jurisdictions instead differ to each other both in the average human capital and in the variance of the distribution of human capital and the poor jurisdiction is more heterogeneous than the rich one, individuals belonging to both jurisdictions would be better off with decentralization.

To sum up if in the economy there are two jurisdictions (a poor one and a richer one) with identical intra-jurisdiction heterogeneity (proxied by the variance of the distribution of income), the poorer jurisdiction would benefit, in terms of human capital accumulation and growth, from a centralized provision of the education-related public good (Case1). On the contrary if the poor jurisdiction is more heterogeneous than the rich one in terms of the distribution of income, then both the jurisdictions would benefit from a federalist system (Case 2). Actually Case 2 shows that both jurisdictions would accumulate more human capital if the education-related public good is provided by the local government.

5.2 The decentralized economy

The final step in our analysis is to study the dynamics of the aggregation of the two jurisdictions. Since, in the previous section, we have analyzed two jurisdictions (Γ and Ω) which may differ to each other in terms of mean (Case 1) and in terms of mean and variance (Case 2), we will now need to compare the centralized world with the aggregation of the two jurisdictions both in Case 1 and in Case 2. Thus, referring to the findings of the previous section, taking the mean of both jurisdictions in Case 1, it follows that at time $t+1$, average human capital is:

$$\overline{h}_{t+1}^{tD}(Case\ 1) = 0.822\,54 \tag{30}$$

and at time $t+2$ average human capital is:

$$\overline{h}_{t+2}^{t+1D}(Case\ 1) = 1.\,773\,6 \tag{31}$$

As far as the mean at time $t+1$ of the two jurisdictions in Case 2 is concerned:

$$\overline{h}_{t+1}^{tD}(Case\ 2) = 0.795821 \tag{32}$$

and at time $t+2$ average human capital is:

$$\overline{h}_{t+2}^{t+1D}(Case\ 2) = 1.\,52778 \tag{33}$$

Since, at time t, both the centralized economy and the decentralized economy are identical in terms of average human capital, we can conclude that, both in case 1 and in case 2, the decentralized economy accumulates more human capital than the centralized economy (recall Table 2), that is $\overline{h}_{t+1}^{tD} > \overline{h}_{t+1}^{tC}$ and $\overline{h}_{t+2}^{tD} > \overline{h}_{t+2}^{tC}$. This result, obviously, holds $\forall w, c, R > 0$.

It is worth saying that such results are of interest since decentralization of the public good under our setting, may allow a poor region to grow at higher rates compared to a scenario in which the public good is uniformly provided regardless the heterogeneity across jurisdictions. Moreover our results also suggest that the more heterogeneous is the poor region, the higher is its rate of growth. This could imply that a poor region would catch up a richer one under a federalist system, reducing inter-jurisdiction inequality along the growth path. The analysis of the dynamics of the inter-regional and inter-individual inequality is left for further research.

6 Conclusions

A result of this paper is that the larger the variance in individuals' demands for public goods, the larger the benefits of decentralization tend to be. This result in the literature on fiscal federalism is very well-known. However, we want to emphasize that such benefit or virtue from decentralization has been developed so far in the literature mainly in a static context, but as Oates argued some years ago "the thrust of the argument should also have some validity in a dynamic setting of economic growth. There surely are strong reasons, in principle, to believe that policies formulated for the provision of infrastructure and even human capital that are sensitive to regional or local conditions are likely to be more effective in encouraging economic development than centrally determined policies that ignore these geographical differences. There is, incidentally, no formalized theory of such a relationship between fiscal decentralization and economic growth; it would probably be useful to work through such a theory (in which investment programs are *jurisdiction-specific)* to determine the parameters on which these gains depend and some idea as to orders of magnitude". (Oates, 1993, p. 240).

In this paper therefore we intended to contribute to this new perspective on fiscal federalism and growth and to this aim, we have considered an endogenous-growth model with overlapping generations of two-period lived, heterogeneous agents in a small open economy. We built on Brueckner's work (2006) but, differently from Brueckner, we have assumed: i) a public good that enhances the human capital and ii) complete heterogeneity among all the agents, in other words human capital is heterogeneously distributed across individuals. This set-up allowed us to study the effects of heterogeneity which is then twofold: inter-jurisdictions and intra-jurisdiction. In particular, intra-jurisdiction heterogeneity is proxied by the variance of the intra- jurisdiction distribution of human capital.

More precisely we have considered the following scenarios: i) a world *à la* Tiebout with one individual in each jurisdiction, which obviously implies that heterogeneity across individuals coincides with heterogeneity across jurisdictions (inter-jurisdiction heterogeneity); ii) the unitary system where all agents belong to a unique jurisdiction (intra-jurisdiction heterogeneity), and iii) a federalist system where heterogeneity is both intra-jurisdiction (across agents of the same jurisdiction) and inter-jurisdictions (across jurisdictions).

Results show that in the idealized Tiebout world the education-related public good levels are tailored on the human capital of heterogeneous agents. This is the engine of a virtuous circle that makes a jurisdiction growing at an increasing rate and, by the same token, allows a jurisdiction with a higher level of human capital to grow faster than another jurisdiction with less human capital. Moreover, when we have considered the aggregation of all the atomistic jurisdictions, we have shown that, thanks to the increasing returns on human capital derived on the efficient provision of the education-related public good in each atomistic jurisdiction, such economy accumulates more human capital (and by the same token grows at a faster rate) the higher the variance of the distribution of human capital with respect to its mean. In the other benchmark case, namely the centralized economy (which supplies a common education-related public good regardless the heterogeneity of population), the provision of the education-related public good departs from the individual first best and inefficiencies arise. This result is very similar to the inefficiency that arises in the static analysis of the decentralization theorem when centralized provision of the public good is assumed. To the best of our analysis this is the first paper that shows this result in a dynamic context. Moreover we have shown that the centralized economy accumulates less human capital and exhibits a lower rate of growth than an atomistic economy.

Finally, when we turned our attention to the last scenario (which is the federalist system) where the complete heterogeneity across agents allowed us to emphasize the role of both intra-jurisdiction heterogeneity and inter-jurisdiction heterogeneity. We have shown that growth rates of each jurisdiction are higher the larger the intra-jurisdiction variance of agents' human capital. Furthermore each jurisdiction benefits from being in the decentralized economy rather than being in the centralized economy the larger the intra-jurisdiction variance in agents' endowment of human capital. When we consider the performance of the aggregation of two jurisdictions, results show unambiguously that the federalist economy grows at a faster rate than the centralized economy.

To conclude, over the last decades we all have witnessed a massive diffusion of federal or decentralized arrangements in many countries (both in developed and developing countries) and decentralization has been promoted mainly to improve the allocative efficiency of the economy. Nowadays an important policy issue, which is emerging in the policy agendas of most OECD countries as well as in developing and transitional countries, is whether such decentralized arrangement also may affect economic growth. In this paper we have been able to show that, under a dynamic setting with complete heterogeneity across agents, the targeting of public spending to citizens' different demands for public goods can be an important source of economic growth. In other words, federalism can be seen as an efficiency enhancing and growth generating process.

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Appendix A. Beyond the growth process: the dynamics of the distribution of human capital in the atomistic economy.

Let us assume that, at time t, the economy is composed by a continuum of agents of mass equal to 1, whose human capital is uniformly distributed on the support $(0,1)$. It follows that the density function of human capital at time t $(f_{h_{it}})$ is $f_{h_{it}} = 1$ and the average human capital is $\overline{h}_t^t = \int_0^1 h_{it}^t dh_{it} = 0.5$ and the variance of the distribution is $Var\left(h_{it}^t\right) = \int_0^1 \left(h_{it}^t - 0.5\right)^2 dh_{it} = 8.3333 \times 10^{-2}$.

Let us make the notation simpler defining $h_{it+1}^t = g\left(h_{it}^t\right)$. It is possible to demonstrate that the density function of human capital at time $t+1$ is

$$f_{h_{it+s}} = f_{h_{it+s-1}} \left[g^{-1}\left(h_{it+s}^t\right)\right] \left| \frac{\partial g^{-1}\left(h_{it+s}^t\right)}{\partial h_{it+s}^t} \right| \tag{34}$$

Therefore

$$f_{h_{it+1}}^A = \frac{1}{\sqrt{\frac{2}{Rc} w h_{it+1}^t + 1}} \tag{35}$$

and

$$f_{h_{it+2}}^A = \frac{1}{\sqrt{\frac{2}{Rc} w \left(\frac{Rc}{w}\left(\sqrt{\frac{2}{Rc} w h_{it+2}^{t+1} + 1} - 1\right)\right) + 1}} \frac{1}{\sqrt{\frac{2}{Rc} w h_{it+2}^{t+1} + 1}} \tag{36}$$

Let us assume, for simplicity, $\frac{w}{2Rc} = 1$. It is possible to demonstrate that as far as time $t+1$ is concerned the average human capital[9], the variance of the distribution of human capital[10] and the rate of growth (eq. 19) are respectively:

$$\overline{h}_{t+1}^{tA} = 0.833 \tag{37}$$

$$Var^A\left(h_{it+1}^t\right) = 0.338 \tag{38}$$

$$\gamma_{t+1}^A \equiv 0.666 \tag{39}$$

As far as time $t+2$ is concerned the average human capital[11], the variance of the distribution of human capital[12] and the rate of growth are respectively:

[9] $\overline{h}_{t+1}^{tA} = \int_0^2 h_{it+1}^t \frac{1}{\sqrt{4h_{it+1}^t + 1}} dh_{it+1}$

[10] $Var^A\left(h_{it+1}^t\right) = \int_0^2 \left(h_{it+1}^t - 0.833\,33\right)^2 \frac{1}{\sqrt{4h_{it+1}^t + 1}} dh_{it+1}^t$

[11] $\overline{h}_{t+2}^{t+1A} = \int_0^6 h_{it+1}^t \frac{1}{\sqrt{4\left(\frac{1}{2}\left(\sqrt{4h_{it+1}^t + 1} - 1\right)\right) + 1}} \frac{1}{\sqrt{4h_{it+1}^t + 1}} dh_{it+1}$

[12] $Var^A\left(h_{it+1}^t\right) = \int_0^6 \left(h_{it+1}^t - 0.833\,33\right)^2 \frac{1}{\sqrt{4\left(\frac{1}{2}\left(\sqrt{4h_{it+1}^t + 1} - 1\right)\right) + 1}} \frac{1}{\sqrt{4h_{it+1}^t + 1}} dh_{it+1}^t$

$$\overline{h}_{t+2}^{t+1A} = 1.8667 \quad (40)$$

$$Var^A\left(h_{it+2}^{t+1}\right) = 2.8695 \quad (41)$$

$$\gamma_{t+2}^A \equiv 1.240 \quad (42)$$

From time $t+2$ to time $t+3$ the economy grows at the rate

$$\gamma_{t+3}^A = 3.4039 \quad (43)$$

Appendix B. Beyond the growth process: the dynamics of the distribution of human capital in the centralized world.

Let us assume that at time t the economy is described, as in Appendix A, by a continuum of agents of mass equal to 1, whose human capital is uniformly distributed on the support $(0,1)$. with mean equal to 0.5 and variance equal to 8.3333×10^{-2}.

It is possible to demonstrate that, assuming again for simplicity $\frac{w}{2Rc} = 1$, at time $t+1$ the density function[13] of human capital is

$$f_{h_{it+1}} = 0.63397 \quad (44)$$

Therefore at time $t+1$ human capital is still uniformly distributed on the support $(0, 1.5773)$.

At time $t+2$ the density function[14] of human capital is

$$f_{h_{it+2}} = 0.33183 \quad (45)$$

Therefore at time $t+2$ human capital is still uniformly distributed on the support $(0, 3.0136)$.

It is possible to verify that as far as time $t+1$ is concerned the average human capital[15], the variance of the distribution of human capital[16] and the rate of growth (eq. 24) are respectively:

$$\overline{h}_{t+1}^{tC} = 0.78863 \quad (46)$$

[13] $f_{h_{it+1}} = \frac{1}{\left(1+\left(\frac{w}{2Rc}\right)\sqrt{Var\left(h_{it}^t\right)+\left(\overline{h}_t^t\right)^2}\right)}$

[14] $f_{h_{it+2}} = \frac{1}{\left(1+\left(\frac{w}{2Rc}\right)\sqrt{Var\left(h_{it}^t\right)+\left(\overline{h}_t^t\right)^2}\right)} \frac{1}{\left(1+\left(\frac{w}{2Rc}\right)\sqrt{Var\left(h_{it+1}^{t+1}\right)+\left(\overline{h}_{t+1}^{t+1}\right)^2}\right)}$

[15] $\overline{h}_{t+1}^{tC} = \int_0^{1.5773} h_{it+1}^t 0.63397 dh_{it+1}$

[16] $Var^C\left(h_{it+1}^t\right) = \int_0^{1.5773}\left(h_{it+1}^t - 0.78863\right)^2 0.63397 dh_{it+1}^t$

$$Var^{C}\left(h_{it+1}^{t}\right)=0.207\,32 \tag{47}$$

$$\gamma_{t+1}^{C}\equiv 0.577\,35 \tag{48}$$

As far as time $t+2$ is concerned the average human capital[17], the variance of the distribution of human capital[18] and the rate of growth are respectively:

$$\overline{h}_{t+2}^{t+1C}=1.\,506\,7 \tag{49}$$

$$Var^{C}\left(h_{it+2}^{t+1}\right)=0.756\,77 \tag{50}$$

$$\gamma_{t+2}^{C}\equiv 0.910\,64 \tag{51}$$

From time $t+2$ to time $t+3$ the economy grows at the rate

$$\gamma_{t+3}^{C}=1.\,739\,8 \tag{52}$$

Appendix C. Beyond the growth process: the dynamics of the distribution of human capital in the two jurisdictions of the decentralized world.

- **Case 1: different mean and the same variance**

Let us firstly assume that jurisdiction Γ is composed by a continuum of agents whose human capital is uniformly distributed on the support $(0,0.5)$. Population of jurisdiction Γ has mass equal to 0.5. As for jurisdiction Ω, let us assume that it is composed by a continuum of agents whose human capital is uniformly distributed on the support $(0.5,1)$. Population of jurisdiction Ω has mass equal to 0.5. It follows that: $\overline{h}_{t}^{t\Gamma}=0.25$ and $Var^{\Gamma}\left(h_{it}^{t}\right)=2.\,083\,3\times 10^{-2}$ as far as jurisdiction Γ is concerned, and $\overline{h}_{t}^{t\Omega}=0.75$ and $Var^{\Omega}\left(h_{it}^{t}\right)=2.\,083\,3\times 10^{-2}$ as for jurisdiction Ω.

Therefore eq. (26) becomes:

$$h_{it+1}^{t\Gamma}=1.\,288\,7h_{it}^{t} \tag{53}$$

for jurisdiction Γ

$$h_{it+1}^{t\Omega}=1.\,763\,8h_{it}^{t} \tag{54}$$

for jurisdiction Ω

Let us now analyze the dynamics of the distribution of human capital in the decentralized world.

[17] $\overline{h}_{t+2}^{t+1A}=\int_{0}^{3.\,013\,6}h_{it+1}^{t}0.331\,83dh_{it+1}$

[18] $Var^{A}\left(h_{it+1}^{t}\right)=\int_{0}^{3.\,013\,6}\left(h_{it+1}^{t}-\mathit{1.\,506\,7}\right)^{2}0.331\,83dh_{it+1}^{t}$

Defining α^J the mass of the population in jurisdiction J, it is possible to demonstrate that at time $t+1$ in each jurisdiction J, the density function is $f^J_{h_{it+1}} = \frac{1}{\alpha^J\left(1+\sqrt{Var^J\left(h^t_{it}\right)+\left(\overline{h}^{tJ}_t\right)^2}\right)}$:Therefore:

$$h^{t\Gamma}_{it+1} \sim U\left(0, 0.644\,34\right) \quad f^{\Gamma}_{h_{it+1}} = 1.\,552 \tag{55}$$

$$h^{t\Omega}_{it+1} \sim U\left(0.881\,88, 1.\,763\,8\right) \quad f^{\Omega}_{h_{it+1}} = 1.\,133\,9 \tag{56}$$

It follows that, as far as jurisdiction Ω is concerned, the mean and the variance of the distribution of human capital are respectively:

$$\overline{h}^{t\Omega}_{t+1} = 1.\,322\,9 \tag{57}$$

$$Var^{\Omega}\left(h^t_{t+1}\right) = 6.\,481\,8 \times 10^{-2} \tag{58}$$

and the rate of growth is:

$$\gamma^{\Omega}_{t+1} = 0.763\,76 \tag{59}$$

Comparing eq. (59) and eq. (48), we conclude that $\gamma^{\Omega}_{t+1} > \gamma^{C}_{t+1}$. This has been obtained assuming $\frac{w}{2Rc} = 1$, but it is easy to verify that the result holds $\forall w, c, R > 0$.

As far as jurisdiction Γ is concerned, the mean and the variance of the distribution of human capital are respectively:

$$\overline{h}^{t\Gamma}_{t+1} = 0.322\,17 \tag{60}$$

$$Var^{\Gamma}\left(h^t_{t+1}\right) = 3.\,459\,8 \times 10^{-2} \tag{61}$$

and the rate of growth is:

$$\gamma^{\Gamma}_{t+1} = 0.288\,67 \tag{62}$$

Comparing eq. (62) and eq. (48), we conclude that $\gamma^{\Gamma}_{t+1} < \gamma^{C}_{t+1}$. Again it is easy to verify that this result holds $\forall w, c, R > 0$.

Actually it easy to verify that eq. (29) holds for jurisdiction Ω whereas it does not hold for jurisdiction Γ.

It is possible to demonstrate that at time $t+2$ eq. (26) becomes:

$$h^{t\Gamma}_{it+1} = 1.\,372 h^t_{it} \tag{63}$$

for jurisdiction Γ

$$h^{t\Omega}_{it+1} = 2.\,347\,2 h^t_{it} \tag{64}$$

for jurisdiction Ω.

It is possible to demonstrate that at time $t+2$ in each jurisdiction J, the density function is $f^J_{h_{it+2}} = \frac{1}{\alpha^J\left(1+\sqrt{Var^J\left(h^t_{it}\right)+\left(\overline{h}^{tJ}_t\right)^2}\right)} \frac{1}{\left(1+\sqrt{Var^J\left(h^t_{it+1}\right)+\left(\overline{h}^{t+1J}_{t+1}\right)^2}\right)}$.

Therefore:

$$h^{t\Gamma}_{it+2} \sim U\left(0, 0.884\,03\right) \;\; f^{\Gamma}_{h_{it+2}} = 1.\,131\,2 \tag{65}$$

$$h^{t\Omega}_{it+2} \sim U\left(2.\,069\,9, 4.\,140\,0\right) \;\;\; f^{\Omega}_{h_{it+2}} = 0.483\,1 \tag{66}$$

It follows that, as far as jurisdiction Ω is concerned, the mean and the variance of the distribution of human capital are respectively:

$$\overline{h}^{t+1\Omega}_{t+2} = 3.\,105\,2 \tag{67}$$

$$Var^{\Omega}\left(h^{t+1}_{t+2}\right) = 0.357\,13 \tag{68}$$

and the rate of growth is:

$$\gamma^{\Omega}_{t+2} = 1.\,347\,2 \tag{69}$$

From time $t+2$ to time $t+3$ the jurisdiction Ω grows at the rate

$$\gamma^{\Omega}_{t+3} = 3.\,162\,2 \tag{70}$$

Comparing eq. (69) with eq. (51) and eq. (70) with eq. (52), we conclude that $\gamma^{\Omega}_{t+2} > \gamma^{C}_{t+2}$ and $\gamma^{\Omega}_{t+3} > \gamma^{C}_{t+3}$. This holds $\forall w, c, R > 0$.

As far as jurisdiction Γ is concerned, the mean and the variance of the distribution of human capital are respectively:

$$\overline{h}^{t+1\Gamma}_{t+2} = 0.442\,01 \tag{71}$$

$$Var^{\Gamma}\left(h^{t+1}_{t+2}\right) = 6.\,512\,6 \times 10^{-2} \tag{72}$$

and the rate of growth is:

$$\gamma^{\Gamma}_{t+2} = 0.372\,01 \tag{73}$$

From time $t+2$ to time $t+3$ the jurisdiction Γ grows at the rate

$$\gamma^{\Gamma}_{t+3} = 0.510\,39 \tag{74}$$

Comparing eq. (73) with eq. (51) and eq. (74) with eq. (52), we conclude that $\gamma^{\Gamma}_{t+2} < \gamma^{C}_{t+2}$ and $\gamma^{\Gamma}_{t+3} > \gamma^{C}_{t+2}$ This holds $\forall w, c, R$.

This numerical example allow us to conclude that, if the two jurisdictions were equally heterogenous, individuals belonging to the poor jurisdiction Γ would be better off in a unitary system, whereas individuals belonging to the rich jurisdiction Ω would be better off in a decentralized world. Actually it easy to verify that eq. (29) holds for jurisdiction Ω but not for jurisdiction Γ.

- **Case 2: different mean and different variance**

Let us assume now that the economy is divided again into two jurisdictions, Γ and Ω, but jurisdiction Γ is composed by agents of mass equal to 0.9, whose human capital is uniformly distributed on the support $[(0, 0.55), (0.65, 1)]$ and jurisdiction Ω is composed by a continuum of agents, of mass equal to 0.1, whose human capital is uniformly distributed on the support $(0.55, 0.65)$. It follows that $\overline{h}_t^{t\Gamma} = 0.488\,89$ and $Var^{\Gamma}(h_{it}^t) = 0.109\,52$ as far as jurisdiction Γ is concerned, and $\overline{h}_t^{t\Omega} = 0.6$ and $Var^{\Omega}(h_{it}^t) = 8.333\,3 \times 10^{-4}$ as for jurisdiction Ω.

Therefore, assuming again for the sake of simplicity $\frac{w}{2Rc} = 1$, eq. (26) becomes:

$$h_{it+1}^{t\Gamma} = 1.590\,4\,5 h_{it}^t \tag{75}$$

for jurisdiction Γ and

$$h_{it+1}^{t\Omega} = 1.600\,7\, h_{it}^t \tag{76}$$

for jurisdiction Ω

Therefore, following the same rationale of case 1, it is possible to demonstrate that at time $t+1$ the population in each jurisdiction is distributed as follows:

$$h_{it+1}^{t\Gamma} \sim U\left[(0, 0.874\,72), (1.033\,8, 1.590\,4)\right] \quad f_{h_{it+1}}^{\Gamma} = 0.698\,66 \tag{77}$$

$$h_{it+1}^{t\Omega} \sim U(0.880\,39, 1.040\,5) \quad f_{h_{it+1}}^{\Omega} = 6.245\,7 \tag{78}$$

It follows that, as far as jurisdiction Ω is concerned, the mean and the variance of the distribution of human capital are respectively:

$$\overline{h}_{t+1}^{t\Omega} = 0.960\,44 \tag{79}$$

$$Var^{\Omega}\left(h_{t+1}^t\right) = 2.136\,3 \times 10^{-3} \tag{80}$$

and the rate of growth is:

$$\gamma_{t+1}^{\Omega} = 0.600\,69 \tag{81}$$

As far as jurisdiction Γ is concerned, the mean and the variance of the distribution of human capital are respectively:

$$\overline{h}_{t+1}^{t\Gamma} = 0.777\,53 \tag{82}$$

$$Var^{\Gamma}\left(h_{t+1}^t\right) = 0.230\,85 \tag{83}$$

and the rate of growth is:

$$\gamma_{t+1}^{\Gamma} = 0.590\,37 \tag{84}$$

Comparing eq. (84) and eq. (48), we conclude that $\gamma_{t+1}^{\Gamma} > \gamma_{t+1}^{C}$. Again it is easy to verify that this result holds $\forall w, c, R > 0$.

At time $t+2$ eq. (26) becomes:

$$h_{it+2}^{t\Gamma} = 1.914 h_{it}^{t} \tag{85}$$

for jurisdiction Γ and

$$h_{it+2}^{t\Omega} = 1.961\,6 h_{it}^{t} \tag{86}$$

for jurisdiction Ω.

Therefore:

$$h_{it+2}^{t\Gamma} \sim U\left[(0, 1.674\,2), (1.978\,7, 3.044)\right] \quad f_{h_{it+2}}^{\Gamma} = 0.365\,03 \tag{87}$$

$$h_{it+1}^{t\Omega} \sim U\left(1.727\,0, 2.041\right) \quad f_{h_{it+1}}^{\Omega} = 3.184\,7 \tag{88}$$

It follows that, as far as jurisdiction Ω is concerned, the mean and the variance of the distribution of human capital are respectively:

$$\overline{h}_{t+2}^{t+1\Omega} = 1.884\,0 \tag{89}$$

$$Var^{\Omega}\left(h_{t+2}^{t+1}\right) = 8.216\,3 \times 10^{-3} \tag{90}$$

and the rate of growth is:

$$\gamma_{t+2}^{\Omega} = 0.961\,55 \tag{91}$$

From time $t+2$ to time $t+3$ the jurisdiction Γ grows at the rate

$$\gamma_{t+3}^{\Gamma} = 1.886\,2 \tag{92}$$

Comparing eq. (91) and eq. (51), we conclude that $\gamma_{t+1}^{\Gamma} > \gamma_{t+1}^{C}$. This holds $\forall w, c, R$.

As far as jurisdiction Γ is concerned, the mean and the variance of the distribution of human capital are respectively:

$$\overline{h}_{t+2}^{t+1\Gamma} = 1.488\,2 \tag{93}$$

$$Var^{\Gamma}\left(h_{t+2}^{t+1}\right) = 0.845\,68 \tag{94}$$

and the rate of growth is:

$$\gamma_{t+2}^{\Gamma} = 0.914 \tag{95}$$

From time $t+2$ to time $t+3$ the jurisdiction Γ grows at the rate

$$\gamma_{t+3}^{\Gamma} = 1.749\,4 \tag{96}$$

Comparing eq. (95) and eq. (51), we conclude that $\gamma_{t+1}^{\Gamma} > \gamma_{t+1}^{C}$. This holds $\forall w, c, R$.