

RAISING THE VOICE:
ACTIVE CITIZENS, OPTIMAL POLICY ADOPTION
AND THE MARKET FOR NEWSPAPERS

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Raising the Voice: Active Citizens, Optimal Policy Adoption and the Market for Newspapers

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Abstract

This article investigates the role of Mass Media as a bottom-up way of communicating dispersed information from citizens to incumbent. Citizens transmit useful information thanks to the newspapers they buy and read. However these newspapers are produced by a third party (a Media Tycoon) that has his own incentives. In particular the Media Tycoon has to decide whether to produce a newspaper that allows the citizens to participate to the public debate (Broadsheet) or does not (Tabloid). Given the fact that this instrument can be bought but not directly produced by the citizens, there exists a tension between the benefit of using a newspaper to express citizens' views and the possibility that this newspaper can be actually produced. Results show that producing a Broadsheet always improves the quality of policy decision making on part of the incumbent. A notable result is that in order to enhance the quality of the public decision making it is better to have *any* Broadsheet than not having one, whatever is the public stance the newspaper takes about the issue at stake. In this article I assume that there is one group of citizens which is interested into having the optimal policy adopted, i.e. the Middle Class and first I assume the Middle Class citizens are the only one who read newspaper. Subsequently I analyse how the results change when citizens from the other classes read newspaper as well. I show how the "partisan readers", committed to buy the Broadsheet supporting the policy they prefer, can ease the production of the Broadsheet. In this case the existence of partisanship and of ideological readers make the implementation of optimal policy easier, not harder, contrary to conventional wisdom.

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1 Introduction

In the literature about the political economy of mass media, the media has been modelled as an institution having either a supervisory role (Besley and Prat 2006) or acting as a megaphone of the (incumbent) politician ((Stromberg 2004a), (Stromberg 2004b)). However, the media might act also as an institutional device reporting useful information to the (incumbent) politician that he might not be aware of, information that the incumbent finds useful when he has to decide on the policy to implement. In fact citizens, in their day to day activities, receive information on the state of the world. Furthermore, some of those citizens might have their preferences that depend on the state of the world and on the information they receive on the state of the world itself. They might be willing to communicate this information and their preferences on the policy to be implemented through their actions, in order to influence the policy to be adopted. However, such an action might be: i) (very) costly; ii) subject to a public good dimension; iii) subject to a coordination problem/failure. A route to escape all these three potential failures is to model the role of mass media as a sort of intermediary in the communication between citizens and incumbent politician. Moreover, I consider the consumption of mass media itself as a way to overcome the problems of public good and coordination failure between agents. In a way the entire society relies on a (sub-)set of citizens who enjoy reading about politics and participating on the political debate and who are willing to do so also because their utility depends on the adopted policy. In turn this policy will be the more efficient the better is the communication between citizens and the incumbent as mediated through mass media.

This article aims at investigating this feature of the media which has not been highlighted by the existing literature. In particular the literature has not considered the role of mass media as a bottom-up way of communicating dispersed information from citizens to the incumbent and has not considered the fact that some of the citizens might enjoy “consuming” politics as others enjoy “consuming” football or arts/literature/cinema. For the sake of concreteness in the following discussion I model the media as being newspapers but the model can be extended to consider the role of other media as well.

Given the fact that the Mass Media can be bought but not directly produced by the citizens, there exists a tension between the benefit of using a newspaper to express citizens’ views and the possibility that this newspaper can actually be produced. I distinguish two possible newspapers that can be produced: Tabloid and Broadsheet, where the former is an uninformative newspaper, while the latter is informative on the state of the world and endorse the implementation of a policy option, of the two available, on part of the incumbent politician. One of these two policy options is optimal, given the state of the world, while the other is not. Results show that the production, and the successive consumption of a Broadsheet, always improves the quality of the policy implementation. Quite interestingly this results of effi-

ciency does not rely on the production of the Broadsheet supporting the right policy option, but on any Broadsheet being produced, instead of a Tabloid.

In this article I first assume that there is just one group of citizens who read newspapers, the Middle Class citizens. Subsequently I analyse how the results change when citizens from the other classes read newspaper as well and they choose between the Broadsheet supporting their views or the Tabloid. Interestingly, results show that the presence of “partisan reader” can make the implementation of optimal policy easier, not harder, contrary to conventional wisdom. Furthermore, in a more polarized society, where the Middle Class shrinks and Poor and/or Rich individuals grow in number, as long as there is a large fraction of partisan readers, a smaller Middle Class can still have the role in the public opinion to transmit useful information to the Incumbent, necessary to implement optimal policy. Finally, there exists a non-empty set of parameters such that if a Broadsheet newspaper is produced, the policy that newspaper supports is implemented more often than the alternative policy, for any state of the world. In such a space of parameters the existence of partisan readers has two roles: i) it allows the preferred policy to be implemented more than if informative newspapers did not exist; ii) it makes it easier for a Broadsheet newspaper to be produced and therefore it increases the probability of optimal policy implementation. Again, these result show that it is not always obvious that a political behaviour which is ideological is at odds with efficiency in the public policy adoption.

This article draws on the small literature on the role of media as an institutional player that helps collect and aggregate dispersed information. Most of the literature ((Besley and Burgess 2001) and (Besley and Burgess 2002)) has dealt with estimating the effectiveness of media in functioning as an institution which communicates useful information on the state of the world to the incumbent. However this literature is mostly empirical. (Piketty 1999) surveys most of the literature on aggregating information and on institutions that have this role. The papers closer to the model in this article are (Lohmann 1993) and (Lohmann 1994). However, in these papers, the citizens interested in policy who receive information communicate it directly, by voting or protesting or going on strike. In this article, instead, citizens transmit useful information thanks to newspapers. However, these newspapers are produced by a third party (a Media Tycoon) that has his own incentives in deciding whether to produce a newspaper that allows the citizens to participate in the public debate (Broadsheet) or does not allow them to do so (Tabloid). This article focuses on the tensions existing between the optimality of producing the informative newspaper for the society as a whole and the optimality for the Media Tycoon.

The structure of the article is the following: the next Section presents the model; Section 3 solves the model and derives the equilibrium of this strategic situation, presenting also some comparative statics exercises . Section 4 analyses the role of ideological citizens/partisan readers in favouring the optimal policy implementation and/or the policy they favour the most. Finally

Section 5 concludes.

2 The Model

The model I employ builds on a simple two states of the world - two actions framework.¹ There are two states of the world $\theta \in \Theta \equiv \{\underline{\theta}, \bar{\theta}\}$ randomly chosen by the Nature with probability $Pr(\underline{\theta}) = \frac{1}{2}$ and $Pr(\bar{\theta}) = \frac{1}{2}$. The Incumbent politician has to take a decision $a \in A \equiv \{\underline{a}, \bar{a}\}$ based on his priors and on the additional information on the state of the world he receives from citizens, through newspapers. For the sake of concreteness I interpret these decisions $a \in A$ as policy choices that the Incumbent has to implement. Although these policy decisions affect different groups of citizens in a different way (see discussion below), they are deemed to be optimal contingent on the state of the world in the following sense: given the state of the world $\bar{\theta}(\underline{\theta})$, the optimal policy choice is $\bar{a}(\underline{a})$. One might think that the optimality of the policy with respect to the state of the world refers to utilitarian welfare: therefore implementing $\bar{a}|\bar{\theta}$ (or $\underline{a}|\underline{\theta}$) maximises citizens' aggregate utility. Let the Incumbent be interested in implementing the optimal policy choice contingent on θ . This can be because he enjoys ego rents from having done "the right thing" or because he is a benevolent social planner whose objective is to maximise aggregate utility. To model this I make the assumption that the Incumbent receives a positive utility if he adopts the optimal policy choice: $u_I(\bar{a}|\bar{\theta}) = u_I(\underline{a}|\underline{\theta}) > 0$; otherwise he receives a utility of zero if he "makes a mistake", i.e.: $u_I(\bar{a}|\underline{\theta}) = u_I(\underline{a}|\bar{\theta}) = 0$.

The polity is made of \hat{N} citizens and consists of three groups indexed by $k \in \{P, R, M\}$. Group *R*(ich) has a numerosity of $N_R = p\hat{N}$. Group *P*(oor) has a numerosity of $N_P = \bar{p}\hat{N}$. Group *M*(iddle class) has a numerosity of $N_M = (1 - \bar{p} - p)\hat{N}$. All the citizens belonging to each group have the same preferences regarding the policy $a \in A$ to be implemented. However, citizens in different groups have different preferences on that same policy options a . In particular: citizens in group *R* always prefer policy \underline{a} to policy \bar{a} regardless of the state of the world; citizens in group *P* always prefer policy \bar{a} to policy \underline{a} regardless of the state of the world. Finally, citizens belonging to *M* maximise their utility when the optimal policy is implemented. Therefore their policy preferences are aligned with the Incumbent's ones and they prefer policy \underline{a} conditional on the state of the world being $\underline{\theta}$, while they prefer \bar{a} conditional on the state of the world being $\bar{\theta}$. Formally, without loss of generality, I assume that $u_i(\bar{a}|\bar{\theta}) = u_i(\underline{a}|\underline{\theta}) > 0$, for each $i \in M$; instead

¹The model I use borrows some elements from Lohmann (1993). However my model on one hand simplifies Lohmann's model by assuming that all the citizens in each group have the same indirect utility from policy. On the other hand, the model employs a multinomial (trinomial) distribution function, instead of a binomial, to describe the pdf of the signals arriving to the citizens. Furthermore it crucial to the model it is the presence of a for-profit-institutional player (Media, i.e. a Newspaper) allowing the citizens to express their signals/preferences/opinion. In Lohmann's articles instead, citizens express their views by means of actions they take directly, although at some cost.

$u_i(\bar{a}|\underline{\theta}) = u_i(\underline{a}|\bar{\theta}) = 0$, for each $i \in M$. Regarding the Poor group, instead, one has that $u_i(\bar{a}|\underline{\theta}) > 0 = u_i(\underline{a}|\underline{\theta})$, for each $i \in P$ and for each $\theta \in \Theta$. Finally, $u_i(\underline{a}|\underline{\theta}) > 0 = u_i(\bar{a}|\underline{\theta})$, for each $i \in R$ and for each $\theta \in \Theta$.

Thanks to their day-to-day activities (for instance due to their experience as users of public good/goods publicly supplied; or because they are dissatisfied with their experiences as constituents of a first-past-the-post uninominal system and they want to reform it in a proportional way; or alternatively because they see from their experience or the experiences of their acquaintances that “death-taxes” are (not) unjust and they do (not) prevent social mobility) citizens may receive a signal on the state of the world. In the examples introduced above the state of the world could be that NHS new foundation hospitals are (not) working; or the Britain transport system is (not) underfunded; or proportional representation is (not) the best electoral system to mobilise citizens towards politics; or inequality in wealth is (not) a threat to social inequality.

Given a state of the world θ there are three possible signals σ that each citizen can receive: in particular: $\sigma \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$. The probability of each citizen receiving any of the three signals, given the state of the world, is the following:

$$\begin{aligned} Pr(\sigma = \bar{\sigma}|\bar{\theta}) &= q \\ Pr(\sigma = \phi|\bar{\theta}) &= 1 - q - q' \\ Pr(\sigma = \underline{\sigma}|\bar{\theta}) &= q' \end{aligned}$$

$$\begin{aligned} Pr(\sigma = \bar{\sigma}|\underline{\theta}) &= q' \\ Pr(\sigma = \phi|\underline{\theta}) &= 1 - q - q' \\ Pr(\sigma = \underline{\sigma}|\underline{\theta}) &= q \end{aligned}$$

with $q \in (0, 1)$ and $q > q'$. Since $q + q' < 1$, then it must be that $q' < \frac{1}{2}$. In fact if not, $q' > \frac{1}{2}$ and, given $q > q' > \frac{1}{2}$, then it would be that $q + q' > 1$ which is impossible.

I assume that all the citizens receive a signal, and these signals are all i.i.d. However, I first focus my attention on the signals received by the M citizens, who will be shown to be the only ones always interested in truthful transmission of the signals.

It is easy to show that a citizen observing a private signal $\bar{\sigma}(\underline{\sigma})$ rationally believes, upon Bayes updating, that the state $\bar{\theta}(\underline{\theta})$ is more likely than the state $\underline{\theta}(\bar{\theta})$. On the other hand, upon observing an empty signal ϕ , the citizen does not learn anything new about the state of the world. This can be proved in the following Lemma:

Lemma 1 *Based on his private information, upon observing a $\bar{\sigma}(\underline{\sigma})$ signal, the Middle Class citizen believes the state of the world $\bar{\theta}(\underline{\theta})$ is more likely than*

$\underline{\theta}(\bar{\theta})$. However, when he privately observes the ϕ signal, the citizen learns nothing.

Proof. To show this, simply apply Bayes's rule to the individual citizen's beliefs. $Pr(\bar{\theta}|\bar{\sigma}) = \frac{Pr(\bar{\sigma}|\bar{\theta})Pr(\bar{\theta})}{Pr(\bar{\sigma}|\bar{\theta})Pr(\bar{\theta})+Pr(\bar{\sigma}|\underline{\theta})Pr(\underline{\theta})} = \frac{q\frac{1}{2}}{q\frac{1}{2}+q'\frac{1}{2}} = \frac{q}{q+q'}$. It follows that $Pr(\bar{\theta}|\bar{\sigma}) > Pr(\bar{\theta}) = \frac{1}{2}$ iff $\frac{q}{q+q'} > \frac{1}{2}$ i.e. iff $q > \frac{1}{2}(q+q')$. Simple algebra shows that $q > \frac{1}{2}q + \frac{1}{2}q'$ iff $\frac{1}{2}q > \frac{1}{2}q'$ iff $q > q'$ which is true given the hypothesis. A similar conclusion can be reached by showing that $Pr(\underline{\theta}|\underline{\sigma}) > Pr(\underline{\theta}) = \frac{1}{2}$ iff

$$q > q'. \text{ Finally } Pr(\phi|\bar{\theta}) = \frac{Pr(\phi|\bar{\theta})Pr(\bar{\theta})}{Pr(\phi|\bar{\theta})Pr(\bar{\theta})+Pr(\phi|\underline{\theta})Pr(\underline{\theta})} = \frac{(1-q-q')\frac{1}{2}}{(1-q-q')\frac{1}{2}+(1-q-q')\frac{1}{2}} = \frac{1}{2}. \blacksquare$$

I identify with $n_{\bar{\sigma}}, n_{\phi}, n_{\underline{\sigma}}$ the number of M citizens receiving (respectively) signals $\bar{\sigma}, \phi, \underline{\sigma}$. This means that $n_{\bar{\sigma}}+n_{\phi}+n_{\underline{\sigma}} = N_M$. To economise on notation, I rewrite $N_M = N$. Finally with i_{σ} I indicate the single citizen $i \in M$ who has observed the signal $\sigma \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$.

It is necessary to spend a few words now in order to justify my modelling of signals, their interpretation and their relation with the state of the world and the implementable policy. For instance let us focus on the state of the world that says "foundation hospitals are working" (say $\bar{\theta}$). In this case each citizen might go to a foundation hospital if available and experience a good treatment (say $\bar{\sigma}$) or a bad one (say $\underline{\sigma}$) or, as a third option, he might not get any information on foundation hospitals because they are not available in his area or because the treatment he has received is similar to what he experienced in the past from any other non-foundation hospital (ϕ). Therefore if two different policies, either of extending foundation hospitals (\bar{a}) or of scrapping this policy (\underline{a}), are being brought forward of the citizen, given his experience, might want to take part in the debate, taking a stance towards supporting the extension of this policy or not.

I want to study how citizens take part in this public policy debate when newspapers are available and they might perform the role of "campaigning newspaper" on behalf of one of the two sides in a policy debate. Alternatively the newspapers can choose not to take any stance in the policy debate. If the newspaper j is a "campaigning newspaper" and takes a position in the policy debate, then $j \in \{\underline{\sigma}, \bar{\sigma}\}$. If the newspaper does not take any stance in the policy debate, I denote this with $j = \phi$.² If the newspaper carries on its front page stories coherent with the signal $\bar{\sigma}(\underline{\sigma})$, this means, following Lemma (1), that the newspaper believes that the state of the world $\bar{\theta}(\underline{\theta})$ is more likely than the alternative one. Of course, given that it regards one of the two states of the world to be more likely than the other one, the newspaper campaigns for the optimal policy contingent on the state of the world to be

²Slightly abusing notation, with the same symbol $\sigma \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$ I indicate both the newspaper $j \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$ and the signal indicative of the state of the world the newspaper has printed on its page.

implemented. I will dub this sort of newspaper a Broadsheet: in this context, Broadsheets are newspapers campaigning for one of the policies $a \in A$ and printing stories explaining the relative merit of one policy over the other. For instance, in my example about foundation hospitals, if the Broadsheet takes a stance favourable to the extension of foundation hospitals ($\bar{\theta}$) it will publish stories about the relative merits of foundation hospitals with respect to NHS-run-hospitals, actively supporting the option of extending this policy option.

The other type of newspaper is a Tabloid one, i.e. an uninformative newspaper which does not take any stance in the policy debate.³ This type of newspaper prints stories about celebrity gossip, football, and sports in general or “useful news” (gardening, motor and cars reviews, food recipes, music and movies reviews, etc.). Consuming them gives some utility to their readers, but does not allow them to take part in the policy debate at large. In the terms of my model a Tabloid carries the signal $\sigma = \phi$.⁴

Let me now define the Middle Class citizen’s utility function as the following:

$$U_i(l, a, j) = l_i + u_i(a|\theta) + u_i(j|\sigma)$$

for each $j \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$ and for each $i \in M$. The function $U_i(l, a, j)$ is separable in the following arguments: the income l_i ; the utility the citizen derives from the implementation of the right policy given the state of the world i.e. $u_i(a|\theta)$; the utility the citizen obtains from reading the newspaper j , given the signal he receives $\sigma \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$, i.e. $u_i(j|\sigma)$. I make the assumption that each citizen belonging to the Middle Class has the same income $l_i = \bar{l}$ for each $i \in M$ and this is enough to buy a single copy of the newspaper, if the citizen wishes to do so. I also assume that each citizen derives the same positive utility \bar{e} from reading a newspaper j coherent with the signal he has observed σ . In symbols this means that $u_i(j|\sigma) = \bar{e}$, for each $j \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$ and for each $i \in M$ group iff $j = \sigma$. Instead $u_i(j|\sigma) = 0$ for each $j \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$ and for each $i \in M$ group iff $j \neq \sigma$.

From the previous discussion it is clear that the action of buying a newspaper accomplishes two things: first, it gives the citizen-reader some utility coming directly from reading a newspaper reflecting the reader’s view of the world; second, it transmits a signal to the incumbent which supplies him with more information about the state of the world than the incumbent himself would have had otherwise.

The argument that the citizen belonging to the Middle Class enjoys read-

³An alternative interpretation of this “Tabloid” is the one of a newspaper giving an equal space to each side of the debate, without taking any strong position in support of either side.

⁴Of course I am aware that there exist Tabloids campaigning for some policy issues or Broadsheet that do not perform any campaigning journalism. However I use the names “Broadsheet” and “Tabloid” to fix ideas regarding the existing difference between campaigning newspapers and not campaigning ones.

ing a newspaper coherent with his view of the world assumes implicitly that he does not hold any bias in favour of one policy or the other before receiving the signal. However, once he receives it, based on his own private information only, he believes that the state of the world coherent with the observed signal is the most likely. This reading behaviour can be justified thanks to the literature on self-serving beliefs and self-serving biases⁵. This literature has for a long time recognized that people tend to confirm their own beliefs and discount, minimise or, even worse, disregard any information opposed to their beliefs. Once they hold a view on the state of the world and on the policy more suitable to this state of the world, view they have derived from the signal they have received, these citizens buy a Broadsheet coherent with the signal they have received. This might be because these citizens enjoy reading about politics and taking part (or feeling they are taking part) in the debate. Although they have received a signal, they might feel they want to know more. Therefore they desire to read about this and, as a consequence, support the policy option in whose favour they have received a private signal.

Similarly to Middle Class citizens' utility, one can define Rich citizen's utility as the following:

$$U_i(r, a, j) = r_i + u_i(a|\theta) + u_i(j|\sigma)$$

for each $j \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$ and for each $i \in R$, with $r_i = \bar{r}$ for each $i \in R$. Likewise Poor citizen's utility is defined as:

$$U_i(m, a, j) = m_i + u_i(a|\theta) + u_i(j|\sigma)$$

for each $j \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$ and for each $i \in P$, with $m_i = \bar{m}$ for each $i \in P$. It is worth repeating that I first analyse the strategic situation when only Middle Class citizens buy the newspaper. Then I extend this analysis to all the citizens in Section (4). Therefore, in this first section, I analyse the strategic situation as if, although the implemented policy $a \in A$ affects all the citizens in each one of the three groups, only Middle Class citizens read the newspaper.

Finally, I introduce an entrepreneur in the mass media industry i.e. a Media Tycoon (MT). He has the task of producing a single newspaper j carrying one of the three possible signals $\sigma \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$. MT's objective function is to maximise his expected profit coming from newspaper production and selling. I make the crucial assumption that the MT does not observe any signal.

This can be rationalized because the MT does not take any interest in the policy the Incumbent will implement, either because he is not affected by the policy, or because the effect on his utility is negligible, compared with the profit coming from his entrepreneurial activity. Also, it might be that the newspaper is owned by a public company and, although the effect of the

⁵For a recent use in economics of idea borrowed from this literature see (S. and A. 2005). For a critical assessment of this literature and its role in recent economic research see (Glaeser 2004).

implementable policy on each shareholder is different depending if they are Poor, Rich or Middle Class, the weighted sum of the welfare effects is equal to zero. Another reason why the MT might not observe any signal is that the MT is a foreigner who does not belong to the polity governed by the incumbent politician and therefore does not observe any signal coming from the environment. Or likewise, it might be that the MT (or the journalists working in the newspaper) observes a plurality of signals but the Bayesian updating is such that he learns nothing on the state of the world.

Following (Gentzkow and Goldin 2006) I assume that the cost function of the newspaper production has constant marginal cost and it is the following:

$$C(y_j) = F_j + c_j y_j$$

where y_j is the number of copies of the newspaper j the MT produces, c_j is the variable cost to produce a copy of the newspaper j and F_j is the fixed cost. In order not to introduce any asymmetry in the cost function which might drive the results, to make comparisons straightforward and to highlight the main determinants of the model, I assume that $c_j = \bar{c}$ for each $j \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$ and $F_j = \bar{F}$ for each $j \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$. The price of the single newspaper copy is labeled with p_j , for each $j \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$.

Finally I assume that the utility each citizen derives from reading a copy of the newspaper is common knowledge as is the distribution of signals Middle Class citizens receive: every player knows this distribution, but does not know the actual realisation of the signals, apart from the signal he observes, if any. Knowing citizens' preferences and their utility from reading the newspaper, MT has to decide which newspaper to produce in order to maximise his expected profit Π_j^E .

2.1 Timing of the game

At $t = 0$, Nature chooses at random the state of the world $\theta \in \Theta \equiv \{\underline{\theta}, \bar{\theta}\}$; at $t = 1$, signals are sent; in $t = 2$, citizens observe signals and MT produces a newspaper without knowing any signals about the state of the world and without knowing citizens' received signals; in $t = 3$, citizens take the buying decision regarding the newspaper; in $t = 4$, the Incumbent observes how many copies of the produced newspaper have been sold, updates his beliefs on the state of the world and implements the policy $a \in A \equiv \{\underline{a}, \bar{a}\}$ accordingly; in $t = 5$, the state of the world is revealed and payoffs accrue to agents.

2.2 Solution concept

The solution concept I employ to solve the model is Perfect Bayesian Equilibrium where each player chooses his optimal strategy given other players' equilibrium strategies and equilibrium beliefs and beliefs are derived along the equilibrium path, whenever possible, given equilibrium strategies.

3 Solving the Game

I solve the game by backward induction. In $t = 5$ the state of the world is revealed and all the players (citizens and Incumbent politician) receive their payoffs depending on the policy implemented in $t = 4$ and on the state of the world. In $t = 4$ the Incumbent observes which newspaper has been produced and how many copies have been sold. Thanks to this information he updates his beliefs on the state of the world $\theta \in \{\underline{\theta}, \bar{\theta}\}$, and implements the policy which he thinks it is more likely to be optimal. Given the beliefs on θ , that is the ex-post probability of θ , which is a function of the number of signals the Incumbent observes by looking at which and how many copies of newspapers have been bought, the Incumbent maximises his expected utility according to the following rule:

Lemma 2 *When the two states of the world have the same probability, the incumbent randomises with any probability between the two policies. When the two states of the world have different probabilities, the incumbent implements with certainty the policy which is more likely to be optimal.*

Proof. To prove the Lemma, assume that $Pr(\bar{\theta}) = \rho > 0$ and $Pr(\underline{\theta}) = 1 - Pr(\bar{\theta}) = 1 - \rho$. Since $u_I(\bar{a}|\bar{\theta}) = u_I(\underline{a}|\underline{\theta}) = 1$, while $u_I(\bar{a}|\underline{\theta}) = u_I(\underline{a}|\bar{\theta}) = 0$ this means that the incumbent taking the strategy \bar{a} obtains an expected utility $u_I^E(\bar{a}|\theta) = u_I(\bar{a}|\bar{\theta})Pr(\bar{\theta}) + u_I(\bar{a}|\underline{\theta})Pr(\underline{\theta}) = u_I(\bar{a}|\bar{\theta})\rho + u_I(\bar{a}|\underline{\theta})(1 - \rho) = \rho$. On the other hand, when he takes the strategy \underline{a} , the incumbent obtains an expected utility equal to: $u_I^E(\underline{a}|\theta) = u_I(\underline{a}|\bar{\theta})Pr(\bar{\theta}) + u_I(\underline{a}|\underline{\theta})Pr(\underline{\theta}) = u_I(\underline{a}|\bar{\theta})\rho + u_I(\underline{a}|\underline{\theta})(1 - \rho) = 1 - \rho$.

Let us define $\Gamma(\bar{a})$ as the probability that the Incumbent implements the policy \bar{a} , while the complementary probability $\Gamma(\underline{a}) = 1 - \Gamma(\bar{a})$ is the probability that the Incumbent implements the policy \underline{a} . Therefore $\Gamma^*(\bar{a}) \in ArgMax_{a \in A} \sum_{\theta \in \Theta} \Gamma(a)u_I^E(a|\theta)Pr(\theta) = \Gamma(\bar{a})\rho + (1 - \Gamma(\bar{a}))(1 - \rho)$. Expanding the previ-

ous expression, one obtains that $\sum_{a \in A} \sum_{\theta \in \Theta} \Gamma(a)u_I^E(a|\theta)Pr(\theta) = 1 - \rho - \Gamma(\bar{a}) + 2\Gamma(\bar{a})\rho =$

$= 1 - \rho - \Gamma(\bar{a})(1 - 2\rho)$. Therefore if $1 - 2\rho > 0$ i.e. $\rho < \frac{1}{2}$ then $\Gamma^*(\bar{a}) = 0$. If $1 - 2\rho < 0$ i.e. $\rho > \frac{1}{2}$ then $\Gamma^*(\bar{a}) = 1$. Finally, if $1 - 2\rho = 0$ i.e. $\rho = \frac{1}{2}$ then $\Gamma^*(\bar{a}) \in [0, 1]$. In particular, $\Gamma^*(\bar{a}) = \frac{1}{2}$. Notice that the incumbent always prefers to have $\rho \neq \frac{1}{2}$ since this gives him a higher expected utility than $\rho = \frac{1}{2}$. ■

From the previous Lemma a simple Corollary follows immediately:

Corollary 3 *When the two states of the world have the same probabilities, the probability of optimal policy implementation is equal to 1/2. If $\rho > 1/2$ ($\rho < 1/2$) the probability of optimal policy implementation is equal to $\rho((1 - \rho))$.*

Proof. From Lemma 2 one knows that when $\rho = \frac{1}{2}$, $\Gamma^*(\bar{a}) \in [0, 1]$. The probability of optimal decision making is equal to the probability of implementing the policy $a \in A \equiv \{\underline{a}, \bar{a}\}$, times the probability that that policy is

optimal, contingent on the state of the world $\theta \in \Theta$. Therefore the probability of optimal policy implementation is equal to:

$$\Gamma^*(\bar{a})Pr(\bar{\theta}) + (1 - \Gamma^*(\bar{a}))Pr(\underline{\theta}) = \Gamma^*(\bar{a})\frac{1}{2} + \frac{1}{2} - \Gamma^*(\bar{a})\frac{1}{2} = \frac{1}{2}$$

for each $\Gamma^*(\bar{a}) \in [0, 1]$. Likewise, from Lemma (2) one knows that when $\rho > \frac{1}{2}$ ($\rho < \frac{1}{2}$), $\Gamma^*(\bar{a}) = 1$ ($\Gamma^*(\bar{a}) = 0$). Then the probability of optimal decision making is equal to the probability of implementing the policy $a \in A \equiv \{\underline{a}, \bar{a}\}$, times the probability that the same policy is optimal, contingent on the state of the world $\theta \in \Theta$. It is easy to find that the probability of optimal policy implementation is equal to:

$$\begin{aligned} \Gamma^*(\bar{a})Pr(\bar{\theta}) &= 1 * \rho = \rho > \frac{1}{2} \\ (1 - \Gamma^*(\bar{a}))Pr(\underline{\theta}) &= 1 * (1 - \rho) = (1 - \rho) > \frac{1}{2} \end{aligned}$$

respectively when $\Gamma^*(\bar{a}) = 1$ ($\Gamma^*(\bar{a}) = 0$). ■

The information about the state of the world $\theta \in \Theta$ and about which optimal policy to implement is given to the Incumbent thanks to the newspaper readership. Since there is one single newspaper produced by the MT, then the Incumbent has to estimate the probability of the state of the world by looking at the number of signals printed and at which signal is printed on that newspaper, i.e. at the copies of the newspaper sold and bought.

In the analysis of the equilibrium of the game one needs to consider both: *a)* the utility the citizen derives directly from consuming the newspaper; and *b)* the effect his buying decision has on the implementable policy and the (expected) utility the citizen could get from that same policy, were it the optimal one. However here I derive the equilibrium of the game considering only the utility the citizen derives directly from reading the newspaper. Implicitly I make the assumption that the single citizen is “too small to matter” in the computation of the expected utility and his action will not manage to change the outcome in the policy adoption decision. This is a good approximation of the truth when the number of Middle class citizens is large. Nevertheless In the Appendix I show how the equilibrium of the game changes when one looks at the more comprehensive effect that buying the newspaper has on citizen’s utility.

Given the solution concept I use is PBE, I need to show that equilibrium strategies are optimal given equilibrium beliefs and viceversa. First I focus on the case of the existence and production of the $j = \bar{\sigma}$ Broadsheet, since the case for $j = \underline{\sigma}$ is similar. The case for Tabloid (i.e. $j = \phi$) is analysed below. The following can be shown:

Lemma 4 *There exists an (separating) equilibrium where Middle Class citizens in $t = 3$ buy the Broadsheet produced if and only if they have received the signal printed on the newspaper and the price of the newspaper is less than the utility the citizens derive from reading it.*

Proof. Assume that in equilibrium a number of signals $n_{\bar{\sigma}}^*$ (to be determined later) is the minimum number of signals sufficient for the Incumbent to believe that $Pr(\bar{\theta}|n_{\bar{\sigma}}^*) > Pr(\bar{\theta}) = 1/2$. Given the monotonicity of the ex-post probability in the number of signals, this means that $Pr(\bar{\theta}|\tilde{n}_{\bar{\sigma}}) > 1/2$ for any $\tilde{n}_{\bar{\sigma}} \geq n_{\bar{\sigma}}^*$, while $Pr(\bar{\theta}|\tilde{n}_{\bar{\sigma}}) \leq 1/2$ for any $\tilde{n}_{\bar{\sigma}} < n_{\bar{\sigma}}^*$. Furthermore remember that, given that there is one single newspaper, if in equilibrium $n_{\bar{\sigma}}$ signals are observed, the Incumbent and the other players have to form expectations on the remaining $N - n_{\bar{\sigma}}$ signals received by the citizens who have not bought any newspaper.

To prove the Lemma I show first that, given a number of newspaper copies $n_{\bar{\sigma}}$ observed in equilibrium and the equilibrium beliefs $Pr(\bar{\theta}|n_{\bar{\sigma}})$, citizens who have received a $\bar{\sigma}$ signal ($\hat{i}_{\bar{\sigma}}$) buy the $j = \bar{\sigma}$ newspaper while citizens ($\hat{i}_{\sigma \in \{\phi, \underline{\sigma}\}}$) never do.⁶

First assume that in equilibrium a number of signals $n_{\bar{\sigma}}^* - 1$ is observed, i.e. there are $n_{\bar{\sigma}}^* - 1$ copies bought of the newspaper $j = \bar{\sigma}$ and the citizen $\hat{i}_{\bar{\sigma}}$ is pivotal. Given what said above, equilibrium beliefs is $Pr(\bar{\theta}|n_{\bar{\sigma}}^* - 1) < 1/2$. In this event there are several possible cases for the single citizen $\hat{i}_{\sigma \in \{\bar{\sigma}, \phi, \underline{\sigma}\}} \in M$ and the Incumbent politician that one needs to analyse.

i) The number $n_{\bar{\sigma}}^ - 1$ is such that $Pr(\bar{\theta}|n_{\bar{\sigma}}^* - 1) < Pr(\bar{\theta})$ and the citizen \hat{i} has privately observed the signal $\bar{\sigma}$.*

The citizen is willing to buy the newspaper $j = \bar{\sigma}$ as long as the charged price $p_{j=\bar{\sigma}}$ is less than or equal to the utility \bar{e} . Again $\hat{i}_{\bar{\sigma}}$ will not read any other newspaper, for any $p_{j \in \{\phi, \underline{\sigma}\}} > 0$, given the signal received and his preferences. Conversely, if the citizen buys the newspaper $j = \bar{\sigma}$, then it must be that the citizen has observed privately the signal $\bar{\sigma}$ and that $p_{j=\bar{\sigma}} \leq \bar{e}$. In fact if it was not so, then this would mean that the citizen buys a newspaper and bears a positive cost equal to the price although he does not receive any utility from reading the newspaper or enjoys a utility inferior to the price. In this case the citizen would be better off by not buying the newspaper.

ii) The number $n_{\bar{\sigma}}^ - 1$ is such that $Pr(\bar{\theta}|n_{\bar{\sigma}}^* - 1) < Pr(\bar{\theta})$ and the citizen \hat{i} has privately observed the signal $\underline{\sigma}$ (ϕ).*

Given the signal $\underline{\sigma}$ (ϕ) he has privately observed, $\hat{i}_{\underline{\sigma}}(\hat{i}_{\phi})$ will not buy the newspaper $j = \bar{\sigma}$ based on his preferences as long as $p_{j=\bar{\sigma}} > 0$. On the other hand, if the citizen does not buy the newspaper $j = \bar{\sigma}$ with $p_{j=\bar{\sigma}} > 0$, then it must be that he has observed a signal different from $\bar{\sigma}$ or that he has observed the signal $\bar{\sigma}$ but $p_{j=\bar{\sigma}} > \bar{e}$.

Now it remains to consider the case when $\hat{i}_{\bar{\sigma}}$ is not pivotal, that is in equilibrium a number $n_{\bar{\sigma}}$ is observed such that $n_{\bar{\sigma}} < n_{\bar{\sigma}}^* - 1$ (or $n_{\bar{\sigma}} > n_{\bar{\sigma}}^*$). Following what said above, equilibrium beliefs will be equal to $Pr(\bar{\theta}|n_{\bar{\sigma}} < n_{\bar{\sigma}}^* - 1) < 1/2$ (or $Pr(\bar{\theta}|n_{\bar{\sigma}} > n_{\bar{\sigma}}^*) > 1/2$).

iii) The number $n_{\bar{\sigma}}$ is such that $n_{\bar{\sigma}} < n_{\bar{\sigma}}^ - 1$ (or $n_{\bar{\sigma}} > n_{\bar{\sigma}}^*$) and the citizen \hat{i} has privately observed the signal $\bar{\sigma}$.*

⁶Abusing notation, with $\hat{i}_{\bar{\sigma}}$ I indicate the signal $\bar{\sigma}$ received by the individual i also. Of course $\hat{i}_{\underline{\sigma}}$ and \hat{i}_{ϕ} have similar meanings.

The citizen is willing to buy the newspaper $j = \bar{\sigma}$ as long as the charged price $p_{j=\bar{\sigma}}$ is less than or equal to the utility \bar{e} . Again $\hat{i}_{\bar{\sigma}}$ will not read any other newspaper, for any $p_{j \in \{\phi, \underline{\sigma}\}} > 0$, given the signal received and his preferences. Conversely, if the citizen buys the newspaper $j = \bar{\sigma}$, then it must be that the citizen has observed privately the signal $\bar{\sigma}$ and that $p_{j=\bar{\sigma}} \leq \bar{e}$. In fact if it was not so, then this would mean that the citizen buys a newspaper and bears a positive cost equal to the price although he does not receive any utility from reading the newspaper or receives a utility inferior to the price. In this case the citizen would be better off by not buying the newspaper.

iv) The number $n_{\bar{\sigma}}$ is such that $n_{\bar{\sigma}} < n_{\bar{\sigma}}^ - 1$ (or $n_{\bar{\sigma}} > n_{\bar{\sigma}}^*$) and the citizen \hat{i} has privately observed the signal ϕ or $\underline{\sigma}$.*

Given the signal $\underline{\sigma}$ or ϕ he has privately observed, again $\hat{i}_{\underline{\sigma}}(\hat{i}_{\phi})$ is strictly better off not buying the newspaper as long as $p_{j=\bar{\sigma}} > 0$. On the other hand, if the citizen does not buy the newspaper $j = \bar{\sigma}$ sold at a strictly positive price $p_{j=\bar{\sigma}} > 0$, then it must be that either he has observed a signal different from $\bar{\sigma}$ or that he has observed the signal $\bar{\sigma}$ but $p_{j=\bar{\sigma}} > \bar{e}$.

The proof for the newspaper $j = \underline{\sigma}$ is similar to the one just seen for $j = \bar{\sigma}$, so I omit it.

After deriving the equilibrium strategies, given the equilibrium beliefs, now it is necessary to do the opposite, i.e. compute the equilibrium beliefs given the equilibrium strategies. Since all the $\hat{i}_{\bar{\sigma}}$ buy the newspaper $j = \bar{\sigma}$ if it is on offer, as long as $p_{j=\bar{\sigma}} \leq \bar{e}$, while all the $\hat{i}_{\sigma \in \{\phi, \underline{\sigma}\}}$ do not, then upon observing a number $n_{\bar{\sigma}}$ of newspaper copies bought, the Incumbent can form the equilibrium beliefs on the probability of the state of the world $\bar{\theta}$, knowing that the remaining $N - n_{\bar{\sigma}}$ citizens have each observed $\sigma \in \{\phi, \underline{\sigma}\}$. Since $Pr(\bar{\sigma}|\bar{\theta}) = q$, therefore $Pr(\sigma \in \{\phi, \underline{\sigma}\}|\bar{\theta}) = 1 - Pr(\bar{\sigma}|\bar{\theta}) = 1 - q$. Of course from $Pr(\bar{\sigma}|\underline{\theta}) = q'$ it follows that $Pr(\sigma \in \{\phi, \underline{\sigma}\}|\underline{\theta}) = 1 - q'$. Then it follows that:

$$\begin{aligned} Pr(\bar{\theta}|n_{\bar{\sigma}}, N - n_{\bar{\sigma}}) &= \frac{Pr(n_{\bar{\sigma}}; N - n_{\bar{\sigma}}|\bar{\theta})Pr(\bar{\theta})}{Pr(n_{\bar{\sigma}}; N - n_{\bar{\sigma}}|\bar{\theta})Pr(\bar{\theta}) + Pr(n_{\bar{\sigma}}; N - n_{\bar{\sigma}}|\underline{\theta})Pr(\underline{\theta})} = \\ &= \frac{\frac{N!}{(n_{\bar{\sigma}})!(N-n_{\bar{\sigma}})!} q^{(n_{\bar{\sigma}})} (1-q)^{N-n_{\bar{\sigma}}} \frac{1}{2}}{\frac{N!}{(n_{\bar{\sigma}})!(N-n_{\bar{\sigma}})!} q^{(n_{\bar{\sigma}})} (1-q)^{N-n_{\bar{\sigma}}} \frac{1}{2} + \frac{N!}{(n_{\bar{\sigma}})!(N-n_{\bar{\sigma}})!} (q')^{(n_{\bar{\sigma}})} (1-q')^{N-n_{\bar{\sigma}}} \frac{1}{2}} = \\ &= \frac{\frac{N!}{(n_{\bar{\sigma}})!(N-n_{\bar{\sigma}})!} q^{(n_{\bar{\sigma}})} (1-q)^{N-n_{\bar{\sigma}}}}{\frac{N!}{(n_{\bar{\sigma}})!(N-n_{\bar{\sigma}})!} q^{(n_{\bar{\sigma}})} (1-q)^{N-n_{\bar{\sigma}}} + \frac{N!}{(n_{\bar{\sigma}})!(N-n_{\bar{\sigma}})!} (q')^{(n_{\bar{\sigma}})} (1-q')^{N-n_{\bar{\sigma}}}} \end{aligned}$$

Finally, to complete the PBE, the following out-of-equilibrium beliefs can be devised in order to sustain the equilibrium. Remember that $n_{\bar{\sigma}}^*$ is the minimum number of signals necessary to be observed to convince the Incumbent that $Pr(\bar{\theta}|n_{\bar{\sigma}}^*) > Pr(\bar{\theta}) = 1/2$ and that I label with $Pr(\bar{\theta}|n_{\bar{\sigma}}, N - n_{\bar{\sigma}})$ the equilibrium belief. If one observes a number of out-of-equilibrium signals $\tilde{n}_{\bar{\sigma}}$, then with $n_{\bar{\sigma}} \geq n_{\bar{\sigma}}^*$, $Pr(\bar{\theta}|\tilde{n}_{\bar{\sigma}}) > 1/2$, for $\tilde{n}_{\bar{\sigma}} \in [n_{\bar{\sigma}}^*, N]$, while $Pr(\bar{\theta}|\tilde{n}_{\bar{\sigma}}) < 1/2$, for $\tilde{n}_{\bar{\sigma}} \in [0, n_{\bar{\sigma}}^*)$. On the other hand, if one observes a number of out-of-equilibrium signals $\tilde{n}_{\bar{\sigma}}$, then with $n_{\bar{\sigma}} < n_{\bar{\sigma}}^*$, $Pr(\bar{\theta}|\tilde{n}_{\bar{\sigma}}) < 1/2$, for $\tilde{n}_{\bar{\sigma}} \in [0, n_{\bar{\sigma}}^*)$, while $Pr(\bar{\theta}|\tilde{n}_{\bar{\sigma}}) > 1/2$, for $\tilde{n}_{\bar{\sigma}} \in [n_{\bar{\sigma}}^*, N]$.

Furthermore if the Incumbent observes any number \tilde{n}_σ of signals $\sigma \in \{\phi, \underline{\sigma}\}$ different from $\bar{\sigma}$, he will implement the policy he believes to be optimal contingent on the “true” state of the world. In order not to allow “crazy” beliefs, I assume that if n_σ^* is the minimum number of signals to convince the Incumbent that the “true” state of the world is $\underline{\theta}$, then if $\tilde{n}_\sigma \geq n_\sigma^*$ ($\tilde{n}_\sigma < n_\sigma^*$) the Incumbent believes that $Pr(\underline{\theta}|\tilde{n}_\sigma) > \frac{1}{2}$ ($Pr(\underline{\theta}|\tilde{n}_\sigma) < \frac{1}{2}$) and implements the policy \underline{a} (\bar{a}). Similarly if the Incumbent observes any number \tilde{n}_ϕ of signals ϕ he will mix between the two policies \underline{a} and \bar{a} with any positive probability.

It is simple to show that these out-of-equilibrium beliefs sustain the equilibrium seen above. In fact, provided that the Incumbent behaves on the off-the-equilibrium path in the same way as he behaves when he observes equilibrium signals, and given that citizen’s utility from reading the newspaper is influenced only by the signal he has observed privately, then he will not change his reading behaviour by buying a newspaper carrying a signal different from the one he has privately observed, as this will give him a utility inferior to the newspaper price. Therefore, the middle class citizen will never change his newspaper buying decision and will never deviate from the equilibrium described.

As the last step of this proof one needs to consider what happens when the newspaper $j = \bar{\sigma}$ is on offer and $p_{j=\bar{\sigma}} > \bar{e}$. In this case the equilibrium strategies are that nobody buys the newspaper, given the preferences of the citizens $\hat{v}_{\sigma \in \{\phi, \underline{\sigma}\}}$ towards the newspaper on offer or the fact that the utility the $\hat{v}_{\bar{\sigma}}$ citizen receives from reading the newspaper $j = \bar{\sigma}$ is less than the price he has to pay for it. Since the incumbent does not observe any signal, this means that $Pr(\bar{\theta}|N_\phi) = Pr(\bar{\theta}) = \frac{1}{2}$, where N_ϕ is the number N of empty signals, where for the Incumbent receiving an empty signal is the same as receiving no signal at all.

In fact:

$$\begin{aligned} Pr(\bar{\theta}|N_\phi) &= \frac{Pr(N_\phi|\bar{\theta})Pr(\bar{\theta})}{Pr(N_\phi|\bar{\theta})Pr(\bar{\theta}) + Pr(N_\phi|\underline{\theta})Pr(\underline{\theta})} = \\ &= \frac{\frac{N_\phi!}{N_\phi!0!}(1-q-q')^{(N_\phi)}(q+q')^0 \frac{1}{2}}{\frac{N_\phi!}{N_\phi!0!}(1-q-q')^{(N_\phi)}(q+q')^0 \frac{1}{2} + \frac{N_\phi!}{N_\phi!0!}(1-q-q')^{(N_\phi!)}(q+q')^0 \frac{1}{2}} = \\ &= \frac{(1-q-q')^{(N_\phi)} \frac{1}{2}}{(1-q-q')^{(N_\phi)} \frac{1}{2} + (1-q-q')^{(N_\phi!)} \frac{1}{2}} = \frac{\frac{1}{2}}{1} = \frac{1}{2} \end{aligned}$$

■

So I have shown that in the case the Media Tycoon has produced a single Broadsheet $j \in \{\bar{\theta}, \underline{\theta}\}$, there is a separating equilibrium where all the citizens who have received a signal equal to the one printed in the only newspaper produced buy a copy of the newspaper, while the others do not.

Now one needs to derive the equilibrium strategy and beliefs when the Media Tycoon has produced a Tabloid. The following holds:

Lemma 5 *There exists an (separating) equilibrium where Middle Class citizens in $t = 3$ buy the produced Tabloid if and only if they have received the signal printed on the newspaper and the newspaper's price is less than the utility the citizens receive from consuming it.*

Proof. Regarding the $j = \phi$ newspaper, assume that the equilibrium strategy is such that each \hat{i}_ϕ buys the newspaper $j = \phi$, while each $\hat{i}_{\sigma \in \{\bar{\sigma}, \underline{\sigma}\}}$ does not buy it. Assuming that there are n_ϕ such citizens buying the newspaper $j = \phi$ it is straightforward to compute the following equilibrium beliefs:

$$\begin{aligned} Pr(\bar{\theta}|n_\phi, N - n_\phi) &= \frac{Pr(n_\phi, N - n_\phi|\bar{\theta})Pr(\bar{\theta})}{Pr(n_\phi, N - n_\phi|\bar{\theta})Pr(\bar{\theta}) + Pr(n_\phi, N - n_\phi|\underline{\theta})Pr(\underline{\theta})} = \\ &= \frac{\frac{N!}{n_\phi!(N - n_\phi)!}(1 - q - q')^{n_\phi}(1 - (1 - q - q'))^{N - n_\phi \frac{1}{2}}}{\frac{N!}{n_\phi!(N - n_\phi)!}(1 - q - q')^{n_\phi}(1 - (1 - q - q'))^{N - n_\phi \frac{1}{2}} + \frac{N!}{n_\phi!(N - n_\phi)!}(1 - q - q')^{n_\phi}(q + q')^{N - n_\phi \frac{1}{2}}} = \\ &= \frac{\frac{N!}{n_\phi!(N - n_\phi)!}(1 - q - q')^{n_\phi}(q + q')^{N - n_\phi \frac{1}{2}}}{\frac{N!}{n_\phi!(N - n_\phi)!}(1 - q - q')^{n_\phi}(q + q')^{N - n_\phi \frac{1}{2}} + \frac{N!}{n_\phi!(N - n_\phi)!}(1 - q - q')^{n_\phi}(q + q')^{N - n_\phi \frac{1}{2}}} = \frac{1}{2}. \end{aligned}$$

This means that if a $j = \phi$ newspaper gets produced, then for any number of copies bought by \hat{i}_ϕ citizens, this is never informative about the state of the world, since the posterior on the state of the world $\bar{\theta}$ is equal to the prior.

Knowing this I now derive citizens' equilibrium strategy regarding buying newspaper $j = \phi$. Each $\hat{i}_{\sigma \in \{\bar{\sigma}, \phi, \underline{\sigma}\}}$ citizen, after receiving the private signals $\sigma \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$, only looks at his preferences towards the Tabloid. This means that each $\hat{i}_{\sigma \in \{\bar{\sigma}, \underline{\sigma}\}}$ never buys the Tabloid $j = \phi$ since he does not receive any (strictly) positive utility from reading it and he would have to bear a (strictly) positive cost $p_\phi > 0$ if he were to buy it. On the other hand, any citizen \hat{i}_ϕ always buys the the Tabloid $j = \phi$, as long as $\bar{e} \geq p_\phi$. Conversely, given that some citizens buy the newspaper $j = \phi$, they must be \hat{i}_ϕ citizens whose utility from reading the Tabloid \bar{e} is more than their price p_ϕ . In fact all the other citizens do not read the Tabloid, either because they are $\hat{i}_{\sigma \in \{\bar{\sigma}, \underline{\sigma}\}}$ or because are \hat{i}_ϕ citizens with $\bar{e} < p_\phi$.

Finally, in order to sustain the equilibrium, the following out-of-equilibrium-beliefs can be devised. Given that the equilibrium belief is equal to n_ϕ , if the Incumbent observes any number \tilde{n}_ϕ of signals ϕ , then he believes that the state of the world is $Pr(\bar{\theta}|n_\phi, N - n_\phi) = \frac{1}{2}$ and therefore mixes between the two policies with any probability. Similarly to the out-of-equilibrium-beliefs devised in Lemma (4), I assume that if the Incumbent observes any number \tilde{n}_σ of signals $\sigma \in \{\bar{\sigma}, \underline{\sigma}\}$ he will adopt any policy he believes to be optimal, contingent on the "true" state of the world. In order not to allow "crazy" beliefs, I assume that if $n_\sigma^*(n_\sigma^*)$ is the minimum number of signals to convince the Incumbent that the "true" state of the world is $\bar{\theta}(\underline{\theta})$, then if $\tilde{n}_\sigma \geq n_\sigma^*$ ($\tilde{n}_\sigma \geq n_\sigma^*$) the Incumbent believes that $Pr(\bar{\theta}|\tilde{n}_\sigma) > \frac{1}{2}$ ($Pr(\underline{\theta}|\tilde{n}_\sigma) > \frac{1}{2}$) and implements the policy $\bar{a}(\underline{a})$.

Nevertheless these out-of-equilibrium beliefs sustain the equilibrium seen above. In fact, provided that the Incumbent behaves on the off-the-equilibrium path in the same way as he behaves on the equilibrium path, and given that citizen's utility from reading the newspaper is influenced only by the signal

he has observed privately, then he will not change his reading behaviour by buying a newspaper carrying a signal different from the one he has privately observed, as this will give him a utility inferior to the newspaper price. Therefore, the middle class citizen will never change his newspaper buying decision and will never deviate from the equilibrium described. ■

Putting together the Lemma 4 and the Lemma 5, the following holds:

Proposition 6 *In $t = 3$ there exists an equilibrium where Middle Class citizens buy the produced newspaper if and only if they have privately observed the same signal printed on the newspaper and the price they pay for buying it is less than the utility they derive from reading it.*

Proof. The proof follows immediately by considering the two Lemmata previously proved. In fact Lemma (4) shows that, provided a Broadsheet $j = \bar{\sigma}(\underline{\sigma})$ is produced and available to consumers, then all the Middle Class citizens who have privately observed the signal $\bar{\sigma}(\underline{\sigma})$ buy the newspaper, while the other do not. Likewise Lemma (5) shows that when a Tabloid (i.e. an uninformative newspaper with a ϕ printed signal) is produced, the Middle Class citizens buy the newspaper if and only if they have privately observed the signal ϕ . The above behaviour holds as long as the price the citizens pay to buy the newspaper is less than the utility they derive from reading it.

Therefore, one can conclude that the same behaviour holds for all the Middle Class citizens, for any signal they receive and any newspaper available to them, provided that it gets produced by the Media Tycoon. ■

The previous Proposition has shown that there exists a unique separating equilibrium where each Middle Class citizen buys the produced newspaper if and only if that newspaper has printed a signal coherent with his preferences, and therefore with the signal he has privately observed, as long as reading the newspaper is “enjoyable” enough. Otherwise citizens do not buy the newspaper if their preferences do not find an outlet to be represented or expressed.

Crucially this consuming behaviour has an important informative content. In fact the reading behaviour shown in Proposition (6) means that an external observer (in particular the Incumbent) can be sure that the quantity of newspapers bought by the citizens is informative about the number of signals they have received and, as a consequence, about the “true”, underlying, state of the world. As a consequence, apart from revealing the tastes of the polity regard the newspaper types, the demand for newspaper can be of great value for learning something more about the state of the world that the Incumbent does not already know.

The existence of the unique equilibrium in the citizens’ behaviour allows to derive the minimum number of signals $\bar{\sigma}(\underline{\sigma})$ such that, upon observing it, the Incumbent believes that the state of the world $\bar{\theta}$ is more likely than $\underline{\theta}$. Needless to say this additional knowledge derived purely from the citizens’ reading behaviour has an important effect on policy implementation, as the following Proposition shows:

Corollary 7 Upon observing a number of signals $n_{\bar{\sigma}} > \frac{\log_{q/q'}\left(\frac{1-q'}{1-q}\right)}{1+\log_{q/q'}\left(\frac{1-q'}{1-q}\right)}N$ the Incumbent believes that the state of the world $\bar{\theta}$ is more likely than the state of the world $\underline{\theta}$ and therefore he implements the policy \bar{a} with probability one. Likewise if the Incumbent observes $n_{\underline{\sigma}} > \frac{\log_{q/q'}\left(\frac{1-q'}{1-q}\right)}{1+\log_{q/q'}\left(\frac{1-q'}{1-q}\right)}N$ he implements the policy \underline{a} with probability one.

Proof. Let $n_{\bar{\sigma}}^*$ be the minimum number of signals $\bar{\sigma}$ such that, upon observing it, an external observer believes that the probability of $\bar{\theta}$ is larger than $\frac{1}{2}$. Formally, given $Pr(\bar{\theta}|n_{\bar{\sigma}}^*, N - n_{\bar{\sigma}}^*)$, by applying the Bayes's rule it follows simply that:

$$\begin{aligned} & Pr(\bar{\theta}|n_{\bar{\sigma}}^*, N - n_{\bar{\sigma}}^*) = \\ &= \frac{\frac{N!}{(n_{\bar{\sigma}}^*)!(N-n_{\bar{\sigma}}^*)!} q^{n_{\bar{\sigma}}^*} (1-q)^{N-n_{\bar{\sigma}}^*} \frac{1}{2}}{\frac{N!}{(n_{\bar{\sigma}}^*)!(N-n_{\bar{\sigma}}^*)!} q^{n_{\bar{\sigma}}^*} (1-q)^{N-n_{\bar{\sigma}}^*} \frac{1}{2} + \frac{N!}{(n_{\bar{\sigma}}^*)!(N-n_{\bar{\sigma}}^*)!} (q')^{n_{\bar{\sigma}}^*} (1-q')^{N-n_{\bar{\sigma}}^*} \frac{1}{2}} \end{aligned}$$

After some simple algebra the previous formula reduces to

$$\frac{q^{n_{\bar{\sigma}}^*} (1-q)^{N-n_{\bar{\sigma}}^*}}{q^{n_{\bar{\sigma}}^*} (1-q)^{N-n_{\bar{\sigma}}^*} + (q')^{n_{\bar{\sigma}}^*} (1-q')^{N-n_{\bar{\sigma}}^*}}.$$

Now it suffices to see when $\frac{q^{n_{\bar{\sigma}}^*} (1-q)^{N-n_{\bar{\sigma}}^*}}{q^{n_{\bar{\sigma}}^*} (1-q)^{N-n_{\bar{\sigma}}^*} + (q')^{n_{\bar{\sigma}}^*} (1-q')^{N-n_{\bar{\sigma}}^*}} \geq \frac{1}{2}$. By multiplying out the denominator of the LHS, and simplifying terms, one obtains that $q^{n_{\bar{\sigma}}^*} (1-q)^{N-n_{\bar{\sigma}}^*} \geq (q')^{n_{\bar{\sigma}}^*} (1-q')^{N-n_{\bar{\sigma}}^*}$ or $\left(\frac{q}{q'}\right)^{n_{\bar{\sigma}}^*} \geq \left(\frac{1-q'}{1-q}\right)^{N-n_{\bar{\sigma}}^*}$. Apply then the increasing monotonic function $\log_{q/q'}$ to both sides of the inequality and obtain that $\log_{q/q'}\left(\frac{q}{q'}\right)^{n_{\bar{\sigma}}^*} \geq \log_{q/q'}\left(\frac{1-q'}{1-q}\right)^{N-n_{\bar{\sigma}}^*}$ or $n_{\bar{\sigma}}^* \geq \log_{q/q'}\left(\frac{1-q'}{1-q}\right)^{N-n_{\bar{\sigma}}^*}$. With further manipulation it follows that $n_{\bar{\sigma}}^* \geq (N - n_{\bar{\sigma}}^*) \log_{q/q'}\left(\frac{1-q'}{1-q}\right)$ and solve finally conclude that $n_{\bar{\sigma}}^* \geq \frac{\log_{q/q'}\left(\frac{1-q'}{1-q}\right)}{1+\log_{q/q'}\left(\frac{1-q'}{1-q}\right)}N$.

Therefore when $\frac{\log_{q/q'}\left(\frac{1-q'}{1-q}\right)}{1+\log_{q/q'}\left(\frac{1-q'}{1-q}\right)}N \notin \mathbb{N}_0$, if the Incumbent observes the number of newspapers $\tilde{n}_{\bar{\sigma}}$ is greater or equal to $n_{\bar{\sigma}}^*$, he updates his posterior beliefs and infers that the probability of the state of the world being $\bar{\theta}$ is larger than $\frac{1}{2}$. Since $Pr(\bar{\theta}|\tilde{n}_{\bar{\sigma}}, N - \tilde{n}_{\bar{\sigma}}) > \frac{1}{2}$, the Incumbent politician will implement the policy \bar{a} with probability one. Of course if $\tilde{n}_{\bar{\sigma}} < n_{\bar{\sigma}}^*$, then $Pr(\bar{\theta}|\tilde{n}_{\bar{\sigma}}, N - \tilde{n}_{\bar{\sigma}}) < \frac{1}{2}$ and the Incumbent politician implements \underline{a} with probability one. Instead when $\frac{\log_{q/q'}\left(\frac{1-q'}{1-q}\right)}{1+\log_{q/q'}\left(\frac{1-q'}{1-q}\right)}N \in \mathbb{N}_0$, and $\tilde{n}_{\bar{\sigma}} = n_{\bar{\sigma}}^*$, $Pr(\bar{\theta}|n_{\bar{\sigma}}^*, N - n_{\bar{\sigma}}^*) = \frac{1}{2}$ and the Incumbent mixes between policies with any non negative probability. Of course when $\tilde{n}_{\bar{\sigma}} > n_{\bar{\sigma}}^*$, since $Pr(\bar{\theta}|\tilde{n}_{\bar{\sigma}}, N - \tilde{n}_{\bar{\sigma}}) > \frac{1}{2}$, the Incumbent politician will implement the policy \bar{a} with probability one.

A similar discussion can be made if a Broadsheet $j = \underline{\sigma}$ is produced. It is easy to see that:

$$\begin{aligned} Pr(\underline{\theta}|n_{\underline{\sigma}}^*, N - n_{\underline{\sigma}}^*) &= \frac{Pr(n_{\underline{\sigma}}^*, N - n_{\underline{\sigma}}^*|\underline{\theta})Pr(\underline{\theta})}{Pr(n_{\underline{\sigma}}^*, N - n_{\underline{\sigma}}^*|\underline{\theta})Pr(\underline{\theta}) + Pr(n_{\underline{\sigma}}^*, N - n_{\underline{\sigma}}^*|\bar{\theta})Pr(\bar{\theta})} = \\ &= \frac{\frac{N!}{n_{\underline{\sigma}}^*(N-n_{\underline{\sigma}}^*)!}q^{n_{\underline{\sigma}}^*}(1-q)^{N-n_{\underline{\sigma}}^*\frac{1}{2}}}{\frac{N!}{n_{\underline{\sigma}}^*(N-n_{\underline{\sigma}}^*)!}q^{n_{\underline{\sigma}}^*}(1-q)^{N-n_{\underline{\sigma}}^*\frac{1}{2}} + \frac{N!}{n_{\underline{\sigma}}^*(N-n_{\underline{\sigma}}^*)!}(q')^{n_{\underline{\sigma}}^*}(1-q')^{N-n_{\underline{\sigma}}^*\frac{1}{2}}} \end{aligned}$$

is the same expression for $Pr(\bar{\theta}|n_{\underline{\sigma}}^*, N - n_{\underline{\sigma}}^*)$, apart from having $n_{\underline{\sigma}}^*$ instead of $n_{\bar{\sigma}}^*$. This means that, without further calculation, it is possible to conclude that the minimum number of signals $n_{\underline{\sigma}}^*$ to be observed in equilibrium by the Incumbent to convince him that the state of the world $\underline{\theta}$ is more likely than

$$\text{the state of the world } \bar{\theta} \text{ is } n_{\underline{\sigma}}^* = \frac{\log_{q/q'}\left(\frac{1-q'}{1-q}\right)}{1+\log_{q/q'}\left(\frac{1-q'}{1-q}\right)}N. \quad \blacksquare$$

Now it remains to analyse what happens in $t = 2$, i.e. at the newspaper production decision stage on part of the Media Tycoon (MT). Remember that the MT does not have any interest in which policy is adopted by the Incumbent. Therefore in the following discussion I focus on the entrepreneur's decision, considering the expected profit he can obtain from the newspaper production only.

The problem the MT is facing is the following: knowing the probability distribution of signals among the Middle Class citizens, he has to estimate the demand for each newspaper type, and then he has to decide whether to produce a Tabloid ($j = \phi$) or a Broadsheet newspaper ($j \in \{\bar{\sigma}, \underline{\sigma}\}$). In this latter case, the MT has to decide which policy side of the debate he wants to give a channel to. Remember that the MT knows that each reader's utility is $u_i(j|\sigma) = \bar{e}$, for each $j \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$ and for each $i \in M$ group iff $j = \sigma$, while $u_i(j|\sigma) = 0$ iff $j \neq \sigma$. Therefore he has to estimate the demand for each newspaper $j \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$ in order to be able to maximise his profit, knowing that the newspaper demand will depend on the realisation of a random variable, that is the probability describing the signal distribution.

While in a context of certainty there is no difference in maximising the expected profit with respect to price or with respect to quantity, this equivalence does not hold necessarily when a random demand is considered. In fact when one looks at the quantity of goods delivered by the MT, it might be that the production is smaller (larger) than the quantity demanded and bought by consumers. For instance the production might be less than what will be demanded and consumed at market prices, because the demand has experienced an unexpected peak. On the other hand, it might be that the production is larger than what will be actually bought and consumed because there is an unanticipated change in consumers' tastes or a slowdown in the economy that makes that good less consumed than it used to be. In cases like the previous ones it is necessary for the MT to decide the right instrument to use in order to maximise his expected profit. In particular, whether price or quantity is the correct instrument to use.

Following (Leland 1972) and (Harris and Raviv 1981) I assume that, if there is no production capacity constraint in the newspaper industry and if the quantity actually demanded is revealed after the price is decided, then it is optimal to maximize expected profit with respect to quantity. Regarding the first assumption, it seems coherent with casual observation and anecdotal evidence that supplementary newspaper production can be easily carried out if the market experiences additional and unexpected demand.⁷ On the other hand, unsold copies of newspapers can be (almost at no cost) freely disposed of. Regarding the second assumption, it seems natural to assume that the price is decided before the quantity is produced and sold, given that the newspaper price is known both to MT and customers before both consumers and MT know if and how many citizens will buy the paper.

After this discussion I can now derive the expression for the Expected Profit Π_j^E for each newspaper $j \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$:

Lemma 8 *The expected profit of producing a Broadsheet newspaper is the same regardless of the policy option the newspaper supports and it is equal to $(\bar{e} - \bar{c})\frac{1}{2}N(q + q') - \bar{F}$, while the expected profit of producing a Tabloid is equal to $(\bar{e}' - \bar{c}')N(1 - q - q') - \bar{F}$.*

Proof. First one needs to derive the demand curve for any newspaper. I start with the $j = \bar{\sigma}$ Broadsheet and then similarly I derive the expression for the other newspapers.

It is straightforward to see that the inverse demand curve for the newspaper $j = \bar{\sigma}$ is constant for any level of newspaper production $y_{j=\bar{\sigma}}$. This follows easily from the fact that each of the middle class citizen has the same willingness to pay for the newspaper. Therefore: $P(Q_{j=\bar{\sigma}}) = p_{j=\bar{\sigma}}(y_{j=\bar{\sigma}}) = \bar{e}$. Having established this, the expected profit expression $\Pi_{j=\bar{\sigma}}^E$ for the $j = \bar{\sigma}$ Broadsheet can now be written as:

$$\begin{aligned}\Pi_{j=\bar{\sigma}}^E &= E_{y_{j=\bar{\sigma}}} [p_{j=\bar{\sigma}}(y_{j=\bar{\sigma}}) * y_{j=\bar{\sigma}} - C(y_{j=\bar{\sigma}})] = \\ &= E_{y_{j=\bar{\sigma}}} [\bar{e} * y_{j=\bar{\sigma}} - c_{j=\bar{\sigma}} y_{j=\bar{\sigma}} - F_{j=\bar{\sigma}}] = \\ &= E_{y_{j=\bar{\sigma}}} [\bar{e} * y_{j=\bar{\sigma}} - \bar{c} * y_{j=\bar{\sigma}} - \bar{F}]\end{aligned}$$

where $E_{y_{j=\bar{\sigma}}}$ is the expectation operator with respect to the random newspaper quantity $y_{j=\bar{\sigma}}$. Since the quantities \bar{e} , \bar{F} , \bar{c} are fixed and do not depend on the realisation of the signals, I can rewrite the expression for the expected profit taking these quantities out of the expectation operator and rewriting $\Pi_{j=\bar{\sigma}}^E$ as: $\Pi_{j=\bar{\sigma}}^E = (\bar{e} - \bar{c})E_{y_{j=\bar{\sigma}}}[y_{j=\bar{\sigma}}] - \bar{F}$. It turns out that $E_{y_{j=\bar{\sigma}}}[y_{j=\bar{\sigma}}] = E_{y_{j=\bar{\sigma}}}[Pr(n_{\bar{\sigma}}, N - n_{\bar{\sigma}}|\theta)] = E_{y_{j=\bar{\sigma}}}[Pr(n_{\bar{\sigma}}, N - n_{\bar{\sigma}}|\bar{\theta})Pr(\bar{\theta}) + Pr(n_{\bar{\sigma}}, N - n_{\bar{\sigma}}|\underline{\theta})Pr(\underline{\theta})] =$

⁷For instance think of a second edition of a newspaper whenever the readership is larger than anticipated. Or think of a special edition of national newspaper that can be printed and sold few hours after some extraordinary and unexpected event has happened, like the death of J.F.K. or Lady D.

$= E_{y_j=\bar{\sigma}}[\frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!}q^{n_{\bar{\sigma}}}(1-q)^{N-n_{\bar{\sigma}}}\frac{1}{2} + \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!}(q')^{n_{\bar{\sigma}}}(1-q')^{N-n_{\bar{\sigma}}}\frac{1}{2}] = \frac{1}{2}N(q+q')$. Plugging the expression for $E_{y_j=\bar{\sigma}}[y_j=\bar{\sigma}]$ in the expected profit expression, one obtains that:

$$\Pi_{j=\bar{\sigma}}^E = (\bar{e} - \bar{c})\frac{1}{2}N(q+q') - \bar{F}$$

Following the same reasoning as the one above, it can be seen easily that $\Pi_{j=\underline{\sigma}}^E$, i.e. the expected profit expression for the $j = \underline{\sigma}$ Broadsheet, is the same as the one for $\Pi_{j=\bar{\sigma}}^E$. Therefore:

$$\Pi_{j=\underline{\sigma}}^E = (\bar{e} - \bar{c})\frac{1}{2}N(q+q') - \bar{F}$$

Finally the same procedure can be repeated to derive the expression for $\Pi_{j=\phi}^E$. Again $P(Q_{j=\phi}) = p_{j=\phi}(y_{j=\phi}) = \bar{e}$ and therefore $\Pi_{j=\phi}^E = E_{y_j=\phi}[p_{j=\phi}(y_{j=\phi}) * y_{j=c} - C(y_{j=\phi})] = \Pi_{j=\bar{\sigma}}^E[\bar{e} * y_{j=\phi} - \bar{c} * y_{j=\phi} - \bar{F}]$. Following the same reasoning already seen, the expression of the expected profit of the Tabloid can be rewritten as $(\bar{e} - \bar{c})E_{y_j=\phi}[y_{j=\phi}] - \bar{F}$. It is straightforward to derive the expression for $E_{y_j=\phi}[y_{j=\phi}]$. In fact $E_{y_j=\phi}[y_{j=\phi}] = E_{y_j=\phi}[Pr(n_{\phi}, N - n_{\phi}|\theta)] = E_{y_j=\phi}[Pr(n_{\phi}, N - n_{\phi}|\bar{\theta})Pr(\bar{\theta}) + Pr(n_{\phi}, N - n_{\phi}|\underline{\theta})Pr(\underline{\theta})] = \frac{1}{2}N(1 - q - q') + \frac{1}{2}N(1 - q - q') = N(1 - q - q')$. Again by plugging the expression for $E_{y_j=\phi}[y_{j=\phi}]$ into the expression for the Tabloid's expected profit, one obtains that:

$$\Pi_{j=\phi}^E = (\bar{e} - \bar{c})N(1 - q - q') - \bar{F}$$

■

Having derived the expression for the expected profit of each of the three newspapers that might be produced, the following Proposition highlights the determinants of the production of the Broadsheet instead of the Tabloid in the newspaper production decision stage of the game:

Proposition 9 *In $t = 2$ the MT produces a Broadsheet iff the total probability of obtaining signals that are informative on the state of the world ($\sigma \in \{\bar{\sigma}, \underline{\sigma}\}$) instead of blank and uninformative signals ($\sigma \in \{\phi\}$) is greater than $2/3$.*

Proof. Knowing that the citizens willingness to pay is constant to \bar{e} for any quantity and any kind of (produced) newspaper, and having estimated the expected profit for each $j \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$ (see Lemma (8)), the MT may now decide which type of newspaper it is optimal to produce, depending on the expected profit.

Given $\Pi_{j=\phi}^E = (\bar{e} - \bar{c})N(1 - q - q') - \bar{F}$ and $\Pi_{j=\bar{\sigma}}^E = \Pi_{j=\underline{\sigma}}^E = (\bar{e} - \bar{c})\frac{1}{2}N(q + q') - \bar{F}$, it turns out that producing a Broadsheet is optimal iff $(\bar{e} - \bar{c})\frac{1}{2}N(q + q') - \bar{F} \geq (\bar{e} - \bar{c})N(1 - q - q') - \bar{F}$ or $\frac{1}{2}(q + q') \geq (1 - q - q')$ from which one obtains that producing a Broadsheet is optimal iff $q + q' \geq \frac{2}{3}$. ■

Finally in $t = 1$, signals are sent and in $t = 0$ Nature chooses the state of the world with equal probability.

It is interesting to comment on Proposition 9. This Proposition says that the Broadsheet production takes place iff the environment is informative enough. Notice that the informativeness of the environment depends on the sum of the probabilities of “correct” signals (i.e. q) and “not correct” signals (i.e. q'). Therefore producing a Broadsheet on part of the MT maximises his expected profit if both q and q' are relatively large. This means that a Broadsheet can be produced even if there is little difference between the probability of “correct” and “wrong” signals (i.e. $q - q'$ is small) as long as both q and q' are large enough (i.e. $q + q'$ is close to 1). Notice also that, whenever a Broadsheet newspaper is produced, if there is no difference in the Broadsheet’s fixed and variable cost or in the utility the citizen derives from either Broadsheet, then the MT is indifferent between the production of any $j \in \{\bar{\sigma}, \underline{\sigma}\}$.

The next Proposition sums up the characteristics of the equilibrium of this game, focussing on the difference and the tension between the two features one can envisage in the Media. In fact, on one hand the Media can be regarded as any other for-profit enterprise: its fundamental objective is profit maximising. However, on the other hand, it has a considerable and important “social” role, given that favours communication between citizens and Incumbent and increases the probability of optimal policy implementation.

Proposition 10 *The presence of a Broadsheet always enhances the optimal policy decision and increases the total probability of implementing good policies. If producing a Broadsheet is profitable then the improvement of optimal policy making is possible, regardless of the policy side the Broadsheet chooses to support.*

Proof. I first compute the probability that, given a Broadsheet is produced, the “right” policy contingent on the state of the world is implemented. Focussing on the $j = \bar{\sigma}$ Broadsheet, this is equal to the cumulative probability that the number of signals $n_{\bar{\sigma}}$ is larger or equal to $n_{\bar{\sigma}}^*$, given that the state of the world is $\bar{\theta}$, plus the cumulative probability that the number of signals $n_{\bar{\sigma}}$ is less than $n_{\bar{\sigma}}^*$, given that the state of the world is $\underline{\theta}$ ⁸. Therefore the

⁸To simplify the discussion I assume that $\log_{q/q'}(\frac{1-q'}{1-q}) \notin \mathbb{N}_0$; see above the implications of this.

probability of optimal policy implementation is equal to:

$$\begin{aligned}
& Pr(n_{\bar{\sigma}} \geq n_{\bar{\sigma}}^* | \bar{\theta}) Pr(\bar{\theta}) + Pr(n_{\bar{\sigma}} < n_{\bar{\sigma}}^* | \underline{\theta}) Pr(\underline{\theta}) = \\
& \sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} q^{n_{\bar{\sigma}}} (1-q)^{N-n_{\bar{\sigma}}} \frac{1}{2} + \sum_{n_{\bar{\sigma}}=0}^{n_{\bar{\sigma}}^*-1} \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}} (1-q')^{N-n_{\bar{\sigma}}} \frac{1}{2} = \\
& \frac{1}{2} \left(\sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} q^{n_{\bar{\sigma}}} (1-q)^{N-n_{\bar{\sigma}}} + \sum_{n_{\bar{\sigma}}=0}^{n_{\bar{\sigma}}^*-1} \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}} (1-q')^{N-n_{\bar{\sigma}}} \right)
\end{aligned} \tag{1}$$

To show that having a Broadsheet increases the probability of implementing the optimal policy contingent on the state of the world, it suffices to compare the eq. (1) with the probability of implementing the optimal policy when no Broadsheet is present, i.e. when $Pr(\bar{\theta}) = Pr(\underline{\theta}) = \frac{1}{2}$. In Corollary (3) it was shown that the probability of implementing the optimal policy in this case is equal to $\frac{1}{2}$. Therefore it suffices to compute when, if a Broadsheet is produced, eq.(1) is greater than $\frac{1}{2}$. Or, likewise,

$$\text{when } \sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} q^{n_{\bar{\sigma}}} (1-q)^{N-n_{\bar{\sigma}}} + \sum_{n_{\bar{\sigma}}=0}^{n_{\bar{\sigma}}^*-1} \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}} (1-q')^{N-n_{\bar{\sigma}}} > 1.$$

$$\begin{aligned}
& \text{Given that } \sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}} (1-q')^{N-n_{\bar{\sigma}}} + \sum_{n_{\bar{\sigma}}=0}^{n_{\bar{\sigma}}^*-1} \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}} (1- \\
& q')^{N-n_{\bar{\sigma}}} = 1, \text{ and } \sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}} (1-q')^{N-n_{\bar{\sigma}}} = 1 - \sum_{n_{\bar{\sigma}}=0}^{n_{\bar{\sigma}}^*-1} \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}} (1- \\
& q')^{N-n_{\bar{\sigma}}} \text{ it follows that eq.(1) is greater than } \frac{1}{2}, \text{ iff}
\end{aligned}$$

$$\sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} q^{n_{\bar{\sigma}}} (1-q)^{N-n_{\bar{\sigma}}} + 1 - \sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}} (1-q')^{N-n_{\bar{\sigma}}} >$$

1, that is iff $\sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} q^{n_{\bar{\sigma}}} (1-q)^{N-n_{\bar{\sigma}}} > \sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}} (1-q')^{N-n_{\bar{\sigma}}}$. But this is always true given it was previously shown that, following the application of Bayes's rule, $\frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} q^{n_{\bar{\sigma}}} (1-q)^{N-n_{\bar{\sigma}}} > \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}} (1-q')^{N-n_{\bar{\sigma}}}$ for each $n_{\bar{\sigma}} \geq n_{\bar{\sigma}}^*$. Since $\sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} q^{n_{\bar{\sigma}}} (1-q)^{N-n_{\bar{\sigma}}}$ is the sum of $N - n_{\bar{\sigma}}^* + 1$ addenda all having the property above, then the thesis follows.

To prove the second part of the Proposition, remember that the improvement in the decision making quality happens when a Broadsheet $j = \bar{\sigma}$ gets produced as it has just been shown. This depends, in turn, on the total probability of receiving the signal $\bar{\sigma}$ being large enough. In fact the MT produces a Broadsheet iff $q + q' > \frac{2}{3}$ as shown in Proposition (9). Therefore, in this case, the probability of optimal policy implementation is larger than when a

Tabloid is produced or when no signal is available. On the other hand, the MT produces a Tabloid if and only if $q + q' < \frac{2}{3}$. Although a Tabloid is produced in this case, a Broadsheet would be beneficial from the perspective of increasing the probability of optimal policy making. In fact when a Tabloid is produced the probability of optimal policy implementation is equal to $\frac{1}{2}$ only. Finally the MT is indifferent between producing a Broadsheet and a Tabloid if and only if $q + q' = \frac{2}{3}$.

Regarding the proof for the $j = \underline{\sigma}$ Broadsheet, remember from Corollary (7) that $n_{\underline{\sigma}}^* = n_{\underline{\sigma}}^*$. It is easy to see that eq. (1) can be rewritten in terms of $n_{\underline{\sigma}}^*$, that is:

$$\begin{aligned}
& Pr(n_{\underline{\sigma}} \geq n_{\underline{\sigma}}^* | \underline{\theta}) Pr(\underline{\theta}) + Pr(n_{\underline{\sigma}} < n_{\underline{\sigma}}^* | \bar{\theta}) Pr(\bar{\theta}) = \\
& \sum_{n_{\underline{\sigma}}=n_{\underline{\sigma}}^*}^N \frac{N!}{n_{\underline{\sigma}}!(N-n_{\underline{\sigma}})!} q^{n_{\underline{\sigma}}}(1-q)^{N-n_{\underline{\sigma}}} \frac{1}{2} + \sum_{n_{\underline{\sigma}}=0}^{n_{\underline{\sigma}}^*-1} \frac{N!}{n_{\underline{\sigma}}!(N-n_{\underline{\sigma}})!} (q')^{n_{\underline{\sigma}}}(1-q')^{N-n_{\underline{\sigma}}} \frac{1}{2} = \\
& \frac{1}{2} \left(\sum_{n_{\underline{\sigma}}=n_{\underline{\sigma}}^*}^N \frac{N!}{n_{\underline{\sigma}}!(N-n_{\underline{\sigma}})!} q^{n_{\underline{\sigma}}}(1-q)^{N-n_{\underline{\sigma}}} + \sum_{n_{\underline{\sigma}}=0}^{n_{\underline{\sigma}}^*-1} \frac{N!}{n_{\underline{\sigma}}!(N-n_{\underline{\sigma}})!} (q')^{n_{\underline{\sigma}}}(1-q')^{N-n_{\underline{\sigma}}} \right)
\end{aligned} \tag{2}$$

Given that $n_{\underline{\sigma}}^* = n_{\underline{\sigma}}^*$ it is straightforward to conclude that the above expression, describing the probability of optimal policy adoption when a $j = \underline{\sigma}$ Broadsheet is produced, is the same as the expression in eq. (1). This means that producing any Broadsheet is the same in terms of optimal policy implementation. Nevertheless producing any Broadsheet improves the quality of policy making with respect to a Tabloid. ■

It is useful to remark that the possibility for a Broadsheet to be produced depends on the sum of both the probabilities of “right” and “wrong” signals arriving, i.e. on the the total informativeness of the environment. Since the policy quality, i.e. the probability of implementing optimal policy contingent on the state of the world, depends on the Broadsheet being produced, it might happen that, given $q + q' < \frac{2}{3}$, an increase in q (i.e. in the probability of signals coherent with the true state of the world) brings about the possibility of the production of a Broadsheet if $q + q'$ becomes greater than $\frac{2}{3}$. More counterintuitively, the same could happen when an increase in q' (i.e. in the probability of signals not coherent with the true state of the world) takes place. In fact, although an increase in q' makes the signals less precise about the state of the world, it does make possible the production of a Broadsheet. The MT’s decision to supply an informative newspaper for reasons of profit maximisation, in turns, increases the probability of optimal policy implementation.

Surprisingly, it does not matter which of the two policy options the only Broadsheet produced favours or if the side of the policy the Broadsheet supports is the optimal one. In fact, all that matters is that an informative newspaper, i.e. a Broadsheet, gets produced; this will increase the dispersed information the Incumbent can receive, as a consequence, it will increase the

probability of optimal policy adoption and the quality of decision making.

This is an interesting result in this model where citizens make themselves heard and are active in the “public sphere” through a “market” intermediary, rather than directly. This modelling leads to different results from the ones usually reached by the literature on “voice” such as the one by (Lohmann 1993) and (Lohmann 1994), and reviewed in (Piketty 1999). In this literature the possibility of “voice” à la (Hirschman 1970), i.e. of citizens expressing their views, preferences, or information in the community they are in, is done through their vote in election or through protests, strikes, petitions and so on. In other words, in this literature, citizens act directly on their own to try and influence the policy decisions of the Incumbent politician. However, to the best of my knowledge, in this literature what happens when there is an institutional intermediary (Media/Newspaper) that gives the citizens the possibility of expressing their views has never been modelled.⁹ Moreover, in this article, I have analysed how this institutional intermediary behaves when it allows the citizens to express their views not for free, or because the institution wants to maximise social welfare, but because in doing so the institution gains some profit from it. Instead of a generic institution, I have chosen to model the role of a Media Tycoon and his incentives regarding which newspaper he chooses to produce (Broadsheet or Tabloid). Furthermore I have shown how the choice between the two newspapers, and therefore their editorial content, may favour or hinder the probability of optimal policy implementation. Finally, given the preferences of the MT, the MT always prefers a Broadsheet to be produced since this increases the probability of optimal policy implementation. However, he is indifferent towards which Broadsheet to produce, since the production of either Broadsheet increases the decision making quality in the same way.

Similarly to Proposition (10), where I have shown that the probability of optimal policy implementation increases when a Broadsheet is produced, one can study the probability of errors in the decision making when either of the two newspaper types is produced:

Corollary 11 *The total weighted probability of error in implementing the optimal policy is larger when a Tabloid is produced than when a Broadsheet is produced.*

Proof. In the Proof of Proposition 10 I have derived the expression for the probability of optimal policy implementation, contingent on the Broadsheet

⁹Of course the literature on lobbying is a notable exception to this. However, in that literature, authors model lobbies as groups of citizens (or entrepreneurs) who contribute to an institution having as its only objective the one of influencing the incumbent politician regarding the implementable policy. Since lobbies are "owned" directly by members they represent their preferences, once members have solved their coordination problem. Furthermore lobbies are "not-market" institutions, since they do not have to make a profit, differently from newspaper.

$j = \bar{\sigma}$ being produced. This is equal to:

$$\begin{aligned} & Pr(n_{\bar{\sigma}} \geq n_{\bar{\sigma}}^*|\bar{\theta})Pr(\bar{\theta}) + Pr(n_{\bar{\sigma}} < n_{\bar{\sigma}}^*|\underline{\theta})Pr(\underline{\theta}) = \\ &= \frac{1}{2} \left(\sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} q^{n_{\bar{\sigma}}}(1-q)^{N-n_{\bar{\sigma}}} + \sum_{n_{\bar{\sigma}}=0}^{n_{\bar{\sigma}}^*-1} \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}}(1-q')^{N-n_{\bar{\sigma}}} \right) \end{aligned} \quad (3)$$

In the same way as computing eq. (3) it is possible to compute the probability of error in the policy implementation. Remember that the statistical theory of hypothesis testing distinguishes two different Error Types: Type I Error and Type II Error. Type I Error is the probability of rejecting a hypothesis which is true, while Type II Error is the probability of accepting (or failing to reject) a hypothesis when the alternative is true (or when one should reject it). In this context, contingent on the $j = \bar{\sigma}$ being produced, Type I Error is the probability of implementing the policy \underline{a} when the optimal policy is \bar{a} . This is equal to:

$$Type I Error : Pr(n_{\bar{\sigma}} < n_{\bar{\sigma}}^*|\bar{\theta}) = \sum_{n_{\bar{\sigma}}=0}^{n_{\bar{\sigma}}^*-1} \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q)^{n_{\bar{\sigma}}}(1-q)^{N-n_{\bar{\sigma}}} \quad (4)$$

Likewise, the Type II Error is the probability of implementing the policy \bar{a} when the optimal policy is \underline{a} . This is equal to: *Type*

$$II Error : Pr(n_{\bar{\sigma}} \geq n_{\bar{\sigma}}^*|\underline{\theta}) = \sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}}(1-q')^{N-n_{\bar{\sigma}}} \quad (5)$$

Again similar expressions for Type I and Type II Errors can be derived when a Broadsheet $j = \underline{\sigma}$ is produced.

Regarding the Tabloid production, since in this case the Incumbent does not learn anything new about the state of the world, and $Pr(\bar{\theta}|n_{\phi}, N-n_{\phi}) = Pr(\bar{\theta}) = \frac{1}{2}$, the probability of optimal policy implementation is equal to $\Gamma^*(\bar{a}|\bar{\theta})Pr(\bar{\theta}) + (1 - \Gamma^*(\bar{a}|\underline{\theta}))Pr(\underline{\theta}) = \Gamma^*(\bar{a})\frac{1}{2} + (1 - \Gamma^*(\bar{a}))\frac{1}{2} = \frac{1}{2}$ with $\Gamma^*(\bar{a})$ being the probability of implementing the policy \bar{a} . With the Tabloid, the expressions for Type I and Type II Errors become:

$$Type II Error : Pr(\text{"implementing policy } \bar{a} \mid \underline{\theta}) = \Gamma^*(\bar{a}) \quad (6)$$

$$Type I Error : Pr(\text{"implementing policy } \underline{a} \mid \bar{\theta}) = 1 - \Gamma^*(\bar{a}) \quad (7)$$

which are indeterminate, given that $\Gamma^*(\bar{a}) \in [0, 1]$ when $Pr(\bar{\theta}) = \frac{1}{2}$. Notice that I have derived the above expressions to make possible comparisons with respect to the previous case of a $j = \bar{\sigma}$ Broadsheet production.

Although the previous results imply that it is not possible to compute univocally Type I and Type II Errors in policy adoption when an uninformative Tabloid is produced, it is easy to see that having a Tabloid always increases

the *total* probability of error in policy implementation with respect to having a Broadsheet. In fact averaging out eqq. (4) and (5), one gets that the total error in optimal policy implementation when a $j = \bar{\sigma}$ is produced, is equal to the following¹⁰:

$$\begin{aligned} \text{Total Error}_{j=\bar{\sigma}} &= \\ &= \frac{1}{2} \left(\sum_{n_{\bar{\sigma}}=0}^{n_{\bar{\sigma}}^*-1} \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q)^{n_{\bar{\sigma}}} (1-q)^{N-n_{\bar{\sigma}}} + \sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}} (1-q')^{N-n_{\bar{\sigma}}} \right) \end{aligned}$$

Regarding the Tabloid instead, averaging out eqq. (6) and (7), the total error in optimal policy implementation is equal to the following:

$$\text{Total Error}_{j=\phi} = \frac{1}{2} [\Gamma(\bar{a}) + 1 - \Gamma(\bar{a})] = \frac{1}{2}$$

To show that $\text{Total Error}_{j=\bar{\sigma}} < \text{Total Error}_{j=\phi} = \frac{1}{2}$, simply one needs to show that

$$\sum_{n_{\bar{\sigma}}=0}^{n_{\bar{\sigma}}^*-1} \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q)^{n_{\bar{\sigma}}} (1-q)^{N-n_{\bar{\sigma}}} + \sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}} (1-q')^{N-n_{\bar{\sigma}}} < 1$$

But this follows immediately from the fact that: $\sum_{n_{\bar{\sigma}}=0}^{n_{\bar{\sigma}}^*-1} \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q)^{n_{\bar{\sigma}}} (1-q)^{N-n_{\bar{\sigma}}} + \sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q)^{n_{\bar{\sigma}}} (1-q)^{N-n_{\bar{\sigma}}} = 1$ and $\sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}} (1-q')^{N-n_{\bar{\sigma}}} < \sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q)^{n_{\bar{\sigma}}} (1-q)^{N-n_{\bar{\sigma}}}$ for each $n_{\bar{\sigma}} \geq n_{\bar{\sigma}}^*$, as shown in the Proof of Corollary (7). ■

3.1 Comparative Statics

In the previous Section I have derived the equilibrium of the model and highlighted the properties of the two types of newspapers in terms of the probability of implementing the optimal policies and relative errors. Having done this, it is useful now to conduct an exercise of comparative statics. Given the complexity of some of the expressions representing the probability of implementing the optimal policy and the relative errors, the exercise of comparative statics will be conducted by changing some values of the parameters in those expressions, instead of using calculus. Then I will conjecture that the results found in this way extend to all the range of values for which the expressions are defined.

Of course in the following discussion I consider the Broadsheet only, given that the Tabloid newspaper has no influence on policy implementation. Again

¹⁰It is useful to repeat that similar expressions can be derived for a $j = \underline{\sigma}$ Broadsheet.

I focus on the $j = \bar{\sigma}$ Broadsheet but similar expressions can be derived for the $j = \underline{\sigma}$ Broadsheet.

Proposition 12 *When a Broadsheet newspaper is produced, whenever q , q' or N increase, the minimum number of signals $n_{\bar{\sigma}}^*$ one has to observe in order to implement the optimal policy increases. On the other hand, the probability of implementing the optimal policy increases when q or N increase, while it decreases when q' increases. Regarding errors in optimal policy implementation, both Type I and Type II Error decrease when q or N increase, while increase when q' increases.*

Proof. First I show what happens to the threshold $n_{\bar{\sigma}}^*(q, q', N)$ when the value of the parameters changes. Tedious but straightforward algebra shows that:

$$\frac{\partial n_{\bar{\sigma}}^*(q, q', N)}{\partial q} = \frac{\partial}{\partial q} \left(\frac{\log_{q/q'} \left(\frac{1-q'}{1-q} \right)}{1 + \log_{q/q'} \left(\frac{1-q'}{1-q} \right)} N \right) = -\frac{1}{q} \frac{\ln \left(\frac{1-q'}{1-q} \frac{q'}{q} \right)}{\left(\ln \frac{1-q}{1-q'} \frac{q'}{q} \right)^2} N$$

It is simple to show that $\frac{\partial n_{\bar{\sigma}}^*(q, q', N)}{\partial q} > 0$. In order to prove it, one has to show simply that $\ln \left(\frac{1-q'}{1-q} \frac{q'}{q} \right) < 0$ for any q and $q' \in (0, 1)$. For this to happen, it has to be that $0 < \frac{1-q'}{1-q} \frac{q'}{q} < 1$. Showing that $0 < \frac{1-q'}{1-q} \frac{q'}{q}$ is trivial, since q and $q' \in (0, 1)$ by assumption and therefore the expression $\frac{1-q'}{1-q} \frac{q'}{q}$ is positive.

Proving that $\frac{1-q'}{1-q} \frac{q'}{q} < 1$ is slightly more complicated. First I show necessity, since proving sufficiency is straightforward. Assume $\frac{(1-q')q'}{(1-q)q} < 1$ and multiply both sides of the inequality by $(1-q)q$. Then expand the result and obtain $q' - (q')^2 < q - q^2$. By rewriting the previous expression as $q^2 - (q')^2 < q - q'$ and factoring the term $q^2 - (q')^2$, the expression rewrites as $(q - q')(q + q') < q - q'$. Simplifying both sides of the inequality by $q - q' > 0$, it follows that $q + q' < 1$, which is true, for each q, q' given that $1 > q > q' > 0$ by assumption.

Having established necessity, sufficiency follows immediately given that $1 > q > q' > 0$ and $q + q' < 1$.

Similarly one can show that:

$$\begin{aligned} \frac{\partial n_{\bar{\sigma}}^*(q, q', N)}{\partial q'} &= \frac{\partial}{\partial q'} \left(\frac{\log_{q/q'} \left(\frac{1-q'}{1-q} \right)}{1 + \log_{q/q'} \left(\frac{1-q'}{1-q} \right)} N \right) = \\ &= \frac{q' \ln \left(\frac{q}{q'} \right) - \ln \left(\frac{1-q'}{1-q} \right) + q' \ln \left(\frac{1-q'}{1-q} \right)}{-q'(1-q') \left[\ln \left(\frac{q}{q'} \right) + \ln \left(\frac{1-q'}{1-q} \right) \right]} N \\ &= \frac{q' \ln \left(\frac{q}{q'} \right) - (1-q') \ln \left(\frac{1-q'}{1-q} \right)}{-q'(1-q') \left[\ln \left(\frac{q}{q'} \right) + \ln \left(\frac{1-q'}{1-q} \right) \right]} N \end{aligned} \quad (8)$$

To show that $\frac{\partial n_{\sigma}^*(q, q', N)}{\partial q'} > 0$, given that the denominator is negative, since it is a product of positive factors multiplied by -1 , it is enough to show that the numerator is negative as well. Therefore one has to prove that $q' \ln(\frac{q}{q'}) - (1 - q') \ln(\frac{1-q'}{1-q}) < 0$ for any q and $q' \in (0, 1)$.

Again I prove necessity first, and then sufficiency. Assuming that $q' \ln(\frac{q}{q'}) - (1 - q') \ln(\frac{1-q'}{1-q}) < 0$ one obtains that $\ln(\frac{q}{q'})^{q'} < \ln(\frac{1-q'}{1-q})^{1-q'}$. By applying the strictly increasing function e^x to both sides of the inequality it follows that $(\frac{q}{q'})^{q'} < (\frac{1-q'}{1-q})^{1-q'}$. Therefore I need to show that $(\frac{q}{q'})^{q'} < (\frac{1-q'}{1-q})^{1-q'}$. Observe that, since $q > q'$ and $q + q' < 1$, then it has to be $q' < \frac{1}{2}$. In fact, if not, it would be $q > q' > \frac{1}{2}$ and $q + q' > 1$ which is a contradiction of the hypothesis. Furthermore, since $q' < \frac{1}{2}$, then $q' + q' < 1$ and therefore $q' < 1 - q'$. Moreover, since $\frac{q}{q'} > 1$ and $\frac{1-q'}{1-q} > 1$, given that $(\frac{q}{q'})^{q'} < (\frac{1-q'}{1-q})^{1-q'}$ it can be either $\frac{q}{q'} < \frac{1-q'}{1-q}$ or $\frac{q}{q'} > \frac{1-q'}{1-q}$.

First consider the case when $\frac{q}{q'} < \frac{1-q'}{1-q}$. Since $q' < q$, one can rewrite $q' = q - \varepsilon$, with $\varepsilon > 0$ and "small". Now substitute $q' = q - \varepsilon$ in the inequality $\frac{q}{q'} < \frac{1-q'}{1-q}$. This rewrites as:

$$\begin{aligned} \frac{q}{q'} &< \frac{1-q'}{1-q} \\ \frac{q}{q - \varepsilon} &< \frac{1 - q + \varepsilon}{1 - q} \\ \frac{q + \varepsilon - \varepsilon}{q - \varepsilon} &< \frac{1 - q + \varepsilon}{1 - q} \\ 1 + \frac{\varepsilon}{q - \varepsilon} &< 1 + \frac{\varepsilon}{1 - q} \\ \frac{\varepsilon}{q - \varepsilon} &< \frac{\varepsilon}{1 - q} \\ 1 - q &< q - \varepsilon \\ 1 + \varepsilon &< 2q \\ q &> \frac{1}{2} + \frac{\varepsilon}{2} \end{aligned}$$

Instead, when $\frac{q}{q'} > \frac{1-q'}{1-q}$, then it follows that it has to be $q < \frac{1}{2} + \frac{\varepsilon}{2}$. Therefore the inequality $(\frac{q}{q'})^{q'} < (\frac{1-q'}{1-q})^{1-q'}$ holds strictly for any $q \in (0, 1) / \{\frac{1}{2} + \frac{\varepsilon}{2}\}$, with ε positive and small.

To prove sufficiency remember that it can be either $q < \frac{1}{2}$ or $q > \frac{1}{2}$, but it has to be $q' < \frac{1}{2}$. First consider the case when $q > \frac{1}{2}$.

Let $q = \frac{1}{2} + \varepsilon$ (with $1 - q = \frac{1}{2} - \varepsilon$) and $q' = \frac{1}{2} - \varphi$ (with $1 - q' = \frac{1}{2} + \varphi$) with $0 < \varepsilon < \frac{1}{2}$ and $0 < \varphi < \frac{1}{2}$ and let $\varphi < \varepsilon$. From this it follows that $\varphi^2 < \varepsilon^2$ and $\frac{1}{4} - \varepsilon^2 < \frac{1}{4} - \varphi^2$ or $(\frac{1}{2} - \varepsilon)(\frac{1}{2} + \varepsilon) < (\frac{1}{2} - \varphi)(\frac{1}{2} + \varphi)$. By cross multiplying the terms, one has that $\frac{\frac{1}{2} + \varepsilon}{\frac{1}{2} - \varphi} < \frac{\frac{1}{2} + \varphi}{\frac{1}{2} - \varepsilon}$. Given that $1 < \frac{\frac{1}{2} + \varepsilon}{\frac{1}{2} - \varphi} < \frac{\frac{1}{2} + \varphi}{\frac{1}{2} - \varepsilon}$, and that $\frac{1}{2} - \varphi < \frac{1}{2} + \varphi$, it follows immediately that $(\frac{\frac{1}{2} + \varepsilon}{\frac{1}{2} - \varphi})^{\frac{1}{2} - \varphi} < (\frac{\frac{1}{2} + \varphi}{\frac{1}{2} - \varepsilon})^{\frac{1}{2} + \varphi}$, for the

monotonicity of the power function. By substituting the values for q and q' one obtains $\left(\frac{q}{q'}\right)^{q'} < \left(\frac{1-q'}{1-q}\right)^{1-q'}$.

Now consider the case when $q < \frac{1}{2}$ and let $q = \frac{1}{2} - \varepsilon$ and $q' = \frac{1}{2} - \varphi$. Since by assumption it has to be $q > q'$, this holds iff $q = \frac{1}{2} - \varepsilon > \frac{1}{2} - \varphi = q'$, that is $\varphi > \varepsilon$. So let $\varphi > \varepsilon$ and $\frac{1}{2} + \varphi > \frac{1}{2} + \varepsilon$, from which it follows that $(\frac{1}{2} + \varphi)^2 > (\frac{1}{2} + \varepsilon)^2$. By expanding the binomial and cross multiplying the terms one obtains that $\frac{\frac{1}{2} + \varphi}{\frac{1}{2} + \varepsilon} > \frac{\frac{1}{2} + \varepsilon}{\frac{1}{2} + \varphi}$. Now multiply by the same factor $\frac{\frac{1}{2} - \varphi}{\frac{1}{2} - \varepsilon} > 0$ both sides of the inequality and then obtain that $\frac{\frac{1}{2} + \varphi}{\frac{1}{2} + \varepsilon} \frac{\frac{1}{2} - \varphi}{\frac{1}{2} - \varepsilon} > \frac{\frac{1}{2} + \varepsilon}{\frac{1}{2} + \varphi} \frac{\frac{1}{2} - \varphi}{\frac{1}{2} - \varepsilon}$. Thanks to the monotonicity of the power function, then it follows that:

$$\left(\frac{\frac{1}{2} + \varepsilon}{\frac{1}{2} + \varphi} \frac{\frac{1}{2} - \varphi}{\frac{1}{2} - \varepsilon}\right)^\varphi < \left(\frac{\frac{1}{2} + \varphi}{\frac{1}{2} + \varepsilon} \frac{\frac{1}{2} - \varphi}{\frac{1}{2} - \varepsilon}\right)^{1/2}$$

given $\varphi < 1/2$. Thanks to some simple algebraic manipulation it is possible to rewrite the previous expression as:

$$\begin{aligned} \left(\frac{\frac{1}{2} + \varepsilon}{\frac{1}{2} + \varphi} \frac{\frac{1}{2} - \varphi}{\frac{1}{2} - \varepsilon}\right)^\varphi &< \left(\frac{\frac{1}{2} + \varphi}{\frac{1}{2} + \varepsilon} \frac{\frac{1}{2} - \varphi}{\frac{1}{2} - \varepsilon}\right)^{1/2} \\ \frac{\left(\frac{\frac{1}{2} - \varphi}{\frac{1}{2} - \varepsilon}\right)^\varphi}{\left(\frac{\frac{1}{2} + \varphi}{\frac{1}{2} + \varepsilon}\right)^\varphi} &< \frac{\left(\frac{\frac{1}{2} + \varphi}{\frac{1}{2} + \varepsilon}\right)^{1/2}}{\left(\frac{\frac{1}{2} - \varepsilon}{\frac{1}{2} - \varphi}\right)^{1/2}} \\ \frac{\left(\frac{\frac{1}{2} - \varepsilon}{\frac{1}{2} - \varphi}\right)^{-\varphi}}{\left(\frac{\frac{1}{2} + \varphi}{\frac{1}{2} + \varepsilon}\right)^\varphi} &< \frac{\left(\frac{\frac{1}{2} + \varphi}{\frac{1}{2} + \varepsilon}\right)^{1/2}}{\left(\frac{\frac{1}{2} - \varepsilon}{\frac{1}{2} - \varphi}\right)^{1/2}} \\ \left(\frac{\frac{1}{2} - \varepsilon}{\frac{1}{2} - \varphi}\right)^{1/2} \left(\frac{\frac{1}{2} - \varepsilon}{\frac{1}{2} - \varphi}\right)^{-\varphi} &< \left(\frac{\frac{1}{2} + \varphi}{\frac{1}{2} + \varepsilon}\right)^{1/2} \left(\frac{\frac{1}{2} + \varphi}{\frac{1}{2} + \varepsilon}\right)^\varphi \\ \left(\frac{\frac{1}{2} - \varepsilon}{\frac{1}{2} - \varphi}\right)^{1/2 - \varphi} &< \left(\frac{\frac{1}{2} + \varphi}{\frac{1}{2} + \varepsilon}\right)^{1/2 + \varphi} \end{aligned} \quad (9)$$

By substituting in the above expression $q = \frac{1}{2} - \varepsilon$, $q' = \frac{1}{2} - \varphi$ and $1 - q = \frac{1}{2} + \varepsilon$, $1 - q' = \frac{1}{2} + \varphi$, it follows that the previous inequality is equivalent to:

$$\left(\frac{q}{q'}\right)^{q'} < \left(\frac{1 - q'}{1 - q}\right)^{1 - q'}$$

Therefore since I have shown that $q' \ln\left(\frac{q}{q'}\right) - (1 - q') \ln\left(\frac{1 - q'}{1 - q}\right) < 0$ (numerator of the fraction in eq. (8)) for any value where q and q' are defined, and that $-q'(1 - q') [\ln\left(\frac{q}{q'}\right) + \ln\left(\frac{1 - q'}{1 - q}\right)] < 0$ (denominator of the fraction in eq. (8)) again for any value q and q' , I can conclude that $\frac{\partial n_{\sigma}^*(q, q', N)}{\partial q'} > 0$ for any value where q and q' are defined.

Finally, given $n_{\bar{\sigma}}^* = \frac{\log_{q/q'}\left(\frac{1-q'}{1-q}\right)}{1+\log_{q/q'}\left(\frac{1-q'}{1-q}\right)}N$, then $\frac{\partial n_{\bar{\sigma}}^*(q,q',N)}{\partial N} > 0$ as $\frac{\log_{q/q'}\left(\frac{1-q'}{1-q}\right)}{1+\log_{q/q'}\left(\frac{1-q'}{1-q}\right)} \in (0, 1)$.

After computing what happens to the threshold $n_{\bar{\sigma}}^*(q, q', N)$, I am interested in analysing the comparative statics properties of the optimal policy implementation probability, with respect to q , q' and N . Remember that the expression of the probability of optimal policy implementation (*POP*) is equal to:

$$\begin{aligned} POP &= Pr(n_{\bar{\sigma}} \geq n_{\bar{\sigma}}^*|\bar{\theta})Pr(\bar{\theta}) + Pr(n_{\bar{\sigma}} < n_{\bar{\sigma}}^*|\underline{\theta})Pr(\underline{\theta}) = \\ &= \frac{1}{2} \left(\sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} q^{n_{\bar{\sigma}}}(1-q)^{N-n_{\bar{\sigma}}} + \sum_{n_{\bar{\sigma}}=0}^{n_{\bar{\sigma}}^*-1} \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}}(1-q')^{N-n_{\bar{\sigma}}} \right) \end{aligned}$$

I conjecture that the following comparative statics properties hold:

$$\begin{aligned} i) &: \frac{\partial POP}{\partial q} > 0 \\ ii) &: \frac{\partial POP}{\partial q'} < 0 \\ iii) &: \frac{\partial POP}{\partial N} > 0 \end{aligned}$$

Instead of computing the partial derivatives of a binomial distribution with respect to the parameters q , q' and N , I show that the above expressions *i*), *ii*) and *iii*) hold for just *one* set of parameters q , q' and N when the values of this set are changed infinitesimally and then I conjecture that they hold for any value of q , q' and N .

First I show that, for particular values of the parameters q , q' and N , when $q \uparrow$ infinitesimally, then $Pr(n_{\bar{\sigma}} \geq n_{\bar{\sigma}}^*|\bar{\theta})Pr(\bar{\theta}) + Pr(n_{\bar{\sigma}} < n_{\bar{\sigma}}^*|\underline{\theta})Pr(\underline{\theta}) \uparrow$. In fact when $(q, q', N) = (0.205, 0.2, 100)$, then the probability of optimal policy implementation is equal to 0.524838752. However, when $q \uparrow$, and in particular when $(q, q', N) = (0.305, 0.2, 100)$, then the probability of optimal policy implementation is equal to 0.887234726.

Secondly I show that, again for particular values of the parameters, when $q' \uparrow$, then $Pr(n_{\bar{\sigma}} \geq n_{\bar{\sigma}}^*|\bar{\theta})Pr(\bar{\theta}) + Pr(n_{\bar{\sigma}} < n_{\bar{\sigma}}^*|\underline{\theta})Pr(\underline{\theta}) \downarrow$. In fact when $(q, q', N) = (0.1, 0.005, 100)$, then the probability of optimal policy implementation is equal to 0.995245123. Instead when $(q, q', N) = (0.1, 0.05, 100)$, and therefore $q' \uparrow$, then the probability of good policy is equal to 0.83299433.

Finally, for a particular set of parameters $(q, q', N) = (0.1, 0.005, 5)$, I calculate what happens to $Pr(n_{\bar{\sigma}} \geq n_{\bar{\sigma}}^*|\bar{\theta})Pr(\bar{\theta}) + Pr(n_{\bar{\sigma}} < n_{\bar{\sigma}}^*|\underline{\theta})Pr(\underline{\theta})$ when $N \uparrow$. In this case the probability of implementing the optimal policy is equal to 0.692379377. However when $N \uparrow$, for instance $(q, q', N) = (0.1, 0.005, 100)$, then the probability of implementing the optimal policy is equal to 0.995245123. Therefore one can conclude that when $N \uparrow$ the probability of good policy implementation increases.

Now I analyse the *Type I Error (TIE)* and I repeat the same exercise of comparative statics for some set of parameters (q, q', N) , by changing marginally those parameters values.

First I compute what happens to *TIE* when $q \uparrow$. I take the first set of parameters I used to compute the $\partial POP / \partial q$, that is $(q, q', N) = (0.205, 0.2, 100)$ and I compute the Type I Error which is equal to 0.509784081. If $q \uparrow$, and in particular $(q, q', N) = (0.305, 0.2, 100)$, then *TIE* is equal to 0.138055164.

Now I calculate how *TIE* changes when $q' \uparrow$. If $(q, q', N) = (0.1, 0.005, 100)$, then *TIE* = 0.007836487. If $q' \uparrow$ and $(q, q', N) = (0.1, 0.05, 100)$, then *TIE* becomes 0.206050862.

Finally, one can easily see how *TIE* varies when $N \uparrow$. If $(q, q', N) = (0.1, 0.005, 5)$, then *TIE* is equal to 0.59049. However with $N = 100$ and the set of parameter is $(q, q', N) = (0.1, 0.005, 100)$, then *TIE* = 0.007836487. Therefore Type I Error decreases when q or N increase, while increases when q' increases. Again I conjecture that these results extend to all the sets of parameters (q, q', N) where q, q' and N are defined.

As a last exercise I want to show, again for some set of parameters, what happens to the *Type II Error (TIIE)* when the values of parameters change marginally. When $(q, q', N) = (0.205, 0.2, 100)$, then *TIIE* = 0.440538415. If q goes to 0.305 and the set of parameters becomes $(q, q', N) = (0.305, 0.2, 100)$, then the *TIIE* is equal to 0.087475385.

On the other hand, when $(q, q', N) = (0.1, 0.005, 100)$, then *TIIE* = 0.001673268. However, when $(q, q', N) = (0.1, 0.05, 100)$, then *TIIE* = 0.127960479.

Finally when N increases, for instance from 5 to 100, with $(q, q') = (0.1, 0.005)$, then *TIIE* goes from 0.024751247 to 0.001673268. Therefore Type II Error decreases when q or N increase, while increases when q' increases. Needless to say, also in this case I conjecture that these comparative statics results extend to all the set of parameters (q, q', N) . ■

3.1.1 Asymmetry in the probability of the policy implementation

Remember that the Middle Class is the only one of the three groups of citizens interested in the optimal policy implementation. Instead the other two groups of citizens, Rich and Poor, want their preferred policy implemented, regardless of the true state of the world. Now, contingent on the production of a Broadsheet, I compute the total probability of implementing the policy supported by the Broadsheet, whether this is optimal or not.

I focus my computation on $j = \bar{\sigma}$ and on the policy \bar{a} . In this case the total probability such that the Incumbent implements the policy \bar{a} regardless

of the true state of the world is the following:

$$\begin{aligned}
Pr(\bar{a}|\theta) &= Pr(n_{\bar{\sigma}} \geq n_{\bar{\sigma}}^*|\bar{\theta})Pr(\bar{\theta}) + Pr(n_{\bar{\sigma}} \geq n_{\bar{\sigma}}^*|\underline{\theta})Pr(\underline{\theta}) = \\
&= \sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} q^{n_{\bar{\sigma}}}(1-q)^{N-n_{\bar{\sigma}}} \frac{1}{2} + \sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}}(1-q')^{N-n_{\bar{\sigma}}} \frac{1}{2} = \\
&= \frac{1}{2} \left(\sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} q^{n_{\bar{\sigma}}}(1-q)^{N-n_{\bar{\sigma}}} + \sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q')^{n_{\bar{\sigma}}}(1-q')^{N-n_{\bar{\sigma}}} \right) = \\
&= \frac{1}{2} \sum_{n_{\bar{\sigma}}=n_{\bar{\sigma}}^*}^N \frac{N!}{n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!} (q^{n_{\bar{\sigma}}}(1-q)^{N-n_{\bar{\sigma}}} + (q')^{n_{\bar{\sigma}}}(1-q')^{N-n_{\bar{\sigma}}}) \quad (10)
\end{aligned}$$

Interestingly it can be proved that the production of the Broadsheet $j = \bar{\sigma}$, rather than of the Tabloid, matters for the probability that the Incumbent implements the policy \bar{a} regardless of the state of the world. In fact the following proposition can be shown:

Proposition 13 *When the MT decides to publish the Broadsheet $j = \bar{\sigma}$ there exists some set of parameters such that the probability of implementing the policy \bar{a} , regardless of the state of the world, is larger (smaller) than $\frac{1}{2}$.*

Proof. Notice that it cannot be decided whether the expression in eq. (10) is larger or smaller than $1/2$. For this reason I have conducted an analysis with some values to show that $Pr(\bar{a}|\theta)$ can be either $>$ or $<$ $\frac{1}{2}$. For instance with $(q, q', N) = (0.4, 0.32, 100)$ it follows that $Pr(\bar{a}|\theta) = 0.522714554$. However with $(q, q', N) = (0.4, 0.385, 100)$ it follows that $Pr(\bar{a}|\theta) = 0.476810756$ ■

The previous Proposition shows that when a Broadsheet is produced, there is some set of parameters such that the implementation of one policy, regardless of whether or not that policy is optimal, happens more (less) often than when a Tabloid is produced. In this sense there is a sort of asymmetry in the policy implementation depending on which newspaper type the MT decides to produce: one policy may be adopted more (less) often than the other when the Broadsheet campaigning for that policy option is produced instead of the Tabloid. All this could be exploited by either of the two groups interested in implementing the policy they favour, if they manage to induce the MT to produce the campaigning newspaper which supports the same policy that benefits them. In order to do so, the role of so called “ideological readers”, i.e. citizens that read the newspaper supporting the policy they like, may become crucial. I investigate this in the following section.

To conclude this section, given the existence of both $n_{\bar{\sigma}}^*$ and $n_{\underline{\sigma}}^*$ and the fact that $n_{\bar{\sigma}}^* = n_{\underline{\sigma}}^*$, a Proposition similar to Proposition 13 can be stated and proved also for $j = \underline{\sigma}$ and \underline{a} .

Proposition 14 *When the MT decides to publish the Broadsheet $j = \underline{\sigma}$ there exists some set of parameters such that the probability of implementing the policy \underline{a} regardless of the state of the world is larger (smaller) than $\frac{1}{2}$.*

Proof. Similar to the proof of Proposition 13. ■

4 The Role of Ideology

In Proposition 13 I have shown that, conditional on producing the Broadsheet newspaper $j = \bar{\sigma}$, there is a non-empty set of parameters such that the cumulative probability of implementing the policy \bar{a} is larger (smaller) than $\frac{1}{2}$, regardless of the state of the world. So it is clear that, for some parameters, producing one Broadsheet rather than the other favours the group who is benefitted by the policy the produced Broadsheet supports. In fact, in this parameter space, the policy is implemented with an ex-ante probability greater than $\frac{1}{2}$, larger than the probability with which that policy is adopted when the MT produces a Tabloid.

To clarify, focus on $j = \bar{\sigma}$ (again the results for the Broadsheet $j = \underline{\sigma}$ are similar). Poor citizens are interested in the implementation of the policy \bar{a} , as they benefit from \bar{a} more than they do from \underline{a} , regardless of the true state of the world. Remember that the polity is made of \hat{N} citizens of whom Rich citizens are $N_R = \underline{p}\hat{N}$, Poor citizens are $N_P = \bar{p}\hat{N}$ while the remaining $N_M = N = (1 - \bar{p} - \underline{p})\hat{N}$ are citizens belonging to the Middle Class. So far in the article I have assumed that no citizen belonging to N_R or N_P buys any newspaper, while the number of individuals belonging to the Middle Class buying a newspaper depends on the realisation of signals. Now I assume that, together with the citizens belonging to the Middle Class, there is a fraction of the other two groups $\gamma_k \in [0, 1]$ with $k \in [R, P]$ who buy the newspaper supporting the policy preferred by that group, regardless of the signal on the state of the world. I call this the “partisan readers”, i.e. citizens who read the Broadsheet if and only if it supports the policy they benefit from, provided that the utility they derive from reading is larger than the price they pay for buying the newspaper. Therefore a fraction γ_P of Poor citizens buy the newspaper $j = \bar{\sigma}$ if that is available, while a fraction γ_R of Rich citizens buy the newspaper $j = \underline{\sigma}$ if it is produced. The complementary fraction $1 - \gamma_k$, $k \in [R, P]$ reads a Tabloid instead, again provided that the utility they derive from reading is larger than the price they pay for buying the newspaper.¹¹

So if a Rich individual is a partisan reader, his utility is equal to $u_i(j, a) = \bar{r} + u_i(a|\theta) + u_i(j|\sigma)$ with $u_i(j) = \bar{e}$ when $j = \underline{\sigma}$, for any $\sigma \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$ the Rich observe privately and $u_i(\underline{a}|\theta) > 0 = u_i(\bar{a}|\theta)$, for any $\theta \in \Theta$. Instead if a Rich individual is not a partisan reader his utility is equal to $U_i(r, a, j) = \bar{r} + u_i(a|\theta) + u_i(j|\sigma)$, with $u_i(j) = \bar{e}$ when $j = \underline{\sigma}$, for any $\sigma \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$ and $u_i(\underline{a}|\theta) > 0 = u_i(\bar{a}|\theta)$. Similarly if a Poor individual is a partisan reader, his utility is equal to $U_i(m, a, j) = \bar{m} + u_i(a|\theta) + u_i(j|\sigma)$ with $u_i(j) = \bar{e}$ when $j = \bar{\sigma}$, for any $\sigma \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$ the Poor observe privately and $u_i(\bar{a}|\theta) > 0 = u_i(\underline{a}|\theta)$, for any $\theta \in \Theta$. On the other hand if a Poor individual is not a partisan reader his utility is equal to $U_i(m, a, j) = \bar{m} + u_i(a|\theta) + u_i(j|\sigma)$, with

¹¹An alternative assumption would be that the complementary fraction $1 - \gamma_k$ does not read any newspaper. In this case the results would be qualitatively similar to the ones in Lemma (15).

$u_i(j) = \bar{e}$ when $j = \phi$, for any σ and $u_i(\bar{a}|\theta) > 0 = u_i(\underline{a}|\theta)$, for any $\theta \in \Theta$. Likewise, if a Rich individual is not a partisan reader his utility is equal to $U_i(r, a, j) = r_i + u_i(a|\theta) + u_i(j|\sigma)$, with $u_i(j) = \bar{e}$ when $j = \phi$, for any σ and $u_i(\underline{a}|\theta) > 0 = u_i(\bar{a}|\theta)$, for any $\theta \in \Theta$.

In order to simplify the signaling game and to focus on the MT's productive decision when the newspaper readership is made of both partisan and Middle Class non-partisan readers, I assume that the Incumbent politician is able to distinguish between the two groups of readers. Technically, in order for this to be true, it is sufficient that the fraction of partisan readers $\gamma_k, k \in [R, P]$ is common knowledge. This means that if the Incumbent observes that the number of copies of Broadsheet bought is larger than γ_k , he knows that the additional readers come from the middle class readership. Given the reading behaviour of Middle Class citizens, this is again informative of the underlying state of the world, as it was shown in the previous Section.

Thanks to the presence of the partisan readers, the Proposition 9 is modified accordingly:

Proposition 15 *The MT produces a Broadsheet supporting the policy favourable to the Poor (Rich) instead of a Tabloid, iff the total probability of obtaining signals informative on the state of the world is greater than $\frac{2}{3}[1 + \frac{(1-\gamma_R)\underline{p} + (1-2\gamma_P)\bar{p}}{1-\bar{p}-\underline{p}}]$ ($\frac{2}{3}[1 + \frac{(1-\gamma_P)\bar{p} + (1-2\gamma_R)\underline{p}}{1-\bar{p}-\underline{p}}]$). Moreover, conditional on producing a Broadsheet, the MT produces the Broadsheet having the larger number of partisan readers.*

Proof. To prove the Proposition above, first I rewrite the expression for the expected profit of the $j = \bar{\sigma}$ Broadsheet. This is equal to $\Pi_{j=\bar{\sigma}}^E = E_{y_{j=\bar{\sigma}}} [p_{j=\bar{\sigma}}(y_{j=\bar{\sigma}}) * y_{j=\bar{\sigma}} - C(y_{j=\bar{\sigma}})]$. Remember that $y_{j=\bar{\sigma}}$ is the number of copies of the $j = \bar{\sigma}$ newspaper produced and bought. Now this number $y_{j=\bar{\sigma}}$ is made up by a random variable, i.e. the number of copies bought by the Middle Class citizens ($y_{j=\bar{\sigma}}^{MC}$) depending on the signals observed by the Middle Class citizens, and by a fixed quantity, the fraction $\gamma_P \bar{p} \hat{N}$, i.e. the number of Poor partisan readers. Extending the reasoning of the proof in Lemma (8) it is possible to see that the inverse demand function for the newspaper $j = \bar{\sigma}$ is constant for any level of newspaper production $y_{j=\bar{\sigma}}$. This follows from the fact that each of the Poor partisan readers has the same willingness to pay for the newspaper, which is, in turn, equal to Middle Class citizens willingness to pay for the newspaper. Therefore one can conclude that: $P(Q) = p_{j=\bar{\sigma}}(y_{j=\bar{\sigma}}) = \bar{e}$.

Now I can rewrite $\Pi_{j=\bar{\sigma}}^E$ as $E_{y_{j=\bar{\sigma}}} [p_{j=\bar{\sigma}}(y_{j=\bar{\sigma}}^{MC}) * y_{j=\bar{\sigma}}^{MC} - C(y_{j=\bar{\sigma}}^{MC})] + \bar{e} * \gamma_P \bar{p} \hat{N} - \bar{e} * \gamma_P \bar{p} \hat{N}$. By expanding the expected profit expression following the proof in Lemma (8) and remembering that $N = (1 - \bar{p} - \underline{p}) \hat{N}$, it follows that:

$$\begin{aligned} \Pi_{j=\bar{\sigma}}^E &= \frac{1}{2}(\bar{e} - \bar{c})(q + q')(1 - p - \underline{p})\hat{N} - F + (\bar{e} - \bar{c}) * \gamma_P \bar{p} \hat{N} = \\ &= (\bar{e} - \bar{c})\hat{N}[\frac{1}{2}(q + q')(1 - \bar{p} - \underline{p}) + \gamma_P \bar{p}] - \bar{F} \end{aligned}$$

Following the same reasoning as above, it is straightforward to see that the expression for the $j = \underline{\sigma}$ Broadsheet's expected profit is:

$$\Pi_{j=\underline{\sigma}}^E = (\bar{e} - \bar{c})\hat{N}\left[\frac{1}{2}(q + q')(1 - \bar{p} - \underline{p}) + \gamma_R\underline{p}\right] - \bar{F}$$

I can also compute the expression for the Tabloid's expected profit. Given the assumption that non-partisan readers read the Tabloid instead of a Broadsheet and they have a willingness to pay equal to \bar{e} , this becomes:

$$\Pi_{j=\phi}^E = (\bar{e} - \bar{c})\hat{N}[(1 - q - q')(1 - \bar{p} - \underline{p}) + (1 - \gamma_R)\underline{p} + (1 - \gamma_P)\bar{p}] - F$$

Now it is possible to prove the Proposition above. For the necessary part, first assume that for the MT producing the Broadsheet $j = \bar{\sigma}$ is more profitable than producing the Tabloid and then derive the condition(s) implied by this. To do so, compare the expected profit of a $j = \bar{\sigma}$ Broadsheet with the expected profit of a Tabloid. Producing the $j = \bar{\sigma}$ Broadsheet rather than a Tabloid is optimal only if the following holds: $\Pi_{j=\bar{\sigma}}^E \geq \Pi_{j=\phi}^E$

$$\begin{aligned} (\bar{e} - \bar{c})\hat{N}\left[\frac{1}{2}(q + q')(1 - \bar{p} - \underline{p}) + \gamma_P\bar{p}\right] - F &\geq (\bar{e} - \bar{c})\hat{N}[(1 - q - q')(1 - \bar{p} - \underline{p}) + (1 - \gamma_R)\underline{p} + (1 - \gamma_P)\bar{p}] - F \\ (\bar{e} - \bar{c})\hat{N}\left[\frac{1}{2}(q + q')(1 - \bar{p} - \underline{p}) + \gamma_P\bar{p}\right] &\geq (\bar{e} - \bar{c})\hat{N}[(1 - q - q')(1 - \bar{p} - \underline{p}) + (1 - \gamma_R)\underline{p} + (1 - \gamma_P)\bar{p}] \\ \left[\frac{1}{2}(q + q')(1 - \bar{p} - \underline{p}) + \gamma_P\bar{p}\right] &\geq [(1 - q - q')(1 - \bar{p} - \underline{p}) + (1 - \gamma_R)\underline{p} + (1 - \gamma_P)\bar{p}] \\ \frac{1}{2}(q + q')(1 - \bar{p} - \underline{p}) &\geq [(1 - q - q')(1 - \bar{p} - \underline{p}) + (1 - \gamma_R)\underline{p} + (1 - \gamma_P)\bar{p}] - \gamma_P\bar{p} \\ \frac{1}{2}(q + q')(1 - \bar{p} - \underline{p}) &\geq [(1 - q - q')(1 - \bar{p} - \underline{p}) + (1 - \gamma_R)\underline{p} + (1 - 2\gamma_P)\bar{p}] \\ \frac{1}{2}(q + q') &\geq (1 - q - q') + \frac{(1 - \gamma_R)\underline{p} + (1 - 2\gamma_P)\bar{p}}{1 - \bar{p} - \underline{p}} \\ \frac{1}{2}(q + q') + (q + q') &\geq \left[1 + \frac{(1 - \gamma_R)\underline{p} + (1 - 2\gamma_P)\bar{p}}{1 - \bar{p} - \underline{p}}\right] \\ \frac{3}{2}(q + q') &\geq \left[1 + \frac{(1 - \gamma_R)\underline{p} + (1 - 2\gamma_P)\bar{p}}{1 - \bar{p} - \underline{p}}\right] \\ q + q' &\geq \frac{2}{3}\left[1 + \frac{(1 - \gamma_R)\underline{p} + (1 - 2\gamma_P)\bar{p}}{1 - \bar{p} - \underline{p}}\right] \end{aligned}$$

Similarly it is straightforward to derive the condition implied by the fact that producing a $j = \underline{\sigma}$ Broadsheet is better than producing a Tabloid for the MT. Following the above derivation, the condition is equal to:

$$\Pi_{j=\underline{\sigma}}^E \geq \Pi_{j=\phi}^E \iff q + q' \geq \frac{2}{3}\left[1 + \frac{(1 - \gamma_P)\bar{p} + (1 - 2\gamma_R)\underline{p}}{1 - \bar{p} - \underline{p}}\right]$$

Now to prove the sufficiency part, assume that $q+q' \geq \frac{2}{3}[1 + \frac{(1-\gamma_R)\underline{p}+(1-2\gamma_P)\bar{p}}{1-\bar{p}-\underline{p}}]$.

It is easy to see that from this it follows that $\frac{3}{2}(q+q') \geq [1 + \frac{(1-\gamma_R)\underline{p}+(1-2\gamma_P)\bar{p}}{1-\bar{p}-\underline{p}}]$

and that $\frac{1}{2}(q+q') \geq (1-q-q') + \frac{(1-\gamma_R)\underline{p}+(1-2\gamma_P)\bar{p}}{1-\bar{p}-\underline{p}}$. From here, by employing some relatively straightforward algebraic manipulations it is easy to conclude that: $[\frac{1}{2}(q+q')(1-\bar{p}-\underline{p}) + \gamma_P\bar{p}] \geq [(1-q-q')(1-\bar{p}-\underline{p}) + (1-\gamma_R)\underline{p} + (1-\gamma_P)\bar{p}]$ and therefore that $(\bar{e} - \bar{c})\widehat{N}[\frac{1}{2}(q+q')(1-\bar{p}-\underline{p}) + \gamma_P\bar{p}] - F \geq (\bar{e} - \bar{c})\widehat{N}[(1-q-q')(1-\bar{p}-\underline{p}) + (1-\gamma_R)\underline{p} + (1-\gamma_P)\bar{p}] - F$ that is $\Pi_{j=\bar{\sigma}}^E \geq \Pi_{j=\underline{\sigma}}^E$.

Having derived the conditions such that the MT prefers to produce a Broadsheet rather than a Tabloid, I now investigate when the MT finds it optimal to produce a $j = \bar{\sigma}$ Broadsheet rather than a $j = \underline{\sigma}$ Broadsheet. This is true iff the following holds:

$$\begin{aligned} \Pi_{j=\bar{\sigma}}^E &\geq \Pi_{j=\underline{\sigma}}^E \\ (\bar{e} - \bar{c})\widehat{N}[\frac{1}{2}(q+q')(1-\bar{p}-\underline{p}) + \gamma_P\bar{p}] - F &\geq (\bar{e} - \bar{c})\widehat{N}[\frac{1}{2}(q+q')(1-\bar{p}-\underline{p}) + \gamma_R\underline{p}] - F \\ (\bar{e} - \bar{c})\widehat{N}[\frac{1}{2}(q+q')(1-\bar{p}-\underline{p}) + \gamma_P\bar{p}] &\geq (\bar{e} - \bar{c})\widehat{N}[\frac{1}{2}(q+q')(1-\bar{p}-\underline{p}) + \gamma_R\underline{p}] \\ [\frac{1}{2}(q+q')(1-\bar{p}-\underline{p}) + \gamma_P\bar{p}] &\geq [\frac{1}{2}(q+q')(1-\bar{p}-\underline{p}) + \gamma_R\underline{p}] \\ \gamma_P\bar{p} &\geq \gamma_R\underline{p} \end{aligned}$$

So producing a Broadsheet $j = \bar{\sigma}$ is better than producing a $j = \underline{\sigma}$ Broadsheet for the MT if and only if the number of Poor “partisan readers” is larger than the number of Rich “partisan readers”. Notice that what counts is both the proportion of “partisan readers” for each group γ_k and the fraction of population that makes up each group (\underline{p} and \bar{p}). ■

The previous Proposition shows that partisan readers have an important role to play when it comes to campaigning newspaper, i.e. Broadsheet newspapers. In fact, since there is a fraction of citizens/readers who read the campaigning newspaper in any case, this allows the MT to have a fixed readership, certain and independent from signal realisation, which, instead, continues to affect the buying decision of the Middle Class citizens. In turn this changes the MT’s relative choice of producing a Broadsheet rather than a Tabloid. The following Corollary compares the trade-off between producing a Broadsheet or a Tabloid when no partisan reader is present versus the same trade-off when partisan readers make up a part of the citizens readership.

Corollary 16 *Producing the Broadsheet $j = \bar{\sigma}$ ($j = \underline{\sigma}$) is more profitable for the MT when there are partisan readers than when there are not iff the fraction of Poor (Rich) partisan readers is greater than $\frac{1}{2} + \frac{1}{2}\frac{\underline{p}}{\bar{p}}(1 - \gamma_R)$ ($\frac{1}{2} + \frac{1}{2}\frac{\bar{p}}{\underline{p}}(1 - \gamma_P)$) and the fraction of Rich (Poor) reading a Tabloid is smaller than the percentage of Poor (Rich) in the population.*

Proof. From Proposition 9 I have found that the Broadsheet is produced instead of a Tabloid iff $q+q' \geq 2/3$. From Proposition (15) the MT prefers

to produce a Broadsheet $j = \bar{\sigma}$ instead of a Tabloid iff $q + q' \geq \frac{2}{3}[1 + \frac{(1-\gamma_R)\underline{p} + (1-2\gamma_P)\bar{p}}{1-\bar{p}-\underline{p}}]$. Now, together with the Middle Class citizens, Rich and Poor citizens read both types of newspapers also. Nevertheless this does not have a clear effect on the relative convenience of producing a Broadsheet rather than a Tabloid with respect to the case when no partisan reader is around. To verify when having a fraction of partisan readers makes the production of a Broadsheet easier than not having partisan readers, simply derive the condition such that:

$$\begin{aligned} \frac{2}{3}[1 + \frac{(1-\gamma_R)\underline{p} + (1-2\gamma_P)\bar{p}}{1-\bar{p}-\underline{p}}] &< \frac{2}{3} \\ 1 + \frac{(1-\gamma_R)\underline{p} + (1-2\gamma_P)\bar{p}}{1-\bar{p}-\underline{p}} &< 1 \\ \frac{(1-\gamma_R)\underline{p} + (1-2\gamma_P)\bar{p}}{1-\bar{p}-\underline{p}} &< 0 \\ (1-\gamma_R)\underline{p} + (1-2\gamma_P)\bar{p} &< 0 \end{aligned}$$

given that $1 - \bar{p} - \underline{p} > 0$ always. It is relatively straightforward to derive the conditions such that $(1 - \gamma_R)\underline{p} + (1 - 2\gamma_P)\bar{p} < 0$. In particular $\gamma_P > \frac{1}{2} + \frac{1}{2}\frac{\underline{p}}{\bar{p}}(1 - \gamma_R)$. Therefore producing the Broadsheet $j = \bar{\sigma}$ is easier when partisan readers are present than when they are not iff the fraction of Poor partisan readers is large enough. In particular that fraction has to be larger than $\frac{1}{2}$. Notice that, since $\gamma_P \in [0, 1]$, then $\frac{1}{2} + \frac{1}{2}\frac{\underline{p}}{\bar{p}}(1 - \gamma_R) < 1$ iff $\frac{\underline{p}}{\bar{p}}(1 - \gamma_R) < 1$ or $\underline{p}(1 - \gamma_R) < \bar{p}$ that is iff the fraction of Rich reading a Tabloid is smaller than the percentage of Poor in the population.

Likewise, one can easily derive the conditions such that producing a Broadsheet $j = \underline{\sigma}$ is easier when there are partisan readers rather than when there are not. In particular $\gamma_R > \frac{1}{2} + \frac{1}{2}\frac{\bar{p}}{\underline{p}}(1 - \gamma_P)$. Therefore producing the Broadsheet $j = \underline{\sigma}$ is easier when the fraction of Rich partisan readers is greater than $\frac{1}{2} + \frac{1}{2}\frac{\bar{p}}{\underline{p}}(1 - \gamma_P)$ and, by definition of γ_R it has to be smaller than 1. Notice that this is possible iff $\frac{\bar{p}}{\underline{p}}(1 - \gamma_P) \leq 1$ or $\bar{p}(1 - \gamma_P) \leq \underline{p}$ that is iff the fraction of Poor citizens reading the Tabloid is smaller than the fraction of Poor in the polity. ■

So far I have analysed the role of partisan readers who, regardless of other circumstances, commit to buy, if available, the Broadsheet supporting their preferred policy. Nevertheless in Proposition 13 I have shown that if a Broadsheet supporting one policy is produced by the MT, there is a space of parameters where that policy is implemented with an ex-ante probability of more than one half, regardless of the true state of the world. Therefore, one could envisage a situation where partisan readers may take advantage strategically of this. For instance, if Poor partisan readers manage to coordinate their buying decisions in the space of parameters where producing the newspaper $j = \bar{\sigma}$ brings about a probability of implementing the policy \bar{a} greater than $1/2$, then it will be optimal for them to do so. In turn this will increase

the probability of optimal policy implementation, since, for this purpose, producing any Broadsheet is better than producing a Tabloid. On the other hand Rich partisan readers, anticipating this, will refrain from committing to buy the newspaper $j = \underline{\sigma}$ in that space of parameters.

Therefore one can conclude that when partisan readers are strategic and manage to coordinate among themselves, their ideological reading behaviour is such that it allows to increase both i) **the probability of their preferred policy being implemented** and ii) **the probability of optimal policy implementation “tout court”**, given that a Broadsheet is produced rather than a Tabloid.

Notice that the fact that the presence of partisan readers can ease the production of a Broadsheet and that this, in turn, increases the probability of implementing the optimal policy runs contrary to the conventional wisdom which sees the ideological behaviour at odds with “rational” (policy) decision making.

4.1 Comparative statics

In this last Section I carry out a final exercise of comparative statics on the results of Proposition (15) i.e. in the scenario where partisan readers commit to read the Broadsheet supporting the policy they prefer, for any value of parameters, without any strategic consideration. I focus again on the case of the production of a $j = \bar{\sigma}$ Broadsheet. Remember from Proposition (15) that in this scenario the Broadsheet production is carried out instead of the Tabloid whenever $q + q' \geq \frac{2}{3}[1 + \frac{(1-\gamma_R)p + (1-2\gamma_P)\bar{p}}{1-\bar{p}-p}]$. This condition is easier to be satisfied the smaller $\frac{(1-\gamma_R)p + (1-2\gamma_P)\bar{p}}{1-\bar{p}-p}$ is. The results of the comparative statics exercise can be summed up in the following proposition:

Proposition 17 *In a scenario where there are partisan readers committed to buy the Broadsheet supporting their preferred policy, the MT prefers to produce a $j = \bar{\sigma}$ Broadsheet rather than a Tabloid whenever the fraction of partisan readers, both Poor and Rich, is large, and whenever the fraction of Poor and Rich citizens increases, contingent on there being a great number of partisan readers. Also the MT prefers to produce a $j = \bar{\sigma}$ Broadsheet rather than a Tabloid whenever the fraction of Poor citizens increases, contingent on there being a small fraction of Rich individuals and whenever the fraction of Rich increases, contingent of there being an increase in the fraction of Poor citizens, as far as there is a majority of Poor partisan readers.*

Proof. To conduct an exercise of comparative statics, I take the derivative of the factor $f = \frac{2}{3}[1 + \frac{(1-\gamma_R)p + (1-2\gamma_P)\bar{p}}{1-\bar{p}-p}]$ with respect to the parameters

$\underline{p}, \bar{p}, \gamma_P, \gamma_R$.

$$\begin{aligned}
f &= \frac{2}{3} \left[1 + \frac{(1 - \gamma_R)\underline{p} + (1 - 2\gamma_P)\bar{p}}{1 - \bar{p} - \underline{p}} \right] \\
\frac{\partial}{\partial \gamma_P} f &= -\frac{4}{3} \frac{\bar{p}}{1 - \bar{p} - \underline{p}} \\
\frac{\partial}{\partial \gamma_R} f &= -\frac{2}{3} \frac{\underline{p}}{1 - \bar{p} - \underline{p}} \\
\frac{\partial}{\partial \bar{p}} f &= \frac{2}{3} \left[\frac{(1 - 2\gamma_P)}{1 - \bar{p} - \underline{p}} + \frac{(1 - \gamma_R)\underline{p} + (1 - 2\gamma_P)\bar{p}}{(1 - \bar{p} - \underline{p})^2} \right] \\
\frac{\partial}{\partial \underline{p}} f &= \frac{2}{3} \left[\frac{(1 - \gamma_R)}{1 - \bar{p} - \underline{p}} + \frac{(1 - \gamma_R)\underline{p} + (1 - 2\gamma_P)\bar{p}}{(1 - \bar{p} - \underline{p})^2} \right]
\end{aligned}$$

By looking at $\partial f / \partial \gamma_P$ and $\partial f / \partial \gamma_R$ one can see that both the partial derivatives are always negative, for any value of \bar{p} and \underline{p} . Therefore the $j = \bar{\sigma}$ Broadsheet production is easier to be carried out whenever the fraction of partisan readers γ_P and/or γ_R increases. In fact this means that, when the fraction of non-Tabloid readers increases, whatever the Broadsheet they want to read and see produced, it will be harder for the MT to produce a Tabloid rather than a Broadsheet.

Regarding the parameters expressing the share of population, by taking the derivative $\frac{df}{d\bar{p}}$ (where \bar{p} is the fraction of polity made up of Poor citizens) one obtains that this is equal to $\frac{df}{d\bar{p}} = \frac{2}{3} \left[\frac{(1 - 2\gamma_P)}{1 - \bar{p} - \underline{p}} + \frac{(1 - \gamma_R)\underline{p} + (1 - 2\gamma_P)\bar{p}}{(1 - \bar{p} - \underline{p})^2} \right]$. This derivative is negative (positive) and then the production of the $j = \bar{\sigma}$ Broadsheet is easier (harder) to carry out when the fraction \bar{p} of Poor in the polity increases, conditional on the proportion of Poor partisan readers being larger (smaller) than a certain threshold, i.e. $\gamma_P > \frac{1}{2} \frac{1 - \underline{p}\gamma_R}{1 - \underline{p}}$. For the consistency of the condition $\gamma_P > \frac{1}{2} \frac{1 - \underline{p}\gamma_R}{1 - \underline{p}}$, one needs to have $\frac{1}{2} \frac{1 - \underline{p}\gamma_R}{1 - \underline{p}} < 1$ which is true iff $1 - \underline{p}\gamma_R < 2(1 - \underline{p})$ and $-\underline{p}\gamma_R < 1 - 2\underline{p}$ from which it follows easily that $\gamma_R > 2 - 1/\underline{p}$. Again since it has to be $2 - 1/\underline{p} < 1$, and this is true for any $\underline{p} < 1$, it is easy to conclude that the condition $\frac{1}{2} \frac{1 - \underline{p}\gamma_R}{1 - \underline{p}} < 1$ holds for any γ_R . By taking the derivative of the expression $\frac{1}{2} \frac{1 - \underline{p}\gamma_R}{1 - \underline{p}}$ with respect to both the parameters γ_R and \underline{p} , one can verify that $\frac{d}{d\gamma_R} \left(\frac{1}{2} \frac{1 - \underline{p}\gamma_R}{1 - \underline{p}} \right) = -\frac{\underline{p}}{2(1 - \underline{p})} < 0$ and that $\frac{d}{d\underline{p}} \left(\frac{1}{2} \frac{1 - \underline{p}\gamma_R}{1 - \underline{p}} \right) = \frac{1 - \gamma_R}{2(1 - \underline{p})} > 0$. Therefore, the larger is the proportion of Poor individuals in the polity, the easier it is to produce the $j = \bar{\sigma}$ Broadsheet, contingent on the fraction of Poor partisan readers being larger than a certain threshold, whenever the fraction of Rich partisan readers is large and the fraction of Rich in the polity is small.

Similarly, by taking the derivative of the factor f with respect to \underline{p} (the fraction of polity made up of Rich citizens), the derivative is equal to $\frac{df}{d\underline{p}} =$

$\frac{2}{3}[\frac{(1-\gamma_R)}{1-\bar{p}-\underline{p}} + \frac{(1-\gamma_R)\underline{p}+(1-2\gamma_P)\bar{p}}{(1-\bar{p}-\underline{p})^2}]$. This derivative is negative (positive) and, therefore, the production of the $j = \bar{\sigma}$ Broadsheet is easier (harder) to carry out when the fraction \underline{p} of Rich in the polity increases, contingent on the proportion of Rich partisan readers being larger (smaller) than a certain threshold, i.e. $\gamma_R > \frac{1-2\gamma_P\bar{p}}{1-\bar{p}}$. For the consistency of the condition $\gamma_R > \frac{1-2\gamma_P\bar{p}}{1-\bar{p}}$, one needs to have $\frac{1-2\gamma_P\bar{p}}{1-\bar{p}} < 1$ which is true iff $\gamma_P > \frac{1}{2}$. Therefore whenever the fraction of Rich individuals increases, the production of the $j = \bar{\sigma}$ Broadsheet is easier iff there are enough Rich partisan readers, conditional on the Poor partisan readers being the majority of all the poor.

Finally to check when the condition $\gamma_R > \frac{1-2\gamma_P\bar{p}}{1-\bar{p}}$ is easier to be satisfied, one needs to take the derivative of the expression $\frac{1-2\bar{p}\gamma_P}{1-\bar{p}}$ with respect to the parameters γ_P and \bar{p} . In this way one obtains easily that $\frac{d}{d\gamma_P}(\frac{1-2\bar{p}\gamma_P}{1-\bar{p}}) = -\frac{2\bar{p}}{1-\bar{p}} < 0$ and that $\frac{d}{d\bar{p}}(\frac{1-2\bar{p}\gamma_P}{1-\bar{p}}) = \frac{1-2\gamma_P}{(1-\bar{p})^2}$, where $\frac{1-2\gamma_P}{(1-\bar{p})^2} < 0$ iff $1-2\gamma_P < 0$, i.e. $\gamma_P > \frac{1}{2}$. One can conclude that the $j = \bar{\sigma}$ Broadsheet production is easier to be delivered the larger is the proportion of Rich individuals in the polity, contingent on the fraction of Rich partisan readers being larger than a certain threshold, which is easier to happen whenever the fraction of Poor partisan readers increases and/or the fraction of Poor in the polity increases, provided there is a majority of Poor partisan readers. ■

So the production of a Broadsheet supporting a specific policy favourable to a group of citizens is more profitable whenever the number of partisan readers (γ_R or γ_P) is large. Furthermore when the number of Poor citizens becomes larger the production of the $j = \bar{\sigma}$ Broadsheet is easier, conditional on the fraction of Poor partisan readers γ_P being larger than a certain threshold. This can be explained quite intuitively. In fact if the number of Poor people grows but they are not committed to supporting their interests by reading the newspaper favouring their policy, then both their non-ideological reading behaviour and the shrinking of an illuminated middle class make the production of an informative newspaper harder. Nevertheless the minimum fraction of Poor partisan people such that the MT prefers to produce a $j = \bar{\sigma}$ newspaper instead of a $j = \emptyset$ decreases the more Rich partisan readers and the less Rich individuals there are. A similar phenomenon happens when the fraction of Rich citizens grows. In this case the production of the $j = \bar{\sigma}$ becomes easier iff the number of partisan Rich citizen is larger than a certain threshold. In fact, in this case, this means that there are many Rich citizens who are not willing to read a Tabloid, making the production of a Tabloid harder. Quite surprisingly this is true also for a Broadsheet which defends the interests different from the one of the Rich citizens. Finally the fraction of Rich partisan citizens necessary to induce a MT to produce a $j = \bar{\sigma}$ newspaper rather than a $j = \emptyset$ decreases when the fraction of Poor partisan readers increases and the fraction of Poor individuals increases, as long as Poor partisan readers form the majority of all the Poor readers.

Therefore it is interesting to observe that a more partisan society, one where both Rich and Poor partisan readers make up a large proportion of all citizens, provided that this makes it easier the production of a Broadsheet, may be conducive to better policy making. Of course the role of the partisan readers is just servant to that of middle class readers: in fact it makes easier for the MT, given that the conditions in Corollary 16 are satisfied, to produce an informative newspaper that the middle class could then consume, so allowing the incumbent to implement high quality policies. The same mechanism is at work with a more polarized society, a society where the middle class shrinks but the Poor grow in number and the Rich are fewer and fewer or both Rich and Poor increase their numbers. Even in this case a smaller but opinionated middle class still keeps its role as long as the partisan members of the Poor (in my example) are the majority. In this model partisanship and informed-open minded citizens are not substitute with each other but rather complementary: the existence of the former makes the rolw of the latter easier to be effective.

4.2 Conclusion

In this article I have described how citizens can take part in the public debate and communicate their views and preferences thanks to the mass media and, in particular, thanks to the newspapers they buy and read. The novelty of this approach is that it models how citizens can express themselves not directly, through their own actions, like in other models of “voice” à la Hirschman, but through the use of an “instrument”, the newspaper, that has been produced and marketed by a for-profit firm. Given the fact that this instrument can be bought but not directly produced by the citizens, I have highlighted how there exists a tension between the benefit of using a newspaper to express citizens’ views and the possibility that this newspaper can actually be produced. I have divided the possible newspapers that can be produced into Tabloid and Broadsheet, where the former is an uninformative newspaper, while the latter is informative on the state of the world. I have highlighted how the presence of a Broadsheet always improves the quality of policy decision making on part of the incumbent. However, this is possible when a Broadsheet is produced, which happens only when the environment is informative enough. If not, the MT produces a Tabloid which does not give any additional information on the state of the world or the optimal policy to be implemented and it does not allow the citizens to express themselves.

Quite interestingly the production of the Broadsheet depends on the total probability of a signal arriving being large, therefore it depends on the sum of the probabilities of correct signal and of wrong one.

In this article I have assumed that there is one group of citizens which is interested in having the optimal policy adopted, i.e. the Middle Class. I first assumed the Middle Class citizens are the only one who read newspapers. Subsequently I analysed how the results change when citizens from the other classes read newspaper as well. In that case, given that they are interested in one policy or the other, regardless of the state of the world, I have assumed

they read the newspaper supporting their views or the tabloid. I have stressed how the “partisan reader” can ease the production of the Broadsheet, instead of hardening it, contrary to conventional wisdom. In this case the existence of partisanship and of ideological readers make the implementation of optimal policy easier, not harder. A similar result has been found in the context of a more polarized society, a society where the Middle Class shrinks and Poor and Rich individuals grow in number. Even in this case, as long as there is a large fraction of partisan readers, a smaller Middle Class can still have the role in the public opinion to transmit useful information to the Incumbent, necessary to implement optimal policy.

Finally, I have found that there exists a non-empty set of parameters such that if a Broadsheet newspaper is produced, the policy that newspaper supports is implemented more often than the alternative policy, for any state of the world. In such a space of parameters the existence of partisan readers who coordinate between themselves and read the informative newspaper supporting the policy they prefer have two roles: i) it allows the preferred policy to be implemented with probability larger than one half; ii) it makes it easier for a Broadsheet newspaper to be produced and therefore it increases the probability of optimal policy implementation. However, the probability of errors in the policy implementation increases with respect to the case when just the Middle Class citizens read.

5 Appendix

In this Appendix I want to show how the results in Proposition 6 change when one considers citizens that are willing to buy newspaper not for their preferences towards reading it, but because they want to make themselves heard. Therefore, in this case, citizens buy newspaper because they want to act strategically and influence the policy implemented by the Incumbent. This means that the only reason the Middle Class citizen possesses to buy the newspaper is to induce the Incumbent politician to implement the policy that he prefers.

In Section 2, the M class’s utility function was assumed to be equal to $U_i(l, a, j) = l_i + u_i(a|\theta) + u_i(j|\sigma)$ for each $j \in \{\bar{\sigma}, \phi, \underline{\sigma}\}$ and for each $i \in M$. Since I now assume that the Middle Class citizen does not derive any utility from reading the newspaper, the utility function rewrites as:

$$\bar{U}_i(l, a) = l_i + u_i(a|\theta)$$

The function $\bar{U}_i(l, a)$ is separable in the following arguments: the income l_i and the utility the citizen derives from the implementation of the optimal policy, given the state of the world, i.e. $u_i(a|\theta)$. I make the assumption that each citizen belonging to the Middle Class has the same income $l_i = \bar{l}$ for each $i \in M$ and this is enough to buy a single copy of the newspaper, if the citizen wishes to do so. I also assume that each citizen $i \in M$ derives the same positive utility $u_i(a|\theta)$ from the implementation of the optimal policy, contingent on the state of the world. As before this means that $u_i(\bar{a}|\bar{\theta}) = u_i(\underline{a}|\underline{\theta}) > 0$,

for each $i \in M$, while $u_i(\bar{a}|\theta) = u_i(\underline{a}|\bar{\theta}) = 0$, for each $i \in M$. Finally remember that, in order to decide whether to buy the newspaper or not, the Middle Class citizen compares the newspaper price and the expected utility he receives when the Incumbent politician adopts the policy the citizen thinks is optimal, given the signal he observes privately and the equilibrium signals he foresees will be observed publicly. To simplify the discussion, I assume that the only available newspaper is the Broadsheet $j = \bar{\sigma}$; nevertheless the discussion works also if the Broadsheet $j = \underline{\sigma}$ is the only produced outlet. The timing of the game when the citizens decide to buy the newspaper only for strategic reason is the same as the previously analysed game. Finally, since I am aware that in games of these kinds there are many equilibria, I limit my analysis to look for symmetric Nash equilibrium in pure strategies where players do not play weakly dominated strategies (see also (Prat and Stromberg 2005)).

The following Lemma substitutes the Lemma (4):

Lemma 18 *There exists an (separating) equilibrium where Middle Class citizens in $t = 3$ buy the Broadsheet produced if and only if they have received the signal printed on the newspaper and the price of the newspaper is less than the expected utility the Middle Class citizens derive from the Incumbent implementing the policy they believe it is more likely to be optimal.*

Proof. Assume that in equilibrium a number of signals $n_{\bar{\sigma}}^*$ (to be determined later) is the minimum number of signals sufficient for the Incumbent to believe that $Pr(\bar{\theta}|n_{\bar{\sigma}}^*) > Pr(\bar{\theta}) = 1/2$. Furthermore remember that, given that there is one single newspaper, if in equilibrium $n_{\bar{\sigma}}$ signals are observed, the Incumbent and the remaining players have to form expectations on the remaining $N - n_{\bar{\sigma}}$ signals received by the citizens who have not bought any newspaper.

I first show that, given a number $n_{\bar{\sigma}}^* - 1$ of bought newspaper foreseen in equilibrium, corresponding to $n_{\bar{\sigma}}^* - 1$ citizens who have received a $\bar{\sigma}$ signal ($\hat{i}_{\bar{\sigma}}$) and bought the $j = \bar{\sigma}$ newspaper, citizens who have received a $\bar{\sigma}$ signal ($\hat{i}_{\bar{\sigma}}$) buy the $j = \bar{\sigma}$ newspaper, while citizens ($\hat{i}_{\sigma \in \{\phi, \underline{\sigma}\}}$) never do.¹²

There are several possible cases that one needs to analyse. In particular I distinguish between cases where $\hat{i}_{\sigma \in \{\bar{\sigma}, \phi, \underline{\sigma}\}}$ is pivotal and cases where $\hat{i}_{\sigma \in \{\bar{\sigma}, \phi, \underline{\sigma}\}}$ is not pivotal. I first begin with analysing the situations when $\hat{i}_{\sigma \in \{\bar{\sigma}, \phi, \underline{\sigma}\}}$ is pivotal:

i) The number $n_{\bar{\sigma}}^* - 1$ is such that $Pr(\bar{\theta}|n_{\bar{\sigma}}^* - 1) < Pr(\bar{\theta})$ and the citizen \hat{i} has privately observed the signal $\bar{\sigma}$.

The citizen is willing to buy the newspaper $j = \bar{\sigma}$ as long as the charged price $p_{j=\bar{\sigma}}$ is less than, or equal to, the expected utility he receives when the policy \bar{a} is implemented and this is optimal, contingent on the state of the world.

¹²Abusing notation, with $\hat{i}_{\bar{\sigma}}$ I will indicate also the signal $\bar{\sigma}$ received by the individual i . Of course $\hat{i}_{\underline{\sigma}}$ and \hat{i}_{ϕ} have similar meanings.

In fact, provided that $Pr(\bar{\theta}|n_{\bar{\sigma}}^* - 1) < Pr(\bar{\theta})$ and $Pr(\bar{\theta}|n_{\bar{\sigma}}^*) > Pr(\bar{\theta})$, then $\hat{i}_{\bar{\sigma}}$ is willing to buy the newspaper $j = \bar{\sigma}$ also to support the policy \bar{a} he believes most likely to be optimal contingent on the state of the world, given his private and public information he foresees will be revealed in equilibrium.

ii) The number $n_{\bar{\sigma}}^* - 1$ is such that $Pr(\bar{\theta}|n_{\bar{\sigma}}^* - 1) < Pr(\bar{\theta})$ and the citizen $\hat{i}_{\bar{\sigma}}$ has privately observed the signal $\underline{\sigma}$.

In this case, $\hat{i}_{\underline{\sigma}}$ will not buy the newspaper based on strategic consideration. In fact suppose that, given $Pr(\bar{\theta}|n_{\bar{\sigma}}^* - 1) < Pr(\bar{\theta})$, it follows that the policy \bar{a} gets implemented by the Incumbent iff $Pr(\bar{\theta}|n_{\bar{\sigma}}^* - 1 + 1) > Pr(\bar{\theta})$. This will be optimal for citizen $\hat{i}_{\underline{\sigma}}$ and will induce him to buy a copy of the newspaper $j = \bar{\sigma}$ iff he believes that $Pr(\bar{\theta}|n_{\bar{\sigma}}^* - 1; \hat{i}_{\underline{\sigma}}) > 1/2$. However this is not so, based on his private and equilibrium public information. In fact given that $\hat{i}_{\underline{\sigma}}$ foresees that there will be other $n_{\bar{\sigma}}^* - 1$ copies of $j = \bar{\sigma}$ in equilibrium, given his own private signal $\underline{\sigma}$, it follows that: $Pr(\bar{\theta}|n_{\bar{\sigma}}^* - 1; \hat{i}_{\underline{\sigma}}; N - n_{\bar{\sigma}}^*) =$

$$\begin{aligned} &= \frac{Pr(n_{\bar{\sigma}}^* - 1; \hat{i}_{\underline{\sigma}}; N - n_{\bar{\sigma}}^* | \bar{\theta}) Pr(\bar{\theta})}{Pr(n_{\bar{\sigma}}^* - 1; \hat{i}_{\underline{\sigma}}; N - n_{\bar{\sigma}}^* | \bar{\theta}) Pr(\bar{\theta}) + Pr(n_{\bar{\sigma}}^* - 1; \hat{i}_{\underline{\sigma}}; N - n_{\bar{\sigma}}^* | \underline{\theta}) Pr(\underline{\theta})} = \\ &= \frac{\frac{N!}{(n_{\bar{\sigma}}^* - 1)! (N - n_{\bar{\sigma}}^*)!} q^{(n_{\bar{\sigma}}^* - 1)} q' (1 - q)^{N - n_{\bar{\sigma}}^* \frac{1}{2}}}{\frac{N!}{(n_{\bar{\sigma}}^* - 1)! (N - n_{\bar{\sigma}}^*)!} q^{(n_{\bar{\sigma}}^* - 1)} q' (1 - q)^{N - n_{\bar{\sigma}}^* \frac{1}{2}} + \frac{N!}{(n_{\bar{\sigma}}^* - 1)! (N - n_{\bar{\sigma}}^*)!} (q')^{(n_{\bar{\sigma}}^* - 1)} q (1 - q')^{N - n_{\bar{\sigma}}^* \frac{1}{2}}} \end{aligned}$$

where $Pr(\bar{\theta}|n_{\bar{\sigma}}^* - 1; \hat{i}_{\underline{\sigma}}; N - n_{\bar{\sigma}}^*)$ is the probability of the state of the world $\bar{\theta}$ given that $\hat{i}_{\underline{\sigma}}$ observes in equilibrium $n_{\bar{\sigma}}^* - 1$ signals of type $\bar{\sigma}$, his own signal $\hat{i}_{\underline{\sigma}}$ and then, regarding the remaining $N - n_{\bar{\sigma}}^*$ signals, $\hat{i}_{\underline{\sigma}}$ knows that they are signals different from type $\bar{\sigma}$, therefore having a $1 - q$ total probability when the state of the world is $\bar{\theta}$ (likewise, each of the other $N - n_{\bar{\sigma}}^*$ signals will have a $1 - q'$ probability when the state of the world is $\underline{\theta}$).

If one assumes that $Pr(\bar{\theta}|n_{\bar{\sigma}}^* - 1; \hat{i}_{\underline{\sigma}}; N - n_{\bar{\sigma}}^*) < \frac{1}{2}$, multiplying both sides of the inequality by the denominator of the fraction and simplifying further, the expression reduces to: $\frac{N!}{(n_{\bar{\sigma}}^* - 1)! (N - n_{\bar{\sigma}}^*)!} q^{(n_{\bar{\sigma}}^* - 1)} q' (1 - q)^{N - n_{\bar{\sigma}}^*} < \frac{N!}{(n_{\bar{\sigma}}^* - 1)! (N - n_{\bar{\sigma}}^*)!} (q')^{(n_{\bar{\sigma}}^* - 1)} q (1 - q')^{N - n_{\bar{\sigma}}^*}$ from which it follows that:

$$q^{(n_{\bar{\sigma}}^* - 1)} q' (1 - q)^{N - n_{\bar{\sigma}}^*} < (q')^{(n_{\bar{\sigma}}^* - 1)} q (1 - q')^{N - n_{\bar{\sigma}}^*} \quad (11)$$

From the starting hypothesis that $Pr(\bar{\theta}|n_{\bar{\sigma}}^* - 1; N - (n_{\bar{\sigma}}^* - 1)) < \frac{1}{2}$, through similar algebraic manipulations to the ones above, it follows that $Pr(\bar{\theta}|n_{\bar{\sigma}}^* - 1; N - (n_{\bar{\sigma}}^* - 1))$ reduces to the following expression:

$$q^{(n_{\bar{\sigma}}^* - 1)} (1 - q)^{N - n_{\bar{\sigma}}^* + 1} < (q')^{(n_{\bar{\sigma}}^* - 1)} (1 - q')^{N - n_{\bar{\sigma}}^* + 1} \quad (12)$$

Now multiply the LHS of 12 by $\frac{q'}{1 - q}$ and the RHS by $\frac{q}{1 - q'}$ to obtain 11. To show that 11 is less than $\frac{1}{2}$, given that $q^{(n_{\bar{\sigma}}^* - 1)} (1 - q)^{N - n_{\bar{\sigma}}^* + 1} < (q')^{(n_{\bar{\sigma}}^* - 1)} (1 - q')^{N - n_{\bar{\sigma}}^* + 1}$ is sure, it suffices to show that $\frac{q'}{1 - q} < \frac{q}{1 - q'}$. By cross multiplying the terms one obtains that $q'(1 - q') < q(1 - q)$, or $q' - (q')^2 < q - q^2$. By rewriting the previous expression as $q^2 - (q')^2 < q - q'$ and factoring the term

$q^2 - (q')^2$, the expression rewrites as $(q - q')(q + q') < q - q'$. Simplifying both sides of the inequality by $q - q' > 0$, it follows that $q + q' < 1$, which is true, for each $1 > q > q' > 0$.

iii) The number $n_{\bar{\sigma}}^ - 1$ is such that $\Pr(\bar{\theta} | n_{\bar{\sigma}}^* - 1) < \Pr(\bar{\theta})$ and the citizen \hat{i} has privately observed the signal ϕ .*

Like in the previous arguments *ii)* and *iii)*, \hat{i}_ϕ knows that $\Pr(\bar{\theta} | n_{\bar{\sigma}}^* - 1; N - (n_{\bar{\sigma}}^* - 1)) < \Pr(\bar{\theta}) = \frac{1}{2}$ and that $\Pr(\bar{\theta} | n_{\bar{\sigma}}^*; N - n_{\bar{\sigma}}^*) > \Pr(\bar{\theta})$. So it is necessary to check whether the \hat{i}_ϕ is going to change his strategy of not buying the $j = \bar{\sigma}$ newspaper, once he foresees that in equilibrium there will be $n_{\bar{\sigma}}^* - 1$ signals of type $\bar{\sigma}$. \hat{i}_ϕ knows that, upon observing $n_{\bar{\sigma}}^* - 1$ signals of type $\bar{\sigma}$ and $N - (n_{\bar{\sigma}}^* - 1)$ citizens who have received the other two signals, the Incumbent rationally assigns $\Pr(\bar{\theta} | n_{\bar{\sigma}}^* - 1, N - (n_{\bar{\sigma}}^* - 1)) < \frac{1}{2}$. \hat{i}_ϕ will change his behaviour if and only if, based on his private knowledge and the one revealed in equilibrium, he believes that the state of the world $\bar{\theta}$ is more likely than the state of the world $\underline{\theta}$. In fact, since his utility depends on the Incumbent implementing the optimal policy contingent on the state of the world, \hat{i}_ϕ will prefer the \bar{a} policy to be implemented if he believes $\bar{\theta}$ being more likely than $\underline{\theta}$. However this cannot be so because, by assumption, $\Pr(\bar{\theta} | n_{\bar{\sigma}}^* - 1, N - (n_{\bar{\sigma}}^* - 1)) < \frac{1}{2}$ for any combination of $n_{\bar{\sigma}}^* - 1$ signals $\bar{\sigma}$ and $N - (n_{\bar{\sigma}}^* - 1)$ signals $(\phi, \underline{\sigma})$. This means that $\Pr(\bar{\theta} | n_{\bar{\sigma}}^* - 1, N - (n_{\bar{\sigma}}^* - 1)) < \frac{1}{2}$ also for the particular combination of $n_{\bar{\sigma}}^* - 1$ signals $\bar{\sigma}$, \hat{i}_ϕ and $N - n_{\bar{\sigma}}^*$ signals different from $\bar{\sigma}$. Since this is true by assumption then it must be true for the player \hat{i}_ϕ , also. Therefore he does not believe that $\bar{\theta}$ is more likely than $\underline{\theta}$ and will not deviate from not buying the newspaper $j = \bar{\sigma}$. Now let us analyse the case when the \hat{i}_σ is not pivotal.

iv) The number $n_{\bar{\sigma}}$ is such that $n_{\bar{\sigma}} < n_{\bar{\sigma}}^ - 1$ (or $n_{\bar{\sigma}} > n_{\bar{\sigma}}^*$) and the citizen \hat{i} has privately observed the signal $\bar{\sigma}$.*

Assume that the number $n_{\bar{\sigma}}$ is greater (strictly smaller) than $n_{\bar{\sigma}}^* (n_{\bar{\sigma}}^* - 1)$. It is easy to see that $\hat{i}_{\bar{\sigma}}$ will not buy the newspaper $j = \bar{\sigma}$. In fact the citizen knows that the policy \bar{a} (\underline{a}) will be implemented in equilibrium whether he buys the newspaper or not. Furthermore given his private information and the information revealed in equilibrium by the other citizens buying the newspaper, $\hat{i}_{\bar{\sigma}}$ knows that adopting the policy the policy \bar{a} (\underline{a}) is optimal given the equilibrium information. Therefore he is indifferent between buying the newspaper or not for strategic reasons. However he is strictly worse off buying the newspaper as long as $p_{\bar{\sigma}} > 0$. In fact, if he does not buy the newspaper the incumbent will implement the optimal policy anyway, but the $\hat{i}_{\bar{\sigma}}$ will save the cost of buying the newspaper copy. Instead, if he buys the newspaper $j = \bar{\sigma}$, the incumbent will implement the same optimal policy, but the citizen has to bear the positive cost $p_{\bar{\sigma}} > 0$. Therefore $\hat{i}_{\bar{\sigma}}$ is better off not buying the newspaper.

v) The number $n_{\bar{\sigma}}$ is such that $n_{\bar{\sigma}} < n_{\bar{\sigma}}^ - 1$ (or $n_{\bar{\sigma}} > n_{\bar{\sigma}}^*$) and the citizen \hat{i} has privately observed the signal ϕ or $\underline{\sigma}$.*

Again let the number $n_{\bar{\sigma}}$ be greater (strictly smaller) than $n_{\bar{\sigma}}^* (n_{\bar{\sigma}}^* - 1)$. It is easy to see that $\hat{i}_{\sigma \in \{\phi, \underline{\sigma}\}}$ will never buy the newspaper $j = \bar{\sigma}$. In fact the

citizen knows already that the policy \bar{a} (\underline{a}) will be implemented in equilibrium and that this is optimal given the equilibrium information and beliefs, no matter what his private information is. Therefore he is indifferent between buying the newspaper or not for strategic reason. However he is strictly better off not buying the newspaper as long as $p_{\bar{\sigma}} > 0$.

After deriving the citizen's equilibrium strategies, given other citizens' equilibrium strategies and equilibrium beliefs, now it is necessary to do the opposite. Since all $\hat{i}_{\bar{\sigma}}$ buy the newspaper $j = \bar{\sigma}$ if it is on offer, while all $\hat{i}_{\sigma \in \{\phi, \underline{\sigma}\}}$ do not, then, upon observing a number $n_{\bar{\sigma}}$ of copies bought, the Incumbent can form the equilibrium beliefs on the probability of state of the world $\bar{\theta}$, knowing that the remaining $N - n_{\bar{\sigma}}$ citizens who have not bought the newspaper $j = \bar{\sigma}$ have observed any $\sigma \in \{\phi, \underline{\sigma}\}$. Since $Pr(\bar{\sigma}|\bar{\theta}) = q$, therefore $Pr(\sigma \in \{\phi, \underline{\sigma}\}|\bar{\theta}) = 1 - Pr(\bar{\sigma}|\bar{\theta}) = 1 - q$. Of course from $Pr(\bar{\sigma}|\underline{\theta}) = q'$ it follows that $Pr(\sigma \in \{\phi, \underline{\sigma}\}|\underline{\theta}) = 1 - q'$. Then it follows simply that:

$$\begin{aligned} Pr(\bar{\theta}|n_{\bar{\sigma}}, N - n_{\bar{\sigma}}) &= \frac{Pr(n_{\bar{\sigma}}; N - n_{\bar{\sigma}}|\bar{\theta})Pr(\bar{\theta})}{Pr(n_{\bar{\sigma}}; N - n_{\bar{\sigma}}|\bar{\theta})Pr(\bar{\theta})Pr(\bar{\theta}) + Pr(n_{\bar{\sigma}}; N - n_{\bar{\sigma}}|\underline{\theta})Pr(\underline{\theta})} = \\ &= \frac{\frac{N!}{(n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!)}q^{(n_{\bar{\sigma}})}(1-q)^{N-n_{\bar{\sigma}}}\frac{1}{2}}{\frac{N!}{(n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!)}q^{(n_{\bar{\sigma}})}(1-q)^{N-n_{\bar{\sigma}}}\frac{1}{2} + \frac{N!}{(n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!)}(q')^{(n_{\bar{\sigma}})}(1-q')^{N-n_{\bar{\sigma}}}\frac{1}{2}} = \\ &= \frac{\frac{N!}{(n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!)}q^{(n_{\bar{\sigma}})}(1-q)^{N-n_{\bar{\sigma}}}}{\frac{N!}{(n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!)}q^{(n_{\bar{\sigma}})}(1-q)^{N-n_{\bar{\sigma}}} + \frac{N!}{(n_{\bar{\sigma}}!(N-n_{\bar{\sigma}})!)}(q')^{(n_{\bar{\sigma}})}(1-q')^{N-n_{\bar{\sigma}}}} = \\ &= \frac{q^{(n_{\bar{\sigma}})}(1-q)^{N-n_{\bar{\sigma}}}}{q^{(n_{\bar{\sigma}})}(1-q)^{N-n_{\bar{\sigma}}} + (q')^{(n_{\bar{\sigma}})}(1-q')^{N-n_{\bar{\sigma}}}}. \end{aligned}$$

Notice that from the condition $Pr(\bar{\theta}|n_{\bar{\sigma}}, N - n_{\bar{\sigma}}) \geq \frac{1}{2}$ it is easy to derive the same threshold point as the one already seen in Corollary 7, that is $n_{\bar{\sigma}} \geq n_{\bar{\sigma}}^* = \frac{int[\log_{q/q'}(\frac{1-q'}{1-q})]}{1+int[\log_{q/q'}(\frac{1-q'}{1-q})]}N$.

Finally, in order to sustain the equilibrium, the following out-of-equilibrium beliefs can be devised. Let $n_{\bar{\sigma}}$ be the number of equilibrium signals, with $n_{\bar{\sigma}} \geq n_{\bar{\sigma}}^*$; if one observes a number of out-of-equilibrium signals $\tilde{n}_{\bar{\sigma}}$, I impose that $Pr(\bar{\theta}|\tilde{n}_{\bar{\sigma}}) > 1/2$, for $\tilde{n}_{\bar{\sigma}} \in [n_{\bar{\sigma}}^*, N]$, while $Pr(\bar{\theta}|\tilde{n}_{\bar{\sigma}}) < 1/2$, for $\tilde{n}_{\bar{\sigma}} \in [0, n_{\bar{\sigma}}^*)$. On the other hand, if one observes a number of out-of-equilibrium signals $\tilde{n}_{\bar{\sigma}}$, with $n_{\bar{\sigma}} < n_{\bar{\sigma}}^*$, $Pr(\bar{\theta}|\tilde{n}_{\bar{\sigma}}) < 1/2$, for $\tilde{n}_{\bar{\sigma}} \in [0, n_{\bar{\sigma}}^*)$, while $Pr(\bar{\theta}|\tilde{n}_{\bar{\sigma}}) > 1/2$, for $\tilde{n}_{\bar{\sigma}} \in [n_{\bar{\sigma}}^*, N]$. This means that the Incumbent's out-of-equilibrium beliefs are the same as his equilibrium beliefs: in fact the Incumbent politician updates his beliefs in the same way upon observing any $\tilde{n}_{\bar{\sigma}}$ or $n_{\bar{\sigma}}$, be it equilibrium or off-equilibrium signals. However the middle class citizen does not change his newspaper buying decision, given that his utility from reading the newspaper is influenced only by the implemented policy. Therefore each citizen does not have any incentive in deviating from the equilibrium¹³.

¹³For a similar off-equilibrium-beliefs see Lohmann (1993) (Lohmann 1993).

The proof for the newspaper $j = \underline{\sigma}$ is similar to the one just seen for $j = \bar{\sigma}$, so I omit it. ■

So I have shown the existence of a separating equilibrium when a Broad-sheet is produced, where all the citizens who have received a signal equal to the one printed on the only produced newspaper buy a copy of the newspaper, while the others do not. With respect to the production of the Tabloid, the following holds:

Lemma 19 *There exists a pooling equilibrium where Middle Class citizens in $t = 3$ never buy the produced Tabloid for any signal they have received, if the Tabloid price is strictly positive.*

Proof. Regarding the $j = \phi$ newspaper, assume that the equilibrium strategy is such that each \hat{i}_ϕ buys the newspaper $j = \phi$, while each $\hat{i}_{\sigma \in \{\bar{\sigma}, \underline{\sigma}\}}$ does not buy it. Assuming that there are n_ϕ such citizens buying the newspaper $j = \phi$, it is straightforward to compute the following equilibrium beliefs:

$$\begin{aligned} Pr(\bar{\theta}|n_\phi, N - n_\phi) &= \frac{Pr(n_\phi, N - n_\phi|\bar{\theta})Pr(\bar{\theta})}{Pr(n_\phi, N - n_\phi|\bar{\theta})Pr(\bar{\theta}) + Pr(n_\phi, N - n_\phi|\underline{\theta})Pr(\underline{\theta})} = \\ &= \frac{\frac{N!}{n_\phi!(N - n_\phi)!}(1 - q - q')^{n_\phi}(1 - (1 - q - q'))^{N - n_\phi \frac{1}{2}}}{\frac{N!}{n_\phi!(N - n_\phi)!}(1 - q - q')^{n_\phi}(1 - (1 - q - q'))^{N - n_\phi \frac{1}{2}} + \frac{N!}{n_\phi!(N - n_\phi)!}(1 - q - q')^{n_\phi}(1 - (1 - q - q'))^{N - n_\phi \frac{1}{2}}} = \\ &= \frac{\frac{N!}{n_\phi!(N - n_\phi)!}(1 - q - q')^{n_\phi}(q + q')^{N - n_\phi \frac{1}{2}}}{\frac{N!}{n_\phi!(N - n_\phi)!}(1 - q - q')^{n_\phi}(q + q')^{N - n_\phi \frac{1}{2}} + \frac{N!}{n_\phi!(N - n_\phi)!}(1 - q - q')^{n_\phi}(q + q')^{N - n_\phi \frac{1}{2}}} = \frac{1}{2}. \end{aligned}$$

This means that if a $j = \phi$ newspaper gets produced, then for any number of copies bought by \hat{i}_ϕ citizens, this is never informative about the state of the world, since the posterior on the state of the world $\bar{\theta}$ is equal to the prior. Knowing this, I can show that citizens \hat{i}_ϕ deviate from their equilibrium strategy regarding buying newspaper $j = \phi$. As I have just shown, the buying decision is not going to affect the implemented policy, since in equilibrium $Pr(\bar{\theta}|n_\phi, N - n_\phi) = 1/2$ for each $n_\phi \in [0, N]$; therefore any $\hat{i}_{\sigma \in \{\bar{\sigma}, \underline{\sigma}\}}$ will not have any strategic motive of inducing the Incumbent to implement either of the two policy for sure, since buying the Tabloid does not change the Incumbent's decision regarding the adopted policy but entails a positive cost, as long as $p_\phi > 0$. Furthermore, $\hat{i}_{\sigma = \phi}$ does not buy the Tabloid, since he knows that the Incumbent will mix between the two policies with any positive probability, whether he buys the newspaper or does not. Therefore no citizen buys the Tabloid which, in turn, does not get produced and the Incumbent mixes between the two policies with any positive probability.

Finally, in order to sustain the equilibrium, I impose the following rule: upon an out-of-equilibrium number of signals $\tilde{n}_\phi \neq n_\phi$, the Incumbent updates his beliefs as if \tilde{n}_ϕ was an equilibrium number of signals. Therefore $Pr(\bar{\theta}|\tilde{n}_\phi, N - \tilde{n}_\phi) = \frac{1}{2}$ and the Incumbent mixes between the two policies. Since citizens anticipate this and know they will not change the implemented policy, but will have to bear a positive cost $p_\phi > 0$ in order to buy the tabloid, they never change their equilibrium strategy of not buying the Tabloid. ■

By comparing the citizens' reading behaviour in the two scenarios, the one where citizens read only because they want to induce the Incumbent

to implement their preferred policy, and the other where citizens read the newspaper because they derive some positive utility from consuming news, it is easy to see that there are some fundamental differences. In fact while both approaches justify the production of a Broadsheet, i.e. a campaigning newspaper that induces the Incumbent to adopt one policy or the other, only the approach where citizens read the newspaper for pleasure is able to justify the production of a Tabloid. Therefore one can conjecture that the right way to model the reading behaviour of (Middle Class) citizens is by having a model encompassing both the behaviours seen separately in this article. In this more comprehensive model citizens would read the newspapers both for pleasure *and* to induce a policy shift in the Incumbent's policy adoption decision. Furthermore if one believes that reading any newspaper, be it Tabloid or Broadsheet, gives the same pleasure to the citizen, then the Broadsheet will give more utility to the reader, given the additional expected utility deriving from the probability of being pivotal and of having implemented the policy he thinks to be more likely to be optimal. Other things being equal this might be one explanation of why there is a price premium to the newspaper of high quality (Broadsheet) with respect to the newspaper of low quality (Tabloid). I leave the exploration of this topic for future research.

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