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FUNDING ELDERLY CARE

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Funding elderly care

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Abstract

This paper provides a theoretical model to analyse public funding of family elderly care when two severity type are present (the *high* and the *low*), under asymmetry of information and increasing costs. The social planner can redistribute between households by means of lump sum transfer, but because of incomplete information he is prevented from observing the type of household. The welfare optimum is characterized both under full and asymmetric information. Under complete information it turns out that the transfer has to be set in such a way to induce equality in the marginal utility of income. Under asymmetric information a second best can be attained by a categorical block grant on care with unconditional block grant.

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1. *Introduction*

Social expenditure for the elderly care is doomed to rapidly increase in next years in most industrialized countries, since population ageing is a widespread phenomenon[1]. When referring to the care for the elderly, we mean a variety of services, both medical and non medical, intended to meet health and personal needs. In fact, that care may be provided by professional workers, but in many cases caregivers are the relatives of the elderly person or volunteers: from this angle we may speak of formal and informal care.

Specifically, the formal care (which incorporates health, nursing and social aspects) is exclusively provided by professional workers. On the opposite, the informal care is usually provided by relatives, volunteer or however non professional workers that provide non medical services aimed at supporting people in their activities of daily living (ADLs). When the informal elderly care is inadequate, then a formal elderly care (skilled care) has to take place, with either a private or a public funding, so that private and public provision coexist.

Family informal care faces monetary costs, firstly depending on the related private expenditure, and then to the purchase of some formal cares if required, when not publicly provided. But the main cost is non monetary, i.e., the cost of the time allocated to the provision of care (opportunity cost of time), and it is determined by the level of attention and the amount of care that the elderly requires according to its type and condition. It follows that family informal care cannot be purchased in a market: we could say that it is produced within the family by a production function whose main input is the time. As we will show in section one, however, modelling informal care using usual production functions with decreasing returns, is tantamount to mimic a virtual market in which the shadow price of elderly care is rising with the amount of care provided.

The industrialized countries have to cope with this challenge of growing elderly care both formal and informal: in fact, in recent years, some north European countries began to pay informal caregivers[2]. The answer may be represented by mechanisms that allow public funding for formal, but even informal, elderly care services; suitable arrangements might consist in financing informal caregivers combining it, where necessary, with formal services that have to vary in quantity and quality according to the severity type of the elderly people.

Thus, population aging should require social intervention for family or informal care, in the specific form of income transfer outlays. The hindrance is that in many circumstances the individual's severity is a private information, that creates favourable conditions for adverse selection and poses a tricky

[1] Among the European Union, Italy is one of the "oldest" country since the percentage of the aged population (over 65) is around 20% of the entire population ; a scenario even more alarming is represented by Liguria (a Region of Italy) in which the aged population counts 27% of the population.

[2] For instance Norway and Denmark allow relatives and neighbours providing regular home care to become municipal employees, complete with regular pension benefits. In Finland, informal caregivers receive a fixed fee from municipalities as well as pension payments.

challenge for the planner intent on social welfare maximization. In fact the individual temptation to lie about the type, in order to result eligible for higher transfers, may avoid a first best outcome to be reached.
In this paper we want to consider a problem that, to our knowledge, the literature has not yet faced, which is the problem of public funding of family elderly care when two severity type are present (the *high* and the *low)*, under asymmetry of information and increasing costs. For instance, in Besley and Coate (1991) the attention focuses on the public provision of private goods and whether this latter can be used to reach allocations not achievable with commodity taxes/subsidies and uniform lump/sum taxes/transfers. They cope with the problem of setting a public expenditure suitable to provide a redistribution towards the poor assumed a different attitude to the quality level for high and low income households.
Kuhn and Nuscheler (2007) consider the role of information when nursing homes is publicly provided. Allocations under full and asymmetric information are investigated in order to assess under which conditions nursing homes improve social welfare. In their model the level of provision is distinguished from the technology of provision. They find that asymmetry in information may be responsible for care provided too often within institutions rather than a family context.
In this paper we are dealing with the transfer design among households that faces different costs for elderly care, suitable to implement the social welfare and the efficient provision, taking into account both asymmetric information and a rising cost structure. The model we will use is closely related to the literature on fiscal federalism that examines, when local jurisdictions face different costs, how decentralized Nash equilibrium might approach Pareto efficiency with appropriate incentive schemes under different information requirements. In fact, the elderly care might have reference to the private provision of public goods, but, as a matter of fact, it could be best approached as a mixed good, in so far as it is a private good to which a public interest (externality) is attached. However, elderly care is considered here a private good in the technical sense, since the public interest is introduced by considering a social welfare function. Thus, we are able to relate to the literature concerning private provision of public goods such as (Cornes and Silva, 2002; Huber and Runkel, 2006) in the limit their results apply to intergovernmental local public goods, which are private goods in the technical sense.
This paper consistently differs with respect to the current literature on long-term elderly care both in terms of settings and results. In comparison with the current literature, the present paper contributes to the topic in two ways. Firstly, it points out that social welfare maximization might require income transfer not always from the high to the low cost household, but at times from the low to the high cost one. Then, we are able to highlight the conditions which call for such a transfer in a fairly general setting, where no particular assumptions on utility and cost function are required. Secondly, in an asymmetric information scenario, we show how the social planner might be limited by incentive compatibility constraints, but also which policy the social planner might implement in order to attain a second best outcome.
The paper is organized as follows: in Section 2 the model is presented, and Section 3 is devoted to the identification of the receiving and giving household in the full information case. In Section 4 asymmetry of information is examined, while Section 5 concerns the analysis of categorical block grant with unconditional block grant in a second best scenario. Finally, in Section 6 some concluding remarks are presented.

2. *The model*

We consider households aiming at utility maximization and assume that to each household belongs a person which requires elderly cares, provided he has problems with activities of daily living (ADLs). We distinguish between two severity type: the *high* and the *low*. To the low severity type is associated a low cost of care whereas to the high severity is associated a high cost, denoting the former by the l index and the latter by the h index. We suppose $L>1$ ($l=1....L$) number of low cost type identical households and $H>1$ ($h=1...H$) of high cost ones.

In modelling informal care, the main difficulty is that it is provided within the families, and no market exists where it may be purchased: yet all the choices should be considered as part of an overall maximising problem. To that end, here it is assumed that each household devotes the time at its disposal both at producing real income, and at caring for the elderly. It gets utility from the consumption of two goods: the care provided to the elderly (*good x*) and the composite *good y* (in which all the real income is spent). The budget constraint is $R = y + w\ell$, where R is the potential income; y, the quantity of private good purchased at the market price normalized to one for simplicity; *w* the wage rate and ℓ the time devoted to the elderly care. Each household produces the quantity of good *(x)* by its own production function $x = \xi(\ell, \theta)$; $x_\ell > 0$, $x_{\ell\ell} < 0$; $x_\theta < 0$, $x_{\theta\theta} \geq 0$, whose only input[3] is the time devoted to the elderly care, and the parameter θ is related to the severity type.
Assuming *x* as a monotonic function, its inverse may be written as $\ell = \zeta(x, \theta)$; $\ell_x > 0$, $\ell_{xx} > 0$; $\ell_\theta > 0$ [4], $\ell_{\theta\theta} \geq 0$: thus, labelling $E = w\ell$, the budget constraint appears as $R = y + E$. Since we may write $E = x\dfrac{w\ell}{x}$, labelling $p(x) = \dfrac{w\ell}{x}$, we can write $E = x\, p(x)$ as well. We know that $\dfrac{\partial p}{\partial x} > 0$ if $\dfrac{\partial \ell}{\partial x} > \dfrac{\ell}{x}$, which is always true if $x = \xi(\ell, \theta) = 0$ for $\ell = 0$, that is to say that *p(x)* is increasing in *x*.
All this boils down to the statement that *(E)* is the virtual expenditure in goods *x* (in care), which might be seen as purchased in a virtual market at a virtual price *p*, the latter rising with the quantity of care provided.
Thus, the cost *E* depends on the quantity of care (*x*) and on the type $i \in \{l, h\}$ of the elder household member. Therefore the virtual expenditure function $E^i(\vartheta_i, x_i)$ for elderly care depends and increases both on the quantity of the care x_i provided, and on the θ_i severity parameter, assuming $\theta_h > \theta_l$. This latter is rendered explicit by the following derivatives: $E^i_x; E^i_\theta > 0$; $E^i_{xx}; E^i_{x\theta} \geq 0$ (the subscript indicates the variable with respect to which the *E* cost function has been derived, either at first or second order).
Introducing a government lump-sum transfer τ^i , either zero, positive or negative, the maximization problem that faces the household according to its severity type $i \in \{l, h\}$ is given by $Max\ U^i(y_i, x_i)$, subject to the budget constraint $R^i + \tau^i = y_i + E^i$. We adopt standard assumptions for the U(·) function: it is increasing in *y* and *x* and strictly quasiconcave, as well as that all goods are normal.
In order to maximize the utility function subject to the budget constraint, the household chooses the amount of (*y*)and (*x*) to be provided, according to the following first order conditions (foc)[5]

$U^i_x = U^i_y E^i_x$ or equivalently $-SMS^i_{x,y} = E^i_x$

$$R^i + \tau^i = y^i + E^i(\vartheta^i, x^i)$$

Thus, using the implicit function theorem, we can define the optimal values[6] as follows[7]:

$$y^{i*} = Y^i(\vartheta^i, R + \tau^i)\,;\ x^{i*} = X^i(\vartheta^i, R + \tau^i) \qquad (1)$$

Shifting our attention from households' utility to social welfare, we are able to define the first best efficiency conditions from the maximization problem faced by the social planner:

$\underset{x_i,\, y_i}{Max}\ W = \sum_{i=1}^{L+H} U^i(x^i, y^i)$ subject to $\sum_{i=1}^{L+H} \left[y^i + E^i(\vartheta^i, x^i)\right] = \sum_{i=1}^{L+H} R^i$

[3] Other inputs should enter the function for the production of care as those goods and factors privately purchased by households. However, for the sake of simplicity, we assume that the only input which enters the production function is the time devoted to care.
[4] See appendix 1 for details.
5 The subscript indicates the derivative with respect to that variable, i.e., for instance $U_x \equiv \partial U(.)/\partial x$
[6] Which represent as well the demand function along the optimal path
[7] $J^i(x^i, y^i) = E^i_x \Rightarrow \begin{cases} y^{i*} = Y^i(\vartheta^i, R + \tau^i) \\ x^{i*} = X^i(\vartheta^i, R + \tau^i) \end{cases}$

As usual the first order conditions (necessary and sufficient for efficiency, given the concave programming problem) are derived:

$$-SMS^i_{x,y} = E^i_x \quad ; \; i = 1,2,\dots,H+L \tag{2}$$

$$U^i_y = U^j_y \quad ; \; i = 1,2,\dots,H \; ; \; j = 1,2,\dots,L \tag{3}$$

where $-SMS^i_{x,y} = U^i_x / U^i_y$

Conditions (3) require to equalize the marginal utility of good *(y)* among the different severity type of households. Assuming that the means at social planner disposal to get its policy goal consist on a lump sum transfer, equal in amount among all the same type households, then the maximization goal entails a solution for the following problem: $\underset{\tau^i}{Max} \sum_{i=1}^{L+H} U^i(x^{i*}, y^{i*})$ subject to $\sum_{i=1}^{L+H} \tau^i = 0$ where x^{i*}, y^{i*} are the household equilibrium values provided by eq. (1).

Analysing the corresponding focs it emerges that the condition

$$\frac{\partial U^h}{\partial \tau^h} = -\frac{\partial U^l}{\partial \tau^l} \tag{4}$$

has to be met, since the constraint $\sum_{i=1}^{L+H} \tau^i = 0$ force the transfer τ to be opposite in sign for each type of household. The economic hint underlying this condition is straightforward: the social planner transfers money from one type of household to the other as long as the marginal utility of the receiving household is higher, in absolute value, with respect to the giver's. The optimal point is reached when eq. (4) is satisfied.

It is possible to prove[8] that the optimum transfer outlays $\tau^{h*} = -\tau^{l*}$, according to equation (4), induce a Nash equilibrium characterized, for all the households, by equal marginal utilities for good *(y)*, or $U^{h*}_y = U^{l*}_y$. That is to say that such a Nash equilibrium is a Pareto equilibrium as well, so that the economic meaning of this statement is that social planner for its maximization goal can use lump-sum income transfers, and in so doing it has simply to control income marginal utilities of the households.

3. *Taxing versus subsiding households*

The direction of the transfer outlays to the households is not defined a priori: the high cost households have to subsidize the low cost one or vice versa, depending both on the cost structures and on the utility functions. To make things simpler, in order to identify the receiving (or conversely the giving) household, we will limit the analysis at only two households: a *high* cost household *h* (with a high severity elderly type) and a *low* cost household *l* (with a low severity elderly type). Thus, for the social planner the problem is to identify the sign of the transfer $\tau^l = -\tau^h$ in order to reach the Pareto optimum[9].

In the case that household have the same utility function, it is easy to find out the right sign of the transfer outlays (see appendix 3). Here we are going to consider the general case when the two households are characterized by different income, utility and cost function, and in particular by $\vartheta^h \neq \vartheta^l$. Suppose that the initial equilibrium (where $\tau = 0$) is not a Pareto optimum; the result is that the two households have different marginal utility with respect to income (or equivalently the good *y*). Let's assume, without loss of generality, $U^{i*}_y > U^{j*}_y$, $i, j \in \{l, h\}, i \neq j$.

Thus, in order to attain the Pareto optimum, the social planner intervention must consist on a positive transfer $\tau > 0$ to the household *i* and on a negative's one to household *j*, while it might be $\vartheta^i > \vartheta^j$ or $\vartheta^i < \vartheta^j$. The positive transfer to severity type *i* (and a negative transfer to severity type *j*), is a mere consequence of the fact that $U^{i*}_y > U^{j*}_y$, which is equivalent to say

[8] The proof is reported in appendix 2

[9] Where $U^{h*}_y[x^{h*}(\vartheta^h, R^h + \tau^h), y^{h*}(\vartheta^h, R^h + \tau^h)] = U^{l*}_y[x^{l*}(\vartheta^l, R^l - \tau^l), y^{l*}(\vartheta^l, R^l - \tau^l)]$

$\Delta U_y^* = U_y^{i*} - U_y^{j*} > 0$

As it is shown in appendix 4, the transfer has to move from *j* towards *i* when $\frac{\Delta U_x^*}{U_x^{j*}} > \frac{\Delta E_x^*}{E_x^{j*}}$, i.e. when the *per cent* difference between utilities is greater than the *per cent* difference between costs ($\Delta E_y^* = E_x^{i*} - E_x^{j*}$).

Defining $\Delta\theta = \theta^i - \theta^j$, we can derive $\varepsilon^\circ = \dfrac{\frac{\Delta U_x^*}{\Delta\theta}}{U_x^{j*}} \bigg/ \dfrac{\frac{\Delta E_x^*}{\Delta\theta}}{E_x^{j*}}$. The result we have obtained boils down to the statement that the transfer has to move from *j* towards *i* if $\varepsilon^\circ > 1$, on the opposite way when $\varepsilon^\circ < 1$ [10] and finally it has to be set equal to zero when $\varepsilon^\circ = 1$.[11]

So far the sign of τ has been identified, but its amount has still to be investigated. To derive this information we can use eq. 3 assuming that the social planner hasn't set any transfer yet and that each household acts according to its individual interest (in other words it moves along the Nash optimal path). Thus assuming $\vartheta^h \neq \vartheta^l$, $\tau^h = \tau^l = \tau$ and different income, utility and cost function between the two households severity type, as consequence it cannot be otherwise that the initial equilibrium point was not a Pareto optimum (eq. 3 is not satisfied).

Setting:

$U_y^{h*}[x^{h*}(\vartheta^h, R^h + \tau), y^{h*}(\vartheta^h, R^h + \tau)] = \Omega^h(\vartheta^h, R^h + \tau)$

$U_y^{l*}[x^{l*}(\vartheta^l, R^l - \tau), y^{l*}(\vartheta^l, R^l - \tau)] = \Omega^l(\vartheta^l, R^l - \tau)$

It is required, to reach the Pareto optimum:

$\Omega^h(\vartheta^h, R^h + \tau) - \Omega^l(\vartheta^l, R^l - \tau) = 0$

Thus using the implicit function theorem and solving for τ, it is possible to obtain the optimal value of transfer, i.e., that τ which enables to meet eq. 3

$\tau^* = \Omega^*(\vartheta^h, \vartheta^l, R^h, R^l)$.

4. *Information asymmetry and transfers*

In this section the incomplete information case is considered. The social planner is aware of the fact that there are low and high type households, but he is prevented from associating the right type to each one. Information concerning potential income, utility function and cost structure are at his disposal, but it is not the level of care provided to the elderly. However he can observe the expenditure on it (E^i) and the expenditure on y^i, for *i=h,l.*

The social planner aims at welfare maximization by means of lump sum transfer.

That is to say that he has to find a value τ° that maximises the social welfare, when both households may opt for receiving the transfer τ° conditional on spending E°, or paying the tax ($-\tau^\circ$) and no auditing.

Note that the social planner cannot offer contracts with the Paretian $[\tau^*, E_h^*]$ or $[\tau^*, E_l^*]$ because of cheating: the household's type which has to pay the transfer, could pretend to be the other.

The social planner maximization problem can be described as follows:

$\underset{\tau}{Max} W[x^i, y^i, x^j, y^j] = U^i[x^i, y^i] + U^j[x^j, y^j]$

subject to :

[10] It should be noted that the conditions just stated are independent in their effectiveness with respect to the initial values assumed by the two cost parameters ϑ^i and ϑ^j.

[11] This latter implies that a Pareto optimum has already been attained at the initial Nash equilibrium.

- budget constraint

$$y^i + E^i(\vartheta^i, x^i) = R^i + \tau \qquad \text{(associated lm: } \lambda^i\text{)} \tag{5}$$

$$y^j + E^j(\vartheta^j, x^j) = R^j - \tau \qquad \text{(associated lm: } \lambda^j\text{)} \tag{6}$$

- incentive compatible constraints[12]

$$U^i[x^i, y^i] \geq U^i\left\{\psi_x^i[E^j(x^j, \vartheta^j), \vartheta^i], y^j\right\}\Big|_{R^i \geq R^j} \qquad \text{(associated lm: } \mu^i\text{)} \tag{7}$$

$$U^j[x^j, y^j] \geq U^j\left\{\psi_x^j[E^i(x^i, \vartheta^i), \vartheta^j], y^i\right\}\Big|_{R^j \geq R^i} \qquad \text{(associated lm: } \mu^j\text{)} \tag{8}$$

- non negativity constraints

$$x^i, y^i, x^j, y^j \geq 0$$

x_j^i is the level of care that the cheating household *j* has to provide in order not to be detected by the social planner. It is possible to show[13] that $x_i^j = \psi_x^j(e^i, \vartheta^j)$ [14].

Considering by hypothesis the case in which τ is positive[15], implies that household *j* is taxed while household *i* is subsidized. This assumption allows us to set $\mu^i = 0$ given that the incentive compatibility constraint of eq.7 is not binding. In fact the receiving household *i* has no advantages to misrepresenting its type declaring to be the other.

Proposition 1: *if* $\varepsilon^\circ = 1$, *second best and first best coincide*
if $\varepsilon^\circ < 1$, *then a second best is attainable by subsidizing the low type*
if $\varepsilon^\circ > 1$, *then a second best is attainable by subsidizing the high type*
In the second best scenario we expect: $U_{x^i}^{i\circ} > U_{x^i}^{i*}$; $U_{y^i}^{i\circ} > U_{y^i}^{i*}$ *and* $x^{i\circ} < x^{i*}$; $y^{i\circ} < y^{i*}$

In the trivial case of $U_y^h = U_y^l$ (or $\varepsilon^\circ = 1$) the first best and the second best allocation coincide and the transfer τ has to be set equal to zero.

But, in general, the initial equilibrium might be characterized either by $U_y^h > U_y^l$ or $U_y^h < U_y^l$, i.e., $\varepsilon^\circ > 1$ or $\varepsilon^\circ < 1$.

In the first case where $U_y^h > U_y^l$, in order to maximise social welfare, it is necessary to rise U^h, i.e. the social planner has to tax *l* and return to *h* the correspondent transfer outlay. The incentive compatible constraint expressed by eq.8 requires that the transfer τ has to be set in such a way to render household *l* indifferent between pay the tax τ and choose its optimal expenditure (E^l, y^l) or to receive the transfer τ conditional to the expenditure (E^h, y^h), which is the optimal expenditure for the other (receiving) household *h*. Actually, as we have noted above, this is tantamount to say that for household *(l)* the positive transfer τ is conditional to the quantities (x_h^l, y^h).

In the opposite case, when it is $U_y^h < U_y^l$, in order to maximise the social welfare it is necessary that U^l rises, i.e., social planner has to tax *(h)* and must give to *(l)* the correspondent transfer outlay.

The incentive compatible constraints have to grant that for *(h)* it is indifferent to pay the tax τ and freely choose the expenditure (E^h, y^h) or to receive the transfer τ conditional to the expenditure (E^l, y^l), in other words for household *(h)* the positive transfer τ is conditional to the quantities (x_l^h, y^l).

[12] The incentive compatible constraints are required to avoid the cheating strategy, that is to mimic the other severity type. These constraints simply state that the utility deriving to the *i* type household when it sincerely reveals its type, has to be not lower than the utility deriving to that household from cheating, i.e. when it falsely declares to be of the other type. The goal is to avoid any incentive to lye.

[13] See appendix 5 for details.

[14] The first order conditions (foc) required for efficiency are reported in appendix 6

[15] See the previous section for details

Analysing eqs. a.2 and a.4[16] we note that, with reference to the *j* severity type which is assumed to be the contributing one, the condition $SMS_{x,y}^{j} = E_{x}^{j}$ has still to be met. This condition coincides with that identified for efficiency in the scenario of complete information.
Eq.a.5 suggests how the transfer τ has to be set by the social planner in order to equalize the shadow price of income of the two households, or in other words to get: $\lambda^{i} = \lambda^{j}$, but this latter in turn implies that the difference in marginal terms between the two type of households with respect to the composite good cannot be settled up.
Indeed, from a.3 and a.4 clearly emerges that: $U_{y}^{i} > U_{y}^{j}$, i.e., the incentive compatible constraint does not permit to join the condition of equality between the marginal utility (with respect to the composite good) for the two household types. The final equilibrium outcome will turn out to be a second best outcome. In fact further improvement might be obtained by a different resource allocation, but this goal is avoided by the incentive compatibility constraint. The efficient equilibrium outcome where $U_{y}^{i} = U_{y}^{j}$ is not attainable in presence of asymmetry of information because this condition creates favourable condition for *j* to lie about its type and thus to adopt a strategic behaviour.

It is interesting to compare the outcome of the first best scenario with this latter characterized by a lack of information. Looking at a.6 and a.8 we note that the marginal utility for the receiver *i* with respect both to good *x* and good *y*, is greater if compared with the complete information scenario, and as a consequence, the quantity for the two goods will be lower. Noting by * (star) the outcome emerging from the complete information case and by ° (circle) the outcome in the incomplete scenario, we can summarize as follows:
$U_{x^i}^{i\circ} > U_{x^i}^{i*}$; $U_{y^i}^{i\circ} > U_{y^i}^{i*}$ and $x^{i\circ} < x^{i*}$; $y^{i\circ} < y^{i*}$.

Proposition 2: if $\dfrac{U_{\psi^j}^{j}\psi_{e^i}^{j}}{U_{y^i}^{j}} \neq 1$ *then the recipient's consumption is distorted and the second best expenditure on care has to be forced*

Lump sum transfer in the second best scenario may avoid the receiving household to meet his efficiency conditions (according to the Nash behaviour) causing distortions in the recipient's spending decision[17]. In particular we may note underconsumption on the level of care (with respect to the efficiency rule) if $\dfrac{U_{\psi^j}^{j}\psi_{e^i}^{j}}{U_{y^i}^{j}} > 1$ but even overconsumption in the opposite case[18]. This result implies that the social planner has to force the recipient to a lower expenditure on care (or consistently to a lower level of care) with respect to his attitude in order to attain a second best outcome. In fact, if not, the incentive compatibility constraint couldn't be met and misrepresentation of type is likely to emerge. Indeed two conclusion regarding the household which receives the subsidy deserve our attention: the first concerns the role of information in bounding its provision of care, the second concerns the subsequent harmful distortion in its behaviour. Therefore it is possible to state that under the condition $\dfrac{U_{\psi^j}^{j}\psi_{e^i}^{j}}{U_{y^i}^{j}} < 1$ the Nash behaviour of the receiving household would enable for a higher level of care

[16] Reported in appendix 6
[17] This result sensibly differs from that of Huber and Runkel (2006), in fact we allow for the particular case of no distortion in the recipient when the condition $U_{\psi^j}^{j}\psi_{e^i}^{j} = U_{y^i}^{j}$ (see a.6 and a.8) is met. On the other hand if this equality is not verified then a distortion emerges.
[18] Also this result is new with respect to Huber and Runkel (2006).

with respect to the social (second best) optimum[19]. The condition $U^j_{\psi^j}\psi^j_{e^i} = U^j_{y^i}$ has a straightforward interpretation: in the limiting case in which the marginal effect on the contributor's utility of the recipient's expenditure on *x* is equal to the contributor's marginal utility with respect to the recipient's expenditure on good *y*, then no room for distortion is left, or evenly, the second best outcome coincides with the outcome coming from individualistic Nash behaviour of households (given the second best transfer).

With reference to the receiving household and with respect to the first best outcome, it is possible to state that underprovision for the goods *x* and *y* is always detectable. That result comes as a direct consequence of the incentive compatible constraint which binds the social planner to a suboptimal amount for the τ : $\tau^{\circ} < \tau^{*}$.
On the other hand eqs. a.7 and a.9 imply the condition that $U^{j\circ}_{x^j} < U^{j*}_{x^j}$; $U^{j\circ}_{y^j} < U^{j*}_{y^j}$ and hence $x^{j\circ} > x^{j*}$; $y^{j\circ} > y^{j*}$. The contributor provision of good *x* and consumption of good *y* in the asymmetric information context exceeds the first best one.

Proposition 3: if the income of the receiving household is not lower wrt the contributing, then the incentive compatible constraints are not binding and a first best policy is a viable way.

In the case that the income of the contributing household is lower than the income of the receiving household, then a first best policy may be implemented by the social planner given that the taxed household would be unable to implement a cheating strategy given its (binding) budget constraint. The constraint that really binds is the budget ($y^j + E^j_x = R^i + \tau$) rather than the incentive compatibility one (eq.5 and eq.6). In the afore mentioned scenario, it happens that the income of the contributing household $(R^i+\tau)$ is not sufficient to match (E^j, y^j). This assertion can be generalized as follows:
if severity type *j* is the cheating household, i.e., the household that misrepresents its type in order to receive the subsidy, then its budget constraint has to meet the condition:

$$R^j + \tau \geq y^i + E^i \qquad (9)$$

where *i* is the receiving household.
The receiving household meets its budget constraint, in order to maximize its utility, by equality, i.e.,

$$y^i + E^i = R^i + \tau \qquad (10)$$

It clearly emerges from eq.9 and eq.10 that the binding budget constraint which allows the donor household to declare to be the other type and meet the individual budget constraint can be simply synthesized by the condition:

$$R^j \geq R^i$$

5. *Categorical block grant on x with unconditional block grant*

The previous sections' analysis shows that if asymmetric information is assumed, then a second best is the only possible outcome, but distortion at recipient household is a likely consequence. The recipient household, when left free to decide about the expenditure on care, will opt either for a lower or a higher level with respect to the second best optimum.
As we have already shown, when adopting a lump sum tax, the social planner is confident that he will not cause any distortion at the contributor. The distortionary policy emerges with reference to the receiver. This point is crucial assuming the household autonomy in the spending decision.

[19] Using CES utility function as: $\left(x_i^{\frac{\beta-1}{\beta}} + y_i^{\frac{\beta-1}{\beta}}\right)^{\frac{\beta}{\beta-1}}$, $\left(x_j^{\frac{\sigma-1}{\sigma}} + y_j^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$ and assuming for instance: β=1,8; θ_i=2; σ=2; θ_j=1; $R_i \leq R_h$; it is possible to check what just stated.

Proposition 4: by unconditional block grant a second best is unattainable if households are autonomous in their spending decision

With incomplete information and by means of a unconditional block grant the first best can't be achieved, except for the trivial case $U_y^h = U_y^l$ in which the first best and the second best allocation coincide and the transfer τ has to be set equal to zero. But in general the initial equilibrium will be characterized either by $U_y^h > U_y^l$ or $U_y^h < U_y^l$, i.e., $\varepsilon^\circ > 1$ or $\varepsilon^\circ < 1$.

The social planner intending to contrast the contributor's cheating strategy is forced to take into account the incentive compatibility constraints. According to this new scenario a second best optimum can be identified but not reached if households are autonomous in their spending decision[20]. In fact there is no reason why the condition $U_{\psi^j}^j \psi_{e^i}^j = U_{y^i}^j$, that allow for no distortion at recipient, should be met and the social planner has no instruments to induce it.

Proposition 5: by open ended matching grant on the expenditure on care with unconditional block grant a second best is unattainable if households are autonomous in their spending decision

Another policy at social planner disposal may consist on a open ended matching grant on the expenditure on good *x* with unconditional block grant. By this policy the social planner provides incentives to the expenditure (and consequently to the consumption) of care by a open ended matching grant, i.e., a subsidy to the expenditure on that good. The amount of the subsidy is function of the rate *r*. In addition to the open ended matching grant, a unconditional block grant τ_u is added. The budget constraint for the recipient household turns to be: $y^j + (1-r)E^j(\vartheta^j, x^j) = R^j + \tau_u$.

However, also in this scenario a second best is not achievable if households are not forced in their spending decision and the only hypothesis "with no distortion" is as usual the limiting case where $U_{\psi^j}^j \psi_{e^i}^j = U_{y^i}^j$.

Proposition 6: by categorical block grant on care with unconditional block grant a second best is attainable if households are autonomous in their spending decision only under the condition $e^{i^\circ} \le e^{j^\circ}$

Adopting a policy consisting on *a categorical block grant on x with unconditional block grant* the social planner is able to implement a second best outcome even if it is true only under specific conditions. In general we can state that a second best is a possible outcome but that result cannot be taken as granted[21]. In our model where households are characterized by different income, utility and cost function, it may happen that the second best optimal expenditure of the subsidized household is lower, greater and even equal to the second best optimal expenditure of the taxed household. In other words, defining the second best optimum by the index °:

$E^{i^\circ}(x^{i^\circ}, \vartheta^i) = e^{i^\circ} >=< E^{j^\circ}(x^{j^\circ}, \vartheta^j) = e^{j^\circ}$.

Assuming *j* the recipient and *i* the contributor, then a second best outcome is implementable only under the condition that $e^{i^\circ} \le e^{j^\circ}$.

Let's start by the new budget constraint that the recipient faces:

$y^j + E^j(\vartheta^j, x^j) = R^j + \tau 1 + \tau 2$ with $e^j \ge \tau 1$

Where τ1 is a categorical block grant on *x* which is received by the household under the condition his expenditure on care is at least as great as the grant, in other words the household is forced to spend at

[20] Except when the condition $U_{\psi^j}^j \psi_{e^i}^j = U_{y^i}^j$ (see a.6 and a.8) is met. On the other hand if this equality is not verified then a distortion emerges.

[21] This result sensibly differ from that obtained by H&R (2006). Because they assume identical utility functions and income, they can state that this policy is able to get a second best. In our scenario where utility functions may vary among households as well as income, this policy may result ineffective in reaching a second best optimum.

least a value equal to the component τ1 on good *x*, otherwise the categorical block grant τ1 is missed out. The residual component τ2 represents a unconditional block grant. The condition $\tau 1+\tau 2=\tau$ has to be met, where τ is the sum of the categorical block grant on *x (τ1)* and the unconditional block grant *(τ2)* that the recipient household gets. τ has to be set by the social planner in order to satisfy the contributor's participation constraint.

Let's assume that the second best optimal expenditure for the recipient is:

$E^{i^\circ}(x^{i^\circ},\vartheta^i)=e^{i^\circ}$ then the second best optimal amount of good *x* is $x^{j^\circ}=F^j(e^{i^\circ},\vartheta^j)$ and the second best optimal amount of good *y* (from the budget constraint) is: $y^{j^\circ}=R^j+\tau 1+\tau 2-e^{j^\circ}$.

Equivalently we can define the optimal values for the contributing household: $x^{i^\circ}=F^i(e^{i^\circ},\vartheta^i)$ and $y^{i^\circ}=R^i+\tau-e^{i^\circ}$.

Thus the participation constraint for the taxed household is:

$$U^{i^\circ}[x^{i^\circ}=F^i(e^{i^\circ},\vartheta^i),y^{i^\circ}=R^i+\tau-e^{i^\circ}]\geq U^{i^\circ}[x^{ic}=F^c(e^{j^\circ},\vartheta^i,\vartheta^j),y^{ic}=R^i+\tau 1+\tau 2-e^{j^\circ}]$$

or

$$U^{i^\circ}[F^i(e^{i^\circ},\vartheta^i),R^i+\tau-e^{i^\circ}]\geq U^{i^\circ}[F^c(e^{j^\circ},\vartheta^i,\vartheta^j),R^i+\tau 1+\tau 2-e^{j^\circ}]$$

Where the *c* index indicates the values corresponding to the *cheating* strategy when a household type mimics the other type. Because we have assumed $e^{i^\circ}\leq e^{j^\circ}$, then the contributing region, when mimicking the other type, can increase its level of care ($x^{ic}>x^{i^\circ}$). What about the amount of good *y* the cheating household is able to purchase? Looking the contributor's budget constraint in the two scenarios (honest behaviour and cheating) and recalling that by hypothesis $\tau=\tau 1+\tau 2$ and $e^{i^\circ}\leq e^{j^\circ}$, it clearly emerges that the disposable income, after the expenditure on *x* is such that $y^{ic}<y^{i^\circ}$.

When the contributing household chooses not to sincerely reveal his type then a gain in terms of good *x* is expected, but at the same time a loss in terms of good *y* is also expected. Because the x^{i° and y^{i° are the values autonomously chosen by the household in a non distorted scenario $U_x^{i^\circ}/U_y^{i^\circ}=E_x^{i^\circ}$, then it is reasonable to expect in the cheating scenario the following inequality to hold $U_x^{ic}/U_y^{ic}<E_x^{ic}$.

If the social planner sets the tax/subsidy mix in a second best scenario, then the contributor's participation constraint should be met and the cheating strategy should be avoided.

On the other hand the receiving household maximizes his utility function under the budget constraint which now is connected to the categorical block grant, i.e., $e^{j^\circ}\geq\tau 1$.

It is straightforward to prove that the utility that the household gets from $e^{j\wedge}<\tau 1$ ($U^{j\wedge}[F^{j\wedge}(e^{j\wedge},\vartheta^j),R^j+\tau 2-e^{j\wedge}]$, where ^ indicates the values the household sets when the requirement to get the categorical matching grant τ1 is not fulfilled) is lower with respect to the utility in the case that the expenditure is set in order to meet the constraint $e^j\geq\tau 1$ ($U^{j^\circ}[F^j(e^{j^\circ},\vartheta^j),R^j+\tau 1+\tau 2-e^{j^\circ}]$). In fact the household decision to set $e^j<\tau 1$ would determine a lower level both of care (good *x*) and of good *y*, and as a consequence a net utility loss.

Summing up it is possible to state that the social planner is able to attain a second best optimum when implementing this policy but only if the condition $e^{i^\circ}\leq e^{j^\circ}$ is met.

6. *Concluding remarks*

The paper considers a social planner aiming at social welfare maximization when households privately provide the elderly care. Assuming that the social planner pursues his purpose by means of lump sum transfer, to this extent we characterize the welfare optimum both under full and asymmetric information. Under complete information it turns out that the lump sum transfer suitable to get the Pareto outcome, by which it is possible to implement the optimal level of elderly care, has to be set in such a way to induce equality in the marginal utility of the composite good *y* which is tantamount to say that equality in marginal utility of income has to be reached. From the analysis emerges that the best resource allocation may require a income transfer not always from the high to the low cost household, but at times from the low to the high cost one. The transfer sign depends on the sign of $\varepsilon°$, where this latter is given by the ratio of the *per cent* difference between utilities and the *per cent* difference between costs.

In the context of information asymmetry, in which the social planner is unable to observe neither the level of care provided nor the household's type, we might expect a second best outcome. However, even a first best is still a possible equilibrium. It occurs under the condition that the subsidized household has a income at least as great as the taxed one. Under the afore mentioned condition the incentive compatible constraint, required to avoid a cheating strategy by the contributing household, does not bind and a first best optimum is attainable, that is to say a first best level of elderly care is achievable. Relaxing the afore income condition it turns out that the second best outcome requires, in order to be implemented, that the level of recipient's expenditure on care has to be forced upwards towards a certain target. This result derives from the fact that lump sum transfer in a second best scenario might avoid the receiving household to meet his Nash efficiency conditions. As a result we obtain distortions in the recipient's spending decision with reference to the elderly care.

However, differently from the existing literature, we allow for an exception to the afore well-established rule. In fact in our very general setting no distortion occurs when the marginal effect on the contributor's utility of the recipient's expenditure on x is equal to the contributor's marginal utility with respect to the recipient's expenditure on good y. Furthermore it could also emerge the result that the social planner had to force the recipient to curb (i.e., to force downwards) the expenditure on elderly care, with respect to his attitude, in order to attain the second best outcome. This result, even if counterintuitive, follows from the risk of type misrepresentation that could be carried on by the taxed household in such a general setting.

Finally, starting from the consideration that lump sum transfer determines the mentioned distortion at the recipient, other policies at social planner disposal are investigated. In particular it is shown that a categorical block grant on the level of care along with a unconditional block grant might be able to reach a second best outcome, avoiding any distortion. This result holds under the condition that the expenditure on elderly care of the receiver is equal or greater to the expenditure on care faced by the contributor. In fact in a model where households are characterized by different income, utility and cost, it may happen that the second best optimal expenditure of the subsidized household turns out to be lower to the second best optimal expenditure of the taxed household. If the latter is true then even a policy consisting on categorical block grant on the expenditure for elderly care with unconditional block grant would be unable to reach a second best outcome without distortions.

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Appendix 1

The quantity of good *(x)* is produced by households according to the production function:

$x = \xi(\ell, \theta)$

where $x_\ell > 0$, $x_{\ell\ell} < 0$; $x_\theta < 0$, $x_{\theta\theta} \geq 0$

Using the implicit function theorem we can write:

$\ell = \zeta(x, \theta)$; $\ell_x > 0$, $\ell_{xx} > 0$; $\ell_\theta > 0$, $\ell_{\theta\theta} \geq 0$

While the first and second derivatives of ℓ with respect to good *x* come as a direct consequence of the fact that $x_\ell > 0$, $x_{\ell\ell} < 0$; the sign of ℓ_θ can be derived in the following way:

$dx = \xi_\ell d\ell + \xi_\vartheta d\vartheta$ and $d\ell = \zeta_x dx + \zeta_\vartheta d\vartheta$

Substituting the former in the latter:

$d\ell = \zeta_x [\xi_\ell d\ell + \xi_\vartheta d\vartheta] + \zeta_\vartheta d\vartheta$

Dividing both sides by $d\vartheta$:

$$\frac{d\ell}{d\vartheta} = \zeta_x [\xi_\ell \frac{d\ell}{d\vartheta} + \xi_\vartheta] + \zeta_\vartheta$$

where $\zeta_x = \dfrac{\partial \ell}{\partial x}$ and $\xi_\ell = \dfrac{\partial x}{\partial \ell}$

Thus: $\zeta_\vartheta = -\zeta_x \xi_\vartheta$ or equivalently: $\dfrac{\partial \ell}{\partial \vartheta} = -\dfrac{\partial \ell}{\partial x}\dfrac{\partial x}{\partial \vartheta} > 0$

Appendix 2

Using (1), we know that $\dfrac{\partial U^i}{\partial \tau^i} = \dfrac{\partial U^i}{\partial x^{i*}}\dfrac{\partial x^{i*}}{\partial \tau^i} + \dfrac{\partial U^i}{\partial y^{i*}}\dfrac{\partial y^{i*}}{\partial \tau^i}$, and, using (2), we can write $\dfrac{\partial U^i}{\partial \tau^i} = U_y^i \left(E_x^i \dfrac{\partial x^{i*}}{\partial \tau^i} + \dfrac{\partial y^{i*}}{\partial \tau^i} \right)$. By differentiating the households' budget constraint $R^i + \tau^i = y^{i*} + E^i(\vartheta^i, x^{i*})$ we get $1 = \dfrac{\partial y^i}{\partial \tau^i} + E_x^i \dfrac{\partial x^i}{\partial \tau^i}$ (it is simply obtained dividing by *dτ* the equation $d\tau^i = \dfrac{\partial y^i}{\partial \tau^i} d\tau^i + E_x^i \dfrac{\partial x^i}{\partial \tau^i} d\tau^i$). It follows that eq. (4) implies eq. (3), that is in the Pareto equilibrium, which is characterized by $U_y^{h*} = U_y^{l*}$, the optimum transfer outlays $\tau^h = -\tau^l$ induces equal marginal utilities for good y for all the households.

Appendix 3

In order to work it out, let's assume provisionally that at the initial equilibrium both utility and welfare are maximized. This hypothesis is admittable, in the simplest scenario, when the two households are characterized by identical income, utility and cost[22] (precisely $\theta^h = \theta^l$). Clearly when the mentioned condition is met, the optimal transfer, that satisfies eq. 3, is equal to zero ($\tau = 0$).

Afterwards, let us assume that an exogenous shock alters the ϑ^h parameter, such that its value exceeds the low cost household's one: $\vartheta^h > \vartheta^l$. The exogenous shock which has occurred to ϑ^h may either increase or decrease the marginal utility of the high severity type with respect to good *y*, i.e.,

$\dfrac{\partial U_y^{h*}}{\partial \vartheta^h}[x^{h*}(\vartheta^h, R^h + \tau^h), y^{h*}(\vartheta^h, R^h + \tau^h)] >, < 0$, which in turn implies (by eq. 2)

$$\frac{\partial \left(\dfrac{U_x^{h*}[\cdot]}{E_x^{h*}[\cdot]} \right)}{\partial \vartheta^h} >, < 0 \quad \Rightarrow \quad \frac{\partial U_x^{h*}}{\partial \vartheta^h} E_x^{h*} - \frac{\partial E_x^{h*}}{\partial \vartheta^h} U_x^{h*} >, < 0 \quad \Rightarrow \quad \varepsilon >< 1$$

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[22] To note that this is not the only possible case and that the identity of income, cost and utility functions of the two type households is not a necessary condition

where $\varepsilon = \dfrac{\partial U_x^{h*}}{\partial \vartheta^h} \dfrac{1}{U_x^{h*}} \Bigg/ \dfrac{\partial E_x^{h*}}{\partial \vartheta^h} \dfrac{1}{E_x^{h*}}$. (*)

Thus, $\varepsilon>1$ if $d\vartheta^h$ let the *per cent* variation of marginal utility be higher than the corresponding variation of *per cent* marginal cost of care (good *x*). In that case $U_y^{h*} > U_y^{l*}$ occurs and the social welfare is maximized by a transfer $\tau>0$, i.e. the low severity household must finance the high severity one.
Vice versa, in the case that $\varepsilon<1$, then the transfer has to move from the high severity to the low severity one; finally if $\varepsilon=1$, then τ has to be set equal to *0*. This result can also be provided in terms of elasticities[23] since the sign of τ depends on the elasticities $\eta_W = \dfrac{\partial(U_x)}{\partial\theta}\dfrac{\theta}{U_x} >,=,< \dfrac{\partial(E_x)}{\partial\theta}\dfrac{\theta}{E_x} = \eta_E$. In fact it is sufficient to multiply both the numerator and the denominator of eq. (*) (which provides the definition of ε) by θ, to realize that condition $\varepsilon>,=,<1$ becomes condition $\eta_W>,=,<\eta_E$.

Appendix 4

In consequence of the fact that the two households behave according to the Nash maximization rule, i.e., they move along the individual optimal path, the individual equilibrium point reflects the (first order) condition (see eq. 2) $U_y^{m*} = \dfrac{U_x^{m*}}{E_x^{m*}}$, $m \in \{l,h\}$, eq. 8 can then be rewritten as follows:

$$\Delta U_y^* = \frac{U_x^{i*}}{E_x^{i*}} - \frac{U_x^{j*}}{E_x^{j*}} = \frac{U_x^{i*}E_x^{j*} - U_x^{j*}E_x^{i*}}{E_x^{i*}E_x^{j*}} \qquad (**)$$

Hence the initial hypothesis $\Delta U_y^* > 0$ turns out to be met when the numerator of equation (**) is greater than zero ($U_x^{i*}E_x^{j*} - U_x^{j*}E_x^{i*} > 0$) given that the denominator is always positive.
Assuming:
$\Delta E_y^* = E_x^{i*} - E_x^{j*} >,< 0$ and $\Delta U_x^* = U_x^{i*} - U_x^{j*} >,< 0$, it follows that:
$\Delta U_y^* > 0$ if $\left(\Delta U_x^* + U_x^{j*}\right)E_x^{j*} - \left(\Delta E_x^* + E_x^{j*}\right)U_x^{j*} > 0 \Rightarrow$ $\Delta U_x^* E_x^{j*} - \Delta E_x^* U_x^{j*} > 0 \Rightarrow \Delta U_x^* E_x^{j*} > \Delta E_x^* U_x^{j*}$ or $\dfrac{\Delta U_x^*}{U_x^{j*}} > \dfrac{\Delta E_x^*}{E_x^{j*}}$.

Appendix 5

$x_i^j = \psi_x^j(e^i, \vartheta^j)$ is derived as follows: $E^i(x^i, \vartheta^i) = e^i$ is the expenditure on care that the receiving household sets in order to maximize its utility. The cheating household *j* will set its expenditure on elderly care $E^j(x_i^j, \vartheta^j) = e^j$ so that its cheating cannot be established. This goal is attained when $E^j(x_i^j, \vartheta^j) = e^j = e^i$. Using the implicit function theorem and naming x_i^j the good provided by household *j* when pretending to be the other type: $x_i^j = \psi_x^j(e^i, \vartheta^j)$

Appendix 6

Social planner maximization problem with asymmetric information; first order conditions (focs):

$$\frac{\partial L}{\partial x^i} = U_{x^i}^i - \lambda^i E_{x^i}^i + \mu^i U_{x^i}^i - \mu^j U_{\psi^j}^j \psi_{e^i}^j E_{x^i}^i \le 0, \qquad x^i \ge 0,\ x^i\left(\frac{\partial L}{\partial x^i}\right) = 0 \qquad \text{(a.1)}$$

$$\frac{\partial L}{\partial x^j} = U_{x^j}^j - \lambda^j E_{x^j}^j - \mu^i U_{\psi^i}^i \psi_{e^j}^i E_{x^j}^j + \mu^j U_{x^j}^j \le 0, \qquad x^j \ge 0, \qquad x^j\left(\frac{\partial L}{\partial x^j}\right) = 0 \qquad \text{(a.2)}$$

$$\frac{\partial L}{\partial y^i} = U_{y^i}^i - \lambda^i + \mu^i U_{y^i}^i - \mu^j U_{y^i}^j \le 0, \qquad y^i \ge 0,\ y^i\left(\frac{\partial L}{\partial y^i}\right) = 0 \qquad \text{(a.3)}$$

$$\frac{\partial L}{\partial y^j} = U_{y^j}^j - \lambda^j - \mu^i U_{y^j}^i + \mu^j U_{y^j}^j \le 0, \qquad y^j \ge 0, \qquad y^j\left(\frac{\partial L}{\partial y^j}\right) = 0 \qquad \text{(a.4)}$$

$$\frac{\partial L}{\partial \tau} = \lambda^i - \lambda^j = 0 \qquad \text{(a.5)}$$

[23] Huber, B. and Runkel, M. (2006).

$$\frac{\partial L}{\partial \lambda^i} = y^i + E^i(\vartheta^i, x^i) - R^i - \tau = 0 \qquad \text{(a.6)}$$

$$\frac{\partial L}{\partial \lambda^j} = y^j + E^j(\vartheta^j, x^j) - R^j + \tau = 0 \qquad \text{(a.7)}$$

$$\frac{\partial L}{\partial \mu^i} = U^i[x^i, y^i] - U^i\left\{\psi_x^i[E^j(x^j, \vartheta^j), \vartheta^i], y^j\right\} \geq 0, \quad \mu^i \geq 0, \quad \mu^i(\frac{\partial L}{\partial \mu^h}) = 0 \qquad \text{(a.8)}$$

$$\frac{\partial L}{\partial \mu^j} = U^j[x^j, y^j] - U^j\left\{\psi_x^j[E^i(x^i, \vartheta^i), \vartheta^j], y^i\right\} \geq 0, \quad \mu^j \geq 0, \quad \mu^j(\frac{\partial L}{\partial \mu^j}) = 0 \qquad \text{(a.9)}$$