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MULTIPLIER DECOMPOSITION, INEQUALITY AND POVERTY IN A SAM FRAMEWORK

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1. Introduction

The relationship among growth, inequality and poverty has been widely explored in the last years with different approaches. International organizations (World Bank, IMF, UN and in particular UNDP), national governments and civil society have been increasingly committed to fight against poverty. All development banks and nearly all multilateral and bilateral aid agencies profess themselves to be principally concerned with reducing the number and proportion of people who live in conditions of absolute poverty. The goal of eradication of poverty is stronger now than ever before. However, in many cases, the concern with poverty reduction has not been followed by adequate and specifically directed policy recommendations. The principal focus of the policies that are pursued is still on promoting economic growth. The main difference with the past is that growth is no longer seen as an end but as a means toward the reduction of poverty. It is still difficult to assess the links between policies and poverty reduction, that is to promote succesfull pro-poor policies

The huge debate within and outside the World Bank following the well-known work by Dollar and Kraay on the relation among growth, inequality and poverty contributed to enlight the complexity of the relations among these variables. The message has been succinctly expressed in the title of Dollar and Kraay (2001, 2002) paper 'growth is good for the poor'. Economic growth is considered the way for reducing poverty and international trade is seen to be the best mean for promoting growth (Dollar, Kraay 2003, 2004). "Policies of deregulating internal markets, providing macroeconomic stability, encouraging private investment through a stable and transparent legal framework, and of course removing barriers to international trade, are recommended as part of the attempt to integrate a local economy into the global economy so that it benefits optimally from this integration" (Kalwij A., Verschoor A., 2005, p. 2).

This exclusive focus on growth and trade has rised doubts and critiques. Many of them were directed towards the methodology, both to the indexes utilised to measure the degree of openness of the so-called globalizers and to the conclusions drawn from the econometric analysis (Rodrik, 2000; Wiesbrot et al., 2000). The evidence for and against this relationship is still objetc of debate (Rodriguez, Rodrik, 2000, Lee, Ricci, Rigobon, 2004)**.** Two concerns, in particular, have been expressed (Jomo, 2003). One is that globalization will promote income growth only in some regions of the world. Furthemore even in regions where globalization does promote growth, the poor could not benefit at all. The evidence supports the view that globalization has contributed to the rise of both between-region and within-region inequality (Cornia, 2003, 2004). There is no doubt that faster economic growth can be associated with faster poverty reduction. It is important, however, to be able to predict with a considerable degree of accuracy the impact of trade-induced income growth and changes in inequality on poverty.

A first strand of literature is aimed to estimate the poverty growth elasticity. This step is important in order to assess the right policies alternatively growth or poverty reduction oriented. "The immediate aim is to shed light on the casual empiricism that contends that the poor appear to benefit much more from income growth, and suffer much more from rising inequality, in some situations than in others" (Kalwij A., Verschoor A., 2005, p. 2, Besley, Burgess, 2003). The causes of poverty changes (changes in mean income and inequality) appear to work out very differently depending on when and where these changes occur.

Ravallion in a pioneering work shows that, other things equal, higher inequality of income at the beginning of a period of income growth *reduces* the extent to which the poor benefit from that income growth (Ravaillon, 1997). The finding that high inequality reduces the effects of pro-poor growth has since been confirmed many times (Hanmer, Naschold 2000; Ravallion 2001; Mosley, Hudson, Verschoor, 2004). The empirical analysis give heterogeneus results. The responsiveness of poverty to income growth and changes in inequality varies widely across regions and (to a lesser extent) over time according to different countries, different periods and "across the various ways of measuring poverty" (Bourguignon, 2003, p. 5). Bourguignon plotting "observations that come from a sample of growth …. 114 spells covering approximately 50 countries" and using a poverty line equal to a $ 1-a-day line observes that "changes in the mean income of the population explain only 26 percent of the variance of observed changes in poverty headcounts" (Bourguignon, 2003, p. 5).

In order to better identify the nature of the remaining 74 percent, the so-called growth-poverty-inequality triangle has been proposed (Bourguignon, 2004). The studies of Bourguignon (2003) and Epaulard (2003) gave theoretical support to this empirical uniformity. "Starting from the common-sense observation that poverty, mean income and inequality are related aspects of one income distribution, they show that the relationship between their changes depends *entirely* on properties of the initial income distribution (both mean income and inequality), which therefore need explicitly to be taken into account when examining the responsiveness of poverty to changes in mean income or income inequality" (Kalwij A., Verschoor A., 2005, p. 2).

A second strand of literature estimates the poverty-growth elasticity starting from a decomposition of the global poverty rate. Following Kakwani it is possible to show that the rate of change of global poverty, measured by a poverty measure additively decomposable, can be decomposed in a change in sectoral poverty rate attributable to changes in sectoral mean incomes and in the sectoral income distribution and by a change in between sector income distribution (Kakwani, 1993). If we accept the assumption that income distribution changes inside socio-economic groups is very low or negligible as long as the composition of groups stay fairly stable it is sufficient to know the growth rate of income in each sector, and the specific poverty elasticity in respect to mean income, in order to assess the change in global poverty.

A third strand of literature not only investigates the role played by growth and inequality in reducing poverty, but tries to identify a causal relationship between micro and macro variables. In order to set up poverty and redistributive policies, the definition of poverty profiles and the measurement of the impacts of economic growth, at an aggregated and sectoral level, should be assessed. Traditionally poverty and inequality are considered essentially as a microeconomic issue. Poverty profiles or inequality determinants are related to individual features. However, the impact of economic policies is related to the macroeconomic and structural policies, i.e. aggregate economic variables. Therefore both microeconomic and macroeconomic approach should be adopted.

A proper understanding of the relationships between income distribution in different socio-economic groups and alternative policies requires to build a system in which the informations on production, intermediate and final demand and income distribution between and inside different socio-economic groups are linked togheter. The Social Accounting Matrix (SAM) is the schema for this goal. The inclusion in the SAM of data related to the production side and of data related to the income distribution and to consumption expenditure allows to consider the SAM not only as a data base and as an accounting tool, but in a wider sense as a macroeconomic model.

This strand of literature can be considered complementary to the previous ones, since it allows relating the formation of individual/family income to the characteristics of the productive structure of each country. "The impact of a sector's output on poverty alleviation can be direct through the increase in incomes accruing to the poor households who contributed through their labour or land to the sector's growth of output. But another part of poverty alleviation results from the indirect effects operating through the interdependence of economic activities, i.e. the closed loop effects familiar in the Social Accounting Matrix (SAM) literature" (Thorbecke, Jung, 1996, p. 280). This kind of effect has been often ignored by current literature on poverty, income distribution and growth.

The approach we propose can be used for structural analysis of the features of the economic system and for analysis of the effects of pro-growth and antipoverty policies. The SAM can be used as a Leontief linear model, once we introduce the hypothesis of constancy for the coefficient of income distribution and of expenditure. The solution of the model brings to a matrix of multipliers which allows to assess the effects of changes of some of the variables (exogenous) on the others (endogenous) of the system. In order to estimate the changes in mean income of different socio-economic groups it is possible to adopt the multiplier decomposition approach based on a Social Accounting Matrix (SAM).

Following the Pyatt and Round's decomposition method of "fixed price multipliers matrix" (Pyatt, Round, 2005), we will determine the multipliers values of different households groups. This decomposition allows measuring the change in the level of mean income of each group as consequence of a change in the values of the exogenous accounts that are included in the SAM. This approach allowes to assess the consequences of real shocks on some variables (the exogenous one) on the equilibrium of the other variables considered as endogenous. The values of sectoral mean incomes obtained applying the mulplier analisys will be linked to the poverty index through sectoral poverty-income elaticities. If the poverty index is an additively decomposable measure as the head-count ratio, the passage from sectoral poverty to global poverty will be easily obtained.

2. The growth effect and the distributional effect.

In order to assess the impact of economic growth on poverty it is necessary to choose a suitable poverty measure. Following Bourguignon (2004) poverty can be measured by the absolute headcount ratio $H = q/N$, that is the number of poor (q), which lies below an absolute poverty line z assumed as fixed, as the proportion of total population (N). The limits of this index are well known. It shows only the wideness and not the intensity of poverty. For measuring how much poor are the poor we should calculate the so called poverty gap. However the headcount ratio has two features which are relevant for the strand of literature we are reviewing and for the accounting multipliers methodology we will discuss (Bourguignon 2004; Kalwij A., Verschoor A., 2005; Pyatt, Round, 2005). The head count ratio is easily linkable to the income density function and to the Lorenz curve. It can be easily calculated without knowing the distribution of incomes of the poor. This information is lacking in many developing countries.

Given a random variable x its distribution can be represented by the frequency density function $f(x)$. The corresponding distribution function is $F(x)$ where $f(x)$ is the derivative of $F(x)$ (Lambert, 2003, p.22):

$$F'(x) = f(x) \qquad\qquad [1]$$

$F(x)$ varies from 0 to 1 and can be interpreted as the proportion of units having an income less than or equal to x.

Figure 1 shows $f(x)$, that is the number of individuals at each level of income represented on a logarithmic scale on the horizontal axis. This function represents the level of inequality, that is the disparities in relative income across the whole population, "disparities in income after normalizing all observations by the population mean so to as to make them indipendent of the scale of incomes"

(Bourguignon, 2004, p. 4). If we choose a poverty line z the head count ratio at time t is simply measured by the area under the curve at the left of the poverty line z and can be written as:

$$H_t = F_t(z) \hspace{4cm} [2]$$

It is possible to show that a change in poverty is a function of growth, measured by the percentage change in mean income, and of changes in income distribution. The shift in the function $f(x)$, which corresponds to a change of the area of poverty H_t, can be decomposed into two effects: a *growth* and a *distributional* effect. A formal statement of the relationship between growth, poverty and distributional parameters is offered in Bourguignon (2003) under the assumption that the distribution function is a log-normal. This is a standard approximation of empirical distributions in the applied literature.

Figure 1 illustrates the decomposition by considering a move from an initial to a final log-normal distribution in two stages: by first shifting its mean and next its dispersion parameter. The initial distribution shifts to the right so that its mean is identical to that of the final distribution but at first it does not change shape: the *relative* distribution remains unchanged. The area between the two identically shaped distributions to the left of the poverty line is the poverty reduction that results from the growth that has actually taken place, under the assumption that the relative distribution of income has not changed. "Because of the logarithmic scale on the horizontal axis, this change corresponds to the same proportional increase of all incomes in the population and thus stands for the pure 'growth effect' with no change taking place in the distribution of relative income" (Bourguignon, 2003, p. 10).

The *distributional* effect corresponds to a change in the distribution of relative incomes which, by definition, is independent of the mean. The movement from curve (I) to the "new distribution" curve occurs at constant mean income, and corresponds to the change in the distribution of 'relative' income, or the 'distribution' effect. The final distribution has a different shape from the initial one.

Figure 1. Decomposition of change in distribution and poverty into "growth effect" and "distributional effect"
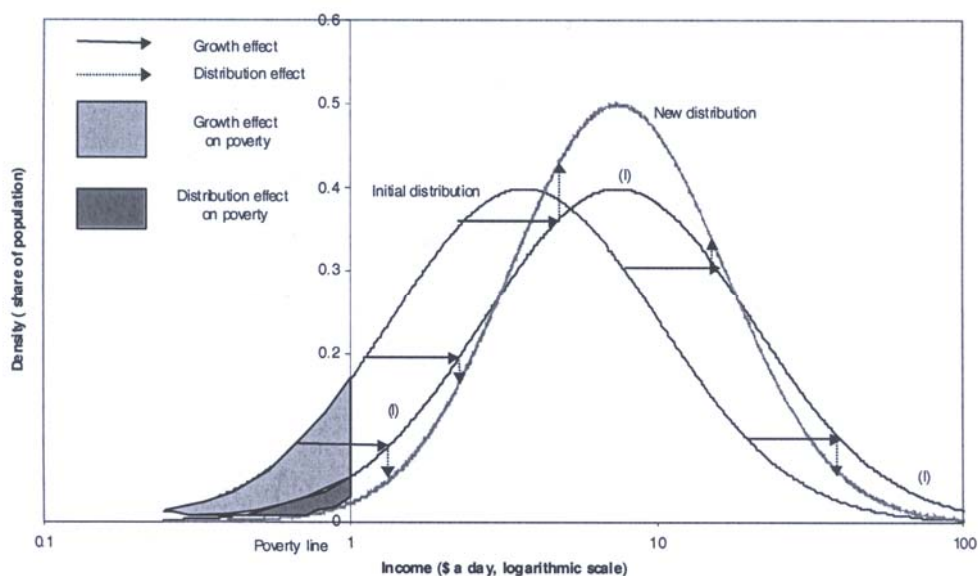
Fonte: Bourguignon (2004, p. 7)

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Of course, there is some path dependence in that decomposition according to what curve move firstly. For sufficiently small changes in mean income and in the distribution, the preceding decomposition corresponds to an identity which expresses the change in poverty as a function of the growth in mean income and changes in the distribution of relative income and we can ignore path dependence problems. What cannot be ignored is that "the impact on a poverty headcount ratio of changes in mean income and Gini depends on the shape and the location of the initial distribution of income" (Kalwij A., Verschoor A., 2005, p. 2).

It is shown by Bourguignon that both the growth and the inequality elasticity of poverty are increasing functions of the level of development and decreasing functions of the degree of relative income inequality. It is also shown how the decomposition identity may be applied to observed growth periods for which distribution data are available at the beginning and at the end of the period. The empirical evidence shows clearly that both growth and inequality changes play a major role in generating changes in poverty. However, the impact of these phenomena will depend both on the initial level of income and of inequality. Moreover, the relative effects of both phenomena may differ quite dramatically across countries (Bourguignon, 2003).

Applying the identity discussed above, it is a rather simple matter to identify in the observed change in poverty what is due to growth – under the assumption of a constant distribution of relative income – and what is due to changes in the distribution of relative income. Observation collected by Bourguignon show that *distribution matters* for poverty reduction. Over the medium-run, distributional changes may be responsible for sizable changes in poverty. In some instances, these changes may even offset the favorable effects of growth.

3. The impact of economic growth on poverty: the mean income poverty elasticity

The change in global poverty can be considered as depending only on the change in the rate of growth when the distributional parameters are supposed fixed and the rate of growth of income is identical in all sectors. This is a strong assumption, but in the short run income distribution can be considered fairly stable so that the only variable that matters for change in poverty is the change in the national average income. In this case the only parameter that matters is the poverty mean income elasticity which measure the *pure growth effect*.

Following Kakwani it is possible to calculate the elasticity of two measures of poverty: the head-count ratio H and the poverty gap starting from the Lorenz curve. The first step is to calculate the change in H in respect to the mean income μ when the Lorenz curve, which represents the degree of inequality, does not shift. The first step will be to differentiate the first derivative of the Lorenz function with respect to μ where:

$$\mu = \int_0^{+\infty} xf(x)dx \qquad [3]$$

The Lorenz curve $L(p)$ is defined as the relationship between the proportional share of total income of the p units having an income less than or equal to x, and the proportion of the units p, given by $F(x)$. For each p there is only one income level x with rank p. In the point $x=z$ the proportion of units having income less or equal to z is $H=F(z)$ and the Lorenz curve is defined as:

$$p = F(z) \rightarrow L(p) = \frac{1}{\mu} \int_0^z xf(x)dx \qquad [4]$$

The first and second derivative of the Lorenz curve with respect to p are obtained differentiating [4] twice (Lambert, 2003, p.32) and using the chain rule. All the passages are in the Appendix.

$$L'(H) = z / \mu \qquad [5]$$

$$L''(H) = \frac{1}{f(z)\mu} \qquad [6]$$

where $f(z)$ is the frequency density function of income x at the point $x = z$.
Assuming that the Lorenz curve does'nt shift (no change in income distribution) we can differentiate [5] with respect to μ to obtain :

$$\frac{\partial H}{\partial \mu} = -\frac{z}{\mu^2 L''(H)}, \qquad [7]$$

Substituting [6] in [7] gives the elasticity of head-count ratio with respect to the mean income of the economic system (Kakwani, 1993, p.123):

$$\eta_H = \frac{\partial H}{\partial \mu}\frac{\mu}{H} = -\frac{zf(z)}{H} < 0, \qquad [8]$$

"which is the percentage of poor who cross the poverty line as a result of a 1 percent growth in the mean income" (Kakwani, 1993, p.123).
It is possible, also, to calculate the mean income elasticity of the poverty gap GP, defined as the product of the two indexes H and I. This measure takes in account not only the number of the poors but also their poverty intensity. The income gap ratio I is defined as the average value of all the poverty gaps (calculated as the distance of the incomes of the poor and the poverty line) as a proportion of the poverty line z.
If $\mu*$ is the average income of the q poors the poverty gap PG can be defined as:

$$PG = \frac{1}{N}\sum_{i=1}^{q}(\frac{z-y_i}{z}) = \frac{1}{N}\frac{qz - \sum_{i=1}^{q} y_i}{z} = \frac{q}{N}\frac{z - \sum_{i=1}^{q} y_i / q}{z}. \qquad [9]$$

$$\frac{q}{N}\frac{z - \mu*}{z} = HI \qquad [10]$$

The mean income elasticity of the *poverty gap* can be calculated after having obtained the relation between the mean income of the poor $\mu*$ and the mean income of all the population μ. If we express the Lorenz curve as

$$L(H) = \frac{H\mu*}{\mu} \qquad [11]$$

"which follows immediatelyfrom the definition of the Lorenz curve"[1].
Differentiating both the members of the [9] in respect to μ, and using equation [6] and equation [8] we obtain[2]:

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[1] Kakwani (1993, p.123).

[2] Bourguignon (2003, p. 14).

$$\frac{d\mu^*}{d\mu} = \frac{\mu^*}{\mu} + \frac{(z - \mu^*)}{\mu}\eta_H \qquad\qquad [12]$$

Looking at the right side of equation [12] we observe that the first term is positive, the second one negative beacause $\eta_H < 0$. So it is not possible to adfirm that the income of the poors will increase with the increase of the average income. Only if no one of the poor cross the poverty line we can adfirm that the average income of the poors will increase togheter with the average income of the entire population. If the richest between the poor cross the poverty line the mean income μ^* will decrease. This is precisely the effect which is caught by the second term in the right side of the equation [12][3].

Utilizing equation [12] and equation [7] into [9] we obtain the elasticity of the poverty gap ratio with respect to μ. All the passages of the analitycal derivation are in the Appendix.

$$\eta_{PG} = -\frac{\mu^*}{z}\frac{H}{HI} = -\frac{\mu^*}{zI} = -\frac{\mu^*}{z\frac{(z - \mu^*)}{z}} = -\frac{\mu^*}{z - \mu^*} < 0, \qquad\qquad [13]$$

The equation [13] showes that the poverty gap ratio always decreases with the growth of the average income of the society[4]. This is a very important result from the theoretical point of view. However it is difficult to estimate because of lack of empirical informations.

4. The sectoral poverty mean income elasticities: the decomposition methodology.

The assumtion that income distribution does'nt matter is very strong. A more realistic approach assumes that the impact of growth on poverty depends on different growth rates of the sectors in which the economic system has been divided. The links between poverty and changes in the mean incomes at a disaggregated level can be explored on the basis of the studies that decompose poverty changes into an effect due to changes in the mean incomes and the effect due to changes in inequality. Studies that pioneered such a decomposition using a parametric specification of the Lorenz curve have been undertaken by Ravallion and Huppi (1991) for Indonesia, Datt and Ravallion (1992) for regions of Brazil and India, and Kakwani (1993) for Côte d'Ivoire. The decomposition methodology introduced in Datt and Ravallion (1992) has become very influential, giving rise to a rich strand of literature.

The Kakwani methodology will be our starting point in order to estimate the sectoral poverty-income elasticities. These elasticities, linked to changes in sectoral mean incomes will allow to estimate the sectoral and global level of poverty (Pyatt and Round, 2005). The choice of the poverty measure matters, of course. An important condition to be satisfied is that this measure is additively decomposable. If we choose a measure of poverty θ additively decomposable across the m groups of households the global poverty index is obtained summing up all θ_i so that

$$\theta = \sum_{i=1}^{m} f_i \theta_i , \qquad\qquad [14]$$

where θ_i is a measure of poverty among households within the particular socio-economic group i

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[3] Kakwani (1993, p. 123).

[4] Kakwani (1993, p. 124)

and f_i is the proportion of individuals in the ith subgroup such that $\sum_{i=1}^{m} f_i = 1$. The poverty index θ_i is a function of three factors: 1) the poverty line z assumed as fixed; 2) the mean per capita income μ_i and 3) the degree of inequality in the distribution of income.

We can put the assumption that income distribution change inside socio-economic groups will be very low or negligible as long as the groups stay fairly homogeneous so that the density distribution function does not changes shape, but shifts toward the right following a rise in the mean income. In this case it is sufficient to know the growth rate of income in each group, and the specific poverty elasticity in respect to the mean income, in order to calculate the impact of growth (sectoral and global) on poverty.

Following Kakwani (1993) we can differentiate [14] with respect to the mean income of the ith subgroup obtaining the elasticity of the total poverty index with respect to the ith subgroup mean income. This elasticity results from the elasticity of the ith subgroup poverty index with respect to the mean income of the ith subgroup $\eta_{\theta_i} = \partial\theta_i / \partial\mu_i (\mu_i / \theta_i)$ and can be expressed as:

$$\eta_{\theta_i}^* = \frac{\theta_i f_i}{\theta} \eta_{\theta_i} \qquad\qquad [15]$$

It can be shown that the elasticity of total poverty index with respect to the mean income of the entire economy η_θ is given by[5]:

$$\eta_\theta = \sum_{i=1}^{m} \frac{\theta_i f_i}{\theta} \eta_{\theta_i} = \sum_{i=1}^{m} \eta_{\theta_i}^* \qquad\qquad [16]$$

Equation [16] shows how the effects of sectoral growth rates on poverty add up to the total effect on poverty. This equation can be rewritten as:

$$\frac{d\theta}{\theta} = \sum \eta * \theta_i \frac{d\mu_i}{\mu_i} \qquad\qquad [17]$$

This relation can be used to measure the proportionate change in the total poverty if we assume that within sector inequality has not changed. This assumption is realistic as long as the individuals belonging to the different sectors are fairly homogeneous. However "Since the sectoral growth rates can differ, the income inequality in the population may change because of between group inequality"(Kakwani, 1993, p.129). This second effect can be significant and must be taken into account by rewriting equation [17] as:

$$\sum \eta * \theta_i \frac{d\mu_i}{\mu_i} = \eta_\theta \frac{d\mu}{\mu} - \sum \eta * \theta_i \left[\frac{d\mu}{\mu} - \frac{d\mu_i}{\mu_i} \right] \qquad\qquad [18]$$

The first term in [18] "is the pure growth effect on poverty and the second measures the effect of change in the between sector inequality caused as a result of different growth rates in various sectors. If every sector has the same growth rate, the second term will be zero"[6].

Then it is possible to show that the rate of change of global poverty measured by an index as the head-count ratio can be decomposed in: 1) a change in sectoral poverty rate attributable to changes in mean incomes and in the sectoral income distribution; 2) a change in between sector income

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[5] Kakwani (1993), p..129.

[6] Kakwani (1993), p..129.

distribution. If n_i is the number of people in socio-economic group i and H_i is the the proportion who are poor, that is the choosen poverty measure additively decomposable, the number of poor Q_i in each group will be

$$Q_i = n_i H_i \qquad\qquad [19]$$

$$Q = \Sigma\, n_i H_i = n\, H \qquad\qquad [20]$$

The change of Q_i can be expressed as

$$dQ_i = n_i\, dH_i + H_i\, d\, n_i \qquad\qquad [21]$$

The change of Q and H will result from the change of each H_i, that is will depend on three factors: 1) the sectoral growth effect related to changes in the mean per capita income μ_i for households in socio-economic group i. These changes could follow a change in prices. In the short run, however, the implications of relative price changes can be set aside. We should take account of them "if only to the extent that such changes will shift the poverty line and hence change the proportion of those in category i who are poor" (Pyatt, Round, 2005, p. 12); 2) the sectoral distribution effect following a change in inequality in sectoral income distribution; 3) the poverty line z assumed as fixed.

5. The multiplier model based on a simplified SAM.

In order to assess the impact on global poverty of changes in mean incomes of each group due to pro-poor policies it is usefull to introduce the multiplier approach, which allow to determinate the values of mean incomes. The starting point is the Social Accounting Matrix (SAM). The SAM can be considered as an extension of the traditional input-output framework. This format adds some matrices, not included in the Leontief schema, which allow taking in account of the relationships between factorial distribution of income, income distribution to Institutions and final demand. The introduction of accounts referred to Institutions (Households, Private Companies, Government, Rest of the World) allows capturing the link between factors of production and the Institutions, which own the different factors of production. The secondary distribution of income is also introduced as the result of transfers between different Institutions, mainly between private Institutions and the Government.

The SAM captures and shows the entire circular flow of income from its production to its distribution and its expenditure. In the original formulation, presented by Brown and Stone in the sixties, this schema can be considered as an analytical presentation of the traditional Keynesian model (Stone,1962, 1985, 1986). The disposable income of Institutions is the starting point for sustaining the final demand. In particular the household, grouped in different socio-economic groups, sustain the demand for consumption. The amount of income, which is not consumed in the current year, is saved and goes into the capital account. In the SAM the values flows of transactions of an economic system are organised in an accounting way starting from elementary flows which link the economic units at different level of aggregation[7].

"If a certain number of conditions are met - in particular, the existence of excess capacity and unemployed or underemployed labour resources - the SAM framework can be used to estimate the effects of exogenous changes and injections, such as an increase in the demand for a given

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[7] "A matrix framework is even optimally suited, as it allows for multiple acting, i.e. distinguishing more than one type of unit within a single accounting system, and multiple sectoring, i.e. distinguishing more than one classification of units within a single accounting system" (Keuning, 1994, p. 22).

production activity, government expenditures or exports on the whole system. As long as excess capacity and a labour slack prevail, any exogenous change in demand can be satisfied through a corresponding increase in output without having any effect on prices. Thus, for any given injection anywhere in the SAM, influence is transmitted through the interdependent SAM system. The total, direct and indirect, effects of the injection on the endogenous accounts, i.e. the total outputs of the different production activities and the incomes of the various factors and socioeconomic groups are estimated through the multiplier process" (Thorbecke, 2000, p. 16).

The income distribution of the Institution Households in the SAM must be considered as an equilibrium one, i.e. the distribution that assure the balance between the final demand for consumption and the supply of different commodities from the productive sectors in a given year. Following a Keynesian approach, we can assume that the total level of income of each group determines the consumption of different commodities by the Institution Households. The multiplier approach allows quantifying the different ways by which an income initially equally earned by each socio-economic group turns into different disposable income levels through the three stages of spending, production and redistribution.

The multipliers approach allow to capture the structural features of income distribution and the interrelations between socio-economic groups each other and with other Institutional Sectors. The resulting inequality can be considered as the minimum inequality compatible with the given productive and spending structures, and hence as a result of the mechanism only explicitly considered in the model. "A main outcome of SAM-based multiplier analysis is to examine the effects of real shocks on the economy on the distribution of income across socio-economic groups of households. One other important feature of SAM-based multiplier analysis is that it lends itself easily to decomposition, thereby adding an extra degree of transparency in understanding the nature of linkage in an economy and the effects of exogenous shocks on distribution and poverty" (Round, 2003, p. 271).

The equilibrium solution through the SAM is obtained once we separate Endogenous Accounts (Households and Private Companies) from the exogenous ones (Government, Rest of the World). The income distribution of the Private Institutions (the *H* groups of Households and the Private Companies aggregated in a single Institution) will be consistent with a given production structure under the assumption that the final demand depends on the disposable income of the Institutions that are stated as Endogenous.

Figure 2 - Exogenous and endogenous accounts in a simplified SAM.

| | Endogenous Accounts | | | Exogenous Institutions | Total |
	Activities	Factors	Private Institutions		
Activies	T_{11}	0	T_1	x_1	y_1
Factors	T_{21}	0	0	x_2	y_2
Institutions	0	T_{32}	T_{33}	x_3	y_3
Exogenous Institutions	l'_1	l'_2	l'_3	x_4	y_4
Total	y'_1	y'_2	y'_3	y'_4	

In order to measure the effects occurring in some variables (the exogenous ones) on the other (the endogenous ones) of the system, a very aggregated SAM (Figure1) must be introduced. In this SAM the endogenous components (Activities, Factors and Private Institutions as Household and Companies) can be isolated from the exogenous ones (Government, Rest of the World and Capital/Saving) by aggregating one or more matrices of the SAM. "A truncated SAM consolidates all exogenous transactions and corresponding leakages and focuses exclusively on the endogenous transactions and transformations" (Thorbecke, 2000, p. 18). Our model, in particular, assumes that the consumption demand comes only by the Households. Private Companies only receive income from Factors and redistribute it to other Private Institutions.

The determination of a multi-sector income multiplier is a distinguishing characteristic of a model based on the SAM. In the SAM model incomes (both total income and groups incomes) have different values depending on the composition of final demand, influenced by the structure of personal income distribution. The traditional input-output analysis, on the contrary, assumes the consumption demand as exogenous and not depending from the personal income distribution of the different Households groups. The equilibrium solution is obtained following the same procedure as in input-output analysis and using the SAM as a linear model. "It is obvious that the SAM formulation contains more information and a higher degree of endogeneity since it captures the endogenously derived effects of income distribution on consumption, which the Leontief national does not" (Thorbecke, 2000, pp. 21-22).

The matrices of average expenditure A_{ij} are obtained dividing each element in the transaction matrices of endogenous accounts T_{ij} by the correspondent column sum vectors y'_j where $\hat{y}_j$ is a diagonal matrix whose elements are the components of y'_j.

$$A_{ij} = T_{ij} (\hat{y}_j)^{-1} \qquad\qquad [22]$$

The hypothesis of fixed expenditure coefficients resulting from A_{jk} is consistent with the assumptions of the linear expenditure system developed by Stone for which there is widespread empirical support. The normalisation of the transaction matrices T_{jk} allows the constraints relating to row and column totals of the SAM in Figure 2 to be rewritten isolating the group of the r (three in our case) endogenous accounts from the exogenous ones. We can, thus, write

$$y = A\, y + x \qquad\qquad [23]$$
$$y_4 = l'_1 y_1 + l'_2 y_2 + l'_3 y_3 + x_4 \qquad\qquad [24]$$

Equation [24] indicates that the equilibrium position of the accounts relating to exogenous Institutions is achieved once endogenous accounts are in equilibrium. This condition allows us that only equation [23] is taken into consideration and it is rewritten as

$$y = (I - A)^{-1} x = Mx \qquad\qquad [25]$$
$$M = (I - A)^{-1} \qquad\qquad [26]$$

Thus, from [25], endogenous incomes y (i.e. production activity incomes y_1, factor incomes y_2, and institution incomes y_3 as shown in Figure 2. 1) can be derived by premultiplying injection x by a multiplier matrix M. This formulation indicates that vector y of receipt totals for each endogenous account can be obtained from vector x, expressing the receipt totals of exogenous Institutions, by the generalised inverse M.

This matrix M, introduced by Pyatt and Round (1979) in a seminal contribution, has been referred to as the "accounting multiplier" matrix "because it explains the results obtained in a SAM and not the process by which they are generated" (Thorbecke, 2000, p. 19). This accounting multipliers matrix can be interpreted as a simplified model of the actual way in which the system is working. From another side the results of the multiplier analysis can be interpreted as a demonstration of how

the economic system is expected to behave when the model assumptions perfectly reflect the real situation: any possible deviation from reality would then indicate both the correct parts and those which must be better calibrated. **M** in equation [25] is the matrix of the global multipliers and shows the overall effects resulting from the direct and indirect transfer processes generated by an initial increase in each of the three exogenous components $\mathbf{x_1}$, $\mathbf{x_2}$, and $\mathbf{x_3}$ on each element of the r (in our case three) endogenous accounts.

6. The decomposition of "fixed prices multiplier" matrix M_c

The estimate of the changes in average income of each household group can be considered a first step toward an assessment of changes in the level of poverty. "Accounting multipliers" are derived in constant prices and they are therefore "fixed-price" in a formal sense. They show average responses of endogenous variables to exogenous injections. One limitation of the accounting multiplier matrix is that "it implies unitary expenditure elasticities (the prevailing average expenditure propensities in **A** are assumed to apply to any incremental injection)" (Thorbecke, 2000, p.19). Average responses could be different from marginal ones. Then 'fixed-price multipliers', based on marginal responses, must be introduced. "The distinction simply recognises that the marginal responses in the system, even in a fixed-price world, may be different from what they are on average" (Round, 2003, p.14).

Following Pyatt and Round (2005) we introduce a SAM where **y** is the vector of row totals. Each element t_{ij} of matrix $\mathbf{T}_{ij}$ can be expressed as:

$$t_{ij} = t_{ij} \ (\mathbf{p}, \mathbf{y}; \theta) \qquad [27]$$

where **p** is a vector of prices for goods and services, and θ is a set of parameters. By summing the values along the rows of the SAM, we can obtain a set of equations

$$y_i = \sum_j \ t_{ij} \ (\mathbf{p}, \mathbf{y}; \theta) + x_i \qquad [28]$$

where y_i is the ith element of **y** and x_i is the element of the ith row of exogenous injections **x**. It then follows from total differentiation of [28] that, for any general equilibrium model of an economy, if the parameters θ are fixed, then

$$d\mathbf{y} = \mathbf{C}d\mathbf{y} + \mathbf{E}d\mathbf{p} + d\mathbf{x} \qquad [29]$$

where the (i, j)th element of **C** and **E** are:

$$c_{ij} = \frac{\partial \sum t_{ij}}{\partial y_j}(p; y; \theta) \qquad [30]$$

$$e_{ij} = \frac{\partial \sum t_{ij}}{\partial p_j}(p; y; \theta)$$

If the matrix of the marginal propensities to espenditure **C** exists and $d\mathbf{p}=0$ then

$$\mathbf{M_c} = (\mathbf{I\text{-}C})^{-1} \qquad [31]$$
$$d\mathbf{y} = \mathbf{M_c} \ (\mathbf{E}d\mathbf{p} + d\mathbf{x}) = (\mathbf{I\text{-}C})^{-1} \ d\mathbf{x} = \mathbf{M_c} \ d\mathbf{x} \qquad [32]$$

The multiplier matrix $\mathbf{M_c}$ is referred to as a fixed price multiplier matrix ..... because $\mathbf{M_c}$ "defines the impact of any change in the injections **x** on each element of the vector **y** when prices are held constant" (Pyatt, Round, 2005, p.4).

$$C = \begin{vmatrix} C_{11} & 0 & C_{13} \\ C_{21} & 0 & 0 \\ 0 & C_{32} & C_{33} \end{vmatrix}$$

C can be easily computed from **A** as follows: $C_{ij} = \eta_{ij} A_{ij}$ where η_{ij} is the elasticity of i with respect to j. "Pyatt and Round (1979) computed both kinds of multipliers in a study for Sri Lanka, by using data on income elasticities for one part of the SAM, namely household expenditures on commodities. All other elasticities were effectively set at unity, so the numerical differences between the two sets of multipliers were very small but, conceptually, this helps to break away from relying on the outlay patterns *per s*e. In most studies accounting multipliers are used as though they are fixed-price multipliers, and equivalently the income elasticities are set at unity" (Round, 2004, p.14).

Equation [32] can be written out in explicit form as

$$\mathbf{dy_1} = \mathbf{C_{11}\,dy_1} \qquad\qquad +\mathbf{C_{13}\,dy_3} + \mathbf{dx_1} \qquad\qquad [33]$$

$$\mathbf{dy_2} = \mathbf{C_{21}\,dy_1} + \qquad\qquad\qquad + \mathbf{dx_2} \qquad\qquad [34]$$

$$\mathbf{dy_3} = \qquad\quad \mathbf{C_{32}\,dy_2} \ \ + \mathbf{C_{33}\,dy_3} + \mathbf{dx_3} \qquad\qquad [35]$$

which yields:

$$\mathbf{dy_1} = \mathbf{(I\text{-}C_{11})^{-1}dx_1} \qquad +\mathbf{(I\text{-}C_{11})^{-1}C_{13}\,dy_3} \qquad\qquad [36]$$

$$\mathbf{dy_2} = \mathbf{C_{21}\,dy_1} \qquad\qquad\qquad\qquad\qquad + \mathbf{dx_2} \qquad\qquad [37]$$

$$\mathbf{dy_3} = \mathbf{(I\text{-}C_{33})^{-1}\,dx_3} \qquad + \mathbf{(I\text{-}C_{33})^{-1}C_{32}\,dy_2} \qquad\qquad [38]$$

This last set of relationships can be represented graphically in Figure 3, which shows clearly and explicitly the mechanisms through which the multiplier process operates as the result of different exogenous injections (Thorbecke, 2000).

Figure 3, Multiplier Process among endogenous accounts
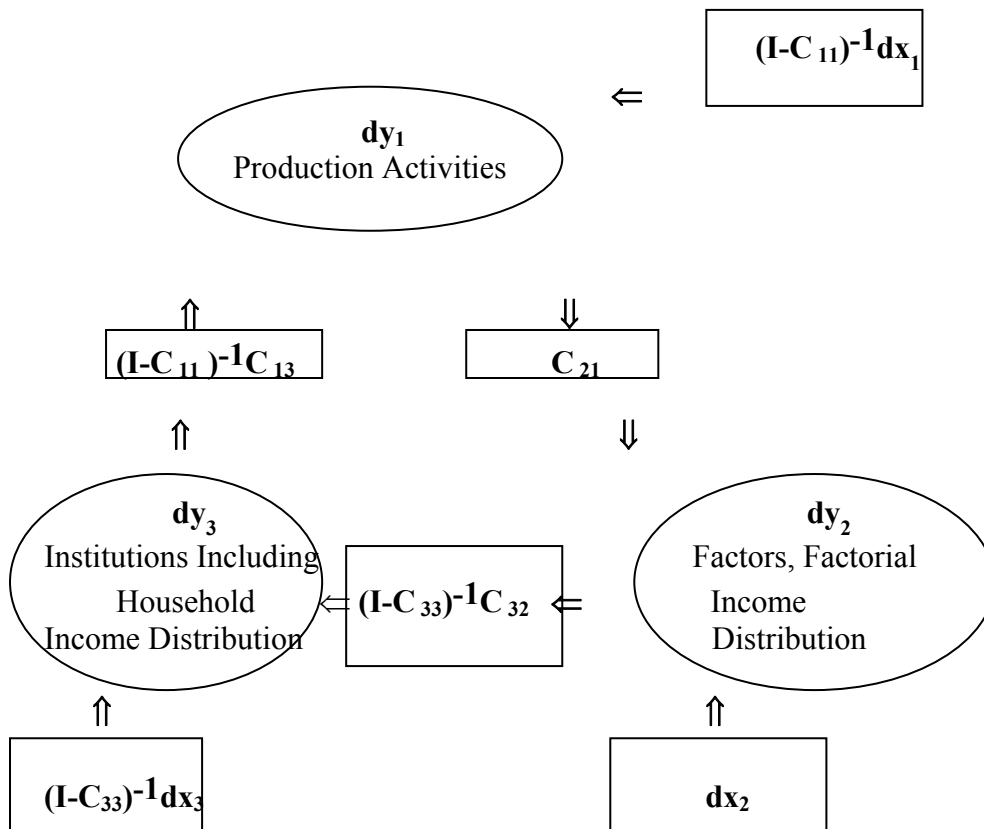
Taking in account that dx_1 is the marginal increase of exogenous final demand from government consumption, export and investment demand; that dx_2 is the marginal increase of exogenous final demand for factors from government consumption, export and investment demand and that dx_3 is the marginal increase of exogenous injection from government transfers, and remittances from abroad toward the Institutions we can represent the loops as in Figure 3. The loop in Figure 3 shows how an exogenous change in the output of any of the production activity translates in a change of household income through some steps.

Thus an exogenous increase (injection) of export, government, or investment demand dx_1 generates a rise equal to $(I-C_{11})^{-1}dx_1$ on the output of the corresponding production activity of. The additional factors of production which have to be employed in order to create the additional output generates a stream of value added $C_{21}dy_1$ which is an income from factors in addition to any other exogenous factor income received from other regions or from abroad and from the government, namely dx_2.

In the next link, households (and companies) receive income based on their resources endowment (C_{32}) and transfers system (C_{33}) as well as exogenous government subsidies and transfer payments and remittances from abroad i.e. $(I-C_{33})^{-1}dx_3$. Finally, the loop is closed through the pattern of household (and companies) expenditures on commodities which translates into new production and an additional flow of income accruing to production activities equal to $dy_1 = (I-C_{11})^{-1} C_{13}$. These "effect aggregate the impact of initial first round of spending and subsequent rounds of responding by the household groups" according to the "degree of integration in the socio economic system on the production and expenditure side" (Thorbecke, Jung, 1996, p. 288).

This formulation generalizes the Leontief model taking in account the effects of an exogenous change in the personal income distribution (dx_3) on the consumption of the various socioeconomic groups through C_{13} which reflects the consumption pattern of each group of households. The open Leontief model, where households' consumption is included in the final demand vector, can be expressed as follows using the same notation $dy_1=(I-C_{11})^{-1}dx_1$ where C_{11} is the input-output coefficient matrix. "It is obvious that the SAM formulation contains more information and a higher degree of endogeneity since it captures the endogenously derived effects of income distribution on consumption, which the Leontief national model does not" (Thorbecke, 2000, pp. 21-22).

Following Pyatt and Round (1979, 2005), Bottiroli Civardi (1988, pp. 94-102) and Timpano (1996) it is possible to decompose the multiplier matrix M_c into three components M_{c1}, M_{c2} and M_{c3}. This decomposition has an interesting economic meaning for a structural analysis of income distribution among and inside the Private Institutions, and above all with reference to the Household sector.

If we introduce the matrix C_0 where:

$$C_0 = \begin{vmatrix} C_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & C_{33} \end{vmatrix}$$

$$(I-C_0) = \begin{vmatrix} (I-C_{11}) & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & (I-C_{33}) \end{vmatrix} \qquad (C-C_0) = \begin{vmatrix} 0 & 0 & C_{13} \\ C_{21} & 0 & 0 \\ 0 & C_{32} & 0 \end{vmatrix}$$

The [32] can be reformulated as

$$dy = Cdy + \mathrm{d}x = Cdy + C_0dy - C_0dy + d\,x = (C - C_0)\,dy + C_0dy + dx$$
$$= (I - C_0)^{-1}\,(C - C_0)\,dy + (I - C_0)^{-1}dx$$
$$= M_{cl}\,(C - C_0)\,dy + M_{cl}dx$$
$$dy = [(I - C^*)^{-1}\,M_{cl}]\,dx \qquad\qquad [39]$$

where

$$(I - C_0)^{-1} = M_{cl} \quad \text{e} \quad C^* = M_{cl}\,(C - C_0)$$

The $\mathbf{M_{cl}}$ multiplier expresses the effects within each account generated by the direct transfers that are independent of the closed-loop process. The multiplier matrix $\mathbf{M_{cl}}$ is a diagonal block matrix where the first diagonal block expresses the multiplier effects of the transfers within the activities and it is precisely the Leontief's inverse matrix. Since it is assumed that no direct transfers between factors take place $\mathbf{M_{cl}}$ second diagonal block is an identity matrix. The third block captures the multiplier effects due to the transfers between endogenous Institutions.

$$\mathbf{M_{cl}} = \begin{vmatrix} (I - C_{11})^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & (I - C_{33})^{-1} \end{vmatrix} = \begin{vmatrix} {}_1M_{c11} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & {}_1M_{c33} \end{vmatrix} \qquad [40]$$

The definition of $\mathbf{M_{cl}}$ allows to introduce matrix $\mathbf{C^*}$ as

$$C^* = M_{cl}(C - C_0) = \begin{vmatrix} 0 & 0 & C^*_{13} \\ C^*_{21} & 0 & 0 \\ 0 & C^*_{32} & 0 \end{vmatrix} \qquad\qquad [41]$$

$C^*_{13} = (I - C_{11})^{-1}\,C_{13}$;
$C^*_{21} = C_{21}$;
$C^*_{32} = (I - C_{33})^{-1}\,C_{32}$ o, se $C_{33} = 0$, $C^*_{32} = C_{32}$

If we assume that $(I - C^*)$ exists we can rewrite the equation [31] as:

$$\mathbf{M_c}\,dx = dy = [(I - C^*)^{-1}\,M_{cl}]\,dx \qquad\qquad [42]$$

Then,
$$\mathbf{M_c} = (I - C)^{-1} = [(I - C^*)^{-1}\,M_{cl}] \qquad\qquad [43]$$

Because:
$$(I - C^*)^{-1} = (I + C^* + C^{*2} + \ldots + C^{*l-1})\,(I - C^{*l})^{-1} \qquad\qquad [44]$$

we can fix $l = 3$ (the endogenous accounts are three). The equation [44] can be rewritten as:

$$dy = (I + C^* + C^{*2})\,(I - C^{*3})^{-1}\,M_{cl}\,dx \qquad\qquad [45]$$

where

$$C^{*2} = \begin{vmatrix} 0 & C^*_{13}C^*_{32} & 0 \\ 0 & 0 & C^*_{21}C^*_{13} \\ C^*_{32}C^*_{21} & 0 & 0 \end{vmatrix}$$

$$C *^{3} = \begin{vmatrix} C^{*}_{13}C^{*}_{32}C^{*}_{21} & 0 & 0 \\ 0 & C^{*}_{21}C^{*}_{13}C^{*}_{32} & 0 \\ 0 & 0 & C^{*}_{32}C^{*}_{21}C^{*}_{13} \end{vmatrix} \qquad [46]$$

Equation [37] becomes

$$dy = M_{c3} \ M_{c2} \ M_{c1} \ dx \qquad [47]$$

where

$$M_{c2} = (I + C * + C *^{2}) = \begin{vmatrix} I & C^{*}_{13}C^{*}_{32} & C^{*}_{13} \\ C^{*}_{21} & I & C^{*}_{21}C^{*}_{13} \\ C^{*}_{32}C^{*}_{21} & C^{*}_{32} & I \end{vmatrix} \qquad [48]$$

M_{c2} captures the multiplier effects resulting from transfers between the various endogenous Sectors and therefore expresses the action of an exogenous shock from any vector **dx_i** over the elements of the other **dy_j** accounts with $i \neq j$.

Finally

$$M_{c3} = (I - C *^{3})^{-1} = \begin{vmatrix} _{3}M_{c11} & 0 & 0 \\ 0 & _{3}M_{c22} & 0 \\ 0 & 0 & _{3}M_{c33} \end{vmatrix} \qquad [49]$$

where

$$_{3}M_{c11} = (I - C *_{13} C *_{32} C *_{21})^{-1} = [I - (I - C_{11})^{-1}C_{13}(I - C_{33})^{-1}C_{32}C_{21}]^{-1} \qquad [50]$$

$$_{3}M_{c22} = (I - C *_{21} C *_{13} C *_{32})^{-1} = [I - C_{21}(I - C_{11})^{-1}C_{13}(I - C_{33})^{-1}C_{32}]^{-1} \qquad [51]$$

$$_{3}M_{c33} = (I - C *_{32} C *_{21} C *_{13})^{-1} = [I - (I - C_{33})^{-1}C_{32}C_{21}(I - C_{11})^{-1}C_{13}]^{-1} \qquad [52]$$

if we assume that **$C_{33} = 0$** equation [44] becomes

$$_{3}M_{c33} = [I - C_{32}C_{21}(I - C_{11})^{-1}C_{13}]^{-1} \qquad [53]$$

Matrix **$_{3}M_{c3,3}$** acquires the meaning of an income multiplier through the consumption expenditure as a result of a four-step "propagation" process. The first step is represented by the matrix **C_{13}** of the consumption coefficients with reference to disposable income of each of the Endogenous Private Institutions. The second step equal to **$(I-C_{11})^{-1}$** corresponds to the passage traditionally captured by the Leontief's inverse matrix which transformes expenditure by sector into intermediate output and which determines the shares of the value added generated in the productive process. The third step, corresponding to the product of matrix **C_{32}** and matrix **C_{21}**, determines the value added received by the Endogenous Private Institutions in connection with their ownership of production factors. The fourth step, finally, given by **$(I-C_{33})^{-1}$**, if different from zero, corresponds to the redistribution of income between Endogenous Institutions. The income thus produced, distributed a redistributed,

turns into new levels of expenditures for consumption and the process occurs again until an equilibrium position is achieved.

7. The disaggregation of the "fixed-prices multipliers" M_c.

Following Pyatt and Round (2005) it is possible to disaggregate the fixed prices global multiplier M_c and to calculate the value of each element m_{cij}. This step allows to better analyze the effects of any exogenous injections dx_1, dx_2, dx_3 on the level of income of different i groups . If we express m_{cij} as:

$$m_{cij} = \mathbf{d'}_i \, \mathbf{M_c} \, \mathbf{d}_j = \mathbf{d'}_i \, \mathbf{M_{c3}} \, \mathbf{M_{c2}} \, \mathbf{M_{c1}} \, \mathbf{d}_j = \mathbf{i'} \, (\mathbf{r} \, \mathbf{A}\hat{\mathbf{s}}) \, \mathbf{i} \qquad [54]$$

where $\mathbf{i}$ is a vector all elements of which are one, while $\mathbf{r}$, $\mathbf{A}$ and $\mathbf{s}$ are given by

$$\mathbf{r'} = \mathbf{d'}_i \, \mathbf{M_{c3}} \quad \mathbf{A} = \mathbf{M_{c2}} \quad \text{and} \quad \mathbf{s} = \mathbf{M_{c1}} \mathbf{d}_j \qquad [55]$$

Each m_{ij} must therefore be equal to the sum of all elements of an $^{\mathbf{r}} \mathbf{A}\hat{\mathbf{s}}$ type transformation of the matrix $\mathbf{M_{c2}}$ when the vector $\mathbf{r'}$ is obtained from the ith row of $\mathbf{M_{c3}}$ and the vector $\mathbf{s}$ is obtained from the j th column of $\mathbf{M_{c1}}$. "In other words, a complete accounting for m_{ij} can be constructed for any i and j from three elements" i.e. the ith row of the matrix $\mathbf{M_{c3}}$, the entire matrix $\mathbf{M_{c2}}$ and the jth column of the matrix $\mathbf{M_{c1}}$ (Pyatt, Round, 2005, p. 10).

In particular, with reference to the Private Institution (sector 3), we take in account m_{cij} as an element of the sub-matrix $\mathbf{M_{c3,1}}$ of $\mathbf{M_c}$ where

$$\mathbf{M_{c3,1}} = {}_3\mathbf{M_{c3,3}} \, {}_2\mathbf{M_{c3,1}} \, {}_1\mathbf{M_{c1,1}} \qquad [56]$$

It is possible to introduce a new matrix as a transformation of the element of the multiplier matrix $\mathbf{M_{c3,1}}$

$$m_{cij} = (\mathbf{d'}_i \, {}_3\mathbf{M_{c3,3}}) \, {}_2\mathbf{M_{c3,1}} \, ({}_1\mathbf{M_{c1,1}} \, \mathbf{d}_j) \qquad [57]$$

which can also be written in the form $\mathbf{i'}(\mathbf{r} \, \mathbf{A}\hat{\mathbf{s}})\mathbf{i}$ where now

$$\mathbf{r'} = \mathbf{d'}_i \, {}_3\mathbf{M_{c3,3}} \quad \mathbf{A} = {}_2\mathbf{M_{c3,1}} \quad \text{and} \quad \mathbf{s} = {}_1\mathbf{M_{c1,1}} \, \mathbf{d}_j \qquad [58]$$

The cell m_{cij} is therefore equal to the sum of all elements of a new $^{\mathbf{r}} \mathbf{A}\hat{\mathbf{s}}$ type transform in which $\mathbf{r'}$ is the i row of ${}_3\mathbf{M_{c3,3}}$, $\mathbf{A}$ is equal to ${}_2\mathbf{M_{c3,1}}$, and $\mathbf{s}$ is the j column of ${}_1\mathbf{M_{c1,1}}$ (Pyatt, Round, 2005, p. 10). This decomposition allows to show in a clear way the consequences of a particular injection in the Activity j on the Institution i. The matrix ${}_2\mathbf{M_{c3,1}}$ will be bordered by two vectors $\mathbf{r}_i$ and $\mathbf{s'}_j$. These are respectively the row i of the matrix ${}_3\mathbf{M_{c3,3}}$ and the column j of the matrix ${}_1\mathbf{M_{c1,1}}$. The initial injection toward a j productive sector generates a multiplier effect $\mathbf{s}_j$ "on the various production activities, the magnitude of which can be read-off from the relevant column of the input-output inverse" equal to $(\mathbf{I} - \mathbf{C_{11}})^{-1} = {}_1\mathbf{M_{c1,1}}$ (Pyatt, Round, 2005, p.10). The consequences of the original injection are then translated by the 'A' part of the $\mathbf{r} \mathbf{A}\hat{\mathbf{s}}$ transform i.e. by the matrix ${}_2\mathbf{M_{c3,1}}$ into increments of income for the various institutions. And, finally, the transmission of these increments right around the system - the complete circular flow - generates the implications for the household i that are captured by the multiplier $\mathbf{r}$, i.e. by the row i of multiplier ${}_3\mathbf{M_{c3,3}}$.

Changes dx_1 in the demand for products will therefore generate increases in the incomes of institutions via the mappings ${}_3\mathbf{M_{c3,3}} \, {}_2\mathbf{M_{c3,1}} \, {}_1\mathbf{M_{c1,1}}$.

$$dy_3 = \mathbf{M_{c3,1}} \, dx_1 = {}_3\mathbf{M_{c3,3}} \, {}_2\mathbf{M_{c3,1}} \, {}_1\mathbf{M_{c1,1}} \, dx_1 \qquad [59]$$

The stimulation of any one or more production activities would rise mean sectoral incomes and the poverty will be reduced according to the value of poverty mean income elasticities of each socio-economic groups. The elasticity index with respect to the mean income of each ith subgroup, which are aggregated in equation [21], are parameters that can be estimated outside of the SAM.

8. Concluding remarks.

The results of the analysis, which can be undertaken with the multiplier approach, strengthen the hypothesis that poverty can be linked to the sectoral growth through the poverty income elasticities. The decomposition of fixed price multiplier matrix allows isolating the value of different groups multipliers, and to assess the linkages between household and sectors of production. This kind of analysis can drive toward a better understanding of the relationships between inequality, poverty and alternative policies.

The examples for the Indonesian case reported by Pyatt and Round (2005, p.11) drive to some interesting observations. Few elements of the $\hat{r} A \hat{s}$ transformation are "sufficiently large to be recorded as contributions to the aggregate multiplier effect" m_{ij} (Pyatt, Round, 2005, p. 10). More powerful linkages are generated by the increased intermediate demand for other sectors as a result of the stimulation of the j sector. This derived demand evidently creates significant extra income for the entire household groups which, in turn, generate extra income for the household i. The closed loop process that is the indirect effects, are stronger than the direct effects.

The extension of SAM multipliers approach to poverty analysis is interesting. This approach, however, will be very informative only if poverty is largely identifiable with some socio-economic groups. Only in this case the multiplier effects, that are confined to determining the income effects of (socio-economic) household groups, are meaningful. "On the other hand it is necessary to try to link the multiplier effects on household group incomes to possible changes in poverty within groups because the intra-group income distributions are not generated directly. To do so usually requires some assumption to be made about the income distribution parameters within household groups (variance or Lorenz parameters)" (Round, 2003, p.278). This further analysis could be explored in a future work.

References

Banca d'Italia (1985), *I bilanci delle famiglie italiane nell'anno 1984*, "Bollettino Statistico", luglio-dicembre, Roma.

Besley, T., Burgess R. (2003), *Halving Global Poverty*, "Journal of Economic Perspectives", 17: 3-22.

Bottiroli Civardi M. (1988), *Le matrici di contabilità sociale per un'analisi strutturale della distribuzione personale dei redditi*, Giuffrè editore, Milano.

Bottiroli Civardi M. (1990), *Income Multipliers In The Household Institutional Sector in Income and Wealth Distribution, Inequality and Poverty*, in: Dagum, M. Zenga (1990), (editors), *Studies in Contemporary Economics*, Springer-Verlag, Berlino.

Bottiroli Civardi M., Targetti Lenti R.(1988), The distribution of Personal Income at the Sectoral Level in Italy: A SAM model, "Journal of Policy Modeling, 10(3), pp. 453-468.

Bottiroli M., Chiappero Martinetti E., Targetti Lenti R. (1990), *Una SAM per l'Italia*, Istituto di Statistica, Università di Pavia, pp. 11-22.

Bottiroli Civardi M., Targetti Lenti R., (1999), *La Matrice di Contabilità Sociale (SAM) ed il Sistema dei Conti Nazionali (SNA): problemi di integrazione e di scelta dei gruppi socio-economici*, Quaderni del Dipartimento di Economia Pubblica e Territoriale, n.5/99, Università di Pavia.

Bottiroli Civardi M., Targetti Lenti R., (2002), *The social accounting matrix (SAM), a framework for building inequality "structural indicators" for analysing the income distribution,* Working paper No. 129, University of Pavia, June.

Bourguignon, F. (2003), *The Growth Elasticity of Poverty Reduction: Explaining Heterogeneity across Countries and Time Periods*, in Eicher T.S., Turnovsky S.J. (eds), *Inequality and Growth: Theory and Policy Implication*s, Cambridge, MA and London: MIT Press, 3-26.

Bourguignon, F. (2004), *The Poverty-Growth-Inequality Triangle*, Mimeo, The World Bank, Washington, D.C.

Cornia G. A. (2003), *The impact of liberalisation and globalisation on income inequality in developing and transitioal economies* , Cesifo Working Paper No.843, January.

Cornia G. A. (2004), *Changes in the distribution of income over the last two decades: extent, sources and possible causes*, "Rivista Italiana degli Economisti", n.3, pp..349-387.

Datt, G., Ravallion M. (1992), *Growth and Redistribution Components of Changes in Poverty Measures: A Decomposition with Applications to Brazil and India in the 1980s*, "Journal of Development Economics", 38: 275-95.

Dollar D., Kraay A. (2001), *Growth is good for the poor*, World Bank Policy Research Working Paper No. 2587.

Dollar, D., Kraay A. (2002), *Growth is Good for the Poor,* "Journal of Economic Growth", 7: 195-225.

Dollar, D., Kraay A. (2003), *Institutions, Trade, and Growth*, "Journal of Monetary Economics", 50: 133-62.

Dollar, D., Kraay A. (2004), *Trade, Growth, and Poverty*, "The Economic Journal", 114: F22-F49.

Epaulard A. (2003), *Macroeconomic Performance and Poverty Reduction,* IMF Working Paper No 03/72. Washington, DC: IMF.

Hanmer L., Naschold F. (2000), *Attaining the International Development Targets: Will Growth Be Enough?*, "Development Policy Review", 18: 11-36.

Lee, H. Y., Ricci L. A., Rigobon R. (2004), *Once Again: Is Openness Good for Growth?*, "Journal of Development Economics", 75: 451-72.

Mosley, P., Hudson J., Verschoor A. (2004), *Aid, Poverty Reduction and the 'New Conditionality'*, "The Economic Journal", 114: F217-43 .

Kalwij A., Verschoor A., (2005), *A Decomposition of Poverty Trends across Regions*, WIDER, Research Paper No. 2005/36, June.

Kakwani N.(1993), *Poverty and economic growth with application to Côte d'Ivoire*, "Review of Income and Wealth" 39, no.2, 121-139.

Keuning S.J. (1994), *The SAM and beyond: Open, SESAME,* Economic Systems Research, vol.6, No.1.

Jomo, K. S. (2003). 'Globalization, Liberalization and Equitable Development—Lessons from East Asia'. Overarching Concerns, Paper No. 3. Geneve: UNRISD.

Pyatt G., Round J. (1979), *Accounting and fixed prices multipliers in a Social Accounting Matrix Framework*, "The Economic Journal", vol. 89, december, pp.850-873

Pyatt G., Round J. I. (2005), *Multiplier effects and the reduction of Poverty*" Chapter 12 in A. de Janvry and R. Kanbur (editors), *Poverty, Inequality and Development: essays in honour of Erik Thorbecke*, Kluwer (in Press).

Ravallion, M. (1997), *Can High-Inequality Countries Escape Absolute Poverty?*, "Economics Letters", 56: 51-7.

Ravallion, M. (2001), *Growth, Inequality and Poverty: Looking Beyond Averages,* "World Development", 29: 1803-15.

Ravallion M., Chen S. (1997), What Can New Survey Data Tell Us about Recent Changes in Distribution and Poverty?, "World Bank Economic Review", 11: 357-32.

Ravallion, M., Huppi M. (1991), *Measuring Changes in Poverty: A Methodological Case Study of Indonesia during an Adjustment Period*, "World Bank Economic Review", 5: 57-82.

Rodigari M. (2005), *L'utilizzo delle SAM (Social Accounting Matrices) nell'analisi della povertà nei Paesi in via di Sviluppo*, Tesi di laurea, Università degli Studi di Pavia, dicembre 2005.

Rodriguez F., Rodrik R. (2000). *Trade Policy and Economic Growth: A Skeptic's Guide to the Cross-National Evidence*, NBER Macroeconomics Annual, 261-325. Cambridge, MA: National Bureau of Economic Research.

Rodrik D. (2000), *Comments on 'Trade, Growth, and Poverty' by D. Dollar and A. Kraay,* Harvard University, ottobre

Round J. (2003), *Social Accounting Matrices and SAM-based Multiplier Analysis*, in: Bourguignon, F., L. Pereira da Silva and N. Stern (2003). *Evaluating the Poverty Impact of Economic Policies: Some Analytical Challenges* (mimeo), World Bank:Washington DC, pp.269-287.

Stone R. (1962), *A Social Accounting Matrix for 1960*, Chapman and Hall, Cambridge.

Stone R.(1985), *The Disaggregation of the Household Sector in the National Accounts*, in: Pyatt G., Round J.(1985), (a cura di), Pyatt G., *Social Accounting Matrices: A Basis for Planning*, The World Bank, Washington D.C.
.

Stone R. (1986), *Social Accounting: The State of Play,* "Scandinavian Journal of Economics", vol.88, n.3.

Timpano F. (1996), *Modelli di contabilità sociale nella teoria e nella politica economica*, Giuffrè editore, Milano.

Thorbecke E., Jung H. S. (1996), *A multiplier decomposition method to analyze poverty alleviation*, "Journal of Development Economics", Vol. 48, pp. 279-300

Thorbecke E. (2000), *The use of Social Accounting Matrices in modeling*, Paper Prepared for the 26th General Conference of The International Association for Research in Income and Wealth Cracow, Poland, 27 August -2 September 2000, pp.1-22.

Weisbrot M., Baker D., Naiman R., Neta G. (2000), *Growth May be Good for The Poor-But are IMF and World Bank Policies Good for Growth?* CEPR, August.

Appendix

A.1 The mean income elasticity of the head count ratio H.

The first and second derivative of the Lorenz curve with respect to p are obtained differentiating $L(p) = \frac{1}{\mu}\int_0^z xf(x)dx$ twice and using the chain rule[8]

$$\frac{\partial F_1(x)}{\partial x} \text{ that is } \frac{\partial}{\partial x}\left\{\frac{1}{\mu}\int_0^x xf(x)dx\right\} = \frac{1}{\mu}\left\{xf(x)\Big|_0^x \frac{\partial x}{\partial x}\right\} = \frac{xf(x)}{\mu} \quad [1A]$$

knowing that $\frac{\partial F(x)}{\partial x} = f(x)$ and that $\frac{\partial F_1(x)}{\partial x} = \frac{xf(x)}{\mu} \Rightarrow dF_1(x) = \frac{xf(x)}{\mu}dx$

$$\frac{\partial F(x)}{\partial x} = f(x) \Rightarrow dF\ (x) = f(x)dx$$

we obtain:

$$\frac{\partial F_1(x)}{\partial x} = \frac{xf(x)}{\mu}dx\frac{1}{f(x)dx} = \frac{x}{\mu} \Rightarrow L^{'}(H) = x/\mu \quad [2A]$$

where L'(H) is the first derivative of the Lorenz curve and $H = F(x)$ is the *headcount ratio*.
The second derivative is obtained differentiating in respect to p as:

$$\frac{\partial^2 F_1(x)}{\partial F(x)^2} = \frac{\partial}{\partial F(x)}\frac{\partial F_1(x)}{\partial F(x)} = \frac{\partial}{\partial F(x)}\frac{x}{\mu} = \frac{\partial x}{\partial F(x)\mu} = \frac{\partial x}{f(x)\partial x\mu} = \frac{1}{f(x)\mu} \quad [3A]$$

Remembering that $x = z$ the result $L^{''}(H) = \frac{1}{f(z)\mu}$ is easily obtained

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[8] For these passages see also: Rodigari (2005), pp. 134-146.

The elasticity of head-count ratio with respect to the mean income $\dfrac{\partial H}{\partial \mu}$ is obtained calculating the total differential of

the first derivative of the Lorenz curve. We know that: $\dfrac{\partial F_1(z)}{\partial F(z)} = \dfrac{\partial F_1(z)}{\partial H} = \dfrac{z}{\mu}$

given z fixed the total differential is:

$$\frac{\partial^2 F_1(z)}{\partial H^2}dH = -\frac{z}{\mu^2}d\mu \qquad \text{so that} \quad \frac{\partial H}{\partial \mu} = -\frac{z}{\mu^2}\frac{1}{\dfrac{\partial^2 F_1(z)}{\partial H^2}} \qquad\qquad \text{[4A]}$$

taking in account that $\dfrac{\partial^2 F_1(z)}{\partial H^2} = \dfrac{1}{f(z)\mu}$ then,

$$\frac{\partial H}{\partial \mu} = -\frac{zf(z)\mu}{\mu^2} = -\frac{zf(z)}{\mu}$$

The elasticity of the *headcount ratio* in respect to the average income will become:

$$\eta_H = \frac{\partial H}{\partial \mu}\frac{\mu}{H} = -\frac{zf(z)}{\mu}\frac{\mu}{H} = -\frac{zf(z)}{H} < 0 \qquad\qquad \text{[5A]}$$

A.2 The mean income elasticity of the poverty gap PG.
The mean income elasticity of the poverty gap GP, defined as product of the two indexes *H* and *I* can be calculated. The *income gap ratio I* is the average value of all the poverty gap as a proportion of the poverty line *z*

$$I = \frac{1}{q}\sum_{i=1}^{q}\left(\frac{z-y_i}{z}\right) = \frac{1}{q}\sum_{i=1}^{q}\frac{g_i}{z}. \qquad\qquad \text{[6A]}$$

This index *I* can be espressed as the average *poverty gap* in respect to the poverty line *z*

$$I = \frac{1}{q}\sum_{i=1}^{q}\left(\frac{z-y_i}{z}\right) = \frac{1}{q}\frac{qz-\sum\limits_{i=1}^{q}y_i}{z} = \frac{q}{q}\frac{z-\sum\limits_{i=1}^{q}y_i/q}{z} = \frac{z-\mu^*}{z}, \qquad\qquad \text{[7A]}$$

where μ^* is the average income of the *q* poors. The *poverty gap PG* can be expressed as:

$$PG = \frac{1}{N}\sum_{i=1}^{q}\left(\frac{z-y_i}{z}\right) = \frac{1}{N}\frac{qz-\sum\limits_{i=1}^{q}y_i}{z} = \frac{q}{N}\frac{z-\sum\limits_{i=1}^{q}y_i/q}{z} = \frac{q}{N}\frac{z-\mu^*}{z} = HI. \qquad\qquad \text{[8A]}$$

how the mean income of the poor changes when the mean income of the whole population increases. Starting from the

Lorenz curve expressed as $L(H) = \dfrac{H\mu^*}{\mu}$ and remembering that

$L'(H) = z/\mu$ ed inoltre $\eta_H = \dfrac{\partial H}{\partial \mu}\dfrac{\mu}{H} = -\dfrac{zf(z)}{H} < 0$ if we differentiate both the members in respect to μ we obtain:

$$z/\mu = \frac{\dfrac{dH}{d\mu}\mu^*\mu + H\dfrac{d\mu^*}{d\mu}\mu - H\mu^*}{\mu^2} \qquad\qquad \text{[9A]}$$

$$z = \frac{dH}{d\mu}\mu^* + H\frac{d\mu^*}{d\mu} - H\frac{\mu^*}{\mu} \qquad\qquad \text{[10A]}$$

and then

$$H \frac{d\mu^*}{d\mu} = H \frac{\mu^*}{\mu} + z - \frac{dH}{d\mu} \mu^* \qquad \text{[11A]}$$

Remembering that $\frac{dH}{d\mu} = \eta_H \frac{H}{\mu}$ we obtain

$$H \frac{d\mu^*}{d\mu} = H \frac{\mu^*}{\mu} + (z - \mu^*)\eta_H \frac{H}{\mu} \qquad \text{[12A]}$$

and finally

$$\frac{d\mu^*}{d\mu} = \frac{\mu^*}{\mu} + \frac{(z - \mu^*)}{\mu}\eta_H \qquad \text{[13A]}$$

Now the the mean income elasticity of the *poverty gap* can be calculated

$$\eta_{PG} = \frac{\partial PG}{\partial \mu} \frac{\mu}{HI} \qquad \text{[14A]}$$

that is

$$\frac{\partial PG}{\partial \mu} = \frac{\partial H}{\partial \mu} I + H \frac{\partial I}{\partial \mu} \qquad \text{[15A]}$$

where

$$\frac{\partial H}{\partial \mu} = \eta_H \frac{H}{\mu} \qquad \text{[16A]}$$

If we introduce the [14A] and the [15A] in the [12A] and we multiply with μ (remembering the definition of *I*) we obtain:

$$\frac{\partial PG}{\partial \mu}\mu = \eta_H H \frac{(z - \mu^*)}{z} - (\frac{\mu^*}{z} + \frac{(z - \mu^*)}{z}\eta_H) = -\frac{\mu^*}{z} H \qquad \text{[17A]}$$

Dividing the [16A] by $PG = HI$ we obtain the poverty gap elasticity:

$$\eta_{PG} = -\frac{\mu^*}{z} \frac{H}{HI} = -\frac{\mu^*}{zI} = -\frac{\mu^*}{z \frac{(z - \mu^*)}{z}} = -\frac{\mu^*}{z - \mu^*} < 0, \qquad \text{[18A]}$$

A3. The Social Account Matrix (SAM).

The simplified SAM, in his matrix form, must satisfy some principles (Figure A1). First of all the basic national accounting principle of balance between entries and expenditures must be satisfied. The crossing of the account on the row *i* and the account on the column *j* is the value of monetary transactions between them. Each transaction is an exit (cost) for the column account and an entry for the row account.

The choice of different types of Institutions must be done taking in account a homogeneity principle based on the nature and the economic behaviour of the unit. The row and column accounts refer to production activities, to factors of production (different types of labour, capital, natural resources), to private and public Institutions and, finally to capital/saving account. The link between the production side and the Institutions is the innovative and the most important feature of the SAM in comparison to the traditional input-output framework and the System of National Accounts (SNA).

Figure A1- A simplified SAM.

Expenditures	Activity	Factors	Institutions			Saving/ Invest.	Indirect Taxes	Rest of the World	Total
Entries			Household	Companies	Government				
Activity	$T_{1,1}$	0	$T_{1,3}$	$T_{1,4}$	$T_{1,5}$	$T_{1,6}$	0	$T_{1,8}$	y_1
Factors	$T_{2,1}$	0	0	0	0	0	0	$T_{2,8}$	y_2
Household	0	$T_{3,2}$	$T_{3,3}$	$T_{3,4}$	$T_{3,5}$	0	0	$T_{3,8}$	y_3
Companies	0	$T_{4,2}$	$T_{4,3}$	$T_{4,4}$	$T_{4,5}$	0	0	$T_{4,8}$	y_4
Governm.	0	$T_{5,2}$	$T_{5,3}$	$T_{5,4}$	$T_{5,5}$	0	$T_{5,7}$	$T_{5,8}$	y_5
Saving/Inv	0	0	$T_{6,3}$	$T_{6,4}$	$T_{6,5}$	0	0	$T_{6,8}$	y_6
Indirect Taxes	$T_{7,1}$	0	$T_{7,3}$	$T_{7,4}$	$T_{7,5}$	0	0	0	y_7
Rest of the World	$T_{8,1}$	$T_{8,2}$	$T_{8,3}$	$T_{8,4}$	$T_{8,5}$	$T_{8,6}$	0	0	y_8
Total	y'_1	y'_2	y'_3	y'_4	y'_5	y'_6	y'_7	y'_8	

(Row block labelled vertically: INSTITUTIONS)

Formally the SAM is a square matrix

$$T=[t_{ij}]$$

Each pair row-column represents the accounting system of the any single unit and it is balanced so that each total row is equal to the corresponding total column.

$$Te = y = T'e$$

e is the unity vector so that the element i of vector y is both the total revenue and the total expenditure of the i account.

The choice of the numbers of accounts depends on the goals of the analysis and on the availability of statistical data. The flexibility of the SAM allows choosing the disaggregation more suitable. In the Figure 1, a simplified SAM shows the main links between the various accounts. The three accounts of the internal Institutions present only the flows of the current side, while the capital flows are aggregated in a single account (column and row 6). The flows of Rest of the World are not disaggregated. Each cell can be a vector or a matrix, not only a scalar. Of course its value is zero in case of no transactions between the two accounts.

The rows show the equilibrium conditions of each unit (Activities, Factors or Institutions) of the economic system. The first row shows the traditional Keynesian identity between aggregated supply (vector y_1) and aggregated demand divided in intermediate ($T_{1,1}$), final demand for consumption of the Households ($T_{1,3}$) and final demand of other Institutions.

The second and third row (vectors y_2 and y_3) refers to process of generation, distribution and redistribution of income to the Households. In a first phase the value added is generated and then distributed to the M factors of production in relationship to their use in the S sectors of activities ($T_{2,1}$) or outside of the economic system ($T_{2,8}$). The second column account assesses the passage from the factorial to the personal income distribution of the Institutions. In particular matrix $T_{3,2}$ shows the passage of income from the factors of production to the Households depending on the ownership of factors by each of the H socio-economic group.

Matrices $\mathbf{T_{3,3}}$, $\mathbf{T_{3,4}}$, $\mathbf{T_{3,5}}$, are related to the moment of redistribution of income between Households, from the Companies (interest and dividends) and from the Government (positive monetary transfers, negative monetary transfers following the payment of social contributions, and of direct and indirect taxes). The matrix $\mathbf{T_{3,8}}$ takes in account the redistribution process from the Rest of the World.

In an analogous way the fourth and fifth row represent the primary and secondary income distribution of the Companies and of the Government. Sixth row refers to the accumulation of capital for the economic system. The matrices at the crossing between the columns of current expenditures of the Institutions represent the saving of each households group (matrix $\mathbf{T_{6,3}}$), the undistributed profits (matrix $\mathbf{T_{6,4}}$) and the saving of Government (matrix $\mathbf{T_{6,5}}$). The matrix $\mathbf{T_{6,8}}$ represents the net capital from the Rest of the World. The eighth row, finally, refers to the Rest of the World account.