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INFORMATION CONTENTS OF EQUIVALENCE SCALES

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Abstract.- The paper examines the problem of equivalence scales identification. It maintains that, provided the attention is limited to a restricted concept of welfare, no identification problem needs to arise. It also shows that commonly used restrictions on preferences are not satisfactory as they mistakenly rely on the idea that observed behaviour can provide some sort of information about the comparability requirements of preference systems. Moreover, it is shown that level comparability is the only restriction on preferences for equivalence scales to be used in social welfare evaluation.

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1. Introduction

Equivalence scales can be arrived at through two different ways. A direct way is that of obtaining people's own assessment as to what extent changes in household characteristics affect family welfare[1]. Instead, the indirect way considers observed consumption behaviour and, by means of revealed preferences theory, estimates changes in family welfare[2]. In this case welfare is defined within the consequentialist approach to individual choices in terms of final consumption[3].

The interest of this paper is with the indirect way of estimating equivalence scales. This approach is capable of representing a large combination of family characteristics based on household budget data, by now available yearly in most developed economies. Moreover, in as much as it describes behaviour from observed choices, it provides a less objectionable method to make welfare comparisons. Despite these interesting features a set of objections has been made to the meaning of equivalence scales estimated from observed consumption choices. The aim of this paper is to examine these objections and some of the major proposals to overcome them. The main objection we deal with is that about identification of equivalence scales which is described in Section 2. Based on the discussion on information constraints presented in Section 3, a possible way out to the problem is described in Section 4. A critical assessment of some contributions aiming at overcoming the identification problem, is presented in Section 5, while. Section 6 deals with the problem of using equivalence scales in social welfare analysis. The main conclusions reached in the paper are summarised in the final section.

2. The identification issue

The indirect way of arriving at equivalence scales can be seen as a by product of the ever lasting econometric effort of providing a satisfactory representation of family consumption choices based on traditional consumer theory. Family characteristics (number of children, their age, other dependants, parents age) and other socio-demographic characteristics (sex, race, education, type of job, area of residence) are highly significant variables in demand equations.

Analytical improvements in the method of introducing demographic characteristics in demand equations have helped to develop a general framework describing the precise role available for characteristics while maintaining to the resulting system the standard properties of any demand system.

Given an expenditure function $c^*(u, p)$ with the usual properties, a

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[1]The term of reference for such literature is Van Praag (1968), see also Hagenaars (1986).

[2] A synthetic presentation of this approach is contained in Deaton and Muellbauer (1980, chp. 8).

[3] This approach to household welfare would therefore fall within the *"commodity fetishist view",* Sen (1985, p. 23), and would exclude any references to the "*functioning"* type of welfare, Sen (1985, 1987). However, for a possible reconciliation see Muellbauer (1987).

demographically modified expenditure function c *(u , p ,* a) can be obtained by applying a set of transformation such that [Lewbel (1985)]:

$$c(u,p,a) = g[c^*(u,h(p,a)), p, a] = g(y^*, p, a). \qquad (1)$$

Where $y^* = c^*(u,h(p,a))$, $p^* = h(p,a)$, and $g(\ )$ are functions to be defined in order for the modified expenditure function to have the same properties as any expenditure function. The system of modifying function can include as special cases almost all the different demographic varying demand systems so long appeared in the literature[1].

Here, though, comes the traditional problem of equivalence scales because a distinction has to be made between two different and both justified uses of modifying function technique. One possible use of (1) is that of simply explaining observed consumption in terms of observed prices, income, and demographic characteristics for each household type. An additional possible use is that of obtaining a set of equivalence scales relating welfare indices of families with different demographic characteristics. If $v(\ )$ is the indirect utility function and a^r is the set of characteristics describing the reference household, then an equivalence scale $s(p,u,a)$ is defined by:

$$v(p,y,a^r) = v[p, s(p,u,a)y, a^h] = u \Rightarrow s(p,u,a) = c(u,p,a^h)/c(u,p,a^r). \qquad (2)$$

If a set of equivalence scales is to be obtained from (1) then at least two elements of arbitrariness can influence the calculation. In order to describe them we find it convenient to refer to a precise system of household preferences often used for econometric purposes.

Assume that the expenditure function for the *h* household *(h* = 1 , .. , *H)* takes the form:

$$\ln c(u^h, p, a^h) = \ln a(p, a^h) + \ln b(p, a^h) u^h, \qquad (3)$$

with:

$$\ln a(p,a^h) = \alpha_0(a^h) + \sum_i \alpha_i(a^h) \ln p_i + \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j, \qquad (4)$$

$$\ln b(p,a^h) = \beta_0(a^h) \prod_i p_i^{\beta_i(a^h)}. \qquad (5)$$

The share demand equation for the *i, (i=1, ..., I),* commodity is:

$$w_i^h = \alpha_i(a^h) + \sum_j \gamma_{ij} \ln p_j + \beta_i(a^h) \ln(y/P), \qquad (6)$$

[1] See Lewbel (1985) to which we refer for the definition of the variables and functions contained in (1).

where $\ln a(p,a^h)=\ln P$, comes to be approximated by Stone's price index $\ln P^s = \sum_i w_i^h \ln p_i$. Equivalence scales at reference prices are:

$$\ln s = \alpha_0(a^h) - \alpha_0(a^r) + [\beta_0(a^h) - \beta_0(a^r)]u. \tag{7}$$

The identification problem arises because observed behaviour, that is (6), does not provide any information about the parameters in (7). That is why in applied work equivalence scales at base prices are implicitly set at some arbitrary value [cfr. Blundell and Lewbel (1991)].

However the difficulties in identifying the parameters in (7) are of different order depending on whether we are concerned with parameters like α_0, or like β_0. Indeed while the latter does not affect behaviour at all, the former does affect behaviour although independently of prices[1]. In order to show this point set $\ln a(.) = \alpha_0(a^h) + \ln P$ and we get a different budget share equation:

$$w_i^h = \delta_i(a^h) + \sum_j \gamma_{ij} \ln p_j + \beta_i(a^h)\ln(y/P). \tag{8}$$

Where $\alpha_i(a^h) - \beta_i(a^h)\alpha_0(a^h) = \delta_i(a^h)$, and for the budget constraint it must be $\sum \delta_i = 1$.

Therefore the first identification problem is that of distinguishing within the components of δ_i between $\alpha_i(.)$ and $\alpha_0(.)$. This is the problem raised with reference to specific functional forms of household preferences in Muellbauer (1975, 1980). General guide-lines in overcoming this problem with reference to some prior information are reviewed and the implications assessed in Deaton and Muellbauer (1986), Deaton et al. (1989), Blackorby and Donaldson (1991b)[2].

The second identification problem, raised by Pollak and Wales (1979), concerns the $\beta_0(.)$ parameters in (7). In this case observed behaviour is of no help at all. The interest in this paper is mainly with this second problem of identification because it constitutes the most serious obstacle in estimating and interpreting equivalence scales. Moreover, some attempts to overcome the problem [see Blundell and Lewbel (1991), B1ackorby and Donaldson (1989, 1991a,b)] have come up with suggestions that at closer scrutinity turn out to be not fully satisfactory.

3. Information constraints

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[1] And because of this it can be referred as the Engel component of equivalence scales.

[2] For a counter-intuitive assumption cfr. Pashardes (1991, p. 204). For comments on preference systems such as (5), see Blundell and Lewbel (1991, p. 58-60).

In order to simplify the discussion in the following sections we need to deal briefly with the information constraints posed by equivalence scales[1]. As it is shown by (2), equivalence scales measure relative household welfare in money terms. They are based on cardinalization of preferences and on comparison of welfare indices. Cardinalization is achieved by means of the expenditure function at given prices, while the comparison is carried out by using as unit of measurement the expenditure level of the reference family.

The meaning of the cardinalization and of the comparison procedure depend on the information contents of the underlying preference system. The information contents can be summarised by reference to the admissible transformation that would guarantee the informative invariance of alternative representation of preferences. If we use $u' = F^h(u)$ as the admissible transformation, some of the more usual combinations in terms of measurability and comparability can be summarised as follows:

- Ordinal Non Comparable preferences (ONC): $F^h(.)$ is a positive monotonic transformation specific to each household;
- Ordinal Level Comparable preferences (OLC): $F^h(.)$ is a positive monotonic transformation common to all household;
- Cardinal Unit Comparable preferences (CUC): $F^h(.)$ is a positive affine transformation $F^h(.) = \alpha^h + \beta(.)$, where β is common to all households, and α^h can be specific;
- Cardinal Full Comparable preferences (CFC): $F^h(.)$ is a positive affine transformation common to all households;
- Ratio Scale Comparable preferences (RSC): $F^h(.)$ is a positive linear transformation common to all households.

For equation (2) to be meaningful ordinality is sufficient in terms of measurability, while in terms of comparability at least level comparability is required. Higher degree of measurability cannot, however, compensate for a reduction in the comparability requirement. For instance cardinal measurability with only unit compatibility (CUC) would not be sufficient for (2) to be meaningful because preference representation could differ from family to family by a specific affine transformation.

Although ordinality is sufficient for (2) to be meaningful, the actual value taken by it carries the same ordinal meaning. As far as comparability is concerned it is straightforward to see that equivalence scales transfer the comparability properties from welfare indices to expenditure indices which would otherwise be non comparable.

For instance, if welfare is only ordinal (OLC), equivalence scales (2) allow to make an ordering of expenditure indices with the same ordinal meaning. If

[1]None of these constraints are mentioned in Deaton and Muellbauer (1980).

instead welfare is CFC, that is in addition to levels also first order differences are meaningful and comparable, by defining a system of equivalence scales (2) it is possible to make expenditure indices have the same CFC properties.

Furthermore, as any transformation admissible by a given preference system does not change preferences, hence not even the equivalence scales would change by the application of such an admissible transformation. Therefore as (2) is defined for preference systems which are at least OLC, it follows that the resulting equivalence scales are invariant with respect to a positive monotonic transformation. The same invariance trivially holds for affine and linear transformations common to all households because these are a sub set of monotonic transformations.

3.1. Information constraints and modifying functions

One first problem to face is that of considering how the alternative methods of introducing demographic characteristics in demand analysis deal with the requirement of OLC in order to get meaningful equivalence scales. From an informational point of view the most important result linked to the development of modifying function (1) is the fact that the modified system is ordinally equivalent to the original system. If $v^h(p,y,a)$ is the modified system and $v^*(p^*,y^*)$ is the original system, we have that [see Lewbel (1985, theorem 4) and equation (1)]:

$$v^h(p,y,a)=v^*(p^*,y^*). \tag{9}$$

However, because the system $v^*(p^*,y^*)$ can be considered as that describing preferences of the reference family, then (9) seems to prove that modifying functions have the uncommon property of granting level comparability of any preference system.

To examine this point and to prove that it cannot possibly hold consider that the modified system is given by:

$$v^*(p^*,y^*)=v^*(p^*,G(y,p,a)). \tag{10}$$

Where symbols and functions are defined as in (1), and $G(.)$ is the inverse function of $g(.)$ with respect to y^*. Substituting for y the expression $y=g(c^*(v^h(p,y,a),p^*),p,a)$ and eliminating the inverse functions one gets the result in (9). However this result does not depend on the properties *of* modifying functions but rather on the assumption for which we have set $u=v^h(p,y,a)$. *If,* instead, we set $u=F(v^h(p,y,a),a)$, then $y=g(c^*(F(v^h(p,y,a),a),p),p,a)$, and because of (10):

$$v^*\left(p^*, y^*\right) = F^h\left(v^h(p, y, a), a\right). \tag{11}$$

Which goes to show that (9) is not a result of the properties of modifying function but rather the direct consequence of an assumption. One probably more direct way to show that (9) is based on a specific assumption is to observe that from (1) we get:

$$c\left(u^h, p, a\right) = g\left(c^*\left(u^h, p^*\right), p, a\right). \tag{12}$$

Setting $y^* = c^*\left(u, p^*\right)$, we have

$$u^h = v^h(p, y, a) = v^*\left(p^*, y^*\right). \tag{13}$$

If the function $v^*(.)$ is specific for each *h,* hence it is not common to households, it cannot be taken as the system describing behaviour of the reference household. This is, however, necessary to obtain a meaningful system of equivalence scales. To solve the problem an assumption on comparability is needed and to this end modifying functions are of no help. The need for such an assumption is clear if one considers that interpreting the function $v^*(.)$ as that of the reference family leads through (13) to

$$v^h(p, y, a) = v^*\left(p^*, y^*\right) = v^k(p, y, a). \tag{14}$$

which in order to make sense requires preferences to be OLC. In terms of (1) this implies that the functions $g(.)$ and $h(.)$ be common to all families.

Therefore although modifying function are capable of transforming a demographic variable preference system into a system defined only in terms of prices *p* * and income y' this does not imply that the transformation carried out through (1) can guarantee that the modified systems of two different families are OLC. The technique of introducing demographic characteristics into preference systems cannot in any way help to guarantee that the modified system is OLC if the original one was not such.

3.2. Preferences and characteristics

Whether in applied analysis we are faced with comparable or non comparable preferences is a matter that comes to be dealt with by means of an assumption. Although we do not have anything better to suggest, nevertheless, in the case of equivalence scales a description of the possible role demographic characteristics can have in household preference systems, turns out to be of help in discussing the identification issue.

A starting point is that of limiting the role of characteristics to the budget constraint. From an informational point of view this is substantially different from the use of modifying function described in (1) because in this way the role of characteristics in the utility function is kept distinct from that on the budget equation. Each family would face the problem of:

$$\max_q \left\{u(q) \,\middle|\, \sum q_i p_i^* = y^*;\ p_i^* = h(p,a);\ y^* = G(y,p,a)\right\}, \tag{15}$$

where symbols and functions are defined as in (1) and (10)[1]. Reference household would have:

$$\max_q \left\{u(q) \,\middle|\, \sum q_i p_i = y\right\}. \tag{16}$$

Comparison of (15) and (16) shows what role characteristics are forced to play: families would all be the same if it wasn't that each is characterised by a budget constraint made specific by given income and family characteristics. As far as the information requirements are concerned they are satisfied by the assumption of ordinality and by granting level comparability as a result of taking each household preference system to be the same that is: $u = u(q), \forall h$. Therefore no identification problem can arise for such a preference system.

Instead, the traditional way of posing the problem is that of treating characteristics as fix modifiers in the utility function, that is to replace $u(q)$ by $u(q,a)$, in (15)[2]. In the logic of the present argument, as characteristics are not a choice variable, the system $u(q,a)$ can be better understood as representing a household specific preference system over goods, that is $u(q,a) = F(u(q),a) = u^h(q)$. The problem posed by (15) becomes:

$$\max_q \left\{u^h(q) \,\middle|\, \sum q_i p_i^* = y^*;\ p_i^* = h(p,a);\ y^* = G(y,p,a)\right\}. \tag{17}$$

Accordingly, from the point of view of the information contents, (17) shows the presence of a non comparability problem, because the system $u^h(q)$ can be seen as a specific monotonic transformation of reference household preference system.

There is the possibility, however, that characteristics could be part of the family's choice set. In this case, their role could no longer be represented only through a modification of the budget constraint, they will have to be included into the preference system as choice variable: $u = u(q,a)$. However such a degree of choice over characteristics is not plausible in most cases.[3] A more suitable

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[1] For example Barten's system would be given by
$$\max_q \left\{u(q) \,\middle|\, \sum q_i p_i^* = y^*; p_i^* = \Delta p; \Delta = diag\left[\lambda_{ij}\left(a^h\right)\right]; y^* = y\right\}.$$

[2] See Deaton and Muellbauer (1980, p. 206-8), Lewbel (1985, p. 6) and Tsakloglou (1991).

[3] *"The number of children is much less endogenous in the short run than are purchases of goods, and i~*

possibility could be that of considering the family decision as referred to a longer time span (life time?), over which is plausible to consider the family capable of making decision about (at least some) characteristics in very much the same way as it is often modelled the decision about durables. Actual behaviour would then be the result of:

$$\max_{q,a}\left\{F(u(q),a) \mid \sum q_i p_i^* = y^*;\ p_i^* = h(p,a);\ y^* = G(y,p,a)\right\}. \tag{18}$$

There is in such a case "weak" separability between choices in terms of consumption goods and choices in terms of characteristics, and expenditure functions are "conditional" upon the choice of each variable[1].

This separability result lends itself to a useful distinction of welfare components: it is possible to see *total welfare* of the family $F(u,a)$ as made up of two distinct components, one being what can be called the *economic welfare* and is represented by the component $u(.)$. It owes its name to the fact that it is the only part subject to the budget constraint. And the other component $F(.,a)$ is the *non-economic welfare* and represents the independent contribution to *total welfare* given by_characteristics. For our purposes this distinction of welfare components turns out to be useful for systems like (17) which are at the base of the identification problem of equivalence scales. In (17) the OLC requirement does not hold because of the direct role of household characteristics on preferences. However the same welfare distinction as in (18) can be applied to (17) and the OLC requirement is meaningful in terms of the concept of *economic welfare*, that is in terms of system $u(q)$. A general formulation including (15) and (17) as special cases is:

$$\max_{q}\left\{F^h\left[u^h(q)\right] \mid \sum q_i p_i^* = y^*;\ p_i^* = h(p,a);\ y^* = G(y,p,a)\right\} \tag{19}$$

The OLC requirement for equivalence scales poses some definite restrictions on the $u^h(.)$ and $F^h(.)$ functions. The following possibilities arise:

a) $u^h \neq u^k$, and $F^h \neq F^k\ \forall h \neq k$, then OLC will not hold neither for *economic* nor for *total welfare*;

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anyone day or year, relatively few households are at the margin of deciding whether to have another child", Muellbauer and Deaton (1980, p.208).

[1] See Deaton and Muellbauer (1980, p. 127). If characteristics were not a choice variable (18) would be the same as (17), moreover if the function $u(q)$ in (18) is replaced by $u(q,a)$ and characteristics are still not a choice variable the problem is once again the same as that in (17), because, as argued in the text, when characteristics are not a choice variable the system $u(q,a)$ can be seen as representing a household specific preference system. Instead, if in (18) we replace u(q) with $u(q,a)$, and characteristics are a choice variable, then separability between goods and characteristics would no longer hold.

b) $u^h \neq u^k\ \forall h \neq k$, but $F^h = F^k\ \forall h,k$, then <u>*economic welfare*</u> is not comparable. The question of whether <u>*total welfare*</u> is comparable is to be left open;

c) $u^h \neq u^k\ \forall h \neq k,\ k$ but $F^h \neq F^k\ \forall h \neq k$, then OLC will hold for <u>*economic welfare*</u>, but it will not hold for <u>*total welfare*</u>, And this is the case described in (17);

d) $u^h \neq u^k$, and $F^h = F^k\ \forall h,k$, then OLC will hold for both <u>*economic*</u> and <u>*total welfare*</u>. *If* $F(.)$ is the identity function it describes case (15).

Therefore, if we can safely assume that the concept of <u>*economic welfare*</u> can bear the OLC assumption, and if characteristics are not a choice variable then the OLC assumption rules out their presence from the direct preference system.

Moreover, considering the case where characteristics do belong to the choice set but it is not plausible to admit over them the same degree of choice as that over goods, although some degree of choice is plausible over a longer time span, then in such a framework a meaningful distinction can be made between <u>*economic*</u> and <u>*total welfare*</u>. This distinction helps to recover some information from cases such as (17) where the OLC assumption does not hold. System (17) can be seen to be in the form of a specific monotonic transform of a OLC system specified in terms of <u>*economic welfare*</u>. Therefore, the distinction proposed for (18) can apply to (17) with the proviso that only the <u>*economic welfare*</u> is comparable.

4. Equivalence scales without apology

Having clarified the information constraints posed by equivalence scales and how they might limit the role played by household characteristics in family decision, we are in a better position to tackle a famous problem, raised by Pollak and Wales (1979) and described in Section 2, about the indentification of equivalence scales. Take two preference systems, say $u = v(p, y, a)$ and $u' = F(v(p, y, a), a)$, with expenditure functions:

$$y = c(u, p, a), \tag{20}$$
$$y = c(f(u', a), p, a) = c'(u', p, a). \tag{21}$$

Where $f(.)$ is the inverse of $F(.)$ with respect to $u(.)$. Of course equivalence scales based on (20) or based on (21) are different. Here then comes the problem because equivalence scales based on (21) cannot be computed without further information in view of the fact that observed behaviour cannot by itself define the function $f(.)$. Indeed, as Blundell and Lewbel (1991) have pointed out, the lack of identification implies that in estimating equivalence scales there comes to be an element of arbitrariness because one would implicitly set a value for the function $f(.,a)$. More explicitly if $q(p,y,a)$ is the estimated system of demand equation and $c(p,u,a)$ and $c(f(u,a),p,a)=c^{'}(p,u^{'},a)$ are two expenditure functions both consistent with the estimated $q(.)$, then the actual functional form given to $c^{'}(.)$ would contain an element of arbitrariness in the choice of the function $f(.)$ which is relevant for the computation of equivalence scales but is not of any importance for the estimation of the demand system.

However this objection needs not to dismantle the equivalence scale apparatus. Indeed, it provides us with a good example to improve the understanding of what equivalence scales based on observed behaviour can actually measure. As shown in the previous section the system $F(v(p,y,a),a)$ can be seen as made up of two distinct components of welfare. One being the *economic* component $v(p,y,a)$, and the other is *non-economic*, and together they define *total welfare*.

Now our simple point is that although from observed behaviour we cannot recover all the elements of system $F(v(p,y,a),a)$, we can nevertheless recover the system $v(p,y,a)$ and from (20) define a set of equivalence scales based on the concept of *economic welfare*. Therefore there is no need for any arbitrariness to interfere with the computation of equivalence scales according to (20)[1].

This simple observation helps to clarify a further point of interest. In most applied works a functional form of market demand system is obtained by Shephard's lemma from a given functional form of expenditure function. Now the question we might be asked is the following: if we are given an expenditure function how could we tell what concept of welfare it embodies? In the present framework the answer is relatively simple. Take any given expenditure function and work out equivalence scales at reference prices. These scales, as shown in Section 2, are made up of two components. The first is the Engel (price independent) scale, the second component relates to the function $f(.)$ in (21). To distinguish between the two is simple because Engel components enter the demand function as income correctors, while the function $f(.)$ does not affect behaviour at all. Therefore we could identify the function $f(.)$ and proceed to isolate the *economic welfare* component.

The main result of this section, if we put it in a rather straightforward manner, says that given a demand system estimated from observed behaviour, the implied expenditure function can be used as it is to compute a set of equivalence scales, and these are a measure of family relative well being in terms of *economic welfare.*

In order to provide an example of the implication of this result in terms of a given and often used demand system, consider the budget share equation:

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[1] This point appears to be implicitly contained in a number of papers [Deaton and Muellbauer (1986), Blackorby and Donaldson (1991a, p. 189), Pashardes (1991, p. 203)], however its underpinnings are not made clear.

$$w_i^h = \alpha_i\left(a^h\right) + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln(y/P). \tag{22}$$

Which can be rationalised by either of the following expenditure functions[1]

$$\ln c\left(u^h p, a^h\right) = \sum_i \alpha_i\left(a^h\right) \ln p_i + \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j + \prod_i p_i^{\beta_i} u^h, \tag{23}$$

$$\ln c\left(u^h p, a^h\right) = \sum_i \alpha_i\left(a^h\right) \ln p_i + \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j + \beta_0\left(a^h\right) \prod_i p_i^{\beta_i} u^h. \quad .(24)$$

For any price vector equivalence scales from (23) and from (24) respectively are:

$$\ln s = \sum_i \alpha_i\left(a^h\right) \ln p_i - \sum_i \alpha_i\left(a^r\right) \ln p_i, \tag{25}$$

$$\ln s = \sum_i \alpha_i\left(a^h\right) \ln p_i - \sum_i \alpha_i\left(a^r\right) \ln p + \prod_i p_i^{\beta_i} \left[\beta_0\left(a^h\right) - \beta_0\left(a^r\right)\right] u. \tag{26}$$

In order to compute (26) we need to know the parameters β_0, but estimation of (22) can tell nothing about them. Moreover if the system (24) is not OLC the knowledge of the parameter β_0 would be of no use for the computation of (26), which in the absence of OLC would have no meaning. However if we do not know the parameters β_0, or system (24) is not OLC, the possibility stays open to compute equivalence scales by using (25). It is also clear that computing (25) starting from (24) means to set $\beta_0\left(a^h\right)u^h = \beta_0\left(a^r\right)u^r = u$, that is to set $f\left(u_h^{'}, a\right) = f\left(u_r^{'}, a\right)$ in (21). This once more contributes to clarify how equivalence scales derived from observed behaviour have a precise although limited meaning of measure of relative *economic welfare*.

5. Base independence and identification

A total different approach put forward in order to overcome the identification problem, observes that if equivalence scales were independent from base index of welfare (IB), then identification could be possible and the data would reveal a proper measure of relative welfare [see Blackorby and Donaldson (1989, 1991a), Blundell and Lewbel (1991)][2]. If this possibility were to come true there would result a great generality of equivalence scales as a welfare indicator. Alas a closer look to the problem reveals that this cannot be the case.

Equivalence scales are IB if they depend only on prices and characteristics, therefore expenditure function must take the form:

$$c(u, p, a) = e(p, u)\lambda(p, a). \tag{27}$$

[1] For simplicity we assume away any Engel component in the expenditure function.
[2] In the words of Blackorby and Donaldson (1989, p. 13) *"the requisite interpersonal comparisons are actually revealed by the data".*

The basic idea behind the proposal to use the IB condition as a device for equivalence scales identification is based on observing that if equivalence scales are IB, then (27) must be the only expenditure function consistent with the demand system $q(p, y, a)$. Any other expenditure function consistent with $q(p, y, a)$ would have to be in the form $y = e[p, f(u', a)]\lambda(p, a)$ which makes equivalence scales dependent from base utility index.

For identification purposes the usefulness of the IB condition fully rests on the possibility of deriving a set of testable restrictions. To show how these restrictions arise Blundell and Lewbel (1991) refer to a specific form of preferences, with expenditure function in the form of equation (3), giving equivalence scales as:

$$\ln s(p,u,a) = \ln a(p,a^h) - \ln a(p,a^r) + [\ln b(p,a^h) - \ln b(p,a^r)]u . \tag{28}$$

For this to be IB the function $b(p,a)$ must be independent of household characteristics. Blundell and Lewbel (1991) maintain that this would pose *"testable restriction on the* (..) *demand equation"* (p. 56), as it implies that *"the price derivatives of* $b(p,a)$ *are independent of a",* (p. 59). In order to show this they define $\ln a(p,a)$ as in (4), and $\ln b(p,a)$ as:

$$\ln b(p,a^h) = \beta_0 \prod_i p_i^{\beta_i(a^h)} . \tag{29}$$

Which after substitution into (3) gives equivalence scales:

$$\ln s = [\alpha_0(a^h) - \alpha_0(a^r)] + [\sum_i \alpha_i(a^h) - \sum_i \alpha_i(a^r)] + [\prod p_i^{\beta_i(a^h)} - \prod p_i^{\beta_i(a^r)}]\beta_0 u , \tag{30}$$

and budget share demand equations:

$$w_i^h = \alpha_i(a^h) + \sum_j \gamma_{ij} \ln p_j + \beta_i(a^h)\ln(y/P) . \tag{31}$$

By comparing (30) to (31) they conclude that if the $\beta_i(a^h)$ parameters were independent of family characteristics then equivalence scale would be IB. Such an independence can be econometrically tested as the parameters $\beta_i(a^h)$ appear in the demand equation (31). Therefore if the IB restriction is not rejected equivalence scales could be fully identified.

The argument must have a faulty point which in general can be uncovered as follows. Take an estimated system of demand equations $q(p, y, a)$, integrate it to recover a consistent expenditure function, say $c(p, u, a)$. Now even if we observe that the expenditure function is in the form of (27), nevertheless we would not be able to observe if a monotonic transformation of preferences with expenditure function $c(p, f(u', a), a)$ is still in the form of (27). This is because behaviour is independent of a monotonic transform of preferences, hence it will never provide any information about the function $f(u', a)$.

In order to make this point more explicit and link it to the exposition made by Blundell and Lewbel (1991), it can be shown that in (3) although the independence of $b(.)$ from characteristics is a sufficient condition to identify equivalence scales, nevertheless:

a) the dependence of $b(.)$ from characteristics does not necessarily pose restrictions on demand equations; and

b) in those cases where a restriction is actually posed, then, contrary to Blundell and Lewbel (1991), no identification problem need to arise. That is independence of $b(.)$ from characteristics is not a necessary condition for identification.

To show these points in a rather simple manner replace (29) with a general functional form such as:

$$\ln b(p, a^h) = \alpha(p)\beta(a^h)\gamma(p(a^h)). \tag{32}$$

Where it is understood that in $\gamma(.)$ characteristics operates only through prices. Now it is easy to show that independence of $b(.)$ from a^h is a sufficient condition for identification. Indeed this means that (32) reduces to $\ln b(.) = \alpha(p)$ and the expenditure function (3) to $\ln y = \ln a(p, a^h) + \alpha(p)u$ from which application of Shephard’s lemma shows that no identification problem arises.

To prove proposition a) we need to show that dependence of $b(.)$ from a^h does not necessarily impose any restriction on demand systems. To see this specify (32) as $\ln b(p, a^h) = \alpha(p)\beta(a^h)$, then the expenditure function is $\ln y = \ln a(p, a^h) + \alpha(p)\beta(a^h)u$. Again Shephard's lemma shows that demand equation would not depend on the $\beta(a^h)$ function. Indeed it is clear that the expenditure function is in the form of $y = c(f(u', a^h), p, a)$ as in (20), that is the function $\beta(a^h)$ constitutes a specific monotonic transform of the preference system.

In terms of the preference system taken as an example by Blundell and Lewbel (1991) this point can be shown by replacing (29) with

$$\ln b(p, a^h) = \beta_0(a^h)\prod_i p_i^{\beta_i}, \tag{33}$$

where the β_i parameters are the same for any household, that is the Blundell and Lewbel condition would be satisfied. Nevertheless equivalence scales do depend on the index of welfare, at reference prices they are:

$$\ln s = \alpha_0(a^h) - \alpha_0(a^r) + [\beta_0(a^h) - \beta_0(a^r)]u . \tag{34}$$

In short, the condition of base independence of equivalence scales does not imply any functional restriction on demand equation, and therefore is an untestable assumption.

To prove proposition b) we need to show that even though $b(.)$ depends on a^h identification is possible. To show this take (32) to be $\ln b(p, a^h) = \gamma(p(a^h))$, then expenditure function is $\ln y = \ln a(p, a^h) + \gamma(p(a^h))u$. Now applying Shephard's lemma would show that the dependence of $\gamma(.)$ form a^h does pose restriction on demand equations. However this will allow us to recover the shape of function $\gamma(.)$ from observed behaviour and no identification problem arises.

In words: the Blundell and Lewbel condition rightly says that if the β_i parameters are not the same for all household, then equivalence scales are not IB. However as the same example made by Blundell and Lewbel helps to show [see system (3), (29)] base dependence needs not be a problem in identifying equivalence scales. In summary the IB condition in itself is of no use in solving the identification problem of equivalence scales.

A slightly different line is followed by Blackorby and Donaldson (1989, 1991a,b) in proposing a solution to the identification problem. Their point can be summarised as follows: suppose that there are two different IB equivalence scales both consistent with the same household behaviour then there is a function $F(.)$ increasing in its first argument such that:

$$v'\left(\frac{y}{\lambda'(p,a)}, p\right) = F\left[v\left(\frac{y}{\lambda(p,a)}, p\right), a\right]. \tag{35}$$

When this happens IB condition cannot discriminate between the original system and a monotonic transform of it, hence identification is not possible. However when (35) does not hold then IB restriction helps overcoming the problem because only one system would be consistent with IB scales.

Even in this case the point begs the question of whether the IB condition is a maintained assumption or a testable restriction. Our claim is that the IB restriction is not testable, and if it is an assumption it is of little use for the identification problem. In order to show this let us sketch the proof of Theorem 6.1 in Blackorby and Donaldson (1989). The inverse of the LHS and of the RHS of (35), respectively, are:

$$e'(v', p)\lambda'(p,a) = y , \tag{36}$$

$$e(f(v', a), p)\lambda(p,a) = y . \tag{37}$$

Then for (35) to hold we need (37) to be written as (36), this requires function $e(.)$ to be homogeneous of first degree in $f(v', a)$, and function $f(.)$ to be in the form:

$$f(v', a) = \phi(v')\psi(a). \tag{38}$$

Then (37) becomes:

$$e(p)\phi(v')\lambda'(p, a) = e'(v', p)\lambda(p, a). \tag{39}$$

Therefore for (37) to be written as (36) preferences have to be homothetic and the function $f(.)$ has to satisfy condition (38). Both these conditions accord with the results obtained by Blackorby and Donaldson (1989, 1991a)[1].

However these results do not help to overcome the identification problem for the simple reason that condition (38) cannot be ascertained from behaviour which is invariant to any monotonic transformation of preferences. Take for instance a homothetic preference system let it be $v(y/\lambda(p,a), p)$ giving IB equivalence scales, now there is no way to check whether the system $v' = F(v,a)$ yields or not IB scales.

Instead IB is a maintained assumption in estimation, as suggested by Blackorby and Donaldson (1989, 1991a), then the question is why should we restrict preferences to be lB. One possible answer would be that suggested by Blackorby and Donaldson (1991a, p. 190-93) of knowing from some other prior sources that the "true" equivalence scale is IB, then if (35) does not hold only one system is consistent with IB scales. But in terms of information required this is non different from assuming straightaway that the estimated preference system is the "true" one.

6. Equivalent income and social welfare

A further point concerning the information contents of equivalence scales arises when they are employed in social welfare analysis. In the process of defining a Social Welfare Function (SWF) two distinct approaches have been followed in the literature. One is that commonly referred as the "single profile" approach, while the second approach is known as the "multi profile". The literature on social choice theory has since long shown that the two approaches make no differences in terms of the fundamental theorems concerning the existence of a SWF [Sen (1977), Roberts (1980)]. In the single profile approach the individual welfare indices $(u^1, u^2, .., u^H)$ included in a welfarist SWF of the general form $W = W(u^1, u^2, .., u^H)$, are *"simply a vector of individual utilities"* Sen (1977, p. 1566). In short this means that each family welfare conditions are summarised in a

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[1] Although they do not seem to require the factorisation of function $f(.)$ as in (38), this is nevertheless implicitly imposed in their equation (A.I5).

single index of welfare. In the multi profile approach, instead, each of the indices $\left(u^1, u^2, .., u^H\right)$ is itself a function that fully describes the household preference system.

This digression on single-multi profile approaches was necessary in order to shed some light on a problem which seems to arise when equivalence scales are used as elements of a SWF. Recently it has been suggested [Lewbel (1989)] that precise restrictions have to hold for equivalence scales, as embodied in the concept of equivalent income, to be used as the informational base for a SWF. From the definition of equivalence scales (2) we get

$$v\left(p, y^r, a^r\right) = v\left(p, sy^h, a^h\right) = u, \tag{40}$$

therefore equivalent income is:

$$\overline{y^h} = sy^h \Leftrightarrow u^r = u^h. \tag{41}$$

If equivalence scales are IB then

$$v\left(p, y^r / \lambda\left(p, a^r\right)\right) = v\left(p, y^h / \lambda\left(p, a^h\right)\right) = u, \tag{42}$$

and

$$y^r / \lambda\left(p, a^r\right) = y^h / \lambda\left(p, a^h\right) \Leftrightarrow u^r = u^h. \tag{43}$$

For reference household we have $\lambda\left(p, a^r\right) = 1$, therefore equivalent income with IB scales is given by:

$$\overline{y}^h = \frac{y^h}{\lambda\left(p, a^h\right)} = e\left(p, u^h\right). \tag{44}$$

The problem posed by Lewbel (1989) is that of defining those conditions preferences have to satisfy for equivalent income to be used as argument of a SWF. Lewbel's argument seems to run as follows: when preferences are IB equivalent income is given by (44), that is by $e(p,u)$ which can be seen as a monotonic transformation of the welfare index *u,* and as such the admissible transformation should depend on the measurability and comparability properties of preferences. Once this way of looking at the problem is accepted it follows as a matter of fact that the form of the function $e(p,u)$ has to conform to the measurability and comparability properties of preferences. That is [see Lewbel (1989)]:

- if preferences are OLC, no restrictions emerge for the function $e(p,u)$.

This is because OLC admits any positive monotonic transformation of preferences;

- if preferences are CFC, then $e(p,u)$ should be in the form of an affine transformation, $e(p,u)=\psi(A(p)+B(p)u)$;

- if preferences are RSC, then $e(p,u)$ should be in the form of a linear transformation, $e(p,u)=\psi(B(p)u)$.

These results in our judgment are based on an improper use, apparently suggested by (44), of the concept of equivalent income. More precisely we think that no restriction needs to hold on preferences for equivalent income to be used as a welfare index in social welfare evaluation. The crucial point on which there seem to be a misunderstanding is the role played by the informative invariance requirement in aggregating individual welfare indices.

To clarify the matter let us recall that utility function and expenditure are two equivalent representation of individual preferences. Therefore given the utility function $u^h = v(p, y^h, a^h)$, and the expenditure function $y^h = c(p,u^h,a^h)$, for any welfarist SWF we would have that:

$$W\left[c(p,u^1,a^1),\ldots,c(p,u^H,a^H)\right]=W\left[v(p,y^1,a^1),\ldots,v(p,y^H,a^H)\right]. \tag{45}$$

This is a straightforward result characterizing the properties of social welfare choice in a multi profile approach also called social welfare functional. In such a framework there is no room to make use of equivalent scales or of the related concept of equivalent income because individual preferences need not be summarised in a single index.

When we come to consider the single profile approach to the SWF we have to summarise each preference system in a welfare index. If we do this according to the expenditure function correspondence we find that (45) becomes:

$$W(y^1, y^2,\ldots, y^H) \neq W(u^1,u^2,\ldots,u^H) \tag{46}$$

Equality between the two sides of (46) cannot hold for the obvious reason that comparability between utility indices when granted by the comparability proprieties of the preference system, cannot be extended to money metric indices of welfare such as expenditure.

For instance, if we have that $v(p,y^h,a^h)=v(p,y^k,a^k)$ we must also have that $y^h \neq y^k$ because $a^h \neq a^k$. This is because the cardinalization of the utility index by mean of y^h is relative to a given price vector p and to a given set of characteristics a^h. Therefore y^h is not comparable to y^k because each is in terms of two different bases of cardinalization, they are (p,a^h) and (p,a^k), respectively.

Equivalence scales overcome this comparability problem because they provide a device to express individual money metric indices all in terms of one single base for cardinalization, that is (p, a^r). The expression for equivalent income makes the point clearer:

$$\bar{y}^h = c(u^h, p, a^h)\frac{c(u^h, p, a^h)}{c(u^h, p, a^r)} = y^h s(u^h, p, a_r^h). \tag{47}$$

This also contributes to clarify that in computing equivalent income y^h the equivalence scale has to be computed at the welfare index u^h given by $v(p, y^h, a^h)$. Moreover this makes clear the operational importance of base independent equivalence scales. However from (47) we can now rewrite (46) as

$$W(\bar{y}^1, \bar{y}^2, ..., \bar{y}^H) = W(u^1, u^2, ..., u^H). \tag{48}$$

This expression sets us in a better position to asses the problem posed by Lewbel (1989), that is whether the comparability and measurability requirements pose any restrictions on the functional form of the functions contained in (47). The first, and to our judgment only restriction for (47) to be meaningful and suitable for social welfare calculation, that is for (48) to hold, is that preferences have to be at least OLC. If they are not comparable then equivalence scales are an empty concept. On the other hand is straightforward matter to see that positive monotonic transformation which characterise OLC preference systems do not alter the right hand side of (48), and as they would not change (47) therefore they will not alter the left hand side of (48). The same results hold for affine and linear transformation, which characterise CFC and RSC preferences respectively.

Therefore none of the conditions highlighted by Lewbel (1989) are required for equivalence scales and equivalent income to be used as a welfare indicator in welfarist social welfare functions.

7. Concluding remarks

Estimation of equivalence scales from demand data poses precise restrictions on the information contents of the underlying preference system. Ordinality and level comparability are required for equivalence scales to be meaningful. Observed behaviour cannot reveal whether this restriction holds or not for the data. However, the assumption that it does hold is at the base of the identification problem of equivalence scales. The examination of the possible role characteristics can play in household preference system shows that a meaningful distinction can be made between *total* and *economic welfare.* The latter is that part of *total welfare* subject to the budget constraint, and if for it is safe to assume that it satisfies the requirement of level comparability then no identification problem needs to arise. Therefore, provided we limit the attention to the *economic welfare,* no identification problem for equivalence scales need to arise, and these have a precise although limited meaning of measure of relative *economic welfare* of different households.

Limiting the meaning of equivalence scales to the *economic* concept of welfare is the only plausible way out we can suggest to the identification problem. However, we have also examined some attempts to overcome the identification problem [see Blundell and Lewbel 1991, Blackorby and Donaldson 1989, 1991a,b)] and we have shown that they are not satisfactory as they rely on the idea that observed behaviour can provide some sort of information about the level comparability assumption.

Moreover, and contrary to Lewbel (1989), the level comparability assumption turns out to be the only restriction on the information contents of the preference system for equivalece scales, as embodied in the concept of equivalent income, to be used in social welfare evaluation.

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