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JEL Classification: H62, H21
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Migration, altruism and capital income taxation*

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Abstract

In this work we tackle the issue of optimal dynamic taxation of capital income in an economy with migration and intra-family altruism. We show that, while in the short run both positive or negative tax rates can be optimal, in the long run there is room only for subsidizing capital. The reason is that the limited altruism brings about a disconnection in the economy since current residents do not care about the welfare of future immigrants: as a consequence, the government finds it optimal to levy corrective taxation by increasing the present rate of consumption growth. In the light of this, as long as the fertility rate is positive, in this model the Chamley-Judd zero tax rate result never applies.

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1 Introduction

Starting from the seminal works by Judd (1985) and Chamley (1986), the issue of dynamic optimal income taxation has been analyzed by a number of

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researchers. In particular, the more recent contributions have, firstly, pointed out that the nature of the zero long run tax result provided by the two mentioned authors has much to do with the more traditional static optimal taxation principles; secondly, they have qualified the conditions under which such result can hold; thirdly, they have extended the analysis under different or more general scenarios.

More precisely, Judd (1999) has shown that the zero tax rate result stems from the fact that a tax on capital income is equivalent to a tax on future consumption: thus, capital income should not be taxed if the elasticity of consumption is constant over time. However, while in infinitely lived representative agent (ILRA) models[1] this condition is necessarily satisfied in the long run, along the transition path, instead, it holds only if the utility function is assumed to be (weakly) separable in consumption and leisure and homothetic in consumption. In fact, both Erosa and Gervais (2001) and De Bonis and Spataro (2002) show that the non zero tax result, applying in the short run when separability is assumed out, can be explained by the traditional Corlett-Hague (1953) rule: since leisure cannot be taxed directly, the second best solution is to tax (subsidize) the good that is more (less) complementary to it, i.e. consumption.

Abandoning the standard ILRA framework in favour of Overlapping Generation models with life cycle (OLG-LC)[2] has delivered the important result of the violation of the zero tax rule also in the long run. This outcome can be understood by reckoning that in such setup optimal consumption and labor, or, more precisely, the elasticity of consumption, are generally not constant over life and even at the steady state, due to life-cycle behavior.

Another source of long run non zero taxation, common to both ILRA and OLG-LC models, occurs if one allows the government and individual discount rates to differ. De Bonis and Spataro (2002), for example, obtain an asymmetric result concerning the long run optimal level of the capital income tax, since this turns out to be negative (even with homothetic in consumption and separable utility functions) if the government is more patient than individuals, while the Chamley-Judd result keeps still valid if the government is less patient[3].

Finally, De Bonis and Spataro (2003) enrich the analysis by focusing on a perpetual youth model *à la* Blanchard (1985) with growing population and

[1]See Atkeson et al. (1999) and Chari et al. (1999).

[2]See Atkinson and Sandmo (1980) and Erosa and Gervais (2002); for a review see Renström (1999) and Erosa and Gervais (2001).

[3]Among other articles focusing on the optimal capital income taxation problem see Jones et al. (1997), modeling human capital accumulation, and Chari et al. (1999), Zhu (1992) and Yakadina (2001), dealing with stochastic frameworks.

with a kind of imperfection affecting the life insurance market[4]. Namely, in this model the authors, besides reproducing the previous results stemming from the shape of the utility function and the government intertemporal discount rate, unveil two further forces determining the violation of the zero tax result: *i)* the difference between the weight the government attaches to each generation and its actual demographic size and *ii)* the probability of death *and* the OLG mechanism[5].

In this work we extend the previous analysis by focusing on the role played by altruism and migration in determining the taxation of capital income: more precisely, we consider an economy with migration combined with overlapping "infinitely lived" dynasties, that is, formed by individuals who are altruistic only toward their own descendants[6].

The main finding is that the presence of migration and dynastic altruism generates a disconnection that gives room to non zero capital income taxation. More precisely, it is optimal for the government to subsidize capital accumulation since this is equivalent to subsidize consumption of future generations of migrants, whose welfare is disregarded by current residents.

The work proceeds as follows: in the first section we present the model and derive the equilibrium conditions for the decentralized economy. Next, we characterize the Ramsey problem by adopting the primal approach. Finally, we present the results by focusing on the new ones. Concluding remarks and a technical appendix will end the work.

2 The model

We consider a neoclassical-production-closed economy in which there is a large number of agents and firms.

Private agents, who are identical in their preferences, differ as for their date of entry into the economy, s; natives are supposed to have entered the economy at some time $s < 0$, while migrants enter at a given rate $\alpha(s)$; both types of individuals live for one period only and have a constant rate of growth n: as a consequence, the population growth rate is equal to $\alpha + n$.

In fact, the whole population at time t has cardinality:

[4] In fact the perpetual youth framework allows to deal with overlapping generations, finite life-time horizon (via a constant probability of death) and life-cycle behavior and to discuss other features which had been studied separately or under special assumptions.

[5] While the first factor descends from intuitive equity arguments, the intuition behind the second one is the following: a tax proportional to the probability of death in the presence of life insurance contracts is second best optimal since it mimics the effects of confiscating wealth upon death in an economy without life insurance.

[6] For a presentation of this model see Barro and Sala-i-Martin (1999), chapter 9.

$$N(t) = N(0)\, e^{nt} e^{\int_0^t \alpha(v)dv}$$

with $N(0)$ the size of population at time 0 and $s \leq t$.

The size of each dynasty (started by the entry of the founder) is:

$$F(s,t) = \alpha(s)\, N(s)\, e^{n(t-s)}$$

Now, by setting $N(0)$ equal to one and supposing α constant, without loss of generality, the size of the whole population, at time t, is:

$$N(t) = e^{(n+\alpha)t}.$$

Moreover, all individuals offer labor and capital services to firms by taking the net-of-tax factor prices, $\widetilde{w}(s,t)$ and $\widetilde{r}(s,t)$ as given. Firms, which are identical to each other, own a constant return to scale technology F satisfying the Inada conditions and which transforms the factors into production-consumption units. Finally, the government can finance an exogenous and constant stream of public expenditure G, by issuing internal debt $B(t)$ and by raising proportional taxes both on interests and wages, referred to as $\tau^k(s,t)$ and $\tau^l(s,t)$ respectively. Notice that taxes can in principle be conditioned on the date of birth of dynasties[7].

2.1 Private agents

Agents' preferences can be represented by the following instantaneous utility function:

$$U(c(s,t), l(s,t))$$

where $c(s,t)$ and $l(s,t)$ are instantaneous consumption and labor supply respectively of the dynasty started in period s, as of instant t. Such a utility function is strictly increasing in consumption and decreasing in labor, strictly concave, and satisfies the standard Inada conditions. Since we assume that individuals care about the well being of their children, agents maximize the following utility function:

$$\max_{\{c(t),l(t)\}_s^\infty} \int_s^\infty e^{-(\beta-n)(t-s)} U(c(s,t), l(s,t))\, dt \tag{1}$$

[7]This strong assumption can be ruled out without loss of generality. In fact, our results do not rely on it.

$$sub\ \dot{a}\,(s,t) = (\widetilde{r}\,(s,t) - n)\,a\,(s,t) + \widetilde{w}\,(s,t)\,l\,(s,t) - c\,(s,t) \tag{2}$$

$$\lim_{t\to\infty} a\,(s,t)\,e^{-\int_s^t(\widetilde{r}(s,v)-n)dv} = 0, \quad a\,(s,s) = \overline{a}$$

where β is the intertemporal discount rate, a the agent's wealth; the notation $\dot{()}$ indicates the derivative with respect to time, while $\widetilde{r}\,(s,t) = r\,(t)\left(1-\tau^k\,(s,t)\right)$ and $\widetilde{w}\,(s,t) = w\,(t)\left(1-\tau^l\,(s,t)\right)$ are the net-of-tax factor prices.

The FOCs of this problem imply:

$$U_{c(s,t)} = p\,(s,t) \tag{3}$$

$$U_{l(s,t)} = -p\,(s,t)\,\widetilde{w}\,(s,t) \tag{4}$$

$$-\left[\tilde{r}\,(s,t) - n\right] p\,(s,t) = \dot{p}\,(s,t) - (\beta - n)\,p\,(s,t) \tag{5}$$

where the expression $U_{i(t)}$ is the partial derivative of the utility function with respect to argument $i = c, l$ at time t and $p\,(s,t)$ is the current value shadow price of wealth. According to such conditions, it can be shown that the growth rates of consumption and labor are:

$$\frac{\dot{c}}{c}\,(s,t) = (\widetilde{r}\,(s,t) - \beta)\,\frac{1}{\theta_c} - \frac{\theta_{cl}}{\theta_c}\frac{\dot{l}}{l} \tag{6}$$

$$\frac{\dot{l}}{l}\,(s,t) = \frac{1}{\theta_l}\,\frac{\left[(\widetilde{r}\,(s,t) - \beta)\left(1 - \frac{\theta_{lc}}{\theta_c}\right) - \frac{\dot{\widetilde{w}}(s,t)}{\widetilde{w}(s,t)}\right]}{1 - \frac{\theta_{cl}\theta_{lc}}{\theta_c\theta_l}} \tag{7}$$

with $\theta_j = -\frac{U_{jj}j}{U_j}$, $j = c, l$, the elasticity of the marginal utility and $\theta_{ij} = -\frac{U_{ij}j}{U_i}$. Notice that, generally, the parameters θ can vary across dynasties; however, if the utility function is additively separable in consumption and labor, the growth rates above are: $\frac{\dot{c}}{c} = (\widetilde{r}\,(s,t) - \beta)\,\frac{1}{\theta_c}$ and $\frac{\dot{l}}{l} = (\widetilde{r}\,(s,t) - \beta)\,\frac{1}{\theta_l}$.

2.2 Firms

Under the assumption of perfect competition, in each instant firms, supposed to be identical, hire capital and labor services according to their market prices (gross of taxes) and in order to maximize current period profits. This means that, for each firm i:

$$\frac{dF\left(K^{i}\left(t\right),L^{i}(t)\right)}{dK^{i}\left(t\right)}=r\left(t\right) \tag{8}$$

$$\frac{dF\left(K^{i}\left(t\right),L^{i}(t)\right)}{dL^{i}\left(t\right)}=w\left(t\right). \tag{9}$$

Assuming a CRS technology, such conditions can also be expressed for the economy as a whole, in per capita terms:

$$f_{k(t)}=r\left(t\right) \tag{8'}$$

$$f_{l(t)}=w\left(t\right), \tag{9'}$$

where $l(t)=\frac{L(t)}{N(t)}=\int_{-\infty}^{t}\nu_{p}\left(s,t\right)l\left(s,t\right)ds$, in which $\nu_{p}\left(s,t\right)=\alpha e^{-\alpha(t-s)}$ is the weight of dynasty s in the whole population at period t.

2.3 The government and market clearing conditions

The government is assumed to finance an amount of exogenous public expenditure by levying taxes on capital and labor income and by issuing debt. In order to rule out the problem of time inconsistency, we suppose that the government has access to a commitment technology that ties it to the announced path of distortionary tax rates whenever the possibility of lump sum taxation arises[8]. Finally, the only constraint on the possibility of debt issuing is the usual no-Ponzi game condition[9]. Thus, one obtains the usual equation of the dynamics of the aggregate debt:

$$\dot{B}\left(t\right)=r\left(t\right)B\left(t\right)+G-T\left(t\right). \tag{10}$$

Finally, since the market clearing condition implies that, at each date, the sum of capital and debt equal the aggregate private wealth, that is:

$$A\left(t\right)=K\left(t\right)+B\left(t\right), \tag{11}$$

[8]In a dynamic setup, as far as capital income is concerned, there exists an incentive for the government to deviate from the announced (ex-ante) second best policy, upon achieving the instant in which it should be implemented; this is so because the stock of accumulated capital ex-post is perfectly rigid and now should be taxed more heavily than announced, since its taxation has a lump sum character. The commitment hypothesis implies also that the capital income tax at the beginning of the policy is given, that is, fixed exogenously at a level belonging to the $(0,1)$ interval.

[9]Namely: $\lim_{t\to\infty}B\left(t\right)e^{-\int_{0}^{t}r(v)dv}=0$, and the initial condition $B\left(0\right)=\overline{B}$.

then, eq. (10) can be also written as

$$\int_{0^-}^{t} \alpha e^{(\alpha s+nt)} \Big[\dot{b}(s,t) - (\widetilde{r}(s,t) - n)\, b(s,t) + \tau^{l}(s,t)\, w(t)\, l(s,t)$$

$$+\tau^{k}(s,t)\, r(t)\, k(s,t) - g\Big]\, ds = 0. \tag{12}$$

3 The Ramsey problem

Since we adopt the primal approach to the Ramsey (1927) problem, a key point is restricting the set of allocations among which the benevolent government can choose, to those that can be decentralized as a competitive equilibrium[10]. Thus, in this paragraph we define a competitive equilibrium and the constraints that must be imposed to the policymaker problem, in order to achieve such a competitive outcome.

The first constraint can be obtained as follows: first, by taking eq. (2) and multiplying both sides by $e^{-\int_s^t [\widetilde{r}(s,v)-n]dv}$, we can write the following expression:

$$\frac{d\left[a(s,t)\, e^{-\int_s^t [\widetilde{r}(s,v)-n]dv}\right]}{dt} = e^{-\int_s^t [\widetilde{r}(s,v)-n]dv} \left[\widetilde{w}(s,t)\, l(s,t) - c(s,t)\right];$$

next, by multiplying both sides by $p(s,t)$ and exploiting the individuals' FOCs (eqs. 3 to 5) we obtain:

$$\begin{aligned} p(s,s)\, e^{-\int_s^t [\widetilde{r}(s,t)-\beta]dv} \frac{d\left[a(s,t)\, e^{-\int_s^t [\widetilde{r}(s,t)-n]dv}\right]}{dt} &= \\ -e^{-\int_s^t [\widetilde{r}(s,t)-n]dv} \left[U_l(s,t)\, l(s,t) + U_c(s,t)\, c(s,t)\right] &\Rightarrow \end{aligned}$$

$$-U_c(s,s) \frac{d\left[a(s,t)\, e^{-\int_s^t [\widetilde{r}(s,t)-n]dv}\right]}{dt} = e^{-(\beta-n)(t-s)} \left[U_l(s,t)\, l(s,t) + U_c(s,t)\, c(s,t)\right];$$

[10] See Atkinson and Stiglitz (1980); on the other hand, the "dual" approach takes prices and tax rates as control variables (see Chamley (1986) and Renström (1999) for some examples).

finally, by integrating out and exploiting the individual's transversality condition, we get:

$$\int_s^\infty e^{-(\beta-n)(t-s)} \left[U_{c(s,t)}c(s,t) + U_{l(s,t)}l(s,t)\right] dt = a(s,s)\, U_{c(s,s)}. \qquad (13)$$

Since this constraint has to be satisfied for the whole economy, it must be

$$\int_{0^-}^t \int_s^\infty \alpha e^{nt} e^{\alpha s} \left\{ e^{-(\beta-n)(t-s)} \left[U_{c(s,t)}c(s,t) + U_{l(s,t)}l(s,t)\right] \right. \qquad (14)$$
$$\left. - e^{-(t-s)} a(s,s)\, U_{c(s,s)} \right\} dtds = 0,$$

which is referred to as the "implementability constraint"[11].

As for the second constraint, writing eq. (2) in the following way

$$\dot{a}(s,t) = [r(s,t) - n]\, a(s,t) + w(t)\, l(s,t) - c(s,t)$$

$$-\tau^k(s,t)\, r(t)\, a(s,t) - \tau^l(s,t)\, w(t) l(s,t), \qquad (15)$$

integrating over the population to get the aggregate wealth,

$$A(t) = \int_{0^-}^t a(s,t)\, \alpha e^{nt} e^{\alpha s} ds$$

then, deriving with respect to time, one gets:

$$\dot{A}(t) = \underbrace{a(t,t)\, \alpha e^{nt} e^{\alpha t}}_{=0} + \int_{0^-}^t \frac{d\left[a(s,t)\, \alpha e^{nt} e^{\alpha s}\right]}{dt} ds,$$

where $a(t,t)$ is the initial wealth of individuals, which is supposed to be zero.

The expression above can be written as:

$$\dot{A}(t) = nA(t) + \int_{0^-}^t \dot{a}(s,t)\, \alpha e^{nt} e^{\alpha s} ds, \qquad (16)$$

so that, including (15) into (16), we obtain:

[11] In the rest of the paper we assume for simplicity that $a(s,s) = \overline{a}$ is equal to zero for each dynasty.

$$\dot{A}(t) = nA(t) + r(t)A(t) - r(t)\int_{0^-}^{t} \tau^k(s,t)\,a(s,t)\,\alpha e^{nt}e^{\alpha s}ds + \qquad (17)$$
$$- C(t) + W(t) - \int_{0^-}^{t} \tau^l(s,t)\,w(t)l(s,t)\,\alpha e^{nt}e^{\alpha s}ds,$$

where $C(t)$ and $W(t)$ are aggregate consumption and gross aggregate wages, respectively. Note that the sum of the two integrals in eq. (17) is the total amount of revenues, $T(t)$.

Finally, recalling the law of motion of aggregate debt, exploiting the market clearing condition and substituting the expression for $T(t)$ of (10) into (17), we get:

$$\dot{K}(t) = r(t)K(t) + W(t) - C(t) - G, \qquad (18)$$

which can also be written as:

$$\int_{0^-}^{t} \alpha e^{\alpha s + nt}\left[\dot{k}(s,t) + (n - r(t))\,k(s,t) - w(t)\,l(s,t) + c(s,t) + g\right] ds = 0. \qquad (19)$$

Such expression is usually referred to as the "feasibility constraint".

We can now give the following definition:

Definition 1 *A competitive equilibrium is: a) an infinite sequence of policies* $\pi = \left\{\tau^k(s,t), \tau^l(s,t), b(s,t)\right\}_0^\infty$, *b) allocations* $\{c(s,t), l(s,t), k(s,t)\}_0^\infty$ *and c) prices* $\{w(t), r(t)\}_0^\infty$ *such that, at each instant* t*: b) satisfies eq.* (1) *subject to* (2)*, given a) and c); c) satisfies eq.* (8′) *and eq.* (9′)*; eqs.* (19) *and* (12) *are satisfied.*

Such allocations are often referred to as "implementable".

In the light of the definition given above, the following proposition holds:

Proposition 1 *An allocation is a competitive equilibrium if and only if it satisfies implementability and feasibility.*

Proof. The first part of the proposition is true by construction. The reverse (any allocation satisfying implementability and feasibility is a competitive equilibrium) is provided in Appendix A. ■

3.1 Solution

Supposing that the policy is introduced at the end of period t_0, the policy-maker's problem is the following:

$$\max_{\{c(s,t),l(s,t),k(s,t)\}_0^\infty} \int_{\max(s,t_0)}^{\infty} \int_{0^-}^{t} \mu_g(s,t)\, e^{-\gamma_g(t-\max(s,t_0))} U\left(c(s,t), l(s,t)\right) ds dt$$

$$sub \int_{\max(s,t_0)}^{\infty} \int_{0^-}^{t} \mu_p(s,t) \left\{ e^{-(\beta-n)(t-\max(s,t_0))} \left[U_{c(s,t)} c(s,t) + U_{l(s,t)} l(s,t) \right] + \right.$$

$$\left. - e^{-(t-\max(s,t_0))} a(s, \max(s,t_0))\, U_{c(s,\max(s,t_0))} \right\} ds dt = 0$$

$$\text{and } \int_{0^-}^{t} \mu_p(s,t) \left[\dot{k}(s,t) + (n - r(t))\, k(s,t) - w(t)\, l(s,t) + \right.$$

$$\left. - (\delta_r - \delta)(b(s,t) + k(s,t)) + c(s,t) + g \right] ds = 0, \ \forall t > t_0,$$

$$\lim_{t\to\infty} k(s,t)\, e^{-\int_{\max(s,t_0)}^{t} (\widetilde{r}(s,v)-n) dv} = 0, \quad a(s,t_0) \quad given, \ \forall s$$

where $\mu_g(s,t)$ and γ_g are the weight that the government attaches to the dynasty born in year s and the government discount rate, respectively[12], and $\mu_p = \alpha e^{\alpha s + nt}$ the size of dynasty s.

Now, by differentiating the feasibility constraint we get:

$$c(s,t) = -\dot{k}(s,t) - (n - r(t))\, k(s,t) + w(t)\, l(s,t) - g;$$

next, by substituting it into the problem, we get[13]:

$$\max_{\{l,k\}_{\max(s,t_0)}^\infty} \int_{\max(s,t_0)}^{\infty} \int_{0^-}^{t} \mu_g e^{-\gamma_g(t-\max(s,t_0))} U\left(c\left(k, \dot{k}\right), l \right) ds dt$$

$$sub \int_{\max(s,t_0)}^{\infty} \int_{0^-}^{t} \mu_p \left\{ e^{-(\beta-n)(t-\max(s,t_0))} \left[U_c c + U_l l \right] + \right.$$

$$\left. - e^{-(t-\max(s,t_0))} a(s, \max(s,t_0))\, U_{c(s,\max(s,t_0))} \right\} ds dt = 0.$$

[12]Note that, in principle, the former parameter may depend also on t. Moreover, we omit the government budget constraint since, by Walras' law, it is satisfied if the implementability and feasibility constraints hold.

[13]From now onward, we omit both the s and t indexes, when it does not generate ambiguity.

By applying the calculus of variations method, the problem can be stated as follows:

$$\max_{\{l,k\}_{\max(s,t_0)}^{\infty}} \int_{\max(s,t_0)}^{\infty} \int_{0^-}^{t} \left\{ \mu_g e^{-\gamma_g(t-\max(s,t_0))} U\left(c\left(k,\dot{k}\right),l\right) + \right.$$

$$\left. + \hat{\lambda}\mu_p \left[(U_c c + U_l l) - e^{(\beta-n-1)(t-\max(s,t_0))} a\left(s, \max\left(s,t_0\right)\right) U_{c(s,\max(s,t_0))} \right] \right\} dsdt$$

where $\hat{\lambda}$ is the current value multiplier associated to the implementability constraint, defined as $\hat{\lambda}(t) = \lambda e^{-(\beta-n)(t-\max(s,t_0))}$. Thus, the solution with respect to k is [14]:

$$e^{-\gamma_g(t-\max(s,t_0))} \left\{ U_c \mu_p \left[\frac{\mu_g}{\mu_p} + \overline{\lambda}\left(1+H_c\right) \right] \left[(r-n) - \gamma_g + \left(\frac{U_{cc}}{U_c}\dot{c} + \frac{U_{cl}}{U_c}\dot{l} \right) \right] + \right. \tag{20}$$

$$\left. + U_c \left[\dot{\mu}_g + (1+H_c)\left(\overline{\lambda}\dot{\mu}_p + \dot{\overline{\lambda}} \right) + \mu_p \overline{\lambda}\dot{H}_c \right] \right\} = 0$$

where $\overline{\lambda} = \hat{\lambda} e^{\gamma_g(t-\max(s,t_0))} = \lambda e^{-\left(\beta-n-\gamma_g\right)(t-\max(s,t_0))}$ and the term $H_i = \frac{U_{ii}i + U_{ji}j}{U_i}$ is what is usually referred to as the "general equilibrium elasticity". Now, by dividing expression (20) by $U_c\mu_p$, and rearranging terms, we get:

$$\frac{\dot{c}}{c} = \frac{1}{\theta_c} \left[\left(r-n-\gamma_g\right) + n \frac{\left[\frac{\dot{\mu}_g}{n\mu_p} + \overline{\lambda}\left(1+H_c\right) \right]}{\left[\frac{\mu_g}{\mu_p} + \overline{\lambda}\left(1+H_c\right) \right]} + \right. \tag{21}$$

$$\left. - \left(\beta-n-\gamma_g\right) \frac{\overline{\lambda}\left(1+H_c\right)}{\left[\frac{\mu_g}{\mu_p} + \overline{\lambda}\left(1+H_c\right) \right]} + \frac{\overline{\lambda}\dot{H}_c}{\left[\frac{\mu_g}{\mu_p} + \overline{\lambda}\left(1+H_c\right) \right]} - \theta_{cl}\frac{\dot{l}}{l} \right].$$

Substituting for the growth rate of consumption stemming from the private agents optimization condition (eq. (6)), we get the expression for the optimal capital income tax:

[14] See Appendix B for the solution conditions of this problem. Note that the interiority of the solution is guaranteed by the Inada conditions. However, the FOCs are necessary but not sufficient due to the possible non convexity of the implementability constraint. The solution for l is omitted for brevity.

$$\tau^k = \frac{1}{f_k}\left\{\left(\gamma_g - (\beta - n)\right) + \frac{\left(\beta - n - \gamma_g\right)\overline{\lambda}\left(1 + H_c\right) - n\left[\frac{\dot{\mu}_g}{n\mu_p} + \overline{\lambda}\left(1 + H_c\right)\right] - \overline{\lambda}\dot{H}_c}{\left[\frac{\mu_g}{\mu_p} + \overline{\lambda}\left(1 + H_c\right)\right]}\right\}. \tag{22}$$

4 Discussion of the results

Before commenting the results, in both the short and the long run, it is worth noting that eq. (22) does not deliver an explicit formula for τ^k, since H_c depends upon the tax rate itself[15].

Next, there are four independent forces determining the level of τ^k: 1) the dynamics of H_c $\left(\dot{H}_c\right)$; 2) the difference between the social intergenerational weight (μ_g) and the one corresponding to the size of each dynasty (μ_p); 3) the difference between the government (γ_g) and individual ($\beta - n$) intertemporal discount factors; 4) the "disconnection" between dynasties.

Since the first three factors have already been widely discussed in the literature, we emphasize the role of the last one[16], i.e. of the disconnection in the economy, along with the third one.

We can now state the following proposition:

Proposition 2 *If the economy converges to a steady state, along the transition path, for $t > 0$, the tax on capital income is in general different from zero unless $\mu_g = \mu_p$, $n = 0$ and $\gamma_g = \beta$.*

Proof. The proof is straightforward by inspection of eq. (22), which, when the equality $\mu_g = \mu_p$ is satisfied, so that $\frac{\dot{\mu}_g}{\mu_p} = n$, becomes:

$$\tau^k = \frac{1}{f_k}\left[\frac{\left(\gamma_g - \beta\right) - n\overline{\lambda}\left(1 + H_c\right)}{1 + \overline{\lambda}\left(1 + H_c\right)}\right];$$

[15] Moreover, we do not have any condition ensuring that the tax rate will be in the $(0, 1)$ interval, while we would suspect capital taxes to get sticking at the interval boundary for a (finite) period of time since the introduction of the policy. However, in the rest of the work we maintain the assumption of interiority of the equilibrium tax rates.

[16] In particular, we rule out the first one by assuming that the utility function is homothetic in consumption and (weakly) separable in consumption and leisure (so that $\dot{H}_c = 0$). As for factor 2), it is sufficient to note that the difference between the social weight on different generations and their actual demographic weight constitutes an additional reason for capital taxation, though stemming from equity rather than efficiency considerations.

note that this expression, if $\gamma_g = \beta$, is zero only if the fertility rate, n, is equal to zero. Note also that, if $\mu_g = \mu_p$ (and $\dot{H}_c = 0$), then optimal taxes will also be constant through dynasties. ■

It is worth recalling that, when the conditions above apply (and given that $\dot{H}_c = 0$), the economy mimics the behavior of an ILRA one, so that the zero tax result applies along the transition path.

We now focus on the steady state path, whose properties can be summarized in the following proposition:

Proposition 3 *If the economy converges to a steady state, at such steady state the capital income tax is different from zero unless a) $\mu_g = \mu_p$, $n = 0$ and $\gamma_g = \beta$ or b) $\gamma_g > \beta$ and $n = 0$.*

Proof. To better understand the implications of the model, we distinguish three cases, according to whether the policymaker discount rate γ_g is equal, higher or lower than the individual one.

1. $\gamma_g = \beta - n$. In this case $\overline{\lambda} \to \lambda$, so that $\tau^k = -\frac{n}{f_k}\left\{\frac{\frac{\dot{\mu}_g}{n\mu_p}+\lambda(1+H_c)}{\frac{\mu_g}{\mu_p}+\lambda(1+H_c)}\right\}$; moreover, in case $\mu_g = \mu_p$, τ^k is negative and equal to -$\frac{n}{f_k}$.

2. $\gamma_g > \beta - n$. In this case $\overline{\lambda} \to \infty$, and, again, $\tau^k = -\frac{n}{f_k}$, (provided that $\dot{\mu}_g$ does not tend to infinity).

3. $\gamma_g < \beta - n$. $\overline{\lambda} \to 0$ and $\tau^k = \frac{1}{f_k}\left[\left(\gamma_g - (\beta - n)\right) - \frac{\dot{\mu}_g}{\mu_p}\right]$. Moreover, if $\mu_g = \mu_p$, $\tau^k = \frac{1}{f_k}\left(\gamma_g - \beta\right)$, which is necessarily negative, since $\gamma_g < \beta - n$ implies $\gamma_g < \beta$.

■

The economic meaning of these results can be grasped by reckoning that the capital income subsidy stems from the difference between the private agents and government optimal rates of consumption growth, i.e. eq. (6) and eq. (21), respectively. This is due to the fact that private agents maximization disregards the welfare of future dynasties of immigrants, while the government takes it into account. In fact, even in the case $\mu_g = \mu_p$ and $\gamma_g = \beta - n$, eq. (21) displays an extra term with respect to eq. (6), i.e. $\frac{n}{\theta_c}$, because of which $r < \widetilde{r}$. This extra term depends crucially on the presence of migration, since it would disappear when $\alpha = 0$: in that case, we would simply have an economy with infinitely lived families and without an overlapping generation mechanism. Therefore, the subsidy to capital income derives from the disconnection generated by the entrance of new individuals who are not linked to residents. Disregarding the welfare of future immigrants' dynasties makes private agents choose a growth rate of consumption, that is a rate of capital accumulation, that would be, in the absence of capital income taxation and *ceteris paribus*, lower than the socially optimal one. Thus the subsidy to capital, i.e. to future consumption, is the appropriate corrective instrument.

Finally, it is worth noting that the zero tax result, when $n > 0$, never comes out to be optimal in this model.

5 Conclusions

We reconsider the issue of optimal capital income taxation in an economy with dynastic altruism and migration and by applying the primal approach to the Ramsey problem.

The thrust of the paper is that, as long as the migration and fertility rates are strictly positive, the Chamley-Judd rule comes out to never apply, in that several forces are at work leading to a non zero tax rate, in both the short and the long run.

Namely, we unveil the presence of four forces: a) the dynamics of the general equilibrium elasticity of consumption (H_c); b) the difference between the weight the government attaches to each generation and its actual demographic size; c) the difference between the government and individual intertemporal discount rates; d) the disconnection between dynasties.

While the first three factors have already been discussed in the literature, the last one is new and can be explained as follows: the combination of limited altruism and migration generates a disconnection in the economy which gives room to corrective taxation, even at the steady state. More precisely, while in the short run both positive or negative taxation can apply, as for the long run, since the welfare of future migrants is not taken into account by current residents, it is optimal for the government to currently subsidize capital income, that is future consumption, so as to increase the current consumption growth rate.

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6 Appendix A: Proof of Proposition 1

Proof. Since a competitive equilibrium (or implementable allocation) satisfies both the feasibility and the implementability constraints by construction, in this Appendix we demonstrate the reverse of Proposition 1: any feasible allocation satisfying implementability is a competitive equilibrium.

Suppose that an allocation satisfies the implementability and the feasibility constraints. Then, define a sequence of after tax prices as follows: $\widetilde{w}(s,t) = -\frac{U_{l(s,t)}}{U_{c(s,t)}}$, $\tilde{r}(s,t) = \beta - \frac{\dot{p}(s,t)}{p(s,t)}$, with $p(s,t) = U_{c(s,t)}, \forall s$ and $\forall t$, and a sequence of before tax prices as: $f_{k(t)} = r(t)$ and $f_{l(t)} = w(t)$. As a consequence, by construction such allocation satisfies both the consumers' and firms' optimality conditions.

The second step is to show that the allocation satisfies the consumer budget constraint. Take the implementability constraint and substitute $U_{c(s,t)}$, $U_{l(s,t)}$ by using the expressions above:

$$\int_s^\infty e^{-(\beta-n)(t-s)} \left[p(s,t)\, c(s,t) - \widetilde{w}(s,t)\, p(s,t)\, l(s,t)\right] dt = a(s,s)\, p(s,s), \quad \forall s$$

then, by exploiting the expression for $\dot{p}(s,t)$ we get[17]:

$$\int_s^\infty p(s,s) e^{-(\beta-n)(t-s)} e^{-\int_s^t [\widetilde{r}(s,v)-\beta]dv} \left[c(s,t) - \widetilde{w}(s,t)\, l(s,t)\right] dt = a(s,s)\, p(s,s).$$

[17] The equations below hold $\forall s$.

Finally, by eliminating $p(s,s)$ and defining $c(s,t) - \widetilde{w}(s,t)\, l(s,t) = \dot{q}(s,t) - \widetilde{r}(s,t)\, q(s,t)$ we get:

$$-\int_s^{\infty} \frac{d\left[q(s,t)\, e^{-\int_s^t [\widetilde{r}(s,v)-n]dv}\right]}{dt} dt = a(s,s)$$

which holds if $q(s,t) = a(s,t)$ and $\lim_{t\to\infty} a(s,t)\, e^{-\int_0^t [\widetilde{r}(s,v)-n]dv} = 0$.

Finally, as for the public sector budget constraint, by substituting the expression for consumption obtainable by the individual budget constraint into the feasibility constraint, we get:

$$\int_{0^-}^t \alpha e^{\alpha s+nt} \left[\dot{k}(s,t) - (r(t)-n)\, k(s,t) - w(t)\, l(s,t)\right.$$
$$\left. -\dot{a}(s,t) + (\widetilde{r}(s,t)-n)\, a(s,t) + \widetilde{w}(s,t)\, l(s,t) + g\right] ds = 0.$$

Finally, by defining $b(t) = k(t) - a(t)$ and exploiting the definition of taxes, the previous expression becomes:

$$\int_{0^-}^t \alpha e^{\alpha s+nt} \left[\dot{b}(s,t) - (\widetilde{r}(s,t)-n)\, b(s,t) + \tau^l(s,t)\, w(t)\, l(s,t)\right.$$
$$\left. +\tau^k(s,t)\, r(t)\, k(s,t) - g\right] ds = 0,$$

which is eq. (12) in the text. ■

7 Appendix B: The "calculus of variations" method

We now sketch the strategy adopted for solving the Ramsey problem presented in Section 3.1.

Following Kamien and Schwartz (1991), suppose the problem has the form

$$\max \int\int F(t,s,x(t,s), x_t(t,s), x_s(t,s))\, dsdt$$

where the symbol x_y indicate the partial derivatives of variable x with respect to y (x can be also a vector of variables). The Euler equation for such a problem is the following:

$$F_x - \partial F_{x_t}/\partial t - \partial F_{x_s}/\partial s = 0.$$

Moreover, in case the problem contains also a (double) integral constraint, such as:

$$\int\int q\left(t,s,x\left(t,s\right),x_{t}\left(t,s\right),x_{s}\left(t,s\right)\right)dsdt=0,$$

this constraint can be appended to the integrand with a multiplier function $\lambda\left(t,s\right)$, so that, if the solution x^{*} maximizing F subject to the constraint does exist, then there is a function $\lambda\left(t,s\right)$ such that x^{*} satisfies the Euler equations for

$$\int\int\left(F+\lambda q\right)dsdt.$$