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THE SOCIAL ACCOUNTING MATRIX (SAM): A FRAMEWORK FOR BUILDING INEQUALITY "STRUCTURAL INDICATORS" FOR ANALYSING THE INCOME DISTRIBUTION

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The Social Accounting Matrix (SAM): a framework for building inequality "structural indicators" for analysing the income distribution.

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1. Introduction

According to Maastricht agreement, the reduction of the public debt/PIL ratio, the control of current deficit and of the inflation ratio, the reduction of the fiscal pressure were the main goals of any macroeconomic policy in the European countries. Other important goals, as the reduction of income inequality or the eradication of any discrimination in the labour market, were not explicitly indicated. The Lisbon summit and, a few months later, the European Council in Nice (December 2000) represent a real change in policies assessment. The European Union has formulated the ambitious goal to transform itself into "the most competitive and dynamic knowledge-based economy in the world capable of sustainable economic growth with more and better jobs and greater social cohesion" [1].
The "distribution of the aggregate among individuals may be as important from the welfare point of view as the aggregate itself"[2]. This change of goals and policies need a change also in the socio-economic indicators with a shift from macro variables to structural ones. For the first time seven indicators of social cohesion have been considered as important components of a system of benchmarking[3]. In particular, they include a measure of inequality in the personal income distribution, a measure of poverty before and after social transfers, a measure of long-term unemployment. These indicators should be calculated and published regularly from now on by all the countries of EU[4].
The new problems to be faced are of different nature. First of all it is important to find some common method of obtaining data and building indicators. In Europe there has been a big effort in harmonisation of national accounts, but a still too little effort in the social indicators field. "The indicators selected should not be seen in isolation but rather as different elements of the same picture"[5]. For analysis and/or benchmarking exercise, the meaning of the structural indicators could improve if they were obtained, necessarily in a synthetic way, as the result of an integrated system.

This paper was presented in a parallel session of the International Workshop *Income Distribution and Welfare*, Università Bocconi, Milano, May 30th, 31 th and June 1st, 2002. It is the result of the research *Schema di integrazione dei conti nazionali nella SAM con dati socio-economici* undertaken under the project *Strumenti di valutazione delle politiche distributive* financed by the C.N.R.

[1] See: Lisbon Summit, 2000. Com (2000) 594 final, Com (2001) 313 final.
[2] See: United Nations (1977).
[3] As a matter of fact, a set of 35 so-called Structural Performance Indicators (SPI) have been calculated for the first synthesis report, presented early 2001. At present, there are several initiatives under way to improve and extend the SPI. For a discussion on this point see: Keuning, Verbruggen (2001).
[4] "The result thus far is a rather incoherent shopping basket with numbers" from different and rarely comparable sources. See: Keuning, Verbruggen (2001), pag. 2.
[5] See: Com (2000) 594 final. The Economic and Social Committee on the SPIs formulated this goal even more explicitly: "The Committee would also stress that, in addition to the high standards of reliability, topicality and uniformity required for each of the proposed indicators, it will be necessary to interpret the indicator-based figures in order to assess progress made in economic, social and structural policy. This can be done with the help of an underlying statistical information system, so that in interpreting the data account is taken of e.g. the economic and demographic characteristics of a Member State".(CES 241/2001).

But, above all, it is very important to learn how to link different indicators each other and with the policy instruments. The present national accounts are inadequate, for instance, to assess the effects of fiscal or monetary policy on personal income distribution. For this purpose it is necessary to build a system in which the information on production, intermediate and final demand and income distribution, between and inside different Institutions (Households, Private Companies, Government, Rest of the World), are integrated. The Social Accounting Matrix (SAM) is the schema for this goal. A SAM is basically a matrix combining in an accounting framework the flows in value of an economic system and showing, in addition, for all transactions, who pays what to whom. The elementary flows, which interrelate the economic units aggregated at different level, are the starting point.
The inclusion in the SAM of data related to the production side (included data on labour market) and of data related to the income distribution and to consumption expenditure allows to built not only a coherent system of socio-economic data, but also to built a set of indicators in an innovative way. This integrated set is very important if we want to assess the progress of European countries with reference to some socio-economic indicators, as these related to well being (structure of consumption) and to social cohesion (employment structure).
Aim of this paper is to show how and why it is possible to consider the SAM not only as a database and as an accounting tool, but also in a wider sense, as a macroeconomic model. Under this respect the SAM must be included in the macroeconomic tradition on which the National Accounts are based. The approach we propose, however, differs from the one suggested by the SNA93 and the SEC95, mainly based on accounting principles, and can be used both for structural analysis and for fiscal and expenditure policies simulations[6].
Firstly we will suggest alternative criteria of grouping different households units in order to build consumption functions and get income multipliers that are meaningful from an economic point of view, according to the traditional Keynesian approach. In particular, we will discuss alternative criterion based on income sources and on income levels. This households grouping taxonomies is different from the one suggested by SEC, which is based on a functional-type criterion as the one of the main income source.
Secondly, the paper aims to build a non-conventional "structural" measure of the inequality in income distribution, following the Pyatt and Round's decomposition method of multipliers matrix[7]. This decomposition allows measuring the change in income, earned by different households groups, affected by an exogenous change in the income of other endogenous accounts (Factors, Activities, Institutions), which are included in the SAM.

2.The Social Account Matrix (SAM).

The SAM captures and shows the entire circular flow of income from its production to its distribution and its expenditure. In the original formulation, presented by Brown and Stone in the sixties, this schema can be considered as an analytical presentation of the traditional Keynesian model[8]. Most of the SAM, in particular, has been oriented towards an analysis of linkages between structural features of an economy and the distribution of incomes and expenditures among households groups.

[6] See: United Nations (1993), Eurostat (1995).
[7] See: Pyatt and Round (1979).
[8] The first SAM has been built by Brown and Stone as a tool for building and simulating the first version of the model related to the "Cambridge Growth Project". See Cambridge University Press (1962-1974), and also: Stone (1951-52, 1962, 1986).

The SAM can be considered as an extension of the traditional input-output framework. This format adds some matrices, not included in the Leontief schema, which allow taking in account of the relationships between factorial distribution of income, income distribution to Institutions and final demand.The introduction of accounts referred to Institutions (Households, Private Companies, Government, Rest of the World) allows capturing the link between factors of production and the Institutions, which own the different factors of production. The secondary distribution of income is also introduced as the result of transfers between different Institutions, mainly between private Institutions and the Government. The disposable income of Institutions is the starting point for sustaining the final demand. In particular the Household, grouped in different socio-economic groups, sustain the demand for consumption. The amount of income, which is not consumed in the current year, is saved and goes into the capital account.

In the SAM the values flows of transactions of an economic system are organised in an accounting way starting from elementary flows which link the economic units at different level of aggregation. "A matrix framework is even optimally suited, as it allows for multiple acting, i.e. distinguishing more than one type of unit within a single accounting system, and multiple sectoring, i.e. distinguishing more than one classification of units within a single accounting system"[9].

Figure 1- A simplified SAM.

[9] See: Keuning S.J. (1994), pag. 22. The advantages of multiple sectoring were spelled out by Stone more than 30 years ago (1962).

	Expenditures	Activity	Factors	Institutions			Saving/ Invest.	Indirect Taxes	Rest of the World	Total
	Entries			Household	Companies	Government				
	Activity	$T_{1,1}$	0	$T_{1,3}$	$T_{1,4}$	$T_{1,5}$	$T_{1,6}$	0	$T_{1,8}$	y_1
	Factors	$T_{2,1}$	0	0	0	0	0	0	$T_{2,8}$	y_2
INS	Household	0	$T_{3,2}$	$T_{3,3}$	$T_{3,4}$	$T_{3,5}$	0	0	$T_{3,8}$	y_3
TI	Companies	0	$T_{4,2}$	$T_{4,3}$	$T_{4,4}$	$T_{4,5}$	0	0	$T_{4,8}$	y_4
TU	Governm.	0	$T_{5,2}$	$T_{5,3}$	$T_{5,4}$	$T_{5,5}$	0	$T_{5,7}$	$T_{5,8}$	y_5
TI	Saving/Inv	0	0	$T_{6,3}$	$T_{6,4}$	$T_{6,5}$	0	0	$T_{6,8}$	y_6
ONS	Indirect Taxes	$T_{7,1}$	0	$T_{7,3}$	$T_{7,4}$	$T_{7,5}$	0	0	0	y_7
	Rest of the World	$T_{8,1}$	$T_{8,2}$	$T_{8,3}$	$T_{8,4}$	$T_{8,5}$	$T_{8,6}$	0	0	y_8
	Total	y'_1	y'_2	y'_3	y'_4	y'_5	y'_6	y'_7	y'_8	

The SAM, in his matrix form, must satisfy some principles. First of all the basic national accounting principle of balance between entries and expenditures must be satisfied. The crossing of the account on the row *j* and the account on the column *k* is the value of monetary transactions between them. Each transaction is an exit (cost) for the column account and an entry for the row account.

The choice of different types of Institutions must be done taking in account a homogeneity principle based on the nature and the economic behaviour of the unit. The row and column accounts refer to production activities, to factors of production (different types of labour, capital, natural resources), to private and public Institutions and, finally to capital/saving account. The link between the production side and the Institutions is the innovative and the most important feature of the SAM in comparison to the traditional input-output framework and the SNA (t-accounts).

Formally the SAM is a square matrix

$$\mathbf{T}=[t_{jk}] \qquad [1]$$

Each pair row-column represents the accounting system of the any single unit and it is balanced so that each total row is equal to the corresponding total column.

$$\mathbf{Te}=\mathbf{y}=\mathbf{T'e} \qquad [2]$$

e is the unity vector so that the element *j* of vector **y** is both the total revenue and the total expenditure of the *j* account.

The choice of the numbers of accounts depends on the goals of the analysis and on the availability of statistical data. The flexibility of the SAM allows choosing the disaggregation more suitable. In the figure 1, a simplified SAM shows the main links between the various accounts. The three accounts of the internal Institutions present only the flows of the current side, while the capital flows are aggregated in a single account (column and row 6). The flows of Rest of the World are not disaggregated.
Each cell can be a vector or a matrix, not only a scalar. Of course its value is zero in case of no transactions between the two accounts.
The rows show the equilibrium conditions of each unit (Activities, Factors or Institutions) of the economic system. The first row shows the traditional Keynesian identity between aggregated supply (vector $\mathbf{y_1}$) and aggregated demand divided in intermediate ($\mathbf{T_{11}}$), final demand for consumption of the Households ($\mathbf{T_{13}}$) and final demand of other Institutions.
The second and third row (vectors $\mathbf{y_2}$ and $\mathbf{y_3}$) refer to process of generation, distribution and redistribution of income to the Households. In a first phase the value added is generated and then distributed to the *M* factors of production in relationship to their use in the *S* sectors of activities ($\mathbf{T_{21}}$) or outside of the economic system ($\mathbf{T_{28}}$). The second column account assesses the passage from the factorial to the personal income distribution of the Institutions. In particular matrix $\mathbf{T_{32}}$ shows the passage of income from the factors of production to the Households depending on the ownership of factors by each of the *H* socio-economic group.
Matrices $\mathbf{T_{33}}$, $\mathbf{T_{34}}$, $\mathbf{T_{35}}$, are related to the moment of redistribution of income between Households, from the Companies (interest and dividends) and from the Government (positive monetary transfers, negative monetary transfers following the payment of social contributions, and of direct and indirect taxes). The matrix $\mathbf{T_{38}}$ takes in account the redistribution process from the Rest of the World.
In an analogous way the fourth and fifth row represent the primary and secondary income distribution of the Companies and of the Government. Sixth row refers to the accumulation of capital for the economic system. The matrices at the crossing between the columns of current expenditures of the Institutions represent the saving of each households group (matrix $\mathbf{T_{63}}$), the undistributed profits (matrix $\mathbf{T_{64}}$) and the saving of Government (matrix $\mathbf{T_{65}}$). The matrix $\mathbf{T_{68}}$ represents the net capital from the Rest of the World. The eighth row, finally, refers to the Rest of the World account.

3. The SAM and the SNA93.

For many years the empirical work of building a SAM, and using it as a model, has been done by some International Organisations as ILO and the World Bank with reference to developing countries[10]. Only recently, the Statistical Offices of United Nations, with the revised System of National Accounts, and the European Union have introduced (SNA93 and SEC95) the rules for building a SAM also for the industrialised countries. In particular, the SNA93 contains a chapter on SAM demonstrating that the input-output approach should be extended to a matrix presentation of a wider set of national accounts[11].

[10] It started as an analytical framework for studying possible trade-offs between growth and equity in developing countries, and gradually evolved to a more general statistical information system for socio-economic policy analysis. See: Pyatt and Thorbecke, (1976), Pyatt and Round (1985), Dervis, De Melo, Robinson (1982), Robinson (1989), Bottiroli Civardi (1988), Caroleo (1989),

[11] The concept of a SAM is elaborated in Chapter XX of the 1993 SNA (United Nations, 1993) and in Chapter VIII of the 1995 European System of National and Regional Accounts (Eurostat, 1995). See also Keuning, de Gijt (1992). At present, a SAM for Europe is developed in a so-called Leadership Group (LEG). A LEG is a task force in which Eurostat co-operates with member states to develop new statistics. LEG's are always co-ordinated by a national statistical institute, in this case Statistics Netherlands. The other member states participating are: Belgium, Finland, Greece, Italy, Norway, Portugal and the United Kingdom. The objective of the LEG is to compile a first draft of a fairly simple and straightforward handbook on the

The use of a SAM for estimating the national accounts is in fact a logical step forward from a method based on supply and use tables (the so-called t-accounts) or on input-output tables. Those tables result from a quite detailed application of the production and expenditure approaches to the estimation of GDP, but they incorporate the income approach only at a very aggregate level. This bias is remedied in the SAM approach[12].

Recording the interactions between economic processes that involve different types of agents requires a matrix format. The alternative, double-entry bookkeeping, serves well in presenting all transactions plus a balance sheet for a single agent (as in company accounts) or group of agents, but is less suitable for portraying the links among transactions of different types of agents[13]. An additional advantage of the matrix format is that the whole accounting system can then be portrayed and analysed by means of a single table, unlike t-accounts. Because of its matrix format, the SAM also elaborates the national accounts "information on income and expenditure distributions"[14].

The revised SNA93 takes in account of different steps, which bring to the production and the distribution of income. The 1993 SNA's central framework, however, is still a hybrid between supply and use tables (input-output) and t-accounts (for the institutional sectors). Also if the Matrix of National Accounts (NAM) translates the traditional accounts in a matrix format, the transactions of the Institutions, however, are registered in more than one account[15].

Following the SNA93 approach, it is possible to obtain a very important accounting tool, but we loose the main feature of the original SAM[16]. As a matter of fact we obtain disaggregated accounts in a matrix form, but we cannot represent primary and secondary income distribution as a circular flow. The relationships between Institutions, and not only disaggregated flows, bring to a SAM, which is and can be used as a real macroeconomic model. In order to simulate the effects of different policies on personal income distribution it is necessary not only to capture the links between different Institutions but also to assume that the different accounts refer to units whose behaviour is meaningful from the economic point of view.

An extension of the central framework of the 1993 SNA is the so-called System of Economic and Social Accounting Matrices and Extensions (SESAME). SESAME is a "detailed statistical information system consisting of sub-modules in matrix format, from which a set of core economic, social and environmental macro-indicators is derived….Consistent indices covering distributional aspects can also be derived for all variables included in the SESAME, because the system registers both the national total value and its distribution among socio-economic households groups and categories of employed persons"[17]. "SESAME is built on the same principles as the national accounts system. Key-indicators are defined and registered within the system in a way that allows information to be extracted at different levels of aggregation: a so-called information-pyramid". Every indicator is obtained from a statistical information system in which single values must be organised in a fully consistent way. Single indicators can be interlinked, through underlying, more detailed accounts. In other words,

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compilation and use of SAMs, taking into account users' priorities, available data sources and staff limitations in all Member States. For all participating countries pilot SAMs should become available. The SAM is already an integral part of the ESA-regulation so that no new regulation is required for its compilation. The work of this LEG is scheduled to be completed in the course of 2002. See: Keuning (2001), pag.10.

[12] According to Keuning, "A Social Accounting Matrix (SAM) is basically a matrix that combines the national accounts (showing, in addition, for all transactions who pays what to whom; cf. flow-of-funds matrices) with detailed labour accounts (both earnings, employment and average wage rate), by industry, by type of labour (male/female, skill level, etc.) and by household subsector". See: Keuning (2001), pag. 9.

[13] For a discussion on the links between t-accounts and the SAM approach see Pyatt (1999).

[14] See: Keuning (2001), pag. 9.

[15] See Battellini, Caricchia, Coli (1997), pag. 8.

[16] See: Ferrari (1998), pag. 4.

[17] See Keuning (2001), pag.6. During the past years, Statistics Netherlands has gained considerable experience with the development of modular sub-systems in most of the policy domains covered by the Structural Indicators. See Kazemier, Keuning and Van de Ven (1998).

there exists a consistent inter-relationship between key-indicators and the information system, which considerably enhances its analytical power[18]. SESAME however is mainly an accounting tool suitable for obtaining structural indicators, but it cannot become a model for simulating the effects of macroeconomic policies.

4. The distribution of income and the choice of socio-economic groups in the Institution Households.

The choice of number and of type of Institutions, and mostly the choice of socio-economic groups of the Institution Households is one of the more important steps of the SAM building process. "Indeed, every institution must be recognised somewhere within the SAM and the modeller's discretion is limited to deciding on the amount of detail that is retained or, alternatively, on the level of disaggregation"[19].
The choice of the single units, the estimate of data (often by survey), the inclusion or exclusion of some variables depend on the researcher goals. These choices are influenced by an underling specific theory. Keynes observed that empirical research "is full of theory"[20].
The accounting principles under the SNA followed, at least at the beginning, the macroeconomic Keynesian approach. Social accounting and modelling are "inextricably interwoven[21]".
The data we collect are the result of decisions of economic units which follow believes and goals[22]. With reference to the Institution Households the classification of different socio-economic groups should be meaningful and homogenous from the point of view of the process of generation and distribution of income. From one side the choice is influenced by the need to obtain meaningful indicators of inequality in personal income distribution and/or in the level of poverty. From the other side the socio-economic groups should correspond to homogenous behaviour from the consumption behaviour. The two criteria, the earning and expenditure side are not always the same.
The links between ownership of the factors of production and the generation and distribution of income to the Institutions is the innovative feature of the SAM. With reference to the Households, the matrix $\mathbf{T_{32}}$ represents this link (Figure 1). The matrix $\mathbf{T_{32}}$, and its meaning, is very important not only from the accounting but also from a theoretical point of view. The variables we choose and the links between them depend on the economic theory of the personal income distribution, which is assumed to underline the economic process and consequently the SAM model.
The main distinctive assumption refer to the saving behaviour of workers, and consequently on their ownership of different factors of production. In a socialist society or in "a Ricardian world of rentiers, who own everything, and tenants, who work for a subsistence wage that precludes them from saving" the factorial distribution is the only one that is considered[23]. On the opposite, if we assume that also workers save and consequently own capital assets, we must model the passage from factorial to Institutional distribution that is the "distribution of income among the various institutions of which the economy is comprised"[24].
With reference, in particular, to the Institution Households we can introduce a very simple relationship between human and physical endowments and the income of each individual

[18] For a more extensive description and applications of this system, see Keuning (1996).
[19] Pyatt (2001), pag. 146.
[20] See De Vecchi (1999), pag 3.
[21] See Pyatt , (2001), pag. 142.
[22] See De Vecchi (1999), pag.1.
[23] For a discussion on this point see: Pyatt (2001), pag 140.
[24] See Pyatt (2001), pag, 139.

and/or household. For any *h* household, the level of income earned in the production activities in every period can be expressed as:

$$y_h = f(c_u, c_p) \qquad [3]$$

Where c_u e c_p refer to the ownership from the household *h* of human capital, from which income of dependent (wages and salaries) and independent work are obtained, and the ownership of physical capital, from which capital income (profits, rents and interests) are obtained. The function *f* is the "income generation function" which transforms personal endowments in personal income given the technologies, the social, institutional and market rules[25].

The matrix $\mathbf{T_{32}}$ can be considered as the product of two matrices:

$$\mathbf{T_{32}} = \mathbf{F.Y} \qquad [4]$$

Which can be defined as:

$$\mathbf{Y} = \begin{matrix} \mathbf{Y^d} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Y^i} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Y^c} \end{matrix} \qquad \mathbf{F} = \begin{matrix} f^{C_{u_1}} \ldots \ldots f^{C_{p_1}} \\ . \quad . \quad . \quad . \\ f^{C_{u_h}} \ldots . f^{C_{p_h}} \\ . \quad . \quad . \quad . \\ f^{C_{u_H}} \ldots . f^{C_{u_H}} \end{matrix}$$

The Households income distribution (matrix $\mathbf{T_{32}}$) can be considered as the result of the linkages between the factorial income distribution (matrix **Y)** and the structure of the ownership of the factors by the Household. The matrix **Y** is a block diagonal matrix whose elements are the total amount of value added earned by different factors of production as dependent workers, independent workers, and physical capital. This matrix **Y** depends on the macroeconomic variables, which influence the factorial income distribution[26]. Each element $f_h^l = y_h^l / Y^l$ of matrix **F**, instead, represents the share of each of the *H* groups of Households to each type of income (wages, salaries, income from autonomous work) according to the different ownership of human and physical capital. These values can be considered as the result of all that variables that influence the earning capability of people as native or acquired personal abilities, level of education, age, and ownership of physical capital. In other words it depends on all the variables that are considered as behind the personal distribution of income in the traditional theories.

"If factor markets are "fair" in the sense that a factor of production receives the same remuneration irrespective of who owns it (a law of one price in factor markets)" then the structure of the matrix $\mathbf{T_{32}}$ will be determined by the distribution across households of the ownership of all the endowments (human and physical capital) that provide factor services"[27].

[25] This income generating function is very similar to the one discussed in Dagum (1980).

[26] For a discussion on this point see: Targetti Lenti (1984).

[27] Pyatt (2001), pag.145.

To assume that matrix $\mathbf{T_{32}}$ is the result of matrix **Y** and **F** allows us to separate two components in the process of the primary distribution of income to the Institution Household. This step is very important in order to capture the different effects on income distribution of changes in the market and/or in public policies.
First of all the income distribution is influenced by a change in the factorial distribution. This happens, for instance, when the technological processes change in the sectors of production. These are macroeconomic variables out of the control of single individuals and/or household. The income distribution can change, however, also because of a change in the ownership of the different factors of production by each group of Households.

5. The multiplier model.

The choice of the level of disaggregation and of the type of socio-economic groups depends above all on the goals of the researcher. If the goal is mostly to build an analytical tool from which to obtain indicators of the labour market, of the employment and unemployment structure, the classification can be based on the "prevailing income" in the household. The classification suggested by the SNA93 and the SEC95 is: blue and white collars, independent workers (employers and autonomous workers), people who has only rent and profits from capital, pensioners and people perceiver of other transfers, others (people who live in a Community)[28]. In the SAM built for the Netherlands, for instance, the Institution Households is divided in 14 socio-economic groups according to the type of income, to the households structure and to the sector of activity[29].
This kind of classification allows showing very well the links between primary distribution of income and structure of employment and/or of the production technology. This classification should allow catching up the effects of policies (fiscal and/or monetary), the level of competitiveness of the system and the equilibrium values in the labour market. For this goal we should disaggregate some groups as blue and white collars according to the level of education, age, sex, and localisation[30].
This taxonomy, however, mainly reproduces at the Households level the factorial distribution of income. If the socio-economic groups are identified by the type of prevailing incomes, it could be difficult to distinguish between the Factor Account and the Institutions Accounts. This is, for instance, the case of those groups of Households classified as "dependent workers" when they receive not only income from capital (interests and/or dividends) but also from capital.[31].
The introduction in the SAM of the Institutions and of the link between factorial and Institutional/personal income distribution suggests grouping the Households according to their level of income. Other criteria, of course, could be chosen. For instance when the economy is simple, as in the developing countries, a good classification is that based on the localisation (urban vs. rural households)[32].

[28] See: Giovannini (1996), pag. 283.
[29] See: Timmerman, Van de Ven, (1994), pag.48.
[30] See: ISTAT (1996), pag.189.
[31] This kind of observation can be done taking in account the arguments discussed in Pyatt (2001).
[32] This is the classification in the SAM for Sri-Lanka. See: Pyatt and Round (1979, Pyatt (2001), pag 143.

The income distribution of the Institution Households in the SAM must be considered as an equilibrium one, i.e. the distribution that assure the balance between the final demand for consumption and the supply of different commodities from the productive sectors in a given year. This equilibrium distribution, and the corresponding level of inequality, can be determined also as a solution of the SAM as a linear model. Following a Keynesian approach, we can assume that the total level of income of each group determines the consumption of different commodities by the Institution Households. This assumption leads to choose the H socio-economic groups so that the propensities to consume are quite homogenous inside of them, but different group from group. Econometric analysis also shows that not only the level but also the composition of consumption is strongly affected by the level of disposable income of the Households[33].

A classification of the Households by class of income (deciles of population) seems, however, the more suitable to assess the effects of any exogenous shocks (fiscal policies for instance) on the income distribution vector of the Households groups. These policies are generally calibrated on the level of total income of each group of Households and not on the source (from the factorial side) of the income. In order to use the SAM as a model, the condition to be satisfied first of all, refers to the homogeneous behaviour of the different groups. Secondly, however, it is necessary to adopt the same classification when the groups are considered both as income receiving units and as consumption units.

Figure 2 - Exogenous and endogenous accounts in a simplified SAM.

	Endogenous Accounts				
	Activities	Factors	Private Institutions	Exogenous Institutions	Total
Activities	$\mathbf{S_{11}}$	**0**	$\mathbf{S_{13}}$	$\mathbf{x_1}$	$\mathbf{t_1}$
Factors	$\mathbf{S_{21}}$	**0**	**0**	$\mathbf{x_2}$	$\mathbf{t_2}$
Private Institutions	**0**	$\mathbf{S_{32}}$	$\mathbf{S_{33}}$	$\mathbf{x_3}$	$\mathbf{t_3}$
Exogenous Institutions	$\mathbf{l'_1}$	$\mathbf{l'_2}$	$\mathbf{l'_3}$	$\mathbf{x_4}$	$\mathbf{t_4}$
Total	$\mathbf{t'_1}$	$\mathbf{t'_2}$	$\mathbf{t'_3}$	$\mathbf{t'_4}$	

[33] With reference to Italy see: ISTAT (1984).

In order to "measure the effects occurring in some variables (the exogenous ones) on the other (the endogenous ones) of the system" a very aggregated SAM (Figure 2) must be introduced[34]. In this SAM the endogenous components (Activities, Factors and Private Institutions) can be isolated from the exogenous ones (Government and Rest of the World) by aggregating one or more matrices of the SAM of Figure 1[35]. The equilibrium solution through the SAM determines the income distribution of the Private Institutions (the H groups of Households and the Private Companies aggregated in a single Institution) consistent with a given production structure under the assumption that the final demand depends on the disposable income of the Endogenous Institutions. Our model assumes that the consumption demand comes only by the Households. Private Companies only receive income from Factors and redistribute it to other Private Institutions.

Following the method advanced by Pyatt and Round it is possible to obtain the "accounting multipliers". These accounting multipliers can be interpreted as a "simplified model of the actual way the system is working". From another side the "results of the multiplier analysis can be interpreted as a demonstration of how the economic system is expected to behave in case the model assumptions perfectly reflect the real situation: any possible deviation from reality would then indicate both the correct parts and those which must be better calibrated".[36]

The determination of a multi-sector income multiplier is a distinguishing characteristic of the models based on the SAM. The traditional input-output analysis assumes the consumption demand as exogenous and the output of various activities depending on the propensities of final demand so that the composition of demand influence that of the value added. The opposite is not true because the model does not include the link between the value added and primary income distribution to the different Households groups. In the SAM model on the contrary "incomes (both total income and groups incomes) have different values depending on the proportions of final demand and this is because our model takes into account the structure of income distribution"[37].

The equilibrium solution is obtained following the same procedure as in input-output analysis and using the SAM as a linear model. The matrices of expenditure $\mathbf{A_{jk}}$ are obtained dividing each element in the transaction matrices of endogenous accounts $\mathbf{S_{jk}}$ by the correspondent column sum vectors $\mathbf{t'_k}$ where $\mathbf{t_k}$ is a diagonal matrix whose elements are the components of $\mathbf{t'_k}$[38].

$$\mathbf{A_{jk}} = \mathbf{S_{jk}} \left(\hat{\mathbf{t}}_\mathbf{k}\right)^{-1} \qquad [5]$$

The hypothesis of fixed expenditure coefficients resulting from $\mathbf{A_{jk}}$ is consistent with the assumptions of the linear expenditure system developed by Stone for which there is widespread empirical support[39]. The normalisation of the transaction matrices $\mathbf{S_{jk}}$ allows the constraints relating to row and column totals of the SAM in Fig.2 to be rewritten isolating the group of the *r* endogenous accounts from the exogenous ones. We can, thus, write

$$\mathbf{t} = \mathbf{A_r}\,\mathbf{t} + \mathbf{x} \qquad [6]$$

$$\mathbf{t_4} = \mathbf{l'_1 t_1} + \mathbf{l'_2 t_2} + \mathbf{l'_3 t_3} + \mathbf{x_4} \qquad [7]$$

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[34] See: Pyatt and Round (1975), Bottiroli Civardi (1990), Pyatt (2001).

[35] This procedure requires that the income of the accounts suppressed by aggregation is then redistributed over the remaining accounts. For the formal aspects connected with this process and for the related bibliography see: Bottiroli Civardi (1988), pag.102 onwards.

[36] See: Bottiroli Civardi (1990), pag. 76.

[37] See: Miyazawa, Misegi (1963), pa.g. 95.

[38] For the implications connected with the simplifying assumptions implicit in the determination of the coefficient of expenditures, which must be fixed in order to make the equations in the model linear, see: Bottiroli Civardi (1988), pagg. 96-97.

[39] See: Stone (1954) and also: Lluch, Powell, Williams (1977).

Equation [7] indicates that the equilibrium position of the accounts relating to exogenous Institutions is achieved once endogenous accounts are in equilibrium. This condition allows us that only equation (6) is taken into consideration and it is rewritten as

$$\mathbf{t} = (\mathbf{I} - \mathbf{A}_r)^{-1}\,\mathbf{x} = \mathbf{M}\,\mathbf{x} \qquad [8]$$

This formulation indicates that vector **t** of receipt totals for each endogenous account can be obtained from vector **x**, expressing the receipt totals of exogenous Institutions, by the generalised inverse

$$\mathbf{M} = (\mathbf{I} - \mathbf{A}_r)^{-1}$$

M is then the matrix of the global multipliers and shows the overall effects resulting from the direct and indirect transfer processes generated by an initial increase in each of the three exogenous components $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ on each element of the *r* (in our case three) endogenous accounts. The three components $\mathbf{M}_1$, $\mathbf{M}_2$ and $\mathbf{M}_3$ resulting from the decomposition of the multiplier matrix **M** has an economic meaning for a structural analysis of income distribution among and inside the Private Institutions, and above all with reference to the Households[40]. Equation [5] can be written as:

$$\mathbf{t} = \mathbf{M}_3\mathbf{M}_2\mathbf{M}_1\mathbf{x} \qquad [9]$$

The multiplier matrix $\mathbf{M}_1$ is a diagonal block matrix where the first diagonal block expresses the multiplier effects of the transfers within the activities and it is precisely the Leontief's inverse matrix. Since it is assumed that no direct transfers between factors take place, $\mathbf{M}_1$ second diagonal block is an identity matrix. The third block captures the multiplier effects due to the transfers between endogenous Institutions. The $\mathbf{M}_1$ multiplier expresses the effects within each account generated by direct transfers that are independent of the closed-loop process.

Also the $\mathbf{M}_2$ captures the multiplier effects resulting from transfers between the various endogenous Private Institutions and therefore expresses the action of an exogenous shock from any vector $\mathbf{x}_j$ over the elements of the other $\mathbf{t}_k$ accounts with $j \neq k$. Finally $\mathbf{M}_3$ multiplier is reflecting the closed-loop structure of the system. Every diagonal element, as a matter of fact, expresses the multiplier effects of an exogenous shock from vector $\mathbf{x}_j$ over the corresponding endogenous account $\mathbf{t}_j$ at the end of the closed-loop process. Finally the implicit assumption behind this decomposition is the existence of matrix **M** and hence of the three $\mathbf{M}_1$, $\mathbf{M}_2$ and $\mathbf{M}_3$ submatrices.

6. The multipliers matrix as a "structural measure" of inequality in personal income distribution.

If we focus our attention on the determination of the income distributed within the endogenous Private Institutions the corresponding $\mathbf{t}_3$ vector is given by:

$$\mathbf{t}_3 = \mathbf{M}_{31}\,\mathbf{x}_3 + \mathbf{M}_{32}\,\mathbf{x}_2 + \mathbf{M}_{33}\,\mathbf{x}_3 \qquad [10]$$

where

$$\mathbf{M}_{31} = {}_3\mathbf{M}_{33}\ {}_2\mathbf{M}_{31}\ {}_1\mathbf{M}_{11}$$
$$\mathbf{M}_{32} = {}_3\mathbf{M}_{33}\ {}_2\mathbf{M}_{32}$$
$$\mathbf{M}_{33} = {}_3\mathbf{M}_{33}\ {}_1\mathbf{M}_{33}$$

[40] See Pyatt (2001), and also Pyatt, Roe et al. (1077), and Pyatt and Round (1979).

Equation [10] allows us to determinate the total income of each group of the Private Institutions by the $\mathbf{M_{31}}$ $\mathbf{M_{32}}$ and $\mathbf{M_{33}}$ multipliers. The sum of the elements of the matrix $\mathbf{M_{31}}$ indicates the increase in the overall income of Private Institutions due to an exogenous injection of one unit in the income of each Activities account. The corresponding sum concerning $\mathbf{M_{32}}$ and $\mathbf{M_{33}}$ matrices indicate the increase in the overall income of Private Institutions due to an exogenous injection of one unit in the income of each Factors or each Private Institutions. The column totals of these matrices are real income multipliers. Each of them, in fact, indicates by how much the overall income of each Private Institutions would rise if the income of the corresponding element in Activities, Factor or Private Institutions account would exogenously increase by one unit. Instead every row total indicates the multiplier effect on the income of every Private Institutions in case the income of each Activity Sector, each Factor or each Private Institution would increase by one unit.
The $\mathbf{M_{33}}$, in particular, can be considered as a "structural" measure of inequality in the personal income distribution since it derives by the product of the component relating to Private Institutions in the $\mathbf{M_1}$ and $\mathbf{M_3}$ multipliers. Matrix $_3\mathbf{M_{33}}$ acquire the meaning of an income multiplier through the consumption expenditure as a result of a four-step "propagation" process. The first step is represented by the matrix $\mathbf{A_{13}}$ of consumption coefficients with reference to disposable income of each of the *H* Households group. The second step corresponds to that traditionally captured by the Leontief's inverse matrix transforming expenditure by sector into intermediate output and determining the shares of value added generated in the productive process. The third step, corresponding to the product of matrix $\mathbf{A_{32}}$ and matrix $\mathbf{A_{21}}$, determines the value added received by the Endogenous Private Institutions in connection with their ownership of production factors. The fourth step, finally, corresponds to the redistribution of income between Endogenous Institutions. The income thus produced, distributed a redistributed, turns into new levels of expenditures for consumption and the process occurs again until an equilibrium position is achieved [41].
The multiplier matrix assumes a precise meaning with reference to a structural analysis of the income distribution of the Institution Households. The elements of the matrix **M** related to this Institution have the meaning, at a disaggregated level, of a Keynesian expenditure multiplier. Its value depends on the linkages built in the SAM (consumption expenditure, input-output relationships, value added distributed to different households groups according to their ownership of the production factors). Therefore it is a "general framework for analysing the relationship between the distribution of income and the structure of production" [42].

7. The determination of multipliers for the Italian economic system. A simulation of a fiscal policy reform.

The multiplier approach allows quantifying the different ways by which an income equally earned by each socio-economic group turns into different disposable income levels through the three stages of spending, production and redistribution. The accounting multipliers obtained using the SAM as a linear model allow to capture the structural features of income distribution and the interrelations between socio-economic groups each other and with other Institutions and Sectors. The resulting inequality can be considered as the minimum

[41] For a discussion about the conditions to be achieved in order to ensure the equilibrium solution see: Bottiroli Civardi (1988), pag 112, and Targetti Lenti (1990).
[42] For a discussion on "some early multiplier models of the relationship between income distribution and production structure" see: Pyatt (2001), pag. 139.

inequality compatible with the given productive and spending structures, and hence as a result of the mechanism only explicitly considered in the model.
The meaning and the relevance of the multiplier approach in the use of the SAM as a simulation model can be illustrated with an application to the Italian economy. The base SAM for Italy has been determined by the authors, in a past research work, for the year 1984[43]. The construction of the Italian SAM required the introduction of very simplifying hypothesis. This exercise must be considered mostly as a useful application to highlight the potentiality of the tool, rather than an unquestionable result. Its meaningful stands mostly in relative values of multipliers and in the direction of changes.
Tables 1 to 3 shows the values of $\mathbf{M_{31}}$, $\mathbf{M_{32}}$ and $\mathbf{M_{33}}$ multipliers determined of the basis of the 1984 SAM for Italy, once some of the accounts have been aggregated in endogenous and exogenous ones.

Table 1 - Matrix $\mathbf{M_{31}}$ Endogenous Institutions' Income Multiplier for variation to $\mathbf{x_1}$
Italy 1984 - Base SAM

	SECTORS								
	Agricul.	Industry	Trade	Tran-sports	Credit. and Insurance	Public Administr.	Others	TOTAL	Averages
1^ decile	0,01486	0,01318	0,01740	0,02040	0,02225	0,02301	0,02253	0,13364	0,01909
2^ decile	0,03932	0,03364	0,04557	0,05294	0,04912	0,05739	0,04942	0,32740	0,04677
3^ decile	0,05284	0,04530	0,06100	0,07169	0,06637	0,07708	0,06556	0,43983	0,06283
4^ decile	0,06468	0,05544	0,07444	0,08801	0,08092	0,09423	0,07807	0,53580	0,07654
5^ decile	0,07397	0,06606	0,08435	0,10383	0,09919	0,11569	0,09031	0,63340	0,09049
6^ decile	0,08590	0,07481	0,09859	0,11819	0,10929	0,12876	0,10472	0,72026	0,10289
7^ decile	0,09892	0,08729	0,11309	0,13717	0,13240	0,15188	0,11803	0,83878	0,11983
8^ decile	0,12574	0,10911	0,14382	0,17356	0,16462	0,18644	0,14925	1,05254	0,15036
9^ decile	0,16479	0,13771	0,19018	0,21993	0,22136	0,22893	0,18980	1,35270	0,19324
10^ decile	0,30793	0,24265	0,35671	0,40079	0,41243	0,37734	0,34079	2,43864	0,34838
Total Households	1,02896	0,86520	1,18516	1,38650	1,35794	1,44075	1,20848	8,47298	1,21043
Companies	0,21571	0,15220	0,23643	0,31450	0,19616	0,17204	0,22582	1,51286	0,21612
TOTAL	1,24467	1,01740	1,42159	1,70100	1,55410	1,61279	1,43430	9,98584	1,42655

The data, which allowed estimating the income distribution of the Insititution Households, were drawn by the annual Survey of the Bank of Italy[44]. The classification of Activities was consequently determined in seven branches (Agriculture, Industry, Trade, Transports, Credit and Insurance, Public Administration, Other Services).
The factor accounts, instead, was disaggregated by authors into five categories (Employed labour, Self-employed labour, Capital in productive Activities, Capital in Housing, Financial Capital). Endogenous Institutions is separated in Companies and 10 groups of Households

[43]The construction of the Italian SAM required an extensive processing of data drawn from different sources. For a description of the methodology see: Bottiroli Civardi, Chiappero Martinetti and Targetti Lenti (1990). The same SAM has been used for applications by other authors. See: Timpano (1996). A process of building a new and updated SAM is underway by ISTAT, but it is not completed yet. A SAM has been recently built also for the Tuscany Region. See: Lattarulo P., Paniccià R., Sciclone N. (2002).
[44] The features of this sample survey are described in Banca d'Italia (1985). The survey collects only the values of disposable income, so that it has been necessary to calculate the vector of the primary income with a fiscal microsimulation model. For the methodology see: Bottiroli Civardi, Chiappero Martinetti and Targetti Lenti (1990).

(deciles of population) according to their level of disposable income. All the other accounts of the base SAM were aggregated into the vector of Exogenous Institutions.

Table 2 - Matrix M32 of Endogenous Institutions' Income Multiplier for variation of $\mathbf{x_2}$.
Italy 1984 - Base SAM

	FACTORS						
	Employees Income	Self-Empl. Income	Product. Capital	Housing Capital	Financial Capital	TOTAL	Averages
1^ decile	0,02588	0,02291	0,00953	0,04095	0,02799	0,12726	0,02545
2^ decile	0,06663	0,06395	0,02760	0,06452	0,05276	0,27546	0,05509
3^ decile	0,08941	0,08283	0,03948	0,08265	0,07136	0,36573	0,07315
4^ decile	0,10961	0,09982	0,04950	0,09123	0,08661	0,43677	0,08735
5^ decile	0,13635	0,10142	0,05163	0,10219	0,10912	0,50071	0,10014
6^ decile	0,15078	0,12764	0,06280	0,12276	0,11653	0,58051	0,11610
7^ decile	0,17844	0,14263	0,06865	0,12716	0,14892	0,66580	0,13316
8^ decile	0,21685	0,18194	0,09932	0,16293	0,18365	0,84469	0,16894
9^ decile	0,25968	0,26836	0,13599	0,20254	0,27195	1,13852	0,22770
10^ decile	0,40552	0,52479	0,33157	0,35644	0,54152	2,15984	0,43197
Total Households	1,63915	1,61629	0,87606	1,35337	1,61041	7,09528	1,41905
Companies	0,14542	0,14084	0,58297	0,23164	0,13967	1,24054	0,24811
TOTAL	1,78457	1,75713	1,45903	1,58501	1,75008	8,33582	1,66716

The calculus of average multipliers (last column in Tables 1 to 3) allows quantifying the effects on the income of Private Institutions of an exogenous injection of income directed alternatively to Activities or to Factors or to Private Institutions accounts. One unit of income exogenously directed toward the Activities account generate an average increase equal to 1.21043 units in the income of the Institution Households. This increase is equal to 1.41905 if the exogenous unit is directed toward the Factors account, and equal to 1.54018 if it is directed toward the Private Institutions. These values are the result of all the mechanisms induced by the closed-loop nature of the process.

The effects produced on each decile by an increase attributed to Factors, and expressed by matrix $\mathbf{M_{32}}$ (Table 2) are only slightly less differentiated. Yet, despite the greater value it displays, employed labour does not seem to play a dominant multiplying effect as compared to self-employment and financial capital[43].

A reading by row of values in table 1 (matrix $\mathbf{M_{31}}$) shows the different ability of various productive branches to generate income for the various deciles. These are considerably differentiated over the various deciles. As expected row totals show monotonically growing values with rather high differences between deciles. The last decile multiplier is 18.2 times higher than the first decile multiplier and 1.8 higher than the 9th multiplier. The contribution of each sector (column total) is, instead, fairly uniform. Public Administration, followed by Transports and Credit, are the sectors showing the higher income multiplier for the Households. These are sectors in which, at least in the 80th before the introduction of Information, Communications Technology (ICT), the share of the value added going to labour, and hence affecting directly the Institution Households is larger.

[43] The determination of multipliers required the estimation of the matrix S_{32} related to the ownership of factors by the Private Institutions. The data were processed starting from the census survey of 1981. See: ISTAT (1982). The estimation of matrix S_{13} related to the consumption of different categories of commodities in each household group to the sectors of activities was obtained according to the correspondence between household budget and the input-output categories. See: ISTAT (1984), and Bottiroli Civardi, Chiappero Martinetti and Targetti Lenti (1990).

Table 3 - Matrix M33 of Endogenous Institutions' Income Multiplier for variation of x_3.
Italy 1984 - Base SAM

	Private Institutions													
	1^ deciIe	2^ deciIe	3^ deciIe	4^ deciIe	5^ deciIe	6^ deciIe	7^ deciIe	8^ deciIe	9^ deciIe	10^ deciIe	Total Househol.	Companies	TOTAL	Averages
1^ deciIe	1,01160	0,01216	0,01217	0,01158	0,01040	0,01117	0,01115	0,01068	0,01007	0,00815	1,10914	0,00297	1,11211	0,10110
2^ deciIe	0,02810	1,02960	0,02970	0,02835	0,02544	0,02734	0,02721	0,02596	0,02447	0,01983	1,26599	0,00705	1,27303	0,11573
3^ deciIe	0,03759	0,03964	1,03970	0,03795	0,03407	0,03661	0,03643	0,03474	0,03275	0,02654	1,35601	0,00940	1,36541	0,12413
4^ deciIe	0,04554	0,04805	0,04820	1,04610	0,04134	0,04442	0,04420	0,04211	0,03970	0,03218	1,43185	0,01147	1,44332	0,13121
5^ deciIe	0,05304	0,05601	0,05620	0,05374	1,04830	0,05186	0,05163	0,04916	0,04635	0,03758	1,50386	0,01467	1,51853	0,13805
6^ deciIe	0,06104	0,06441	0,06461	0,06175	0,05543	1,05960	0,05929	0,05648	0,05325	0,04316	1,57902	0,01609	1,59511	0,14501
7^ deciIe	0,07008	0,07403	0,07429	0,07105	0,06379	0,06856	1,06820	0,06495	0,06124	0,04965	1,66581	0,01968	1,68549	0,15323
8^ deciIe	0,08835	0,09330	0,09361	0,08952	0,08037	0,08637	0,08595	1,08180	0,07715	0,06254	1,83894	0,02382	1,86276	0,16934
9^ deciIe	0,11316	0,11948	0,11986	0,11458	0,10286	0,11054	0,10992	0,10469	1,09870	0,07999	2,07378	0,02932	2,10310	0,19119
10^ deciIe	0,20420	0,21550	0,21615	0,20660	0,18543	0,19928	0,19798	0,18866	0,17784	1,14410	2,93574	0,04743	2,98317	0,27120
Total Households	1,71269	1,75218	1,75447	1,72121	1,64742	1,69574	1,69196	1,65923	1,62153	1,50371	16,76014	0,18189	16,94203	1,54018
Companies	0,14610	0,16351	0,16559	0,16240	0,14911	0,15810	0,15583	0,15007	0,14241	0,11863	1,51175	1,01610	2,52785	0,22981
TOTAL	1,85879	1,91569	1,92006	1,88361	1,79653	1,85384	1,84779	1,80930	1,76394	1,62234	18,27189	1,19799	19,46988	1,76999

Table 4 - Matrix $\mathbf{M_{31}}$ of Endogenous Institutions' Income Multiplier for variation of $\mathbf{x_1}$.
Italy 1984 – Simulation

	SECTORS								
	Agricul.	Industry	Trade	Transport	Credit/ Insurance	Public Administr.	Others	TOTAL	Averages
1^ deciIe	0,01485	0,01316	0,01738	0,02036	0,02223	0,02296	0,02251	0,13344	0,01906
2^ deciIe	0,03928	0,03358	0,04552	0,05285	0,04906	0,05726	0,04936	0,32689	0,04670
3^ deciIe	0,05277	0,04521	0,06093	0,07157	0,06628	0,07690	0,06547	0,43914	0,06273
4^ deciIe	0,06460	0,05534	0,07436	0,08787	0,08082	0,09401	0,07796	0,53496	0,07642
5^ deciIe	0,07388	0,06594	0,08425	0,10366	0,09907	0,11544	0,09018	0,63242	0,09035
6^ deciIe	0,08580	0,07467	0,09848	0,11800	0,10916	0,12847	0,10458	0,71916	0,10274
7^ deciIe	0,09880	0,08714	0,11296	0,13695	0,13224	0,15155	0,11787	0,83751	0,11964
8^ deciIe	0,12559	0,10891	0,14366	0,17328	0,16442	0,18602	0,14905	1,05093	0,15013
9^ deciIe	0,16459	0,13746	0,18998	0,21958	0,22110	0,22840	0,18955	1,35066	0,19295
10^ deciIe	0,30758	0,24219	0,35634	0,41600	0,41197	0,37638	0,34032	2,45078	0,35011
Total Households	1,02773	0,86360	1,18387	1,40012	1,35633	1,43738	1,20685	8,47588	1,21084
Companies	0,21545	0,15185	0,23615	0,31401	0,19581	0,17128	0,22546	1,51001	0,21572
TOTAL	1,24318	1,01545	1,42002	1,71413	1,55214	1,60866	1,43231	9,98589	1,42656

Table 5 - Matrix $\mathbf{M_{32}}$ of Endogenous Institutions' Income Multiplier for variation of $\mathbf{x_2}$.
Italy 1984 - Simulation

	FACTORS						
	Employees Income	Self-Emp. Income	Product. Capital	Housing Capital	Financial Capital	TOTAL	Averages
1^ deciIe	0,02581	0,02291	0,00954	0,04095	0,02798	0,12718	0,02544
2^ deciIe	0,06645	0,06394	0,02764	0,06449	0,05274	0,27525	0,05505
3^ deciIe	0,08917	0,08281	0,03952	0,08259	0,07133	0,36543	0,07309
4^ deciIe	0,10932	0,09981	0,04955	0,09116	0,08657	0,43641	0,08728
5^ deciIe	0,13600	0,10141	0,05169	0,10211	0,10907	0,50028	0,10006
6^ deciIe	0,15039	0,12764	0,06287	0,12268	0,11648	0,58006	0,11601
7^ deciIe	0,17798	0,14263	0,06873	0,12706	0,14887	0,66527	0,13305
8^ deciIe	0,21627	0,18193	0,09943	0,16282	0,18358	0,84403	0,16881
9^ deciIe	0,25894	0,26834	0,13613	0,20239	0,27187	1,13767	0,22753
10^ deciIe	0,40420	0,52475	0,33182	0,35617	0,54136	2,15830	0,43166
Total Households	1,63452	1,61617	0,87693	1,35242	1,60985	7,08989	1,41798
Companies	0,14438	0,14084	0,58320	0,23141	0,13958	1,23941	0,24788
TOTAL	1,77890	1,75701	1,46013	1,58383	1,74943	8,32930	1,66586

Table 6 - Matrix $\mathbf{M_{33}}$ of Endogenous Institutions' Income Multiplier for variation of $\mathbf{x_3}$.
Italy 1984 - Simulation

	PRIVATE INSTITUTION													
	1^ deciIe	2^ deciIe	3^ deciIe	4^ deciIe	5^ deciIe	6^ deciIe	7^ deciIe	8^ deciIe	9^ deciIe	10^ deciIe	Total Households	Companies	TOTAL	Averages
1^ deciIe	1,01210	0,01302	0,01229	0,01182	0,01027	0,01068	0,01063	0,01033	0,00982	0,00846	1,10941	0,00296	1,11237	0,10112
2^ deciIe	0,02930	1,03170	0,02999	0,02892	0,02512	0,02613	0,02594	0,02511	0,02386	0,02056	1,26662	0,00703	1,27365	0,11579
3^ deciIe	0,03919	0,04244	1,04010	0,03872	0,03364	0,03499	0,03473	0,03360	0,03193	0,02751	1,35685	0,00938	1,36622	0,12420
4^ deciIe	0,04748	0,05145	0,04867	1,04700	0,04082	0,04246	0,04214	0,04073	0,03871	0,03336	1,43281	0,01145	1,44425	0,13130
5^ deciIe	0,05529	0,05997	0,05674	0,05483	1,04760	0,04957	0,04922	0,04755	0,04519	0,03896	1,50491	0,01464	1,51955	0,13814
6^ deciIe	0,06363	0,06897	0,06524	0,06310	0,05474	1,05690	0,05652	0,05463	0,05192	0,04475	1,58040	0,01605	1,59645	0,14513
7^ deciIe	0,07305	0,07926	0,07501	0,07249	0,06299	0,06553	1,06510	0,06282	0,05970	0,05148	1,66743	0,01964	1,68706	0,15337
8^ deciIe	0,09210	0,09989	0,09452	0,09133	0,07936	0,08255	0,08194	1,07910	0,07522	0,06484	1,84085	0,02376	1,86461	0,16951
9^ deciIe	0,11796	0,12793	0,12103	0,11691	0,10157	0,10566	0,10480	0,10126	1,09620	0,08294	2,07626	0,02924	2,10550	0,19141
10^ deciIe	0,21287	0,23075	0,21825	0,21080	0,18311	0,19047	0,18874	0,18248	0,17338	1,14940	2,94025	0,04730	2,98755	0,27160
Total Households	1,74295	1,80539	1,76183	1,73592	1,63923	1,66493	1,65976	1,63762	1,60591	1,52225	16,77577	0,18144	16,95721	1,54156
Companies	0,15227	0,17508	0,16719	0,16570	0,14726	0,15113	0,14857	0,14516	0,13884	0,12301	1,51421	1,01600	2,53021	0,23002
TOTAL	1,89522	1,98047	1,92902	1,90162	1,78649	1,81606	1,80833	1,78278	1,74475	1,64526	18,28998	1,19744	19,48742	1,77158

The effects produced on each decile by an increase attributed to Factors, and expressed by matrix $\mathbf{M_{32}}$ (Table 2) are only slightly less differentiated. Yet, despite the greater value it displays, employed labour does not seem to play a dominant multiplying effect as compared to self-employment and financial capital[43].

Matrix $\mathbf{M_{33}}$ (Table 3) diagonal elements represent the income multiplier within each Private Institution (deciles of population and Firms) generated by an additional unit of disposable income exogenously attributed to the group itself. With reference to Households, they are obviously all higher than one and show a monotonically growing trend from the first to the last decile. This means that, an exogenous injection of additional income equally done, the final effect within the poorest group is always weaker than within the richest. The poorest decile has a lower ability to generate income for themselves than for generating income for the aggregate Institution Household. The column totals indicate that the first four deciles, and particularly the second and the third, have the greatest income generating ability.

The row totals of $\mathbf{M_{33}}$ reflect the degree of inequality in the income distribution over Private Institutions which can be considered structural, i.e. the distribution related to the values of the coefficients of expenditure, of intermediate production, of value added distribution and redistribution among the Private Institutions. All these values show a monotonically upward trend. The value for the 1st decile is rather small and it indicates the reduced potential of the system to distribute income to the poorest, while the multiplier effect in favour of the last decile appears to be particularly strong. The contribution of the internal multiplier effect is only 38 per cent of the total for the 10th decile, while for the 1th decile it covers 91% of the total. The global multiplier $\mathbf{M_{33}}$ for the Institution Households is quite completely determined (98,7%) by $_3\mathbf{M_{33}}$ that is by the closed-loop nature of the process.

These observations bring us to affirm that in our economic system, and probably in any market economy, the benefits produced by an increase in disposable income, initially equally earned by all the deciles, propagate through ways of spending so as to increasingly favour the upper-middle deciles, and particularly the last ones. The labour market, the ownership of factors, the technological features of the production process stay behind the level of the inequality. The closed-loop process is something strictly interwoven with the operating of the market and can be considered a special feature of every system.

It is possible, also, to evaluate the sensitiveness of multipliers to capture the changes observed in the Households income distribution by any economic policy measure, for instance by quantifying the impact of a new taxation policy on the income multipliers for the Institution Households. The policy challenge is to simulate the effects of any macroeconomic policy (fiscal, directed toward full employment and so on) on the level of income inequality.

In order to calculate the new multiplier matrix, that results in the new equilibrium settlement, after all policies effects have been exhausted, we must use the base SAM as a data base for the determination and the solution of a Transaction Value Model (TV model), that is a computable general equilibrium (CGE) model.

The solution consists in obtaining the base SAM as the result of a model in which every cell of the SAM is substituted by a behavioural function. This solution allows determining the parameters and the elasticities that can be used for the simulation. In this case the mechanism of transmission are the prices. The multipliers we obtain therefore are not any more simple accounting multipliers. Their value, in other words, depends on the hypothesis we have to assume about the behaviour of different units, which are collected in the Institutions[44].

The result we obtained can be considered as one of the many applications it is possible to undertake by a SAM model. In this case we used the software package (HERCULES) developed by Drud and Kendrick at the World Bank[45]. In particular, we simulated a change in the tax system and the adoption of a "long rate" structure with only three brackets (zero for taxable income below 20 millions; 7 per cent for taxable income between 20 and 30 millions; 14 per cent for taxable income higher than 30 millions). The

[43] The determination of multipliers required the estimation of the matrix S_{32} related to the ownership of factors by the Private Institutions. The data were processed starting from the census survey of 1981. See: ISTAT (1982). The estimation of matrix S_{13} related to the consumption of different categories of commodities in each household group to the sectors of activities was obtained according to the correspondence between household budget and the input-output categories. See: ISTAT (1984), and Bottiroli Civardi, Chiappero Martinetti and Targetti Lenti (1990).

[44] These hypothesis and the results of the simulations are widely discussed in Bottiroli Civardi, Chiappero Martinetti, Targetti Lenti (1994).

[45] See Drud et. Al. (1986).

values of the SAM obtained after the simulation allow calculating the new multipliers matrix[46]. Matrix $\mathbf{M_{31}}$ (table 4) indicate that the new tax policy would not produce any variation in both the global multiplier for all Institutions following an exogenous increase of one unit in the income of the Activities sector. Also the differences between deciles would be unchanged while the contribution of each branch of Activity to the global multiplier would be lightly different.
A slight reduction, instead, would occur in the $\mathbf{M_{32}}$ multiplier (table 5) with respect to the average multiplier effect of one-unit increase in the income of Factors. It occurs in both the global multiplier for the Households overall income and for the Private Institutions. However the relative positions of the Factors, as well as of deciles, would remain unchanged. The tax reform, would slightly reduce the global multiplier (1.41798 Vs 1.41905), but leave unchanged the differences, and hence the inequality, between the deciles.
As expected, the $\mathbf{M_{33}}$ multiplier (shown in table 6), linked to the income increases directly attributed to Private Institutions, would slightly increase. What it is interesting to note, however, is that this manoeuvre, which was mainly meant to achieve an equalizing effect, at least in favour of the first four deciles (income under 20 millions), actually determines a marked internal effect in them (see the value on the diagonal), but also ends up by unchanging the difference, and hence the inequality, between the deciles. The weaker ability of the richest deciles (except the 10^{th} that would be improved by this tax manoeuvre) to activate income in the Households, as a whole, is in this case more marked.

8. Concluding remarks

The results of the analysis, which can be undertaken with a SAM, strengthen the hypothesis that inequality in the personal income distribution is determined first of all by mechanisms operating in connection with primary income distribution. In particular the multipliers approach allows quantifying the structural and endogenous level of inequality. The $\mathbf{M_{33}}$ component of the multiplier matrix makes it possible to quantify the different ways by which an income initially equally earned by each group turns into different levels of disposable income through the steps of spending and producing in a market economy. The resulting inequality can be considered as that equilibrium level compatible with the existing structure of the productive activities, with the composition of private consumption expenditure, and with the links that interwoven the Private Institutions. These links depend on factors as the demographic structure of the Households, the structure of their ownership of factors of production, the technological features of the production process.
The inequality in the disposable income, of course, can be and should be different from that resulting from the loop process related to the production and the primary distribution of income. Our analysis and simulations show that in the Italian system the benefits originating from an increase in the disposable income that is initially equal for all deciles increasingly favour the upper-middle income deciles, particularly the top ones, through their "ways of spending". The productive sector seems to have a very low power to generate income for the poorest groups.
The redistributive action of Government could and should reduce the "endogenous" inequality through fiscal and transfers policies. Our analysis, however, allow to observe that the market mechanisms toward inequality are very strong and powerful. The results of any fiscal reform could be, and probably are, small in terms of reducing the inequality in the personal income distribution. Sometimes they could be even "adverse", as in our simulation and they could end up by precisely favouring the richest.
Also the transfer policy in Italy has benefit relatively more the middle-income classes[47]. Monetary transfers are mostly pensions whose level is proportional to the value of income earned during the working life. Poverty, inequality and exclusion must be fight not only with traditional macroeconomic policies, but also with more targeted instruments. These policies can be undertaken, and their effects can be better assessed, with the help of structural indicators based on information and schema as the SAM.

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[47] For an analysis of the features of personal income distribution in Italy see: Bottiroli Civardi, Targetti Lenti (2001).

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