Società italiana di economia pubblica

WORKING PAPERS
No. 122/2002

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March, 2002

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Abstract

This note provides a counter-example to existing literature. The latter establishes the dominance of ad valorem over per unit taxation in oligopoly frameworks. On the contrary, the present note shows that per unit taxation may welfare dominate ad valorem taxation, when the number of consumers is relatively large with respect to that of oligopolists. Our analysis uses a strategic market game formulation which allows to study strategic behaviour within a productive economy cast into a general equilibrium context.

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I am deeply grateful to Jean J. Gabszewicz, Maurice Marchand and Pierre Pestieau for their helpful comments and suggestions.

1 Introduction

When commodities are taxed in imperfectly competitive economies, two sources of distortion arise. Firstly, there are distortions generated by the market mechanism, i.e. the use of market power.[1] Secondly, when taxation serves to collect resources to finance a given public budget requirement, additional distortions are introduced via commodity taxation, and these can be more or less severe depending on the chosen form of taxation.[2]

To compare the welfare properties of different forms of taxation, the public finance literature has concentrated the attention on ad valorem *versus* per unit taxes. Under perfect competition, the two types of tax are equivalent (Suits and Musgrave (1953), Bishop (1968)). However, under monopoly, an ad valorem tax is unambiguously welfare superior to a per unit tax that raises the same yield (Suits and Musgrave (1953), Skeath and Trandel (1994)). The dominance of ad valorem over per unit taxation usually extends also to the oligopoly case. This result is established by Delipalla and Keen (1992) for the case of symmetric Cournot oligopoly, with both a fixed number of firms and entry, and by Denicolò and Matteuzzi (2000) for the asymmetric case.[3] More recently, the relative efficiency of the two forms of taxation has been investigated by Anderson et al. (2001) for a wide range of market conduct. The superiority of ad valorem taxation arises under Cournot competition with homogeneous products, both in the symmetric and asymmetric case; but, unit taxation may be welfare-superior under Bertrand competition with product differentiation.[4] Anderson et al. also suggest that it is the mode of competition that is responsible for this result and not product differentiation.

This note provides a counter-example to previous literature. The latter establishes the dominance of ad valorem over per unit taxation in oligopoly frameworks. On the contrary, in a Cournot oligopoly model with an homogeneous good and symmetric costs, we show that a per unit tax may welfare dominate an ad valorem tax, when the number of consumers is relatively large with respect to that of oligopolists. To perform our analysis, we use a strategic market game formulation (Shapley (1976), Shapley and Shubik (1977)) which allows to study strategic behaviour within a productive economy cast into a general equilibrium context (Gabszewicz and Michel (1997)).[5]

The next section provides the model of homogeneous oligopoly with ad valorem and per unit taxes, and section 3 compares the welfare properties of the two forms of taxation. Section 4 contains a short conclusion.

[1] Such distortions may be corrected via suitable fiscal instruments. See for example Guesnerie and Laffont (1978), Myles (1996) and Gabszewicz and Grazzini (1999).

[2] Lump-sum taxes are supposed not available for the well-known reasons.

[3] See Keen (1998) for a survey on this topic.

[4] See also Colangelo and Galmarini (1997).

[5] All papers cited above, except Delipalla and Keen (1992), are cast into a partial equilibrium set-up.

2 The model

Consider a productive economy with two goods, 1 and 2, and including $n+m$ agents, falling into two types.[6] Agents i, $i = 1, ..., n$, -the *consumers*- behave competitively on the market and their initial endowment consists only of good 1. Agents j, $j = 1, ..., m$, -the *oligopolists*- do not own initially any good, but own each a linear technology allowing to produce good 2 using good 1 as input. More precisely, consider the following economy. All agents have the same utility function U defined by

$$U(x^1, x^2) = x^1 x^2,$$

while initial endowments are defined by

$$a_i = (\frac{1}{n}, 0), \qquad i = 1, ..., n \tag{1}$$

and

$$a_j = (0, 0), \qquad j = 1, ..., m. \tag{2}$$

Furthermore, agents of type 2 own a linear technology, defined as

$$y_j = \frac{1}{\alpha} z_j, \qquad \alpha > 0 \tag{3}$$

where y_j denotes the amount of good 2, which can be produced out of an amount z_j of good 1. More specifically, notice that agents of type 2 have to take two distinct decisions. Firstly, they have to decide how much of good 2 to produce, which also determines via (3) the amount z_j of good 1 to buy from agents of type 1. Secondly, they have to choose which share q_j of the amount y_j produced of good 2 to send to the market for trade (and the resulting amount $y_j - q_j$ to keep for private consumption). Clearly the equilibrium exchange rate between good 1 and good 2 depends on the amount q_j of good 2 sent by each oligopolist j to the market. This amount influences the total supply $\sum_{k=1}^{m} q_k$ of good 2, compared with the fixed total supply $\sum_{k=1}^{n} a_k^1$ of good 1. Consequently, each oligopolist j can individually manipulate the exchange rate by choosing the share q_j. This gives rise to a game whose *players* are the oligopolists, with *strategies* for oligopolists j, $j = 1, ..., m$, defined by pairs (q_j, y_j) with $q_j \leqslant y_j$.

Now consider that a commodity tax is levied on good 2. This tax can take the form of an ad valorem tax at a (tax-inclusive) rate t, $0 < t < 1$, or a per unit tax τ, $0 < \tau < \frac{1}{\alpha}$.[7] In the case of an ad valorem tax, the producer price for good 2 obtains as $P^2 = p^2(1-t)$, where p^2 is the consumer price for good 2, consequently, the total tax product is given by $R_t = tp^2 \sum_{k=1}^{m} q_k$. If, in contrast, a per unit tax is imposed, the producer price is defined by $P^2 = p^2 - \tau$, and accordingly the total tax product obtains as $R_\tau = \tau \sum_{k=1}^{m} q_k$.

[6] This model of homogeneous oligopoly has been proposed by Gabszewicz and Michel (1997).

[7] In this model, taxes are expressed in real terms. Since the maximum amount of good 2 which can be produced out of good 1 is $1/\alpha$ via the linear technology in (3), we assume that $\tau < 1/\alpha$.

Given a price vector (p^1, p^2), a competitive agent i, $i = 1, ..., n$, solves the problem

$$\begin{aligned} \max_{x^1, x^2} \quad & x^1 x^2 \qquad s.t. \\ & x^1 + p x^2 \leqslant \tfrac{1}{n}, \end{aligned}$$

giving rise to individual demand

$$x_i(p) = \left(\frac{1}{2n}, \frac{1}{2np} \right), \qquad i = 1, ..., n \tag{4}$$

where $p = \frac{p^2}{p^1}$, and total demand for good 2 equals to $\frac{1}{2p}$. Thus, the indirect utility function S of consumers obtains as

$$S(p) = \left(\frac{1}{2n} \right) \left(\frac{1}{2np} \right). \tag{5}$$

Now we proceed to the definition of the *payoffs* of the game among the oligopolists. To this end, assume that producer j has selected the strategy (q_j, y_j), $j = 1, ..., m$. At a price vector p, the profit of oligopolist j obtains as

$$p(1-t)q_j - z_j,$$

in the case of ad valorem taxation, and as

$$(p - \tau)q_j - z_j,$$

in the case of per unit taxation. With this profit, he can buy an amount of good 1 equal to $p(1-t)q_j - \alpha y_j$, in the case of an ad valorem tax and $(p-\tau)q_j - \alpha y_j$, in the case of a per unit tax, yielding resulting utility payoffs

$$\left(p(1-t)q_j - \alpha y_j \right) (y_j - q_j), \tag{6}$$

and

$$\left((p-\tau)q_j - \alpha y_j \right) (y_j - q_j), \tag{7}$$

respectively. Given these strategies (q_j, y_j), $j = 1, ..., m$, the value of p at which supply equals demand on market for good 2 is given by

$$\sum_{k=1}^{m} q_k = \frac{1}{2p};$$

or

$$p = \frac{1}{2 \sum_{k=1}^{m} q_k}. \tag{8}$$

By substituting this equilibrium exchange rate in the utility payoffs (6) and (7), we finally obtain the payoffs of the game, namely

$$V(q_j, y_j) = \left(\frac{1-t}{2 \sum_{k=1}^{m} q_k} q_j - \alpha y_j \right) (y_j - q_j); \qquad j = 1, ..., m, \tag{9}$$

in the case of ad valorem taxation, and

$$V(q_j, y_j) = \left(\left(\frac{1}{2\sum_{k=1}^{m} q_k} - \tau \right) q_j - \alpha y_j \right) (y_j - q_j); \qquad j = 1, ..., m, \tag{10}$$

in the case of per unit taxation. At an *oligopoly equilibrium*, V must be maximal with respect to q_j and y_j, given the strategies (q_k, y_k) chosen by the oligopolists k, $k \neq j$, and this must be satisfied for all j, $j = 1, ..., m$. The optimality conditions with respect to q_j and y_j give

$$\frac{1}{y_j - q_j} = \frac{\frac{1-t}{2Q} - \tau - \frac{(1-t)q_j}{2Q^2}}{\left(\frac{1-t}{2Q} - \tau\right) q_j - \alpha y_j} = \frac{\alpha}{\left(\frac{1-t}{2Q} - \tau\right) q_j - \alpha y_j}, \tag{11}$$

with $Q = \sum_{k=1}^{m} q_k$, and where $0 < t < 1$ and $\tau = 0$, in the case of wholly ad valorem taxation, while $0 < \tau < \frac{1}{\alpha}$ and $t = 0$, in the case of wholly per unit taxation. From the second equality of the above equation, we obtain that for all j, $j = 1, ..., m$, the equality

$$1 - \frac{q_j}{Q} = 2\frac{\alpha + \tau}{1 - t} Q, \qquad j = 1, ..., m. \tag{12}$$

must hold at equilibrium. Summing up equations (12), we get, at equilibrium,

$$Q_h^* = \frac{(m-1)(1-t)}{2m(\alpha + \tau)}, \qquad h = t, \tau; \tag{13}$$

and

$$p_h^* = \frac{m(\alpha + \tau)}{(m-1)(1-t)}, \qquad h = t, \tau; \tag{14}$$

where the subscript h, $h = t, \tau$, denotes hereafter a variable obtained under ad valorem or per unit taxation. Accordingly, total tax revenue is equal to

$$R_t^* = t p_t^* Q_t^* = \frac{1}{2} t, \tag{15}$$

under ad valorem taxation, and

$$R_\tau^* = \tau Q_\tau^* = \frac{(m-1)\tau}{2m(\alpha + \tau)}, \tag{16}$$

under per unit taxation.[8] Furthermore, using (12) and (13), we obtain

$$q_{jh}^* = \frac{\alpha + \tau}{2m^2(\alpha + \tau)^2}(m-1)(1-t), \qquad j = 1, ..., m; \qquad h = t, \tau; \tag{17}$$

and

$$y_{jh}^* = \frac{(\alpha + \tau)(2m\alpha + \tau - \alpha)}{4m^2\alpha(\alpha + \tau)^2}(1-t), \qquad j = 1, ..., m; \qquad h = t, \tau. \tag{18}$$

Since $m > 1$ the oligopoly equilibrium has m "active" firms both in the case of ad valorem and per unit taxation.

[8] Notice that in (15) total tax revenue under ad valorem taxation does not depend on the number m of firms which potentially can be active at equilibrium.

Finally, notice that, from (14), it is easy to check that per unit taxes are over-shifted: consumer price rises more than the increase in tax, i.e. $\frac{\partial p_\tau^*}{\partial \tau} > 1$. On the contrary, the result on the incidence of ad valorem taxation shows that consumer price may rise less than the increase in tax (under-shifting). Furthermore, from (13), it is easily checked that an increase in either a per unit tax or an ad valorem tax leads to a reduction in the total quantity of good 2, Q_h^*, $h = t, \tau$, which oligopolists are willing to exchange on the market, thus reinforcing the distortion already generated by their strategic behaviour, i.e. $\frac{\partial Q_\tau^*}{\partial \tau} < 0$ and $\frac{\partial Q_t^*}{\partial t} < 0$.[9]

3 A comparison between ad valorem and per unit taxes

In this section, we compare the welfare effects of ad valorem and per unit taxation. To perform this analysis, as a basis of comparison, we use ad valorem and per unit taxes that are *revenue-neutral*, at oligopoly equilibrium. More precisely, we consider a shift from an ad valorem tax t to a per unit tax τ which raises an equal amount of tax revenue.[10] Specifically, from equating (15) and (16), the value of τ which is used as a basis of comparison obtains as

$$\tau = \frac{m\alpha t}{m(1-t)-1}, \tag{19}$$

with $0 < \tau < \frac{1}{\alpha}$, under the assumption that $m > \frac{1}{1-t(1+\alpha^2)}$. By substituting (19) into (14), it is easily checked that, under per unit taxation, the price at equilibrium obtains as

$$p_\tau^* = \frac{m\alpha}{m(1-t)-1}, \tag{20}$$

which is strictly greater than the price under ad valorem taxation obtained in (14).

To compare the welfare properties of ad valorem and per unit taxes, we consider their effects on the *aggregate welfare*, namely the sum of the utility levels of consumers i, $i = 1, ..., n$, and oligopolists j, $j = 1, ..., m$ at equilibrium. For that case, we state the following proposition.

Proposition 1 *A revenue-neutral shift from an ad valorem tax t to a per unit tax τ increases aggregate welfare whenever $n > \frac{4m^2}{2-t}$.*

Proof. By substituting (17) and (18) into (9) for the case of ad valorem taxation, and into (10) with τ given in (19) for the case of per unit taxation, the difference in the utility level for each oligopolist j, $j = 1, ..., m$, under ad valorem and per unit taxes obtains as

$$V(q_{jt}^*, y_{jt}^*) - V(q_{j\tau}^*, y_{j\tau}^*) = -\frac{(2-t)t}{16m^4\alpha}. \tag{21}$$

[9] Notice that these results are specific to oligopolistic models and in line with previous literature (see for example Delipalla and Keen (1992)).

[10] Different basis of comparison may be used. For example, Delipalla and Keen (1992) consider a small tax shift that leaves total tax payments unchanged at the initial equilibrium price, but which is not fully revenue-neutral. See also Suits and Musgrave (1953) for a discussion on this point.

Similarly, by substituting (14) into (5) for the case of ad valorem taxation, and (20) into (5) for the case of per unit taxation, the difference in the utility level for each consumer i, $i = 1, ..., n$, is given by

$$S(p_t^*) - S(p_\tau^*) = \frac{t}{4n^2 m\alpha}. \tag{22}$$

Finally, from (21) and (22), the difference in aggregate welfare with ad valorem and per unit taxes, obtains as

$$n \cdot \frac{t}{4n^2 m\alpha} + m \cdot \left(-\frac{(2-t)t}{16m^4\alpha} \right) = \frac{t}{16nm^3\alpha}(4m^2 - n(2-t)),$$

which is strictly negative if $n > \frac{4m^2}{2-t}$. ■

Proposition 1, shows that a conflict of interests arises between consumers i, $i = 1, ..., n$, who are in favor of ad valorem taxation, and oligopolists j, $j = 1, ..., m$, which, on the contrary, are in favor of per unit taxation. However, when the two forms of taxation are compared with respect to aggregate welfare, Proposition 1 shows that per unit taxation welfare dominates ad valorem taxation that raises an equal amount of tax revenue, whenever the number of consumers is sufficiently high. Notwithstanding each consumer prefers ad valorem taxes, when their number is sufficiently high aggregate welfare is higher under per unit taxes. The reason for this result is due to the fact that when the number of consumers increases, for each of them the difference in the utility level between ad valorem and per unit taxation decreases (see (22)). Furthermore, notice that the assumption underlying proposition 1 is not very demanding since it is in adequacy with the assumption that consumers behave as price-takers.[11] Accordingly, our main result provides a counter-example to previous literature arguing instead the superiority of ad valorem taxation.

4 Concluding remarks

This note compares the different welfare properties of ad valorem and per unit taxation in a general equilibrium model of oligopolistic interaction. Our main result intends to provide a counter-example to previous literature. When the number of consumers is sufficiently high, we show that per unit taxation welfare dominates ad valorem taxation that raises an equal amount of tax revenue.

Finally, notice that our analysis has been cast into a particular oligopoly model, in which oligopolists are simultaneously producers and consumers. This stylized description is suitable to represent a world in which each firm would be owned by a single individual, but not that of enterprises owned by shareholders who do not have the same preferences (Gabszewicz and Michel (1997)). Nevertheless, with the latter formulation, several difficulties arise in modelling imperfect competition into a general equilibrium framework, namely the oligopoly equilibrium is not invariant

[11] Under the assumption of a single representative consumer, who is the sole owner of all oligopolistic firms, Delipalla and Keen (1992) show the dominance of ad valorem over per unit taxation. It is easy to check that, under this assumption, i.e. $n = 1$, we would also obtain the superiority of an ad valorem tax. However, in our context, this event has not any economic meaning since consumers represent the competitive side of the market, and for this reason their number has to be sufficiently high compared with the number of oligopolists which describe the strategic side.

with respect to the normalization rule used to normalize price, and the profit maximization criterion may not be optimal from the shareholders' viewpoint (Gabszewicz and Vial (1972)).

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