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MEASURING AUTONOMY FREEDOM

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Abstract

In this paper we propose an alternative perspective on the measurement of an individual's autonomy freedom. We adhere to the view (dominant in the literature) that a decision maker is autonomous when he faces relevant choices, i.e., choices among equally choosable alternatives. Yet, we depart from the available contributions in that, to decide when options are equally choosable, we make use of `the context in which the decision is taken' instead of reasonability.

Information about the context is captured in our framework by moving the analysis in the space of `opportunity situations', i.e., couples of opportunity sets and sets of potential preference relations. We then identify some necessary and sufficient conditions for for the existence of a ranking of autonomy freedom on the space of opportunity situations and characterize our result with respect to those available in the literature.

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1 Introduction

In the search for axiomatic measures of individual liberty, three approaches may be distinguished within the Freedom of Choice Literature (FCL). The first, due to Pattanaik and Xu (1990), interprets liberty as availability of opportunities and measures its extent on the basis of the cardinality of the decision maker's opportunity set. The second, advanced by Sen (1988, 1992, 1993), defends the view that freedom is doing what one wants. Accordingly, its measure should take into account at the same time the extension of an agent's set of opportunities as well as her actual preference relation over those alternatives. The third approach is due to Jones and Sugden (1982), Sugden (1999) and Pattanaik and Xu (1998); it argues for the view that to be free is to enjoy autonomy in choosing and that a measure of autonomy freedom should look at both the options that an individual may select and at her potential preference relations.

In this paper we move our steps right from the third approach. Though we agree with its main contention, i.e., that freedom should be assessed by looking at options and potential preferences, nonetheless we settle for an alternative point of view on potential preferences according to which all preference relations available to an agent in a choice situation should be taken into account in the assessment of her own autonomy freedom. We believe that our proposal is a more faithful representation of the Millian notion of autonomy as the affirmation of one's own individuality. Furthermore, by pursuing our perspective, some other gains may be secured. First, some difficulties with the current approach to autonomy measures within FCL can be overcome. In particular, we are referring here to the fact that, in some instances, the current approach reliance on a `sociological' interpretation of reasonability leads to unsatisfactory conclusions about autonomy freedom. Second, following our line of treatment, new light can be shed on both intra and inter-personal comparisons of the extent of an agent's autonomy freedom. This is because new information is brought to bear on the assessment of an agent's autonomy freedom. Such information is captured by constructing freedom rankings over the domain of opportunity situations instead of opportunity sets.

The paper is organized as follows. Section 2 explores the Millian roots of the current interpretation of autonomy freedom within the literature and reviews its main characteristics. Section 3 starts with pointing at the limits of FCL's treatment of autonomy; in particular, at its unsatisfactory treatment of Mill's view of autonomy. Then, it makes the case for our alternative framework by linking autonomy to some idea of distance among options, first, and presenting a framework for capturing the context of a choice in the construction of an autonomy freedom ranking, then. Section 4 departs from a purely narrative way of argumenting and lies the ground for the construction of our ranking by introducing the notation and the axioms. Section 5 proves the uniqueness of a ranking of opportunity situations which assesses the extent of autonomy freedom. Section 6 illustrates the interpretive gains of our ranking, on the one hand, and its relation with other results proposed in the literature, on the other. Finally, section 7 recapitulates and suggests lines for further research.

2 Autonomy and the Freedom of Choice Literature

2.1 Millian roots

As gray:milldefence writes, \[d]espite the absence in his writings of any explicit use of the jargon of autonomy [...], we are on ¯rm ground if we include an ideal of personal autonomy among Mill's most fundamental commitments" (p. 199). Such an ideal fails if human beings do not have the opportunity for developing a will of their own and for acting on it. The development of such a will is connected in Mill to the exercise of choosing.

Having options to choose from fosters the development of individuality since certain fundamental qualities of an agent which render him autonomous, such as \perception, judgement, discriminative feeling, mental activity, and even moral preference" [?, p. 122], can only be exercised and developed when he makes choices, i.e., in the deliberation process. Choosing is therefore valuable because it is expression of a person's autonomy, irrespective of the correctness of the choice.

Of course, not all acts of choosing signal the exercise of autonomy but only those which share a speci¯c quality, i.e., that of inducing the decision maker to resort to those personal traits of character that make up his own individuality. Choices with such a quality are called relevant. Consider, to start with, the case of a decision that should be taken between two identical options, say two cans of coke that only di®er for their bar-code. In such a circumstance, we can hardly claim that the decision maker needs to resort to his discriminative power or to his mental activity to opt in favour of either of them. Since the two options cannot be distinguished in any meaningful way to the end of the speci¯c decision that it is going to be taken, they cannot make up a relevant choice.

Similarly, consider the case of a choice that has to be made between options such that one of them is undoubtedly better, as in the case of a chess player who has to exchange queens to avoid mate. Again, it is hardly conceivable that, in the circumstances of his choice, he needs to resort to his fundamental qualities to make a decision about how to move. Since one of the options dominates all the others, no relevant choice emerges.

2.2 Relevant choices, potential preferences and reasonability

Within the freedom of choice literature some authors [?] have moved from the relation between relevant choices and autonomy for constructing a measure of autonomy freedom. Grounding their framework on Millian roots, they suggest that autonomy is enhanced as the number of relevant choices increases.

To capture the quantitative aspect of autonomy freedom, they shift the domain over which freedom rankings are constructed. Within FCL this is in general de¯ned over the set of options available to a decision maker once his preferences are formed. In the words of werth:coercion, FCL measures individual freedom at the post-deliberation stage of a choice where preferences are given. But, if preferences are formed (and therefore given), then the measure of the extent of freedom cannot take into account the deliberation process and,

as a consequence, no room is provided for autonomy. To anyone interested in autonomy, assessing the extent of choice at the post-deliberation stage is a strategy that must be abandoned altogheter.

The obvious amendment is constructing freedom rankings on the basis of measures of the extent of choice taken at the pre-deliberation stage. At this stage options do not present themselves already ranked in terms of the decision maker's preference relation. On the contrary, it is conceivable that, since the process of choice is still to be accomplished, any given option may be considered as the best, depending on which actual preference relation the chooser will end up with upholding. At the pre-deliberation stage then, an individual faces a set of potential preferences and deliberation serves the goal of selecting the actual one among the many.

Potential preferences are important in the assessment of the extent of an individual's autonomy freedom because they elicit relevant choices since the latter emerge whenever two options can be chosen on the basis of two alternative actual preference relations that a decision maker may uphold. The di±culty with potential preferences is that they need to be interpreted to make sense of a measure of autonomy. Suppose that all options available to a decision maker are eligible and the problem of assessing how many relevant choices are available coincides with that of measuring the extent of the agent's opportunity set. But then resorting to potential preferences becomes meaningless as we would be back to the problem of measurign freedom at the post-deliberation stage. Similarly, suppose that what a person prefers is entirely determined by \psychological and social in°uences" and therefore the set of potential preferences is a singleton. Then it would become impossible to regard any counterfactual preference relation as potential, making the idea empty once again.

sug:metric is aware of this problem. He suggests that, in constructing the notion of potential preference,

> \[t]here seems to be no way of avoiding appeal to contestable ideas of `normal', `reasonable' or `natural' preferences. For example, I take size 42 shoes. Is my range of opportunity increased if a given style of shoes is available in size 38 as well as size 42? I think not. Given the size of my foot, a preference for size 38 shoes seems too perverse to be taken seriously as a potential preference" (p. 324).

In Sugden's view, reasonability is consistent with a `sociological' interpretation of potential preference. Take a group of individuals identi¯ed on the basis of some characteristics other than what they prefer (say, the middle-aged British men, to use Sugden's example). Though each individual possesses his own preference relations, they may di®er from person to person. Nonetheless, it is likely that, as these relations are expressions of individuals sharing some common pool of characteristics, then every person will consider each of them as a preference relation that he may hold. The set of potential preferences is therefore composed by all the actual preference relations of all individuals of a given reference class of people. As Sugden (1999) writes,

> \[t]he idea here is that if someone who is su±ciently like me in terms of non-preference characteristics has a particular preference

ordering, then that preference ordering is to be regarded as one that I might have had" (p. 325).

On the basis of such a conceptual framework, pattaxu:prefandfree ranks states of a®airs on the basis of autonomy freedom by counting the number of relevant choices open to a decision maker, given his potential preference relations.

2.3 Autonomy and reasonability: a dissenting view

In the interpretation of autonomy developed within FCL reasonability works as some sort of screening device which excludes from the set of potential preferences those preference orderings which are not normally upheld by individuals belonging to a certain reference class. Sugden's appeal to reasonability is motivated by the search for a screening device that is \neutral between di®erent conceptions of the good", in the sense that it could be agreed on by individuals in search for a \public measure" of people's opportunity. Reasonability satis¯es this requirement since it aggregates over a certain class of people each individual's view of the goodness and, in so doing, it establishes some `public standard of opportunity' in which the possibility that any speci¯c conception could prevail is ultimately ruled out.

Although we recognize the need for a screening device in the analysis and measurement of autonomy freedom, yet we regard reasonability as unsatisfactory. The following example reveals the underlying intuitive motivations for our discomfort. In a given town the local two-screen cinema features quite a diverse program. In screen one patrons can enjoy the latest release of Schwarzenegger's ¯ght for rescuing the globe and its special e®ects (S). On the contrary, screen two proposes one of the many Tru®aut's cerebral analysis of women (T). Two persons, i and j, approach the counter for buying a ticket. Their opportunity set is the same: $\{S;T\}$. Yet, mister i is only aware of Hollywood productions and, therefore, he cannot express a preference for Tru®aut, despite Bardot playing in the leading role. On the contrary, mister j is a regular ¯lm-goer fully aware of both the re¯ned direction and acting that characterize Tru®aut's movies, as well as the breath-taking plots of Schwarzenegger's adventures.

Suppose we use reasonability as a screening device in the assessment of autonomy freedom. Then, either we claim that i and j belong to the same reference class of individuals or we do not. In the ¯rst instance reasonability would lead us to say that since j may express a preference for T, then such a preference has to be in i's set of potential preferences. Accordingly, a relevant choice is available to both people despite the contrary evidence that i is in fact unaware of one kind of movies and therefore can hardly face a relevant choice. The use of reasonability is here at odds with intuition about autonomy comparisons. Alternatively, we may say that di®erent awareness signals di®erent belonging, namely that the two individuals belong to two di®erent reference classes; in which case the extent of autonomy freedom that they enjoy would be di®erent | as intuition would suggest | but still dependent on the selected public standards of opportunity. On a more general track, then, the use of

reasonability is here at odds with the view that autonomy is concerned with shaping one's own life.

3 Opportunity situations and the measurement of autonomy freedom

3.1 The context of a choice

Autononomy | especially in its Millian sense | is a kind of achievement: a person is autonomous if he is author of his own life, if his life is \of his own making". This is quite common an interpretation among philosophers, albeit not the only one. Consider raz:morality, for example.

> [T]he autonomous person's life is marked not only by what it is but also by what it might have been and by the way it became what it is. A person is autonomous only if he has a variety of acceptable options available to him to choose from, and his life became as it is through his choice of some of these options. A person who has never had any signi¯cant choice, or was not aware of it, or never exercised choice in signi¯cant matter but simply drifted through life is not an autonomous person (p. 204).

Sugden's interpretation of autonomy is certainly compatible with such a view. By introducing potential preferences he necessarily takes into account information about \what it might have been". Yet, the appropriateness of this counterfactual depends on the public standard of opportunity as it is de¯ned through the screening device of reasonability. We suggest that an alternative and more appropriate screening device is given by information about the contextual circumstances of a choice. Translated in terms of the ¯lm-goer example this means that, since being unaware of French ¯lms (the circumstance of his choice), i cannot express a preference for T, then he does not enjoy autonomy.

Information about the contextual circumstances of a choice can be attained by looking at the distance among options or conducts.

> \When the distance between alternatives is such that one choice is obviously correct, it is as if, in Mill's words, the `world' has chosen for us. Such decisions are my decisions to a lesser extent than decisions that do not involve an obvious choice. Obvious choices may still be `dictated by reason', but it is anyone's reason, and not especially mine. There is a choice, but there is no (complex process of) choosing" [?, p. 196].

Wertheimer's thrust is that when two alternatives are either too distant or too close, anyone's reason (as he calls it) or contextual conditions (as we call them), prevail over deep reasoning in the deliberation process and become powerful enough a motivation for pursuing a course of action or for selecting a speci¯c opportunity. On the contrary, when options are not too far away from each other, autonomy is called in because the decision maker is required

to implement a process of deliberation which is evidence of the affirmation of his individuality.

The characterization and assessment of autonomy freedom depends therefore on the available options, on the one hand, and on potential preferences, on the other. Our problem is that of transforming the information about these two elements into affirmation of a decision maker's individuality. Reliance on a ranking constructed over the domain of opportunity sets cannot fulfill such a requirement because it would not be possible to attribute some beareance on the assessment of autonomy to potential preference, unless an external screening device is introduced, as in the case of reasonability. We therefore define an alternative space: that of opportunity situations. An opportunity situation is a pair composed by the set of options available to a decision maker and the set of potential preference orderings that he he is confronting in a specific choice situation.

The interplay of the information conveyed by accessible potential preferences and options delivers then conclusions about relevant choices which can be translated into assessments of an agent's autonomy freedom. As the next two sections show, this is amenable to formal representation and, in particular, it allows the construction of a ranking of opportunity situations with some useful properties and which generalizes some of the results achieved so far in the literature.

4 The analytical framework

4.1 Notation

In this paper we make use of the following notation. We denote by X the universal set of options, interpreted as feasible opportunities for choice or actions, and by $N = \{1, \ldots, n\}$ the set of agents. In general, the agent may access a subset of the universe X of possibilities; we denote by $A, B, \ldots \subseteq X$ these subsets. $P(X) = 2^X - \emptyset$ represents the set of opportunity sets. In this paper we are often interested on the number of elements contained in a given set, i.e., its cardinality. If A is the set under examination, its cardinality will be denoted by $|A|$.

To capture the idea of autonomy, we make use of information beyond availability of opportunities. In particular, we introduce the decision maker's preference relation (ordering) and denote it by R. As any well-behaved preference relation, R is a complete, reflexive and transitive binary relation defined over X. But we also assume that the agent may hold different preference relations. The set of all possible preference relations is denoted by $\Re = \{R_1, \ldots, R_m\}$. Thus, $\forall x, y \in X; \forall h \in \{1, \ldots, m\}$; xR_hy means that "x is preferred to y according to the preference ordering R_h": In our framework each of the n individuals in the society holds a set $\Re_i, i \in N$; of preference orderings, where $\Re_i \subseteq \Re$. $P(\Re) = 2^{\Re} - \emptyset$ represents the set of all subsets of preference relations.

Another element of our construction is the choice set of an individual, i.e., the set of his most preferred opportunities as elicited by the set of preference

relations that he holds. So, $\forall A \in P(X); \forall \succsim_i \in P(\succsim)$, we denote this set by $\max_i(A) = \{x \in A : \nexists y \in A \text{ such that } yRx \text{ for some } R \in \succsim_i\}$.

We are interested in ranking opportunity situations. Formally, an opportunity situation is a pair $(A; \succsim_i) \in P(X) \times P(\succsim)$. Hence our ranking is represented by a binary relation $\succeq$ over $P(X) \times P(\succsim)$. The expression $(A; \succsim_i) \succeq (B; \succsim_j)$ should then be read as "the opportunity situation $(A; \succsim_i)$ offers at least as much autonomy freedom as the opportunity situation $(B; \succsim_j)$". We also assume that the binary relation $\succeq$ is transitive.

Next we impose some axioms on $\succeq$, which capture our intuition on the extent of autonomy freedom enjoyed by the decision maker under alternative situations.

4.2 Axioms

The first axiom establishes the circumstances under which an individual enjoys no autonomy freedom.

Axiom 4.1 *Indifference between no-freedom situations (INF).*
$\forall A, B \in P(X); \forall \succsim_i, \succsim_j \in P(\succsim); [\max_i(A) = \{x\} \text{ and } \max_j(B) = \{y\}] \Rightarrow (A; \succsim_i) \sim (B; \succsim_j)$.

Axiom INF is related in spirit to the principle of no choice, introduced by Jones and Sugden (1982) and subsequently used by Pattanaik and Xu (1990), which can be stated as follows: if two opportunity sets are singleton, then the degree of fredom offered by them is identical. The intuition advanced for the property was that, since singleton opportunity sets do not offer any choice at all, then they are indifferent in terms of freedom, though the options they contain can be ranked differently in terms of utility. We believe that this is a very reasonable intuition; however, the principle of no choice, as formulated by Jones and Sugden (1982), fails to detect situations in which the absence of freedom is due to contextual circumstances, rather than to absence of options to choose from.

This consideration leads us to rephrase this property in a context where not only opportunity sets, but also individual circumstances, play a role. The idea can be illustrated by the following example. Consider three sets of options, A, B and C, where $A = \{x, y, z\}$, $B = \{s\}$ and $C = \{t\}$, with associated circumstances represented by the sets of preference profiles $\succsim_i$, $\succsim_j$ and $\succsim_h$ respectively. Assume now that the agent endowed with the set A, individual i, has access to a single (and linear) preference ordering; it follows that, although her opportunity set contains several options, her preference profile leads her to a choice set that is a singleton. On the other hand, however rich the sets of preference orderings $\succsim_j$ and $\succsim_h$ can be, the choice sets available to individuals j and h are constrained to be singleton, given the opportunity sets B and C. In this case, we believe that the three situations $(A; \succsim_i)$, $(B; \succsim_j)$ and $(C; \succsim_h)$ are freedom-wise indistinguishable (because they do not offer any choice at all). Whereas Jones and Sugden's principle of no choice, reformulated in our framework, would require indifference between $(B; \succsim_j)$ and $(C; \succsim_h)$; but not between

$(B; \succsim_j)$ and $(A; \succsim_i)$ or between $(A; \succsim_i)$ and $(C; \succsim_h)$: Hence we introduce axiom INF, which states that, if two opportunity situations lead to as many choice sets which are singletons, then the extent of autonomy freedom they offer is the same (being nil). Which implies that the cardinality of the opportunity sets is irrelevant whenever the preference profiles lead to choice sets that are singleton.

The next two properties impose some restrictions on the effect of adding (subtracting) an option to (from) a given opportunity set, over the ranking of opportunity situations.

Axiom 4.2 Addition of undominated alternatives (AUA).
$\forall A \in P(X); \forall \succsim_i \in P(\succsim); \forall x \in \max_i(A \cup \{x\}); (A \cup \{x\}; \succsim_i) \hat{A} (A; \succsim_i)$.

Axiom 4.3 Addition of dominated alternatives (ADA).
$\forall A \in P(X); \forall \succsim_i \in P(\succsim); \forall x \in X \setminus \max_i(A \cup \{x\}); (A; \succsim_i) \succ (A \cup \{x\}; \succsim_i)$.

Axioms AUA and ADA describe what happens when an option is added to a given opportunity set. Does the addition of a new option enhance the degree of freedom enjoyed by an individual? Different answers have been provided in the literature to this question. Pattanaik and Xu (1990) introduces a monotonicity axiom which leads to an answer in the affirmative: for all $x; y \in X$, $\{x; y\}$ offers strictly more freedom than each of the two sets $\{x\}$ and $\{y\}$. Sen (1988, 1991, 1993) conditions the answer to what the decision maker actually prefers. If x is preference-wise better than y, then adding the option of doing x to the set $\{y\}$ enhances the extent of individual freedom. Sugden (1999) makes a step further and refine the property as follows: the addition of an option is freedom-enhancing if, and only if, it makes a relevant choice, where relevance is evaluated in terms of the preference orderings of a reasonable person. If reasonability is abandoned, as we have argued it should be, the addition of a new option enlarges the chooser's extent of freedom if it makes a relevant choice in the light of the set of potential preference relations that he may uphold. So, for example, if the president is confronted with a fourth course of action consisting of surrendering to the claims of the other country and loosing his face, this is probably going to be an ineligible possibility and therefore should not enlarge his autonomy freedom. Once again, the idea is that the "irrelevance" of an option or a course of action can be determined by the limited set of available preference orderings; i.e., by the inability to choose that option. This is the idea expressed by axioms AUA and ADA.

Each of the principles introduced so far | the indifference between no-freedom situations, the addition of relevant alternatives, the addition of irrelevant alternatives and the transitivity of the relation $\succ$ | seems reasonable and normatively appealing; however, as noted by Jones and Sugden (1982) and Sugden (1999), jointly considered they are logically inconsistent. This inconsistency is robust enough to extend to rankings defined over the space of opportunity situations as we now show.

Proposition 4.1 No transitive binary relation $\succ$ over $P(X) \times P(\succsim)$ exists that satisfies INF, AUA and ADA.

Proof. Consider any $x;y \in X$ and suppose that xRy for all $R \in \¦$: Then, by AUA,

$$8\¦_i \in P(\¦);(fx;yg;\¦_i) \hat{A} (fyg;\¦_i)$$

and, by ADA,

$$8\¦_i \in P(\¦);(fx;yg;\¦_i) \succ (fxg;\¦_i)$$

hence, by transitivity,

$$8\¦_i \in P(\¦);(fxg;\¦_i) \hat{A} (fyg;\¦_i):$$

But, by INF,

$$8\¦_i \in P(\¦);(fxg;\¦_i) \succ (fyg;\¦_i):$$

■■

What is the root of this incompatibility? Consider the example given in the proof. If there is an option x which is universally considered as better than another option y; axiom AUA states that the addition of x to the set fyg is freedom enhancing. But this applies even if the distance between x and y is as large as to make the choice problem trivial, making reliance on one's own judgement and personal characteristics unnecessary. So, this is in contrast with the intuition about autonomy freedom that this paper is supporting. In other words, it would be like saying that giving to a chess player the possibility of moving the knight when he has to exchange queens to avoid a mate is enlarging his possibilities, while it is clearly not.

Sugden (1999) makes this point. However, as a way of overcoming the impossibility result, he instead proposes to reject the principle of no-choice, expressed in our context by axiom INF. On the contrary, we choose to reformulate axioms AUA in order to accommodate for the objection expressed above. The idea is the following: an option is relevant if it expands the choice set open to the individual, given the contextual circumstances. That is, in order to be freedom enhancing, the new option not only has to be preferred to the existing ones by some preference ordering available to the individual - as in axiom AUA; it also should not be \so good" as to make the choice problem trivial. Hence, the addition of x to the set fyg is freedom enhancing if the following two conditions are satis¯ed: (i) x is preferred to y by some preference ordering available to the individual, which ensures that x is in the choice set elicited from the opportunity set fx;yg; (ii) the distance between x and y is not as large as to make the choice problem trivial, which implies that y, after the addition of x; is still an eligible option . We formulate this requirement in the next axiom, by stating that the addition of an option is freedom enhancing if, and only if, it expands the choice set open to the individual.

Axiom 4.4 Addition of relevant alternatives (ARA).

$8A \in P(X);8\¦_i \in P(\¦);8x \in X \setminus \max_i(A);[\max_i(A) \succ \max_i(A \cup fxg)] \Rightarrow (A \cup fxg;\¦_i) \hat{A} (A;\¦_i)$.

An inspection of the argument used in the previous theorem readily shows that this axiom is compatible with the other properties introduced so far. Moreover, as we shall see, it leads to the characterization of a unique ranking of opportunity situations.

Our last property is intuitively related to the independence axiom originally introduced in Suppes (1987) and to the composition property introduced in Sen (1991).

Axiom 4.5 Composition (COM).

$8A;B;C;D \in P(X);8\succsim_i;\succsim_j \in P(\succsim)$, such that $B \cap D = \emptyset$; $A \cap C = Z$; $Z \cap \max_i(A \cup C) = \emptyset$ and $\max_j(B \cup D) = (B \cup D)$,

$$[(A;\succsim_i) \circ (B;\succsim_j) \text{ and } (C;\succsim_i) \circ (D;\succsim_j)] \Rightarrow (A \cup C;\succsim_i) \circ (B \cup D;\succsim_j):$$

Composition requires that, under certain circumstances, joining sets together does not alter their relative ranking in terms of autonomy freedom.

5 Measuring autonomy freedom

5.1 A ranking of opportunity situations

On the basis of these axioms we are able to prove the following proposition which establishes a unique ranking for opportunity situations.

Definition 5.1 Autonomy Ordering (AUT)

For all $A;B \in P(X)$; for all $\succsim_i;\succsim_j \in P(\succsim)$;

$$(A;\succsim_i) \circ_{AUT} (B;\succsim_j) \Leftrightarrow |\max_i(A)| \geq |\max_j(B)|:$$

Proposition 5.1 $\circ$ satisfies INF, ARA and COM if and only if $\circ = \circ_{AUT}$

According to proposition 5.1, an individual i enjoys more autonomy freedom than another individual j if and only if the choice set that his preference profiles elicit from his own opportunity set A has at least as many elements as the choice set that j can elicit by means of his own preference profiles from B.

To prove proposition 5.1 we first state and prove the following lemma.

Proof.

Lemma 5.1 If $\circ$ satisfies INF and COM, then, $8A \in P(X);8\succsim_i \in P(\succsim)$,

$$(A;\succsim_i) \sim (\max_i(A);\succsim_i):$$

Proof. If $\max_i(A) = A$; then the result clearly follows. If not, suppose $|\max_i(A)| = \{a_1;...;a_g\}$ and let $\max_i(A) = \{a_1;...;a_g\}$ and $A \setminus \max_i(A) = \hat{A}$: Now, $\max_i\{a_1\} \cup \hat{A} = \max_i(\{a_1\}) = \{a_1\}$ and $\max_i\{a_2\} \cup \hat{A} = \max_i(\{a_2\}) = \{a_2\}$: Hence by INF,

$$\{a_1\} \cup \hat{A};\succsim_i \sim (\{a_1\};\succsim_i)$$

and

$$\{a_2\} \cup \hat{A}, \succsim_i \sim (\{a_2\}, \succsim_i).$$

Clearly, $\{a_1\} \cap \{a_2\} = \emptyset$, $\max_i(\{a_1\} \cup \{a_2\}) = \{a_1\} \cup \{a_2\}$, $\{a_1\} \cup \hat{A} \cap \{a_2\} \cup \hat{A} = \hat{A}$ and $\hat{A} \cap \max_i \{a_1\} \cup \hat{A} \cup \{a_2\} \cup \hat{A} = \emptyset$. Hence we can apply axiom COM and obtain,

$$\{a_1\} \cup \{a_2\} \cup \hat{A}, \succsim_i \sim (\{a_1\} \cup \{a_2\}, \succsim_i).$$

By considering successively $a_3, a_4, \ldots, a_g$, and applying INF and COM repeatedly, we finally obtain

$$\max_i(A) \cup \hat{A}, \succsim_i \sim (\max_i(A), \succsim_i)$$

or

$$(A, \succsim_i) \sim (\max_i(A), \succsim_i).$$

■ ■ ■

We are now in the position to prove proposition 5.1.

Proof. Necessity is straightforward. We therefore prove sufficiency. To start with, we show that

$$|\max_i(A)| = |\max_j(B)| \Rightarrow (A, \succsim_i) \sim (B, \succsim_j). \quad (1)$$

Suppose $|\max_i(A)| = |\max_j(B)| = g$. It follows that, $\max_i(A) = \{a_1, \ldots, a_g\}$ and $\max_j(B) = \{b_1, \ldots, b_g\}$. Using INF, $(\{a_1\}, \succsim_i) \sim (\{b_1\}, \succsim_j); (\{a_2\}, \succsim_i) \sim (\{b_2\}, \succsim_j)$ and $\{a_1\} \cap \{a_2\} = \emptyset$. Now, $\max_i\{a_1, a_2\} = \{a_1, a_2\}$, so we can use axiom COM to yield:

$$(\{a_1, a_2\}, \succsim_i) \sim (\{b_1, b_2\}, \succsim_j).$$

By INF, $(\{a_3\}, \succsim_i) \sim (\{b_3\}, \succsim_j)$; by COM,

$$(\{a_1, a_2, a_3\}, \succsim_i) \sim (\{b_1, b_2, b_3\}, \succsim_j)$$

and so on. Finally we have:

$$(\{a_1, \ldots, a_g\}, \succsim_i) \sim (\{b_1, \ldots, b_g\}, \succsim_j),$$

i.e.,

$$(\max_i(A), \succsim_i) \sim (\max_j(B), \succsim_j). \quad (2)$$

Since $\succsim$ satisfies INF and COM, we can apply Lemma 4.1 and obtain

$$(A, \succsim_i) \sim (\max_i(A), \succsim_i). \quad (3)$$

and

$$(B; \succsim_j) \succ (\max_i(B); \succsim_j): \tag{4}$$

Now, (2), (3), (4), and transitivity of $\succ$ imply $(A; \succsim_i) \succ (B; \succsim_j)$:

Now we show that

$$|\max_i(A)| > |\max_j(B)| \Rightarrow (A; \succsim_i) \succ (B; \succsim_j):$$

Suppose $|\max_i(A)| = g + t$ and $|\max_j(B)| = g$. So, $\max_i(A) = \{a_1; \ldots; a_{g+t}\}$ and $\max_j(B) = \{b_1; \ldots; b_g\}$. Now, $\max_i(\{a_1; \ldots; a_g\}) = \{a_1; \ldots; a_g\}$. Hence, by (1),

$$(\{a_1; \ldots; a_g\}; \succsim_i) \succ (B; \succsim_j) \tag{5}$$

Now, $\max_i(\{a_1; \ldots; a_{g+1}\}) = \{a_1; \ldots; a_{g+1}\}$. By ARA,

$$(\{a_1; \ldots; a_{g+1}\}; \succsim_i) \succeq (\{a_1; \ldots; a_g\}; \succsim_i)$$

and, by (5) and transitivity of $\succeq$,

$$(\{a_1; \ldots; a_{g+1}\}; \succsim_i) \succ (B; \succsim_j):$$

By adding $a_{g+2}; \ldots; a_{g+t}$ successively, and by using ARA repeatedly, we have

$$(\{a_1; \ldots; a_{g+t}\}; \succsim_i) \succeq (B; \succsim_j)$$

i.e.,

$$(\max_i(A); \succsim_i) \succeq (B; \succsim_j): \tag{6}$$

We know from Lemma 4.1 that

$$(A; \succsim_i) \sim (\max_i(A); \succsim_i):$$

Clearly, $(A; \succsim_i) \sim (\max_i(A); \succsim_i)$, (6) and transitivity of $\succ$ imply $(A; \succsim_i) \succeq (B; \succsim_j)$. ■■

6 On some useful properties of the autonomy ranking

6.1 Comparability made easier

Opportunity situations deliver greater information than opportunity sets. This is not without consequences, in particular when issues of comparability are confronted. Comparisons of this sort are quite common among scholars interested on freedom. In fact, they go well back in the past. Hobbes, for example, writes that

> "liberty is in some places more, and in some less, and in some times more, in other times less, according as they that have the sovereignity shall think most convenient" (chp. 21).

If freedom rankings are constructed on the space of opportunity sets, unless we agree on some objective way of counting alternatives, both intertemporal (intrapersonal) and international (interpersonal) comparisons are hard to make. Even in this case, difficulties stem from the fact that some options which are available at a given point in time (space) are not available at another, making the interpretation of the comparison even murkier. Using opportunity situations solves these difficulties. The reason is that the comparison is now between pairs of opportunity set and sets of potential preferences. The latter work as some sort of common denominator which standardizes the comparison between opportunity sets in terms of whether the person who has to take his decision may shape his own preferences, irrespective of time and space considerations. In other words, rankings over opportunity situations compare the position of each decision maker from his own perspective, at the pre-deliberation stage.

6.2 Generalizing some previous results

Notice the following corollaries:

Corollary 6.1 Let ^{o} satisfy INF, ARA and COM.
Then, $\forall A \in P(X); \forall \succsim_i, \succsim_j \in P(\succsim)$,

$$(A; \succsim_i) \, ^{o} \, (A; \succsim_j) \Leftrightarrow |max_i(A)| \geq |max_j(A)|.$$

Corollary 6.2 Let ^{o} satisfy INF, ARA and COM.
Then, $\forall A \in P(X); \forall \succsim_i, \succsim_j \in P(\succsim)$,

$$(A; \succsim_i) \, ^{o} \, (A; \succsim_j) \Leftrightarrow |(A - max_i(A))| \leq |(A - max_j(A))|.$$

On the basis of these results, some comparisons with what it has already been proved in the literature can be established.

Remark 6.1 Suppose that the set of preference orderings $\succsim$ satisfies a "richness" assumption, such that $\forall A \in \succsim(X); max(A) = \{x \in A : \neg \exists y \in A$ such that yRx for some $R \in \succsim\} = A$. Then, if $\forall i \in N; \succsim_i = \succsim$, all possible preference profiles can be hold by the individuals, and the ranking established in theorem 5.1 coincides with the Simple Cardinality-based Ordering of Pattanaik and Xu (1990).

A second remark concerns the relationship between our result and a ranking of opportunity sets based on the reasonable preference profiles.

Remark 6.2 If $\forall i \in N; \succsim_i = \succsim^{*}$, where $\succsim^{*}$ stands for the set of reasonable preference profiles *à la* Sugden (1999) and Pattanaik and Xu (1998), then the ranking established in proposition 5.1 coincides with the rule characterized by Pattanaik and Xu (1998).

The last remark applies to the case in which the agents may access one preference pro¯le only.

Remark 6.3 If $\forall i, \succsim_i$ is a singleton and the preference relation is linear, then $\forall A \in P(X), \forall \succsim_i \in P(\succsim), \max_i(A) = \{x\}$ and $\forall A, B \in P(X), \forall \succsim_i, \succsim_j \in P(\succsim)$) $(A, \succsim_i) \succ (B, \succsim_j)$.

7 Conclusion

Some attempt has been made within the freedom of choice literature at measuring freedom as autonomy. The main ranking which has been constructed so far adheres to the view that autonomy has to do with having relevant choices. In this paper we have proposed a di®erent point of view: autonomy freedom should not be linked with having to make relevant choices but with having access to a number of preference pro¯les.

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