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PRESUMPTIVE TAXATION, MARKETS, AND REDISTRIBUTION

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Abstract

Aim of this paper is to make a comparison between redistribution policies based on taxation of *effective* income and those based on taxation of *presumptive* income. To this end, we set up an occupational choice model in which two kinds of taxpayers exist: entrepreneurs, who are able to engage in costly avoidance of the income tax, and workers (employees), who cannot avoid taxation. In comparing effective and presumptive taxation, we focus on a particular kind of trade-off. As for effective income taxation, we recognize, on the one hand, that it is costly to enforce, since it gives room to tax dodging. On the other hand, the market plays a role in smoothing the tax avoidance induced inequities: since taxpayers can choose between the two types of jobs, then the labor market tends to adjust gross earnings in the two sectors so as to take account for different levels of tax dodging. On the contrary, presumptive income taxation is less demanding than effective taxation in terms of administrative costs. It nevertheless poses relevant, albeit different, equity issues, for "systematic" errors are introduced in the assessment of tax liabilities. Interestingly, it emerges that the market tends to "punish" presumptive taxation, by limiting its ability of affecting income distribution, since the market tends to adjust gross earnings so as to counterbalance the errors (inequities) generated by presumptive taxation.

Keywords: Tax avoidance, Optimal taxation, Tax enforcement, Presumptive taxation.

JEL: H21, H26

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1 Introduction

There is a strong feeling, among citizens and policy makers, that tax evasion and tax avoidance frustrate the efforts to attain equity goals through income taxation. This concern is particularly serious when the opportunities for tax evasion or avoidance are not uniformly distributed among taxpayers, for tax dodging is then likely to bring about both vertical and horizontal inequities in the distribution of tax burdens. The empirical evidence shows indeed that there are some "hard to tax" groups, such as self-employed workers, professionals, small business, for which tax dodging is a relatively lucrative and riskless activity. On the other hand, there are taxpayers, such as employees of large corporations, pensioners, that are entirely unable to escape their tax obligations.

To face this problem, policy makers may try to improve the efficiency of the tax collection system. This goal can be achieved by increasing the penalties for tax evasion and/or the frequency and the quality of tax audits, or by reforming the tax code so as to reduce the opportunities of exploiting the so called "tax loopholes". Of course, any potential gain so obtained in terms of a more equitable distribution of tax burdens must be confronted with the increased costs in tax administration. In some cases, however, these traditional "weapons" are almost entirely ineffective in controlling tax evasion or avoidance. In fact, there are circumstances in which tax dodging is impossible to detect, monitor and verify. For instance, a sale "under the counter" for cash does not leave any direct evidence in the accounting books; at the most, indirect evidence can be obtained if the taxpayer is unable to make "under the counter" purchases of inputs.

The awareness of these problems has lead some governments, in both industrialized and developing countries, to rely on various forms of *presumptive* taxes, in which tax liabilities are computed using indirect methods for assessing the effective tax bases. In the real world, presumptive taxes are used in lieu of taxes on actual income in many situations; for instance, when income is difficult to measure —as in the agricultural sector— or when taxpayers cannot be obliged to keep records of their activity —as in the case of small individual business— or finally when the taxing agency lacks the expertise to administrate an income tax.[1] One can conceive many types of presumptive taxes; however, the basic idea is simple. A set of variables that (a) are highly correlated

[1] Presumptive taxes are used in France, Israel and in some Latin American and developing countries. See Tanzi and Casanegra (1989) for a survey.

with the effective tax base, (b) tax authorities can monitor at low cost and (c) taxpayers may find it difficult to "manipulate", are used to estimate the effective tax base. For instance, wages, telephone and electricity bills, the number of employees, the size of the store, can be used to estimate revenues and thus income (profits). Of course, presumptive taxes are not a panacea, as they are likely to reintroduce horizontal and vertical inequities from the back door, for the obvious reason that presumptive tax bases are imperfectly correlated with effective tax bases. Indeed, presumptive taxation turns out to be a kind of effective taxation with systematic measurement errors (Stern, 1982).

Aim of this paper is to make a comparison between redistribution policies based on taxation of *effective* income and redistribution policies based on taxation of *presumptive* income. To this end, we set up an occupational choice model in which two kinds of taxpayers exist: entrepreneurs, who are able to engage in costly avoidance of the income tax, and workers (employees), who cannot avoid taxation. In the design of tax policy, both equity and efficiency arguments are considered; as for the latter, however, attention is restricted to administrative and tax avoidance costs; incentive effects of taxation on labor supply are ignored.[2]

In comparing effective and presumptive taxation, we focus on a particular kind of trade-off. As for effective income taxation, we recognize, on the one hand, that it gives room to tax dodging, and for this reason is costly to administer and to enforce. On the other hand, we show that the market may play a role in smoothing, at least to some extent, the tax avoidance induced inequities. In a nutshell, the idea is that since taxpayers can choose between the two types of jobs, one in which tax avoidance is feasible, and one in which tax avoidance is unfeasible, then the labor market will tend to adjust gross earnings in the two sectors so as to take account for different levels of tax dodging.[3] On the contrary, presumptive income taxation is less demanding than effective taxation in terms of administrative and enforcement costs. It nevertheless poses relevant, albeit different, equity issues, for "systematic" errors are introduced

[2] A recent contribution on optimal linear income taxation with endogenous occupational choice and labor supply is by Parker (1999); however, he assumes that tax avoidance rates are exogenous and does not consider presumptive taxation.

[3] The idea that markets may drive out the inequities associated to tax dodging has been stressed by Kaplow (1996).

in the assessment of tax liabilities. Interestingly, and contrary to effective taxation, it emerges that the market tends to "punish" presumptive taxation, by limiting its ability of affecting income distribution. The reason is simple: market forces tend to adjust gross earnings so as to counterbalance the errors (inequities) generated by presumptive taxation.

Our analysis of optimal income taxation with tax avoidance is closely related to Slemrod (1994), though our framework is wider as, building on Boadway *et al.* (1991), it allows for two groups of taxpayers and for endogenous occupational choice.[4] Other relevant contributions on optimal income taxation with tax avoidance or evasion are Sandmo (1981), Pestieau and Possen (1991), Cremer and Gahvari (1994, 1995) and Schroyen (1997); none of them, however, addresses the issue of presumptive taxation.[5]

One important fact to bear in mind about our model is that we model tax dodging in terms of what is usually known as *tax avoidance*, as opposed to *tax evasion*. Tax avoidance is costly and riskless, whereas tax evasion is risky because taxpayers face the possibility of an audit and, in case of discovered tax evasion, are subject to the payment of a fine. This distinction, due to Cowell (1990, pp. 10–14), differs from the one popularly used, which is based on a *legal* criteria: tax evasion is an illegal activity whereas tax avoidance is a legitimate activity. With the former, the taxpayer breaks the law; with the latter, he or she takes advantage of special provisions or loopholes in the tax code. According to Cowell's definition, therefore, tax avoidance can be either legal or illegal: what only matters is that the taxpayer can get away with it without taking any risk.[6]

In our model the government is supposed to solve a standard optimal taxation problem, by which it chooses tax parameters in order to maximize a social welfare function under the constraint of raising a certain revenue amount.[7] Our main goal is

[4] Related contributions on optimal taxation in occupational choice models, but without tax avoidance or evasion, are by Kanbur (1981), Kihlstrom and Laffont (1983) and Moresi (1998).

[5] Watson (1985), Kesselman (1989), Jung, Snow and Trandel (1994) and Trandel and Snow (1999) examine under different perspectives the interaction between labor market, income taxation, tax dodging and the size of the underground economy. There is however no optimal taxation analysis.

[6] The standard model of tax evasion as a choice under uncertainty is due to Allingham and Sandmo (1972). For an early theoretical contribution on tax avoidance, see Cross and Shaw (1982).

[7] An interesting element of our framework is the explicit consideration of tax enforcement activity. We suppose that tax enforcement causes a direct deadweight loss, as people employed by the government

to investigate whether presumptive taxation can be used for redistributive purposes. To this end, we focus on two kinds of presumptive taxes. In the first, which is labelled *Occupational Choice Presumptive Taxation* (**OC-PT**), tax liabilities are based on the type of occupation; **OC-PT** is thus a very simple form of presumptive taxation. In the second, which is labelled *Input Costs Presumptive Taxation* (**IC-PT**), tax liabilities are assessed on production costs that are correlated (on average) with income; **IC-PT** is thus a more sophisticated tax instrument than **OC-PT**.

The comparison between **OC-PT** and **IC-PT** brings about a couple of interesting results. The first is that **OC-PT** is useful for redistributive purposes no matter whether an income tax is levied; on the contrary, for **IC-PT** to be effective, it must be accompanied by a traditional income tax that taxpayers' are able to avoid partially. Hence, *simple* presumptive taxes can be used intead of income taxes (their redistributive power is likely to be, however, very low), whereas *complex* presumptive taxes cannot represent a substitute for an income tax. The second result shows that, when effective, **IC-PT** is on average regressive, as it takes the form of a subsidy, not a tax, on production costs.

The paper is organized as follows. Section 2 introduces the model, the tax instruments and then derives market equilibrium and the comparative statics results. The tax problem is illustrated in Section 3. The role of presumptive taxation in the presence of income taxation is examined in Section 4. Section 5 offers some concluding remarks.

2 The model

The model we employ is based on Galmarini (2000). On the one hand, we extend that model by assuming that the individuals differ along two dimensions; on the other hand, for reasons of tractability, we restrict our attention to full employment equilibria only.[8]

There is a large population of individuals, subdivided into two groups of equal size (normalised to unity), identified by a parameter $s = h, l$, $h \geq l > 0$. Each individual in

to enforce the tax code cannot be hired by the private sector and produce consumption goods. On the contrary, current literature simply assumes that tax enforcement is costly and absorbs part of government's gross revenue. In the present version of the paper, however, we take the tax administration activity as given.

[8] Galmarini (2000) discusses in some detail the question of the existence of the equilibrium and allows also for unemployment equilibria. In the present paper, we will briefly refer to the latter only when it seems relevant to do so, without offering a systematic treatment.

each group is characterized by a parameter n. We take n to be a continuous variable ranging from 0 to $N > 0$ (both included), with density $g(n) > 0$; the distribution of n is the same in both groups. We can think of n as representing entrepreneurial skill and of s as representing the size of the firm, which in turn may depend on some endowment of capital goods: two individuals with the same entrepeneurial ability may differ in terms of endowments, and therefore may own firms of different size.

Each agent can choose among two options: to become an *entrepreneur* (**e**) or to become a *worker* (**w**). An individual that chooses **e** produces n units of a consumption good hiring one unit of labor, for which it pays a wage $w \geq 0$. Thus, if a type-n agent belonging to the s-group becomes an entrepreneur, her gross income is $s\,(n - w)$, where the skill times the size of the firm, sn, represents revenues (the output price is normalized to unity) and sw represents labor costs.[9] An individual that chooses **w** supplies one unit of labor to an entrepreneur. Workers can be hired also by the public sector to enforce taxes. Wages in the private and the public sector are the same. There is no labor-leisure choice on the part of both entrepreneurs and workers. Also, shifting between occupations is costless.

Within this setting, there are three possible *tax bases* that the government can use for redistributive purposes: occupational choice, income and input costs. As tax authorities are assumed to observe the occupational choice (**e** or **w**) of each individual, they can levy (income unrelated) poll taxes to workers and entrepreneurs. This is a very simple (and crude) form of presumptive taxation that assesses the ability to pay on the basis of the type of occupation only; hence we label it **Occupational Choice Presumptive Taxation** (**OC-PT**). As for income, tax authorities observe workers' wage directly, whereas they have no direct knowledge of the skill level n, and hence of income, of each entrepreneur. Hence, while workers are unable to avoid income taxation, taxation of entrepeneurs' income must rely on *reported* income, which may differ from *actual* income. As a result, entrepeneurs will engage in tax avoidance, i.e. will not report their actual income level, so as to gain a more favourable tax treatment. **Income taxation** (**IT**) is restricted to be proportional, and we also assume that the marginal tax rate cannot be conditioned on the type of occupation, i.e. it must be uniform for workers and entrepreneurs. Finally, since we assume that government

[9] The idea is that an n-entrepeneur can activate s production processes, each yielding n units of output and requiring one unit of labour.

can observe labour costs, sw, a second, and more sophisticated, form of presumptive taxation can be used to separately tax type-l and type-h entrepreneurs, provided that some correlation between sw and $s(n-w)$ exists. This type of taxation, which is also resticted to be proportional, is labelled **Input Costs Presumptive Taxation** (**IC-PT**).

2.1 Agents' payoffs

Let $y^{\mathbf{i}}$, $\mathbf{i} \in \{\mathbf{w}, \mathbf{e}\}$, be net income (consumption) associated to option **i**. The following assumption characterizes agents' preferences.

Assumption 1 (**i**) *If* $y'^{\mathbf{i}} > y''^{\mathbf{i}}$, $\mathbf{i} \in \{\mathbf{w}, \mathbf{e}\}$, *then* $y'^{\mathbf{i}} \succ y''^{\mathbf{i}}$. (**ii**) *If* $y^{\mathbf{i}} > y^{\mathbf{j}}$, $\mathbf{i}, \mathbf{j} \in \{\mathbf{w}, \mathbf{e}\}$, $\mathbf{i} \neq \mathbf{j}$, *then* $y^{\mathbf{i}} \succ y^{\mathbf{j}}$. (**iii**) *If* $y^{\mathbf{w}} = y^{\mathbf{e}}$, *then* $y^{\mathbf{w}} \succ y^{\mathbf{e}}$.

A1(i) simply says that, within each option, a higher income is preferred to a lower one. A1(ii) says that whenever two options give a different income then the one with the higher income is preferred. A1(iii) strikes a choice whenever the two options give the same income: for equal income, **w** is preferred to **e** (but, nothing of substance changes if we made the opposite assumption).

To define entrepreneurs' net income, let $1-a$ be the fraction of reported income; then $a \in [0,1]$ is the fraction of avoided income. Let $b \in [0,1)$ be the proportion of people employed as tax collectors, i.e. the size of the bureaucracy. Tax avoidance costs are assumed to be linear in gross income: the per-unit-of-gross-income tax-avoidance-costs, $c(a,b)$, depend on the fraction of concealed income and on the level of tax enforcement.[10] $c(.)$ satisfies the following restrictions (subscripts denote partial derivatives):

Assumption 2 (**i**) $c(0,b) = 0$, $c > 0$ *for* $a > 0$. (**ii**) $c_a(0,b) = 0$, $c_a > 0$ *for* $a > 0$, $c_a(1,b) > 1$, $c_{aa} > 0$. (**iii**) $c_b(0,b) = c_{ab}(0,b) = 0$, $c_b > 0$, $c_{ab} > 0$ *for* $a > 0$.

A2(i): Full reporting is costless, whereas tax avoidance is always costly (even when $b = 0$). A2(ii): The marginal cost of concealing the first unit of income is zero; then the marginal cost is positive and increasing in the level of concealed income; the marginal cost of concealing the last unit of income is greater than one. A2(iii):

[10] The implicit assumption is that the tax avoidance technology exhibits constant returns to scale — *cf.* Boadway *et al.* (1994). On tax avoidance costs, see also Slemrod (1998).

Tighter enforcement of the tax code makes tax avoidance more costly (provided that tax avoidance is positive), as it increases both unit and marginal costs.

Provided that $n - w \geq 0$,[11] net income of a type-n entrepreneur is thus equal to

$$\alpha + [1 - t + ta - c(a, b)]\, s(n - w) - \tau sw, \tag{1}$$

where $t \in [0, 1]$ is the rate of **IT**, α the poll subsidy (a tax when negative) related to **OC-PT**, and τ the rate of **IC-PT**.

By A1(i), tax avoidance is chosen by maximizing (1) with respect to a.[12] The first order condition for an interior solution is

$$t = c_a(a, b). \tag{2}$$

Solving (2), let $\hat{a}(t, b)$ be the optimal choice for a and let $\hat{c} = c(\hat{a}, b)$. The following Lemma characterises the agent's behaviour and notes some results which we will use later on.

Lemma 1 *Under A2:* (**i**) $\hat{a}(0, b) = 0$, $\hat{a} \in (0, 1)$ *for* $t \in (0, 1]$. (**ii**) $\hat{a}_t = 1/\hat{c}_{aa} > 0$, $\hat{a}_b = -\hat{c}_{ab}/\hat{c}_{aa} < 0$ *for* $\hat{a} > 0$. (**iii**) $t\hat{a} \geq \hat{c}$, $1 - t + t\hat{a} - \hat{c} > 0$.

Proof. Part (i) comes directly from A2(ii) and the foc (2). Part (ii) comes from totally differentiating (2) and using A2(ii)–(iii). Part (iii) comes from the fact that, if $t > 0$ then $\int_0^{\hat{a}} [t - c_a(a, b)] da = t\hat{a} - c(\hat{a}, b) > 0$, since, from (2), $t > c_a$ for $a \in [0, \hat{a})$; if $t = 0$ then $\hat{a} = \hat{c} = 0$. Finally, $1 - t + t\hat{a} - \hat{c} > 0$, since $1 - t > 0$ for $t < 1$ and $t\hat{a} - \hat{c} > 0$ for $t > 0$. ■

Tax avoidance is zero when income is not taxed at the margin, and positive when taxed; 100% tax avoidance never occurs. The proportion of avoided income is increasing in the tax rate and is decreasing in the level of tax enforcement. Part (iii) shows that the *effective* marginal tax rate, $t - t\hat{a} + \hat{c}$, is smaller than the *statutory* tax rate, t.

With a optimally chosen, net income associated to $\mathbf{e}$ is thus equal to

$$y^{\mathbf{e},s} = \alpha + (1 - t + t\hat{a} - \hat{c})s(n - w) - \tau sw. \tag{3}$$

[11] Since we assume that losses are not subsidized at the margin, we may assume that a type-n never chooses $\mathbf{e}$ whenever $n - w < 0$. See also the remarks about the non-negativity constraints on gross income introduced in the tax problem (22) below.

[12] Entrepreneurs take w as given when maximizing net income. That is, they are price takers in the labour market.

Workers and tax collectors cannot avoid paying the income tax. Letting β be the poll subsidy related to **OC-PT**, their net income is

$$y^{\mathbf{w}} = \beta + (1-t)w. \tag{4}$$

Notice that when $\alpha = \beta$ **OC-PT** becomes ineffective in taxing entrepreneurs and workers differently; with $\alpha = \beta > 0$ and $t > 0$, we obtain a progressive linear income tax.[13]

2.2 Occupational choice and market equilibrium

To shorten notation in the following formulae, let $\hat{\theta} \equiv 1 - t + t\hat{a} - \hat{c}$ and $\theta \equiv 1 - t$. Occupational choices are then defined by the following pair of *arbitrage equations:*

$$\alpha + \hat{\theta}s(n^s - w) - \tau sw = \beta + \theta w, \quad s = h, l. \tag{5}$$

where n^s is the marginal skill within group s. These conditions establish that, for each s-group, the net income of the marginal worker (the most able among them) must equal the net income of the marginal entrepreneur (the least skilled along the n dimension). Thus, by 1(ii) and 1(iii), those individuals whose ability goes from 0 to n^s (included) choose **w**, while those whose ability goes from n^s (excluded) to N choose **e**.

The *market clearing equation* is written

$$G(n^h) + G(n^l) = [1 - G(n^h)]h + [1 - G(n^l)]l + b \tag{6}$$

where the l.h.s. represents total labour supply, and the r.h.s. total labour demand (from the private and public sectors).

[13] Since all workers earn the same income, notice also that there is no loss of generality in using the same marginal tax rate, t, to tax entrepreneurs' and workers' income. Formally, suppose that the marginal tax rate is $t^{\mathbf{e}}$ and $t^{\mathbf{w}}$, for entrepreneurs and workers respectively. One can then show that it is always possible to reduce (or increase) $t^{\mathbf{w}}$ and β while leaving agents' net incomes, as well as government's revenue, unaffected. This means that there are several equivalent linear tax structures that could have been employed in place of (α, t) for entrepreneurs and (β, t) for workers. An alternative is $(\alpha, t^{\mathbf{e}})$ for entrepreneurs and $(\alpha, t^{\mathbf{w}})$ for workers, i.e. a uniform lump sum subsidy with differentiated tax rates. Another one is $(\alpha, t^{\mathbf{e}})$ for entrepreneurs and $(\beta, 0)$ for workers, i.e. an income tax on the former and a lump sum tax on the latter. While formally equivalent to (α, β, t), these alternatives do not allow a clear distinction between taxation of income (**IC**) and taxation of occupational choice (**OC-PT**).

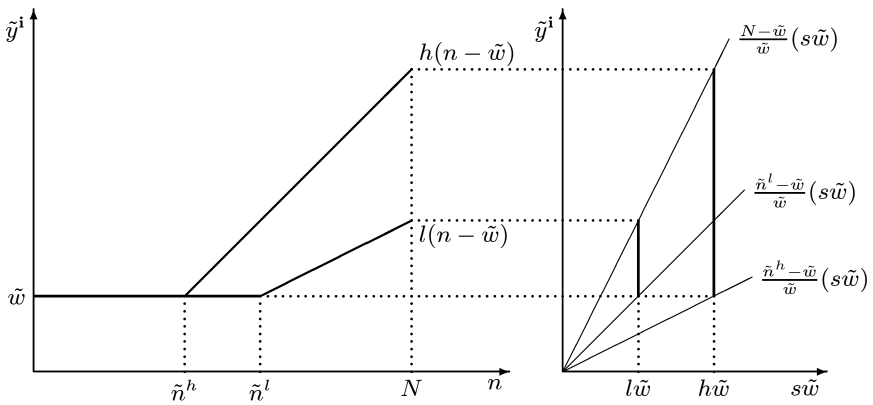

Figure 1: Income distribution: Model with two-dimensional skill

Eqs. (5) and (6) determine the equilibrium wage rate, $\tilde{w}$, and the equilibrium values of the marginal skills, $\tilde{n}^s$, $s = h, l$, as a function of the policy variables α, β, t, τ and b. Market equilibrium is depicted in Figure 1 in the case of no taxation. For the time being, consider the left-graph only. This shows agents' net income as a function of skill level (n) and firm size (s). More type-h than type-l individuals become entrepreneurs ($\tilde{n}^h < \tilde{n}^l$) and conversely more type-l than type-h individuals are workers, as type-h, for any given n, are more productive than type-l.

It is also immediate to see that, with no government intervention, the market outcome is Pareto efficient, since aggregate output is maximized. Let $Y = \int_{n^l}^{N} l\, ng(n)dn + \int_{n^h}^{N} lhg(n)dn$ be aggregate output. By maximizing Y with respect to n^l and n^h under the constraint of full employment, $G(n^l) + G(n^h) = l[1 - G(n^l)] + h[1 - G(n^h)]$ one obtains that at the optimum (n_0^l, n_0^h)

$$\frac{hn_0^h}{1+h} = \frac{l\, n_0^l}{1+l} \tag{7}$$

This condition says that (marginal) type-l entrepreneurs are as productive as (marginal) type-h entrepreneurs, since $\dfrac{sn_0^s}{1+s}$ is the ratio between output, sn_0^s, and labor inputs, 1 entrepreneur and s workers. In a market equilibrium without taxation, i.e. $\alpha = \beta = \tau = t = 0$ (the latter implies $\hat{\theta} = \theta = 1$), from (5) one gets $l(\tilde{n}^l - \tilde{w}) = \tilde{w}$ and $h(\tilde{n}^h - \tilde{w}) = \tilde{w}$, which imply (7). Hence, with no government intervention, market equilibrium is efficient.

2.3 Comparative statics

We now determine the comparative statics derivatives. While a more detailed derivation is given in the Appendix, we state here the results.[14] As for $\tilde{w}$, we get

$$\frac{\partial \tilde{w}}{\partial \tau} = -\frac{(1+l)\tilde{g}^l + (1+h)\tilde{g}^h}{\Delta}\tilde{w} < 0 \tag{8}$$

$$\frac{\partial \tilde{w}}{\partial \beta} = -\frac{\partial \tilde{w}}{\partial \alpha} = -\frac{(1+l^{-1})\tilde{g}^l + (1+h^{-1})\tilde{g}^h}{\Delta} < 0 \tag{9}$$

$$\frac{\partial \tilde{w}}{\partial t} = -\frac{[(1-\hat{a})l(\tilde{n}^l - \tilde{w}) - \tilde{w}](1+l^{-1})\tilde{g}^l + [(1-\hat{a})h(\tilde{n}^h - \tilde{w}) - \tilde{w}](1+h^{-1})\tilde{g}^h}{\Delta} \tag{10}$$

where

$$\Delta = (h^{-1}\theta + \hat{\theta} + \tau)(1+h)\tilde{g}^h + (l^{-1}\theta + \hat{\theta} + \tau)(1+l)\tilde{g}^l > 0 \tag{11}$$

Since τ is levied on entrepreneurs, its increase will result in a lower wage because, were $\tilde{w}$ to stay fixed, some entrepreneurs (the least skilled among them) would try to shift to a salaried job, causing thus an excess of labor supply.

While β reduces $\tilde{w}$, α increases it. For a given wage rate, an increase in the lump sum subsidy to workers would result in an excess of labor supply as some entrepreneurs (the least skilled among them) would now prefer to shift to a salaried job; $\tilde{w}$ thus lowers so as to clear the labor market. Conversely, for a given wage rate, an increase in the lump sum subsidy to entrepreneurs would result in an excess of labor demand as some workers (the most skilled among them) would now prefer to shift to entrepreneurship; in this situation, $\tilde{w}$ must increase to clear the labor market.

The income-tax marginal-tax-rate, t, has an ambiguous effect on $\tilde{w}$. From (10), we see that the direction of the impact is related to the sign of $(1-\hat{a})s(\tilde{n}^s - \tilde{w}) - \tilde{w}$, $s = l, h$. To interpret this, notice that $(1-\hat{a})s(\tilde{n}^s - \tilde{w})$ is income reported by type-s marginal entrepreneurs, while $\tilde{w}$ is income reported by workers. Thus, a sufficient condition for t to increase $\tilde{w}$ is that the effective tax base of both type-l and type-h marginal entrepreneurs be lower than the tax base of workers. When this is the case, since the effective marginal tax rate for entrepreneurs is lower than the tax rate for workers ($\hat{\theta} > \theta$ if $\hat{a} > 0$), an increase in t reduces workers' income by more than entrepreneurs'

[14] For the time being, in this version of the paper we take the size of the tax administration agency, b, as given.

income, and hence, in order to preserve market equilibrium, workers (who cannot avoid taxation) are "compensated" by the market in terms of higher wages. Conversely, a sufficient condition for (10) to be negative is that the effective tax base of of both type-l and type-h marginal entrepreneurs be greater than the tax base of workers. Instead, when $(1-\hat{a})l(\tilde{n}^l-\tilde{w})-\tilde{w}$ and $(1-\hat{a})h(\tilde{n}^h-\tilde{w})-\tilde{w}$ have opposite sign, the sign of (10) remains undetermined.

Turning next to the impact of the tax system on occupational choices, we obtain

$$\frac{\partial\tilde{n}^l}{\partial\tau} = \frac{h^{-1}-l^{-1}}{\Delta}\theta(1+h)\tilde{g}^h\hat{\theta}^{-1}\tilde{w} < 0 \tag{12}$$

$$\frac{\partial\tilde{n}^l}{\partial\beta} = -\frac{\partial\tilde{n}^l}{\partial\alpha} = -\frac{\hat{\theta}+\tau}{\theta\tilde{w}}\frac{\partial\tilde{n}^l}{\partial\tau} > 0 \tag{13}$$

$$\begin{aligned}\frac{\partial\tilde{n}^l}{\partial t} = {} & \frac{\hat{\theta}^{-1}(1+h)\tilde{g}^h}{\Delta}l^{-1}(h^{-1}\theta+\hat{\theta}+\tau)[(1-\hat{a})l(\tilde{n}^l-\tilde{w})-\tilde{w}] + \\ & -\frac{\hat{\theta}^{-1}(1+h)\tilde{g}^h}{\Delta}h^{-1}(l^{-1}\theta+\hat{\theta}+\tau)[(1-\hat{a})h(\tilde{n}^h-\tilde{w})-\tilde{w}]\end{aligned} \tag{14}$$

$$\frac{\partial\tilde{n}^h}{\partial z} = -\frac{(1+l)\tilde{g}^l}{(1+h)\tilde{g}^h}\frac{\partial\tilde{n}^l}{\partial z}, \quad z=\tau,\alpha,\beta,t. \tag{15}$$

Since technology is Leontief (one type-s entrepreneur and s workers in each firm), whenever a tax parameter increases (reduces) $\tilde{n}^l$, it then reduces (increases) $\tilde{n}^h$. Taxation may thus cause production inefficiency, even though in a market equilibrium with taxation there is still full employment. To see this formally, let

$$Y = \int_{\tilde{n}^l}^{N} l\,ng(n)dn + \int_{\tilde{n}^h}^{N} lhg(n)dn \tag{16}$$

be aggregate output. Taking the derivative with respect to a generic tax parameter z and then using (15) one gets

$$\frac{\partial Y}{\partial z} = -l\tilde{n}^l\tilde{g}^l\frac{\partial\tilde{n}^l}{\partial z} - h\tilde{n}^h\tilde{g}^h\frac{\partial\tilde{n}^h}{\partial z} = \left(\frac{h\tilde{n}^h}{1+h}-\frac{l\tilde{n}^l}{1+l}\right)(1+l)\tilde{g}^l\frac{\partial\tilde{n}^l}{\partial z} \tag{17}$$

To illustrate, consider the tax instrument τ and suppose that the other tax parameters are zero. We know that, with no taxes, production is efficient; hence, starting from $\tau=0$ a small increase in τ does not affect Y, for the term in brackets in (17) is zero, see (7). However, since $\frac{\partial\tilde{n}^l}{\partial\tau}<0$ and $\frac{\partial\tilde{n}^h}{\partial\tau}>0$, a further increase in τ would make $\frac{h\tilde{n}^h}{1+h}$ to become bigger than $\frac{l\tilde{n}^l}{1+l}$, with the result that, as (17) becomes negative, aggregate production falls. Hence, each tax instrument causes, when used in isolation (i.e. with the other instruments set to zero), a reduction of aggregate output below its Pareto efficient level.

In most cases, however, two or more tax instruments are used at a time, and in this case the sign of $\dfrac{h\tilde{n}^h}{1+h} - \dfrac{l\tilde{n}^l}{1+l}$ is not known a priori. To continue the illustration with τ, an increase in τ increases aggregate production if and only if $\dfrac{h\tilde{n}^h}{1+h} < \dfrac{l\tilde{n}^l}{1+l}$, i.e. if (marginal) type-l entrepreneurs are more productive than (marginal) type-h entrepreners. If this condition holds, an increase in τ, by increasing $\tilde{n}^h$ and reducing $\tilde{n}^l$, brings about an efficiency gain that increases Y. The same sort of considerations can be made for the impact of α and β, since (13) is positive. More difficult is to assess the impact of t, for (14) is difficult to sign.

3 The tax problem

In the economy described in the previous section an equity-oriented government has certainly some scope for action. As the left-graph of Figure 1 shows, the income distribution is far from equal, with workers (all earning the same income) at the bottom, and entrepreneurs at the top. The three taxation schemes described in section 2 have some obvious merits and demerits as far as the achievement of equity objectives is concerned. **OC-PT** is simple and efficient, as no tax enforcement is needed; however, it taxes rich and poor entrepreneurs all in the same way. **IC** taxes the rich more than the poor; however, it leaves room for (costly) tax avoidance and some resources must be spent on tax enforcement. The redistributive role of **IC-PT** is better understood by looking at the right-graph in Figure 1. We then see that type-l entrepreneurs have (observable) input costs of $l\tilde{w}$, and (unobservable) income ranging from $\tilde{w}$ to $l(N-\tilde{w})$, while type-h entrepreneurs input costs are $h\tilde{w}$ with income ranging from $\tilde{w}$ to $h(N-\tilde{w})$. Hence, since type-h entrepreneurs are, on average, richer than type-l, a proportional tax on input costs can be useful to do some redistribution. However, notice that **IC-PT** is horizontally inequitable, as there are some equal-income type-l and type-h individuals that end up paying a different tax since production costs are different.

Suppose that the government's objective is given by a social welfare function

$$V = \sum\nolimits_s \left(\int_0^{\tilde{n}^s} \psi(\tilde{y}^{\mathbf{w}})dG + \int_{\tilde{n}^s}^{N} \psi(\tilde{y}^{\mathbf{e},s})dG \right) \tag{18}$$

where $\psi(\cdot)$ is a strictly concave function expressing the social valuation of income and the "tilde" over the net income symbol y indicates that we are considering equilibrium payoffs. The government problem will be that of choosing the tax instruments in order to maximise (18) under the constraint that some pre-determined revenue target is satisfied. Since we will mostly be interested in the redistributive properties of the tax system, we will assume in what follows that such revenue target is zero.

The equilibrium payoffs and the revenue constraint are

$$\tilde{y}^{\mathbf{w}} = \beta + \theta\tilde{w} \tag{19}$$

$$\tilde{y}^{\mathbf{e},s} = \alpha + \hat{\theta}s(n - \tilde{w}) - \tau s\tilde{w} \tag{20}$$

$$R = \sum\nolimits_s \left(\int_0^{\tilde{n}^s} (t\tilde{w} - \beta)dG + \int_{\tilde{n}^s}^N \{(1 - \hat{a})ts(n - \tilde{w}) + \tau s\tilde{w} - \alpha\}dG \right) - b\tilde{w} = 0 \tag{21}$$

The optimal tax problem can then be stated as

$$\max_{\alpha,\beta,\tau,t} V \quad \text{s.t.} \quad R = 0, \quad \tilde{n}^l - \tilde{w} \geq 0, \quad \tilde{n}^h - \tilde{w} \geq 0, \quad \tilde{w} \geq 0 \tag{22}$$

We impose the contraints that before-tax incomes, for both entrepreneurs and workers, be non-negative. In fact, with appropriate combinations of tax parameters, entrepreneurs gross income may become negative, although their after tax income is nonetheless positive. To make a rigorous analysis of such cases, however, we should allow the individuals a third option, that is the choice to stay unemployed. In this way, we would not need to impose the non-negativity constraints on gross incomes, as these would emerge endogenously under unemployment equilibria. We do not deal with unemployment equilibria in this paper since, as shown in Galmarini (2000), they are never optimal.

To derive the optimal tax rules, it is useful first to manipulate the revenue function (21) as follows. Using (19) to substitute $t\tilde{w} - \beta = -\tilde{y}^{\mathbf{w}} + \tilde{w}$ and (20) to substitute $(1 - \hat{a})ts(n - \tilde{w}) + \tau s\tilde{w} - \alpha = (1 - \hat{c})s(n - \tilde{w}) - \tilde{y}^{\mathbf{e},s}$, we get

$$R = \sum\nolimits_s \left(\tilde{G}^s\tilde{w} + \int_{\tilde{n}^s}^N (1 - \hat{c})s(n - \tilde{w})dG - \int_0^{\tilde{n}^s} \tilde{y}^{\mathbf{w}}dG - \int_{\tilde{n}^s}^N \tilde{y}^{\mathbf{e},s}dG \right) - b\tilde{w} \tag{23}$$

Using the market clearing equation (6) to substitute $(\tilde{G}^l + \tilde{G}^h - b)\tilde{w} = \sum_s \int_{\tilde{n}^s}^N s\tilde{w}dG$ into (23) and then simplifying, we finally get

$$R = \sum\nolimits_s \left(\int_{\tilde{n}^s}^N (1 - \hat{c})sndG + \int_{\tilde{n}^s}^N \hat{c}s\tilde{w}dG - \int_0^{\tilde{n}^s} \tilde{y}^{\mathbf{w}}dG - \int_{\tilde{n}^s}^N \tilde{y}^{\mathbf{e},s}dG \right) \tag{24}$$

By differentiating (24) with respect to a generic tax instrument z, we get

$$\frac{\partial R}{\partial z} = \underbrace{-\sum\nolimits_s (1-\hat{c}) s\tilde{n}^s \tilde{g}^s \frac{\partial \tilde{n}^s}{\partial z}}_{A(z)} \underbrace{-\sum\nolimits_s \hat{c} s\tilde{w}\tilde{g}^s \frac{\partial \tilde{n}^s}{\partial z}}_{B(z)} + \underbrace{\sum\nolimits_s (1-\tilde{G}^s)\hat{c}s\frac{\partial \tilde{w}}{\partial z}}_{C(z)} + \underbrace{-\sum\nolimits_s \left(\int_0^{\tilde{n}^s} \frac{\partial \tilde{y}^{\mathbf{w}}}{\partial z} dG + \int_{\tilde{n}^s}^{N} \frac{\partial \tilde{y}^{\mathbf{e},s}}{\partial z} dG\right)}_{D(z)} \tag{25}$$

As for social welfare[15]

$$\frac{\partial V}{\partial z} = \sum\nolimits_s \left(\int_0^{\tilde{n}^s} \psi_y^{\mathbf{w}} \frac{\partial \tilde{y}^{\mathbf{w}}}{\partial z} dG + \int_{\tilde{n}^s}^{N} \psi_y^{\mathbf{e},s} \frac{\partial \tilde{y}^{\mathbf{e},s}}{\partial z} dG\right) \tag{26}$$

where, to shorten notation, we let $\psi_y^{\mathbf{w}} \equiv \psi_y(\tilde{y}^{\mathbf{w}})$ and $\psi_y^{\mathbf{e},s} \equiv \psi_y(\tilde{y}^{\mathbf{e},s})$.

The optimal tax problem (22) is difficult to solve at a general level. Therefore, in the next section we limit the analysis to tax reforms.[16]

4 Presumptive taxation rules

We take income taxation as exogenous and we focus on presumptive taxation. We start with **OC-PT** in the following section and then move on to **IC-PT**.

4.1 Occupational Choice Presumptive Taxation (OC-PT)

Let τ, t and b be fixed at given values and consider the possibility of adjusting α and β. In particular, we will examine revenue-neutral welfare-improving tax-reforms, i.e. tax policies consisting of *small* changes in α and β such that social welfare increases for given tax revenue. Formally, we look for a $(d\alpha, d\beta)$ pair such that

$$dV = \frac{\partial V}{\partial \alpha} d\alpha + \frac{\partial V}{\partial \beta} d\beta > 0 \quad \text{s.t.} \quad dR = \frac{\partial R}{\partial \alpha} d\alpha + \frac{\partial R}{\partial \beta} d\beta = 0 \tag{27}$$

Firstly, we compute $\dfrac{\partial R}{\partial \beta}$. From (25) we get

$$A(\beta) = -(1-\hat{c})\left(l\tilde{n}^l \tilde{g}^l \frac{\partial \tilde{n}^l}{\partial \beta} + h\tilde{n}^h \tilde{g}^h \frac{\partial \tilde{n}^h}{\partial \beta}\right) = (1-\hat{c})\left(\frac{h\tilde{n}^h}{1+h} - \frac{l\tilde{n}^l}{1+l}\right)(1+l)\tilde{g}^l \frac{\partial \tilde{n}^l}{\partial \beta} \tag{28}$$

[15] Note that we used the fact that the least-skilled entrepeneur has the same income of the marginal worker to cancel out the effects of the variables on the limits of integration in the social welfare function.

[16] Of course, we are planning to solve the optimal tax problem in a future version of the paper.

$$B(\beta) = -\hat{c}\tilde{w}\left(l\tilde{g}^l\frac{\partial \tilde{n}^l}{\partial\beta} + h\tilde{g}^h\frac{\partial \tilde{n}^h}{\partial\beta}\right) = \hat{c}\tilde{w}\left(\frac{h}{1+h} - \frac{l}{1+l}\right)(1+l)\tilde{g}^l\frac{\partial \tilde{n}^l}{\partial\beta} > 0 \quad (29)$$

$$C(\beta) = [(1-\tilde{G}^l)l + (1-\tilde{G}^h)h]\hat{c}\frac{\partial \tilde{w}}{\partial\beta} < 0 \quad (30)$$

In computing $A(\beta)$ and $B(\beta)$, we have used (15). To derive $D(\beta)$, we first notice that

$$\frac{\partial \tilde{y}^{\mathbf{w}}}{\partial\beta} = 1 + \theta\frac{\partial \tilde{w}}{\partial\beta} = (\hat{\theta}+\tau)\Delta^{-1}[(1+h)\tilde{g}^h + (1+l)\tilde{g}^l] > 0 \quad (31)$$

$$\frac{\partial \tilde{y}^{\mathbf{e},l}}{\partial\beta} = -(\hat{\theta}+\tau)l\frac{\partial \tilde{w}}{\partial\beta} = -(\hat{\theta}+\tau)(h-l)(1+h^{-1})\tilde{g}^h\Delta^{-1} + \frac{\partial \tilde{y}^{\mathbf{w}}}{\partial\beta} \quad (32)$$

$$\frac{\partial \tilde{y}^{\mathbf{e},h}}{\partial\beta} = -(\hat{\theta}+\tau)h\frac{\partial \tilde{w}}{\partial\beta} = (\hat{\theta}+\tau)(h-l)(1+l^{-1})\tilde{g}^l\Delta^{-1} + \frac{\partial \tilde{y}^{\mathbf{w}}}{\partial\beta} \quad (33)$$

so that we obtain

$$D(\beta) = -2\frac{\partial \tilde{y}^{\mathbf{w}}}{\partial\beta} + E(\beta) \quad (34)$$

where

$$E(\beta) = (\hat{\theta}+\tau)(h-l)\Delta^{-1}[(1-\tilde{G}^l)(1+h^{-1})\tilde{g}^h - (1-\tilde{G}^h)(1+l^{-1})\tilde{g}^l] \quad (35)$$

Hence

$$\frac{\partial R}{\partial\beta} = A(\beta) + B(\beta) + C(\beta) + E(\beta) - 2\frac{\partial \tilde{y}^{\mathbf{w}}}{\partial\beta} \quad (36)$$

Using the fact that $\frac{\partial \tilde{n}^s}{\partial\alpha} = -\frac{\partial \tilde{n}^s}{\partial\beta}$, $\frac{\partial \tilde{w}}{\partial\alpha} = -\frac{\partial \tilde{w}}{\partial\beta}$ and that

$$\frac{\partial \tilde{y}^{\mathbf{w}}}{\partial\alpha} = \theta\frac{\partial \tilde{w}}{\partial\alpha} > 0 \quad (37)$$

$$\frac{\partial \tilde{y}^{\mathbf{e},l}}{\partial\alpha} = 1 - (\hat{\theta}+\tau)l\frac{\partial \tilde{w}}{\partial\alpha} = (\hat{\theta}+\tau)(h-l)(1+h^{-1})\tilde{g}^h\Delta^{-1} + \frac{\partial \tilde{y}^{\mathbf{w}}}{\partial\alpha} \quad (38)$$

$$\frac{\partial \tilde{y}^{\mathbf{e},h}}{\partial\alpha} = 1 - (\hat{\theta}+\tau)h\frac{\partial \tilde{w}}{\partial\alpha} = -(\hat{\theta}+\tau)(h-l)(1+l^{-1})\tilde{g}^l\Delta^{-1} + \frac{\partial \tilde{y}^{\mathbf{w}}}{\partial\alpha} \quad (39)$$

we immediately obtain

$$\frac{\partial R}{\partial\alpha} = -A(\beta) - B(\beta) - C(\beta) - E(\beta) - 2\frac{\partial \tilde{y}^{\mathbf{w}}}{\partial\alpha} \quad (40)$$

Hence, using (36) and (40), from the revenue constraint in (27) we get

$$\left.\frac{d\beta}{d\alpha}\right|_{dR=0} = -\frac{\partial R/\partial\alpha}{\partial R/\partial\beta} = \frac{A(\beta)+B(\beta)+C(\beta)+E(\beta)+2\frac{\partial \tilde{y}^{\mathbf{w}}}{\partial\alpha}}{A(\beta)+B(\beta)+C(\beta)+E(\beta)-2\frac{\partial \tilde{y}^{\mathbf{w}}}{\partial\beta}} \quad (41)$$

Next we compute $\frac{\partial V}{\partial \beta}$. From (26) and (31)–(33) we get

$$\begin{aligned}
\frac{\partial V}{\partial \beta} &= \int_0^{\tilde{n}^l} \psi_y^{\mathbf{w}} \frac{\partial \tilde{y}^{\mathbf{w}}}{\partial \beta} dG + \int_0^{\tilde{n}^h} \psi_y^{\mathbf{w}} \frac{\partial \tilde{y}^{\mathbf{w}}}{\partial \beta} dG + \int_{\tilde{n}^l}^N \psi_y^{\mathbf{e},l} \frac{\partial \tilde{y}^{\mathbf{e},l}}{\partial \beta} dG + \int_{\tilde{n}^h}^N \psi_y^{\mathbf{e},h} \frac{\partial \tilde{y}^{\mathbf{e},h}}{\partial \beta} dG = \\
&= (\overline{\psi_y^l} + \overline{\psi_y^h}) \frac{\partial \tilde{y}^{\mathbf{w}}}{\partial \beta} + F(\beta) \qquad (42)
\end{aligned}$$

where

$$\begin{aligned}
F(\beta) &= (\hat{\theta} + \tau)(h - l)\Delta^{-1}[(1 + l^{-1})\tilde{g}^l (1 - \tilde{G}^h)\overline{\psi_y^{\mathbf{e},h}} - (1 + h^{-1})\tilde{g}^h (1 - \tilde{G}^l)\overline{\psi_y^{\mathbf{e},l}}] \\
\overline{\psi_y^s} &= \int_0^{\tilde{n}^s} \psi_y^{\mathbf{w}} dG + \int_{\tilde{n}^s}^N \psi_y^{\mathbf{e},s} dG \\
\overline{\psi_y^{\mathbf{e},s}} &= (1 - \tilde{G}^s)^{-1} \int_{\tilde{n}^s}^N \psi_y^{\mathbf{e},s} dG
\end{aligned}$$

As for $\frac{\partial V}{\partial \alpha}$, from (26) and (37)–(39) we get

$$\frac{\partial V}{\partial \alpha} = (\overline{\psi_y^l} + \overline{\psi_y^h}) \frac{\partial \tilde{y}^{\mathbf{w}}}{\partial \alpha} - F(\beta) \qquad (43)$$

Finally, using (42), (43) and (41), using the fact that $\frac{\partial \tilde{y}^{\mathbf{w}}}{\partial \alpha} + \frac{\partial \tilde{y}^{\mathbf{w}}}{\partial \beta} = 1$, after some tedious manipulations we get

$$\begin{aligned}
\left.\frac{dV}{d\alpha}\right|_{dR=0} &= \frac{\partial V}{\partial \alpha} + \left.\frac{\partial V}{\partial \beta} \frac{d\beta}{d\alpha}\right|_{dR=0} = \\
&= \frac{(\overline{\psi_y^l} + \overline{\psi_y^h})[A(\beta) + B(\beta) + C(\beta) + E(\beta)] + 2F(\beta)}{\partial R / \partial \beta} \qquad (44)
\end{aligned}$$

(44) is the term that gives the *direction* of the tax reform. If (44) is positive, a welfare-improving revenue-neutral tax reform consists of $d\alpha > 0$ and $d\beta < 0$ such that (41) is satisfied. If (44) is negative, a welfare-improving revenue-neutral tax reform consists of $d\alpha < 0$ and $d\beta > 0$. Finally, no welfare improving tax reform exists if (44) is zero.

The tax reform terms (44) and (41) are rather difficult to sign. At a general level, we can expect (44) to be different from zero and hence that **OC-PT** is an effective tool for redistribution; the direction of the tax reform, however, as well as its impact on $\tilde{w}$, $\tilde{n}^l$ and $\tilde{n}^h$, may be difficult to determine. In some particular cases, however, the results are neat and are stated in the following

Proposition 1 *(a) If $l = h$ and $t = 0$, then (44) is zero.*

(b) If $l = h$ and $t > 0$, then (44) is positive and (41) is less than one.

(c) If $l < h$ and $t = 0$, then (44) is likely to be different from zero.

Proof. (a) If $l = h$ and $t = 0$, then $A(\beta) = B(\beta) = C(\beta) = E(\beta) = F(\beta) = 0$.

(b) If $l = h$ and $t > 0$, then $A(\beta) = B(\beta) = E(\beta) = F(\beta) = 0$ and $C(\beta) < 0$. Since $\partial R/\partial\beta < 0$, then (44) is positive.

(c) If $l < h$ and $t = 0$, then $B(\beta) = C(\beta) = 0$ but $A(\beta)$, $E(\beta)$ are different from zero. ■

Parts (a) and (b) of the proposition refer to the case in which entrepreneurs are not differentiated along the s dimension. In this case, when no income tax is levied ($t = 0$), **OC-PT** is useless. It becomes instead useful for redistribution when $t > 0$. In particular, since (41) is less than one, $d\alpha > 0$ is larger, in absolute value, than $d\beta$; hence the tax reform makes redistribution by inducing and increase in $\tilde{w}$.[17] Finally, part (c) shows that **OC-PT** becomes useful even in the absence of **IT** when there is differentiation along the s dimension.

4.2 Input Costs Presumptive Taxation (IC-PT)

Now we take α, t and b as fixed and consider the possibility of adjusting τ and β. In this case, intuition suggests that we may be able to increase social welfare by taxing entrepreneurs on their input costs ($d\tau > 0$) and then use the revenue to subsidize workers ($d\beta > 0$). As in the previous section, we analyse the impact on welfare of revenue-neutral tax-reforms.

Firstly, we compute $\dfrac{\partial R}{\partial \tau}$. From the definitions of $A(z)$, $B(z)$ and $C(z)$ in (25), noticing that $\dfrac{\partial \tilde{n}^l}{\partial \tau} = -\dfrac{\theta\tilde{w}}{\hat{\theta}+\tau}\dfrac{\partial \tilde{n}^l}{\partial \beta}$ from (13), and using (28)–(30) we get

$$A(\tau) = -\frac{\theta\tilde{w}}{\hat{\theta}+\tau}A(\beta)$$

$$B(\tau) = -\frac{\theta\tilde{w}}{\hat{\theta}+\tau}B(\beta)$$

$$C(\tau) = [(1-\tilde{G}^l)l + (1-\tilde{G}^h)h]\hat{c}\frac{\partial \tilde{w}}{\partial \tau} = -[(1-\tilde{G}^l)l + (1-\tilde{G}^h)h]\hat{c}\left(\frac{\tilde{w}}{\hat{\theta}+\tau} + \frac{\theta\tilde{w}}{\hat{\theta}+\tau}\frac{\partial \tilde{w}}{\partial \beta}\right) =$$

$$= -\frac{\theta\tilde{w}}{\hat{\theta}+\tau}C(\beta) - \frac{\theta\tilde{w}}{\hat{\theta}+\tau}[(1-\tilde{G}^l)l + (1-\tilde{G}^h)h]\hat{c}\theta^{-1}$$

[17] For a throughout discussion of results (a) and (b), see Galmarini (2000).

To derive $D(\tau)$, we first notice that [see (8) and (9) to derive (45)–(47)]

$$\frac{\partial \tilde{y}^{\mathbf{w}}}{\partial \tau} = \theta \frac{\partial \tilde{w}}{\partial \tau} = -\frac{\theta \tilde{w}}{\hat{\theta}+\tau}\frac{\partial \tilde{y}^{\mathbf{w}}}{\partial \beta} \tag{45}$$

$$\frac{\partial \tilde{y}^{\mathbf{e},l}}{\partial \tau} = -(\hat{\theta}+\tau) l \frac{\partial \tilde{w}}{\partial \tau} - l\tilde{w} = \theta \tilde{w} l \frac{\partial \tilde{w}}{\partial \beta} = -\frac{\theta \tilde{w}}{\hat{\theta}+\tau}\frac{\partial \tilde{y}^{\mathbf{e},l}}{\partial \beta} \tag{46}$$

$$\frac{\partial \tilde{y}^{\mathbf{e},h}}{\partial \tau} = -(\hat{\theta}+\tau) h \frac{\partial \tilde{w}}{\partial \tau} - h\tilde{w} = \theta \tilde{w} h \frac{\partial \tilde{w}}{\partial \beta} = -\frac{\theta \tilde{w}}{\hat{\theta}+\tau}\frac{\partial \tilde{y}^{\mathbf{e},h}}{\partial \beta} \tag{47}$$

and then we obtain, using (34)

$$D(\tau) = 2\frac{\partial \tilde{y}^{\mathbf{w}}}{\partial \beta} - E(\beta)$$

Hence

$$\frac{\partial R}{\partial \tau} = -\frac{\theta \tilde{w}}{\hat{\theta}+\tau}\frac{\partial R}{\partial \beta} - \frac{\theta \tilde{w}}{\hat{\theta}+\tau}[(1-\tilde{G}^l)l + (1-\tilde{G}^h)h]\hat{c}\theta^{-1} \tag{48}$$

Hence, using (36) and (48), we get

$$\left.\frac{d\beta}{d\tau}\right|_{dR=0} = -\frac{\partial R/\partial \tau}{\partial R/\partial \beta} = \frac{\theta \tilde{w}}{\hat{\theta}+\tau} + \frac{\theta \tilde{w}}{\hat{\theta}+\tau}\frac{[(1-\tilde{G}^l)l + (1-\tilde{G}^h)h]\hat{c}\theta^{-1}}{\partial R/\partial \beta} \tag{49}$$

Next we compute $\frac{\partial V}{\partial \tau}$. From (26) and (45)–(47) we get

$$\frac{\partial V}{\partial \tau} = -\frac{\theta \tilde{w}}{\hat{\theta}+\tau}\frac{\partial V}{\partial \beta} \tag{50}$$

Finally, using (42), (50) and (49), after some manipulations we get

$$\begin{aligned} \left.\frac{dV}{d\tau}\right|_{dR=0} &= \frac{\partial V}{\partial \tau} + \frac{\partial V}{\partial \beta}\left.\frac{d\beta}{d\tau}\right|_{dR=0} = \\ &= \frac{\tilde{w}}{\hat{\theta}+\tau}\frac{\partial V/\partial \beta}{\partial R/\partial \beta}[(1-\tilde{G}^l)l + (1-\tilde{G}^h)h]\hat{c} \end{aligned} \tag{51}$$

(51) tells us two important facts about **IC-PT**. Firstly, if $t = 0$, then $\hat{c} = 0$ and (51) is zero: without income taxation (**IT**), **IC-PT** is useless. Presumptive taxation based on input costs works only when income taxation is also used. Secondly, since we expect that $\frac{\partial R}{\partial \beta} < 0$, with $t > 0$ ($\hat{c} > 0$) we have that (51) is negative. This means that the revenue-neutral welfare-improving tax reform consists of *subsidizing* input costs ($d\tau < 0$) and taxing workers ($d\beta < 0$), which is exactly the opposite of what we expected. The intuition for this result is the following. The combination ($d\tau < 0, d\beta < 0$) may benefit workers, since the impact of τ may offset that of β so that the net effect makes $\tilde{w}$ and $\tilde{y}^{\mathbf{w}}$ to increase. As for entrepreneurs, however, a subsidy $\tau < 0$ is on average regressive, since it gives a premium to high costs firms, i.e. those with larger income (on average). The reason is related to production efficiency: in fact, the subsidy on input costs is given to the more productive entrepreneurs.

5 Conclusions

We have examined the role of presumptive taxes in a framework in which one group of taxpayers engages in tax avoidance of the income tax, whereas another group of taxpayers is unable to avoid income taxation. We obtain two interesting results.

The first is that presumptive income taxes are not a substitute for effective income taxes. At most, presumptive taxation on tax dodgers can be a useful complement to effective taxation. The second result is that simple presumptive taxes may perform better than complex presumptive taxes.

Since this is a preliminary work, we conclude with the warning that these results need some further investigation, which we are currently undertaking.

Appendix

Recalling that $\dfrac{\partial \hat{\theta}}{\partial t} = -1 + \hat{a} + (t - \hat{c}_a)\hat{a}_t = -(1-\hat{a})$ from (2), by totally differentiating (5) and (6) with respect to α, β, τ, and t one gets

$$d\alpha + \hat{\theta} l d\tilde{n}^l - l(\hat{\theta} + \tau)d\tilde{w} - l\tilde{w}d\tau - l(\tilde{n}^l - \tilde{w})(1-\hat{a})dt = d\beta + \theta d\tilde{w} - \tilde{w}dt$$

$$d\alpha + \hat{\theta} h d\tilde{n}^h - h(\hat{\theta} + \tau)d\tilde{w} - h\tilde{w}d\tau - h(\tilde{n}^h - \tilde{w})(1-\hat{a})dt = d\beta + \theta d\tilde{w} - \tilde{w}dt$$

$$\tilde{g}^h d\tilde{n}^h + \tilde{g}^l d\tilde{n}^l = -h\tilde{g}^h d\tilde{n}^h - l\tilde{g}^l d\tilde{n}^l$$

These can be rearranged as

$$\hat{\theta} d\tilde{n}^l = l^{-1}(d\beta - d\alpha) + (l^{-1}\theta + \hat{\theta} + \tau)d\tilde{w} + \tilde{w}d\tau + l^{-1}[(1-\hat{a})l(\tilde{n}^l - \tilde{w}) - \tilde{w}]dt \tag{52}$$

$$\hat{\theta} d\tilde{n}^h = h^{-1}(d\beta - d\alpha) + (h^{-1}\theta + \hat{\theta} + \tau)d\tilde{w} + \tilde{w}d\tau + h^{-1}[(1-\hat{a})h(\tilde{n}^h - \tilde{w}) - \tilde{w}]dt \tag{53}$$

$$\frac{d\tilde{n}^h}{d\tilde{n}^l} = -\frac{(1+l)\tilde{g}^l}{(1+h)\tilde{g}^h} \tag{54}$$

Inserting the (52) and (53) into (54) and then rearranging we obtain

$$\begin{aligned}\Delta d\tilde{w} = & -[(1+l^{-1})\tilde{g}^l + (1+h^{-1})\tilde{g}^h](d\beta - d\alpha) - [(1+l)\tilde{g}^l + (1+h)\tilde{g}^h]\tilde{w}d\tau + \\ & -(1+l^{-1})\tilde{g}^l[(1-\hat{a})l(\tilde{n}^l - \tilde{w}) - \tilde{w}]dt - (1+h^{-1})\tilde{g}^h[(1-\hat{a})h(\tilde{n}^h - \tilde{w}) - \tilde{w}]dt\end{aligned} \tag{55}$$

with Δ defined in (11). From (55), partial derivatives (8), (9) and (10) immediately follow.

As for $\frac{\partial \tilde{n}^l}{\partial \tau}$, from (52) we get

$$\frac{\partial \tilde{n}^l}{\partial \tau} = \hat{\theta}^{-1}(l^{-1}\theta + \hat{\theta} + \tau)\frac{\partial \tilde{w}}{\partial \tau} + \hat{\theta}^{-1}\tilde{w}$$

Substituting $\frac{\partial \tilde{w}}{\partial \tau}$ from (8) and Δ from (11) we get

$$\begin{aligned}
\frac{\partial \tilde{n}^l}{\partial \tau} &= -\frac{\hat{\theta}^{-1}(l^{-1}\theta + \hat{\theta} + \tau)[(1+l)\tilde{g}^l + (1+h)\tilde{g}^h]}{\Delta}\tilde{w} + \hat{\theta}^{-1}\tilde{w} = \\
&= \frac{-(l^{-1}\theta + \hat{\theta} + \tau)(1+l)\tilde{g}^l - (l^{-1}\theta + \hat{\theta} + \tau)(1+h)\tilde{g}^h + \Delta}{\Delta}\hat{\theta}^{-1}\tilde{w} = \\
&= \frac{-(l^{-1}\theta + \hat{\theta} + \tau)(1+h)\tilde{g}^h + (h^{-1}\theta + \hat{\theta} + \tau)(1+h)\tilde{g}^h}{\Delta}\hat{\theta}^{-1}\tilde{w} = \\
&= \frac{h^{-1} - l^{-1}}{\Delta}\theta(1+h)\tilde{g}^h\hat{\theta}^{-1}\tilde{w} < 0
\end{aligned}$$

As for $\frac{\partial \tilde{n}^l}{\partial \beta}$ and $\frac{\partial \tilde{n}^l}{\partial \alpha}$, from (52) we get

$$\frac{\partial \tilde{n}^l}{\partial \beta} = -\frac{\partial \tilde{n}^l}{\partial \alpha} = \hat{\theta}^{-1}l^{-1} + \hat{\theta}^{-1}(l^{-1}\theta + \hat{\theta} + \tau)\frac{\partial \tilde{w}}{\partial \beta}$$

Substituting $\frac{\partial \tilde{w}}{\partial \beta}$ from (9) and Δ from (11) we get

$$\begin{aligned}
\frac{\partial \tilde{n}^l}{\partial \beta} &= \frac{\Delta l^{-1} - (l^{-1}\theta + \hat{\theta} + \tau)(1+l^{-1})\tilde{g}^l - (l^{-1}\theta + \hat{\theta} + \tau)(1+h^{-1})\tilde{g}^h}{\Delta}\hat{\theta}^{-1} = \\
&= \frac{l^{-1}(h^{-1}\theta + \hat{\theta} + \tau)(1+h)\tilde{g}^h + l^{-1}(l^{-1}\theta + \hat{\theta} + \tau)(1+l)\tilde{g}^l}{\Delta}\hat{\theta}^{-1} + \\
&\qquad + \frac{-(l^{-1}\theta + \hat{\theta} + \tau)(1+l^{-1})\tilde{g}^l - (l^{-1}\theta + \hat{\theta} + \tau)(1+h^{-1})\tilde{g}^h}{\Delta}\hat{\theta}^{-1} = \\
&= \frac{l^{-1}(h^{-1}\theta + \hat{\theta} + \tau)(1+h) - (l^{-1}\theta + \hat{\theta} + \tau)(1+h^{-1})}{\Delta}\hat{\theta}^{-1}\tilde{g}^h = \\
&= \frac{l^{-1}h^{-1}\theta(1+h) + l^{-1}(\hat{\theta} + \tau)(1+h) - l^{-1}\theta(1+h^{-1}) - (\hat{\theta} + \tau)(1+h^{-1})}{\Delta}\hat{\theta}^{-1}\tilde{g}^h = \\
&= \frac{l^{-1}(\hat{\theta} + \tau)(1+h) - (\hat{\theta} + \tau)(1+h^{-1})}{\Delta}\hat{\theta}^{-1}\tilde{g}^h = \\
&= \frac{(\hat{\theta} + \tau)(l^{-1} - h^{-1})}{\Delta}(1+h)\hat{\theta}^{-1}\tilde{g}^h
\end{aligned}$$

Finally, using (12), (13) is obtained.

As for $\frac{\partial \tilde{n}^l}{\partial t}$, from (52) we get

$$\frac{\partial \tilde{n}^l}{\partial t} = \hat{\theta}^{-1}(l^{-1}\theta + \hat{\theta} + \tau)\frac{\partial \tilde{w}}{\partial t} + \hat{\theta}^{-1}l^{-1}[(1-\hat{a})l(\tilde{n}^l - \tilde{w}) - \tilde{w}]$$

Substituting $\dfrac{\partial \tilde{w}}{\partial t}$ from (10) and Δ from (11) we get

$$\begin{aligned}\frac{\partial \tilde{n}^l}{\partial t} = -&\frac{\hat{\theta}^{-1}(l^{-1}\theta+\hat{\theta}+\tau)[(1-\hat{a})l(\tilde{n}^l-\tilde{w})-\tilde{w}](1+l^{-1})\tilde{g}^l}{\Delta}+\\ &-\frac{\hat{\theta}^{-1}(l^{-1}\theta+\hat{\theta}+\tau)[(1-\hat{a})h(\tilde{n}^h-\tilde{w})-\tilde{w}](1+h^{-1})\tilde{g}^h}{\Delta}+\\ &+\frac{\hat{\theta}^{-1}l^{-1}[(1-\hat{a})l(\tilde{n}^l-\tilde{w})-\tilde{w}](h^{-1}\theta+\hat{\theta}+\tau)(1+h)\tilde{g}^h}{\Delta}+\\ &+\frac{\hat{\theta}^{-1}l^{-1}[(1-\hat{a})l(\tilde{n}^l-\tilde{w})-\tilde{w}](l^{-1}\theta+\hat{\theta}+\tau)(1+l)\tilde{g}^l}{\Delta}\end{aligned}$$

Since the terms in the first and fourth row cancel out, from the remaining terms (14) is obtained.

As for $\tilde{n}^h$, partial derivatives (15) are immediately obtained from (54).

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