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WORKING PAPERS

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JEL Classification: D13 – H21 – H31 – J13
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July 2000

ENDOGENOUS FERTILITY AND THE DESIGN OF FAMILY TAXATION

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July 26, 2000

Abstract

The paper innovates on the existing optimal taxation literature by taking fertility as endogenous, and allowing for households to be differentiated by ability to raise children, as well as wage rates. In a context where the government cannot observe personal abilities, fertility behaviour conveys a great deal of information about those characteristics, that helps relax the self-selection constraints on the design of policy. One of the results is that, in the absence of explicit inequality aversion, re-distribution is likely to be in favour of low-wage households, even if the latter happen to have higher *laissez-faire* utility than high-wage households. Another is that children should be a tax asset to their parents only if market and domestic skills are positively correlated. Even in that case, however, the optimal policy may include an unusual mix of taxes on number of children, subsidies on child-specific commodities, income support for low-wage households, and positive marginal rates of income tax for all.

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JEL Code: D13, H21, H31, J13

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1 Introduction

One of the problems in the design of optimal income taxation is that certain personal or household characteristics of interest to the fisc are either not observable by the policy maker, or observable only at high cost. The latter must then either give up the idea of taxing different categories of individuals at different rates, or be ready to distort private incentives to such an extent, that it is no longer worthwhile for the tax payer to conceal his or her true characteristics. This informational asymmetry raises the efficiency cost of re-distributing income or welfare. Various ways of getting round this obstacle have been devised, from designing indirect taxation so as to make it more costly for certain categories of tax payers to conceal their true identities, to using indicators correlated with the personal characteristics that the policy maker cannot observe ("tagging").[1] Curiously, however, nobody seems to have thought of the obvious fact that reproductive behaviour conveys a great deal of information about the characteristics of a person or couple, and that the number of children is observable at no cost for the fiscal authority.[2]

To make the point as simply as possible, suppose that households are differentiated by wage rates only (in the rest of the paper, we shall examine the case in which households are differentiated also by other characteristics). Imagine that the government wishes to re-distribute in favour of low-wage households, but can only observe incomes, not wage rates or time worked. If the government were to tax high incomes more severely than low incomes, that could make it advantageous for high-wage households to reduce their labour supply until their incomes are as low, as those of low-wage households. The possibility of this kind of behaviour ("mimicking"),[3] prevents the government from taking re-distribution as far as it is deemed to be desirable.[4] We shall argue that a powerful means of discouraging mimicking is to make a household's total tax bill depend not only on income, but also on number of children.[5] Assuming that parents have

[1] *cf.,* Akerlof (1978).

[2] Monitoring and keeping a register of births and deaths, keeping track of who bears responsibility for the upbringing of a child, etc. does have a cost, but it is one of the basic functions of the State. The marginal cost of retrieving this information for fiscal purposes may be taken to be zero.

[3] *cf.,* Stiglitz (1982).

[4] *cf.,* Boadway and Keen (2000).

[5] Contrary to the finding of Nerlove *et al.* (1993), there may thus be a (second-best) argument for taxing (or subsidizing) the number of children for purposes of horizontal (within-generations) re-distribution, even if fertility is endogenous.

some control over the size of their progeny, a high-wage household wanting to pass for low-wage would then have to give itself not only the same income level, but also the same number of children, as a low-wage household. Since the number of children is costlessly observable by the government, while the allocation of time between labour and other activities is not, that would impose a more stringent requirement on would-be-mimickers, and thus make mimicking more costly, than just having to hit some target level of income.

In real life, child-related subsidies and tax breaks of one kind or another combine to make a household's net tax bill dependent on number of children. The logic of these benefits and allowances is not, however, that of inducing the household to reveal its true colours. Their intended purpose is, rather, to either compensate parents for the cost of having children, or to convey resources to the children themselves. Neither of these justifications stands up to close scrutiny,[6] but we shall not go into that here. Our concern, in the present paper, is the design of an optimal system of direct and indirect taxes, in a context of endogenous fertility, under the assumption that the government wishes to maximize some convex combination of the utilities of current tax payers, without questioning their inter-generational preferences and, therefore, the way in which resources are allocated between parents and children.[7]

2 The setup

In what follows, we apply the self-selection approach (Mirrlees, 1971; Stiglitz, 1982) to the optimal choice of an extended set of policy instruments, that includes not only a (non-linear) income tax, and (linear) commodity taxes, but also money transfers conditional on income and number of children. With rare exceptions,[8] the determination of such transfers is not usually regarded as an integral part of the optimal taxation problem. We will show that its inclusion raises welfare, and makes it less unlikely that a first best can be reached.

Household behaviour is described by a domestic production model with endogenous fertility (Becker, 1991; Cigno, 1991). Households may differ in their ability to raise money, represented by the wage rate, w, but also in their ability to raise children, represented by a domestic

[6] The first because it presupposes that an extra child necessarily reduces the maximized utility of his or her parents. The second because it assumes that any resources given to parents for a child's benefit are automatically and entirely passed on to the intended beneficiary. *Cf.* Cigno (1996).

[7] For example, children count more for society than for their parents in Cigno (1983).

[8] See Cigno (1983, 1986), Nerlove *et al.* (1993), Cigno and Pettini (1999).

productivity parameter, k.[9] We shall assume that there are only two possible values of w, and two possible values of k, so that w and k are always perfectly correlated (positively or negatively) across households. Whatever the value of k, we refer to high-wage households as type-2, and low-wage households as type-1. The government knows individual preferences (assumed to be the same for every household) and the co-distribution of abilities (equal number of households of each type), but cannot tell who is who. All the government can see is the income and the number of children each household has. Pre-tax prices are assumed to be invariant with respect to government policy, and normalized to unity. Market transactions are anonymous, but it is possible to distinguish between child-specific commodities (baby food, children's books and clothes, etc.) and adult-specific commodities.

The government sets the rate of tax on child-specific commodities, s (the one on adult-specific commodities is normalized to zero), and a household tax schedule relating after-tax income to pre-tax income and number of children. Given that there are only two household types, that is the same as offering households a menu $\left(s, t^i, B^i, Y^i\right)$, $i = 1, 2$, where t^i, Y^i and B^i are, respectively, the tax rate on number of children, and household income before and after income tax (but gross of taxes on children and commodities), intended for household type i. The income tax intended for that type of household is, of course, $T^i = Y^i - B^i$. Any, but not all, of these taxes could be negative. A negative (positive) value of s means that child-specific commodities are taxed less (more) than adult-specific commodities. A negative (positive) value of t^i means that type-i households receive a subsidy (pay a tax) on the number of children they have. Given these taxes, households choose their behaviour, which may involve concealing their true characteristics in order to benefit from the more favourable tax treatment intended for another household type. In choosing the policy mix, the government takes into account these behavioural responses.

[9] Other taxation models in which households differ both in market and non-market abilities are discussed by Sandmo (1990), Beaudry and Blackorby (1997), Apps and Rees (1999), Balestrino *et al.* (1999).

2.1 Households

Household preferences are described by a concave utility function[10]

$$U(X, Q, N), \tag{1}$$

where X is adult consumption, Q an index of the children's quality of life ("quality" for short),[11] and N the number ("quantity") of children. We may think of Q as of a composite consumption good, specific to children of that particular household, domestically produced by the child's parents with inputs of own time and child-specific commodities bought from the market. Alternatively, we may think of Q as of (the parental perception of) a child's lifetime utility, conditional on how much time and money the parents have spent on the child. If we favour this second interpretation, U becomes a kind of household-level social welfare function. Either way, Q will depend on the quantity of child-specific commodities, z, and parental time ("attention"), h, provided to each child, as well as on the domestic ability parameter, k,

$$Q = Q(z, h; k). \tag{2}$$

Denoting by (z_0, h_0) the minimum levels of z and h necessary to bring a child into the world, and to keep him or her alive, we can set the scale of Q so that $Q(z_0, h_0; k) = 0$.[12] The function $Q(.; k)$ will be taken to be homogeneous of degree one in $(z - z_0, h - h_0)$.[13]

The household budget constraint is

$$X + (1 + s)Z + tN = B, \tag{3}$$

[10] To avoid unnecessary complexities, we treat adult household members as a single agent. In other words, we abstract from the problem of aggregation of the preferences of adult members. That is legitimate so long as one is not interested in intra-generational distribution effects. For an approach that, overlooking children, focusses on the effects of taxation on the intra-generational allocation of resources, in particular on the allocation between husband and wife, see Apps and Rees (1999).

[11] By assigning the same Q to all children born in the same household, we are implicitly assuming that they have all the same hereditary characteristics, and are equally treated by their parents.

[12] Alternatively, if we prefer to think of Q as utility, we may interpret (z_0, h_0) as the child's standard of living, possibly higher than mere subsistence, to which parents subjectively assign utility zero (Meade, 1976). Put another way, parents-to-be will not give birth, if they cannot guarantee the child at least (z_0, h_0).

[13] In the absence of decisive empirical evidence on the subject, that seems less arbitrary than assuming either increasing or decreasing returns to scale in the "production" of Q. Keep in mind, however, that if parental time is fully occupied by children, the marginal cost of Q (as of N) will still rise with Q (or N) as z is substituted for h at a diminishing marginal rate.

where $Z \equiv Nz$. Normalizing the time endowment (of adult household members) to unity, we write the time-budget constraint as

$$H + L = 1, \tag{4}$$

where $H \equiv Nh$, and L is the labour supply.

Adult household members choose expenditures and time allocation. Within bounds dictated by nature,[14] they also choose how many children to have.[15] Notice that, if labour were the household's only source of finance and the government were absent ($B = Y = wL$), complete specialization in domestic activities would be prevented by the budget constraint, because the household would have to sell some of its time to the labour market to pay for Z. The same restriction does not apply in the opposite direction (Cigno, 1991). We will see that this asymmetry has important policy implications.

Since $L = Y/w$, choosing Y is the same as choosing L. Since government policy effectively restricts household choice to a finite number of alternatives (actually two, one intended for type 1, the other intended for type 2), we can describe household choice as a two-step decision procedure. First, the household finds the (X, N, z, h) that maximizes household utility for any given (s, t, B, Y). Second, it selects, from the menu offered by the government, the (s, t, B, Y) with the higher maximized utility (*i.e.*, effectively, it declares to be either type 1, or type 2).

A household choosing the (s, t, B, Y) that was intended for its own type maximizes (1), subject to (2)−(4). The solution satisfies

$$U_X = \alpha, \tag{5}$$

$$U_Q Q_z = \alpha N (1 + s), \tag{6}$$

$$U_N = \alpha [(1 + s) z + t] + \beta h, \tag{7}$$

$$U_Q Q_h = \beta N, \tag{8}$$

where α and β are the marginal utilities of, respectively, income and time, and subscripts denote partial differentiation. Solving these first-order conditions together with the constraints, gives us the household demands for (X, N, z, h) as functions of the policy instruments. Substituting

[14] The choice of N will have to satisfy $0 \leq N \leq n$, where n is natural fecundity. We assume, however, that this restriction is never binding (otherwise the problem would be equivalent to one with exogenous fertility).

[15] A more realistic assumption would be that parents condition, by their choice of fertility controls, the probability distribution of births, but that would make no difference to the point at issue.

back into the utility function, gives us the indirect utility function, $V(s,t,B,Y;w,k)$. The latter is the pay-off to being truthful.

The marginal utilities of income after and before income tax are, respectively, $V_B = \alpha$ and $V_Y = -\beta/w$. The negative sign of the second expression reflects the fact that a rise in Y, holding w constant, implies a labour supply rise. Using $T \equiv Y - B$, and the first-order conditions, we can write the marginal income tax rate as

$$T'(Y) \equiv 1 + \frac{V_Y}{V_B} = 1 - \frac{\beta}{\alpha w}. \tag{9}$$

The right-hand-side of (7) represents the marginal cost of N. Denoting by $\tau \equiv sz + t$ the effective tax on money spent for each child, and using (9), we may re-write this marginal cost as

$$\pi = z + \tau + \left[w - T'(Y)\right] h, \tag{10}$$

where $(z + \tau)$ represents the out-of-pocket cost, and $[w - T'(Y)]\, h$ the opportunity-cost (different from forgone earnings if the marginal rate of income tax is different from zero) of raising an extra child. A child of zero quality (*i.e.,* a child to whom parents give only the bare necessities of life) costs $\pi_0 = (1+s)\, z_0 + t + [w - T'(Y)]\, h_0$. We may view π_0 as the fixed cost of having a child. If we think of π as the "price" of children, it becomes clear that fertility decisions depend on *all* policy instruments.

For future reference, we derive the duality properties of this optimization problem. Due to the non-linearity of the budget constraint, standard results cannot be applied directly. It is possible to show, however, that the Slutsky relations apply to the demand for children, N, and to the demand for child-specific commodities, Z,

$$\begin{aligned} \overline{N}_t &= N_t + NN_B, \quad \overline{Z}_t = Z_t + NZ_B, \\ \overline{N}_s &= N_s + ZN_B, \quad \overline{Z}_s = Z_s + ZZ_B, \end{aligned} \tag{11}$$

where the upper bar denotes a Hicksian demand. The own-price Slutsky terms are negative ($\overline{N}_t < 0$, $\overline{Z}_s < 0$), and the symmetry property,

$$\overline{Z}_t = \overline{N}_s, \tag{12}$$

holds as in the standard consumer model. As in all self-selection problems, we have also the property[16] that

$$\overline{N}_Y = N_Y + \frac{V_Y}{V_B} N_B \quad \text{and} \quad \overline{Z}_Y = Z_Y + \frac{V_Y}{V_B} Z_B. \tag{13}$$

[16] See, *e.g.*, Anderberg and Balestrino (2000).

2.2 Mimickers

If different household types receive different tax treatment, it may be in the interest of a type-i household to pretend to be type-j ($i, j = 1, 2$). If that is the case, we call this household an "ij-mimicker". In conventional optimal income taxation models, where households are differentiated only by earning ability, the standard ("single-crossing") assumption that, in the (B, Y)-plane, the indifference curves are everywhere flatter for high-wage than for low-wage households, combined with the assumption that the policy maker's preferences are a convex combination of household preferences, so that re-distribution will always be in favour of low-wage households, allows one to rule out the possibility that a low-wage household will ever want to be taken for high-wage. In our more general set-up, with households differing by more than just wage rates, this shortcut is not available, and we have thus to allow for the possibility that either household type could have an interest in concealing its true identity.

An i-type household wanting to mis-represent its type will set the choice variables that the government can observe equal to those of type-j households. If it is assumed that the government can only observe incomes, adjusting the labour supply is then enough to make the mimicker indistinguishable from the mimicked. If the government can also observe the number of children, however, the mimicker will give away its true identity if it does not also choose the same number of children as the mimicked. Therefore, an ij-mimicker will have not only the same gross income, Y^j, but also the same fertility, N^j, as the mimicked. What this means is that, where time allocation is concerned, a mimicker has no choice at all: it must devote $\left(Y^j/w^i\right)$ hours to the labour market, and $\left[\left(w^i - Y^j\right)/w^i\right]$ hours to looking after children. That is what makes mimicking so much arduous in an endogenous fertility context.[17]

The mimicker's optimization,

$$\begin{array}{ll} \max\limits_{X,z} & U^{ij} = U\left[X^{ij}, Q^{ij}\left(z^{ij}, \frac{\left(w^i - Y^j\right)}{w^i N^j}\right), N^j\right] \\ \text{s.t.} & X^{ij} + \left[(1+s)\, z^{ij} + t^j\right] N^j = B^j, \end{array} \tag{14}$$

has first-order conditions

$$U_X^{ij} = \alpha^{ij}, \tag{15}$$

$$U_Q^{ij} Q_z^{ij} = \alpha^{ij} N^j (1+s). \tag{16}$$

[17] A mimicker's life is particularly hard in our model, because there are only two possible uses of time, making money or raising children, and both have an observable output. Were there other possible uses of time – *e.g.,* X, too, were domestically produced using time ("leisure") – the mimicker would have a little more leeway, but not very much, because children are notoriously time-intensive.

The indirect utility function that emerges from this optimization, $V^{ij} = V(s, t^j, B^j, Y^j; w^i, k^i)$, $i \neq j$, represents the pay-off to being untruthful.

3 Optimal tax rules

The government's aim is assumed to be that of maximizing a Paretian, quasi-concave function, $W(.)$, of the utilities of the two household types. The choice of policy instruments is restricted by the government budget constraint, but also by the self-selection constraints that neither household type must be better-off mimicking, than behaving according to type. Assuming, for simplicity, that the government has no revenue requirement (purely re-distributive taxation) the problem is to choose $\left(s, t^1, t^2, B^1, B^2, Y^1, Y^2\right)$ so as to

$$\begin{array}{lll} \max & W\left[V\left(s, t^1, Y^1, B^1; w^1, k^1\right), V\left(s, t^2, Y^2, B^2; w^2, k^2\right)\right] & \\ \text{s.t.} & V\left(s, t^1, Y^1, B^1; w^1, k^1\right) \geq V\left(s, t^2, Y^2, B^2; w^1, k^1\right) & \left[\sigma^{12}\right] \\ & V\left(s, t^2, Y^2, B^2; w^2, k^2\right) \geq V\left(s, t^1, Y^1, B^1; w^2, k^2\right) & \left[\sigma^{21}\right] \\ & \sum_i \left(sZ^i + t^i N^i + Y^i - B^i\right) = 0. & [\mu] \end{array} \tag{17}$$

The Lagrange-multipliers are indicated in square brackets.

Denoting by $W_i = \frac{\partial W}{\partial v^i} > 0$ the welfare weight of a type-i household, we can write the first-order conditions[18] for the policy optimization as

$$\left(W_i + \sigma^{ij}\right) V_Y^i - \sigma^{ji} V_Y^{ji} + \mu\left[1 + sZ_Y^i + t^i N_Y^i\right] = 0, \quad i, j = 1, 2,\ i \neq j \tag{18}$$

$$\left(W_i + \sigma^{ij}\right) V_B^i - \sigma^{ji} V_B^{ji} + \mu\left[-1 + sZ_B^i + t^i N_B^i\right] = 0, \quad i, j = 1, 2,\ i \neq j \tag{19}$$

$$\left(W_i + \sigma^{ij}\right) V_t^i - \sigma^{ji} V_t^{ji} + \mu\left[sZ_t^i + N^i + t^i N_t^i\right] = 0, \quad i, j = 1, 2,\ i \neq j \tag{20}$$

$$\sum_i \left(W_i + \sigma^{ij}\right) V_s^i + \sum_i \sigma^{ij} V_s^{ji} + \mu \sum_i \left[Z^i + sZ_s^i + t^i N_s^i\right] = 0 \tag{21}$$

The self-selection constraints cannot both be stringent. If neither of them is binding (*i.e.,* households reveal their true characteristics), the government can carry out the desired re-distribution by lump-sum transfers, and the solution is a first best. If either of them is binding, the government needs to distort marginal incentives in order to induce households to reveal their true characteristics, an the solution is a second best. Which, if any, of the self-selection constraints will be binding is determined, jointly with the direction of re-distribution, by the

[18] As usual, this second-best policy problem is not necessarily well-behaved. However, we follow the whole literature on the subject in assuming that a solution exists.

optimal choice of policy instruments. If re-distribution is from high to low-wage households, σ^{12} is zero. If it is the other way round, σ^{21} is zero.

Given that the direction of re-distribution is important, not only for its own sake, but also for self-selection purposes, some general considerations will help us in the more detailed analysis that follows. If households were differentiated by labour market ability (wage rate) only, as in conventional optimal taxation models, re-distribution would come about for "equity" reasons. If households are differentiated also by skill in the domestic production of a non-tradeable good, and this skill is negatively correlated with labour market ability, however, there can also be an "efficiency" motive for re-distributing. Efficiency requires that households allocate their time according to comparative advantage. As we saw in the last section, however, the budget constraint may prevent households with a comparative advantage in domestic production from specializing completely in raising children (but nothing would stop households with a comparative advantage in making money from specializing completely in market activities if they so wished). Re-distributing from high to low-wage households could then raise welfare, quite apart from any equity consideration, simply because it allows the beneficiaries to pursue their comparative advantage more fully.

If low-wage households have lower *laissez-faire* utility than high-wage households, equity and efficiency then pull in the same direction. We can then be sure that σ^{21} is zero, and may expect σ^{12} to be positive. If the opposite is true, equity and efficiency considerations pull in opposite directions, and the outcome depends on how strongly the policy maker dislikes inequality (how convex the social indifference curves are). If the social welfare function is Benthamite (additive), implying that the policy maker is only interested in the size of the pie, and cares about its division only insofar at it affects size, the "efficiency" motive may well predominate (Balestrino *et al.* 1999). Re-distribution will then be in favour of low-wage households, and σ^{12} will be zero. Otherwise, re-distribution will be in favour of high-wage households, and σ^{21} will be zero. In either case, however, the amount re-distributed is likely to be modest, and there is thus a chance that neither of the self-selection constraints will be binding ($\sigma^{12} = \sigma^{21} = 0$). If that occurred, the government would then be able to design lump-sum transfers specific for each household type, and the outcome would be a first best. Given sufficient inequality aversion (certainly if the welfare function were Rawlsian), however, re-distribution in favour of high-wage households would be substantial, and σ^{12} would then be positive.

3.1 Taxes on income

Denoting by $\Theta^i\left(Y^i, N^i\right) \equiv \left(sZ^i + t^i N^i + Y^i - B^i\right)$ the total tax bill of a type-i household, and using (9), we find how this is affected by gross income,

$$\Theta_Y^i = T'\left(Y^i\right) + sZ_Y + t^i N_Y \equiv 1 + \frac{V_Y}{V_B} + sZ_Y + t^i N_Y. \tag{22}$$

Using (22) and (13) to manipulate (18)-(19), we obtain expressions for the optimal values of the marginal income tax rates,[19]

$$T'\left(Y^1\right) = \frac{\sigma^{21} V_B^{21}}{\mu}\left(\frac{V_Y^{21}}{V_B^{21}} - \frac{V_Y^1}{V_B^1}\right) - \left[s\overline{Z}_Y^1 + t^1\overline{N}_Y^1\right]; \tag{23}$$

$$T'\left(Y^2\right) = \frac{\sigma^{12} V_B^{12}}{\mu}\left(\frac{V_Y^{12}}{V_B^{12}} - \frac{V_Y^2}{V_B^2}\right) - \left[s\overline{Z}_Y^2 + t^2\overline{N}_Y^2\right]. \tag{24}$$

To interpret these rules, suppose for a moment that no taxes on children or commodities are at hand, so that the second r.h.s. term in each expression is identically zero. Suppose, also, that $\sigma^{21} > 0$ and $\sigma^{12} = 0$, meaning that, at the optimum, type-2 households are potential[20] mimickers. Under the standard assumption that, at the optimal (B^1, Y^1), the indifference curve of a low-wage household is steeper than that of a high-wage mimicker, imposing a positive marginal income tax rate on the former would deter the latter from mimicking. Conversely, since type-1 households have no interest in mimicking, there is no point in distorting the decisions of type-2 households by imposing a positive marginal rate of income tax on them too. Indeed, for $\sigma^{21} > 0$ and $\sigma^{12} = 0$, (23)$-$(24) imply $T'(Y^1) > 0$ and $T'(Y^2) = 0$.

If, in addition to taxing income, the government taxes (or subsidies) child-specific commodities or the number of children, there is also a revenue effect, reflected by the term in square brackets in (23) and (24). Suppose, for instance, that $\left[s\overline{Z}_Y^2 + t^2\overline{N}_Y^2\right] < 0$, meaning that the revenue from taxing commodities and the number of children falls, as the labour supply of type-2 households goes up. Even if nobody were interested in mimicking them $\left(\sigma^{12} = 0\right)$, imposing a positive marginal rate of income tax on high-wage households would then raise tax revenue. There could thus be an additional and independent reason for distortionary income taxation.

[19] We are adapting the procedure in Edwards *et al.* (1994).

[20] Not actual, because mimicking is deterred by an appropriate choice of policy instruments.

3.2 Taxes on commodities and number of children

Now, consider (20) and (21). Using Roy's identities, substituting from (19), applying (11) and (12), and noting that $N^{ji} = N^i$, we find

$$s\overline{N}_s^i + t^i\overline{N}_t^i = 0, \quad i = 1, 2, \tag{25}$$

and

$$\sum_i \left(s\overline{Z}_s^i + t^i\overline{Z}_t^i\right) = N^1\frac{\sigma^{21}V_B^{21}}{\mu}(z^1 - z^{21}) + N^2\frac{\sigma^{12}V_B^{12}}{\mu}(z^2 - z^{12}). \tag{26}$$

The l.h.s. of (26) measures the cost of distorting the demand for child-specific commodities through s and t^i as (a first-order approximation to) the compensated effect on Z . If the effect is negative (positive) we say that the demand for child-specific commodities is "discouraged" ("encouraged"). The r.h.s. represents the corresponding gain. To see the intuition behind the rule, suppose, for instance, that $\sigma^{21} > 0$, $\sigma^{12} = 0$ and $\left(z^1 - z^{21}\right) < 0$ (true type-1 households buy less commodities for each of their children, than 21-mimickers). Since V_B^{21} and μ are positive, it is clear that, in this case, distorting prices in favour of adult-specific commodities would harm mimickers more than genuine low-wage households. Therefore, the relevant self-selection constraint can be relaxed by discouraging Z. Analogous considerations apply to the other possible cases.

The l.h.s. of (25) measures the cost of, and the r.h.s. the gain from distorting fertility decisions through s and t^i. Since the gain is zero, it is clear that these taxes must *not* distort fertility decisions. That is so because, as fertility is observable by the government, and the mimicker must thus have the same number of children as the mimicked, distorting fertility choices has no "screening power". As it cannot be used to discourage mimicking, there is then no point in distorting fertility decisions by these means.

This rule does not imply that t^i should be zero.[21] Since there is a second-best rationale for taxing or subsidizing child-specific commodities in accordance with (26), the policy prescription is in fact to set t^i so that it totally offsets the distortionary effect of s. More specifically, (25) tells us that s and t^i must have *opposite signs* if Z and N are Hicksian *complements*, the *same sign* if Z and N are Hicksian *substitutes*.[22]

[21] This would be true only if differential commodity taxation were ruled out (*e.g.*, because the planner cannot distinguish between adult and child-specific goods). If that were the case, (25) would reduce to $t^i\overline{N}_t^i = 0$, which implies $t^i = 0$.

[22] Note that this conclusion cannot be reached by inspection of (26) alone.

Furthermore, since s and t^i are not the only policy instrument affecting the cost of children, (25) does not imply that fertility will be undistorted at a second-best optimum. We can see by simply looking at the price of N (10) that, even if $s = t^i = 0$, the post-tax price of children will differ from the pre-tax price so long as $T'(Y) \neq 0$ (by taxing income at the margin, the government reduces the opportunity-cost of childbearing). The same is likely to be true if $T'(Y) = 0$, but children and child-specific commodities are either taxed or subsidized, because τ will then be different from zero if s and t^i have the same sign, and does not need to be zero even if s and t^i have opposite signs (because, in general, $\left|sz^i\right| \neq \left|t^i\right|$).[23] To sum up, there is no point in distorting fertility decisions for the sake of discouraging mimicking, because it would not work, but there may well be a point in doing it for distributional reasons, or in order to counter the effects of other distortions.

Differentiating the total tax bill of a type-i household with respect to N^i,

$$\Theta_N^i = \tau^i - T'(Y^i)w^i h^i, \tag{27}$$

we find out whether a child is a tax asset or a tax liability for that household. At a first best, where there is no distortionary taxation, children are clearly tax neutral (Nerlove *et al.*, 1993). If households are differentiated by the wage rate only, as in the standard optimal taxation model, then, at a second best with income taxation only, children are tax neutral for potential mimickers, but a tax asset for low-wage households. With child-specific commodities and the number of children taxed or subsidized, however, anything is possible because, as we have seen, $\tau^i \equiv sz^i + t^i$ can have any sign.

4 The design of family taxation

There is a presumption, in the political and institutional debate, that children should always be a tax asset to their parents, and that this should be especially true for low-income households. We have shown that, in the standard optimal income taxation case with households differentiated by wage rates only, children are indeed a tax asset for low-wage households, but not for high-wage ones. We could not say, however, what happens when households are differentiated by domestic, as well as labour market ability, and the government can tax child-specific commodities and quantity of children, as well as income, because we could not establish whether the tax on

[23] The paradox is only apparent, because s affects not only the price of the intermediate good z, but also the marginal costs of the final goods N and Q.

number of children, while specific to household type, should be higher or lower for low-income parents. And, even if we did, we still could not establish whether a child should be a tax asset or a tax liability for low-income households, because that depends also on all the other policy instruments.[24] Employing specific functional forms, and using numerical simulations,[25] we will show that children need not be a tax asset for anybody, not even for low income parents. The examples will also serve to illustrate the general proposition that (*i*) a tax/subsidy on number of children may be welfare improving, and (*ii*) endogenous fertility makes it less unlikely that a first best can be reached by fiscal means.

4.1 Special assumptions

Suppose that the household utility function is log-linear,

$$u = \delta \ln(X) + \varepsilon \ln(Q) + \eta \ln(N). \tag{28}$$

and the quality function Stone-Geary,

$$Q = (z - z_0)^{\psi}(h - h_0)^{(1-\psi)}. \tag{29}$$

Substituting (29) into (28), we can see that labour supplied $(1 - H)$ is separable from commodities demanded. That is the special case in which, with exogenous fertility, and households differentiated by w only, there is no point in using other tax instruments if an optimal income tax is in place − the well-known Atkinson-Stiglitz (1976) theorem. It will be interesting to see whether that remains true, in an endogenous fertility context, when households differ in k, as well as w.

Due to the non-linearity of the budget constraint, we need some restrictions on the parameters to make sure that the household optimization problem is globally concave. Sufficient[26]

[24] In optimal taxation models with operative self-selection constraints, it is usually difficult to characterize the properties of the tax system in general terms. Ours is no exception.

[25] There is a long-established tradition of numerical simulations in optimal tax theory. A recent example is Myles (2000), which also contains a list of relevant references.

[26] The assumption that the subsistence quantity of attention is zero is stronger than necessary: the problem is well-behaved also for small positive values of h_0. However, calculations are much simpler if we put $h_0 = 0$. Since the absolute minimum, below which parental attention cannot be substituted with purchased child care (included in z), must be very small in relation to the total lifetime endowment, it seemed reasonable to approximate this minimum to zero.

conditions are

$$\eta > \varepsilon > \delta, \tag{30}$$

$$h_0 = 0. \tag{31}$$

The fixed cost of a child is then $\pi_0 = (1+s)z_0 + t$. We shall identify higher skill in domestic production with lower fixed cost of reproduction, $k \equiv 1/z_0$.

Household demands for N and z are given by

$$z = \frac{\Gamma}{(\eta - \varepsilon)\, k} + \frac{t\varepsilon\psi}{(1+s)\,(\eta - \varepsilon)}, \tag{32}$$

$$N = \frac{(1+s)\, B}{(1+s) + tk}, \tag{33}$$

where $\Gamma \equiv \delta - (1-\psi)\,\varepsilon + \eta$. The marginal utility of disposable income is

$$V_B = \frac{\Gamma}{B}. \tag{34}$$

Note that, thanks to the separability of (28), neither the demands, nor the marginal utility of disposable income, depend on w. The marginal utility of disposable income is decreasing in B. The per-child demand for child-specific commodities is decreasing in the domestic productivity parameter, k, and independent of B. The demand for quantity of children is increasing in both k and B. It can be easily checked that N and Z are always Hicksian complements $(\overline{Z}_t < 0, \overline{N}_s < 0)$.

The welfare function is assumed to be additive,

$$W = V^1 + V^2. \tag{35}$$

Combined with diminishing marginal utilities of income, this would imply re-distribution in favour of low-wage households, if households were differentiated by w only.

4.2 Taxing incomes

Since the marginal utility of income does not depend on either k or w, the tendency to equalize marginal utilities of income, that arises because of the utilitarian social welfare function, implies a tendency to equalize disposable incomes. If neither of the self-selection constraints is binding (first best), the process goes as far as equalizing adult consumption. If either of them is binding (second best), re-distribution is taken to the point, independent of the distribution of k, where

	$k^1 < k^2$	$k^1 > k^2$
$\sigma^{21} > 0,\ \sigma^{12} = 0$	$s < 0,\ t^i > 0$	$s > 0,\ t^i < 0$
$\sigma^{21} = 0,\ \sigma^{12} > 0$	$s > 0,\ t^i < 0$	$s < 0,\ t^i > 0$

Table 1: Possible patterns of indirect taxation and taxation of children

one of the self-selection constraints prevents further subsidization of low-wage households. The marginal income tax rate is

$$T'(Y) = 1 + \frac{(1-\psi)\,\varepsilon B}{(Y-w)\,\Gamma}. \tag{36}$$

In principle, this can have any sign. Later in this section, we will employ numerical examples to shed more light on the issue. We now turn to our main task, of characterizing the optimal choice of s and t^i.

4.3 Taxing child-specific commodities and the number of children

Our first step is to deduce from (26) whether child-specific commodities are taxed or subsidized at a second-best optimum. Since the demand for z does not depend on w, differences in k are all that matters when making comparisons between the demand of the mimicker, and that of the mimicked. Whether the use of z should be encouraged or discouraged depends only, therefore, on whether the mimicker has higher or lower domestic productivity than the mimicked. Given that Z and N are Hicksian complements, implying that s and t^i must have opposite signs, the possible second-best choices of policy mix are as shown in Table 1. The first column pertains to the case in which market and domestic skills are positively correlated, the second to the one in which the correlation is negative. Re-distribution is from high to low-wage households (the former are the potential mimickers) in the first row, from low to high-wage households in the second.

It is thus clear that the Atkinson-Stiglitz theorem does not apply when households differ in k as well as w: taxes on commodities or number of children may raise social welfare even if income is optimally taxed, and the utility function is separable in consumption and labour. With the functional forms that we have adopted, we can be even more specific in characterizing the relation between t^i and s,

$$t^i = -\frac{(1+s)s}{[\varepsilon\psi + (1+s)\eta]\,k^i}. \tag{37}$$

This says that the second-best tax (subsidy) on quantity of children is decreasing (increasing) in k^i, but independent of w.[27] Therefore, it is not necessarily true that parents should be directly subsidized for having children. And, if it is true, the size of the direct subsidy does not depend on the wage rate.

Whichever is the case, t^i is only part of the story. Although s is the same for everybody, the effective tax rate on the number of children in household i depends on z^i,

$$\tau^i = \frac{[2\eta - (1-\psi)\,\varepsilon]\,\varepsilon\psi s}{(1+s)\,(\eta-\varepsilon)\,\eta - (1-\psi)\,\varepsilon^2\psi + 2\varepsilon\eta\psi} z^i. \tag{38}$$

It is thus clear that the sign of τ^i does not depend on $\left(w^i, k^i\right)$. It is also clear that the *ad valorem* effective tax rate on the out-of-pocket cost of a child $\left(\tau^i/z^i\right)$ is constant across households. Therefore, whatever the sign of the correlation between skills, and whatever the direction of re-distribution, the combined effect of taxing commodities and number of children is distributionally neutral.[28]

4.4 A numerical simulation

Even with specific functional forms, the signs of Θ^i nor Θ^i_N depend on the numerical values of the parameters. To establish the direction of re-distribution, and ascertain whether an extra child should be a tax asset or a tax liability, we assume

$$\eta = .5, \quad \varepsilon = .3, \quad \delta = .2, \quad \psi = .5, \tag{39}$$

but the results are qualitatively the same for all $(\delta, \varepsilon, \eta, \psi)$ satisfying (30)−(31). Regarding the values of (k, w), we consider three cases.

4.4.1 Domestic and market skills are positively correlated.

Assume that type-1 households are worse at everything,

$$k^1 = .1, \quad w^1 = 50, \quad k^2 = .2, \quad w^2 = 100, \tag{40}$$

so that their *laissez-faire* utility is clearly lower than that of type-2 households.

[27] Recall that, although (k, w) is not directly observable, policy is designed to induce households to reveal their true type.

[28] Although its sharpness is due to the specific functional forms used, this result reflects a more general principle, namely the prevalence of the income tax as a redistributive device.

	s	t^i	$T'(Y^i)$	$\frac{T(Y^i)}{Y^i}$	Θ^i	Θ^i_N	V^i
$i=1$	-0.05	0.28	0.12	-0.44	-0.16	-1.61	0.92
$i=2$	-0.05	0.14	0.02	0.22	0.16	-0.39	1.12

Table 2: Tax rates and tax liabilities when skills are positively correlated

Re-distribution is then from high to low-wage households, and the self-selection constraint that prevents the former from mimicking the latter is binding. Policy optimization yields a *second best* (Table 2). In this case, child-specific commodities are subsidized (or taxed less than adult-specific commodities). Fertility is taxed, and the tax rate is *higher* for low-wage households. The income tax schedule is designed to transfer money from high to low-wage households, but both face a positive marginal rate of income tax (for the latter, this means that the subsidy is reduced as their income rises). At the margin, a child is a tax asset for both types of household, but more for the one with lower wage rate. Despite re-distribution, high-wage households remain better off.

4.4.2 Skills are negatively correlated, low-wage households have higher *laissez-faire* utility than high-wage households

Assume that the wage rate of type-2 households is not sufficiently larger than that of type-1 households to compensate for the lower domestic productivity:

$$k^1 = .2, \quad w^1 = 50, \quad k^2 = .1, \quad w^2 = 75. \tag{41}$$

Hence, low-wage households have higher *laissez faire* utility than high-wage households. Re-distribution is now in favour of the latter, but not large enough for the former to have an interest in mimicking. As neither of the self-selection constraints is binding, the policy optimization yields a *first best* (Table 3). Re-distribution (by lump-sum transfers) reduces the utility advantage of low-wage households, but does not eliminate it, because efficiency considerations require these households to specialize in child raising activities.[29]

[29] See the general discussion in the introductory part of section 3.

	s	t^i	$T'(Y^i)$	$\frac{T(Y^i)}{Y^i}$	Θ^i	Θ^i_N	V^i
$i=1$	0.00	0.00	0.00	-0.34	-12	0.00	1.02
$i=2$	0.00	0.00	0.00	0.20	12	0.00	0.82

Table 3: Tax rates and tax liabilities when skills are negatively correlated and low-wage households have higher $laissez-faire$ utility

	s	t^i	$T'(Y^i)$	$\frac{T(Y^i)}{Y^i}$	Θ^i	Θ^i_N	V^i
$i=1$	0.05	-0.15	0.00	-0.70	-23	0.27	1.13
$i=2$	0.05	-0.29	0.02	0.26	23	0.73	0.93

Table 4: Tax rates and tax liabilities when skills are negatively correlated and low-wage hosehols have lower $laissez-faire$ utility

4.4.3 Skills are negatively correlated, high-wage households have higher *laissez-faire* utility than low-wage households

Skills are again negatively correlated, but the domestic productivity of type-1 households is not sufficiently larger than that of type-2 households to compensate for the lower wage rate:

$$k^1 = .2, \quad w^1 = 50, \quad k^2 = .1, \quad w^2 = 100. \tag{42}$$

Type-1 households have thus lower *laissez-faire* utility than type-2 households. Re-distribution is again in favour of low-wage households, and large enough for high-wage households to have to be deterred from mimicking. The solution is a *second best (*Table 4*).* Fertility is directly subsidized in both household types (at a higher rate in high-wage households). However, as child-specific commodities are taxed (more than adult-specific ones), children are a tax liability for both high and low-wage parents (more for the former). The income tax schedule is again designed to transfer money from high lo low-wage households, but the marginal rate is now close to zero for high earners (commodity taxation is thus used for revenue purposes, rather than to deter mimicking). Re-distribution is so large, that low-wage households end up having higher utility than high-wage households.

In all these examples, re-distribution is in favour of low-wage households, even in the one where the latter have higher *laissez-faire* utility than high-wage households. Clearly, in that case, equity is not the reason for re-distributing. As pointed out earlier, when households are differentiated by their skill in the production of a non-tradeable good, as well as by their ability to raise money, and the two types of skill are negatively correlated, there is then an efficiency

motive for re-distributing in favour of those who have a comparative advantage in domestic production. As we are using a Benthamite social welfare function, the efficiency motive will more or less always prevail. If equity and efficiency considerations pull in opposite directions, as in the second of the cases considered, re-distribution will be "small" (making it more likely that the policy will hit a first best), but will still be giving to the better off. If they pull in the same direction, as in the third of our cases, re-distribution may be so massive as to overturn the *laissez-faire* situation.

If domestic and labour market skills are negatively correlated, children are never a tax asset. An extra child reduces the tax bill only if, as in the first of the cases considered, the two types of skill are positively correlated. In that case, it is also true (of course) that a child is worth more to a low income family, than to one with a high income. If domestic and labour market skills are positively correlated, the outcome is then the same as if households differed for their wage rates only, but with the important qualification that the tax system achieves this conventional result in a most unconventional manner, by simultaneously taxing fertility, distorting prices in favour of child-specific commodities, and distorting labour decisions more in low, than in high-wage households.[30]

5 Conclusions

This paper innovates on the existing literature in two ways. The first is to allow for the possibility that households are differentiated by their ability in the domestic production of a non-tradeable good, as well as by their ability in the labour market. The second is to make fertility endogenous, and to identify domestic ability with a lower cost of raising children.

In general, bi-dimensional differentiation introduces the possibility that re-distribution will be from low to high-wage households, because the latter could be the ones who, in the absence of policy, have lower utility. In our particular context, however, it turns out that, unless the policy maker is sufficiently averse to inequality, re-distribution is always in favour of low-wage households, even if that means taking from the worse-off and giving to the better-off. The reason is that, while utility depends on absolute advantage (in the absence of policy, those who are better at everything are also better off), allocative efficiency requires households to specialize according to comparative advantages. As the domestically produced good (children) is not

[30]Note that, in line with a large part of the literature, we are ignoring administration costs. If the cost of tax collection is sufficiently high, all this "tax churning" may well be sub-optimal.

tradeable, specialization in domestic activities is restricted by the need to raise income in order to pay for the commodities used as an input in domestic production. Therefore, re-distribution in favour of households with a comparative advantage in raising children may well be welfare improving, even if the beneficiaries happen to have higher *laissez-faire* utility than the rest.

Endogenous fertility gives the policy maker a considerable advantage in the design of an optimal system of taxes and transfers. In a context where the government does not observe, or can only discover at high cost, the characteristics of adult household members, fertility behaviour conveys a great deal of information about those characteristics, which helps to relax the self-selection constraints on the design of policy. Indeed, it can say so much, that it is not worthwhile for any of the households to try and hide their true identities. Where that is the case, a first best can be achieved by lump-sum transfers specific to household type.

Combined with bi-dimensional household differentiation, endogenous fertility leads to some unconventional policy prescriptions. One is that, with a Benthamite social welfare function, re-distribution is likely to be in favour of low-wage households, whatever their ability in raising children; or, if it is in favour of high-wage households, to be small enough for a first best to be a real possibility. Another is that children should be a tax asset to their parents only if market and domestic skills are positively correlated. Even in that case, however, the optimal policy may include an unusual mix of taxes (higher for low-wage families!) on the number of children, subsidies (or lower taxes) on child-specific commodities, income support for low-wage households, and positive marginal rates of income tax for all.

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