Società italiana di
economia pubblica

WORKING PAPERS

Alberto Cavaliere

COORDINATION AND PROVISION OF DISCRETE PUBLIC GOODS BY CORRELATED EQUILIBRIA

Pavia, February 2000

Coordination and Provision of Discrete Public Goods by Correlated Equilibria

Alberto Cavaliere*
(Department of Public Economics, University of Pavia-Italy)
cavalier@unipv.it

February 2000

Abstract

The strategic analysis of the private provision of discrete public goods has shown the existence of multiple Nash Equilibria with the efficient number of players voluntarily contributing. However the coordination issue is left unexplained by this literature. The experimental evidence shows that communication among players is helpful to achieve ccoperation.We claim that, from the theoretical point of view, this is equivalent to playing correlated equilibria in an extended public good game with communication, modelled as Chicken. We characterise such equilibria as feasible cordination mechanisms to achieve public goods provision in the general contribution game.We further introduce a second kind of game characterised by pay-off externalities that may persist after the minimal threshold of contributors is achieved. While it is easy to show the existence of Pareto efficient correlated equilibria in the first game, in the second one players face incentive problems such that a first best cannot always be an equilibrium. Nevertheless there exist correlated equilibria that can be qualified as incentive efficient mechanisms, once free riding is seen as

*Aknowledgements: I would like to thank Guido Ascari, Paolo Bertoletti and especially Enrico Minelli for useful discussions and two anonymous referees for their comments on a first version of this paper. I am also grateful to the Departmentof Economics of University College London for its hospitality. Financial support by CNR is aknowledged.

a moral hazard issue. Finally, with an example, we discuss the impact of coalition formation in our framework.

JEL Classification:C72,D79,D89,H41

Key words: private provision of public goods, contribution games, games with communication, incentives.

1 Introduction

In their seminal contribution concerning the private provision of discrete public goods, Palfrey and Rosenthal (1984) point out that the free rider problem has two sides: the demand revelation question and the participation issue. Even if preferences for the public good were perfecly known and/or the willingness to pay for the public goods were the same for each agent, one cannot exclude strategic behaviour with respect to participation: each member of a group will be tempted to free ride, hoping that the other members will pay the cost of the public good.

However voluntary participation can be observed not only in economic reality but also considering the great deal of experimental evidence that we dispose of (Ledyard, 1992). From the theoretical point of view, Palfrey and Rosenthal show that voluntary participation is feasible, in a game where a discrete public good is to be provided. In their model there are in fact both pure strategy and mixed strategy Nash equilibria characterised by the private provision of a pure public good. Moreover, the efficient number of players contributing is an equilibrium.

One problem with these results is the multiplicity of equilibria. When the number of players is large, mixed strategy equilibria can be excluded by Palfrey and Rosenthal, but still a plethora of pure strategy Nash equilibria remain in their model.This represents a relevant issue to the extent that it is unclear why agents should play one equilibrium rather than another one. The authors state in fact that "Some form of coordination is probably very important (possibly necessary) for the attainment of a Nash equilibrium"[1].

The experimental evidence shows that the degree of cooperation among players in a public good game is greatly enhanced when players are allowed to

[1] Palfrey and Rosenthal, 1984, p.191.

communicate (see for example Ledyard 1992, and Dawes and Thaler, 1988). Communication can be helpful to achieve coordination among players'actions. As to the way to introduce communication in public good games, one can suppose that players communicate first and then play the game[2]. But dealing with communication in the extensive form would be too costly an option from the modelling point of view[3]. From the theoretical point of view the idea to play a game with communication can also be dealt with using solution concepts, particularly the concept of correlated equilibrium (Aumann, 1974 and 1987). The idea of a correlated equilibrium is that players find communication helpful to correlate their strategies by using common random devices (like coin flips or sunspots) and then play a Nash equilibrium in the extended game with communication. Thanks to communication, players can agree on the probability that anyone of them will pick up one action rather than another one. Correlation thus creates then a built in consensus about the fact that one game will be played in one specified way. This idea seems appealing in our framework, being supported also by experimental results showing that players, when allowed to communicate, often adopt lotteries as random devices for the distributional decision about the allocation of the cost of a public good characterised by a threshold (Van de Kragt, Orbell and Dawes,1983).

In section one we start from the original contribution game[4] of Palfrey and Rosenthal and extend equilibrium analysis to consider also the notion of correlated equilibrium.We shall provide the basic intuitions concerning the feasibility and efficiency of the voluntary provision of a public good in a correlated equilibrium, starting from a two-players example. We shall then give a definition of correlated equilibrium for the general game and discuss

[2] One approach is to compare a game in the usual sense with an augmented version of the game in which some additional strategies are added that have no apparent effect on payoffs: communication is just cheap talk. A game can then be played twice: one time the decisions result in no pay-off but players may observe others "hypothetical" decisions, and then a second time the game is played for real. This is the approach followed by Palfrey and Rosenthal (1991) to test the effect of comunication on a public good game with incomplete information.

[3] For example to formally model player one'sopportunity to say one word to player two, player one must have a move and player two must have an information state for every word in the dictionary. Cfr Myerson (1994)

[4] We remind that in contribution games contributions are not refunded if the project is not completed. We shall not deal at all with subscription games, in which contributions can instead be refunded.

the efficiency issue. In section two we shall further extend our analysis to a slightly different case in which not only the benefits due to the consumption of the public good but also the contribution amount grows with the number of contributing players. With respect to the first case, we assume that pay-off externality among players can persist even after the threshold for the provision of the public good has been reached. In this last case not only the characterisation of a correlated equilibrium changes (section 3.1) but also our results concerning efficiency are modified (section 3.2). We point out that a Pareto efficient allocation can no longer be achieved with a correlated equilibrium. Constrained Pareto effcient allocations can however be reached when considering free riding as a moral hazard problem and correlated equilibria as incentive efficient mechanisms (Myerson, 1985) to deal with it. Finally we also provide an example about the fact that players could form coalitions on the basis of asymmetric information concerning the outcome of the random device(section 3.3). In this case not only the individual payoff of the better informed players increases but also collective welfare further improves with respect to the symmetric correlated equilibrium that is generated by the incentive efficient mechanism. This last result leads us to derive some conclusions about the economic incentives of those agents who creates private institutions to provide public goods.Some conclusions and suggestions for further research are given in section four.

2 Correlated Equilibria in the Contribution Game

The formal specification of the contribution game follows directly Palfrey and Rosenthal (1984). There are M players i, each with pure strategies denoted s_i of either contributing a fixed amount c ($s_i = 1$) or not contributing at all ($s_i = 0$). The number of contributors that is necessary to produce the public good is denoted by w and the actual number of contributors by m. If $m < w$ the good is not provided. We assume that utility functions are linear in the benefits of the public good and the cost of participation. The rules of the game are the following:

If $m \geq w$ and $s_i = 1$, i receives $1 - c$
If $m \geq w$ and $s_i = 0$, i receives 1
If $m < w$ and $s_i = 1$, i receives $-c$

If $m < w$ and $s_i = 0$, i receives 0

We can start equilibrium analysis with an example in which $M = 2$ and $w = 1$. In this case the strategic form is the following:

$1-c, 1-c$	$1-c, 1$
$1, 1-c$	$0, 0$

The ranking of payoffs is that of the game of Chicken[5]. Actually this game has two Nash equilibria in pure strategies. In each equilibrium one player contributes to the public good, given that the other is not contributing. At equilibrium we thus observe both free riding by one player and the provision of the public good through the contribution of the other one[6]. Besides the two equilibria in pure strategies, another equilibrium in mixed strategies also exists in which each player contributes with probability $1 - c$ and gets an expected pay-off equal to $1 - c$. Given the multiplicity of equilibria (none of which Pareto dominates the other ones) the feasibility of voluntary provision appears here as a coordination problem: which player will contribute to the public good and who will have a free ride instead?

The previous literaure dealing with the private provision of public goods as the game of Chicken has neglected to consider that players have a further opportunity. If they are allowed to communicate, they can correlate their strategies by using common random devices and then play the game accordingly. Concerning our example, let players flip a fair coin and then agree to let player one not contribute if heads occurs and contribute if tails does, while player two contributes if heads occurs and does not contributes if tails occurs. Actually, one can check that no player will deviate from the prescribed strategy, given that its opponent also follows it. The distribution 1/2 (0, 1), 1/2 (1, 0) represents a correlated equilibrium: players are jointly randomizing over their pure strategy Nash Equilibriua. What follows is the conventional representation of the same correlated equilibrium as a probability distribution over the cells of the strategic form:

[5] Previous applications of this kind of game to the theory of public goods include also Taylor and Ward (1982), Lipnowsky and Maital (1982), Bliss and Nalebuff (1984), Cornes and Sandler (1984) and Palfrey and Rosenthal (1988).

[6] This result is a direct consequence of the typical structure of the Chicken game implying that : 1) The contribution of one agent is suffcient to provide the public good 2) The provision of the public good gives to each agent such a benefit to justify his contribution, even when his opponent does not contribute to it. In fact if no provision would occur the agent would suffer a loss that overcomes the benefits of cost avoidance when both players defect

0	1/2
1/2	0

As a result, in equilibrium both players could achieve an expected payoff equal to $1-0.5c$. This correlated equilibrium is an example of a more general result (Aumann, 1974) which states that, by making reference to any random variable that is publicly observable, players can coordinate their moves and attain any pay-off vector in the convex-hull of Nash equilibrium pay-offs (figure 2). What is interesting to notice is that in the last correlated equilibrium, not only is the public good is provided but players can also attain the symmetric and efficient allocation, as figure 1 clearly shows.

Correlated strategies might be either the result of pre-play communication (players design and implement a procedure for obtaining correlated private signals) or the result of observing exogenous random signals (like sunspsots) if players do not meet. One can also take the view that there is a mediator who just sends recommendation of how each player should play, like in mechanism design theory. In this last case, and considering our example, some agents are actually told to contribute to the public good by the mediator while others are not.

We can now characterise a correlated equilibrium in our public good game. Before giving general results let us consider again our contribution game with two players and any probability distribution that is a candidate to be a correlated equilibrium:

p_{11}	p_{12}
p_{21}	p_{22}

Following the original definition of Aumann (1987) and the game being symmetric, if player A is suggested $s_i = 1$ he will not deviate if $(1-c)p_{11} + (1-c)p_{12} \geq (1)p_{11} + (0)p_{12}$. If the same player is suggested $s_i = 0$, he will not deviate if $(1)p_{21} + (0)p_{22} \geq (1-c)p_{21} + (1-c)p_{22}$. Rearranging, we obtain that the following condition should be satisfied in any correlated equilibrium of the contribution game:

$$p_{22}/(p_{21}+p_{22}) \leq c \leq p_{12}/(p_{11}+p_{12}) \tag{1}$$

Considering now player B, if he is suggested $s_i = 1$ he will not deviate if $(1-c)p_{11} + (1-c)p_{21} \geq (1)p_{11} + (0)p_{21}$. If the same player is suggested $s_i = 0$, he will not deviate if $(1)p_{12} + (0)p_{22} \geq (1-c)p_{12} + (1-c)p_{22}$. Rearranging, we obtain that the following condition should also be satisfied in any correlated equilibrium of the contribution game:

$$p_{22}/(p_{12}+p_{22}) \leq c \leq p_{21}/(p_{11}+p_{21}) \tag{2}$$

One can check that for any given c, both (1) and (2) have more chances to be satisfied the lower are p_{22} and p_{11}. If we take the lowest values: $p_{22} = 0$, $p_{11} = 0$, we obtain that voluntary provision of the public good is both feasible and efficient. Actually we have just a convex combination of Nash equilibria. If in addition the random device is also symmetric: $p_{21} = p_{12}$ then (1) and (2) coincide and players can voluntary assure efficient provision by playing correlated equilibria for any value of c no lower than zero and no greater than one. For any other random device the set of admissible contribution amounts shrinks.

Let us consider any random device still implying $p_{21} = p_{12}$and $p_{22} = 0$, but with $p_{11} > 0$. In this case one can show that the upper limit of c decreases with p_{11}. In fact as the probability that both players contribute increases, there will not be any correlated equilibria for contribution games with high contribution amounts (the risk of waste of individual contributions is high and to deviate from the recommendation becomes profitable when each player is asked to contribute since the other one may also have received the same recommendation). On the contrary for any random device implying $p_{21} = p_{12}$and $p_{22} > 0$, but with $p_{11} = 0$, the lower limit of c increases with p_{22}. In this case since players fear that the provision of the public good risks not being feasible, there will not be any correlated equilibrium for contribution games with low contribution amounts(any player when recommended not to contribute may find it more profitable to deviate from this recommendation, since the other player may also have received the same recommendation). Of course the set of admissible contribution amounts shrinks further for any (inefficient) random device implying both $p_{22} > 0$ and $p_{11} > 0$[7].

We can now qualify the results given in the last example in the general game with M players. Let us recall that in this game, if $w > 1$, there are $\binom{M}{w}$pure strategy Nash equilibria, each with w contributors and one equilibrium with no contributors[8]. Palfrey and Rosenthal (op.cit) also characterise

[7]Moreover the set of admissible contribution amounts shrinks also for efficient and asymmetric correlated equilibria that are convex combination of Nash equilibria. Let us take for example $p_{21} = 0.8$, $p_{12} = 0.2$. In this case one can check that the upper limit of c will be equal to the probability of contribution, and then will be different for each player. If we maintain the assumption that c should be equal across players then the upper limit of c will then coincide with the lowest probability (0.2 in our example).

[8]Cfr.Proposition 2 in Palfrey and Rosentahal, op.cit., p. 176

the set of admissible mixed strategy Nash equilibria. Concerning correlated equilibria, we shall limit ourselves to a definition embracing the simmetric case. A definition that would also consider the asymmetric case would be in fact too cumbersome.

Proposition 1 *In the contribution game any symmetric correlated equilibrium is characterised by the following inequalities:*
$\frac{\binom{M-1}{M-w}}{\sum_{k=0}^{M-1}\binom{M-1}{M-(k+1)}} \leq c \leq \frac{\binom{M-1}{M-w}}{\sum_{K=1}^{M}\binom{M-1}{M-k}}$

Pr oof.the proof is a straigthforward application of the definition of correlated equilibrium to the general contribution game. Any correlated equilibrium can be characterised by a set of linear inequalities. In the case of our contribution game and for $\forall i$, let us consider a generic probability distribution and denote with $pr(m = k|s_i)$, $k = 0, 1, ..., M$, the conditional probability that the number of contributors is k, given the strategy of player i. As we consider simmetric correlated equilibria, then $pr(m = k|s_i)$ will be equal for $\forall i$. Then if any player is told $s_i = 1$, he will not deviate if :

$$\begin{aligned}
(1-c)pr(m &= M|s_i = 1) + (1-c)pr(m = M-1 > w|s_i = 1) + \quad (3)\\
+(1-c)pr(m &= M-2 > w|s_i = 1) + ... + (1-c)pr(m = w|s_i = 1) + \\
-cpr(m &= w-1|s_i = 1) - cpr(m = w-2|s_i = 1) - .... + \\
-cpr(m &= 1 < w|s_i = 1) \geq (1)pr(m = M|s_i = 1) + \\
+pr(m &= M-1 > w|s_i = 1) + pr(m = M-2 > w|s_i = 1) + \\
+.... + (0)pr(m &= w|s_i = 1)
\end{aligned}$$

If any player i is told $s_i = 0$, he will not deviate if:

$$\begin{aligned}
pr(m &= M-1 > w|s_i = 0) + pr(m = M-2 > w|s_i = 0) + \quad (4)\\
+...... + pr(m &= w|s_i = 0) + (0)pr(m = w-1|s_i = 0) \\
&\geq (1-c)pr(m = M-1 > w|s_i = 0) + \\
+(1-c)pr(m &= M-2 > w|s_i = 0) + ... + (1-c)pr(m = w|s_i = 0) + \\
+(1-c)pr(m &= w-1|s_i = 0) - cpr(m = w-2|s_i = 0) + \\
-......... - cpr(m &= 1 < w|s_i = 0) - cpr(m = 0|s_i = 0)
\end{aligned}$$

We can rewrite (1) to obtain:

$$(1-c)\sum_{k=w}^{M} pr(m = k \geq w|s_i = 1) - c\sum_{k=1}^{w-1} pr(m = k < w|s_i = 0) \geq \quad (5)$$

$$\sum_{k=w+1}^{M} pr(m = k > w|s_i = 1)$$

and rewriting (2) we obtain:

$$\sum_{k=w}^{M-1} pr(m = k \geq w|s_i = 0) \geq (1-c) \sum_{k=w}^{M-1} pr(m = k \geq w|s_i = 0) +$$
$$-c \sum_{k=0}^{w-1} pr(m = k < w|s_i = 0) \tag{6}$$

Considering together (3) and (4) and rearranging results:

$$\frac{pr\left(m = w-1|s_i = 0\right)}{\sum_{k=0}^{M-1} pr\left(m = k|s_i = 0\right)} \leq c \leq \frac{pr\left(m = w|s_i = 1\right)}{\sum_{k=1}^{M} pr\left(m = k|s_i = 1\right)} \tag{7}$$

Now let $pr\left(m = w-1|s_i = 0\right) = \binom{M-1}{M-w} / \sum_{j=0}^{M-1} \binom{M-1}{j}$ and $\sum_{k=0}^{M-1} pr\left(m = k|s_i = 0\right) = \sum_{k=0}^{M-1} \binom{M-1}{M-(k+1)} / \sum_{j=0}^{M-1} \binom{M-1}{j}$ and $pr\left(m = w|s_i = 1\right) = \binom{M-1}{M-w} / \sum_{j=0}^{M-1} \binom{M-1}{j}$ and $\sum_{k=1}^{M} pr\left(m = k|s_i = 1\right) = \sum_{k=1}^{M} \binom{M-1}{M-k} / \sum_{j=0}^{M-1} \binom{M-1}{j}$ to get:

$$\frac{\binom{M-1}{M-w}}{\sum_{k=0}^{M-1} \binom{M-1}{M-(k+1)}} \leq c \leq \frac{\binom{M-1}{M-w}}{\sum_{K=1}^{M} \binom{M-1}{M-k}} \tag{8}$$

Lemma 2 *In the contribution game efficient and symmetric correlated equilibria exist for* $0 \leq c \leq 1$.

Proof: Just look at (7) and then notice that efficient correlated equilibria are characterised by: $pr\left(m = w-1|s_i = 0\right) = 0$ and $\sum_{k=1}^{M} pr\left(m = k|s_i = 1\right) = pr\left(m = w|s_i = 1\right)$.

Lemma 3 *In the contribution game convex combinations of pure strategy Nash equilibria are efficient correlated equilibria.*

Proof: convex combinations of Nash equilibria in pure strategies are correlated equilibria, such that (3) reduces to $(1-c)pr(m = w|s_i = 1) \geq 0$ and (4) reduces to $pr(m = w-1|s_i = 0) \geq (1-c)pr(m = w|s_i = 0)$, both are verified for $0 \leq c \leq 1$ .In turn these inequalities imply that $\sum_{k=1}^{M} pr\left(m = k|s_i = 1\right) = pr\left(m = w|s_i = 1\right)$ and $pr\left(m = w-1|s_i = 0\right) = 0$

3 A Contribution Game with Externalities over the Threshold

What the contribution game of section two fails to consider is that the benefits deriving from the public good may be increasing with the number of contributors, even beyond the threshold: each added contribution may increase either the quantity or the quality of the public good to be provided. Let us think, for example, of fundraising for medical research or to financing independent radio and newspapers.Funds not only are collected to keep some institution alive but also to promote new research activities (in the medical research case) or expand the scope of independent information provision (in the case of radio and newspapers). One more feature of this kind of fundraising is that people are frequently informed about the total amount of funds that are collected[9]. From this observation we derive the assumption that people may be led to increase the amount of their contribution when observing that a lot of people are contributing[10].Of course the reverse can also be true: people may find free riding more attractive when observing that a lot of funds have already been collected.

3.1 Equilibrium analysis

The rules of the contribution game described in the last section may thus be modified accordingly. Once the threshold of contributors that is necessary to provide the public good is reached we suppose that both the consumption benefits and the contribution amount increase with the number of contributors:

If $m \geq w$ and $s_i = 1$, i receives $m(1-c)$
If $m \geq w$ and $s_i = 0$, i receives m
If $m < w$ and $s_i = 1$, i receives $-c$

[9] For example in Italy fundraising for medical research is provided each year through a campaign called "Telethon". Funds are collected non stop for 24 hours with banks that are kept open for this scope. During the evening and the night a television show not only provides information about the results already achieved by medical resarch but keep people continuosly informed about he amount of money that is beieng collected.

[10] Of course one could question about the rationality of this kind of behavior. Pay-off externality may provide a useful explanation. However rational conformity of mass behaviour may also find an alternative foundation in the theory of informational cascades.Cfr. Bikhchandani, Hirshleifer and Welsh, 1992.

If $m < w$ and $s_i = 0$, i receives 0

Depending on the value of c we can distinguish two cases: a) Joint contribution by all players results in collective welfare being higher with respect to any combination of strategies where the number of contributing players is just w; b) Joint contribution by all players contributing to the public good result in collective welfare being lower with respect to any combination of strategies where the number of contributing players is just w. The cut-off contribution amount $c^* = \frac{M(M-w)}{M^2-w^2}$ discriminates betwen the two cases and is obtained by the following inequality: $M^2(1-c) \geq w^2(1-c) + (M-w)w$. For $c \leq c^*$ one can say that the spillover effects due to joint contribution overcomes the aggregate benefits that non contributing agents derive from free riding. For $c \geq c^*$ exactly the opposite is true

Let us start equilibrium analysis again with the case in which $M = 2$ and $w = 1$.In this case we get the following strategic form game:

$2(1-c), 2(1-c)$	$(1-c), 1$
$1, (1-c)$	$0, 0$

One can check that the equilibria of this game depend on c. For $c \leq 0.5$, to contribute is a dominant strategy for each player . For $0.5 < c \leq 1$, the structure of the game is again that of Chicken. Thus we again have two pure strategy Nash equilibria and a Nash equilibrium in mixed strategies in which each player contributes with probability $p = (1-c)/c$. In this equilibrium the expected pay-off of players is $(1-c)/c$. In order to reach coordination players can play a correlated equilibrium. For example, with a coin flip players obtain an expected payoff equal to $1 - 0.5c$, as they did in the example of the previous section.

Concerning effciency we must compute the cut-off contribution amount to distinguish between the two cases mentioned above. Since $c^* = \frac{2}{3}$, we can further distinguish the sub-case in which $c \leq 0.5$ from the sub-case in which $0.5 < c \leq \frac{2}{3}$. In the first sub-case the game has just an equilibrium in dominant strategies that is Pareto efficient because of joint contribution by all players. If $0.5 < c \leq \frac{2}{3}$,then to contributing is no longer a dominant strategy, even if joint contribution continues to represent the best combination of strategies (the first best) from the collective point of view. In this second sub-case the resulting correlated equilibrium cannot be Pareto-efficient. As figure 2 in fact shows, the symmetric and efficient outcome is outside the convex hull of Nash equibrium pay-offs and thus cannot be obtained as a correlated equilibrium. If $\frac{2}{3} < c < 1$,joint contribution is no longer Pareto-efficient and the efficient outcomes are again on the convex hull of Nash equilibrium pay-offs, as shown in figure 3.

As for the case in which $0.5 < c \leq \frac{2}{3}$, it is interesting to show that players can achieve a pay-off that is outside the convex hull of Nash equilibrium pay-offs, by using a common random device that sends different but still correlated signals to each one of them (Fudenberg and Tirole, 1992). Let us consider a random device that will have three equally likely states: a, b and c and let Ω be the information partition of players. For example let player one be perfectly informed if state a occurs, while he cannot distinguish state b from state c. Its information partition will then be the following:

$\Omega_1 = \{(a)(b,c)\}$

Player two is perfectly informed if the state is c, but cannot distinguish state a from state b. Then his information partition will be the following:

$\Omega_2 = \{(a,b)(c)\}$

Let us suppose that players use this random device to correlate their strategies. It is then possible to show that the following is a Nash equilibrium: player 1 plays $s_i = 0$, when the state is a and plays $s_i = 1$ when the state is b or c; player 2 plays $s_i = 0$ when the state is c, and plays $s_i = 1$, when the state is a or b. One can actually check that no player will deviate from these strategies, provided that $\frac{1}{2} < c \leq \frac{2}{3}$. The following probability distribution represents then a correlated equilibrium of the contribution game:

1/3	1/3
1/3	0

Concerning this correlated equilibrium it is worthwhile showing that not only is the public good provided, but that each player can also average a payoff equal to $\frac{4}{3} - c$ that is outside the convex hull of Nash equilibrium pay-offs, though it is not Pareto efficient: the public good is provided at a suboptimal level.

As to the generalisation of our result, we again characterise symmetric correlated equilibria in the general contribution game and put off the discussion of the efficiency issue to the next section.

Proposition 4 *In the contribution game with spillover effects a symmetric correlated equilibrium is characterised by the following inequalities:*

$$\frac{(w-1)\binom{M-1}{M-w}+\sum_{k=0}^{M-1}\binom{M-1}{M-(k+1)}}{\sum_{k=w-1}^{M-1}k\binom{M-1}{M-(k+1)}+\sum_{k=0}^{M-1}\binom{M-1}{M-(k+1)}} \leq c \leq \frac{w\binom{M-1}{M-w}+\sum_{k=1}^{M}k\binom{M-1}{M-k}}{\sum_{k=1}^{w-1}\binom{M-1}{M-k}+\sum_{k=w}^{M}k\binom{M-1}{M-k}}$$

Proof: In the general contribution game with spillover effects, for $\forall i$, if a player is told $s_i = 1$, he will not deviate if :

$$
\begin{aligned}
M(1-c)pr(m &= M|s_i=1)+(M-1)(1-c) & (9)\\
pr(m &= M-1>w|s_i=1)+(1-c)(M-2)\\
pr(m &= M-2>w|s_i=1)+.......+\\
+(1-c)wpr(m &= w|s_i=1)-cpr(m=w-1<w|s_i=1)-\\
-cpr(m &= w-2<w|s_i=1)-..........-cpr(m=1|s_i=1)\geq\\
&\geq (M-1)pr(m=M|s_i=1)+(M-2)\\
pr(m &= M-1>w|s_i=1)+........+wpr(m=w+1>w|s_i=1)
\end{aligned}
$$

If a player is told $s_i=0$, he will not deviate if :

$$
\begin{aligned}
(M-1)pr(m &= M-1>w|s_i=0)+ & (10)\\
+(M-2)pr(m &= M-2>w|s_i=0)+.........+\\
+wpr(m &= w|s_i=0)+(0)pr(m=w-1|s_i=0)\geq\\
M(1-c)pr(m &= M-1>w|s_i=0)+\\
(M-1)(1-c)pr(m &= M-2>w|s_i=0)+..........+\\
+(w+1)(1-c)pr(m &= w|s_i=0)+w(1-c)pr(m=w|s_i=0)-\\
-cpr(m &= w-2|s_i=0)-.....-cpr(m=1|s_i=0)-\\
-cpr(m &= 0|s_i=0)
\end{aligned}
$$

We can write (6) as:

$$
\begin{aligned}
\sum_{k=w}^{M} k(1-c)pr(m &= k|s_i=1)-c\sum_{k=1}^{w-1} pr(m=k<w|s_i=1)\geq & (11)\\
&\geq \sum_{k=w+1}^{M}(k-1)pr(m=k>w|s_i=1)
\end{aligned}
$$

And write (7) as:

$$
\begin{aligned}
\sum_{k=w}^{M-1} kpr(m &= k>w|s_i=0)\geq(1-c)\sum_{k=w-1}^{M-1}(k+1)pr(m=k>w|s_i=0) & (12)\\
-c\sum_{k=1}^{w-2} pr(m &= k<w|s_i=1)
\end{aligned}
$$

Rearranging (8) and (9) and considering them toghether we get

$$\frac{(\sum_{k=w-1}^{M-1}(k+1)pr(m=k|s_i=0)-\sum_{k=w}^{M-1}kpr(m=k>w|s_i=0)}{(\sum_{k=w-1}^{M-1}(k+1)pr(m=k|s_i=0)+\sum_{k=0}^{w-2}pr(m=k|s_i=0)} \leq c \quad (13)$$

and

$$c \leq \frac{(wpr(m=w|s_i=1)+\sum_{k=w+1}^{M}pr(m=k>w|s_i=1)}{(\sum_{k=1}^{w-1}pr(m=k<w|s_i=1)+\sum_{k=w}^{M}pr(m=k\geq w|s_i=1)}$$

If we substitute for the probability values then (13) becomes:

$$\frac{(w-1)\binom{M-1}{M-w}+\sum_{k=0}^{M-1}\binom{M-1}{M-(k+1)}}{\sum_{k=w-1}^{M-1}k\binom{M-1}{M-(k+1)}+\sum_{k=0}^{M-1}\binom{M-1}{M-(k+1)}} \leq c \qquad (14)$$

$$c \leq \frac{w\binom{M-1}{M-w}+\sum_{k=1}^{M}k\binom{M-1}{M-k}}{\sum_{k=1}^{w-1}\binom{M-1}{M-k}+\sum_{k=w}^{M}k\binom{M-1}{M-k}}$$

3.2 Incentive efficient mechanisms

The example we have presented in last section shows that even if public good provision may not be Pareto efficient, nevertheless, with some correlated equilibrium players can achieve a payoff that is outside the convex hull of Nash equilibrium pay-offs. This result suggests that there may exist correlated equilibria that represent not only a feasible mechanism for the private provision of a public good, but also the optimal one.

Myerson (1985) has shown that the set of inequalities defining a correlated equilibria are equivalent to the incentive compatibility constraints of an optimisation problem whose solution is given by an incentive efficient mechanism that is precisely a correlated equilibrium.

Actually the contribution game we are representing is a game of complete information with respect to the contribution amount and the value assigned by players to the public good (adverse selection issues are excluded). But each player is uncertain about the participation of its opponents. Before playing the game, contribution is a hidden action from the point of view of each single agent and thus free riding can be analysed as a moral hazard problem. The issue is to give players the correct incentives for contribution: a social planner would like them to be insured at the lowest cost against the event that the public good is not provided at the optimal level. Thus every player should be asked contribute in order that the externality is fully internalised. However we cannot give them complete insurance because any player should find it profitable to free ride trusting on the contribution of others. Any correlated equilibrium can be seen as an optimal solution to this trade-off.We are now interested in investigating these optimal solutions: they can be represented by those correlated equilibria that maximise collective welfare, subject to the incentive compatibility constraints.

We then take the view that there is a mediator sending recommendations on how each player should play the contribution game, with the aim of maximising an objective function given by the sum of players' expected utilities and subject to a set of constraints requiring that players should not deviate from the mediator recommendation. Considering our last example with two agents, the problem of finding an incentive efficient mechanism takes the following form:

$$\begin{array}{l} Max_p 4(1-c)p_{11} + (2-c)p_{12} + (2-c)p_{21} \\ s.t. 2(1-c)p_{11} + (1-c)p_{12} \geq p_{11} \\ p_{21} \geq 2(1-c)p_{21} + (1-c)p_{22} \\ 2(1-c)p_{11} + (1-c)p_{21} \geq p_{11} \\ p_{12} \geq 2(1-c)p_{12} + (1-c)p_{22} \\ p_{11} + p_{12} + p_{21} + p_{22} = 1 \\ p_{11} \geq 0, p_{12} \geq 0, p_{21} \geq 0, p_{22} \geq 0 \end{array}$$

The solution to this linear programming problem is a correlated equilibrium that depends on the value of c. For $0 \leq c \leq 0.5$ the incentive efficient mechanism is an equilibrium in dominant strategies with $p_{11} = 1$. For $0.5 < c \leq 0.66$ any incentive efficient mechanism is a symmetric correlated equilibrium[11] with $p_{12} = p_{21}$, $p_{11} > p_{12} > 0$, $p_{22} = 0$. If we take for example $c = 0.655$, the incentive effcient mechanism is given by the following correlated equilibrium distribution

0.3571	0.32124
0.32124	0

[11]Please, notice that even if the solution is simmetric, the incentive constraints are equivalent to the inequalities defining a correlated equilibrium that is not necessariliy simmetric.

Moreover one can check that p_{11}decreases with c and goes to zero for $c = 0.67$.For $0.67 < c \leq 1$, any incentive effcient mechanism can be characterised as a correlated equilibrium with p_{12} or p_{21} equal to one. If we impose symmetry in addition, any incentive effcient mechanism is characterised by $p_{12} = p_{21} = 0.5$. Thus free riding is excluded for low values of the contribution amount but becomes unavoidable for high values. When the contribution amount is too high the incentive for each player to free ride on the other player's contribution is also very high and overcomes the benefits of joint contribution.

The incentive effcient mechanisms of the general contribution game can be characterised as correlated equilibria that are solutions to the following optimisation problem:

$$\begin{aligned}
MaxW &= M^2(1-c)pr(m=M)+ \\
\sum_{k=w}^{M-1}[M-kc]\,pr(m &= k \geq w)- \\
-c\sum_{k=1}^{w-1}kpr(m &= k < w) \\
\text{s.t.}\frac{(w-1)\binom{M-1}{M-w}+\sum_{k=0}^{M-1}\binom{M-1}{M-(k+1)}}{\sum_{k=w-1}^{M-1}k\binom{M-1}{M-(k+1)}+\sum_{k=0}^{M-1}\binom{M-1}{M-(k+1)}} &\leq c \leq \frac{w\binom{M-1}{M-w}+\sum_{k=1}^{M}k\binom{M-1}{M-k}}{\sum_{k=1}^{w-1}\binom{M-1}{M-k}+\sum_{k=w}^{M}k\binom{M-1}{M-k}} \\
\sum_{k=0}^{M}pr(m &= k)=1
\end{aligned}$$

Since $pr(m=M) = 1/\sum_{k=0}^{M}\binom{M}{M-k}, pr(m=k) = \binom{M}{M-k}/\sum_{k=0}^{M}\binom{M}{M-k}$
we can substitute in the expression of collective expected utility to get an optimisation problem, depending on c for any fixed M and w.

3.3 Coalitions

A further question concerning the private provision of public goods relates to the incentive of small groups of people to create private institutions that are devoted to the provision of the public good. Of course there may be social and historical reasons that can explain this decision, but one wonders if there may also be good economic ones. In our framework we suppose that those agents who establish the institution for the provision of the public good will be entitled to decide the random device to be used to coordinate all agents participating in the mechanism, given that other agents agree on that. We shall not provide general results about this issue and limit ourselves to discussing the following example.

Example 5 *Let us consider the same game already presented in this section with* $M = 3$ *and* $w = 2$ *, letting player one choose row, player two choose column and player three choose the matrix (if he contributes to the public good, let him choose the first matrix)*

$3(1-c), 3(1-c), 3(1-c)$	$2(1-c), 2, 2(1-c)$
$2, 2(1-c), 2(1-c)$	$0, 0, -c$

$2(1-c), 2(1-c), 2$	$-c, 0, 0$
$0, -c, 0$	$0, 0, 0$

One can check that if $c \leq \frac{1}{3}$ *there is an efficient equilibrium in dominant strategies where all players contribute. Let us exclude this case and suppose that* $\frac{1}{3} < c \leq \frac{3}{5}$ *to get a Chicken game with three Nash Equilibria in pure strategies where* $w = 2$ *and one equilibrium where no agents contribute. The efficient combination of strategies cannot be an equilibrium in this case*[12] *Let us suppose that players arrange to build a correlating device with three equally likely states and all agree that the true state will be revealed only to player one and two, while player three remains completely uninformed about it. We are making reference again to the common random device already presented in section 3.1, which sends different but still correlated signals to player one and two and generates the following correlated equilibrium distribution* $\frac{1}{3}\ (1,1), \frac{1}{3}\ (1,0), \frac{1}{3}(0,1)$*. In this case the strategy of contributing is always a best response for player three (as the three state are always equally likely for him) provided that* $c \leq \frac{5}{7}$*. This asymmetric coordination device can be helpful to achieve the provision of the public good as player three always contributes to it, given his uncertainty about the behaviour of his opponents which makes him fear that one of the first two players may not contribute.One can also say that it is the commitment of the first two players to contribute only with a certain (optimal) frequency that forces player three to always contribute*[13]*.It is interesting to make some comments about individual and collective welfare, considering this last result. It*

[12] If $\frac{3}{5} < c < 1$ the game has again three Nash Equilibria in pure strategies with $w = 2$ but they are also Pareto efficient and the combination of strategies with each player contributing is inefficient.

[13] This can also be seen as one of the cases where one player commits himself to non contribution in order to force contribution by the other ones as suggested by Taylor and Ward (op.cit.) in the first analysys of the public good issue as the game of Chicken:"An individual can commit himself by manipulating lack of information and uncertainty about

is easy to show that the asymmetric information between the coalition formed by the first two player and player three creates additional advantages for the players that are more informed. In fact their utility is higher with respect to the utility of player three, for any value of c.But it is above all interesting to point out that in this case collective welfare (given by $7-\frac{17}{3}c$*) not only increases, but is also greater with respect to the collective welfare the players could achieve with a symmetric incentive efficient mechanism. For example if* $c=\frac{1}{2}$ *, the incentive efficient mechanism is represented by the following correlated equilibrium*

0.57143	0.14286
0.14286	0
0.14286	0
0	0

and thus expected collective welfare will be equal to 1.428585,*while with the coalition between the first two players it will amount to* 4.16.

This example could then provide a rationale for the behavior of agents who further private institutions devoted to the provision of public goods. Indeed agents who are first comers will be able to choose the appropriate mechanism to implement the provision of the public good and create an asymmetry that could be exploited from the economic point of view. Then one can imagine that it is this opportunity that creates the appropriate incentives for some agents to create the collective institution. Moreover the example suggests that this behaviour is also worthwhile from the social point of view.

4 Conclusions

In this paper we have found some theoretical foundations for the experimental observations concerning the impact of communication on the degree of cooperation in public good games. This impact has been captured through the notion of correlated equilibrium. The adoption of this solution concept is also consistent with the fact that players involved in experiments often use lotteries as coordination devices. Considering correlated equilibria in the

the incentives facing him in such a way that it appears highly likely that he will find cooperation relatively desirable".Cfr. Taylor and Ward (op.cit), p.361.

original public good game of Palfrey and Rosenthal we can explain how people can attain one of the multiple pure-strategy Nash equilibria and prove that in equilibrium the private provision of the public good is both feasible and efficient.

Further, we introduce a modification of the original game to account for payoff externalities persisting even after the threshold has been reached and get a second type of public good game characterised by spillover effects. For low values of the contribution amount, agents always find it optimal to contribute to the public good and completly internalise the externality. For higher values of the contribution amount this new public good game is a game of Chicken as well, but we obtain different results concerning both the characterisation of equilibria and efficiency. Particularly, in a given range of the contribution values, agents must trade off the net benefit of contribution with the net benefits of free riding. Actually in this framework we find that free riding is equivalent to a moral hazard problem: agents cannot completely assure each other against the event that the public good is provided at a sub-optimal amount in order to maintain the individual incentive to contribute. The solution to this problem is an incentive efficient mechanism in the sense of Myerson (1985) and is equivalent to a correlated equilibrium. Playing such an equilibrium, agents only partially internalise the externality. Moreover for very high values of the contribution amount the collective benefits of free riding for players that do not contribute is so high that joint contribution by all players is no longer efficient. In this case the incentive effcient mechanism is such that the public good is produced at the threshold level.

One further question that is connected to our results is why should agents choose one random device rather than another as a coordination mechanism. In this paper we do not provide a general answer to this question. Rather, with the help of one example we point out that players could form coalitions that are helpful in generating asymmetric information among players about the outcome of the random device chosen for coordination. The highest payoff obtained by players who belong to the coalition can be seen as an economic incentive for these players to further private institutions that are devoted to public purposes in order to fix the rules to their own advantage. Nevertheless, with a coalition collective expected utility is also higher with respect to the case in which agents play the symmetric correlated equilibrium that represents the incentive efficient mechanism.

Of course our analysis is limited by the fact that we consider only the participation issue and do not discuss the preference revelation problem. In economic reality adverse selection issues add to moral hazard problems in the framework of the private provision of public goods. One should then consider a public good game with incomplete information in order to account for the fact that the willingness to pay for a public good is not only heterogeneous but also private information. Actually most part of the recent literaure devoted to the private provision of public goods[14] deals with adverse selection issues in Bayesian games and does not consider moral hazard problems connected to participation. We think that future research should integrate both dimensions in the same model. Developments in the theory of Bayesian games with communication and the generalisation of the concept of correlated equilibrium to these same games (Forges,1986 and Myerson,1994) might add further insights into the feasibility and efficiency of the private provision of public goods.

References

[1] Aumann, R.J.(1974), "Subjectivity and Correlation in Randomized Strategies", *Journal of Mathematical Economics*, 1, pp.67-96.

[2] Aumann, R.J.(1987),"Correlated Equilibria as an Expression of Bayesian Rationality", *Econometrica*, 55, pp.1-18

[3] Bikhchandani, S., D.Hirshleifer and I.Welsh (1992)"A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades", *Journal of Political Economy*, 100, pp. 992-1026.

[4] Bliss, C. and B. Nalebuff (1984)"Dragon Slaying and Ballroom Dancing: the Private Supply of a Public Good" *Journal of Public Economics*, 25, pp. 1-12

[5] Cornes, R. and T. Sandler (1986) *The Theory of Externalities, Public Goods and Club Goods*, Cambridge, Cambridge University Press.

[6] Dawes, R.M. and R.H.Thaler (1988), "Cooperation", *Journal of Economic Perspectives*, 3, pp.187-197

[14] Concerning discrete public goods see for example the seminal paper of Palfrey and Rosenthal (1988); Gradstein (1994) and Menezes, Monteiro and Temini (1998).

[7] Forges, F.(1986) "An Approach to Communication Equilibria"*Econometrica* 54, pp.1375-1386

[8] Fudemberg, D. and J.Tirole (1992), *Game Theory* , Boston, The MIT press

[9] Gradstein, M.(1994), "Efficient Provision of a Discrete Public Good" *International Economic Review*, 35, pp. 877-897.

[10] Ledyard, J. (1992), "Public Goods: a Survey of Experimental Research" in Handbook of Experimental Economics

[11] Lipnowsky, I. and S.Maital (1983)"Voluntary Provision of a Pure Public Good as the Game of Chicken", *Journal of Public Economics* 20, pp.381-386

[12] Menezes, F.M, P.K.Monteiro, A.Temini (1998)"Discrete Public Goods with Incomplete Information"mimeograph

[13] Myerson, R.B.(1985) "Bayesian Equilibrium and Incentive Compatibility" in L.Hurwictz, D. Schmeidler and H. Sonnenschein, eds.*Social Goals and Social Organisation*. Cambridge, Cambridge University Press, pp.229-259.

[14] Myerson, R.B.(1994) "Communication, Correlated Equilibrium and Incentive Compatibility" in R.J.Aumann and S.Hart, eds. *Handbook of Game Theory*, vol.II, Amsterdam, Elsevier, pp. 827-847.

[15] Palfrey, T.R. and H.Rosenthal (1984) "Participation and the Provision of Discrete Public Goods: a Strategic Analysis", *Journal of Public Economics* 24,pp.171-193

[16] Palfrey, T.R. and H.Rosenthal (1988) "Private Incentives in Social Dilemmas", *Journal of Public Economics* 24,pp.171-193.

[17] Palfrey, T.R. and H.Rosenthal (1991)"Testing for Effects of Cheap Talk in a Public Goods Game with Private Information", *Games and Economic Behavior* 3, pp.183-220.

[18] Taylor, M. and H.Ward (1982) "Chickens, Whales and Lumpy Goods: Alternative Models of Public Goods Provision" *Political Studies*, 30, pp. 350-370.

[19] Van de Kragt A., J.M.Orbell, R.M. Dawes (1983)"The Minimal Contributing Set as a Solution to Public Goods Problems" *The American Political Science Review*, 77, pp. 112-121.

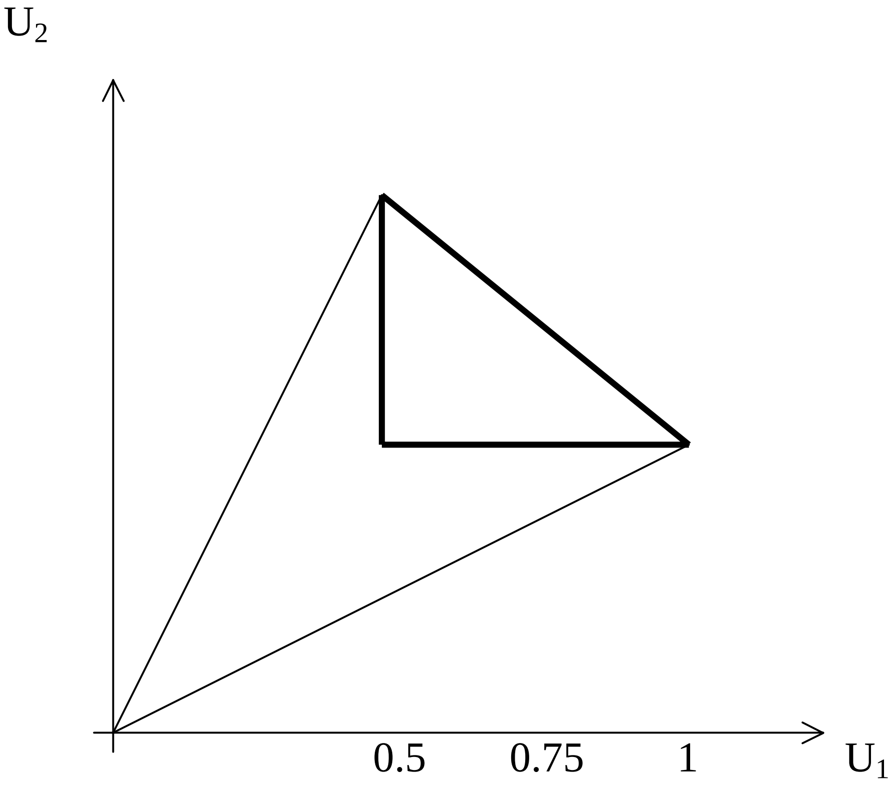

Figure 1: The convex hull of Nash Equilibrium Pay-offs in bold face for the contribution game (assuming M=2, w=1 and c=1/2)

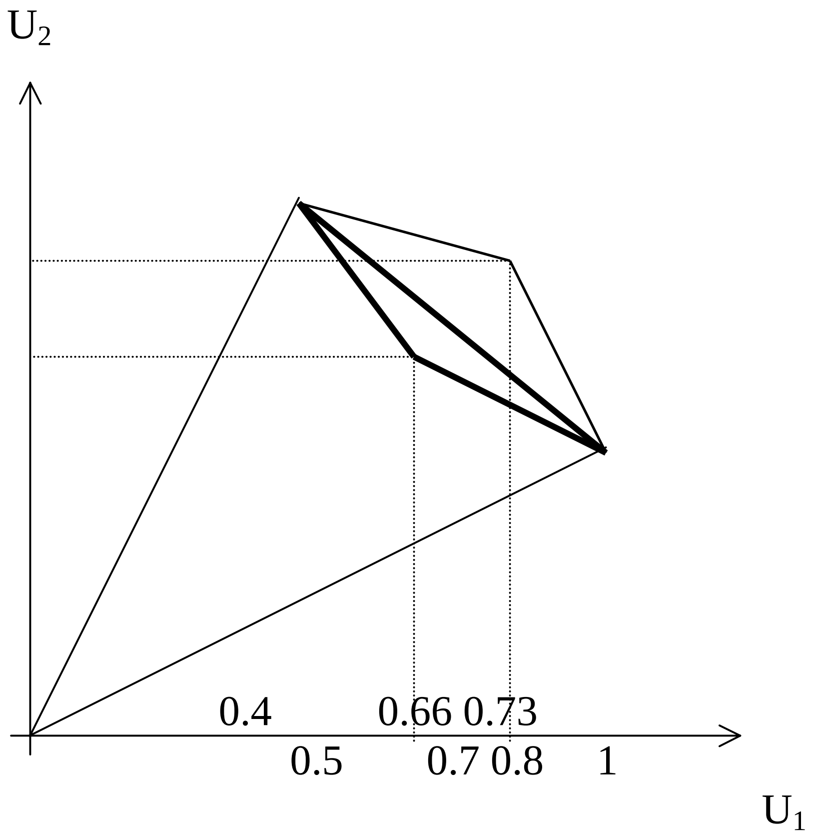

Figure 2: The convex hull of Nash Equilibrium Pay-offs in bold face for the contribution game with externalities (assuming M=2, w=1 and c=0.6)

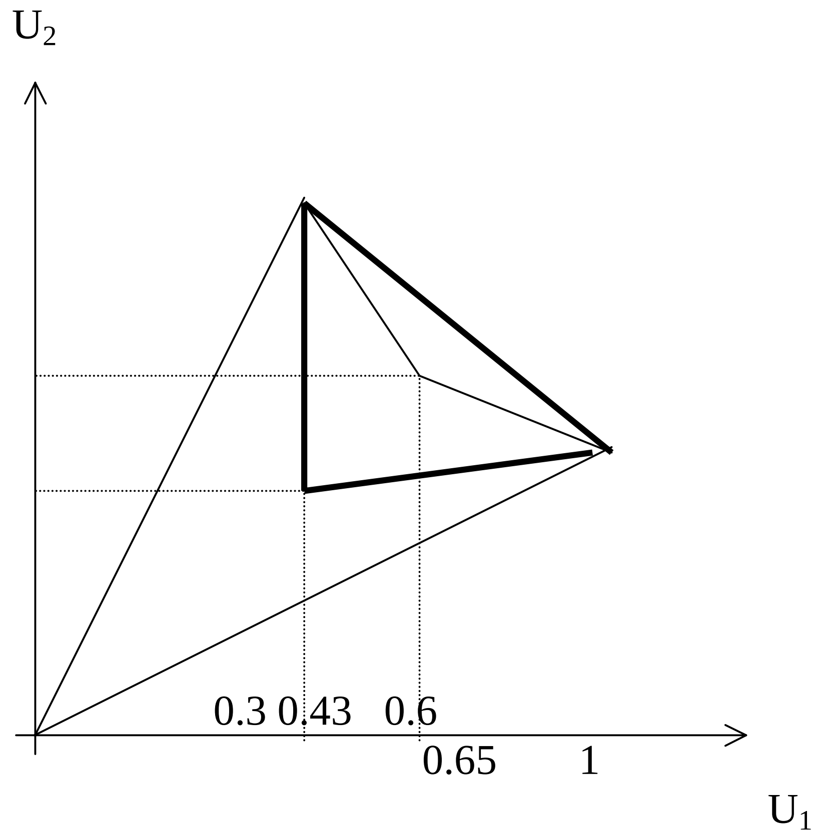

Figure 3: The convex hull of Nash Equilibrium Pay-offs in bold face for the contribution game with externalities (assuming M=2, w=1 and c=0.7)