

MIXED DUOPOLY, PRIVATIZATION AND THE SHADOW COST OF PUBLIC  
FUNDS: EXOGENOUS AND ENDOGENOUS TIMING

CARLO CAPUANO, GIUSEPPE DE FEO

pubblicazione internet realizzata con contributo della



# Mixed duopoly, privatization and the shadow cost of public funds: exogenous and endogenous timing

Carlo Capuano\*

Department of Economics, University of Naples Federico II

Giuseppe De Feo†

CORE, Université Catholique de Louvain and

Department of Economics, University of Naples Federico II

August 31, 2006

## Abstract

The purpose of this article is to investigate how the introduction of the shadow cost of public funds in the utilitarian measure of the economy-wide welfare affects the behavior of a welfare maximizer public firm in a mixed duopoly. We prove that when firms play simultaneously, the mixed-Nash equilibrium can dominate any Cournot equilibria implemented after a privatization, with or without efficiency gains. This can be true both in terms of welfare and of public firm's profit.

When we consider endogenous timing, we show that either mixed-Nash, private leadership or both Stackelberg equilibria can result as subgame-perfect Nash equilibria (SPNE). As a consequence, the sustainability of sequential equilibria enlarges the subspace of parameters such that the market performance with an inefficient public firm is better than the one implemented after a full-efficient privatization. Absent efficiency gains, privatization always lowers welfare.

*JEL codes:* H2, H42, L13, L32, L33

*Keywords:* mixed oligopoly, privatization, endogenous timing, distortionary taxes.

---

\*Dipartimento di Economia, Complesso Universitario di Monte S. Angelo, Via Cintia, Napoli, 80126; E-mail: carlo.capuano@unina.it; www.carlocapuano.it

†CORE – Center for Operations Research and Econometrics, Voie du Roman Pays 34, B-1348 Louvain-la-Neuve, Belgium. E-mail: defeo@core.ucl.ac.be

# 1 Introduction.

Starting from the second half of the last century we measure an increasing interest by economic theory for the role played by a non-monopolistic public firm that competes with private-owned ones. The presence of public firms in oligopolist market characterized several industries in continental Europe, UK, North America as well as developing countries all over the world. In 1984 state-owned enterprises accounted for 8.5% of the GDP in industrialized countries (*Sheshinski and López-Calva, 1999*).

Despite the structure of any industry, in general the presence of a public firm can be used as a policy instrument to improve resource allocation in any imperfectly competitive market. Moreover, merit goods or products and services which confer significant positive externalities, such as housing, education, health, pensions, are often provided by the public together with the private sector. Sometimes the strategic importance of some activities, such as banking, transportation, military production has justified nationalization of some, if not all, firms in the industry. In some cases the government has stepped in to rescue a failing firm to maintain employment or to create "national champions" in strategic, especially technologically advanced, sectors (*Anderson et al., 1997*).

Conversely, starting from the eighties a vast process of privatization and liberalization has characterized the industrialized economies and soon was extended to developing and former socialist countries. The size of state sector shrank rapidly and reached the 5% of GDP in 2000 (*Meggison and Netter, 2001*). The motivations for this program were essentially linked to the general perception of poor performance of public firms and to the idea that private discipline and profit motivation could enhance efficiency.<sup>1</sup>

However, empirical findings are more ambiguous in this respect.<sup>2</sup> Most of the works report an increase in profitability after the privatization, but the evidence of productive efficiency improvements is less clear and the variance of the results is substantial (*Cuervo and Villalonga, 2000*). Besides, the allocative efficiency effect is even more puzzling since there is not a general evidence of positive welfare effects of privatization.<sup>3</sup>

---

<sup>1</sup>Theoretical support was provided by the theory of incentives that demonstrated how agency problems in state-owned enterprises can cause larger inefficiencies than in private-owned firms. In fact, profit maximizing owners subject to threats of bankruptcy and takeover have stronger incentives to reduce costs than politicians or bureaucrats. They would motivate and monitor appointed managers more efficiently than when there are wider and sometimes distorted objectives, soft budget constraints and complicated chains of command (*Gadal and Shirley, 1995*). In addition, *Sappington and Stiglitz (1987)* presents the "fundamental privatization theorem" (analogously to the fundamental theorem of welfare economics) providing conditions under which government production cannot improve upon private production.

<sup>2</sup>See for example the metareviews of *Villalonga, 2000; Meggison and Netter, 2001; Willner, 2001*; that report the results of hundreds of empirical papers on privatization.

<sup>3</sup>Also modern theory is less dogmatic on ownership (see *Estrin and Pérotin, 1991*). For example, an owner-manager would indeed have strong incentives to cut costs. But to privatize a managerial firm may increase costs, because the profit motive reduces the incentive to pay for lower managerial slack (*de Fraja, 1993; Willner 2003*). Public ownership may in some models mean excessive labour intensity and private ownership the opposite, with ambiguous consequences for overall productivity (see *Pint, 1991*). The ranking of ownership is sensitive

Actually, privatization processes have reduced the public sector in general, and the presence of public firm in oligopolistic markets in particular. Nevertheless, liberalization processes have opened previous state controlled monopolies to competition of the private sector without full privatization; evidences occur in water, transportation, telecommunications and energy industries. For example, in spite of regulating a privatized monopoly, governments could enforced a facility-based competitions in order to achieve a so-called dynamic efficiency<sup>4</sup>. As a consequence, we face many *mixed oligopoly* where public firms (often the former monopolists) compete with private ones.

The aim of early theoretical works on mixed oligopoly has been to analyze whether it is possible to reduce the allocative inefficiency raised by imperfect competition, through the use of a public firm as an internal regulation mechanism. One of the first articles is *Merrill and Schneider (1966)*, where the mixed oligopoly setting is defined as the "*fourth way*" of industry control and property assessments<sup>5</sup>. Under symmetry constant return to scale and industry capacity constraints, it proves that the presence of a public firm, facing a joint (collusive) oligopoly, can improve welfare guaranteeing a lower equilibrium price, an higher total output, positive profits for any firms, and that the demand does not exceed the industry total capacity. Similar results are obtained in *Harris and Wiens (1980)*, where it is examined how a government enterprise could be used to promote static economic efficiency within a imperfectly competitive market structure. This is true when policy instruments are limited to the set of variables under the control of the government-owned firm. It considers a quantity competition among firms with increasing marginal costs. The setting is characterized by a strange "*dominant*" public firm that announces its output strategy, i.e. to produce the difference between the socially optimal output and the private one.<sup>6</sup>

*Hagen (1979)*, *Rees (1984)* and *Bös (1986)* extended the result of the second best literature to the mixed oligopoly framework and prove that, in an imperfectly competitive market, a public firm should depart from the marginal cost pricing rules in order to maximize social welfare. The optimal departure implies that the public firm plays as Stackelberg leader. In *Beato and Mas-Colell (1984)*, the analysis involves a duopoly quantity competition in an industry characterized by increasing marginal costs. It gives new strength to the marginal cost pricing rule by reversing the role of the firms with respect to what is

---

to details in the objective function and reward schedule if low performance means that the manager is fired (*Willner and Parker, 2003*). Also, privatization often requires regulation, and hence an additional agency problem that may sometimes cost more than public ownership (*Shapiro and Willing 1990; Laffont and Tirole, 1991*).

<sup>4</sup>For deeper viewpoints on the role played by facility-based competition in EU and US Telecommunications liberalization and regulation processes see *Taschdjian (1997)* or *Stehmann and Borthwick (1994)*.

<sup>5</sup>They consider that an industry structure can be characterized by (i) public properties and controls; (ii) private properties and controls; (iii) private properties but constrained controls by regulation and antitrust, (IV) public and private firms that compete, i.e. the mixed oligopoly.

<sup>6</sup>Actually, the public leader plays as a Stackelberg follower that maximizes industry-wide welfare.

assumed in the second best literature. In fact, the authors provide conditions so that welfare is higher when public firm plays as follower than when it plays as leader.

In *Cremer et al. (1989)* the public firm is explicitly viewed as an instrument for regulating an oligopolistic market: they analyze the case of an  $n$ -firm quantity competition in a simultaneous setting. They find conditions under which it is social improving to nationalize one or more private firms, assuming that public firms will act as consumer surplus maximizer and its budget constraint is always binding.<sup>7</sup> They conclude that the nationalization of at least one private firm is always welfare improving. *Garvie and Ware (1996)* obtain the same policy implications in the case of cost uncertainty, i.e. private but correlated information about industry costs.<sup>8</sup>

All these works consider that the public firm does not play as a profit maximizer but its objective function is represented either by the consumer surplus, or by the industry total output, or by the utilitarian measure of the industry-wide welfare. However, given the strategical interaction, the best way to maximize welfare in oligopoly may be that public firm maximizes something different. In *de Fraja and Delbono (1989)* is shown that at Nash equilibria social welfare may be higher when the public firm is instructed to maximize profits rather than welfare. In particular they study the case of an  $n+1$  oligopoly composed by a public firm and  $n$  profit maximizer private competitors<sup>9</sup>. They conclude that nationalization is always socially better than public leadership, which is in turn socially better than Cournot-Nash. But, when public leadership is not implementable, if  $n$  is high enough, it would be better that public firm maximize profits rather than welfare. The powerful extension of these results is that without any efficiency gain privatizing the public firm can be welfare improving. The same result can be found in *Haskel and Sanchis (1995)* where in presence of X-inefficiency and a process of workers' effort bargaining between union and firms, privatization could achieve efficient wage levels. More generally, in literature privatization is considered as a solution of the so-called *no-market failures*.

More recent works focus on manipulation of the public firm's objective function. For example, dealing with partial privatization, in *Fershman (1990)* the reaction function of the public firm is "*a compromise*", i.e. a linear combination, between the reaction function of a profit maximizer with the one of a welfare

---

<sup>7</sup>In particular, they consider that firms produce a commodity and all of them are characterized by constant marginal costs and fixed costs. Even though the public firm is characterized by an higher marginal cost, this is not a case of X-inefficiency since the differential is treated as a neutral transfer from government to workers.

<sup>8</sup>However, when cost differential between public and private firms is too large or demand for a product is too weak, the mixed oligopoly setting could be welfare inferior to a private oligopoly.

<sup>9</sup>They assume a quantity competition for a commodity, in a complete information context. All firms are symmetric, characterized by an increasing marginal cost, no fixed cost and no capacity constraints are considered. These assumptions allow the presence of a finite number of small competitors. They exploit only private symmetric equilibria comparing the firm and industry performance in the case of (i) public leadership (ii) public nationalization of the industry (iii) mixed Nash competition, i.e. public firm maximize industry-wide welfare; (iv) privatization, i.e. Cournot Nash competition.

maximizer. In *Matsumura (1998)* the public firm is assumed to maximize the weighted average of payoff of the government, i.e. a linear combination between welfare and consumer surplus, and its own profit. The weights are strictly correlated with the share-holding owned by the government. In a easier way, in *Claude and Hindriks (2005)*, the objective function is directly a convex combination of welfare and profit, where the endogenous weights derive from the degree of privatization. Moreover, in *White (2002)* it is demonstrated how the government can maximize its true objective by assigning the public firm a different objective function.<sup>10</sup>

Our paper deals with a mixed duopoly where a private and a public firm are involved in a homogeneous good quantity competition. Since we refer to former public monopoly opened to competition, we consider firms characterized by increasing returns to scale (with fixed and constant marginal costs). In particular, the public firm is assumed to be less inefficient than its private competitor, but, differently from *Cremer et al. (1989)*, the public firm's higher cost is not a neutral transfer from firm to workers belonging to the same economy but, as an  $X$ -inefficiency, it reduces any utilitarian measure of welfare. We derive the mixed oligopoly equilibria and define conditions under which a privatization, with or without efficiency gains, leads to a reduction of allocative efficiency and profitability.

The novel contribution of this paper is twofold.

First, we characterize the objective of the public firm taking into account the shadow cost of public funds as initially analyzed in *Meade (1944)* and exploited in *Laffont and Tirole (1986,1993)*. Usually this analysis has been used to characterize public monopolies running a deficit.

[M]any public enterprises are natural monopolies, i.e. firms that exhibit increasing returns to scale. Once it has been proved desirable to run such an enterprise at all, its product should be priced at marginal cost provided the resulting deficit can be financed through lump-sum taxes. If there are not lump-sum, discrepancies between consumer and producer taxes will result in inefficiencies in the rest of the economy. (...) This has been taken as an argument for requiring the public enterprise to cover, by its own means, at least part of its deficit. (*Marchand et al., 1984*)

Here we apply the same reasoning to a public firm competing in an oligopoly. In our framework, with fixed cost and constant marginal cost, the idea is that public firm's profit – whether positive or negative – have to be considered as a (positive or negative) transfer from the government to the public firm. In fact, when public firm deficit occurs, government must resort to distortionary taxes on income, capital or consumption. If government rises 1 Euro, society pays

---

<sup>10</sup>All the previous articles consider markets for commodities. Mixed oligopolies with differentiated products are analyzed in *Cremer et al. (1991)*, *Grilo (1994)* and *Anderson et al. (1997)*.

$(1+\lambda)$  Euro. Consequently, public profits, when positive, avoid an equivalent public transfer, reducing distortionary taxes. Public firm's profit and deficit are not a neutral transfer among agents of the same economy. They ought not to be weighted as private firm's profits or consumer net surplus in the utilitarian measure of welfare, but they should be weighted  $1+\lambda$ . Even though there is no general agreement on the objective of the public firm (*Vickers and Yarrow, 1991*), we believe that our proposal, in a partial equilibrium setting meets the intuition of *Marchand et al. (1984)*. Moreover, there is large empirical evidence of prices above marginal cost with soft budget constraint and also positive profits by public firm, where the latter occurs more frequently in the mixed oligopoly context than in natural public monopolies. Thus, we do not require the public firm to act under *hard* budget constraint and the profit will be positive or negative depending on the importance of the distortion due to taxation and the amount of fixed costs.

Second, after having analyzed the simultaneous moving game, we consider the issue of endogenizing the timing of competition; that is, the order of play in a given two player game. It is worth noting that in *Harris and Wiens (1980)*, *Beato and Mas-Colell (1984)*, *Cremer et al. (1989)* and *de Fraja and Delbono (1989)*, the choice of the timing of competition (either Nash-Cournot or Stackelberg) is exogenously given. On the contrary, the determination of simultaneous (Nash-Cournot) versus sequential (Stackelberg) games and the assignment of leader and follower roles should be the result of preplay independent and simultaneous decisions by the players.

To endogenize the timing of the game, we apply the *Hamilton and Slutsky (1990)* model to our framework. In their insightful paper, the authors construct an extended game adding to the basic game a preplay stage at which players simultaneously and independently decide whether to move early or late in the basic game. Then, the basic game is played according to these timing decisions: if both players decide to move at the same time, simultaneous play endogenously arises; while sequential play – with the order of moves as decided by players – otherwise. As a consequence, the subgame-perfect Nash equilibria of the extended game induce an endogenous sequencing of moves in the basic game. Hamilton and Slutsky refer to it as a *game with observable delay* since both players choose the action in the basic game after having observed the time decision of the other. *Amir and Grilo (1999)* apply this model to a private duopoly showing that, in a quantity setting with strategic substitutability, the simultaneous game emerges as the endogenous timing and the Cournot equilibrium as the unique subgame-perfect Nash equilibrium of the extended game. *Pal (1998)* addresses the issue of endogenous order of moves in a mixed oligopoly by adopting the same game structure. It shows that sequential playing emerges as the endogenous timing and both Stackelberg solutions as the subgame-perfect Nash equilibria of the extended game.

Applying the Hamilton and Slutsky's model to our setting allows us to characterize the different equilibria emerging in a mixed market, to analyze whether the privatization leads to a change of the endogenous timing, and to define the

overall welfare effect of privatization.

The main results of our analysis can be summed up as follows. In the simultaneous setting:

1. There exists a parameters space such that welfare is larger when the (inefficient) public firm is a welfare maximizer than after privatization. Moreover, when the shadow cost of public funds is not negligible, public firm's profit are larger, too. That is, there is no trade-off between welfare and public firm's profit.
2. The market performance with an inefficient public firm may be better than the one implemented after a full-efficient privatization.

Extending the analysis to the endogenous timing:

3. Differently from *Pal (1998)*, in our model setting either mixed-Nash, private leadership or both Stackelberg outcomes can result as subgame-perfect Nash equilibria (SPNE) of the endogenous timing game.
4. Absent efficiency gains, privatization always lowers welfare.
5. The endogenous timing setting enlarges the parameters space such that the market performance with an inefficient public firm is better than the one implemented after a full-efficient privatization.

Last results rely on the fact the presence of a public firm let sequential equilibria be part of a SPNE of the game. Conversely, with private-owned firms, only simultaneous equilibria can be implemented.

It is worth noting that our results are obtained under the assumption that government has the bargaining power to extract all the rent of the privatized firm; that is, the price paid for the former public firm is equal to the profit gained in the new (Cournot) equilibrium. This assumption drives the results in favor of privatization, since it overweights the revenue from privatization by  $\lambda$  in any welfare comparisons.

In what follows, the next Section sets up the model. Section 3 presents the mixed-Nash equilibrium of the game and exploits the presence or the lack of advantages of privatization, even though full efficient. Section 4 is focused on the issue of endogenous timing in mixed oligopoly, while Section 5 is devoted to analysis of privatization in the same framework. Our conclusions are delegated to Section 6.

## 2 The theoretical model.

We consider the simplest setting of a private and a public firm, respectively labelled with  $i = 1, 2$ , producing a homogenous commodity. Demand preferences

are described by a linear function where intercept and slope are normalized to one.

$$P(q_1, q_2) = 1 - q_1 - q_2 \quad (1)$$

Both firms are characterized by constant marginal costs<sup>11</sup>,  $c_i \geq 0$ , and fixed costs,  $K_i \geq 0$ , sustained only in case of producing. But, while the private firm's marginal cost is normalized to zero, the public firm's one is positive, equal to  $c \in (0, \frac{1}{2})$ . This assumption avoids the cases in which the duopoly degenerates in a monopoly<sup>12</sup>

We adopt a static, partial equilibrium analysis and we assume complete information for all the agents. Both firms play a quantity-competition game. We assume that public funds have a shadow cost equal to  $\lambda > 0$  and that public profits, when positive, avoid an equivalent public transfer, reducing distortionary taxes. Then, we assume that the public firm does not maximize the industry-wide welfare but, taking into account the distortionary effect of the avoided taxes, it maximizes an utilitarian measure of the economy-wide welfare.

In particular, let  $S(Q)$  denote the consumer gross surplus, where  $Q = \sum_{i=1}^2 q_i$  is the industry total output.  $P(Q)$  denotes the inverse demand function. In the presence of a shadow cost of public funds the utilitarian measure of welfare is

$$\begin{aligned} W &= S(Q) - P(Q)Q - (c_1 q_1 + K_1 - P(Q)q_1) - (1 + \lambda)(c_2 q_2 + K_2 - P(Q)q_2) \\ &= CS(Q) - (-\Pi_1) - (1 + \lambda)(-\Pi_2) \\ &= CS(Q) + \Pi_1 + (1 + \lambda)\Pi_2 \end{aligned} \quad (2)$$

where  $CS(Q)$  denotes the consumer net surplus.

The industry-wide welfare is

$$V = \Pi_1 + \Pi_2 + \frac{(q_1 + q_2)^2}{2} \quad (3)$$

Then, the public firm's objective function can be rewritten as follows.

$$W = V + \lambda\Pi_2 \quad (4)$$

where the profit levels are

$$\Pi_1 = (1 - q_1 - q_2)q_1 \quad (5)$$

$$\Pi_2 = (1 - c - q_1 - q_2)q_2 \quad (6)$$

---

<sup>11</sup>Although the assumption of increasing marginal cost is popular in the literature, many mixed markets have occurred in industrial production where it would be more realistic to assume constant marginal costs. For the papers adopting increasing marginal costs, see Beato and Mas-Colell (1984), de Fraja and Delbono (1989), Fjell and Pal (1996), Pal and White (1998); while for the papers adopting constant marginal costs, see Cremer et al. (1989) and Martin (2004).

<sup>12</sup>In fact, absent fixed costs, when the public firm is not less efficient than the private one, the unique equilibrium in simultaneous and sequential timing is a public monopoly. Analogously, when the public firm is too inefficient ( $c \geq \frac{1}{2}$ ), a private monopoly always arises. When we introduce asymmetric fixed costs, a monopoly result holds again. But, its public or private nature depends on the levels of the fixed costs.

Function 2, or equivalently 4, is derived considering that a public transfer occurs in order to guarantee the public firm's budget balance. This transfer is positive (negative) when public firm's profits are negative (positive). Notice that without fixed costs, our model setting guarantees always non-negative profits for both firms. That means, the public transfer is always non-positive.

Otherwise, when there exist fixed costs, the private firm's reaction function ought to be truncated in the point it crosses the zero-isoprofit curve and on-the-boundary solutions can occur in equilibrium. The public firm's best reply, even though public transfers guarantee the sustainability of negative profits, is truncated. This depends on the duplication of fixed costs that occurs when public firm produces. In fact, when its production is too small, the fixed cost outweighs the allocative benefits.

Therefore, in what follows we provide the assumptions on the admissible sets in the parameters space.

**Assumption 1.** *The parameters  $c$  and  $\lambda$  belong to the subspace  $A \subset \mathbb{R} \times \mathbb{R} = \{(c, \lambda) | c \in (0, \frac{1}{2}) \vee \lambda \in [0, \bar{\lambda}]\}$ , where  $\bar{\lambda}$  is a finite, reasonable value of the shadow cost of public funds.*

**Assumption 2.** *The private firm's fixed cost  $K_1$  belongs to the subspace  $B \subset \mathbb{R} = [0, \bar{K}[$ , where  $\bar{K}$  is the minimum among the private firm surplus derived in any one-shot equilibrium of the model (simultaneous or sequential).*

**Assumption 3.** *The public firm's fixed cost  $K_2$  belongs to the subspace  $C \subset \mathbb{R} = [0, \bar{K}[$ , where  $\bar{K}$  is lower than the allocative improvement due to the presence of the public firm in any one-shot equilibrium of the model (simultaneous or sequential).*

Given our model setting, the firms' reaction functions are:

$$r_1(q_2) = \begin{cases} \frac{1}{2}(1 - q_2) & \text{if } q_2 < \bar{q}_2 \\ 0 & \text{if } q_2 \geq \bar{q}_2 \end{cases} \quad (7)$$

$$r_2(q_1) = \begin{cases} \frac{1+\lambda}{1+2\lambda}(1 - c - q_1) & \text{if } q_1 < \bar{q}_1 \\ 0 & \text{if } q_1 \geq \bar{q}_1 \end{cases} \quad (8)$$

where

$$\begin{aligned} \bar{q}_2 &\equiv q_2 : \Pi_1(r_1(q_2), q_2) = 0 \\ \bar{q}_1 &\equiv q_1 : W(q_1, r_2(q_1)) = W(q_1, 0) \end{aligned}$$

In Figure 1, the reaction functions are depicted. Coherently with the Assumptions 1-3, the discontinuity occurs above the monopolistic quantity of the rival avoiding multiple or corner equilibria.

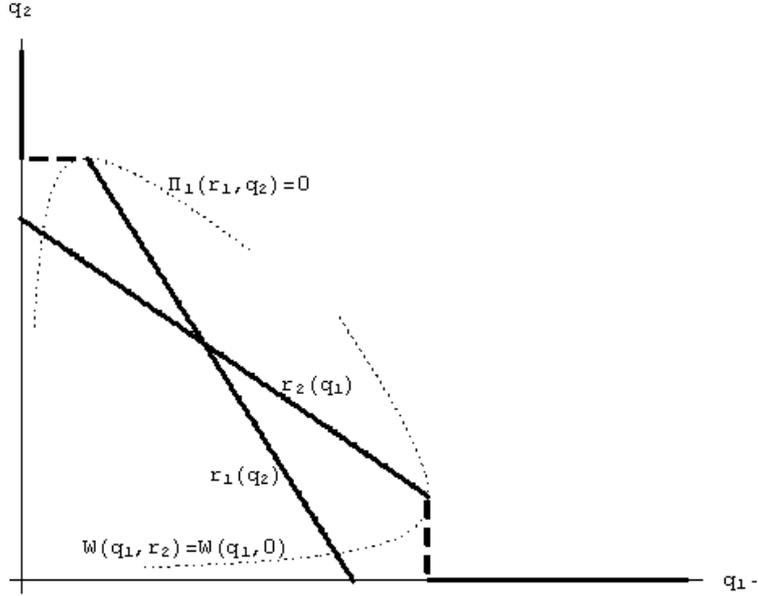


Figure 1: Firms' reaction functions.

### 3 Simultaneous equilibria: mixed oligopoly versus privatization.

When we assume firms play simultaneously, the output levels solve the system of the reaction functions 7 and 8. We refer to this equilibrium as a *mixed-Nash equilibrium* and all the derived variables are labeled by *MN*. The output levels are

$$q_1^{MN} = c + \lambda \frac{(1-2c)}{(3\lambda+1)} \quad (9)$$

$$q_2^{MN} = (1-2c) - \lambda \frac{2(1-2c)}{(3\lambda+1)} \quad (10)$$

Notice that when  $\lambda = 0$ , the public firm's output level is computed such that in equilibrium the price is equal to the public firm's marginal cost. That means, in a simultaneous mixed duopoly, the public firm implements a total output level equal to the one derived in the case of a welfare maximizer (but inefficient) monopoly; but now the welfare is higher<sup>13</sup>. Moreover, when public firm is at least as efficient as the private one, the first best solution is implemented.

<sup>13</sup>This is because the same total output is partially produced by the more efficient private competitor.

Increasing  $\lambda$  decreases the equilibrium public firm's output level,  $q_2$ , and increases the private firm's one,  $q_1$ ; then, the industry total cost decreases enhancing *technical efficiency*. This is because the public firm's concern for public transfers serves as a credible commitment to decrease output under the industry-wide welfare maximizing level. Moreover, since the reaction functions are contractions, the market output level,  $Q = q_1 + q_2$ , decreases and the price increases. It is obvious that the effect on consumer surplus is negative, raising in this way the *allocative inefficiency*. There exists a clear trade off between technical and allocative efficiency, and the net effect on industry-wide welfare is ambiguous. In fact, the net surplus generated in the market is

$$V^{MN} = \frac{(6\lambda - 2c - 10c\lambda + 3c^2 + 8\lambda^2 - 8c\lambda^2 + 14c^2\lambda + 11c^2\lambda^2 + 1)}{2(3\lambda + 1)^2} - K_1 - K_2$$

where only if

$$\lambda \leq \tilde{\lambda} = \frac{c}{1 - 5c}$$

the industry-wide welfare increase with  $\lambda$ , i.e.

$$\frac{\partial V}{\partial \lambda} = (c - \lambda + 5c\lambda) \frac{(1 - 2c)}{(3\lambda + 1)^3} \geq 0$$

Given  $c$ , the industry-wide welfare presents a inverse U-shape as  $\lambda$  increases; raising  $c$ , the threshold value  $\tilde{\lambda}$  such that  $V^{MN}$  is maximal becomes larger.

The next step of the analysis is a comparative statics exercise involving privatization. In fact, if we consider the case in which the public firm is sold to a private management that uses the same technology, i.e. no efficiency gain by privatization occurs, then the new firm will play as a profit maximizer and the implemented equilibrium is à la Cournot. We prove that, under our model setting, even if the government has the bargaining power to extract all the rent from the buyer<sup>14</sup>, the industry-wide and the economy-wide measures of welfare may be lower after the privatization. A similar result holds when we introduce the case of a fully efficient privatization. That is, the privatized firm achieves the same efficiency level of its competitor. In Table 1 we present quantities, profits and welfare in the cases of mixed Nash (MN), privatization without efficiency gains (CN), and fully efficient privatization (FE).

The following theorems hold<sup>15</sup>.

**Theorem 1** *Introducing the shadow cost of the public funds  $\lambda$ , there exists a subspace*

$$H = \left\{ (c, \lambda) \mid c \leq c^* = \frac{4\lambda + 6\lambda^2 + 1}{26\lambda + 12\lambda^2 + 8} \right\} \subseteq A$$

<sup>14</sup>That means the latter pays for the public firm a price equal to Cournot profits. This assumption drives the results in favor of privatization, since it overweights the revenue from privatization by  $\lambda$  in any welfare comparisons.

<sup>15</sup>Proofs are omitted since theorems come from comparisons of values presented in Table 1.

|      | $q_1$   | $q_2$                                   | $\Pi_1$  | $\Pi_2$  |
|------|---|---|--|--|
| $MN$ | $c + \lambda \frac{1-2c}{3\lambda+1}$   | $(1 + \lambda) \frac{1-2c}{3\lambda+1}$ | $\left(\frac{c+\lambda(1+c)}{3\lambda+1}\right)^2 - K_1$ | $(\lambda + 1) \lambda \left(\frac{1-2c}{3\lambda+1}\right)^2 - K_2$ |
| $CN$ | $\frac{1+c}{3}$   | $\frac{1}{3}(1-2c)$                     | $\left(\frac{1}{3}c + \frac{1}{3}\right)^2 - K_1$        | $\left(\frac{2}{3}c - \frac{1}{3}\right)^2 - K_2$                    |
| $FE$ | $\frac{1}{3}$   | $\frac{1}{3}$                           | $\frac{1}{9} - K_1$                                      | $\frac{1}{9} - K_2$  |
|      | $W$   |   |  |  |
| $MN$ | $\frac{(6\lambda-2c-10c\lambda+3c^2+10\lambda^2+2\lambda^3-16c\lambda^2+14c^2\lambda-8c\lambda^3+19c^2\lambda^2+8c^2\lambda^3+1)}{2(3\lambda+1)^2} - K_1 - K_2$ |   |  |  |
| $CN$ | $\frac{1}{18}(2\lambda + 11c^2 + 8 - 8c(1 + \lambda - c\lambda)) - K_1 - K_2$   |   |  |  |
| $FE$ | $\frac{1}{9}(\lambda + 4) - K_1 - K_2$  |   |  |  |

Table 1: The equilibrium values of quantities, profits and welfare in the cases of mixed oligopoly (MN), privatization with no efficiency gain (CN) and full efficient privatization (FE).

such that the economy-wide welfare is larger with a public firm playing as an economy-wide welfare maximizer than with an (inefficient) privatized firm, i.e.,  $W^{MN} \geq W^{CN}$ .

Moreover, there may not exist a trade off between welfare and profits as shown by the following corollary.

**Corollary 2** *Introducing the shadow cost of the public funds  $\lambda$ , there exists a subspace*

$$\underline{H} = \left\{ (c, \lambda) \mid c \leq c^* = \frac{4\lambda+6\lambda^2+1}{26\lambda+12\lambda^2+8}, \lambda \geq \frac{1}{3} \right\} \subset H$$

such that not only the economy-wide welfare but also the public firm profit in the mixed-Nash equilibrium is larger than the one obtained by an (inefficient) privatized firm, i.e.,  $W^{MN} \geq W^{CN}$  and  $\Pi_2^{MN} \geq \Pi_2^{CN}$ .

In figure 2, the subspaces  $H$  and  $\underline{H}$  are depicted.

**Theorem 3** *Introducing the shadow cost of the public funds  $\lambda$ , there exists a subspace*

$$Y = \{(c, \lambda) \mid c \leq c'\} \subseteq A \quad (11)$$

$$\text{with } c' = \frac{15\lambda+24\lambda^2+12\lambda^3-\sqrt{2}\sqrt{(4\lambda+3\lambda^2+1)^2(8\lambda+8\lambda^2+3)+3}}{3(\lambda+1)^2(8\lambda+3)}$$

such that public firm's profit level, economy-wide welfare and industry-wide welfare are higher when public firm is an economy-wide welfare maximizer than when a full efficient privatization occurs.

Moreover, as before, there may not exist a trade off between welfare and profits as shown by the following corollary.

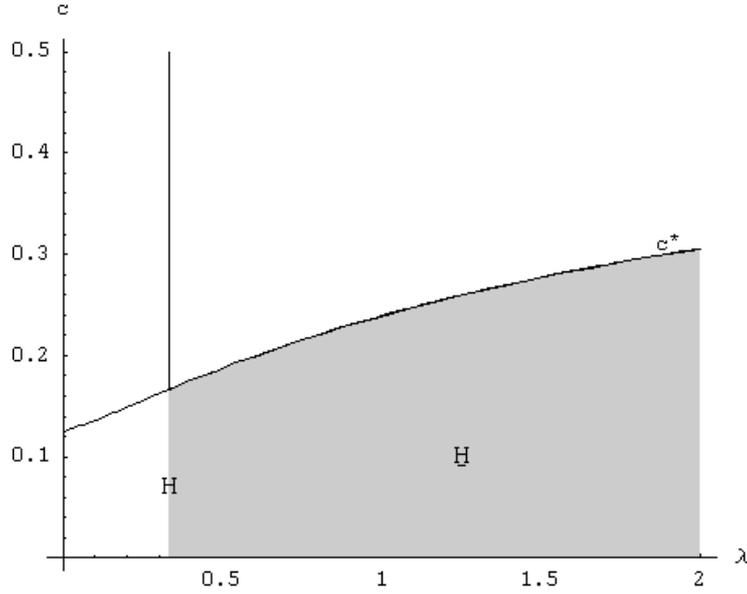


Figure 2: The subspaces  $H$  and  $\underline{H}$  in the parameter space  $(c, \lambda)$ .

**Corollary 4** *Introducing the shadow cost of the public funds  $\lambda$ , there exists a subspace*

$$\underline{Y} = \left\{ (c, \lambda) \mid c \leq c'', \lambda \geq \frac{1}{3} \right\} \subset Y$$

$$\text{with } c'' = \frac{(3\lambda + 3\lambda^2 - \sqrt{\lambda + 7\lambda^2 + 15\lambda^3 + 9\lambda^4})}{6\lambda + 6\lambda^2}$$

*such that not only the economy-wide welfare but also the public firm profit in the mixed-Nash equilibrium is larger than the one obtained by an efficient privatized firm, i.e.,  $W^{MN} \geq W^{FE}$  and  $\Pi_2^{MN} \geq \Pi_2^{FE}$ .*

In Figure 3 we graph the subspaces  $Y$  and  $\underline{Y}$  in the space of parameters  $(c, \lambda)$ . If we analyze the private firm's profit the following theorem holds.

**Theorem 5** *Introducing the shadow cost of the public funds  $\lambda$ , the private competitor is always better off when the public firm is privatized without efficiency gains.*

The intuition for the last theorem is that with an inefficient privatization the former public firm reduces its output and in the new equilibrium the private competition end up producing more with an higher price. Starting from this situation, if the privatization achieves efficiency gains the private competitor is

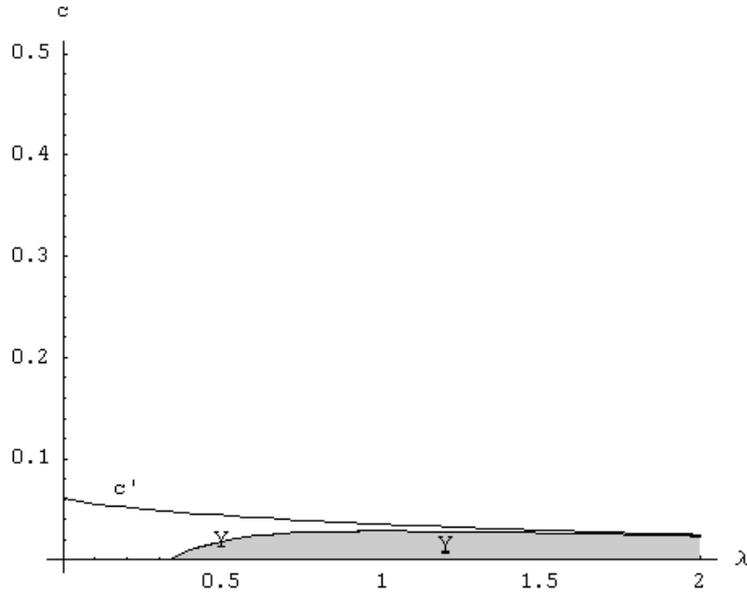


Figure 3: The subspaces  $Y$  and  $\underline{Y}$  in the parameter space  $(c, \lambda)$ .

of course worse off in the new equilibrium. Summing up the results in terms of economy-wide welfare and firms' profits, there exists a subspace of the parameters where the mixed-Nash equilibrium Pareto dominates the full-efficiency one.

**Corollary 6** *Introducing the shadow cost of the public funds  $\lambda$ , there exists a subspace*

$$X \subseteq Y = \left\{ (c, \lambda) \mid \lambda \geq \frac{1}{3}, c \leq c'' \right\} \subseteq A$$

*such that both public and private firms prefer the equilibrium in which public firm maximizes a measure of the economy-wide welfare with respect the case of full-efficient privatization.*

## 4 Endogenous timing.

In the previous section the timing of the game was exogenous and we assumed that firms play simultaneously. In the following, we investigate how the determination of simultaneous (Nash-Cournot) versus sequential (Stackelberg) games and the assignment of leader and follower roles in the latter case is the result

of preplay independent and simultaneous decisions by the players.<sup>16</sup> Then, in order to endogenize the timing of the game, we use the game with observable delay defined by *Hamilton and Slutsky (1990)*.

In the first stage firms simultaneously and independently choose the timing of action and then, once observing each other timing, they play the basic quantity game. The extensive form of the extended game is represented in Figure 4. The

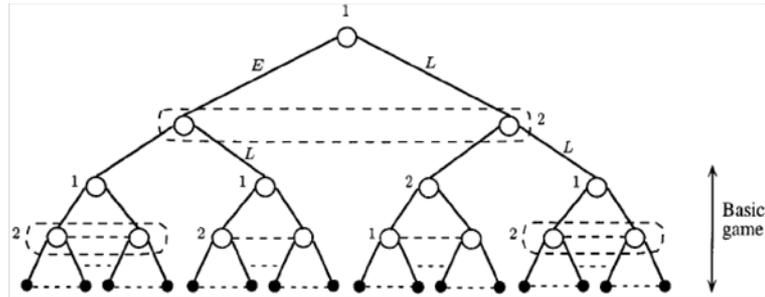


Figure 4: The extensive form of the extended game (Hamilton and Slutsky, 1990).

relevant equilibrium concept is the subgame-perfect Nash equilibrium and each player decides the timing of action according to the outcomes in the second stage, the basic game. Of course none of the firms can choose the type of competition by itself, but it can only eliminate some outcome. For example, if one firm decides to move early two outcomes are possible according to the decision of the other; only the Stackelberg outcome where this firm is follower is ruled out by its decision.

Assuming existence and uniqueness of equilibria in each basic game, the following Proposition summarizes the results obtained in Hamilton and Slutsky (1990) for any two-player game.

**Proposition 7** *Consider a two-player game for which the Nash and the two Stackelberg equilibria exist. Given that both players prefer always to be Stackelberg leader than simultaneous player, the set of pure strategy subgame-perfect Nash equilibria of the extended game is defined in the following way:*

*i) if for each firm Stackelberg follower's payoff is lower than the simultaneous player's payoff, then the unique SPNE of the extended game is the Nash equilibrium where both firms decide to move early;*

*ii) if each firm's Stackelberg follower payoff is strictly larger than the simultaneous player's payoff, the two Stackelberg equilibria are pure strategy Nash equilibria of the extended game;*

<sup>16</sup>Notice that in formal game-theoretical terms, Stackelberg's proposal is not to be construed as a new solution concept for one-shot games, but rather as a subgame-perfect Nash equilibrium of a two-stage game of perfect information with exogenously given first and second movers.

iii) if firm  $i$ 's Stackelberg follower payoff is strictly larger than the simultaneous player's payoff and firm  $j$  prefers to play simultaneously than to be Stackelberg follower, the unique SPNE of the extended game is the Stackelberg equilibrium with firm  $j$  being the leader.

**Proof.** The proof of this Proposition follows from Theorems II, III and IV in Hamilton and Slutsky (1990)<sup>17</sup>. ■

We use Proposition 7 in order to determine the endogenous timing in our model in which existence and uniqueness of each basic game are assured by Assumptions 1-3.

In our mixed duopoly game, we can write the normal form of the extended game with payoffs resulting from equilibria in each possible basic game, as represented in Table 2.

| Public Firm | Private Firm                         |                                      |
|-------------|--------------------------------------|--------------------------------------|
|             | Early                                | Late                                 |
| Early       | $W^{MN}(\cdot), \Pi_1^{MN}(\cdot)$   | $W^{PuL}(\cdot), \Pi_1^{PuL}(\cdot)$ |
| Late        | $W^{PrL}(\cdot), \Pi_1^{PrL}(\cdot)$ | $W^{MN}(\cdot), \Pi_1^{MN}(\cdot)$   |

Table 2: The normal form of the extended game in mixed oligopoly. MN, PrL and PuL stand respectively for mixed-Nash, Private Leadership and Public leadership equilibria.

In order to solve the game we need to compare the equilibrium payoffs in each basic game. Since the simultaneous equilibria have already been derived in Section 3, in what follows we derive the sequential ones.

#### 4.1 Sequential outcomes of the mixed oligopoly.

In the following we analyze the case of (i) public leadership, i.e. when the public firm moves first; and (ii) private leadership, i.e. when the public firm moves second. Notice that from now we assume that  $K_1 = K_2 = 0$ .

*Public Leadership (PuL).*

By backward induction, when public firm is an economy-wide welfare maximizer and moves before its private competitor, we have to distinguish two cases. In fact, there exists a threshold value of the marginal cost of the public firm,  $\bar{c}$ ,

$$\bar{c} \equiv \frac{2\lambda + 1}{4\lambda + 4}$$

---

<sup>17</sup>The intuition behind these results is the following. Given that firms prefer to be leader than to be simultaneous player, if simultaneous player's payoff is higher than follower's payoff, then this firm has a dominant strategy to move early. But if the firm prefers its follower payoff to the simultaneous player payoff, it has no dominant strategy: when the other player moves early it prefers to move late and vice versa. This reasoning explains the 3 possible results listed in Proposition 7.

|                                | $q_1^{PuL}$   | $q_2^{PuL}$                                      | $\Pi_1^{PuL}$                                     | $\Pi_2^{PuL}$   |
|--------------------------------|---|--|---|---|
| $c \in (0, \bar{c})$           | $\frac{(2c+\lambda+2c\lambda)}{(4\lambda+1)}$   | $\frac{(2\lambda-4c-4c\lambda+1)}{(4\lambda+1)}$ | $\frac{(2c+\lambda+2c\lambda)^2}{(4\lambda+1)^2}$ | $\frac{(2c\lambda-\lambda-c)(4c-2\lambda+4c\lambda-1)}{(4\lambda+1)^2}$ |
| $c \in [\bar{c}, \frac{1}{2})$ | $\frac{1}{2}$   | 0  | $\frac{1}{4}$                                     | 0   |
|                                | $W^{PuL}$   |  |   |   |
| $c \in (0, \bar{c})$           | $\frac{1}{2} \frac{(4\lambda-2c-6c\lambda+4c^2+\lambda^2-4c\lambda^2+8c^2\lambda+4c^2\lambda^2+1)}{(4\lambda+1)}$ |  |   |   |
| $c \in [\bar{c}, \frac{1}{2})$ | $\frac{3}{8}$   |  |   |   |

Table 3: The Public Leadership (PuL) equilibrium values of quantities, profits and welfare.

such that  $\forall c \in (0, \bar{c})$  the public firm produces in equilibrium. When  $c \in [\bar{c}, \frac{1}{2})$ , the public firm prefers not to produce and the private firm plays as a monopolist: its quantity, market price, and any measure of welfare are the same of a private monopoly. Economy-wide and industry-wide welfare are higher under private monopoly than in any combination with a positive quantity produced by the public firm.

Moreover, since

$$\forall \lambda \geq 0, \quad \bar{c} \in \left[ \frac{1}{4}, \frac{1}{2} \right] \quad \text{and} \quad \frac{\partial \bar{c}}{\partial \lambda} = \frac{1}{4(\lambda+1)^2} > 0 \quad (12)$$

when we introduce the shadow cost  $\lambda$ , increasing  $\lambda$  increases the upper limit of the public firm's marginal cost such that it produces: an higher level of public inefficiency is compatible with a sequential duopoly equilibrium. In Table 3 quantities, profits and welfare in the public leadership equilibrium are summarized.

#### *Private Leadership (PrL).*

Assume that private firm moves before its public competitor, that is, it behaves as a leader in the Stackelberg game. As before, we have two different subgame-perfect equilibria depending on the value of  $c$ . There is a threshold value  $\underline{c}$

$$\underline{c} \equiv \frac{\lambda}{3\lambda+1} \quad (13)$$

such that  $\forall c \in (0, \underline{c})$  the public firm produces in equilibrium. More precisely, for the private leader is optimal to choose a quantity such that the public firm's best response is positive. When  $c \in [\underline{c}, \frac{1}{2})$  the public firm does not produce in equilibrium and the private firm plays as a public (inefficient) monopolist that maximizes the industry-wide welfare: its quantity, as limit level, is such that the market price is equal to the marginal cost of the public firm.<sup>18</sup> Of course the welfare is higher because the private competitor produces more efficiently.

<sup>18</sup>This is the standard case when  $\lambda = 0$ : the public follower can always produce the quantity that needed to achieve this target and the best action for the private firm is directly producing that level.

|                                      | $q_1^{PrL}$  | $q_2^{PrL}$   | $\Pi_1^{PrL}$  | $\Pi_2^{PrL}$  |
|--------------------------------------|--|---|--|--|
| $c \in (0, \underline{c})$           | $\frac{1}{2} \frac{(c+\lambda+c\lambda)}{\lambda}$   | $\frac{(\lambda-c-3c\lambda)(\lambda+1)}{2(2\lambda+1)\lambda}$ | $\frac{1}{4} \frac{(c+\lambda+c\lambda)^2}{(2\lambda+1)\lambda}$ | $\frac{1}{4} \frac{(c-\lambda+3c\lambda)^2(\lambda+1)}{(2\lambda+1)^2\lambda}$ |
| $c \in [\underline{c}, \frac{1}{2})$ | $1 - c$  | $0$   | $c(1 - c)$   | $0$  |
|                                      | $W^{PrL}$  |   |  |  |
| $c \in (0, \underline{c})$           | $\frac{(4\lambda-4c\lambda+4c^2+8\lambda^2+\lambda^3-10c\lambda^2+17c^2\lambda-6c\lambda^3+22c^2\lambda^2+9c^2\lambda^3)}{8(2\lambda+1)\lambda}$ |   |  |  |
| $c \in [\underline{c}, \frac{1}{2})$ | $\frac{1}{2} - \frac{1}{2}c^2$   |   |  |  |

Table 4: The Private Leadership (PrL) equilibrium values of quantities, profits and welfare.

Moreover, since

$$\forall \lambda \geq 0, \underline{c} \in \left[0, \frac{1}{3}\right] \quad \text{and} \quad \frac{\partial \underline{c}}{\partial \lambda} = \frac{1}{(3\lambda + 1)^2} > 0 \quad (14)$$

when we introduce the shadow cost  $\lambda$ , increasing  $\lambda$  increases the upper limit of the public firm's marginal cost such that it produces: again, an higher level of public inefficiency is compatible with a sequential duopoly equilibrium. In Table 4 quantities, profits and welfare in the private leadership equilibrium are summarized.

## 4.2 Endogenous timing in mixed oligopoly.

In this section we derive the endogenous timing of our extended game. To do this, we define payoff rankings needed for applying Proposition 7. In particular, in Lemma 8 we compare the private firm's profit under public leadership (i.e., the follower payoff) and under mixed-Nash equilibria, while in Lemma 9 we compare the economy-wide welfare under private leadership (again the follower payoff) and mixed-Nash equilibria. It is worth noting that these Lemmas are sufficient. In fact, any player always prefers to be leader than to play simultaneously, by the property of Stackelberg equilibria. Moreover, the comparison between leader and follower payoffs is useless since in this framework no firm can unilaterally switch from one sequential equilibrium to the other: at most each player can switch to the simultaneous one.

**Lemma 8** *Introducing the shadow cost of public funds  $\lambda$ , there exists a subspace  $F_1 = (c, \lambda) \subseteq A$ , such that the private firm is better off in the case of public leadership than in the case of mixed-Nash equilibrium. In the subspace  $\widehat{F}_1 = A - F_1$  the reverse is true.*

**Proof.** Comparing values in Tables 1 and 3, it easy to check that  $\forall \lambda \geq 0$ :

(i)  $\forall c \in (0, \bar{c})$

$$\Pi_1^{PuL} - \Pi_1^{MN} > 0 \quad \forall c \leq \bar{c}$$

where

$$\bar{c} \equiv \frac{\lambda^2}{3\lambda + 2\lambda^2 + 1} \leq \bar{c} \quad (15)$$

(ii)  $\forall c \in (\bar{c}, \frac{1}{2})$ ,

$$\Pi_1^{PuL} - \Pi_1^{MN} > 0 \quad (16)$$

Thus, we define the subspace  $F_1 = (c, \lambda) \subseteq A$  as follows.

$$F_1 = \{(c, \lambda) \subseteq A | \bar{c} < c\} \quad (17)$$

and

$$\hat{F}_1 = \{(c, \lambda) \subseteq A | \bar{c} \geq c\}$$

■

**Lemma 9** *Introducing the shadow cost of public funds  $\lambda$ , there exists a subspace  $F_2 = (c, \lambda) \subseteq A$ , such that the public firm is better off in the case of private leadership than in the case of mixed-Nash equilibrium.  $\hat{F}_2 = A - F_2$  the reverse is true.*

**Proof.** Comparing values in Tables 1 and 4, it easy to check that  $\forall \lambda \geq 0$ :

(i)  $\forall c \in (0, \underline{c})$ ,

$$W^{PrL} - W^{MN} > 0 \quad \forall c \in (\underline{c}, \underline{c}) \quad (18)$$

where<sup>19</sup>

$$\underline{c} \equiv \frac{3\lambda^2 + 7\lambda^3}{21\lambda + 34\lambda^2 + 17\lambda^3 + 4} < \underline{c} \quad \forall \lambda \in (0, \bar{\lambda}) \quad (19)$$

(ii)  $\forall c \in (\underline{c}, \frac{1}{2})$ ,

$$W^{PrL} - W^{MN} > 0 \quad \forall \lambda \in (0, \bar{\lambda}) \quad (20)$$

Thus, we define the subspace  $F_2 = (c, \lambda) \subseteq A$  as follows.

$$F_2 = \{(c, \lambda) \subseteq A | \underline{c} < c\} \quad (21)$$

and

$$\hat{F}_2 = \{(c, \lambda) \subseteq A | \underline{c} \geq c\} \quad (22)$$

■

The intuition behind the previous lemma is the following. The subspace  $F_2$  is the set of parameter values such that the objective function of the public firm

<sup>19</sup>The inequality  $\underline{c} < \underline{c}$  holds until  $\lambda < 5.37228$ . Since  $\lambda$  is a measure of the distortion by taxation, we are comfortable assuming that  $\bar{\lambda}$  is lower than 5.37228.

is increasing in the private firm's output in the Nash equilibrium. This implies that moving from mixed-Nash equilibrium to public leadership, the public firm's output decreases while the private's one increases. It follows that the private firm is better off. Of course, the same happens for the public firm that plays as leader.

So, in the following theorems the different SPNE of the extended game in the subspace  $A$ .

**Theorem 10** *When  $(c, \lambda) \in \widehat{F}_2$ , the mixed-Nash equilibrium is the unique SPNE of the extended game.*

**Proof.** When  $(c, \lambda) \in \widehat{F}_1$  the private firm prefers to implement the mixed-Nash equilibrium than to play as Stackelberg follower. When  $(c, \lambda) \in \widehat{F}_2$  the public firm prefers to implement the mixed-Nash equilibrium than to play as Stackelberg follower. Then, in the intersection space  $\widehat{F}_1 \cap \widehat{F}_2$ , none want to be follower. Since  $\underline{c} < \bar{c}$ , It follows that  $\widehat{F}_2 \subset \widehat{F}_1$ , then  $\widehat{F}_1 \cap \widehat{F}_2$  coincides with  $\widehat{F}_2$ . Given that both players prefer always to be Stackelberg leader than simultaneous player, both firms have dominant strategy to move early. Then, the unique SPNE is mixed-Nash equilibrium. ■

**Theorem 11** *When  $(c, \lambda) \in \widehat{F}_1 \cap F_2$ , the unique SPNE of the extended game is the Stackelberg outcome with the private firm as leader.*

**Proof.** When  $(c, \lambda) \in \widehat{F}_1$  the private firm prefers to implement the mixed-Nash equilibrium than to play as Stackelberg follower. When  $(c, \lambda) \in F_2$  the public firm prefers to play as Stackelberg follower than to implement the mixed-Nash equilibrium. Since the private firm prefers always to be Stackelberg leader than simultaneous player, it has dominant strategy to move first. Then, the unique SPNE of the extended game is the Stackelberg outcome with the private firm as leader. ■

**Theorem 12** *When  $(c, \lambda) \in F_1$ , both Stackelberg outcomes are the (pure strategy) SPNE of the extended game.*

**Proof.** When  $(c, \lambda) \in F_1$  the private firm prefers to play as Stackelberg follower than to implement the mixed-Nash equilibrium. When  $(c, \lambda) \in F_2$  the public firm prefers to play as Stackelberg follower than to implement the mixed-Nash equilibrium. Then, in the intersection space  $F_1 \cap F_2$ , none want to play the simultaneous game. Since  $\underline{c} < \bar{c}$ , It follows that  $F_1 \subset F_2$ , then  $F_1 \cap F_2$  coincides with  $F_1$ . Given that both players prefer always to be Stackelberg leader than simultaneous player, then both Stackelberg outcomes are the (pure strategy) SPNE of the extended game. ■

Figure 5 describe the three possible outcomes of the endogenous timing game in the space of the parameters.

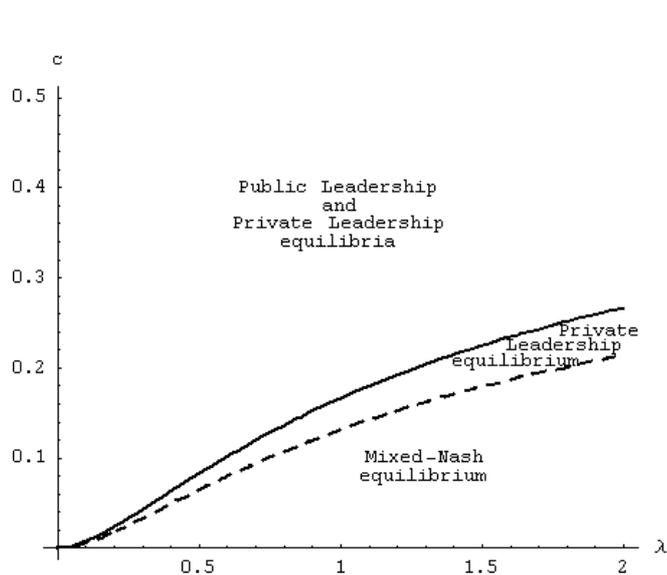


Figure 5: SPNE of the endogenous timing game in the space of parameters  $(\lambda, c)$ .

## 5 Privatization with endogenous timing

Until now we have assumed the public firm to be an economy-wide welfare maximizer. In this Section we carry out some comparisons in order to analyze the effects of privatization on welfare. As before, by privatization we consider the case in which the public firm is sold and management is instructed by the new owners to maximize profits. This change in ownership could also have the effect of reducing technical inefficiency. We consider the extreme cases in which either no efficiency gain or full efficiency are achieved. In the first case the privatized firm keeps the same technology as before; in the second it is able to produce at the same marginal cost of its competitor, here normalized to zero.

In order to compare economy-wide welfare before and after the privatization, the price paid to the government for buying the firm matters. As we are taking into account the shadow cost of public funds, it is not indifferent whether profits are public or private, and if the government is able to raise enough money from the privatization. The more money the government is able to raise by selling the public firm, the higher the welfare after the privatization. In what follows we give full bargaining power to the government; i.e., it is able to extract the whole profit from the privatized firm.

To analyze the welfare effect of privatization we have to compare the endogenous timing equilibria in the mixed and in the private duopoly. In the former case, when multiple equilibria arise, we assume the private leadership equilibrium as the relevant outcome. In the latter case, we can apply the results

in Amir and Grilo (1999) in order to define the equilibrium outcomes of the endogenous timing game when two private firms compete.

**Proposition 13** *Consider a private duopoly quantity game with strategic substitutes. When the value of the parameters are in the admissible set  $A$ , no Nash equilibrium lies on the boundary, i.e. no firm produces zero output. Then, both firms prefer always to be simultaneous player than Stackelberg follower. So, according to Proposition 7, the unique SPNE of the extended game is the Cournot-Nash equilibrium where both firms decide to move early.*

**Proof.** See Theorem 2.2 in Amir and Grilo (1999) ■

In Section 3 we have found the space of parameters' value such that in the simultaneous setting if the public firm maximized profits – or if it were privatized – the economy-wide welfare would increase, absent efficiency gains. In the following theorem, we obtain a (much stronger) result.

**Theorem 14** *Consider a mixed duopoly game in which the order of moves is endogenous. A privatization that does not achieve any efficiency gain always reduce economy-wide welfare.*

**Proof.** By Theorem 10 when  $(c, \lambda) \in \widehat{F}_2$ , so that the Mixed-Nash is the unique SPNE of the mixed oligopoly endogenous timing game. Comparing the economy-wide welfare before and after the privatization, it is straightforward to see that:

$$W^{MN} > W^{CN} \quad \forall (c, \lambda) \in \widehat{F}_2$$

When  $(c, \lambda) \in F_2$ , the Stackelberg outcome with the private firm as leader is either the unique SPNE or one of the two SPNE of the extended game (see Theorems 11 and 12). Again, comparing the economy-wide welfare before and after the privatization we have:

$$W^{PrL} > W^{CN} \quad \forall (c, \lambda) \in F_2$$

■

The difference with the results in Section 3 is clear. When the simultaneous play is exogenously given there is room for a welfare improving privatization even when it does not achieve efficiency gains. This is no longer the case when endogenous timing is considered: whenever welfare after the privatization is larger than welfare in the mixed-Nash equilibrium, the latter is not the SPNE of the endogenous timing game of the mixed oligopoly. And when the Stackelberg outcome with private leadership is a SPNE, it is welfare superior to the Cournot-Nash equilibrium that arise after the privatization.

Now we move to the analysis of the other extreme case: full efficient privatization and the following Theorem formalizes the result.

**Theorem 15** *Consider a mixed duopoly game in which the order of moves is endogenous. In addition, assume that by privatization the firm achieves full*

efficiency and all the profits are extracted by the government. Then there exists a subset of the parameter  $J$ , with  $Y \subset J$ , such that the privatization reduces welfare.

**Proof.** In order to prove the theorem we need to compare the welfare under full efficient privatization with the one derived in the proper endogenous timing outcome of the duopoly game. We consider: (i) the case in which the mixed-Nash is the equilibrium of the extended game; (ii). the case in which the Stackelberg outcome is either the unique SPNE or one of the two SPNE of the extended game, distinguishing when (iia) the public firm produces or (iib) it does not produce.

- (i) By Theorem 10, the subset  $\widehat{F}_2$  contains values of  $(c, \lambda)$  for which the mixed-Nash outcome is the equilibrium of the endogenous timing. By Theorem 3, the subset  $Y$  contains values of  $(c, \lambda)$  for which economy-wide welfare in the mixed-Nash equilibrium is larger than the one achieved after a full efficient privatization. Then we identify the set  $J^1$ .

$$J^1 = \left\{ (c, \lambda) \in \widehat{F}_2 \cap Y \Leftrightarrow W^{MN} - W^{FE} \geq 0 \right\}$$

for which an efficient privatization reduces welfare when the endogenous timing equilibrium of the mixed oligopoly game is the Mixed-Nash equilibrium. Comparing (11) and (22), it is easy to check that  $J^1$  is not an empty set.

- (ii) By Theorems 11 and 12, when  $(c, \lambda) \in F_2$ , the Stackelberg outcome with the private firm as leader is either the unique SPNE or one of the two SPNE of the extended game. In order to analyze this case we have to distinguish the values of  $(c, \lambda)$  such that the public firm produces in equilibrium from those for which it does not. First of all we define these sets:

$$\begin{aligned} F_{2a} &= \left\{ (c, \lambda) \mid c < \underline{c} \Leftrightarrow q_2^{\text{Pr}L} > 0 \right\} \\ F_{2b} &= \left\{ (c, \lambda) \mid c \geq \underline{c} \Leftrightarrow q_2^{\text{Pr}L} = 0 \right\} \end{aligned}$$

Then:

- (iia) We compare the economic-wide welfare when the private leadership is the SPNE of the mixed oligopoly game and the public firm produces positive quantity; i.e.,  $(c, \lambda) \in F_{2a}$ . Comparing the values presented in Tables 1 and 4, it is easy to check that there exists a set  $J^{2a} \subseteq F_{2a}$  such that the privatization reduces welfare, and it is not empty.

$$J^{2a} = \left\{ (c, \lambda) \in F_{2a} \mid 4\lambda - 7\lambda^3 - 18c\lambda(1 + \lambda)(2 + 3\lambda) + 9c^2(1 + \lambda)^2(4 + 9\lambda) > 0 \right\}$$

- (iib) Looking at the case in which the public follower does not produce in the SPNE; i.e.,  $(c, \lambda) \in F_{2b}$ , there exists a set  $J^{2b} \subseteq F_{2b}$  such that the privatization reduces welfare, and it is not empty.

$$J^{2b} = \left\{ (c, \lambda) \in F_{2b} \mid c < \frac{1}{3}\sqrt{1 - 2\lambda} \Leftrightarrow W^{\text{Pr}L} - W^{FE} \geq 0 \right\}$$

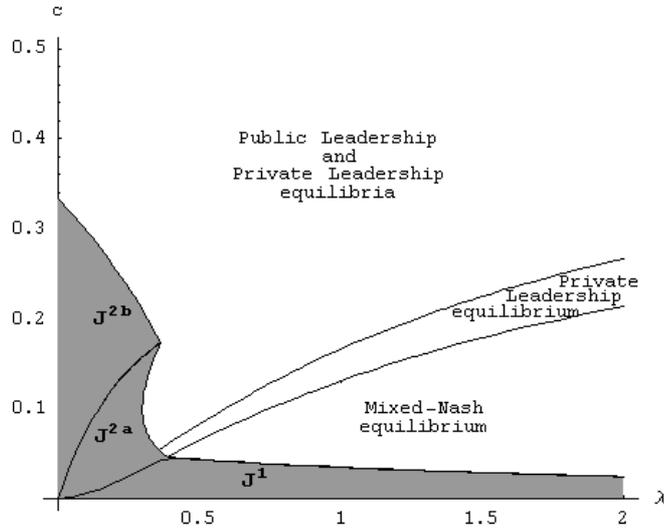


Figure 6: The subset  $J$  that collect all the values of the parameters such that a full efficient privatization reduces welfare.

Finally, the subset of parameters' values such that a full efficient privatization with full bargaining power to the government reduces economy-wide welfare is the following.

$$J = J^1 \cup J^{2a} \cup J^{2b}$$

In Figure 6 we graph the set  $J$  in the parameters' space. It is easy to check that  $Y \subset J$ . ■

## 5.1

## 6 Conclusions

Privatization not only can increase the efficiency of a market where an inefficient public firm competes with private-owned ones, but often it is treated as powerful instrument to raise money. In fact, in contexts where economic policies are constrained by budget balance requirements, the prices paid for public firms reduces the use of distortionary taxes. In this work, we start internalizing this target, "*getting money for reducing distortionary taxes*", in the objective function of the public management and we prove that this can be sufficient to implement an economy performance superior to the one obtained by privatization. This may be true even if the privatized-firm becomes efficient as its competitors (*full efficient privatization*). This is true both in terms of welfare and in terms of public firm's profit.

Moreover, when we analyze the issue of endogenous timing, while with private-owned firms only simultaneous equilibria can be implemented, we prove that sequential equilibria are part of the SPNE of the model. This property enlarges the set of the parameters where the mixed oligopoly dominates the full-efficient privatization.

### *References*

- Anderson, S. P., de Palma, A. and Thisse J. -F., 1997, "Privatization and Efficiency in a Differentiated Industry.", *European Economic Review* 41: 1635—1654.
- Amir, R. and I. Grilo, 1999, Stackelberg versus Cournot equilibrium, *Games and Economic Behavior*, 26, 1-21.
- Barros, F., 1994, Delegation and efficiency in a mixed oligopoly. *Annales d'Économie et de Statistique* 33, 51–72.
- Barros, F., 1995, Incentive schemes as strategic variables: an application to a mixed duopoly. *International Journal of Industrial Organization* 13, 373–386.
- Beato, P., Mas-Colell, A., 1984, "The Marginal Cost Pricing as a Regulation Mechanism in Mixed Markets", In M. Marchand, P. Pestieau and H. Tulkens (eds.): *The Performance of Public Enterprises*, North-Holland, Amsterdam.
- Bös, D., 1986, *Public Enterprise Economics*, North Holland, 1986.
- Bös, D., 1991, *Privatization: A Theoretical Treatment*. Clarendon Press, Oxford.
- Claude, D., and J. Hindriks, 2005, Strategic privatization and regulation policy in mixed markets, CORE Discussion Paper, 2005/15.
- Cremer, H., Marchand, M., Thisse, J. F., 1989, "The Public Firm as an Instrument for Regulation of an Oligopolistic Market", *Oxford Economic Papers* (39) pp. 283-301.
- Cremer, H., M. Marchand, and J. F. Thisse, 1991, "Mixed Oligopoly with Differentiated Products." *International Journal of Industrial Organization*, 9(1991):43-54.
- Cuervo, A., Villalonga, B., 2000. Explaining the variance in the performance effects of privatization. *The Academy of Management Review* 25 (3), 581—590.
- de Fraja, G., 1993, Productive efficiency in public and private firms, *Journal of Public Economics* 50, 15–30.
- de Fraja, G., Delbono, F., 1989, Alternative strategies of a public enterprise in oligopoly. *Oxford Economic Papers* 41, 302–311.
- de Fraja, G., Delbono, F., 1990, Game theoretic models of mixed oligopoly. *Journal of Economic Surveys* 4, 1–17.
- Estrin, S., Pérotin, V., 1991, Does ownership always matter?. *International Journal of Industrial Organization* 9 (1), 55–72.
- Fershtman, C., Judd, K.L., 1987, Equilibrium incentives in oligopoly. *American Economic Review* 77, 927–940.
- Fershtman, C. ,1990, The interdependence between ownership status and market structure: the case of privatization, *Economica*, 57, 319–28.

- Fjell, K. and Heywood, J., 2002, Public Stackelberg leadership in a mixed oligopoly with foreign firms, *Australian Economic Papers* 41, 267–281.
- Fjell, K. and Pal, D., 1996, A mixed oligopoly in the presence of foreign private firms, *Canadian Journal of Economics*, 29, 737-43.
- Garvie, D. and R. Ware, 1996, "Public Firms as Regulatory Instruments with Cost Uncertainty," *Canadian Journal of Economics*, vol. 29(2), pages 357-78.
- Gadal, A., Shirley, M., 1995, *Bureaucrats in Business: The Economics and Politics of Government Ownership*, The World Bank, Washington: Oxford University Press.
- Grilo, I., 1994, Mixed duopoly under vertical differentiation, *Annales de Économie et de Statistique*, 33, 91-112.
- Hagen, K.P., 1979, "Optimal Pricing in Public Firms in an Imperfect Market Economy", *Scandinavian Journal of Economics*, 81, 475-93.
- Haskel, J. and Sanchis, A., 1995, "Privatisation and X-Inefficiency: A Bargaining Approach," *Journal of Industrial Economics*, Blackwell Publishing, vol. 43(3), pages 301-321.
- Hamilton, J.H. and Slutsky, S.M., 1990. "Endogenous timing in duopoly games: Stackelberg or cournot equilibria," *Games and Economic Behavior*, vol. 2(1), pages 29-46.
- Harris, R. G., Wiens, E. G., 1980, "Government Enterprise : An Instrument for the Internal Regulation of Industry", *Canadian Journal of Economics*, (13), pp. 125-132.
- Laffont, J.-J., and J Tirole, 1986, Using cost observation to regulate firms, *Journal of Political Economy*, 94, 614-641.
- Laffont, J.-J. and J Tirole, 1991, Privatization and Incentives, *Journal of Law, Economics and Organization*, vol. 7, Spring, pp. 84-105.
- Laffont, J.-J., and J Tirole, 1993, *A Theory of Incentives in Procurement and Regulation*. MIT Press, Cambridge,MA
- Levy, B., 1987, A theory of public enterprise behavior. *Journal of Economic Behavior and Organization* 8, 75–96.
- Marchand, M., Pestieau P., Tulkens H., 1984, The performance of public enterprises: normative, positive and empirical issues, in: M. Marchand, P. Pestieau and H. Tulkens (Hrsg.), *The Performance of Public Enterprises*, Amsterdam: North-Holland, 3-42.
- Martin, S., 2004, Globalization and the Natural Limits of Competition, in *Handbook of Competition*, edited by M. Neumann and J. Weigand. Cheltenham: Edward Elgar.

- Matsumura, T., 1998, Partial privatization in mixed duopoly, *Journal of Public Economics*, 70, 473–83.
- Matsumura, T., 2003, Stackelberg mixed duopoly with a foreign competitor, *Bulletin of Economic Research* 55, 275–287.
- Meade, J., 1944, Price and output policy of state enterprise, *Economic Journal*, 54, 321–328.
- Meggison, W.L., Netter, J.M., 2001, From State to Market: A survey of empirical studies on privatization, *Journal of Economic Literature* 39 (2), 321–389.
- Merril, W., Schneider, N., 1966, Government firms in oligopoly industries: A short run analysis, *Quarterly Journal of Economics* 80, 400–412.
- Myles, G., 2002, Mixed oligopoly, subsidization and the order of firms' moves: an irrelevance result for the general case. *Economics Bulletin* 12, 1–6.
- Nett, L., 1993. Mixed oligopoly with homogeneous goods. *Annals of Public and Cooperative Economics* 64, 367–393.
- Pal, D., 1998. Endogenous timing in a mixed oligopoly. *Economic Letters* 61, 181–185.
- Pal, D., White, M., 1998. Mixed oligopoly, privatization and strategic trade policy. *Southern Economic Journal* 65, 264–282.
- Pint, E. M., 1991, 'Nationalization vs. regulation of monopolies: the effects of ownership on efficiency', *Journal of Public Economics*, vol. 44, pp. 131–64.
- Poyago-Theotoky, J., 2001. Mixed oligopoly, subsidization and the order of firms' moves: an irrelevance result. *Economics Bulletin* 12, 1–5.
- Rees, R., 1984, *Public Enterprise Economics*, Weinfeld and Nicolson, London, 2nd Ed.
- Sappington, D. E., Stiglitz, J. E., 1987, Privatization, information and incentives, *Journal of Policy Analysis and Management* 6, 567–582.
- Sepahvand, M., 2002. Privatization in a Regulated Market Open to Foreign Competition. University of Nottingham. Discussion Paper 02/04.
- Shapiro, C. and R. D. Willig, 1990, Economic Rationales for the Scope of Privatization, pp. 55–87 in Suleiman, E.N. and J. Waterbury (eds.), *The Political Economy of Public Sector reform and Privatization*, Boulder CO: Westview Press; also pp. 39–71 in Parker, D.(ed.), *Privatisation and Corporate Performance*, International Library of Critical Writings in Economics 123, Cheltenham: Edward Elgar, 2000.
- Sheshinski, E., López-Calva, L.F., 1999. Privatization and its benefits: Theory and Evidence. Harvard University. HIIID Discussion Paper 698.

- Short, R.P., 1984. The role of public enterprises: an international statistical comparison. In: Robert, H.F., et al.(Eds.), *Public Enterprise in Mixed Economies: Some Macroeconomic Aspects*. The International Monetary Fund, Washington, D.C., pp. 110–196.
- Skliwas, S.D., 1987. The strategic choice of managerial incentives. *RAND Journal of Economics* 18, 452–458.
- Stehmann, O. and Borthwick, R., 1994, Infrastructure competition and the European Union's telecommunications policy, *Telecommunications Policy*, 18(8), 601-615.
- Taschdjian, M., 1997, Alternative Models of Telecommunications Policy: Services Competition versus Infrastructure Competition, Twenty-Fifth Annual Telecommunications Policy Research Conference, Alexandria, September 27–29.
- Vickers, J. and Yarrow, G.K., 1991, Economic Perspective on Privatization, *Journal of Economic Perspectives* 5(2) pp.111-132
- Vickers, J. and Yarrow, G.K., 1988, *Privatization: An Economic Analysis*. MIT Press, Cambridge, MA.
- Villalonga, B., 2000. "Privatization and efficiency: Differentiating ownership effects from political, organizational, and dynamic effects" *Journal of Economic Behavior and Organization* 42, 43-74.
- Ware, R. (1986): "A Model of Public Enterprise with Entry" *Canadian Journal of Economics*, (19), pp. 642-655.
- White, M., 1996. Mixed oligopoly, privatization and subsidization. *Economics Letters* 53, 189–195.
- White, M.D., 2001, Managerial incentives and the decision to hire managers in markets with public and private firms. *European Journal of Political Economy* 17, 877–896.
- White, M.D., 2002, "Political manipulation of a public firm's objective function," *Journal of Economic Behavior & Organization*, Vol.49, pp.487-499.
- Willner, J., 2001, "Ownership, Efficiency, and Political Interference", *European Journal of Political Economy*, vol. 17, n. 4, pp. 723-748.
- Willner, J., 2003, "Privatization – A skeptical analysis", pp 60-86 (Ch. 4) in Parker, D. and D.Saal (Eds.), *International Handbook of Privatization*, Cheltenham: Edward Elgar.
- Willner, J. and D. Parker, 2003, 'The Relative Performance of Public and Private Enterprise under Conditions of Active and Passive Ownership', mimeo, Åbo akademi University and Aston BusinessSchool/The Centre on Regulation and Competition, IDPM, University of Manchester.
- Yarrow, G., 1999, A Theory of Privatization, or Why Bureaucrats are Still in Business, *World Development*, 27, 1, pp. 157-168.