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ENDOGENOUS INFORMATION AND SELF-INSURANCE IN INSURANCE MARKETS: A WELFARE ANALYSIS

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Endogenous Information and Self-Insurance in Insurance Markets: a Welfare Analysis

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Abstract

We develop a model where consumers *do not* have ex-ante private information on their risk but can decide to acquire such information before insurance policy purchase. Adverse selection can arise endogenously in the insurance market. We focus on the case where information has decision-making value: information allows consumers to optimally choose a self-insurance action. We analyze insurance market response to endogenous information and consumers' incentive to search for such information. Welfare costs caused by the lack of coverage against the risk to be a high risk are analyzed. The case of genetic testing serves as an illustration.

1 Introduction

The standard assumption in insurance models is that consumers are perfectly informed about their probability to incur a loss. In other words, individuals perfectly observe their risk (type), while insures do not. In insurance markets characterized by adverse selection, insurance firms offer self-selecting contracts: the well-known Rothschild/Stiglitz equilibrium allows insurers to separate the high- from the low-type consumers.

In many situations, however, consumers have only a vague perception of their probability of incurring a loss: they do not have ex-ante superior information. This is the case, for example, of health related risk. Nevertheless, recent developments in medical science makes genetic tests for many diseases available to consumers: whenever consumers choose to undertake a test, they decide to acquire precise information about their morbidity. This means that individuals *can* learn information about their risk of illness before purchasing the insurance contract: information is endogenous.

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Consumers' decision to learn information on their risk is influenced by the reaction of insurance market to such information. Market response to endogenous information is clearly essential in understanding consumers' incentives to search for information and strictly depends on whether consumers' information status is observable by insurers. Despite its importance, very few papers investigate the issue. Crocker and Snow (1992) show that, if insurers can observe whether or not consumers are informed and if consumers have no private prior information, then the private value of information is negative and consumers prefer to remain ignorant. The reason is that, when not informed, consumers have access to a full insurance contract based on the average probability of loss in the population. On the contrary, if consumers decide to acquire information on their types, insurers can write contracts that depend on the consumers' information status: they offer different policies to the informed and uninformed (whether or not the *test result* is observed by the insurer). Ex-ante, risk-averse consumers obviously prefer the first scenario.

In the same vein Doherty and Thistle (1996) show that information has positive private value only when insurers cannot observe consumers' information status, or if consumers can conceal their information status. In this case all consumers learn their type (at zero cost) and the outcome is again the self-selecting Rothschild/Stiglitz equilibrium: high-risk consumers hide the test results and receive a full-insurance contract, low-risk ones show the test result and receive partial insurance. Doherty and Thistle (1996) examine the existence and characterization of equilibria under different configurations of information costs and benefits; however they focus on the case in which information has no decision-making value. In other words, consumers only choose whether to become perfectly informed on their risk or to stay ignorant: information does not create new opportunity and no (preventative) action can be taken.

However, in the case of genetic tests as in many other situations, precise information on morbidity allows consumers to make more efficient decisions: primary and secondary prevention measures are often available and these measures are more effective the higher is the precision of information about the individuals' characteristics. As an example, let us consider the BRCA1 genetic mutation which is implicated in many hereditary breast cancer cases, and carries with it a very high risk of ovarian cancer. A woman who is positive to the BRCA1 test can undertake effective preventive measures to detect the illness at an early stage.

When information has decision-making value, consumers choose whether to become informed not only evaluating the consequences of information on the insurance premium but also taking into account the benefit of information in terms of more efficient actions. Moreover, as it is well known, when self-insurance actions are considered the standard trade-off between incentives and optimal risk sharing arises.

In this paper we analyze endogenous adverse selection in insurance markets where information has decision-making value. We focus on self-insurance (secondary prevention) for which the assumption of observable action is plausible. We analyze both the case where insurers observe consumers' information status and the case where consumers' information status is not observable.

The model more closely related to our is Doherty and Posey (1998). Also in their paper information has decision-making value, however the authors analyze the case of self-protection, we instead consider the case of self-insurance. A second important difference between the two papers is in that our simple model allows welfare analysis to be performed: we are able to investigate the welfare losses due to the lack of insurance against the risk to learn to be a high risk and due to endogenous information asymmetry.

The paper is organized as follows. Section 2 describes our assumptions and analyses the decision-maker's problem. Section 3 shows the first-best of the model and discuss how to decentralize such allocation in the market. Section 4 describes how (second-best) insurance affects consumers' choice of prevention. Finally, in section 5, the equilibrium in the insurance market is obtained allowing for different informational structures: first the case where information is symmetric and then the case where insurers do not observe decision-makers' information status are analyzed. Section 6 provides some final remarks.

2 The model

Decision-makers are endowed with a fixed amount of wealth w, and are characterized by the von Neumann-Morgenstern utility function $u(w)$, increasing and concave. With probability p_i, $i = L, H$, the decision-maker faces the monetary loss $L(a)$, where $0 < L(a) < w$. The action a is a self-insurance measure affecting the monetary loss. When the loss $L(.)$ is interpreted as the monetary equivalent of a negative *health* shock, the action a refers to secondary prevention or early detection of disease. It can take only two values, 0 and 1, and makes the loss decrease such that $L(1) = l < L(0) = L$. The action a is taken *before* the realization of the risk and implies a utility cost $\Psi(a)$, with $\Psi(0) = 0$ and $\Psi(1) = \Psi$.

We consider two consumers' types, the high- and the low-risk ones, respectively characterized by the probabilities p_L and p_H, with $0 < p_L < p_H < 1$. We assume that the probabilities p_L and p_H are fixed, so that no *ex-ante* moral hazard problem exists. The population proportions of high- and low-risk types are λ and $(1 - \lambda)$ respectively. These parameters are assumed to be common knowledge.

Consumers do not know their type *ex-ante*. The loss probability of uninformed individuals is $p_U = \lambda p_H + (1-\lambda) p_L$. Information can be gathered without cost by performing a diagnostic test (i.e. a genetic test). Risk neutral insurance companies can propose insurance contracts to consumers.

Note that, in the model, consumers face two different risks: the first risk is standard and is related to the loss $L(a)$. The second one is associated to the risk of being a high-risk and, thus, corresponds to the risk of paying a high premium. While insurance contracts designed to cover the risk of monetary losses of different nature and entities are really common, in the real world we do not observe "premium insurance". These policies have been called "genetic insurance" by Tabarrok (1994). Despite they would obviously increase consumers' welfare and ten years after the policy debate started and genetic tests are made available, the market seems not able to provide such insurance policies. Thus, it is worthwhile to analyze the reasons and the possible solutions for this market failure related to the lack of contingent market for insurance.

A crucial element in our analysis is the beneficial effect of information in terms of more efficient actions: when informed about his probability loss, the decision-maker is able to target his preventative effort. As will be clear in the next section, the optimal action is contingent on the loss probability. Ignorance can lead to under- or over-prevention. On the other hand, from an ex-ante perspective and since information leads to the premium risk, consumers can be worse off because of information.

2.1 The decision-maker's problem without insurance

Let us first examine the case where no insurance is available. An individual characterized by loss probability $p \in \{p_L, p_U, p_H\}$ and choosing preventive action a, achieves the following expected utility level:

$$V_0(p, a) = pu(w - L(a)) + (1 - p)u(w) - \Psi(a)$$

The individual chooses a positive amount of prevention if $V_0(p, 1) \geq V_0(p, 0)$, that is if $pu(w - l) + (1 - p)u(w) - \Psi \geq pu(w - L) + (1 - p)u(w)$, or:

$$p \geq \frac{\Psi}{u(w - l) - u(w - L)} = \frac{\Psi}{\Delta_0} \qquad (1)$$

The term Δ_0 is positive and measures the benefit from prevention. Obviously, when the benefit from prevention is large and its cost Ψ is low, inequality (1) is easily verified. For our purpose, inequality (1) is important because it shows that consumers choose prevention only when their loss probability is sufficiently high.

Remark 1 *Without insurance, riskier types perform prevention more often.*

Definition 1 $\widehat{a}(p)$ *is the action chosen by an individual characterized by probability of loss* p. $\widehat{V}_0(p)$ *is the individual's indirect expected utility when the probability is* p *and the chosen action* $\widehat{a}(p)$.

The uninformed individual deciding whether to acquire information on his type should compare $\widehat{V}(p_U)$ to $\lambda\widehat{V}_0(p_H) + (1 - \lambda)\widehat{V}_0(p_L)$. Note that, in general, the individual faces a trade-off: on the one hand, by learning his type he faces the premium-risk, on the other hand he is able to target his preventive effort to his personal characteristics.

Obviously, the case $\widehat{a}(p_L) = 1$ is not very interesting since it implies $\widehat{a}(p_U) = \widehat{a}(p_H) = 1$, and, $\widehat{V}_0(p_U) = \lambda\widehat{V}_0(p_H) + (1 - \lambda)\widehat{V}_0(p_L)$. In this case the individual is indifferent between acquiring and not acquiring information. We assume that, when indifferent, the individual chooses no-information gathering. Here the preventive action is always taken. Similarly, if $\widehat{a}(p_H) = 0$, $\widehat{a}(p_U) = \widehat{a}(p_L) = 0$; the individual remains uninformed and doesn't take preventive action. These two cases are summarized in the remark below.

Remark 2 *(i) When $\widehat{a}(p_L) = 1$, then $\widehat{a}(p_U) = \widehat{a}(p_H) = 1$. The test is not performed and prevention is positive.*

(ii) When $\widehat{a}(p_H) = 0$, then $\widehat{a}(p_U) = \widehat{a}(p_L) = 0$. The test is not performed and prevention is not taken.

More interesting are the cases where $\widehat{a}(p_L) = 0$ and $\widehat{a}(p_H) = 1$, that is $p_L \leq \frac{\Psi}{\Delta_0} \leq p_H$. Here, the high-risks choose positive prevention, whereas the low-risks do not. Thus:

$$\begin{aligned}\lambda\widehat{V}_0(p_H) + (1-\lambda)\widehat{V}_0(p_L) &= \lambda\left(p_H u(w-l) + (1-p_H)u(w) - \Psi\right) \\ &\quad + (1-\lambda)\left(p_L u(w-L) + (1-p_L)u(w)\right) \\ &= \lambda p_H u(w-l) + (1-\lambda)p_L u(w-L) + (1-p_U)u(w) - \lambda\Psi\end{aligned}$$

$$\text{Assumption 1: } p_L \leq p_U \leq \frac{\Psi}{\Delta_0} \leq p_H$$

Note that, without insurance and under Assumption 1, when they are uninformed, individuals do not undertake prevention; whereas, when they are informed, only high-types choose a positive level of prevention.

Remark 3 *Without insurance and under Assumption 1, uninformed individuals acquire information on their risk-type: information has a positive value for the decision-maker.*

Proof. Under assumption 1, $\widehat{V}_0(p_U) = p_U u(w-L) + (1-p_U)u(w)$ whereas $\lambda\widehat{V}_0(p_H) + (1-\lambda)\widehat{V}_0(p_L) = \lambda p_H u(w-l) + (1-\lambda)p_L u(w-L) + (1-p_U)u(w) - \lambda\Psi$. It is easy to verify that $\widehat{V}_0(p_U) < \lambda\widehat{V}_0(p_H) + (1-\lambda)\widehat{V}_0(p_L)$. ■

The previous remark shows that, without insurance, the benefit of information in terms of more efficient prevention choice prevails over its cost in terms of increased risk: uninformed consumers undertake the test.[1]

3 The first-best

As a benchmark we have to define the optimal allocation in this economy. We focus on the "*ex-ante* optimal allocation" as the one that maximizes *ex-ante* expected utility under feasibility constraint. Here the social planner covers both the premium-risk and the risk of the loss. All decision-makers perform the test *after* the contract is designed (the social planner designs the contract "under

[1] Such result is robust to the introduction of a cost for the test, provided the cost is sufficiently low.

the veil of ignorance"). Everything is observable and contractible. However, since it is defined in utility terms, the cost of the action a_i is not insurable.

Let us define P_i and I_i, $i = L, H$, the premium and the indemnity respectively. The social planner maximizes:

$$\begin{cases} \max_{P_H, I_H, P_L, I_L, , a_H, a_L} & \lambda \left(p_H u(w - P_H - L(a_H) + I_H) + (1 - p_H) u(w - P_H) - \Psi(a_H) \right) + \\ & (1 - \lambda) \left(p_L u(w - P_L - L(a_L) + I_L) + (1 - p_L) u(w - P_L) - \Psi(a_L) \right) \\ \\ \text{s.t.:} & \lambda P_H + (1 - \lambda) P_L = \lambda p_H I_H + (1 - \lambda) p_L I_L \end{cases}$$

Note that the previous program also corresponds to the utilitarian optimum: the utility functions of the two decision-makers' types are summed up and weighted by the proportion of each type in the whole population. What is crucial is the timing: in first-best expected utility is maximized under the veil of ignorance, in the utilitarian optimum expected utility is maximized *interim*, that is after the information on the type is revealed to decision-makers.

Obviously the first-best implies full insurance: $I_i = L(a_i)$, $i = L, H$. Moreover, the optimal premium is uniform and equal to $P^* = \lambda p_H L(a_L) + (1 - \lambda) p_L L(a_L)$. Since both types pay the same premium irrespective of their loss $L(a_i)$ and get utility $u(w - \lambda p_H L(a_H) - (1 - \lambda) p_L L(a_L))$, whenever the action chosen by the two types is different, the social planner attributes different utility levels to the two groups. In particular, the type performing more prevention suffers the higher disutility and, thus, is characterized by the lower utility.

We saw from Remark 1 that high-types are more likely to perform prevention than the low-types. This is because their probability to benefit from prevention is higher. As a consequence we expect that, whenever the two groups act differently, the high-risks are worse off.

The optimal values of a_i is the solutions of :

$$\max_{a_H, a_L} W^e(a_H, a_L) = u(w - \lambda p_H L(a_H) - (1 - \lambda) p_L L(a_L)) - \lambda \Psi(a_H) - (1 - \lambda) \Psi(a_L) \quad (2)$$

Note that, according to who performs prevention, four possible values for the welfare function are possible:

$$W_1^e = u(w - p_U l) - \Psi \quad (3)$$
$$W_2^e = u(w - \lambda p_H l - (1 - \lambda) p_L L) - \lambda \Psi \quad (4)$$
$$W_3^e = u(w - \lambda p_H L - (1 - \lambda) p_L l) - (1 - \lambda) \Psi \quad (5)$$
$$W_4^e = u(w - p_U L) \quad (6)$$

Welfare is W_1^e (W_4^e) when both types (no type) perform prevention. W_2^e and W_3^e correspond to the case where only high-types and only low-types respectively choose positive prevention.

As it was discussed before, when only one decision-makers' type performs prevention, the most natural case to analyze is the one where prevention is performed by high-types. As a consequence we assume that, $\forall \Psi$, $W_2^e \geq W_3^e$.

It can be easily checked that such inequality is always verified if the following assumption holds:

$$\text{Assumption 2:} \quad \begin{cases} \text{a) } \lambda p_H \geq (1-\lambda) p_L \\ \text{b) } \lambda \leq (1-\lambda) \end{cases}$$

Inequalities 2a and 2b are sufficient conditions such that it is socially optimal that only high-type decision-makers perform prevention (thus, later on we exclude the case expressed by the welfare function W_3^e). Note that, according to assumption 2b, high-type decision-makers are less frequent (ex-post) than low-type ones: $\lambda \leq 1/2$. Assumption 2a and 2b together indicate that the loss probability p_H must be sufficiently higher than p_L, in particular $p_H \geq \frac{1-\lambda}{\lambda} p_L$ where $\frac{1-\lambda}{\lambda} \geq 1$.[2]

Proposition 1 *Under assumption 2, first-best is such that:*

- *if* $\Psi \leq \frac{u(w-p_U l)-u(w-\lambda p_H l-(1-\lambda)p_L L)}{1-\lambda} = \Psi_2$ *then both types choose positive prevention.*
- *if* $\Psi_2 \leq \Psi \leq \frac{u(w-\lambda p_H l-(1-\lambda)p_L L)-u(w-p_U L)}{\lambda} = \Psi_1$ *then only high-types choose positive prevention.*
- *if* $\Psi_1 \leq \Psi$ *then none prevent.*

Proposition 1 shows that, when the cost of prevention is low, it is optimal to have both types performing prevention. As the cost of prevention increases, only high-types choose positive prevention. Finally, when the cost is sufficiently high, no prevention is performed. Figure 1 below describes social welfare in first-best as a function of the cost of prevention Ψ and offers a description of Proposition 1.

insert figure 1 here

Note that, for $\Psi_2 \leq \Psi \leq \Psi_1$, the high-types reach utility $u(w - \lambda p_H l - (1 - \lambda)p_L L) - \Psi$, whereas the low-types gain $u(w - \lambda p_H l - (1 - \lambda)p_L L)$.

Remark 4 *When* $\Psi_2 \leq \Psi \leq \Psi_1$*, in first-best high-types decision-makers are worse off.*

When they are fully insured, decision-makers benefit from prevention only because prevention allows to pay a lower premium (whereas, without insurance, we showed that the benefit from prevention was in terms of decreased loss). However, here the premium is uniform, thus it can be that cross subsidization arises between different types. It is interesting to ask whether the first-best does redistribute resources from the high- to the low-types. To see that, let

[2] When the decision-makers choose among a continuum of possible actions, no assumption 2 is required. The optimal action always increases with the loss probability p_i. In other words, provided that $p_H > p_L$, the high-types always choose a higher amount of prevention than the low-types.

us consider the premium high-types would pay for *fair insurance* when $\Psi_2 \leq \Psi \leq \Psi_1$: $P_H = p_H l$. Comparing P_H and $P^* = \lambda p_H l + (1-\lambda) p_L L$ we see that $P^* \geq p_H l$ implies $p_H l \leq p_L L$. Thus, high-types pay more than their fair premium in the first-best when prevention leads to a large fall in the monetary loss. We can state the following remark:

Remark 5 *When $\Psi_2 \leq \Psi \leq \Psi_1$ and the benefit from prevention is sufficiently high $\left(l \leq \frac{p_L}{p_H} L\right)$, first-best redistributes resources from the high- to the low-types.*

In such a case, not only the high-types pay a disutility cost because of prevention, they also pay a higher premium than they would pay with fair insurance. In particular, while only high-types pay the cost of prevention, both types receive its benefit. Interestingly, with secondary prevention, the general result that low-risks subsidize high-risks does not hold anymore.

As a final remark, note that without insurance decision-makers perform prevention more often since the marginal benefit from prevention is higher than without insurance. In particular, without insurance positive prevention is compatible with a lower value of the probability loss p_i and/or with a higher cost of prevention Ψ.[3] This is explicitly shown in section 4 when treating *ex-post* optimal insurance.

3.1 Premium insurance

Tabarrok (1994) proposes to decentralize the optimal allocation by creating an explicit market for insurance against the possibility to be a high-risk. The insurance policy should be mandatory: information acquisition is possible only after genetic insurance has been purchased.[4] This is necessary to avoid adverse-selection problems (we will discuss this point more in details later).

Let us consider our model. Suppose, as before, that the decision-maker's action is observable, such that full insurance can be implemented in a competitive insurance market. If genetic insurance is available and all decision-makers purchase it, they pays the premium $P_{GI} = \lambda p_H L(a_H) + (1-\lambda) p_L L(a_L)$. After genetic insurance has been bought, decision-makers acquire information performing the test and exhibit their test result to insurers in the competitive market. Those who learn that they type is high receive $p_H L(a_H)$ from the genetic insurer and, with that amount, purchase fair insurance in the market; those who learn that they type is low receive $p_L L(a_L)$ and purchase fair insurance as well.

[3] Again this can be easily shown in the case where the decision-maker chooses among a continuum of actions. Here, to verify that without insurance conditions such that *both* types perform prevention are less stringent, we must compare the threshold values $\frac{u(w-p_U l)-u(w-\lambda p_H l-(1-\lambda)p_L L)}{1-\lambda} = \Psi_2$ and $\Psi = p_L [u(w-l) - u(w-L)]$ (see Proposition 1 and inequality 1). We expect that:

$$p_L [u(w-l) - u(w-L)] \geq \frac{u(w-p_U l) - u(w-\lambda p_H l - (1-\lambda) p_L L)}{1-\lambda}.$$

[4] This can be enforced by making it illegal for physicians and laboratories to run tests without proof that genetic insurance has been bought.

Decision-makers purchase genetic insurance if their utility having performed the test is higher than their utility without information:

$$u\left(w-P_{GI}\right)-\Psi\left(a_{GI}\right)\geq u\left(w-p_{U}L\left(a\right)\right)-\Psi\left(a_{U}\right) \qquad (7)$$

Under Assumption 2 the l.h.s. of (7) can be W_1^e, W_2^e or W_4^e. Note that, when both types choose the same preventative action, then $P_{GI}=p_{U}L\left(a\right)$. Thus, the left- and the right-hand side of (7) are different only for $\Psi_2 \leq \Psi \leq \Psi_1$. However, in such a case, since with information acquisition the action a is targeted on the decision-makers' loss probability, utility under genetic insurance weakly dominates utility when decision-makers are uninformed. We can conclude that genetic insurance does allow the utilitarian optimum to be decentralized.

Note that genetic insurance presents some similarities with Cochrane's (1995) "time-consistent insurance". As Cochrane writes, time-consistent insurance provides premium insurance *as well as* insurance against the uncertain component of one period health expenditures. Moreover, the key feature for time-consistent insurance contracts is a severance payment: a person whose premium increases (for example because a long-term illness is diagnosed) receives a lump sum equals to the increased present value of his premium. The severance payment compensates for changes in premium and every consumer always purchases insurance at his actuarially fair premium.

What is different with respect to severance payments in time-consistent insurance is that, in our context, decision-makers face the problem of endogenous information acquisition and the insurance market must anticipate consumers' choice when designing insurance policies. Adverse selection becomes a crucial issue.

4 *Ex-post* optimal insurance

We analyze here the optimal insurance contract from an *ex-post* perspective, that is when decision-makers are informed and their type and preventative action are observable. For each type the social planner maximizes:

$$\begin{cases} \max\limits_{P_i, I_i, a_i} & p_i u\left(w-P_i-L\left(a_i\right)+I_i\right)+\left(1-p_i\right)u\left(w-P_i\right)-\Psi\left(a_i\right) \\ \text{s.t.:} & P_i = p_i I_i \end{cases}$$

where $i=L,H$. Obviously the optimal contract provides full-insurance: $I_i = L\left(a_i\right)$. Under full actuarial insurance the level $W_i(a)$ of expected utility achieved by a consumer with risk p_i and action a is:

$$W_i(a)=u(w-p_iL(a))-\Psi(a)$$

Prevention is positive if:

$$u(w-p_il)-\Psi\geq u(w-p_iL)$$

or:

$$\Delta(p_i)=u(w-p_il)-u(w-p_iL)\geq\Psi \qquad (8)$$

Remark 6 *(i) Higher risks are more likely to perform prevention. (ii) Under full-insurance, the optimal level of prevention is lower than the optimal level without insurance.*

Proof. (i) We have to prove that $\Delta(p_i)$ is an increasing function. In fact, $\frac{\partial \Delta(p_i)}{\partial p_i} = -p_i u'(w - p_i l) + p_i u'(w - p_i L) = p_i [u'(w - p_i L) - u'(w - p_i l)] > 0$. (ii) Recall that inequality (1) indicates the threshold value for positive prevention choice without insurance. We have to prove that $\Delta(p_i) \leq p_i \Delta_0$. This inequality can be rewritten as $u(w - p_i l) - u(w - p_i L) \leq p_i [u(w - l) - u(w - L)]$ $= p_i [u(w - l) - u(w - L)] + (1 - p_i) [u(w) - u(w)]$ or $u(w - p_i l) - u(w - p_i L) \leq$ $[p_i u(w - l) + (1 - p_i) u(w)] - [p_i u(w - L) + (1 - p_i) u(w)]$ which is true given that $l < L$ and $u(\cdot)$ concave. ■

From the previous remark:

Remark 7 *Insurance discourages prevention for a given risk: when $\Delta(p_i) < \Psi \leq p_i \Delta_0$, the fully insured decision-maker does not prevent although the uninsured one does.*

Let us consider social welfare in the *ex-post* optimal allocation:

Definition 2 *The ex-post optimal allocation is such that all decision-makers are informed and social welfare is:*

$$\begin{aligned} W_T^* &= \max_{a_H, a_L} W_T(a_H, a_L) \qquad (9) \\ &= \lambda u(w - p_H L(a_H)) + (1 - \lambda) u(w - p_L L(a_L)) - \lambda \Psi(a_H) - (1 - \lambda) \Psi(a_L) \end{aligned}$$

As we will show in the next section, the *ex-post* optimal allocation cannot be decentralized thought an insurance market where insurers could observe information status, risk type and decision-makers' action; that is in a market with symmetric information. In fact, when the *ex-ante* choice to acquire information is taken into account, an insurance market with symmetric information does not provide enough incentives to gather information such that inefficient prevention choices are taken.

5 The insurance market

Let us consider a competitive insurance market. The timing of actions is the following: first, insurance companies propose contracts which can depend on decision-makers' information status, type and level of prevention according to their observability; then insurees choose whether to perform the test, accept a contract and decide their level of prevention.

5.1 Endogenous choice of information acquisition with symmetric information

We suppose now that the decision-maker can remain uninformed or perform a test. Insurance firms observe the test result. Thus, insurers observe decision-makers' risk as well as their action. Full-insurance is provided.[5] In particular, three different contracts are offered: the contract for uninformed decision-makers, that for high-types and that for low-types.

If the decision-maker chooses to remain uniformed he achieves the following level of utility:

$$W_U^* = \max_{a_U} W_U(a_U) = u(w - p_U L(a_U)) - \Psi(a_U)$$

If he chooses to perform the test he obtains:

$$\begin{aligned} W_T^* &= \max_{a_H, a_L} W_T(a_H, a_L) \\ &= \lambda u(w - p_H L(a_H)) + (1-\lambda) u(w - p_L L(a_L)) - \lambda \Psi(a_H) - (1-\lambda)\Psi(a_L) \\ &= \lambda \max_{a_H} W_H(a_H) + (1-\lambda) \max_{a_L} W_L(a_L) \\ &= \lambda W_H^* + (1-\lambda) W_L^* \end{aligned}$$

Let us denote $a_H^T, a_L^T \equiv \arg\max W^T(a_H, a_L)$.

From (8), when $\Psi \leq \Delta(p_U)$ uninformed decision-makers choose $a = 1$ and expected utility is $W_U^*(a = 1) = u(w - p_U l) - \Psi$. When $\Psi \geq \Delta(p_U)$ uninformed decision-makers choose $a = 0$ and expected utility is $W_U^*(a = 0) = u(w - p_U L)$.

We can state the following lemma:

Lemma 1 *Under full information, when $\Psi \leq \Delta(p_L)$ and $\Psi \geq \Delta(p_H)$, expected utility without the test dominates expected utility with the test: decision-makers prefer to stay uninformed.*

Proof. For $\Psi \leq \Delta(p_L)$ and $\Psi \geq \Delta(p_H)$, high- and low-types' optimal action after information acquisition is the same: $a_H^T = a_L^T$. Since decision-makers are risk-averse, this implies $W_U^* > W_T^*$. ■

The previous lemma shows that, when information disclosed by the test has no decision-making value, the test is not performed since it increases decision-makers' risk.

When the test is performed three cases can arise: for $\Psi \leq \Delta(p_L)$ both types choose $a = 1$ and expected utility becomes $W_T^* = \lambda u(w - p_H l) + (1 - \lambda) u(w - p_L l) - \Psi$. For $\Psi \geq \Delta(p_H)$ both types choose $a = 0$ and expected utility becomes $W_T^* = \lambda u(w - p_H L) + (1 - \lambda) u(w - p_L L)$. Finally, when $\Delta(p_L) \leq \Psi \leq \Delta(p_H)$, only high-types choose positive prevention and expected utility is $W_T^* = \lambda u(w - p_H l) + (1 - \lambda) u(w - p_L L) - \lambda \Psi$.

As it was stated in Lemma 1, for $\Psi \leq \Delta(p_L)$ and $\Psi \geq \Delta(p_H)$, expected utility without the test dominates expected utility with the test. Whereas, for $\Delta(p_L) \leq \Psi \leq \Delta(p_H)$, it can be that expected utility without the test dominates expected utility with the test, or the opposite. In particular, according to the

[5] In this model self-insurance imposes a utility cost that is not insurable. However, secondary prevention is generally (also) characterized by monetary costs. Since the informational structure of the present and the following subsection allows to consider contracts providing full insurance for the monetary loss, we think that the lack of coverage for the disutility costs of prevention is plausible.

level of decision-makers' risk-aversion, two possible cases arise, as it can bee seen in the following graphs where the levels of expected utility is a function of Ψ.

insert figure 2 here

In figure 2, since risk-aversion is low, the intercept $(0, u(w - p_U l))$ is close to $(0,\ \lambda u(w - p_H l) +\ (1 - \lambda)u(w - p_L L))$. Thus, in the interval $[\Delta(p_L), \Delta(p_H)]$ values of Ψ such that expected utility with the test dominates expected utility without it exist.

insert figure 3 here

On the contrary, in figure 3, whatever the value of Ψ expected utility without the test always dominates expected utility with the test. It can be easily verified that:

Lemma 2 *Under full information, when $\Delta(p_L) \leq \Psi \leq \Delta(p_H)$, expected utility with the test can be higher or lower than expected utility without the test. In particular, when the following sufficient condition is satisfied:*

$$\lambda u(w - p_U l) + (1 - \lambda)u(w - p_U L) \leq \lambda u(w - p_H l) + (1 - \lambda)u(w - p_L L) \quad (10)$$

expected utility with the test dominates expected utility without the test in the interval $\Psi_3 \leq \Psi \leq \Psi_4$, where:

$$\begin{aligned}\Psi_3 &= \frac{1}{1-\lambda}\left[u(w - p_U l) - \lambda u(w - p_H l) - (1 - \lambda)u(w - p_L L)\right] \\ \Psi_4 &= \frac{1}{\lambda}\left[\lambda u(w - p_H l) + (1 - \lambda)u(w - p_L L) - u(w - p_U L)\right]\end{aligned}$$

Proof. For $\Delta(p_L) \leq \Delta(p_U) \leq \Psi \leq \Delta(p_H)$, only high-types choose positive prevention. Expected utility with the test dominates expected utility without the test if $\lambda u(w-p_H l) + (1-\lambda)\ u(w-p_L L) - \lambda\Psi \geq u(w-p_U L)$. For $\Delta(p_L) \leq \Psi \leq \Delta(p_U) \leq \Delta(p_H)$, both the uninformed and the high-type decision-makers choose positive prevention. Expected utility with the test dominates expected utility without the test if $\lambda u(w-p_H l) + (1-\lambda)\ u(w-p_L L) - \lambda\Psi \geq u(w-p_U l) -\Psi$. Putting together the previous inequalities, expected utility with the test dominates expected utility without the test if:

$$\begin{aligned}&\frac{1}{1-\lambda}\left[u(w - p_U l) - \lambda u(w - p_H l) - (1 - \lambda)u(w - p_L L)\right] = \Psi_3 \leq \Psi \quad (11)\\ &\leq \Psi_4 = \frac{1}{\lambda}\left[\lambda u(w - p_H l) + (1 - \lambda)u(w - p_L L) - u(w - p_U L)\right]\end{aligned}$$

which gives inequality 10. ■

From Lemma 1 and Lemma 2:

Proposition 2 *Under full information, (i) When the opposite of inequality (10) holds, decision-makers always remain uninformed. (ii) When inequality (10) holds, decision-makers perform the test in the interval $\Psi_3 \leq \Psi \leq \Psi_4$.*

Proposition 2 shows that, under symmetric information, even if insurance against the premium risk is not available, decision-makers may prefer to acquire information. This can be the case when aversion to risk is sufficiently low, such that decision-makers do not suffer too much because of increased risk. Moreover, this is possible for intermediate values of prevention cost Ψ, that is when it is efficient for the high-risk to perform prevention. In particular, when prevention cost Ψ is close to $\Delta(p_U)$, ignorance can impose excessive costs to uninformed decision-makers: for $\Psi_3 \leq \Psi \leq \Delta(p_U)$ uninformed low-types perform prevention even if its cost is too high, for $\Delta(p_U) \leq \Psi \leq \Psi_4$ uninformed high-types do not perform prevention even if its cost is sufficiently low.

It is interesting to compare the first-best to the allocations described in Proposition 2. Note that $u(w - \lambda p_H l - (1-\lambda)p_L L) \geq \lambda u(w - p_H l) + (1-\lambda)u(w - p_L L)$, thus, the intercept of the line describing expected utility when the test is performed and high-types choose positive prevention lies below the point $(0,\ u(w - \lambda p_H l - (1-\lambda)p_L L))$. We can state the following proposition:

Proposition 3 *When premium insurance is not available and information is symmetric: (i) if risk-aversion is high such that decision-makers always prefer to remain uninformed, over-prevention arises for* $\Psi_2 < \Psi \leq \Delta(p_U)$ *whereas under-prevention arises for* $\Delta(p_U) < \Psi < \Psi_1$*. (ii) if risk-aversion is low such that uninformed decision-makers perform the test for* $\Psi_3 \leq \Psi \leq \Psi_4$*, prevention choice is optimal in such an interval, whereas over-prevention arises for* $\Psi_2 < \Psi < \Psi_3$ *and under-prevention arises for* $\Psi_4 < \Psi < \Psi_1$*. Welfare losses are lower than in the previous case. (iii) First-best is always reached for* $\Psi \leq \Psi_2$ *and* $\Psi \geq \Psi_1$*. (iv) The allocation is not ex-post efficient.*

Proof. See figure 4. ■

Take the case where the opposite of inequality (10) holds such that decision-makers always prefer to stay uninformed, Proposition 3 shows that the lack of coverage for the premium risk leads to a welfare cost also when information in the market is symmetric. In particular, for $\Psi_2 \leq \Psi \leq \Delta(p_U)$ uninformed low-types perform prevention even if its cost is *too high*, for $\Delta(p_U) \leq \Psi \leq \Psi_1$ uninformed high-types do not perform prevention even if its cost is *sufficiently low*. Prevention choices are optimal for $\Psi \leq \Psi_2$ and for $\Psi \geq \Psi_1$, in such cases the first-best is reached.

Let us consider now the case where inequality (10) holds, this corresponds to the situation in which decision-makers prefer to acquire information in the interval $\Psi_3 \leq \Psi \leq \Psi_4$. In such an interval prevention choices are now optimal, however, since no coverage against premium-risk exists, decision-makers' utility is lower than in first-best. Note that here welfare losses are lower than in the case before. Thus, decision-makers are better off when they are characterized by low risk-aversion because they can benefit of more efficient choices at least for certain values of prevention cost Ψ.

Figure 4 below illustrates Proposition 3.

insert figure 4 here

5.2 Endogenous choice of information acquisition when decision-makers' information status is not observable

In the previous subsection information was symmetric. Here, we assume that decision-makers can secretly take the test before insurance purchase and are then free to show the test result or to conceal it. As in the previous subsection, if decision-makers show the test result to insurers, the latter can offer contracts contingent on such information. Moreover, as before prevention is observable such that insurance contracts can be contingent on decision-makers' action too.

The following proposition can be derived:

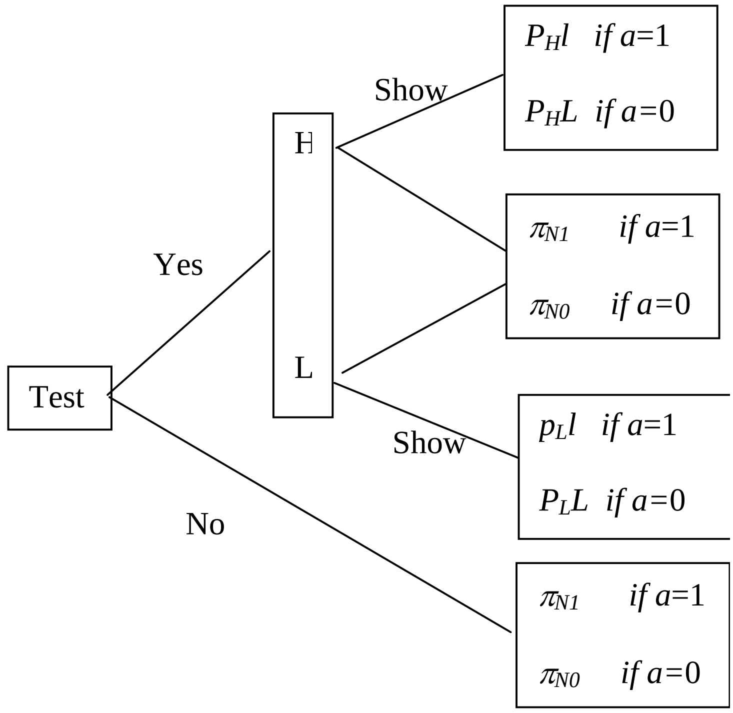
Proposition 4 *When the information status is not observable and the insuree can conceal the test result, at the equilibrium decision-makers perform the test and show the test-result to the insurer when they learn to be low-risk. The equilibrium is ex-post efficient and the level of expected utility achieved is:*

$$\begin{aligned} W_T^* &= \max_{a_H, a_L} W_T(a_H, a_L) \quad (12) \\ &= \lambda u(w - p_H L(a_H)) + (1-\lambda)u(w - p_L L(a_L)) - \lambda\Psi(a_H) - (1-\lambda)\Psi(a_L). \end{aligned}$$

Proof. (i) **Full-insurance contracts**. Suppose first that companies are constrained to offer full-insurance contracts. If prevention is contractible, insurance companies propose *ex-ante* 6 full-insurance contracts contingent on the test result (possibly observed) and on the decision-maker's action. The insurance premiums are:

	prevent	don't prevent
Show L	$\pi_{L1} = p_L l$	$\pi_{L0} = p_L L$
Show H	$\pi_{H1} = p_H l$	$\pi_{H0} = p_H L$
Don't show	π_{N1}	π_{N0}

The uninformed consumer faces the following decision-tree:



At equilibrium, we have necessarily $p_H l \geq \pi_{N1} \geq p_L l$ and $p_H L \geq \pi_{N0} \geq p_L L$. So that, when the test result is L (respectively H), it is optimal to show (respectively conceal) it.

When deciding whether to perform the test or not, the consumer must compare:

$$\lambda \max \{u(w - \pi_{N1}) - \Psi, u(w - \pi_{N0})\} + (1 - \lambda) \max_{a_L} (u(w - p_L L(a_L)) - \Psi(a_L)) \tag{13}$$

with:

$$\max(u(w - \pi_{N1}) - \Psi, u(w - \pi_{N0})) \tag{14}$$

where (13) is expected utility when the test is performed: with probability λ the decision maker is high-risk, does not show the test, and chooses the maximum between full-insurance with prevention and full-insurance without prevention; with probability $1 - \lambda$ the decision maker is low-risk, shows the test, and maximizes his (full-insurance) utility with respect to the action.

Now, suppose that:

$$\max(u(w - \pi_{N1}) - \Psi, u(w - \pi_{N0})) \geq \max_{a_L} u(w - p_L L(a_L)) - \Psi(a_L)$$

then nobody performs the test and $\pi_{N1} = p_U l$, $\pi_{N0} = p_U L$. This is impossible since:

$$\max_{a_L} u(w - p_L L(a_L)) - \Psi(a_L) > \max_{a_U} u(w - p_U L(a_U)) - \Psi(a_U)$$

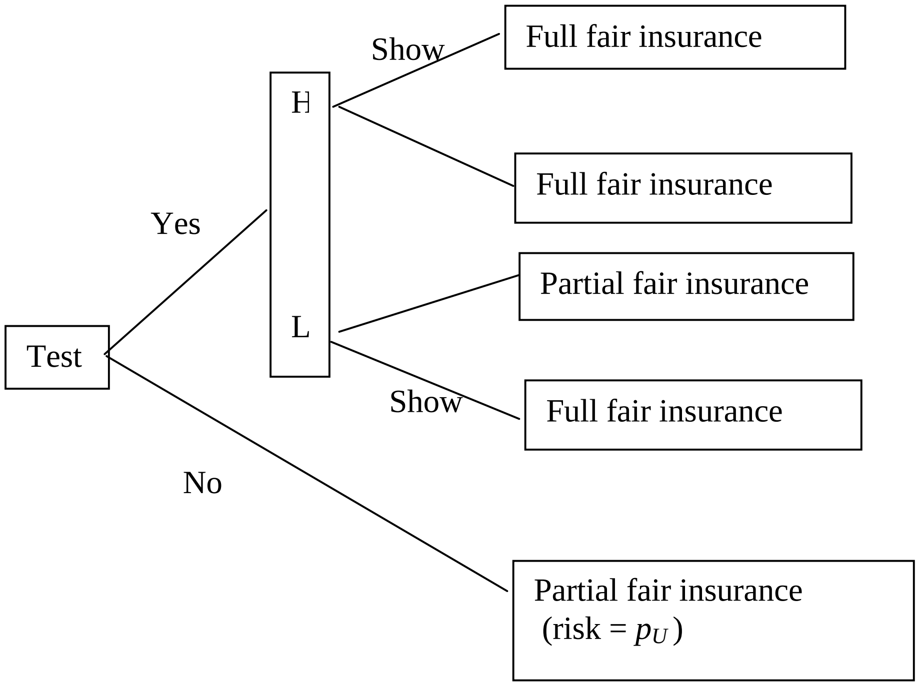
We can conclude that the only possible equilibrium is such that uninformed decision-makers perform the test and show it to the insurer only when the result is L. Thus, all decision-maker not showing the test are high-risk and $\pi_{N1} p_H l =$ and $\pi_{N0} = p_H L$.

(ii) **Menu of partial-insurance contracts** Suppose now that Insurance companies can propose *ex-ante* self selective contracts with partial coverage. In this case, companies will propose full-insurance contracts if the insurees show the test result, and a menu of self-selective contracts for those who don't. We do not model here the competition scenario that leads insurance companies to self-selective fair (actuarial) contracts, we simply suppose that competition is such that only "fair contracts" are sustainable. Assuming as before that prevention is observable, we obtain the set of contracts depicted in the following table.

	positive prevention	no prevention
Show L	$\pi_{L1} = p_L l$, full coverage	$\pi_{L0} = p_L L$, full coverage
Show H	$\pi_{H1} = p_H l$, full coverage	$\pi_{H0} = p_H L$, full coverage
Don't show	M_1 partial/full coverage	M_0 partial/full coverage

where M_i is a set of 3 self-selective contracts designed for the 3 possible types H, L, and U. These contracts correspond obviously to the Rothschild and Stiglitz allocation where L- and U-types are partially insured while H-types obtain full actuarial insurance.

The uninformed consumer now faces the following decision-tree:



When the test is taken it is optimal to show the result L . When the result is H, the insuree is indifferent since he obtains the same full fair insurance in the self-selective menu. Performing the test gives:

$$W_T^* = \max_{a_H, a_L} W_T(a_H, a_L) = \lambda u(w - p_H L(a_H)) + (1-\lambda) u(w - p_L L(a_L)) - \lambda \Psi(a_H) - (1-\lambda)\Psi(a_L)$$

Thus:

$$\begin{aligned} W_T^* &= \lambda \max(u(w - p_H L), u(w - p_H l) - \Psi) + (1-\lambda) \max(u(w - p_L L), u(w - p_L l) - \Psi) \\ &= \lambda W_H^* + (1-\lambda) W_L^* \end{aligned}$$

If, on the other hand, the insuree decides to remain uniformed, he obtains a partial insurance coverage (y or Y according to whether he chooses prevention or not) which correspond to the binding self-selective contract:

$$\begin{aligned} u(w - p_H L) &= p_H u(w - p_U Y + Y - L) + (1 - p_H) u(w - p_U Y) = U_H(Y) \\ u(w - p_H l) &= p_H u(w - p_U y + y - l) + (1 - p_H) u(w - p_U y) = U_H(y) \end{aligned}$$

Self-selective constraints give:

$$W_H^* = \max(U_H(Y), U_H(y) - \Psi)$$

We also have:

$$\begin{aligned}
U_L(y) &= p_L u(w - p_U y + y - l) + (1 - p_L) u(w - p_U l) \\
&< u(p_L(w - p_U y + y - l) + (1 - p_L)(w - p_U l)) \\
&= u(w - p_L l - (p_U - p_L) y) \\
&< u(w - p_L l) \\
\text{and similarly } U_L(Y) &< u(w - p_L L)
\end{aligned}$$

This implies:

$$W_L^* > \max(U_L(Y), U_L(y) - \Psi)$$

Which finally gives:

$$\begin{aligned}
\lambda W_H^* + (1-\lambda) W_L^* &> \max(\lambda U_H(Y) + (1-\lambda) U_L(Y), \lambda U_H(y) + (1-\lambda) U_L(y) - \Psi) \\
\lambda W_H^* + (1-\lambda) W_L^* &> \max(U_U(Y), U_U(y) - \Psi)
\end{aligned}$$

So that uninformed decision-makers strictly prefer to take the test. ■

Proposition 4 shows that, when the information status is not observable by insurers, all decision-makers acquire information irrespective of the value of prevention cost Ψ. This proves that, with adverse selection, the insurance market provides good incentives for information acquisition. Since decision-makers learn their risk, here prevention choices are always optimal. In the equilibrium allocation welfare losses are exclusively due to the lack of premium insurance. First-best is not reached for any value of prevention cost Ψ.

The following proposition compares social welfare in the allocation with symmetric and asymmetric information:

Proposition 5 *When premium insurance is not available, for $\Psi \leq \Psi_2$ and $\Psi \geq \Psi_1$ social welfare is higher under symmetric than under asymmetric information. When inequality (10) holds and for $\Psi_3 \leq \Psi \leq \Psi_4$, social welfare is the same under symmetric and asymmetric information. In all the other cases, the ranking between the two allocations is ambiguous.*

Note that, whether social welfare is higher under symmetric or asymmetric information for $\Psi_2 \leq \Psi \leq \Psi_1$ depends on the relative magnitude of inefficiency costs caused by inaccurate prevention choices and costs due to the lack of coverage for premium risk and risk-aversion. In particular, the higher risk-aversion, the higher social welfare is under symmetric information.

6 Conclusion

To be written....

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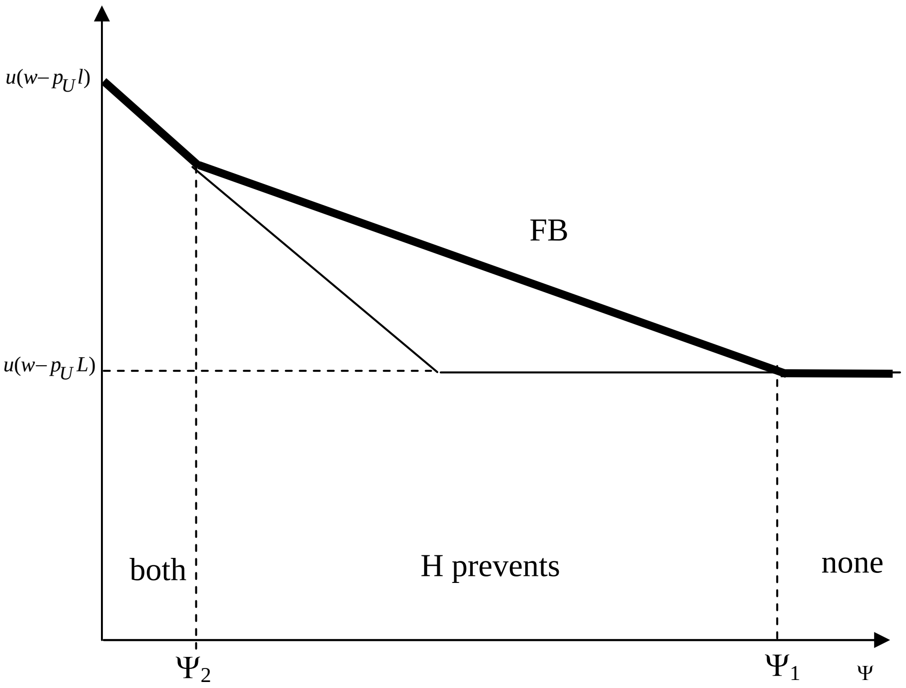

Figure 1: first-best allocation.

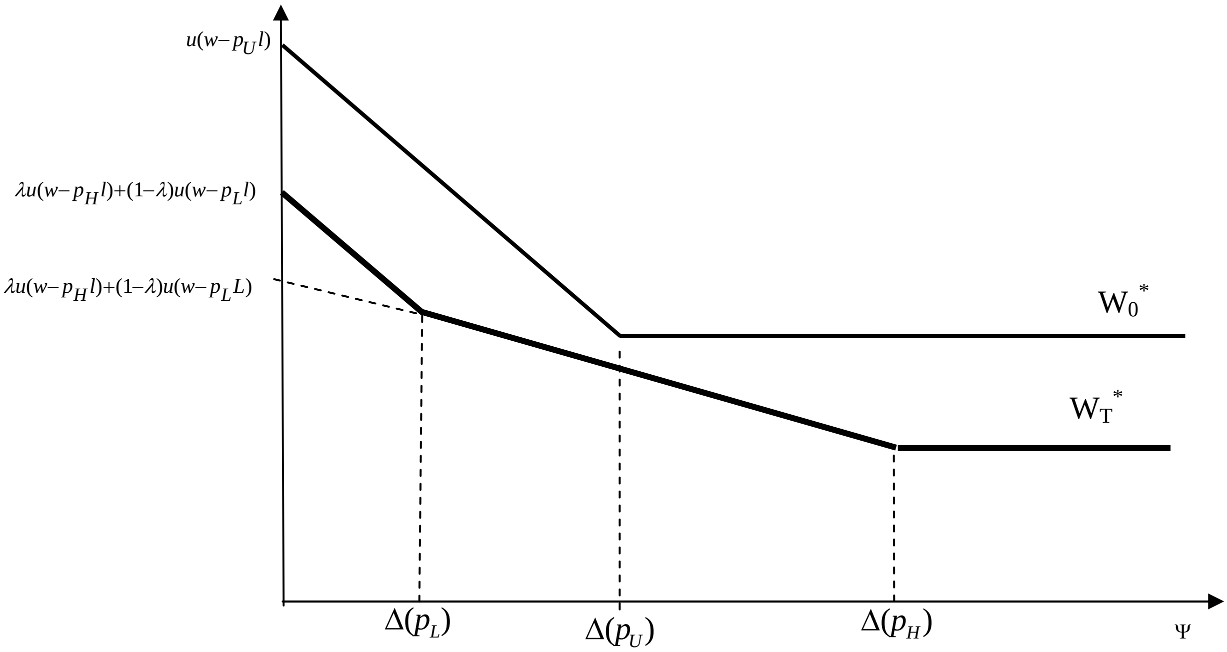

Figure 2: social welfare with and without the test
when risk-aversion is high.

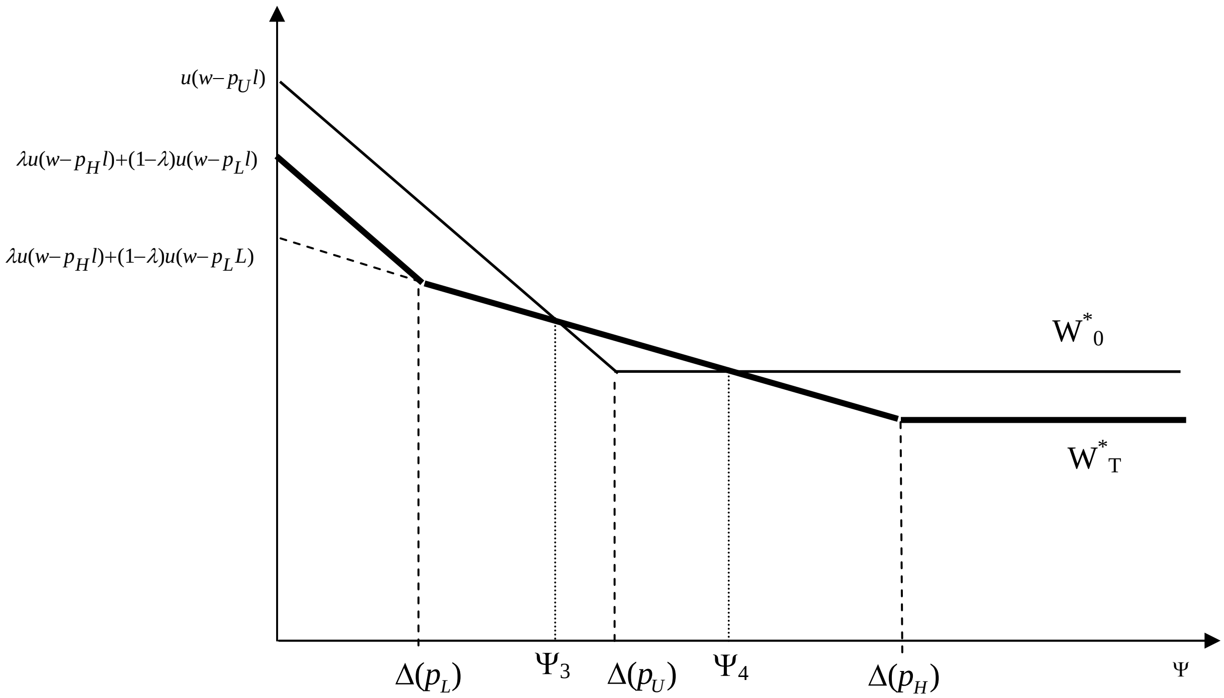

Figure 3: social welfare with and without the test when risk-aversion is low.

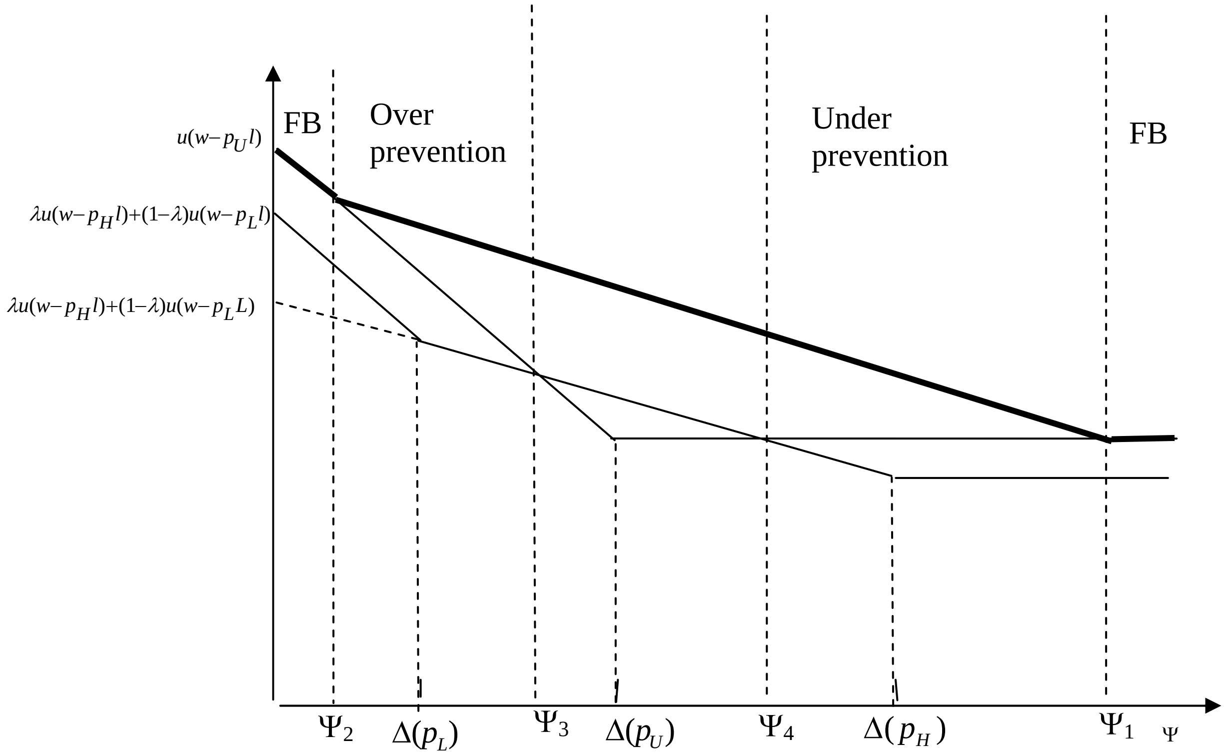

Figure 4: comparison between first-best and social welfare with observable information status and low risk-aversion.