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BIDDING TO LOSE? AUCTIONS WITH RESALE

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Bidding to Lose? Auctions with Resale

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Abstract

A losing bidder can still purchase the prize from the winner after the auction. We show why a strong bidder may prefer to drop out of the auction before the price has reached her valuation, and acquire the prize in the aftermarket: a strong bidder may be in a better bargaining position in the aftermarket if her rival won at a relatively low price. So it can be common knowledge that, in equilibrium, a weak bidder will win the auction and, even without uncertainty about relative valuations, resale will take place. Moreover, the possibility of reselling to a strong bidder attracts weak bidders to participate in the auction, and raises the seller's revenue. We explore how the seller can manipulate the conditions under which wealth-constrained bidders can finance their bids in order to induce a resale equilibrium which raises the auction price.

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1. Introduction

Before the UK "third-generation" (3G) mobile-phone licenses auction in 2000, it was known that one of the bidders, Orange, was going to be sold after the auction.[1] All other potential buyers knew that, provided Orange was among the winning firms, even if they lost the auction, they could still obtain a license by acquiring Orange. This is indeed what happened: NTL, a consortium controlled by France Telecom, first raised the auction price and then dropped out, allowing Orange to win one of the licenses on sale.[2] After the auction, France Telecom took over Orange. Similarly, after the European 3G auctions, Telia, the biggest telecom company in northern Europe, took over Sonera, a smaller and debt burdened telecom company, and obtained the licenses that Sonera had won in Germany, Italy, Spain, and Norway.

Winning an auction is not the only chance for a potential buyer to acquire the object on sale. A losing bidder can also obtain the prize after the auction, by purchasing it from a winning bidder. A weak (i.e., low-value) bidder then has an incentive to bid more than his valuation of the auction prize, in order to win and later resell to a strong (i.e., high-value) bidder. And a strong bidder has a choice between overbidding her competitor during the auction, or letting him win the auction and then purchasing the prize in the aftermarket.[3]

It may be expected that a stronger bidder always prefers to raise the auction price in order to weaken her competitor, and so be able to purchase the prize cheaply after the auction. Furthermore, since a weak bidder knows that the price will rise until his surplus is reduced to zero, it may be expected that he never wants to participate in the auction at all. But neither of these statements is necessarily true. We will show that, when wealth constraints matter, a strong bidder is in a better bargaining position in the resale market if the weak bidder has won at a low rather than a high price. Therefore, even the weak bidder has an incentive to participate in the auction and bid aggressively, since he knows that the strong bidder will let him win at a low price, rather than overbid him.[4]

[1] Orange was required to be sold by the European competition authority. Before the auction, Vodafone took over Mannesman, which had previously taken over Orange. Both Vodafone and Orange were incumbent mobile-phone operators in the UK and were willing to bid for a 3G license. The UK government allowed them to do so, because Vodafone was obliged to sell Orange after the auction and appropriate "Chinese-wall" requirements forbade the coordination of their bidding strategies during the auction. For an analysis of the UK 3G auction, see Binmore and Klemperer (2002).

[2] When Telefonica quit the UK auction in round 133, there were only 6 bidders left, including Orange and NTL, for the 5 licenses on sale. At that point, NTL could have ended the auction making sure that Orange obtained a license (since each bidder could win at most one license). Instead NTL kept on bidding until the price increased by almost 10%, and only then dropped out in round 150.

[3] We adopt the convention of using feminine pronouns for a strong bidder and masculine pronouns for a weak bidder.

[4] In a standard ascending auction with complete information, a strong bidder is indifferent between buying in the resale market and winning the auction at the same price at which he can buy in the resale market. So there is an

The reason is that, when a project with uncertain value is on sale, a wealth-constrained bidder enjoys limited liability (since he cannot lose more than his wealth) and treats the auction prize as an option: if the project turns out to be unprofitable, instead of continuing to invest in it, the bidder can declare bankruptcy and liquidate his wealth.[5] But then a very high auction price, by increasing the potential loss from bad projects, reduces the expected profit of a strong bidder more than the expected profit of a weak and wealth-constrained bidder, and hence reduces the strong bidder's surplus in the resale market. Therefore, in order to purchase in the aftermarket, during the auction the strong bidder does not bid above a certain price, thus allowing the weak bidder to win.

So it can be common knowledge that resale will take place after the auction, even if the order of bidders' valuations — and the fact that the order will not change — is commonly known.

However, there are also reasons why a strong bidder may prefer to raise the auction price at least some distance before dropping out. If a wealth-constrained bidder has to pay a borrowing cost to finance his bid, a higher auction price reduces his profit by a greater amount, and improves the strong bidder's bargaining position in the resale market. So the presence of a borrowing cost pins down a particular price at which the strong bidder chooses to drop out of the auction.

Our broader point is that, when bargaining in the resale market is affected by the price paid by the auction winner, the share of the resale surplus that a strong bidder can appropriate depends on the auction price, and hence a strong bidder is *not* indifferent about the price her rival pays in the auction.[6] The reasons we explore for this are limited liability and borrowing cost, but the point is more general. For example, when bidding against a risk-averse rival, a strong bidder may want to raise the auction price if, by reducing the winner's residual wealth, this reduces her rival's bargaining power in the resale market. Or, if the managers of a weak firm are willing to resell the prize at a fixed mark-up over the auction price (to justify their strategy with shareholders), then a

equilibrium with resale in which the weak bidder bids up to the resale price and the strong bidder drops out at zero. However, this is only one among many possible equilibria, and (unlike the resale equilibrium in our model) it is not robust to slight changes that make the model more realistic — for example, an arbitrarily small cost of resale, or bidders discounting (by even an arbitrarily small amount) the future surplus from the resale market. Moreover, another problem with this equilibrium (but not the equilibrium of our model) is that, if the strong bidder follows her weakly dominant strategy of bidding up to the resale price, the weak bidder cannot obtain a positive profit and, hence, has no incentive to participate in the auction.

[5] The effects of limited liability on bidding strategies in auctions without resale have been analyzed by Che and Gale (1998), Board (1999) and Zheng (2001). Board (1999) and Zheng (2001) argue that bidders with limited liability and lower wealth bid relatively more aggressively, and this can raise the seller's revenue. Che and Gale (1998) prove that when bidders face a budget constraint (and there is no uncertainty about profits), first-price auctions yield higher revenue than second price auctions, because the budget constraint is more likely to bind in a second-price auction.

[6] Other papers (e.g., Bikhchandani and Huang, 1989, Haile, 1999, 2003, and Zheng, 2002) show that, when bidders have incomplete information, the resale market can be affected by the auction because bids can signal a bidder's valuation to his opponents and, hence, can affect the division of the resale surplus. Notice that, in contrast, we do not require incomplete information for the auction to have effects on the outcome of the resale market.

strong bidder may want to drop out of the auction at a lower price, in order to purchase the prize more cheaply from the winner.

Of course, if valuations change after the auction, resale can occur when another potential buyer turns out to have a higher valuation than the winner. This may happen because additional buyers appear after the auction (as in Bikhchandani and Huang, 1989, Haile, 1999, and Milgrom, 1987), or because bidders' valuations change after the auction (as in Haile, 2000, 2001, 2003).[7] By contrast, in our model the uncertain component of the prize's value is common to all bidders, and all potential buyers can participate in the auction. Therefore, the ex-post efficient allocation is known before the auction starts. So resale in our model is not due to unexpected gains from trade: even with complete information about bidders' valuations, resale may take place in equilibrium.

Garratt and Tröger (2002) show that, even without valuations changing after the auction, in a second-price auction there are equilibria in which a weak bidder who has no value for the prize bids a high price (expecting not to pay it) and induces a strong bidder to bid zero, so that the weak bidder wins the auction at price zero and then resells at a profit to the strong bidder.[8] However, their equilibria are not unique (and there are also many equilibria without resale) and rely upon the weak bidder bidding a price higher than the maximum price he would be happy to pay, even after taking into account the surplus he can obtain in the resale market. By contrast, in our model, resale can be the unique equilibrium and does not require the weak bidder to bid a price higher than the maximum price he would be happy to pay. Moreover, our results do not depend on any bidder entering the auction or dropping out of the auction when indifferent about doing so — in our model, resale arises even with bidding and resale costs.

Resale increases the seller's revenue by giving even bidders who know they are weak a chance to win the auction, and hence an incentive to participate and bid aggressively. By contrast, resale was not allowed in some of the European 3G mobile-phone license auctions, possibly costing the governments billions of dollars.[9]

[7] Haile (1999, 2003) argues that the possibility of resale can induce a common-value element in bidders' valuations, and shows that bidding strategies are determined by the option value of buying and selling in the resale market, while Haile (2000) analyzes the effects of a reserve price. Haile (2001), using data from auctions of timber contracts, confirms that bidders' willingness to pay increases with the expected level of competition in the aftermarket, because this increases the option value to resell. Bikhchandani and Huang (1989) analyze multiple-unit common-value auctions in which bidders participate only to resell, and compare discriminatory to uniform-prize auctions.

[8] A similar equilibrium is discussed in Zheng (2000). See also note 4. Garratt and Tröger (2002) also show that resale can lead to an inefficient allocation and that, with resale, first-price and second-price auctions are not revenue equivalent.

[9] For example, in the Netherlands auction 5 licenses were on sale in a market with exactly 5 incumbents (which had a higher valuation than new entrants because of their recognizable brand-name, their familiarity with the market and their customer base). Other bidders did not show up, because they realized they had no chance of winning against incumbents, and the revenue was 70% lower than the government had forecast based on the comparable UK auction (Klemperer, 2002). The Swiss 3G auction, as well as some of the 1995 US PCS auctions (Klemperer and Pagnozzi, 2004), resulted in similar outcomes.

The seller can induce resale by modifying the terms on which a weak bidder can finance his bid.[10] A high borrowing cost makes a weak bidder unwilling to bid aggressively because it reduces his expected profit. This makes resale harder. But a high borrowing cost also increases the effect of limited liability, because it makes bankruptcy more likely, and so induces the strong bidder to drop out of the auction sooner. This makes resale easier. Depending on which effect prevails (which in turn depends on the bidder's wealth), the seller should either improve or worsen the weak bidder's borrowing conditions to induce a resale equilibrium.

Moreover, given that resale takes place, the seller can increase the auction price by inducing a bidder with higher wealth to participate, and by reducing his borrowing cost. The reason is that these strategies reduce the effects of limited liability, and make the strong bidder willing to bid more aggressively. However, for the reasons discussed above, if these strategies are pushed too far, they may prevent resale and actually reduce competition in the auction.

Section 2 presents the model. Section 3 discusses the effects of a wealth constraint on a bidder's expected profit, and Section 4 analyzes bargaining in the resale market. Section 5 proves that a strong bidder may prefer to drop out of an auction against a weaker competitor and derives conditions under which resale takes place in equilibrium. The strategy that may be adopted by the seller to increase revenue is analyzed in Section 6. Sections 8 discusses possible extensions and the last section concludes. All proofs are in Appendix A.

2. The Model

Consider an ascending auction for a project (for example, a license to provide mobile-phone services) with two potential buyers, bidder A and bidder B.[11] Bidder i has initial wealth w_i and values the project v_i, $i = A, B$. Bidders' wealths and valuations are common knowledge. We assume $v_A > v_B$ and $w_A > w_B$: A is a *strong* bidder with both a high valuation and a high initial wealth, while B is a *weak* bidder with both a low valuation and a low initial wealth.[12]

[10] There is a recent literature that studies the optimal seller's mechanisms in the presence of resale. Zheng (2002) analyzes when the optimal allocation in an auction without resale can be achieved when repeated resale is permitted, if the auction's winner has all the bargaining power in the resale market. Calzolari and Pavan (2003), on the other hand, assume that the bargaining power in the resale market depends on the identity of the auction's winner, and prove that resale reduces the seller's revenue compared to the revenue-maximizing mechanism without resale. Ausubel and Cramton (1999) argue that, in multi-unit auctions with perfect resale, the seller maximizes his revenue by assigning the prizes efficiently, so that no resale takes place.

[11] In an ascending auction the price is raised continuously by the auctioneer and bidders who wish to be active at the current price depress a button. When a bidder releases it, he is withdrawn from the auction (and cannot become active again). The price level and the number of active bidders are continuously displayed and the auction ends when only one active bidder is left.

[12] We model wealth-constrained bidders as in Zheng (2001), who analyzes auctions without resale.

In order to run the project, the owner pays an *operating* cost c, drawn from a uniform distribution on $[0, \overline{c}]$, and realized after the auction is over. Therefore, to obtain his valuation of the project, a bidder has to pay the *total* cost of the project, defined as the sum of the auction price p and the operating cost. If his initial wealth is less than the total cost, to pay for the exceeding cost a bidder borrows money at an additional cost $\beta > 0$ per unit of capital. The borrowing cost can be interpreted as either the interest rate on loans determined exogenously by banks, or as a cost set by the seller for accepting a bid higher than a bidder's wealth. The *operating* profit is the difference between the project's value and the total cost. But bidders are only liable for the total cost up to their initial wealth, because they cannot end up with negative final wealth. Therefore, if paying the total cost would generate a loss higher than his wealth, a bidder prefers to declare bankruptcy instead.[13] For simplicity, we assume that if the winner declares bankruptcy, the project returns to the seller and is never sold again.[14]

After the auction, and before the operating cost is realized and the auction price is paid, the winner can resell the project to the loser, if both bidders agree.[15] So the auction price is not paid immediately after the auction, but only after resale takes place.[16] This assumption fits many real life auctions (as well as our example in the introduction). For instance, winners were required to pay the auction price in yearly installments in the Danish, Italian and French 3G mobile-phone license auctions, as well as in many early FCC spectrum auctions in the US. And after the UK 3G mobile-phone licenses auctions, it was France Telecom that paid the price of the license won by Orange, when it took over Orange, because Orange was only required to pay after being sold by Vodafone.[17]

The resale price is determined by the bargaining between the two bidders, and we will assume that they share equally the surplus generated by resale. The timing of the game is the following:

[13] However, if the total cost is lower than his wealth, a bidder always pays for it, even if this implies obtaining negative profit. Hence, as is typically the case in spectrum auctions, bidders cannot simply return the project to the seller if it is not profitable to operate it. See also Section 8.2.

[14] In reality, the seller would probably re-auction the project after a delay. But our results only require that, after the weak bidder wins the auction, the strong bidder prefers to buy in the resale market, rather than wait for the weak bidder to go bankrupt and for a new auction to take place (e.g., because waiting for a new auction is costly, or because a new auction would attract other weak bidders, so the strong bidder would be unable to buy the project cheaply anyway).

[15] An additional reason for resale arises if resale takes place after the operating cost is realized: a strong bidder may choose to let a weak bidder win and bear the risk of a high cost, waiting until the uncertainty about the project's profitability is resolved before acquiring it. For example, big firms often leave R&D races to small firms with less to lose in case of failure, and then obtain the innovation by taking over the winner.

[16] Our results do not hinge on this assumption — an alternative model in which the auction price is paid before resale yields similar qualitative results. In particular, wealth effects may still induce a strong bidder to drop out of the auction and resale may still take place in equilibrium. See Section 7 and Appendix B.2.

[17] Because both Orange and Vodafone won a license and a single conglomerate firm was not allowed to own two licenses, the UK government required that Orange was divested before the two firms could claim their licenses.

(1) Bidder i wins the auction at price p, which is due to be paid in stage (4);

(2) Resale can take place;

(3) The operating cost c is realized;

(4) The owner of the project, bidder j (which is different from the auction winner if there is resale in stage (2)), can:

   (i) obtain v_j by paying for $(p + c)$ out of his wealth w_j and, if this is not sufficient, by paying the borrowing cost β on $(p + c) - w_j$, or

   (ii) declare bankruptcy and liquidate his wealth.

So a bidder has a chance of obtaining his valuation of the project even if he loses the auction. We assume that bidder A has infinite wealth, i.e. that $w_A = \infty$, so that she can pay any total cost and any resale price. Hence, bidder A never pays the borrowing cost if she owns the project, and she also never finds it profitable to declare bankruptcy in stage (4).[18] Her profit if she owns the project, net of the auction price (and neglecting the possible resale price), is:

$$\pi_A(p, c) = v_A - (p + c),$$

and her expected profit is:

$$\mathbb{E}_c[\pi_A(p, c)] = v_A - p - \mathbb{E}[c].$$

We assume that v_A is sufficiently high that, for any equilibrium auction price, bidder A's expected profit from the project is positive and higher than bidder B's one.[19]

We assume bidder B's initial wealth is $w_B = w < v_B$, so that bidder B faces a wealth constraint but enjoys *limited liability*. Bidder B's profit from owning the project depends on the relation between the total cost, his valuation and his initial wealth. If the total cost is lower than bidder B's wealth, his operating profit is $v_B - (p + c)$, the same as without a wealth constraint. If the total cost is higher than bidder B's wealth, then bidder B pays the borrowing cost to operate the project, and his operating profit is lower than without a wealth constraint, and equal to:

$$\underbrace{v_B}_{\text{value}} - \underbrace{(p+c)}_{\text{total cost}} - \underbrace{\beta(p+c-w)}_{\text{borrowing cost}} = v_B + \beta w - (1+\beta)(p+c).$$

Finally, if c is so high that the above profit is lower than $-w$, then bidder B declares bankruptcy, obtains an actual profit of $-w$ and is left with zero final wealth.

[18] Since the total cost is only paid in stage (4), if a bidder with low wealth wins the auction but resells to bidder A, then in stage (4) the latter does not have to pay the borrowing cost.

[19] Details are provided in Section 3.

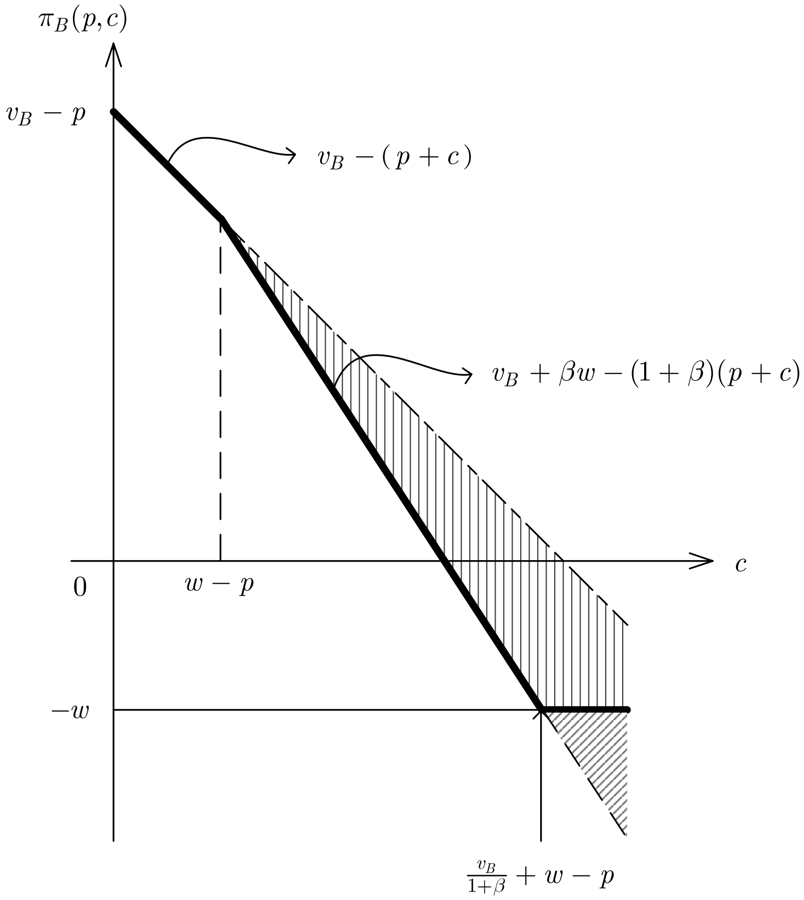

Figure 2.1: Bidder B's profit with a wealth constraint.

Summing up, bidder B's profit from owning the project is:

$$\pi_B(p, c) = \begin{cases} v_B - (p + c) & \text{if } c \leq w - p, \\ v_B + \beta w - (1 + \beta)(p + c) & \text{if } w - p < c < w - p + \frac{v_B}{1+\beta}, \\ -w & \text{if } w - p + \frac{v_B}{1+\beta} \leq c. \end{cases}$$

Figure 2.1 represents bidder B's profit as a function of the realized cost (for $w - \overline{c} < p < w$).[20]

We define a *resale equilibrium* as a Nash equilibrium of the auction with the following properties: (i) bidder B bids up to the highest price he is happy to pay, taking into account the surplus he obtains in the resale market, and (ii) bidder A strictly prefers to drop out of the auction and purchase the project from bidder B. (We will show that, for a wide range of parameters, this is the only equilibrium in undominated strategies.)

[20] This framework includes, as special cases, a (standard) model without a wealth constraint (when $\beta = 0$ and w is large) and a model with a strictly binding wealth constraint (when $\beta = \infty$ and hence bidder B cannot borrow above his initial wealth). While if $w = 0$, bidder B never obtains negative profit. This happens, for example, if the project can be returned to the seller if it is unprofitable to operate it.

3. Effects of a Wealth Constraint

A low initial wealth is like a budget constraint since it limits how much a bidder can pay for the project on sale, but it also limits how much a bidder can lose if the projects turns out to be unprofitable. A wealth constraint has two effects (a positive one and a negative one) on bidder B's expected profit from owning the project:

(i) *Borrowing Cost Effect:* if the total cost is higher than the bidder's wealth, profits on good projects (i.e., with a low operating cost) are reduced;

(ii) *Limited Liability Effect:* losses from bad projects (i.e., with a high operating cost) are reduced, because they are limited by the initial wealth (e.g., Zheng, 2001).

Compared to a situation without a wealth constraint, the borrowing cost effect makes a bidder worse off, while the limited liability effect may make a bidder better off. A bidder with high initial wealth does not face a wealth constraint and never pays the additional borrowing cost, but she also always has to pay any realized operating cost. In Figure 2.1, the vertical shaded area represents the reduction in expected profits due to the borrowing cost effect, while the diagonal shaded area represents the reduction in expected losses due to the limited liability effect.

For $p \geq \frac{v_B}{1+\beta} + w$ bidder B's expected profit is zero, while for $p < \frac{v_B}{1+\beta} + w$ bidder $\acute{B}$'s expected profit is:

$$\begin{aligned}\mathbb{E}_c[\pi_B(p,c)] &= \int_0^{w-p} [v_B - (p+c)]\, dF(c) + \int_{w-p}^{\frac{v_B}{1+\beta}+w-p} [v_B + \beta w - (1+\beta)(p+c)]\, dF(c) \\ &\quad -w\left[1 - F\left(\tfrac{v_B}{1+\beta} + w - p\right)\right],\end{aligned}$$

and:

$$\begin{aligned}\tfrac{\partial}{\partial p}\mathbb{E}_c[\pi_B(p)] &= -(1+\beta)\, F\left(\tfrac{v_B}{1+\beta} + w - p\right) + \beta F(w-p) \\ &= -1 + \Pr[B \text{ goes bankrupt}] - \beta \cdot \Pr[B \text{ borrows}],\end{aligned}$$

where the last equality follows because $\left[1 - F\left(\frac{v_B}{1+\beta} + w - p\right)\right]$ is the probability of bidder B going bankrupt and $\left[F\left(\frac{v_B}{1+\beta} + w - p\right) - F(w-p)\right]$ is the probability of bidder B borrowing to pay the total cost.

$$\left|\tfrac{\partial}{\partial p}\mathbb{E}_c[\widetilde{\pi}_B(p)]\right| \geq 1 \quad \Leftrightarrow \quad \beta \cdot \Pr[B \text{ borrows}] \geq \Pr[B \text{ goes bankrupt}].$$

An increase in the auction price reduces bidder B's expected profit relatively to bidder A's expected profit if and only if the borrowing cost effect is stronger than the limited liability effect.

For simplicity, we are going to discuss bidder B's expected profit from the project when $\frac{v_B}{1+\beta} > \overline{c}$. This assumption only affects the mathematical expression for bidder B's expected profit, but it does not affect any other part of our analysis.[21]

Lemma 1. *After winning the auction at price p, the expected profit of bidder B (for $\frac{v_B}{1+\beta} > \overline{c}$) is:*

$$\mathbb{E}_c[\pi_B(p,c)] = \begin{cases} v_B - \left(p + \frac{\overline{c}}{2}\right) & \text{for } p < w - \overline{c}, \\ v_B - (1+\beta)\left(p + \frac{\overline{c}}{2}\right) + \beta w - \frac{\beta(w-p)^2}{2\overline{c}} & \text{for } w - \overline{c} \le p < w, \\ v_B - (1+\beta)\left(p + \frac{\overline{c}}{2}\right) + \beta w & \text{for } w \le p < \frac{v_B}{1+\beta} + w - \overline{c}, \\ \frac{1}{2\overline{c}(1+\beta)}\left[v_B - (1+\beta)(p-w)\right]^2 - w & \text{for } \frac{v_B}{1+\beta} + w - \overline{c} \le p < \frac{v_B}{1+\beta} + w, \\ -w & \text{for } \frac{v_B}{1+\beta} + w \le p. \end{cases}$$

Bidder B's expected profit is decreasing in β because of the borrowing cost effect. The effect of an increase in w, however, is ambiguous: on the one hand, an increase in w reduces the need to obtain outside financing, which increases bidder B's expected profit for a relatively low auction price; on the other hand, an increase in w increases the liability of bidder B and his loss in case of bankruptcy, which reduces his expected profit for a relatively high auction price.

As shown in Figure 7.1, bidder B's expected profit function is (weakly) decreasing in p.[22] In the absence of a wealth constraint, the marginal negative effect of an increase in the auction price is equal to 1 — expected profit falls linearly as p increases. On the other hand, when B faces a wealth constraint:

$$\left|\frac{\partial \mathbb{E}_c[\pi_B]}{\partial p}\right| \ge 1 \quad \Leftrightarrow \quad p \le \frac{v_B - \overline{c}}{1+\beta} + w.$$

Starting from a relatively low price, an increase in the auction price reduces the expected profit of a wealth-constrained bidder more than the expected profit of a standard bidder because of the borrowing cost effect. However, for a high auction price the limited liability effect dominates, and the marginal negative effect of an increase in the auction price eventually becomes lower for a wealth-constrained bidder.[23]

By assumption, v_A is sufficiently higher than v_B so that, for any equilibrium auction price, $\mathbb{E}_c[\pi_A(p)] > 0$ and $\mathbb{E}_c[\pi_A(p)] > \mathbb{E}_c[\pi_B(p)]$.[24] Hence, without the possibility of resale, bidder A

[21] Appendix B.1 discusses the case $\frac{v_B}{1+\beta} < \overline{c}$ and shows that all our results are unchanged.

[22] Indeed:

$$\frac{\partial \mathbb{E}_c[\pi_B(p,c)]}{\partial p} = \begin{cases} -1 & \text{for } p < w - \overline{c}, \\ -(1+\beta) - \frac{\beta}{\overline{c}}(p-w) & \text{for } w - \overline{c} \le p < w, \\ -(1+\beta) & \text{for } w \le p < \frac{v_B}{1+\beta} + w - \overline{c}, \\ -\frac{1}{\overline{c}}\left[v_B - (1+\beta)(p-w)\right] & \text{for } \frac{v_B}{1+\beta} + w - \overline{c} \le p < \frac{v_B}{1+\beta} + w, \\ 0 & \text{for } \frac{v_B}{1+\beta} + w \le p. \end{cases}$$

For $p < \frac{v_B}{1+\beta} + w - \overline{c}$ the expected profit function is concave in p, while for $p > \frac{v_B}{1+\beta} + w - \overline{c}$ it is convex.

[23] When the total cost is so high that bidder B always chooses to declare bankruptcy, an increase in the auction price does not affect his profit at all.

[24] As the auction price increases, B's expected profit, being bounded by $-w$, eventually becomes higher than A's expected profit, which can be arbitrary small. But in equilibrium, the auction price is never higher than $\left(v_A - \frac{\overline{c}}{2}\right)$,

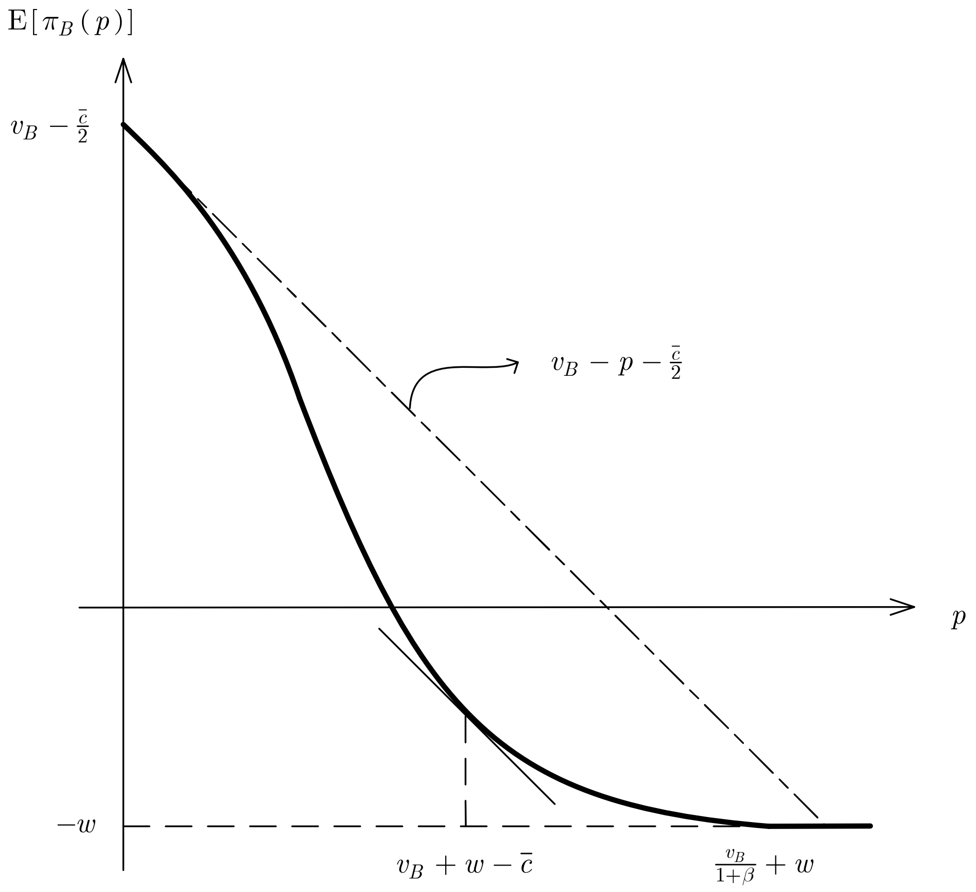

Figure 3.1: Bidder B's expected profit from owning the project.

would always win an ascending auction. Moreover, if bidder B is the winner, there are gains from trade between the two bidders in the resale market.[25]

4. Bargaining in the Resale Market

Assume bidder B wins the auction at price p. Before c is realized, bidder A can purchase the project form bidder B in the resale market, and then obtain the project's value by paying the total cost. Hence, her valuation of the project is $\mathbb{E}_c[\pi_A(p)]$. We normalize bidder A's outside option in the resale market to zero. Bidder B can keep the project and obtain the project's value by paying the total cost. So his valuation of the project, and hence his outside option in the resale market,

the price at which bidder A's expected profit is zero. So, in order to always have $\mathbb{E}[\pi_A] > \mathbb{E}[\pi_B]$ in equilibrium, it is sufficient to assume that, at price $\left(v_A - \frac{\bar{c}}{2}\right)$, bidder B's expected profit is negative. For example, this is true if $v_A > \frac{v_B}{1+\beta} + w + \frac{\bar{c}}{2}$. And this condition also implies that, at price $\frac{v_B - \bar{c}}{1+\beta} + w$, bidder A's expected profit is positive (see Section 5).

[25] As we are going to argue in Section 5, the highest price at which bidder B can win the auction is $\frac{v_B - \bar{c}}{1+\beta} + w$. At this price, $\mathbb{E}_c[\pi_B(p)] < v_B - p - \frac{\bar{c}}{2} < \mathbb{E}_c[\pi_A(p)]$. Therefore, whenever bidder B wins the auction, his expected profit is lower than bidder A's one.

is $\mathbb{E}_c[\pi_B(p)]$. The gains from trade resulting from resale are equal to the difference between these two valuations, that is, $\mathbb{E}_c[\pi_A(p)] - \mathbb{E}_c[\pi_B(p)]$.

We assume the outcome of bargaining between the two bidders in the resale market is given by the Nash bargaining solution, where the disagreement point is represented by bidders' outside options. This can be interpreted as the limit, as the length of the bargaining periods goes to zero, of a strategic model of alternating offers where parties face a small exogenous risk of breakdown of negotiations, that induces them to take their outside options (Binmore et al., 1986, and Sutton, 1986). So bidder B's valuation of the project is relevant in the resale market, and measures his bargaining power.[26]

Therefore, bidders share equally the gains from trade in the resale market,[27] and the resale price is equal to half the gains from trade plus bidder B's outside option, that is, $\frac{1}{2}(\mathbb{E}_c[\pi_A(p)] + \mathbb{E}_c[\pi_B(p)])$.

Letting S_i be the surplus obtained by bidder i if resale takes place, bidders' resale surplus is equal to:

$$S_A = S_B = \frac{1}{2}(\mathbb{E}_c[\pi_A(p)] - \mathbb{E}_c[\pi_B(p)]).$$

But when evaluated *before* the start of the auction, the total surplus of bidder B if he wins the auction and resells the project is equal to his resale surplus *plus* his outside option in the resale market, that is, the resale price.

Since $\mathbb{E}_c[\pi_A(p)] > \mathbb{E}_c[\pi_B(p)]$, resale is always efficient if bidder B wins the auction, and it is profitable for both bidders as long as $\mathbb{E}_c[\pi_A(p)] > 0$. The resale surplus depends on the auction price through its effect on bidders' expected profits from owning the project: because bidder B is wealth constrained while bidder A is not, an increase in the auction price affects the two bidders' expected profit differently.

5. Resale Equilibrium

In order to have a resale equilibrium two conditions must be satisfied:

[26] This assumption may be justified because, for example, governments auctioning spectrum licenses usually require the winner to start operating the license (and building the necessary infrastructure) by a predetermined date after the auction. So the winner's valuation of the license is relevant in the resale market because, while bargaining, bidders know that, if resale does not take place, the winner will have to take his outside option regardless of his will to do that. Sometimes, while trying to resell a project, the winner is even obliged to start operating it, so that his valuation of the project actually represents his "impasse point" (or "inside option") in case bargaining in the resale market continues forever. (See Binmore et al., 1989.)

[27] There are other ways to model bargaining between bidders — e.g., assuming unequal sharing of resale surplus or modelling the outside option differently. Many different assumptions yield results similar to those presented in this paper, as long as the auction price affects the outcome of bargaining in the resale market. For example, all results hold if bidder B obtains a share of resale surplus equal to k, for $0 < k < 1$.

- Condition (A): bidder A must strictly prefer to drop out of the auction and then purchase the project in the resale market, instead of winning the auction;
- Condition (B): bidder B must find it profitable to bid high enough to win the auction and then resell the project.

In this section, we first analyze how bidders' strategies in the auction are affected by the possibility of resale, and then we obtain conditions on the model's parameters for a resale equilibrium.

5.1. Weak Bidder Bids Aggressively

After winning the auction (at any equilibrium price), bidder B always prefers to resell to bidder A. Hence, bidder B is willing to remain active in the auction as long as his total surplus from winning and reselling the project (i.e., the resale price) is positive, and he drops out when his total surplus is equal to zero. The following lemma describes bidder B's bidding behaviour when he can resell the project and compares it with his bidding behaviour when he cannot resell the project, but still participates in the auction.

Lemma 2. *When resale is possible, it is a weakly dominant strategy for bidder B to bid up to price p_B such that $\mathbb{E}_c[\pi_A(p_B)] + \mathbb{E}_c[\pi_B(p_B)] = 0$. Bidder B bids more aggressively than when resale is not possible.*

This captures the idea that, during an auction, a weak bidder does not drop out as soon as the price reaches his valuation of the prize, because even if he wins at a higher price he can still resell to a bidder with a higher valuation, obtaining a positive surplus.[28] Hence, a weak bidder is always willing to pay a price included between his own valuation and a strong bidder's valuation.

Moreover, if bidders do not participate in the auction when indifferent (because, for example, they have to pay a small bidding cost to do so), without resale bidder B does not enter an ascending auction at all, because he knows he has no chance of winning against a stronger competitor. By contrast, if he expects to resell the prize, bidder B is willing to enter the auction, even under our extreme assumptions about bidders' valuation and wealth. Therefore, resale induces a weak bidder both to participate in the auction *and* to bid more aggressively.

5.2. Strong Bidder may Drop Out

Bidder A has a choice between two strategies: overbidding bidder B to win the auction, or dropping out of the auction and then purchasing the project from bidder B.

[28] In the terminology of Haile (2003), bidder B bids more aggressively because of the "resale seller effect."

Assume bidder A allows bidder B to win at price p and purchases the project from him. Then bidder A obtains a surplus equal to $S_A(p)$, which depends on the gains from trade in the resale market $(\mathbb{E}_c[\pi_A(p)] - \mathbb{E}_c[\pi_B(p)])$. The effect of an increase in the auction price on bidder A's surplus is given by:

$$\begin{aligned} \frac{\partial S_A}{\partial p} &= \frac{1}{2}\left(\frac{\partial \mathbb{E}_c[\pi_A]}{\partial p} - \frac{\partial \mathbb{E}_c[\pi_B]}{\partial p}\right) > 0 \quad \Leftrightarrow \quad \left|\frac{\partial \mathbb{E}_c[\pi_A]}{\partial p}\right| < \left|\frac{\partial \mathbb{E}_c[\pi_B]}{\partial p}\right| \\ &\Leftrightarrow \quad p < \frac{v_B - \bar{c}}{1+\beta} + w. \end{aligned}$$

So bidder A's resale surplus is increasing in the auction price if and only if $p < \frac{v_B - \bar{c}}{1+\beta} + w$. A further increase in the auction price reduces bidder A's expected profit more than bidder B's expected profit and, hence, reduces the gains from trade and makes bidder A worse off.

Therefore, when bidder A buys the project in the resale market, she is not indifferent about the auction price paid by bidder B and she raises the auction price only up to:[29,30]

$$p^* \equiv \frac{v_B - \bar{c}}{1+\beta} + w.$$

This is due to the two effects of a wealth constraint on bidder B's expected profit. As the auction price rises, the borrowing cost effect, by reducing bidder B's expected profit more than bidder A's, worsens the bargaining position of bidder B and increases the gains from trade; while the limited liability effect, by limiting bidder B's loss relative to bidder A's, improves the bargaining position of bidder B and reduces the gains from trade. For a low auction price, the borrowing cost effect dominates and an increase in price raises bidder A's resale surplus; while for a high auction price, the limited liability effect dominates and an increase in price lowers her surplus.[31]

But does bidder A prefer to win the auction at price p_B or drop out at price p^* and purchase from bidder B? Clearly bidder A prefers to purchase in the aftermarket if her resale surplus (after B wins the auction at price p^*) is higher than her profit from winning the auction at price p_B — i.e., if $S_A(p^*) > \mathbb{E}_c[\pi_A(p_B)]$.

Lemma 3. *Bidder A strictly prefers to drop out of the auction at price p^* and purchase the project in the resale market if and only if $p_B > p^*$.*

[29] By assumption, v_A is such that bidder A expected profit at p^* is positive (see note 24).

[30] This rules out equilibria in which the weak bidder can bid an arbitrarily high price because the strong bidder drops out of the auction at zero, being indifferent about the auction price — bidder A never drops out at a price lower than p^* when bidder B bids a higher price.

[31] An increase in w raises p^* because it makes it more costly for bidder B to declare bankruptcy, and hence reduces the limited liability effect. This increases the incentive for bidder A to raise the auction price and worsens B's outside option. On the other hand, an increase in β reduces p^* because it reduces bidder B's expected profit and makes him more likely to declare bankruptcy. This increases both the borrowing cost and the limited liability effects, but the latter dominates so that bidder A is less willing to raise the auction price. (If $\beta = 0$, only the limited liability effect is present and bidder A drops out of the auction as soon as the price is such that bidder B's operating profit may become lower than $-w$, and so he may declare bankruptcy.)

As shown in the proof of Lemma 3, at price p_B bidder A is exactly indifferent between winning the auction and dropping out to purchase in the resale market. (And this is true regardless of how bidders share the gains from trade in the resale market.) Since her resale surplus is maximized at p^* (and it is strictly lower at a higher price), if $p_B > p^*$ bidder A strictly prefers to purchase the project in the resale market, because overbidding her rival to win the auction is more costly than sharing the resale surplus with him.

5.3. Conditions for Resale

For a resale equilibrium, Condition (B) requires p_B (the price up to which bidder B bids anticipating resale) to be greater than p^* (the price at which bidder A drops out of the auction to purchase in the resale market). And because of Lemma 3, if Condition (B) is satisfied, then Condition (A) is satisfied too and, hence, resale takes place in equilibrium. The following proposition clarifies for which values of the borrowing cost and of B's initial wealth this is the case.

Proposition 1. *Bidder A dropping out at price p^* and bidder B bidding up to price $p_B > p^*$ (i.e., a resale equilibrium) is the unique equilibrium in undominated strategies if and only if:*

(i) $v_A > 2w + \frac{\bar{c}}{2}$ and $\beta > \frac{v_B - v_A + 2w - \bar{c}}{v_A - 2w - \frac{\bar{c}}{2}}$, or

(ii) $v_A < 2w + \frac{\bar{c}}{2}$ and $\beta < \frac{v_B - v_A + 2w - \bar{c}}{v_A - 2w - \frac{\bar{c}}{2}}$.

So resale takes place if: (i) bidder B's initial wealth is low compared to v_A, and his borrowing cost is high, or (ii) bidder B's initial wealth is high compared to v_A, and his borrowing cost is low. In both cases, bidder B is prepared to bid up to the price at which bidder A prefers to let him win the auction and obtain the project in the resale market, rather than overbid him.

Notice that the resale equilibrium does not depend on any bidder entering the auction or dropping out of the auction when indifferent about doing so — resale is robust to the addition of both a (small) bidding cost and a (small) cost to trade in the resale market.

Interpretation

In order to buy the project from bidder B in the resale market, bidder A prefers to drop out of the auction at price p^*, instead of bidding up to her expected profit. The reason is that, because of the limited liability effect, after winning at a higher price bidder B has a relatively stronger bargaining position in the resale market.

So resale takes place if bidder B is prepared to bid up to p^* during the auction (i.e., if Condition (B) holds), which depends on how much he can obtain in the resale market. Specifically, bidder B bids up to p^* if his total surplus when he wins the auction at price p^* and then resells the project

— i.e., $\frac{1}{2}\left(\mathbb{E}[\pi_A(p^*)] + \mathbb{E}[\pi_B(p^*)]\right)$, which is his outside option plus half the gains from trade — is greater than zero. Therefore, he is willing to bid aggressively if the joint bidders' surplus in case of resale (i.e., $\mathbb{E}[\pi_A(p^*)]$) and his share of it, which depends on his bargaining power (i.e., his outside option $\mathbb{E}[\pi_B(p^*)]$), are large.

As shown in Proposition 1, resale arises in two opposing cases, that depend on β and w. This is because a high β has two contrasting effects on Condition (B), and the magnitude of these effects depends on w. A high β increases the limited liability effect, since it makes it more likely that bidder B chooses to declare bankruptcy if he owns the project. This induces bidder A to drop out of the auction sooner and, hence, lowers p^*, the auction price in case of resale. And a reduction in p^* increases the joint bidders' surplus and increases B's outside option. Therefore, a high β, through its effect on p^*, makes resale easier, and this is the main effect if w is small. In this case, resale takes place in equilibrium if β is high.[32]

But a high β has also another, direct, effect on bidder B: it reduces B's expected operating profit and, hence, his outside option in the resale market.[33] This effect is larger the larger is w, since a high initial wealth increases bidder B's liability and hence his loss in case of bankruptcy, which makes the potential reduction in B's outside option larger. And for a high w, the direct effect can overcome the positive indirect effect (through p^*) of a high β on resale. In this case, resale takes place in equilibrium if β is low.[34]

6. The Seller's Strategy

If resale is not allowed, or if it does not take place in equilibrium, the natural assumption is that bidder B does not participate in an ascending auction against bidder A because he knows he would lose. Strictly speaking, without resale bidder B is indifferent between participating in the auction or not and, hence, there are also equilibria in which he participates. However, these other equilibria are not robust to the introduction of an arbitrarily small bidding cost. Therefore, in this section we assume bidders have to pay an (arbitrarily small) bidding cost.

On the other hand, in a resale equilibrium bidder A prefers to let bidder B win and then

[32] If bidder B cannot borrow money to finance his bid ($\beta \to \infty$), in order to maximize her resale surplus bidder A drops out of the auction at price w, so that bidder B goes bankrupt for sure (and loses w) if he does not resell. At price $p^* = w$, bidder B's resale surplus is equal to $\frac{1}{2}\left(\mathbb{E}[\pi_A(w)] + \mathbb{E}[\pi_B(w)]\right) = \frac{1}{2}\left(v_A - 2w - \frac{\overline{c}}{2}\right)$, and bidder B finds resale profitable if and only if this surplus is greater than 0, which is consistent with Proposition 1.

[33] Due to this direct effect, the net effect of an increase in β is always to reduce bidder B's outside option, even if it reduces p^* too. Indeed, $\frac{\partial}{\partial\beta}\mathbb{E}[\pi_B(p^*)] < 0$.

[34] But if bidder B is not wealth constrained, resale is never the unique equilibrium because bidder A has no reason to strictly prefer to drop out of the auction (see footnote 4). Indeed, if w is very high, Proposition 1 requires a negative β. (Notice also that Condition (B) is more easily satisfied as β decreases if and only if v_B is large ($v_B > \frac{3}{2}\overline{c}$), because in this case the effect of a change in β on p^* is small and, hence, the indirect effect is small too.)

purchase in the aftermarket. This provides bidder B with an incentive to participate in the auction and bid aggressively, thus raising the auction price. Hence resale raises the seller's revenue without reducing efficiency since, even if the auction is won by the weak bidder, the project is eventually obtained by the bidder with the highest valuation.

The seller can manipulate the conditions under which bidder B finances his bid in order to induce resale after the auction.[35,36] Suppose bidders' borrowing cost is determined by the interest rate exogenously chosen by banks. Then the seller can reduce bidder B's financing cost by committing, before the auction, to finance a bid higher than his wealth with a loan at a borrowing rate lower than the bank interest rate. From Proposition 1, if bidder B has a relatively high initial wealth, this allows him to bid aggressively, hence making resale easier and increasing the competitiveness of the auction.[37]

But, if bidder B has a low initial wealth, the seller should increase his financing cost to increase the gains from trade in the resale market and induce resale in equilibrium. The seller can achieve this by, for example, requiring bidder B to provide additional collateral when bidding more than his wealth. The seller can also relax the condition for resale by reducing bidder B's initial wealth, because this increases the limited liability effect and induces bidder A to bid less aggressively.

However, given that resale takes place, the seller's revenue is given by p^*, the price at which bidder A drops out of the auction. Hence, to increase his revenue in case of resale, the seller should reduce β and increase w, since both these strategies, by reducing the limited liability effect, induce bidder A to drop out of the auction later. But, as argued above, precisely because they increase p^*, these strategies also make it less profitable for bidder B to win the auction and, if pushed too far, may prevent resale. Summing up, we have the following result.

Proposition 2. *When bidder B is going to win the auction and resell the prize, the seller should reduce β and increase w in order to raise the auction price. But if the seller wants to induce resale, then he should reduce w and, depending on bidder B's initial wealth, either reduce or increase β.*

[35] In our analysis, we assume the seller's only available strategy is to change the borrowing cost. This is an extreme assumption. If the seller knows the exact bidders' valuations and can set a reserve price, his optimal strategy is to set a reserve price equal to the strong bidder's value minus the expected operating cost ($v_A - \frac{\bar{c}}{2}$), and obtain the whole bidders' surplus. But, in reality, the seller's information is much more uncertain. Even if the seller only knows the distribution of bidders' valuations, there are perhaps more complex mechanisms, which in theory could extract more of the bidders' surplus. However, in the real world, setting a reserve price is often extremely difficult, and more complex mechanisms are even harder to implement.

[36] Because bidder A never allows bidder B to win the auction at a price lower than $p^* = \frac{v_B - \bar{c}}{1+\beta} + w$, resale never takes place if bidder B cannot bid more than his wealth. Therefore, to make resale possible, the seller should always allow a financially constrained bidder to bid a price higher than his wealth.

[37] Zheng (2001) obtains a similar result in a model without resale. However, in our model committing to lend money to a weak bidder to induce resale entails no cost for the seller because, after resale, it is bidder A who pays the auction price and, hence, bidder B does not actually need to borrow money.

So, contrary to the common intuition, when resale is possible the seller may prefer to worsen the borrowing conditions of a weak bidder or reduce his wealth in order to induce bidders to trade in the resale market.

6.1. No Entry Cost

Contrary to what is assumed in Section 6, assume now that bidder B always enters the auction and bids up to his expected profit because, for example, entry and bidding are both costless. Then bidder B participates in the auction even when resale is not allowed (and, hence, he has no chance of winning). In this case the seller's revenue is not necessarily higher with resale.

Proposition 3. *If bidder B always participates in the auction, resale raises the seller's revenue if and only if $2w(1+\beta) > \bar{c}$.*

Hence, provided bidder B's wealth and the borrowing cost are sufficiently high, resale raises the seller's revenue even if bidding is costless.[38]

7. Auction Price Paid before Resale

We analyze an alternative model in which the auction price is paid in stage (1) — i.e., right after the auction and before resale takes place — and show that the qualitative results of our main model still hold. In particular, we show that bidder A may still want to drop out of the auction at a relatively low price and that resale may take place in equilibrium. The reason is that, if bidder B wins the auction, the price he pays changes his residual wealth and, hence, affects the probability of bankruptcy. Therefore, because of the limited liability effect, bidder A may be in a better bargaining position in the resale market if her opponent wins the auction at a relatively low price.

For simplicity, we assume bidder B cannot pay an auction price higher than his wealth (i.e., borrowing during the auction is not allowed).[39] After winning the auction and paying the auction price, bidder B is left with residual wealth $(w-p)$. If bidder B retains the project and the operating cost is higher than his residual wealth, in order to pay the cost and obtain v_B he has to borrow at cost β per unit of capital. All other assumptions are as in our main model.

The new timing of the game is the following:

[38] A high w reduces the limited liability effect and increases p^*, while its effect on the auction price without resale is more ambiguous. And a high β reduces bidder B's profit and, hence, reduces the price at which bidder B drops out without resale. (A high β reduces p^* too because it increases the limited liability effect, but this effect is mitigated by the borrowing cost effect, which raises p^* for a high β.)

[39] At the end of this section, we discuss how this assumption affects our results.

(1) Bidder i wins the auction and pays the auction price;

(2) Resale can take place;

(3) The operating cost is realized;

(4) The owner of the project, bidder j, can:

  (i) obtain v_j by paying for c out of his residual wealth and, if this is not sufficient, by borrowing at cost β, or

  (ii) declare bankruptcy and liquidate his residual wealth.

Bidder A's operating profit if she owns the project is:

$$\widetilde{\pi}_A(c) = v_A - c,$$

and her expected profit (after paying the auction price) is:

$$\mathbb{E}_c[\widetilde{\pi}_A(c)] = v_A - \mathbb{E}[c].$$

After winning the auction and paying the auction price, bidder B's profit from operating the project is equal to $v_B - c$ if his residual wealth is higher than c and is equal to $v_B - c - \beta[c - (w-p)]$ if his residual wealth is lower than c. However, bidder B declares bankruptcy if his operating profit is lower than $-(w-p)$. Summing up, bidder B's profit from owning the project is:

$$\widetilde{\pi}_B(p,c) = \begin{cases} v_B - c & \text{if } c \leq w-p, \\ v_B - c - \beta(p+c-w) & \text{if } w-p < c < \frac{v_B}{1+\beta} + w - p, \\ -(w-p) & \text{if } \frac{v_B}{1+\beta} + w - p \leq c. \end{cases}$$

Figure 7.1 represents bidder B's profit as a function of c. As in our main model, a wealth constraint has both a borrowing cost effect (which reduces profit on good projects) and a limited liability effect (which reduces losses from bad projects).

Bidder B's expected profit is:

$$\begin{aligned} \mathbb{E}_c[\widetilde{\pi}_B(p,c)] &= \int_0^{w-p} (v_B - c)\, dF(c) + \int_{w-p}^{\frac{v_B}{1+\beta}+w-p} [v_B - c - \beta(p+c-w)]\, dF(c) \\ &\quad - (w-p)\left[1 - F\left(\tfrac{v_B}{1+\beta} + w - p\right)\right]. \end{aligned}$$

In the absence of a wealth constraint, an increase in the auction price has no effect on a bidder's expected profit in stage (2). By contrast, because bidder B faces a wealth constraint:

$$\tfrac{\partial}{\partial p}\mathbb{E}_c[\widetilde{\pi}_B(p)] = -(1+\beta)F\left(\tfrac{v_B}{1+\beta} + w - p\right) + \beta F(w-p)$$

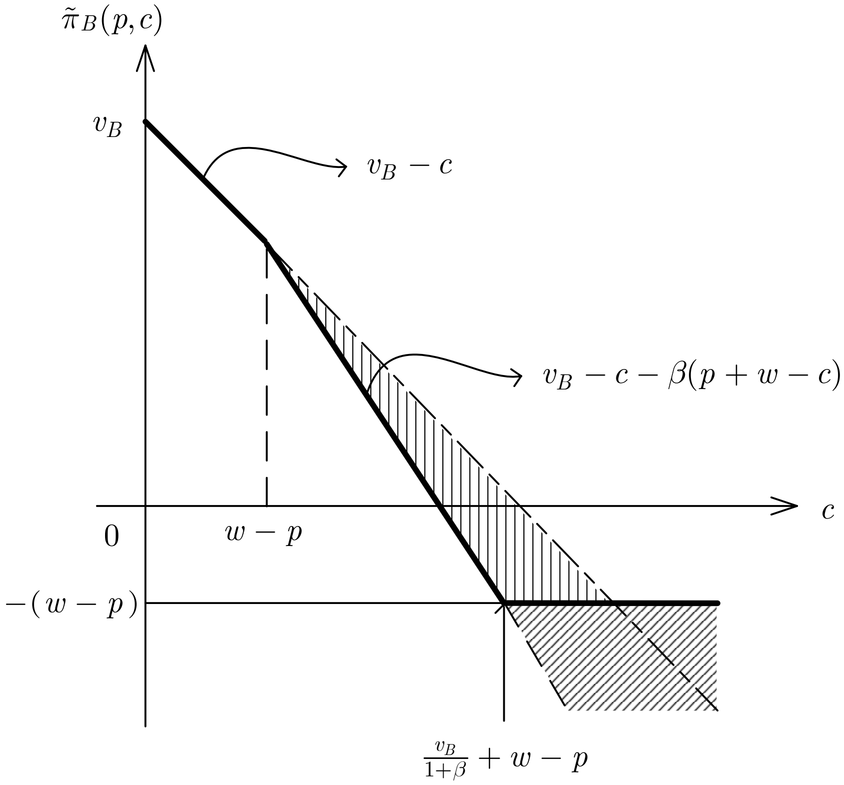

Figure 7.1: Bidder B's profit after paying the auction price.

and, hence:

$$\frac{\partial}{\partial p}\mathbb{E}_c\left[\widetilde{\pi}_B\left(p\right)\right] \leq 0 \quad \Leftrightarrow \quad \beta \cdot \Pr\left[B \text{ borrows}\right] \geq \Pr\left[B \text{ goes bankrupt}\right].$$

Therefore, as in our main model, an increase in the auction price reduces bidder B's expected profit relatively to bidder A's expected profit if and only if the borrowing cost effect is stronger than the limited liability effect.

To obtain a closed form solution we assume that c is drawn from a uniform distribution on $[0, \overline{c}]$, where $\overline{c} > v_B$ for limited liability to matter.[40] To simplify the analysis, we also assume $v_B + w > \overline{c}$, as in our main model. Our results do not hinge on this assumption.

Figure 7.2 represents bidder B's expected profit.[41] Starting from a relatively low price, an increase in the auction price reduces bidder B's expected profit because he has to pay the additional borrowing cost. However, a high auction price also reduces bidder B's residual wealth and, therefore, because of the limited liability effect, starting from a relatively high price, an increase in the auction price increases bidder B's expected profit.

[40] Limited liability is only relevant if bidder B's operating profit (excluding the auction price) can be negative for some realizations of the operating cost.

[41] The details of the analysis of the uniform distribution case are in the appendix.

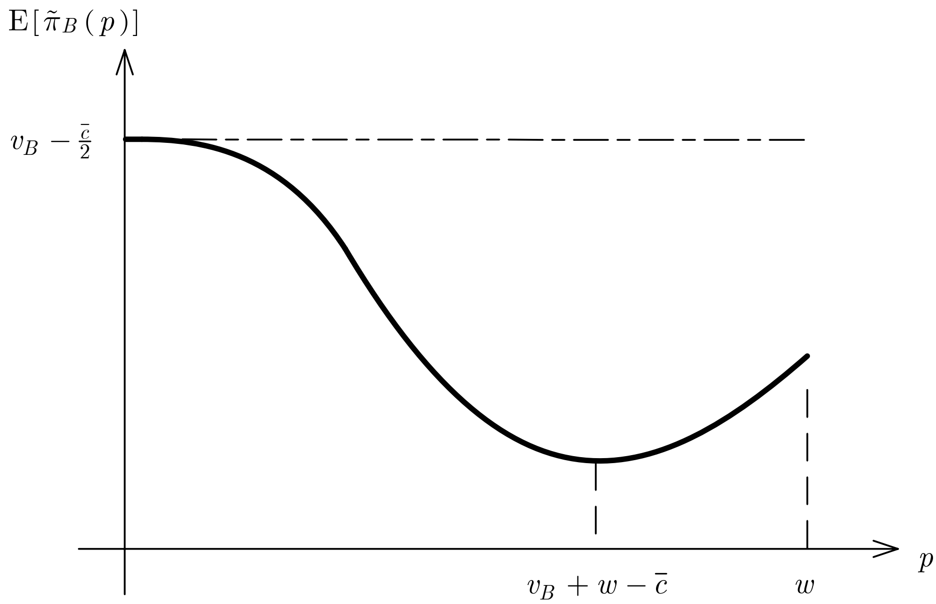

Figure 7.2: Bidder B's expected profit from owning the project, after paying the auction price.

If bidder A buys the project from bidder B, she wants to maximize the resale surplus $\frac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A(c)] - \mathbb{E}_c[\widetilde{\pi}_B(p, c$
Because of the effect of the auction price on bidder B's expected profit, bidder A is not indifferent about the price paid by her opponent in the auction and, in order to buy in the resale market, she drops out of the auction at price:

$$p^* = \arg\max_{p \geq 0} \left(\mathbb{E}_c[\widetilde{\pi}_A(c)] - \mathbb{E}_c[\widetilde{\pi}_B(p, c)]\right) \quad \Leftrightarrow \quad p^* = v_B + w - \bar{c}.$$

Our general point is that a high auction price can harm bidder A in the resale market because of the limited liability effect. When the auction price is paid after resale (as in our main model), a high auction price increases the limited liability effect for bidder B because it raises the total cost of the project $(p + c)$. By contrast, when the auction price is paid before resale, a high auction price increases the limited liability effect for bidder B because it lowers his residual wealth $(w - p)$.

In order to have a resale equilibrium, bidder B must be willing to bid more than p^* and bidder A must prefer to let bidder B win the auction at price p^* and buy in the resale market, rather than win the auction herself at the price at which bidder B drops out, which can be at most equal to w.

Proposition 4. *When the auction price is paid before resale takes place, bidder A dropping out at price p^* and bidder B bidding up to a price greater then p^* is an equilibrium if and only if:*

$$-\left(\bar{c} - v_B\right) < v_A - 2w - \frac{v_B^2 \beta}{2\bar{c}(1+\beta)} < \left(\bar{c} - v_B\right).$$

Therefore, as in our main model, bidder A may strictly prefer to drop out of the auction rather than overbid bidder B. So resale may take place in equilibrium. As in our main model, β and w must be such that ... However, ...

Once the auction price is higher than w, an increase in the auction price does not affect the limited liability effect — because at this point bidder B has no residual wealth anyway.[42] If bidder B can pay a price higher than w, even if bidder A wants to buy in the resale market, she may prefer to raise the auction price above p^* and w (in order to reduce bidder B's expected profit further). In this case, there is still resale equilibrium in which bidder A drops out at price p^* when ...

7.1. Endogeneous Timing

TO BE COMPLETED

8. Extensions

8.1. More Bidders

Our results do not depend on the presence of only two bidders. Assume there are n bidders and let A be the bidder with the highest valuation, B the bidder with the second-highest valuation and C the bidder with the third-highest valuation. For simplicity, assume that bidder A is the only bidder who is not wealth constrained and that all other bidders have the same initial wealth.

During the auction, the weak bidders (i.e., all bidders who do not have the highest value) compete for a chance to resell to the strong bidder (i.e., bidder A). Because a weak bidder's surplus in the resale market is increasing in his valuation, the price at which he drops out of the auction is also increasing in his valuation. Therefore, bidder B overbids all other weak bidders.[43] And, exactly as with only two bidders, provided bidder B is willing to bid up to p^* (i.e., up to the price that maximizes bidder A'a surplus in the aftermarket), bidder A strictly prefers to drop out of the auction and purchase the prize from bidder B.

But because there are other bidders, in a resale equilibrium, bidder B may need to pay more than p^* to win the auction. Specifically, bidder B pays a price equal to the highest of p^* and the price at which bidder C drops out of the auction. Hence, apart from hurting bidder B, competition among weak bidders may also hurt bidder A, because bidder A's surplus in the resale market is maximized when bidder B wins the auction at price p^*. The seller, on the other hand, is better off since the auction price may be higher. However, because all weak bidders other than bidder B

[42] In Figure 7.2, for $p > w$ bidder B's expected profit decrease monotonically to zero as p increases, because at that point the limited liability effect disappears.

[43] If weak bidders have different initial wealths, the bidder who is willing to bid higher may not be the one with the highest valuation. But the rest of our argument remains unchanged.

know they have no chance of winning the auction, if there is an arbitrarily small bidding cost they do not participate in the auction at all, leaving only two active bidders.

Notice also that we assumed bidders cannot merge before the auction, or reach an agreement to prevent the participation of the weak bidder in the auction. With such an agreement, bidders could obtain the project at price zero and share the whole surplus. However, this possibility rests on the assumption that there is no other potential buyer for the project on sale. If more buyers are present, an agreement with a weak bidder does not help the strong bidder, since other competitors would participate and raise the auction price anyway.[44]

8.2. Returning the Prize

Assume a winning bidder can return the project to the seller without paying the operating cost, if the operating cost turns out to be too high and, hence, the project not to be profitable. This, in practice, introduces limited liability for both bidders, regardless of their initial wealth, because bidders can avoid losses on bad projects when the operating profit is negative.

But if bidders are asymmetric — i.e., if v_A is higher than v_B — limited liability becomes relevant at a lower auction price for bidder B than for bidder A (because, for a given auction price, B's profit is lower than A's profit and, hence, he is more likely than bidder A to want to return the project to the seller). So, even if both bidders have infinite wealth, there is a range of prices (in which bidder B may prefer to return the project to the seller, while bidder A does not) where raising the auction price reduces bidder A's expected profit more than it reduces bidder B's expected profit. Therefore, similarly to our model, in order to purchase the project from bidder B, bidder A may prefer to drop out of the auction at a low price to maximize the gains from trade in the resale market.

9. Conclusions

A strong bidder may prefer to drop out of an auction before the price has reached her valuation, and then purchase the prize from a competitor with limited liability in the aftermarket. And the possibility of reselling the prize to a strong bidder makes a weak and wealth-constrained bidder

[44] For example, if in the UK 3G mobile-phone licenses auction the number of bidders had been only one more than the number of licenses on sale, bidders would have probably tried to merge before the auction to reduce competition. But since the number of potential buyers was larger, this was much more difficult. So only after the auction had been run did the marginal loser take over Orange. On the other hand, before other 3G auctions that were run later (e.g., the Netherlands auction), the number of potential buyers shrank and bidders merged before the auction, reducing competition and the seller's revenue.

willing to participate in the auction and to bid aggressively. So resale can take place in equilibrium, and a weak bidder can win an auction even against a clearly advantaged competitor.

Thanks to resale, competition in the auction can be greater and the price higher. The seller can induce resale by reducing weak bidders' initial wealth and by manipulating the conditions at which they can finance their bids, sometimes even worsening these conditions. But when resale is going to take place, in order to increase the auction price, the seller should improve the borrowing conditions and increase the initial wealth of weak bidders, provided this does not prevent resale.

When European governments auctioned 3G mobile-phone licenses, it was not clear whether winners would be allowed to resell the license they acquired. In many countries resale was explicitly forbidden.[45] This may have induced new entrants, who had a lower valuation of the licenses than incumbents, to bid less aggressively, and may even have discouraged them from participating at all. Indeed, many 3G auctions lacked sufficient competition and generated disappointingly low revenues. Our analysis suggests that, by allowing winners to resell their licenses, governments could have encouraged new entrants and small bidders to participate and bid aggressively, without affecting the efficient allocation of the spectrum. Moreover, by committing to finance new entrants' bids, governments could have induced incumbents to bid aggressively, even if they expected to purchase in the resale market.

Our broader point is that there may be a variety of reasons why bargaining in the aftermarket can be affected by the price paid by the auction winner. When this occurs, a strong bidder is not indifferent about the price paid by her competitor during the auction, and may prefer to drop out of the auction in order to purchase the prize in the resale market. We explored some of these reasons, and leave to further research the analysis of more of them.

[45] Even if direct resale is not allowed, a losing bidder can perhaps still obtain the auction's prize by taking over the winner. But this is clearly more problematic than just acquiring the prize.

Appendix: Proofs

Proof of Lemma 1. Suppose bidder B wins the auction at price p. His expected profit depends on the relation between p, c and w, and on whether he may choose to declare bankruptcy when the auction price is lower than his initial wealth.

When $\frac{v_B}{1+\beta} > \overline{c}$, $w < \frac{v_B}{1+\beta} + w - \overline{c}$ and, hence, bidder B never declares bankruptcy after winning the auction at a price lower than w. Then there are five cases. Firstly, if $p + \overline{c} < w$, then B has enough wealth to pay the total cost of the project, regardless of the realized operating cost. In this case (like for a bidder without a wealth constraint) his expected profit is:

$$\mathbb{E}_c[\pi_B(p,c)] = v_B - p - \tfrac{\overline{c}}{2}.$$

Secondly, if $w - \overline{c} < p < w$, bidder B has to pay the additional borrowing cost if the total cost is high (i.e., for $c + p > w$). Moreover, in this case B does not declare bankruptcy and always pays the whole total cost, since his operating profit is never lower than $-w$. Therefore, letting $F(c)$ be the operating cost's distribution function, in this case B's expected profit is:

$$\begin{aligned}\mathbb{E}_c[\pi_B(p,c)] &= \int_0^{w-p} [v_B - (p+c)]\, dF(c) + \int_{w-p}^{\overline{c}} [v_B + \beta w - (1+\beta)(p+c)]\, dF(c) \\ &= v_B - (1+\beta)\left(p + \tfrac{\overline{c}}{2}\right) + \beta w - \tfrac{\beta(w-p)^2}{2\overline{c}}.\end{aligned}$$

Thirdly, if $w < p < \frac{v_B}{1+\beta} + w - \overline{c}$, bidder B always pays the additional borrowing since the auction price is higher than his wealth, and he never declares bankruptcy since his operating profit is still never lower than $-w$. Therefore, in this case B's expected profit is:

$$\begin{aligned}\mathbb{E}_c[\pi_B(p,c)] &= \int_0^{\overline{c}} [v_B + \beta w - (1+\beta)(p+c)]\, dF(c) \\ &= v_B - (1+\beta)\left(p + \tfrac{\overline{c}}{2}\right) + \beta w.\end{aligned}$$

Fourthly, if $\frac{v_B}{1+\beta} + w - \overline{c} < p < \frac{v_B}{1+\beta} + w$, bidder B pays the additional borrowing cost, but he declares bankruptcy and limits his actual loss to w when the operating profit is lower than $-w$ (i.e., for $c > \frac{v_B}{1+\beta} + w - p$). Therefore, in this case B's expected profit is:

$$\begin{aligned}\mathbb{E}_c[\pi_B(p,c)] &= \int_0^{w-p+\frac{v_B}{1+\beta}} [v_B + \beta w - (1+\beta)(p+c)]\, dF(c) - \int_{w-p+\frac{v_B}{1+\beta}}^{\overline{c}} w\, dF(c) \\ &= \tfrac{1}{2\overline{c}(1+\beta)} [v_B - (1+\beta)(p-w)]^2 - w.\end{aligned}$$

Finally, if $p > \frac{v_B}{1+\beta} + w$, bidder B declares bankruptcy and loses the initial wealth regardless of the operating cost and, hence, his expected (and actual) profit is $-w$. Rearranging the five cases yields the result. ■

Proof of Lemma 2. The value that bidder B attaches to winning the auction is given by the total surplus he can obtain in the resale market, that is, the resale price. In an ascending auction, it is a weakly dominant strategy for bidder B to bid up to the maximum price he is happy to pay. Because with resale bidder B does not pay the auction price, it is a weakly dominant strategy for him to bid up to a price p_B such that the resale price is equal to zero — i.e., such that $\mathbb{E}_c[\pi_A(p_B)] + \mathbb{E}_c[\pi_B(p_B)] = 0$.

On the other hand, when resale is not allowed, if bidder B participates in the auction, he bids up to his expected profit from operating the project, i.e., up to $\widehat{p}$ such that $\mathbb{E}_c[\pi_B(\widehat{p})] = 0$.[46] Since by assumption $\mathbb{E}_c[\pi_A(p_B)] > \mathbb{E}_c[\pi_B(p_B)]$, $\mathbb{E}_c[\pi_B(p_B)]$ must be less than zero. And because $\partial\mathbb{E}_c[\pi_B(p)]/\partial p \leq 0$, it follows that $p_B > \widehat{p}$. ■

Proof of Lemma 3. We prove the result for an arbitrary division of the resale surplus among bidders. Assume bidder B obtains a share k of the gains from trade in the resale market and bidder A obtains a share $(1-k)$, for $0 < k < 1$.

Bidder A can win the auction at the price at which bidder B drops out. This is price p_B such that the resale price is equal to 0, i.e.:

$$k\left(\mathbb{E}_c[\pi_A(p_B)] - \mathbb{E}_c[\pi_B(p_B)]\right) + \mathbb{E}_c[\pi_B(p_B)] = 0$$
$$\Leftrightarrow \quad \mathbb{E}_c[\pi_A(p_B)] = -\frac{1-k}{k}\mathbb{E}_c[\pi_B(p_B)].$$

If, on the other hand, bidder A drops out at price p_B and purchases the project from bidder B, she obtains a surplus equal to:

$$\begin{aligned} S_A(p_B) &= (1-k)\left(\mathbb{E}_c[\pi_A(p_B)] - \mathbb{E}_c[\pi_B(p_B)]\right) \\ &= (1-k)\left(\mathbb{E}_c[\pi_A(p_B)] + \frac{k}{1-k}\mathbb{E}_c[\pi_A(p_B)]\right) \\ &= \mathbb{E}_c[\pi_A(p_B)]. \end{aligned}$$

Therefore, at price p_B, bidder A is exactly indifferent between winning the auction and purchasing in the aftermarket.

Because, by definition, p^* is the price that maximizes bidder A's resale surplus, at any price higher than p^* bidder A's resale surplus is lower than at price p^*. Hence, if $p_B > p^*$ (i.e., if Condition (B) is satisfied):

$$S_A(p^*) > S_A(p_B) = \mathbb{E}_c[\pi_A(p_B)],$$

and so bidder A strictly prefers to let bidder B win at price p^* and purchase in the resale market, rather than win the auction at price p_B.

On the contrary, if $p_B < p^*$, bidder A has no incentive to drop out of the auction and purchase in the resale market. The reason is that bidder A strictly prefers to win the auction at price p_B, rather than purchase in the resale market after letting bidder B win the auction at a price lower than p_B; and bidder A is indifferent between winning the auction at price p_B and buying in the resale market, after letting bidder B win the auction at price p_B. ■

Proof of Proposition 1. By Lemma 3, bidder A drops out of the auction at price p^* and resale is an equilibrium if Condition (B) is satisfied — i.e., if bidder B is willing to bid more than p^*.

[46] In the working paper version of this article we explicitly derive $\widehat{p}$ and show that, in an auction without resale, bidders with low wealth bid more aggressively than bidders with high wealth because of the limited liability effect, provided the borrowing cost is sufficiently low. This confirms the results obtained by Zheng (2001) for first-price auctions.

Since bidders' expected profits are decreasing in the auction price, this is true if and only if bidder B's surplus if he wins the auction at price p^* and resells the project is positive, that is:

$$
\begin{aligned}
& \mathbb{E}_c[\pi_A(p^*)] + \mathbb{E}_c[\pi_B(p^*)] > 0 \\
\Leftrightarrow \quad & \left\{ v_A - p - \tfrac{\overline{c}}{2} + \tfrac{1}{2\overline{c}(1+\beta)} \left[v_B - (1+\beta)(p-w) \right]^2 - w \right\} \Big|_{p=\frac{v_B-\overline{c}}{1+\beta}+w} > 0 \\
\Leftrightarrow \quad & v_A > \frac{v_B - \overline{c}}{1+\beta} + \frac{\overline{c}\beta}{2(1+\beta)} + 2w \\
\Leftrightarrow \quad & \left(v_A - \tfrac{\overline{c}}{2} - 2w \right) \beta > v_B - \overline{c} - v_A + 2w \\
\Leftrightarrow \quad & \begin{cases} \beta > \frac{v_B - \frac{3}{2}\overline{c} - \left(v_A - \frac{\overline{c}}{2} - 2w\right)}{v_A - \frac{\overline{c}}{2} - 2w} & \text{if } v_A - \frac{\overline{c}}{2} - 2w > 0 \\ \beta < \frac{v_B - \frac{3}{2}\overline{c} - \left(v_A - \frac{\overline{c}}{2} - 2w\right)}{v_A - \frac{\overline{c}}{2} - 2w} & \text{if } v_A - \frac{\overline{c}}{2} - 2w < 0. \end{cases}
\end{aligned}
$$

Rearranging yields the conditions in the statement.

It is straightforward to check that, under these conditions, a resale equilibrium is the only equilibrium in undominated strategies. On the other hand, when the conditions in the statement are not satisfied (i.e., when $p^* > p_B$), bidder A bidding up to p^* and bidder B dropping out at p_B is an equilibrium without resale in undominated strategies.

(The condition on v_A has an intuitive interpretation. If v_A is sufficiently larger than v_B and w, then the gains from trade that bidders can obtain in the resale market are large and bidder B's loss if he declares bankruptcy is small. Therefore, because of the limited liability effect, bidder A prefers to drop out of the auction relatively early in order to purchase the project from bidder B, and bidder B can obtain a high surplus from bidding aggressively and reselling to bidder A, and hence is willing to do so. Notice also that, if $v_A > 2w + \frac{\overline{c}}{2} + \frac{v_B}{1+\beta}$, the gains from trade in the resale market can be so high that bidder B is even willing to bid up to a price such that, if he wins the auction and keeps the project, he has to declare bankruptcy for sure, since at a price higher than $\frac{v_B}{1+\beta} + w$ his operating profit is always lower that $-w$.) ■

Proof of Proposition 3. When resale takes place the auction price is p^*. Without resale, bidder B drops out of the auction at price $\widehat{p}$ such that $\mathbb{E}_c[\pi_B(\widehat{p})] = 0$. Therefore, the auction price and the seller's revenue are higher with resale if and only if $p^* > \widehat{p}$, that is:

$$
\mathbb{E}_c[\pi_B(p^*)] < 0 \quad \Leftrightarrow \quad 2w(1+\beta) > \overline{c}.
$$

In this case, without resale bidder B drops out of the auction before the price has reached p^*. ■

Analysis of the Case $\frac{v_B}{1+\beta} < \overline{c}$.

Consider bidder B's expected profit from the project when $\frac{v_B}{1+\beta} < \overline{c}$. Compared to the case $\frac{v_B}{1+\beta} > \overline{c}$, the only difference is that bidder B may now declare bankruptcy also after winning the auction at a price lower than w.

Lemma 4. *If $\frac{v_B}{1+\beta} < \overline{c}$, after winning the auction at price p, the expected profit of bidder B is:*

$$\mathbb{E}_c[\pi_B(p,c)] = \begin{cases} v_B - \left(p + \frac{\overline{c}}{2}\right) & \text{for } p < w - \overline{c}, \\ v_B - (1+\beta)\left(p + \frac{\overline{c}}{2}\right) + \beta w - \frac{\beta(w-p)^2}{2\overline{c}} & \text{for } w - \overline{c} \le p < \frac{v_B}{1+\beta} + w - \overline{c}, \\ \frac{v_B}{\overline{c}}(w-p) + \frac{(w-p)^2}{2\overline{c}} + \frac{v_B^2}{2\overline{c}(1+\beta)} - w & \text{for } \frac{v_B}{1+\beta} + w - \overline{c} \le p < w, \\ \frac{1}{2\overline{c}(1+\beta)}\left[v_B - (1+\beta)(p-w)\right]^2 - w & \text{for } w \le p < \frac{v_B}{1+\beta} + w, \\ -w & \text{for } \frac{v_B}{1+\beta} + w \le p. \end{cases}$$

Proof. Suppose that $\frac{v_B}{1+\beta} < \overline{c}$. It follows that $w > \frac{v_B}{1+\beta} + w - \overline{c}$ and, hence, bidder B may choose to declare bankruptcy after winning the auction at a price lower than w. Now, for $\frac{v_B}{1+\beta} + w - \overline{c} < p < w$, in order to obtain v_B bidder B pays the additional borrowing cost if the total cost is higher than his wealth, but, at the same time, his loss is limited to w when the total cost is very high (i.e., when $p + c > w + \frac{v_B}{1+\beta}$). Therefore, in this case B's expected profit is:

$$\begin{aligned} \mathbb{E}_c[\pi_B(p,c)] &= \int_0^{w-p} \left[v_B - (p+c)\right] dF(c) + \\ &\quad \int_{w-p}^{w-p+\frac{v_B}{1+\beta}} \left[v_B + \beta w - (1+\beta)(p+c)\right] dF(c) - \int_{w-p+\frac{v_B}{1+\beta}}^{\overline{c}} w\,dF(c) \\ &= \frac{v_B}{\overline{c}}(w-p) + \frac{(w-p)^2}{2\overline{c}} + \frac{v_B^2}{2\overline{c}(1+\beta)} - w. \end{aligned}$$

The computations for the other cases are identical to those for $\frac{v_B}{1+\beta} > \overline{c}$. ■

Bidder B's expected profit function is (weakly) decreasing in p. It is straightforward to check that (exactly as in the case $\frac{v_B}{1+\beta} > \overline{c}$):

$$\left|\frac{\partial \mathbb{E}_c\left[\pi_B(p)\right]}{\partial p}\right| \ge 1 \quad \Leftrightarrow \quad p \le \frac{v_B - \overline{c}}{1+\beta} + w.$$

Therefore, in order to buy the project in the resale market, bidder A still prefers to drop out of the auction at price $p^* \equiv \frac{v_B - \overline{c}}{1+\beta} + w$. And the rest of our analysis remains unchanged.

Lemma 5. *After winning the auction and paying the auction price p, in stage (2) the expected profit of bidder B is:*

$$\mathbb{E}_c[\widetilde{\pi}_B(p,c)] = \begin{cases} v_B - \frac{\overline{c}}{2} & \text{for } p < w - \overline{c}, \\ v_B - \frac{(1+\beta)\overline{c}}{2} - \beta(w-p) - \frac{1}{2\overline{c}}\beta(w-p)^2 & \text{for } w - \overline{c} \le p < \frac{v_B}{1+\beta} + w - \overline{c}, \\ \frac{1}{2\overline{c}}\left[(w-p)^2 - 2(w-p)(\overline{c} - v_B) + \frac{v_B^2}{1+\beta}\right] & \text{for } \frac{v_B}{1+\beta} + w - \overline{c} \le p < w. \end{cases}$$

Proof of Lemma 5. Suppose bidder B wins the auction at price p. His expected profit depends on whether he may choose to declare bankruptcy and, hence, on the relation between the realized operating cost c and his residual wealth $(w-p)$.

Firstly, if $w-p>\overline{c}$, bidder B has enough residual wealth to pay for any realized cost. In this case (like for a bidder without a wealth constraint) his expected profit is $v_B-\frac{\overline{c}}{2}$. Secondly, if $\overline{c}-\frac{v_B}{1+\beta}\leq w-p<\overline{c}$, bidder B has to pay the additional borrowing cost β if the operating cost is high (i.e., for $c>w-p$). Moreover, B never declares bankruptcy because his operating profit is always higher than $-(w-p)$. Therefore, letting $F(c)$ be the operating cost's distribution function, in this case bidder B's expected profit is:

$$\begin{aligned}\mathbb{E}_c[\widetilde{\pi}_B(p,c)] &= \int_0^{w-p}(v_B-c)\,dF(c)+\int_{w-p}^{\overline{c}}[v_B-c-\beta(p+c-w)]\,dF(c)\\ &= v_B-\tfrac{(1+\beta)\overline{c}}{2}-\beta(w-p)-\frac{1}{2\overline{c}}\beta(w-p)^2.\end{aligned}$$

Thirdly, if $0\leq w-p<\overline{c}-\frac{v_B}{1+\beta}$, bidder B pays the borrowing cost if c is high, but he declares bankruptcy and limits his actual loss to his residual wealth when the operating profit is lower than $-(w-p)$ (i.e., for $v_B-c-\beta(p+c-w)<p-w$). Therefore, in this case bidder B's expected profit is:

$$\begin{aligned}\mathbb{E}_c[\widetilde{\pi}_B(p,c)] &= \int_0^{w-p}(v_B-c)\,dF(c)+\\ &\quad\int_{w-p}^{\frac{v_B}{1+\beta}+w-p}[v_B-c-\beta(p+c-w)]\,dF(c)-\int_{\frac{v_B}{1+\beta}+w-p}^{\overline{c}}(w-p)\,dF(c)\\ &= \tfrac{(w-p)^2}{2\overline{c}}-\tfrac{(w-p)(\overline{c}-v_B)}{2\overline{c}}+\tfrac{v_B^2}{2\overline{c}(1+\beta)}.\end{aligned}$$

Finally, by assumption bidder B cannot bid more than w. ■

Proof of Proposition 4. After bidder B wins the auction at price p^*, the resale price is equal to:

$$\tfrac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A]+\mathbb{E}_c[\widetilde{\pi}_B(p^*)]\right)=\tfrac{1}{2}\left[v_A+v_B-\overline{c}-\tfrac{v_B^2\beta}{2\overline{c}(1+\beta)}\right].$$

This is the lowest price that bidder A can pay to buy the project in the resale market.

There are two possible cases in which there is a resale equilibrium: (i) bidder B bids up to a price higher the p^* and lower than w and bidder A prefers to drop out at price p^*; (ii) bidder B bids w and bidder A prefers to drop out at price p^*.

Firstly, bidder B bids up to a price higher the p^* and lower than w if and only if: (a) the resale price at which he can resell the project after winning the auction at price p^* is higher than p^*, and (b) the resale price at which he can resell the project after winning the auction at price w is higher than w, i.e.:

$$\begin{aligned} &\begin{cases}\tfrac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A]+\mathbb{E}_c[\widetilde{\pi}_B(p^*)]\right)>p^*\\ \tfrac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A]+\mathbb{E}_c[\widetilde{\pi}_B(w)]\right)<w\end{cases}\\ \Leftrightarrow &\begin{cases}\tfrac{1}{2}\left[v_A+v_B-\overline{c}-\tfrac{v_B^2\beta}{2\overline{c}(1+\beta)}\right]>v_B+w-\overline{c}\\ \tfrac{1}{2}\left[v_A-\tfrac{\overline{c}}{2}-\tfrac{v_B^2}{2\overline{c}(1+\beta)}\right]<w\end{cases}\\ \Leftrightarrow\ &2v_B-\tfrac{3}{2}\overline{c}+\tfrac{(\overline{c}-v_B)}{2\overline{c}}<v_A-2w+\tfrac{v_B^2}{2\overline{c}(1+\beta)}<\tfrac{1}{2}\overline{c}.\end{aligned}$$

In this case, bidder B bids up to a price p_B such that $\frac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A]+\mathbb{E}_c[\widetilde{\pi}_B(p_B)]\right)=p_B$. It follows that, at price p_B, bidder A is indifferent between winning the auction and dropping out to buy in the resale market. And since $\frac{\partial}{\partial p}\mathbb{E}_c\left[\widetilde{\pi}_B(p)\right]>0$ for $p>p^*$, bidder A strictly prefers to drop out at price p^* rather than win the auction at price p_B.

Secondly, bidder B bids up to w if and only if:

$$\tfrac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A]+\mathbb{E}_c[\widetilde{\pi}_B(w)]\right)>w \quad\Leftrightarrow\quad v_A+\tfrac{v_B^2}{2\overline{c}(1+\beta)}>2w+\tfrac{\overline{c}}{2}.$$

In this case, bidder A prefers to drop out of the auction at price p^* and buy in the resale market if and only if the resale price is lower than w, i.e.:

$$\tfrac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A]+\mathbb{E}_c[\widetilde{\pi}_B(p^*)]\right)>w \quad\Leftrightarrow\quad v_A+v_B<2w+\overline{c}+\tfrac{v_B^2\beta}{2\overline{c}(1+\beta)}.$$

Notice that, if bidder B is happy to win the auction at price w, then he is also happy to win at price p^* since:

$$\tfrac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A]+\mathbb{E}_c[\widetilde{\pi}_B(w)]\right)>w \quad\Rightarrow\quad \tfrac{1}{2}\left(\mathbb{E}_c[\widetilde{\pi}_A]+\mathbb{E}_c[\widetilde{\pi}_B(p^*)]\right)>p^*.$$

Summing up, there is a resale equilibrium in which bidder B bids w and bidder A prefers to drop out at price p^* if and only if:

$$\tfrac{1}{2}\overline{c}<v_A-2w+\tfrac{v_B^2}{2\overline{c}(1+\beta)}<\tfrac{1}{2}\overline{c}+\tfrac{(\overline{c}-v_B)}{2\overline{c}}.$$

Rearranging conditions ??? and ??? yields the result. ■

Indeed:

$$\frac{\partial}{\partial p}\mathbb{E}_c\left[\widetilde{\pi}_B(p,c)\right]=\begin{cases}0 & \text{for } p<w-\overline{c},\\ -\beta+\frac{\beta(w-p)}{\overline{c}} & \text{for } w-\overline{c}\le p<\frac{v_B}{1+\beta}+w-\overline{c},\\ \frac{1}{\overline{c}}\left(p-w+\overline{c}-v_B\right) & \text{for } \frac{v_B}{1+\beta}+w-\overline{c}\le p<w.\end{cases}$$

For $p<\frac{v_B}{1+\beta}+w-\overline{c}$ the expected profit function is concave in p, while for $p>\frac{v_B}{1+\beta}+w-\overline{c}$ it is convex.

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