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# Optimal Rate Base Reviews With Price-Cap Regulation

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## Abstract

This paper demonstrates, in a dynamic model of monopoly regulation with price-cap, that a periodical price review may increase productive efficiency. When the firm's choice of cost-reducing effort depends on the output supplied, a revision allows the regulator to set more binding prices thus inducing the monopolist to exert more cost-reducing effort in the future. In a continuous-time setting, the model obtains the optimal timing for the review from a cost-efficiency point of view and the conditions under which, within a given concession period, a single full rate base review improves cost-efficiency. We find that a rate base review may be optimal on pure cost grounds, depending on the length of the concession period in relation to the slope of the demand function and the intensity of the disutility of effort. This result adds a theoretical argument in favour of the practice of periodical reviews in price-cap regulation and a basis for calculation of the optimal regulatory lag.

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## 1. Introduction\*

Building on the “Arrow effect” (Arrow 1962), some authors have argued that, in a natural monopoly static environment, price regulation may not only improve allocative efficiency but also productive efficiency: whenever the regulated price is binding for the monopolist, then effort supplied by a regulated firm must also be larger than the monopolist effort (Cabral and Riordan 1989 and Clementz 1991; and also, in a context of optimal regulation, Laffont and Tirole 1986). The intuition for this is that if effort reduces marginal cost then the benefit of supplying effort for the firm will be larger the more output it produces. Starting from this basic result Coco and De Vincenti (2004) found conditions under which price-cap regulation strictly increases productive efficiency, in a two-period model characterised by repeated choice of the effort by the firm and by a permanent effect of the effort on the cost function. Coco and De Vincenti (2004) subsequently used the Arrow effect in order to discuss the relative merits of purely fixed price schemes and periodical rate base reviews<sup>1</sup>. The received wisdom on this topic runs like this: revising the base to set a new cap reduces incentives to cost-reduction because it reduces the time horizon over which the firm appropriates the benefits of cost reduction; on the other side a revision is necessary to redistribute gains from cost-reduction to consumers and to achieve allocative efficiency (Green and Rodriguez Pardina 1999, Ch. 4). Thus the longer the regulatory lag the better the incentive properties and the worse the distributive and allocative properties of the regulatory scheme (see Armstrong and Sappington 2003 and Armstrong, Rees and Vickers 1995). In Coco and De Vincenti (2004) instead we showed, in a two period framework, that a partial rate base review at the beginning of the second period induces the firm to supply more effort in that period, due to the ‘Arrow effect’. Of course, in deciding whether to review prices, the regulator has to balance these positive effects on the future levels of effort, with the adverse consequences of the review on the current level of effort. We found that a (partial) review is beneficial on pure cost-efficiency grounds whenever the elasticity of demand exceeds a certain threshold value.

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<sup>1</sup> The issue of the frequency of the rate base change was already signalled by Acton and Vogelsang (1989) in the Introduction to a Symposium on price-cap regulation in the *RAND Journal of Economics*, as one of the most important on which to focus in forthcoming research. The amount of work devoted to the issue, however, has been disappointing.

This paper develops the idea in a more general continuous-time setting, in order to obtain not only conditions under which a full rate base review improves cost-efficiency but also to establish the optimal timing for a rate base review within a given concession period from a cost-efficiency point of view. We find that a rate base review may be optimal on pure cost grounds, depending mainly on the length of the concession period in relation to some parameters of the model. Notably the revision is more likely to be optimal, the higher the slope of the (direct) demand function and the lower the intensity of the disutility of effort. In our simplified environment, the optimal timing for a (single) rate base review within a concession period of length  $T$  is  $T/2$ .

## 2. The model

The case we discuss is one of a regulated natural monopoly for which a concession for a period of length  $T$  has been granted. We assume a single-product firm with a production function characterised by constant returns to scale for any given level of its effort, and by a permanent effect of effort on the marginal cost. In this sense the model best describes a situation where a manager has to decide whether to invest on the upgrading of productive processes. Once the investment has been made the marginal cost is permanently lower. We assume that time is a continuous variable and that the marginal cost at time  $t$ ,  $c_t$ , is a function of total effort  $S_t$  spent by the firm in the interval  $[0, t]$ :  $c_t = c(S_t)$ , with  $c'(\cdot) < 0$  and  $c''(\cdot) \geq 0$ . Total effort  $S_t$  is assumed continuous and differentiable on the concession period  $[0, T]$ . The firm is supposed to be risk neutral and to maximise profits, net of the disutility from supplying cost-reducing effort at every point in time,  $\varphi(s_t)$ , where  $s_t$  is the derivative  $\dot{S}_t$  of total effort function  $S_t$  at time  $t$ . The disutility of effort is assumed increasing and convex in  $s_t$ . The effort spent is strictly sector-specific and hence its disutility is a sunk cost for the firm. The regulator knows the demand function and sets prices for the concession period  $T$ . He does not know the firm's disutility function  $\varphi(s_t)$ ; moreover he cannot directly observe the effort exerted by the firm but at every point in time he observes the cumulated reduction in the cost due to the total effort  $S_t$  spent by the firm in the interval  $[0, t]$ . Suppose also that the regulator can credibly commit itself to an *ex-ante* specified pricing pattern for the entire concession period, thanks to an appropriate institutional framework and/or to its reputation. Costs

of renegeing on a specified pattern of price regulation (reputation loss) are sufficiently high to rule out this possibility. Hence we can focus on the optimal ex-ante price regulation strategy for the regulator. We will in particular focus on the choice to implement a rate base review.

Assuming a demand function  $y(p)$  constant over time and, for simplicity, the absence of any stochastic shock on demand function and cost function, the firm maximises the following utility function:

$$U = \int_0^T [p_t y(p_t) - c(S_t) y(p_t) - \varphi(s_t)] \cdot e^{-\rho t} dt \quad (1)$$

where  $\rho$  is the time discount rate.

### 2.1. The pure fixed price regime

Under the no-review regime (that is pure fixed price regime with  $p_t = p_0 \forall t$ ), the firm maximizes  $U$  by choosing the optimal level of effort over the whole length of the concession period. To further simplify the problem we assume that:

$$c(S_t) = k - S_t \quad (2)$$

$$[\varphi(s_t)] = \frac{a}{2} s_t^2 \quad (2')$$

Hence  $c'(\cdot) = -1$ ,  $[\varphi'(s_t)] = a s_t$ , and  $[\varphi''(s_t)] = a$ . Denoting with  $y_0$  the quantity demanded for the fixed price  $p_0$ , the resulting conditions for maximization (Euler equation plus initial condition  $S_0 = 0$  and transversality condition for a fixed-time-horizon problem) are:

$$\begin{cases} a\dot{s}_t - (a s_t)\rho = -y_0; \\ S_0 = 0; \\ -(a s_T) e^{-\rho T} = 0; \end{cases} \quad (3)$$

Integrating two times the first condition and substituting for the other two, we find the following effort spent by the firm in the interval  $[0, t]$ :

$$S_t = \frac{y_0}{a\rho} \left[ t + \frac{1}{\rho} e^{-\rho T} - \frac{1}{\rho} e^{-\rho(T-t)} \right] \quad (4)$$

The total effort supplied by the firm over the whole concession period will be:

$$S_T = \frac{y_0}{a\rho} \left[ T + \frac{1}{\rho} (e^{-\rho T} - 1) \right] \quad (5)$$

Equations (4) and (5) find again the “Arrow effect” in our dynamic optimization framework: the higher the quantity  $y_0$ , the greater is the effort spent by the firm because the benefit of supplying effort for the firm is larger the more output it produces. Therefore, the more the regulated price is binding for the monopolist, the higher the productive efficiency which is reached by the firm. Of course, the lower the intensity  $a$  of the disutility of effort and the lower the rate  $\rho$  which discounts the benefits of effort, the greater is the level of effort chosen by the firm.

Moreover, as expected, the effort  $S_t$  spent by the firm until time  $t$  is increasing and concave in  $T$ . Hence the intuition and common view given in the introduction is confirmed in our dynamic optimization framework: the longer the concession period - equivalent in a fixed price regime to the regulatory period - the larger the effort spent by the firm. It is interesting nonetheless that the gains from a longer period are decreasing in  $T$ .

## 2.2. *The rate base review regime*

The result that a longer period leads to higher effort overlooks the possible gains from a price review which occurs at some point  $t_1$  during the concession period. By reducing price in the light of the reduction which has been obtained in the cost until  $t_1$ , the review may boost the firm’s optimal effort thanks to the increase in the quantity produced by the firm (and in the benefit of its effort) from  $t_1$  onwards. To investigate this possibility we keep  $T$  constant, and divide it in two sub-periods,  $[0, t_1]$  and  $[t_1, T]$ , where at  $t_1$  the price review occurs. We assume that the derivative  $s_t$  of total effort  $S_t$  is continuous on each sub-interval  $[0, t_1]$  and  $[t_1, T]$ , and admits two finite limits at  $t_1$  one from the left and the other from the right<sup>2</sup>. Moreover, we will assume that for the first regulatory period  $[0, t_1]$  the regulator sets a price equal to the initial marginal cost, that is  $p_0 = k$ , and for the second period  $[t_1, T]$  a price equal to the marginal cost at time  $t_1$ , that is  $p_1 = k - S_{t_1}$  (full rate base review). Therefore, the firm knows that the profits obtainable by spending effort in the first regulatory period will be entirely offset by the price review at time  $t_1$ .

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<sup>2</sup> In mathematical terms, we are assuming that  $S_t$  is a piecewise-smooth function of time: it is continuous and differentiable on  $[0, T]$  and its derivative  $\dot{S}_t$  is piecewise-continuous on  $[0, T]$ .

The optimal solutions for the total effort supplied by the firm over the first and the second sub-period have the same shape of equation (5), but the periods' lengths are now respectively  $t_1$  and  $T - t_1$  and the quantities produced in the two sub-periods are  $y_0 = y(p_0)$  and  $y_1 = y(p_1)$ . Hence:

$$S_{t_1} = \frac{y_0}{a\rho} \left[ t_1 + \frac{1}{\rho} (e^{-\rho t_1} - 1) \right] \quad (6)$$

$$S_{T-t_1} = \frac{y_1}{a\rho} \left[ T - t_1 + \frac{1}{\rho} (e^{-\rho(T-t_1)} - 1) \right] \quad (6')$$

and the total effort over the whole concession period under the review regime is:

$$S_T^R = S_{t_1} + S_{T-t_1} \quad (6'')$$

### 2.3. Optimal rate base review

To make manageable the problem of comparing the total effort  $S_T^R$  under the review regime with the total effort  $S_T$  under the fixed price regime, we introduce now some additional simplifying assumptions. Firstly, we assume  $\rho = 0$ , that is the firm does not discount the future. The set of conditions (3) simplifies to:

$$\begin{cases} a\dot{s}_t = -y_0; \\ S_0 = 0; \\ -as_T = 0; \end{cases} \quad (7)$$

As with the more general case, in order to find the optimal path for  $S_t$ , we integrate two times the first condition and substitute for the others to find:

$$S_t = \frac{y_0}{a} t(T - t/2) \quad (8)$$

In a regime with fixed price for the whole concession period, the overall effort spent at the end of the period is therefore:

$$S_T = \frac{y_0}{2a} T^2 \quad (9)$$

When we consider the possibility of a rate base review at time  $t_1$ , the total effort has to be computed as a sum of effort exerted before the review and after. From the equivalent of condition (9) we find effort supplied by the firm over the first and the second sub-period:

$$S_{t_1} = \frac{y_0}{2a} t_1^2 \quad (10)$$

$$S_{T-t_1} = \frac{y_1}{2a} (T - t_1)^2 \quad (10')$$

Of course, total effort over the whole concession period under the review regime is:

$$S_T^R = S_{t_1} + S_{T-t_1} \quad (10'')$$

Suppose now that the demand has a standard linear form,  $y = \alpha - \beta p$ . In the case of a full rate base review, where  $p_0 = k$  and  $p_1 = k - S_{t_1}$ , it follows necessarily that  $y_1 = y_0 + \beta S_{t_1}$ . Substituting for  $y_1$  in (10'), the overall effort exerted in the review regime according to equation (10'') is:

$$S_T^R = \frac{y_0}{2a} t_1^2 + \frac{[y_0 + \beta S_{t_1}]}{2a} (T - t_1)^2 = \frac{y_0}{2a} [t_1^2 + (T - t_1)^2] + \frac{\beta S_{t_1}}{2a} (T - t_1)^2 \quad (11)$$

Note that the first term in the right hand side of (11) bears some similarity with (9), but is necessarily smaller. The difference between the two,  $L = \frac{y_0}{a} t_1 (T - t_1)$ , can be interpreted as the lower effort exerted for the break-up of the concession period in two regulatory periods due uniquely to the shorter horizon over which the firm maximizes. The second term on the right hand side on the contrary,  $G = \frac{\beta S_{t_1}}{2a} (T - t_1)^2$ , is the gain in terms of effort due to the increase in output,  $\beta S_{t_1}$ , linked to the price review. Whether a price review is beneficial for the overall effort exerted during the concession period depends on the net balance of these two effects. Hence it is beneficial depending on the difference  $\Delta = S_T^R - S_T = G - L \stackrel{>}{<} 0$ . Substituting equation (10) in  $G$ , this condition in turn reduces to:

$$\Delta = \left[ \frac{\beta}{4a} t_1 (T - t_1) - 1 \right] \cdot \left[ \frac{y_0}{a} t_1 (T - t_1) \right] \stackrel{>}{<} 0 \quad (12)$$

Studying (12) we can easily check that:

1) Both terms in square brackets are concave in  $t_1$  and reach a maximum at  $t_1 = T/2$ , so that also the difference  $\Delta$  reaches a maximum at  $t_1 = T/2$ . Therefore, the optimal timing for a rate base review is the middle of the concession period<sup>3</sup>.

2) The second term in square brackets is always positive for  $0 < t_1 < T$ , so that the sign of  $\Delta$  depends on the sign of the first term. Given the result found under 1) and substituting  $t_1 = T/2$  in (12), the first term in square brackets turns out to be positive - so that a price review is beneficial for the overall effort exerted during the concession period - when:

$$T > 4\sqrt{\frac{a}{\beta}} \quad (13)$$

Condition (13) specifies the minimum length of the concession period that makes a review worthwhile for cost-efficiency: a price review at time  $t_1 = T/2$  is beneficial for the overall effort exerted by the firm when  $T > 4\sqrt{a/\beta}$ , whereas a pure fixed price regime is better when  $T < 4\sqrt{a/\beta}$ . The news is that, when the concession period gets over a certain threshold, a rate base review is beneficial in order to improve not only allocative but also productive efficiency. From (13) we can also infer that the concession length which makes the review beneficial depends on the slope  $\beta$  of the (direct) demand curve and on the intensity  $a$  of the disutility of effort: the steeper the demand curve and the lower the intensity of effort's disutility, the shorter turns out to be the length of the concession period that makes a review worthwhile for cost-efficiency.

3) More in general, as condition (12) points out, the difference  $\Delta$  between the gain and the loss of the review in terms of effort is an increasing function of  $\beta$  and a decreasing function of  $a$ .

3.1) In particular the review is more beneficial the steeper the (direct) demand curve. This is fairly simple to explain. The gains from the review are due to the increase in quantity supplied following the price revision. The larger the output increase, the larger the ensuing incentives for cost reduction for the firm. For a given price revision, the boost in output is determined uniquely by the slope of the (direct) demand curve. The steeper the demand curve the larger the output gain. This result as well, calling ultimately for more frequent price revision in markets where demand is sensitive to price conditions, is at odds with the conventional wisdom. Distributional concerns usually are invoked to call for stricter (ie more adherent to cost conditions) regulation of natural

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<sup>3</sup> Obviously this discussion overlooks the possibility of multiple reviews (see below the conclusion section).

monopolies with low elasticity of demand. Our argument for the opposite policy recipe, however, is entirely based on productive efficiency considerations.

3.2) The review is more beneficial also if the parameter  $a$ , which represents both the intensity and convexity of the disutility of effort, is lower. To understand why, we need to look at the  $G$  and  $L$  functions again. Both the loss and the gain functions depend negatively on  $a$  for the standard effect of the disutility of effort on effort itself. But the gain function depends also on the output gain,  $\beta S_{t_1}$ , that is itself adversely related to the intensity of disutility of effort. Indeed the price review will be influenced by the cost savings realized up to  $t_1$ , hence is negatively related to  $a$  (see eqn. (10)).

4) Once the concession length  $T$  satisfies condition (13), the difference  $\Delta$  between  $G$  and  $L$  proves to be increasing in  $T$ . Hence the longer the concession period the more advantageous a price review. This is not surprising once we observe that the gain  $G$  from the review grows faster than the corresponding loss  $L^4$ . Still the result is far from obvious since it states that the longer a concession (regulatory) period, the more grounded the argument for rate base review not only on allocative and distributional grounds but even on pure cost-efficiency grounds.

### 3. Conclusion

This paper demonstrates, in a dynamic model of monopoly regulation with price-cap, that a periodical price review may increase productive efficiency. Our result adds a theoretical argument in favour of the practice of periodical reviews in price-cap regulation and a basis for calculation of the optimal regulatory lag for cost-efficiency purposes. The basic idea is that a revision allows the regulator to set more binding prices thus inducing the monopolist to exert more cost-reducing effort in the future. Therefore, in order to set the optimal regulatory lag, the regulator has to balance the expected costs which arise not only from allocative but also from productive inefficiency due to high prices with the well known adverse consequences on the current level of effort deriving from the rate base review. The price review, far from being a pure instrument to

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<sup>4</sup> To check, simply substitute in  $L$  and  $G$  for  $t_1 = T/2$  and for  $S_{t_1}$ , differentiate them and take  $T > 4\sqrt{a/\beta}$  into account.

achieve allocative efficiency at the cost of moulding incentives to cost reduction, may itself perform a role in increasing productive efficiency.

We find, in a simplified environment, that the optimal timing for a single price review is in the middle of the concession period. Our discussion overlooks the possibility of multiple reviews. The optimal number of reviews is an excellent topic for further work.

The review is found to be beneficial for productive efficiency when the concession length exceeds a certain threshold, whereas a pure fixed price regime is preferable for a shorter concession. The concession length which makes the review beneficial is an increasing function of the slope of the (direct) demand curve and a decreasing function of the intensity of the disutility of effort. In particular a price review is more beneficial the longer the concession period. This result complies with and reinforces an already commonly held view. The underlying argument however was up to now based only on the usual trade-off between allocative and productive efficiency.

The review is more beneficial the more reactive the demand to prices and the lower the intensity of the disutility of effort. In both cases the reason must be found in the fact that the relative advantages of a review depend, in this model, on the output gain it causes. The output gain itself depends entirely on:

- a) The cost savings realized before the review, which are inversely related to the intensity of effort. This entails that a price review is more beneficial when the unobservable disutility borne by the firm for cost reduction is relatively low. Hence more frequent reviews are preferable when incentives motives are less binding.
- b) The measure in which those cost savings, through the price review, boost output, hence the slope of the (direct) demand function. The consequent policy recipe would call for more frequent reviews in markets with higher reactivity of demand to prices.

Therefore a smaller intensity of effort's disutility and a larger reactivity of demand to prices, both increase the output gain consequent to the price review and finally boost incentives for cost reduction after the review. While the first result generally conforms comfortably with the conventional wisdom, the second result is quite at odds with it. It is nonetheless useful to remind once again that our result focuses only on the effect of the review on cost-efficiency. Hence the result under b) can be interpreted, in a more general picture, as stating that while more frequent reviews are mainly beneficial for allocative efficiency and distributional purposes in monopolistic markets with low elasticity of demand, they are more beneficial for productive efficiency reasons in monopolistic markets with a more sensitive demand to price conditions.

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