OPTIMAL COPAYMENT STRATEGIES IN A PUBLIC HEALTH CARE SYSTEM

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Abstract

Health care usually represents a so called merit good, i.e. a good whose consumption should be promoted and given that in most cases it might be essential to restore health or to stop its decay, most countries have implemented a public health care system where care is supplied to anybody needing it for free and its cost is borne by all the community. Inappropriate use of health care. To curb this expenditure waiting lists and copayments have been introduced in the public health care system.

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Key Words: Copayment, treatment, tax and expenditure ceiling

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1 Introduction

When a market works according to the rules of perfect competition there is no need for regulation because a first best allocation of resources, where consumers maximise their utility and firms their profits, is reached automatically. Prices on these markets reflect the marginal benefit from consumption and the equilibrium is found at a point where the marginal cost of production is equal to marginal utility of consumption. This condition also implies zero profit for the firms and cost minimisation. The markets for health care is not a perfect competitive structure for several reasons related to the ignorance and irrationality of consumers that cannot evaluate the marginal benefit of the service, to the high proportion of cost in R&D which means that the price is usually much higher than the cost of production.

Health care usually represents a so called merit good, i.e. a good whose consumption should be promoted and given that in most cases it might be essential to restore health or to stop its decay, most countries have implemented a public health care system where care is supplied to anybody needing it for free and its cost is borne by all the community. The rules for allocating such a cost are quite variegated, ranging from pure social insurance to pure tax instruments to mixed health care markets where a range of services are financed by the public health care system and for everything else the consumer can either pay out of pocket or stipulate a supplementary health insurance contract.

This means that in most countries a substantial proportion of health care is financed by the public sector and since the first oil crisis, the objective to rationalise and control expenditure has become a priority for any effective government policy. Improved efficiency in health care has been the main objective of the reforms proposed and implemented in Europe in the last few decades with the creation of internal markets for health care and with the adoption of other form of regulation both on the price of health services and on the cost borne by the consumer.

In particular, it has been observed that free access to health care might create an inappropriate use of health care. To curb this expenditure waiting lists and copayments have been introduced in the public health care system.

Waiting list create a sort of implicit price for health care (Martin and Smith, 1999) that can be used to improve the distributional effects of health care or to induce the richest to address their demand to the private health care system, with a positive effect on public health care expenditure and a double dividend in terms of income distribution (Hoel and Saeter, 2003;
Copayments have been introduced as an instrument to rationalise the use of the service, but in some cases they produce a substantial revenue that can be used to finance health care or to reduce the level of taxation or for equity reasons. The supplementary payment is usually means tested and this means that the richest are made pay more for the service they receive.

As a general rule, copayments are used for services whose cost is relatively low and for services whose consumption is likely to be repeated through time. The most interesting example is pharmaceutical drugs for which several countries have a form of copayment, either in a lump-sum form or as a share of the full price of the drug.

Copayments have been extensively studied from an empirical point of view to understand to what extent they reduce the consumption of specific health care services while only few papers have attempted to study them from a theoretical point of view. These exceptions are represented by Jelovac (2002), Smith (2005), Ma and Riordan (2002).

Jelovac studies the relationship between copayment and price setting in the market for prescribing drugs while Ma and Riordan study the problem of moral hazard in the presence of full insurance against health risks. In both cases the demand depends on the copayment paid: in the first case the service can be considered non essential while in the second paper copayment is specifically used to reduce moral hazard. Smith considers instead the problem of using copayment in third world countries where the tax revenue is not sufficient to match the cost.

The aim of our paper is to study copayment from a theoretical point of view, as a policy instrument that can be used to improve equity in the access and finance of a public health care system. For this reason, we assume that the demand for health care is appropriate, i.e. the care required is all necessary to restore health. In our approach copayment can then be seen as an additional tax instrument that the policy maker can use to maximise the welfare of the community it represents.

Our model shows that if there is no ceiling on expenditure and tax revenue, the copayment should be used only if the treatment offered is not so effective as to restore health care completely. In this case, if the tax system is optimal the copayment is independent of the income of the receiver and it is proportional to the health that cannot be restored.

If a ceiling exists on expenditure or on tax revenue, the copayment should be used as an extra source of income to increase the provision of health care.

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1 See, for example Van Der Vilt (2001) and Lexchin, 2004
The alternative use of the revenue to reduce the overall tax bill is not optimal from a welfare point of view and it might be complicated to be implemented.

2 The model

The model used in this paper represents a very stylized economy where the decision maker has to determine the optimal copayment system for health care. To simplify the model we assume that each individual has a fixed, uniform probability of being ill and if so it needs to receive a uniform treatment whose price is determined outside the model. To start with, we assume that only public health care exists, i.e. the consumer can receive care only by the public sector. A form of general taxation is the general instrument used to finance health care that in the model is assumed to be the only source of public expenditure, but it can be topped up by a form of copayment on the use of health care. The income of each consumer is fixed, but health dependent.

The policymaker, which is assumed to be a benevolent regulator, has to find the optimal tax and copayment mix in this framework.

2.1 The environment

The community is made of up of \( N \) individuals, normalised to one. Each of them has a fixed probability \( p \) of being ill and if so he can benefit from a specific health care treatment. To simplify the model, we assume that one unit of treatment is sufficient to treat the patient. The distribution of income is assumed to follow a known distribution in the support \((0; Y)\) with \( f(Y) \) representing the density and \( F(Y) \) the cumulative distribution.

Consumers are assumed to be utility maximisers. The literature often assumes that the utility of consumers depends on the quantity of a composite commodity they can buy and on the health care they receive when sick. Utility is then assumed to be additive in \( c \) and \( h \) and to respond to the usual axioms of utility theory. Given that our model deals with the financial side of health care rather than on demand determination, we simplify the model by assuming that utility depends on the consumption of a composite good that is equal to their income, but it is state contingent, i.e. it depends on whether the consumer will be healthy or sick.

Several modelling approaches are possible in this context. Health might be assumed to be a productivity parameter that allows the consumer to increase his income. In this case his income (hence the consumption of the
composite commodity) would be equal to \( Y_i \) but it would decrease to \( \delta Y_i \) ill. The expected utility would then be written as:

\[
U_i = (1 - p)U(Y_i) + pU(\delta Y_i)
\]

Alternatively, we can think that health care allows the consumer to derive an higher level of satisfaction from the activities he does. In this case, we can think of health are as a productivity parameter as concerns utility and the expected utility of the individual can be written as:

\[
U_i = (1 - p)U(Y_i) + p\delta U(Y_i)
\]

For our analysis the main difference between the two approaches are the implication for health care finance. In the first case, the consumer suffers a decrease in his income as a consequence of his ailment which induces a reduction in the tax revenue. In the second case, income (hence the tax revenue) is independent of the health state. For the time being and to keep the problem as simple as possibile, the second approach will be used. The treatment can improve the productivity (in terms of income or utility) of each individual by restoring a fraction \( \epsilon \leq \delta \) of the lost health care. His expected utility after receiving care will then be written as:

\[
U_i = (1 - p)U(Y_i) + p\alpha U(Y_i)
\]

### 2.2 Optimal tax and subsidy

We model a public health care system that supplies care in exchange for the payment of a fraction of its cost \( (s_i \text{ : the copayment or user charge}) \) and finances expenditure through general taxation. We study the problem from the point of view of a benevolent regulator that has to decide how to finance the provision of this treatment. We assume that it can use a mix of two polices: an income tax \( t(Y) \) and a copayment \( s(Y) \). To finance the provision of health care the revenue raised by using both instrument must be at least equal to the cost of health care. If we assume that each treatment has a fixed cost \( c \) the total amount that is necessary to raise is equal to \( E = pc \) and the budget constraint can be written as:

\[
\int_0^{\bar{Y}} t(Y) f(Y)dY + p \int_0^{\bar{Y}} s(Y) f(Y)dY = E
\]
It is interesting to note that while taxation is compulsory, the copayment is optional, i.e. the patient, if offered the possibility, accepts to receive care only if it improves his utility. In formal terms we can write that:

\[
U(Y - t(Y) - s(Y)) \geq \delta U(Y - t(Y)) \\
\]
\[
s(Y) \leq Y - t(Y) - U^{-1}(\delta U(Y - t(Y)))
\]

The problem for the benevolent regulator, if we assume that there is no deadweight loss from taxation, can be written as:

\[
\text{Max } W = \int_0^{\tilde{y}} (1 - p)U(Y - t(Y)) + p\alpha U(Y - t(Y) - s(Y))f(Y)dY \quad (3)
\]
\[
s.t. \\
\int_0^{\tilde{y}} t(Y) f(Y)dY + p \int_0^{\tilde{y}} s(Y) f(Y)dY = E \\
s(Y) \leq Y - t(Y) - U^{-1}(\delta U(Y - t(Y)))
\]

From the solution of this problem we can derive the following proposition:

**Proposition 1** *If there is no ceiling on expenditure, i.e. if the resources that can be raised through taxation are sufficient to pay for the overall provision, the optimal copayment is positive unless the treatment can restore health completely, i.e \( \epsilon = (1 - \delta) \). Its shape depends on the tax rate. If the taxation is optimal, the copayment is equal for all the individuals and depends on the loss of utility caused by the health that cannot be restored.*

**Proof.** See appendix one □

This result might seem counterintuitive; in a model like this one where income is fixed and independent of the health status, one might expect the consumer whose expected utility decreases because of bad health and the inability of the treatment to restore it completely should receive a compensation. Proposition one shows instead a sort of risk sharing property of the model, i.e. if the treatment is not perfectly effective, those falling ill are asked to pay a fraction of the cost of care so to compensate (through a tax
reduction) those that are not ill and have still to contribute to the cost of a
treatment that is not very effective. This is the reason why if the tax instru-
ment is chosen optimally the copayment is independent of income. When
the tax rate is not chosen optimally, the interpretation of \( s(Y) \) becomes more
complicated since the instrument is used to improve income redistribution.
In this latter case the redistributive objective might become predominant so
that the copayment might lose its primary objective of being a risk sharing
instrument.

2.3 Optimal copayment with expenditure or tax revenue ceil-
ings

In the actual health care systems, copayments are seldom used in an optimal
cost. One of the main problems that arise in the real economic systems
is that, for a series of reasons, it is not possible to implement an optimal
tax schedule. Furthermore, given the increase that the public sector has
experienced in the past few years, the need to reduce and rationalise public
expenditure has become a priority of any effective government policy. This
implies the use of instruments such as ceiling on public expenditure and (or)
tax revenue. In this context the copayment can be used as a form to increase
the resources or to improve the equity of the system. Which of the two roles
it plays, it depends on the use that the Central Government decides it is
optimal for the revenue from the copayment.

This result is quite standard in the optimal taxation literature. If there
are no constraints, the tax instrument is sufficient to reach any distribution
and equity objective. Such an instrument is however quite costly: from
an economic point of view, taxation creates important deadweight losses
and it might not be sustained from a political point of view. For these
reasons, a ceiling on the revenue that can be raised through taxation might
be imposed. In our model we assume that the policy maker fixes a ceiling
on the public expenditure for drugs to \( B < E \). In this case only a fraction

\[ \rho = \frac{B}{E} \]

of patients can be cured and since there is no private alternative in
this model, those falling ill and not receiving care will suffer a permanent
loss in their utility. Given that we want to study the effect of ceilings on the
optimal copayment schedule, it will be assumed that the treatment offered is
perfectly effective, i.e. health care is completely restored\(^2\). From proposition
1 we know that in this case the copayment should not be used; we can then

\(^2\)This implies that \( \epsilon = (1 - \delta) \)
isolate the effects of this instrument in the presence of financial constraint from those deriving from risk sharing on the effectiveness of the treatment.

### 2.3.1 Total expenditure constraint

Let us start by assuming that $\rho$ is fixed, i.e. in equilibrium demand will be rationed since the number of treatments asked is higher than what can be supplied with the budget. In this case, if a private market alternative does not exist, each individual is now confronted with a probability $p(1 - \rho)$ of not being cured which means a loss of utility equal to $p(1 - \rho)(1 - \delta)U(Y)$ and the welfare utility becomes:

$$W = \int_0^Y (1 - p)U(Y - t(Y)) + ppU(Y - t(Y)) + p(1 - \rho)\delta U(Y - t(Y))f(Y)dY$$

(4)

This result can be improved upon by introducing a copayment $s(Y)$ whose revenue is used to reduce the total tax bill. In this case the copayment is introduced for equity reasons. Those benefitting from health care pay an higher price that reduces the difference in utility between being cured and not receiving health care and expected utility also increases because the tax bill is reduced. As before, the copayment is optional, i.e. the consumer, if offered the possibility, receives care only if it improves his utility.

The problem for Central government is as before to find a tax scheduled and a copayment scheduled so to maximise:

$$\text{Max } W = \int_0^Y (1 - p)U(Y - t(Y)) + ppU(Y - t(Y) - s(Y)) + p(1 - \rho)\delta U(Y - t(Y))f(Y)dY$$

(5)

$$s.t. \int_0^Y t(Y) f(Y)dY + pp \int_0^Y s(Y) f(Y)dY = B$$

$$s(Y) \leq Y - t(Y) - U^{-1}(\delta U(Y - t(Y)))$$

In appendix two it is shown that the optimal condition can be written as:
\[
\frac{[(1 - p) + p(1 - \rho)\delta]U'(Y - t(Y))}{p\rho U'(Y - t(Y) - s(Y))} = \frac{1 - p\rho}{p\rho}
\] (6)

which can be interpreted as follows: the right hand side is the ratio between the expected utility for not being cured (either because healthy or because the treatment was not offered) and the expected utility from health care while the left hand side is the ratio of the probability of the two event. This optimal condition can then be interpreted in terms of marginal rate of substitution between being cured or not and their virtual price that in this model is represented by the probability of the two events.

If the tax bill is optimal, the copayment is fixed and independent of income; if this is not the case, the copayment can be used partially used to improve income redistribution.

The function \(s(Y)\) is positive with \(s(Y) > 0\). To show this we can rewrite (6) as:

\[
\frac{U'(Y - t(Y))}{U'(Y - t(Y) - s(Y))} = \frac{1 - p\rho}{(1 - p) + p(1 - \rho)\delta} > 1
\]

which implies \(s(Y)\) positive. In this latter case, which represent the norm in the actual health care systems, the copayment loses its equity function and becomes a second best instrument for income distribution.

### 2.3.2 Tax revenue constraint

If a ceiling exists on the tax revenue instead of on expenditure, the revenue from copayment can be used to increase the number of treatment that the public sector is able to supply. In this case \(r\) is no longer fixed and depends on the revenue that can be raised using the copayment:

\[
\rho = \frac{B + p\rho \int_{\tilde{Y}} s(Y) f(Y) dY}{E}
\]

\[
\rho = \frac{E}{B}
\]

\[
E - p \int_{\tilde{Y}} s(Y) f(Y) dY
\]

As before, the copayment is optional, i.e. the consumer, if offered the possibility, receives care only if it improves his utility. This means that:
\[ U(Y - t(Y) - s(Y)) \geq \delta U(Y - t(Y)) \]
\[ s(Y) \leq Y - t(Y) - U^{-1}(\delta U(Y - t(Y)) \]

from which we can derive the following function:

\[ \int_{0}^{\hat{Y}} s^{\text{MAX}}(Y) f(Y) dY = Z \]

The two constraints have a different interpretation. The first one, as before represents the individual participation constraint and has to be met for any income class. The second one instead represents the maximum revenue that can be raised using copayment. From (7) it follows that if \( Z < E - B \), it will never be possible to provide health care to all those in need.

When a tax ceiling exists, the problem faced by the policy maker can be written as.

\[ \text{Max} \int_{0}^{\hat{Y}} (1 - p)U(Y - t(Y)) + p\rho U(Y - t(Y) - s(Y)) + p(1 - \rho)\delta U(Y - t(Y)) f(Y) dY \]

\[ s.t. \]
\[ \rho = \frac{B}{E - p \int_{0}^{\hat{Y}} s(Y) f(Y) dY} \]
\[ \int_{0}^{\hat{Y}} s^{\text{MAX}}(Y) f(Y) dY = Z \]
\[ \rho \leq 1 \]
\[ s(Y) \leq Y - t(Y) - U^{-1}(\delta U(Y - t(Y)) \]

From the solution of this problem we can derive the following proposition:

**Proposition 2** If the revenue that can be raised through copayment is sufficient to finance the provision that the tax revenue cannot meet, it is always
optimal to use the revenue to increase the provision of health care. In this case, the copayment can be used to improve income redistribution, provided the individual constraints are met. If the revenue is not sufficient, the best Central Government can do is to impose a copayment according to the function $s^{\text{max}}$.

**Proof.** see appendix 3. ■

Proposition 2 shows that if the public sector cannot meet demand completely, the best policy would be to impose a ceiling on the tax revenue rather than on expenditure. In the latter case in fact it is possible to use the copayment to increase the number of treatment offered and this policy is preferred to use such a revenue to reduce the tax burden.

## 3 A numerical simulation

In the previous sections we have shown that copayment and tax bills have a strict relationship and that the objectives that can be reached using the latter instrument can be quite different according to the type of tax structure. For this reason we will be now presenting a numerical simulation of the theoretical results just presented which also shows how the nature of the copayment changes in the presence of tax and expenditure ceilings.

Our community is made of 100 individuals, each of them falling into one of the following 10 income brackets:
The average and the median income are to equal to 30 and 35 respectively. This distribution might well represent income in an industrialised country. In order to avoid problems relating to negative utility, the income distribution does not start from Y=10, but to an income level which is 1/10 the highest income. \( p \), the probability of falling ill, is equal to 0.6 and the cost for each treatment is assumed to be equal to 5. so that \( E = 0.6 \times 5 \times 100 = 300 \). If ill, the consumer experiences a reduction utility equal to \( \delta = 0.8 \); when the treatment is not perfectly effective it allows to increase the productivity of health by 0,1 so that \( \alpha = 0.9 \). His utility function is assumed to be log linear.

The first case we present is the optimal tax and copayment system when there are no ceilings on expenditure and tax revenue. For a generic consumer \( i \) his expected utility can be written as:

\[
U_i = (1 - p)U(Y_i - t(Y_i)) + p\alpha U(Y_i - t(Y_i) - s_i)
\]
Given that the optimal copayment depends on the tax scheduled, two assumption have been made on the latter:

- an optimal tax bill, which for the present model means that income after tax is equal for all income brackets;
- a linear income tax which has been used here as an example of non optimal taxation.

For both cases the optimal copayment scheduled and the individual participation constraint has been evaluated. The results are presented in figure two.

In both cases the copayment is positive as the theoretical model would predict. If the tax rate is optimal, the optimal copayment is constant and equal to $3.3810828$ which is exactly equal to 9.
When the tax rate is not optimal, the copayment increases the income redistribution by making copayment increase with income. In this case the copayment is anyway below the individual participation constraint. Welfare is maximum for the first case as might expect; in this case the use of copayment improves on a model where only the tax instrument is used (welfare increases from 328,38 to 328,516). For the linear tax case welfare decreases to 316,44. This first model shows the role of copayment from an optimal point of view: it should be used as an instrument to improve the risk sharing properties relating to health loss. However, when the tax schedule is not optimal, the copayment changes its nature and becomes a second best tool to redistribute income.

Let us now turn to the case where specific ceilings exist on tax and revenue. To start with, we examine the case where a ceiling exist on tax revenue. In particular we assume that $B = 225$ so that without copayment $\rho = 0, 75$.

In this example, the tax schedule is given and we assume two different shapes:

- a linear income tax $t$;
- an income tax proportional to the square of income, i.e. $t(Y_i) = tY^2$.

The individual participation constraints and the maximum amount that can be raised using copayment depend on the tax schedule. For a linear income tax, $Z$ is equal to 1008,8 while for the other case it is equal to1006,6 which means that in both cases $\rho = 1$. The optimal copayment schedule can then be evaluated using optimal control theory; in this case, given that our income distribution is discrete it has been evaluated using the optimisation routine of Maple. The results are presented in figure 3.
The individual participation constraint is always satisfied and the copayment is here used as an instrument to improve the distribution of income. For the linear case, the first six income brackets do not pay any user charges while if the taxation is proportional to the square of the income it is only the first five income brackets that receive treatment for free and the schedule is in general less steep.

Copayment is a second best instrument: this can be shown by comparing the total welfare which is equal to 332.56 for the linear case and 333.44 for the quadratic one.

If the revenue from copayment is used to reduced the income tax, the numerical simulations show the difficulty in using this approach. In this case the only way to use copayment as an equity instrument would be to
apply an optimal income tax, which in this case would mean to have also negative bills for the people in the lowest income brackets. However, given that in this model we have assumed that there is a ceiling on expenditure, i.e. on the policy maker intervention, it has to be assumed that the tax schedule is not optimal and in particular that the tax bill should not be negative. In this case, the numerical simulations show that the best policy to be implemented is to use copayment instead of taxes to finance health care.

As in the previous case, the optimal copayment schedule has been evaluated for a linear and a quadratic income tax. In both case the optimal $t$ is equal to zero and the copayment raises all the income necessary to finance health care. This is shown in figure four.

Figure four: Expenditure constraint, $\rho$ fixed
In both cases, the use of a copayment system increases welfare, but in this latter case we have a value of 327 which is lower than when the copayment is used for increasing the provision of health care.

4 Conclusions

Most industrialised countries foresee the provision of health care using a public health care system where the most important source of finance is general taxation, each individual has the same access to the service and the service is usually (almost) free at the point of use. Most system of health care financing currently includes elements of income redistribution, i.e. the provision of health care and its finance is used as an in-kind transfer to the poor. From the financial side there are two further instruments that can be used to improve such a distribution, namely means tested copayments and delay. Delay can be used in this context as an incentive for the individuals in the higher brackets of the income distribution to seek private provision. In this way, the still contribute to the public health care system with their income tax bill, but they do not use the service and end up paying twice for the service.

Another instrument is copayment by which it is meant that a fraction of the cost of treatment has to be met by the patient at the point of use. The use of copayment has often been advocated for efficiency reasons, i.e. to avoid the unappropriate use of health care, but such an instrument is now so much widespread to represent an important source of finance to the health care system.

In this paper we have studied its role in a context where all the health care demanded is appropriate. We have shown that in the context of pure public health care market, the optimal role of a copayment system would be to share the cost among those falling ill and the rest of the community of a treatment that is not perfectly effective.

In the presence of other constraints, such as ceilings on tax and/or expenditure, the copayment changes its role and becomes more a tool to redistribute income or to raise extra resource. In general and especially from the point of view of the actual implementation, it would be best to use the revenue to increase the provision of health care rather than to reduce the tax bill. In the latter case, in fact, the copayment becomes an income redistribution instrument and the policy maker has to fix a lower bound for the tax revenue anyway.
The model presented so far does not consider a private market alternative which creates from one side more restrictions to the action of the policy maker, but that might also create opportunities to improve welfare and income redistribution.

Appendix 1

\[
\begin{align*}
\text{Max} & \int_0^Y \left[ (1-p)U(Y - t(Y)) + paU(Y - t(Y) - s(Y)) \right] f(Y) dY \\
\text{s.t.} & \int_0^Y t(Y)f(Y) dY + p \int_0^Y s(Y)f(Y) dY = E
\end{align*}
\]

Computing the Fréchet derivatives of the associated Lagrangean we get the following FOC system:

\[
\begin{align*}
0 &= \frac{\partial L}{\partial s} = -pa \int_0^Y U'(Y - t(Y) - s(Y))v(Y)f(Y)dY + \lambda \int_0^Y f(Y)v(Y)dY \\
0 &= \frac{\partial L}{\partial t} = -(1-p) \int_0^Y U'(Y - t(Y))v(Y)f(Y)dY \\
&\quad -pa \int_0^Y U'(Y - t(Y) - s(Y))v(Y)f(Y)dY + \lambda \int_0^Y f(Y)v(Y)dY
\end{align*}
\]

for any test function \(v\). From the first equation we get

\[
\alpha U'(Y - t(Y) - s(Y)) = \lambda
\]

and substituting in the second one we obtain

\[
(1-p)U'(Y - t(Y)) + paU'(Y - t(Y) - s(Y)) = \alpha U(Y - t(Y) - s(Y))
\]

that is

\[
U'(Y - t(Y)) = \alpha U'(Y - t(Y) - s(Y)). \quad (8)
\]
Now, since $\alpha < 1$ and $U'$ is strictly decreasing we must have $Y - t(Y) > Y - t(Y) - s(Y)$, so that $s(Y) > 0$ for all $Y$. In fact, using the invertibility of $U'$ we obtain from (8) a rather explicit formulation of $s$

$$s(Y) = Y - t(Y) - (U')^{-1}\left[\frac{1}{\alpha}U'(Y - t(Y))\right].$$

(9)

Let us specialize the result to some particular cases. Suppose for example that $U$ satisfies the following condition $U(aY) = a^sU(Y)$ for all $a > 0$ and some non-negative $s$. Then since $U'(aY) = a^{s-1}U'(Y)$ we have

$$(U')^{-1}\left[\frac{1}{\alpha}U'(Y - t(Y))\right] = (U')^{-1}(U'((Y - t(Y))\alpha^{-1/(s-1)})) = (Y - t(Y))\alpha^{-1/(s-1)}$$

Then if for example $U(Y) = \log Y$ from the above equation and (9) we get

$$s(Y) = (1 - \alpha)(Y - t(Y)).$$

Then the expenditure constraint can be written as

$$\int_0^{Y} t(Y)f(Y)dY = \frac{E - p(1 - \alpha)Y_T}{1 - p + \alpha p},$$

where $Y_T$ is the total population revenue. If we then restrict the taxation to be linear i.e. $t(Y) = tY$, from the above equation we get the optimal $t$

$$t = \frac{E - p(1 - \alpha)Y_T}{Y_T(1 - p + \alpha p)}.$$

**Appendix 2**

$$\text{Max} \int_0^{Y} \left[(1 - p)U(Y - t(Y)) + ppU(Y - t(Y) - s(Y)) + p(1 - \rho)\delta U(Y - t(Y))\right]f(Y)dY$$

s.t.

$$\int_0^{Y} t(Y)f(Y)dY + p\rho \int_0^{Y} s(Y)f(Y)dY = B$$

$$s(Y) \leq Y - t(Y) - U^{-1}(\delta U(Y - t(Y)))$$
To simplify the exposition of the solution to this problem and to show the pure equity effect of the copayment, we assume that the second constraint is always satisfied. After obtaining the optimal solution it will then be shown that it satisfies it.

The F.O.C for the problem can be written as:

\[
0 = \frac{\partial L}{\partial s}v = -p\rho \int_{0}^{\bar{Y}} U'(Y - t(Y) - s(Y))v(Y)f(Y)dY + \lambda pp \int_{0}^{\bar{Y}} f(Y)v(Y)dY
\]

\[
0 = \frac{\partial L}{\partial t}v = -[(1 - p) + p(1 - \rho)\delta] \int_{0}^{\bar{Y}} U'(Y - t(Y))v(Y)f(Y)dY
\]

\[
-pp \int_{0}^{\bar{Y}} U'(Y - t(Y) - s(Y))v(Y)f(Y)dY + \lambda \int_{0}^{\bar{Y}} f(Y)v(Y)dY
\]

for any test function \(v\). Therefore from the first equation we get

\[
U'(Y - t(Y) - s(Y)) = \lambda
\]

and plugging this relation into the second one we obtain the following condition

\[
[(1 - p) + p(1 - \rho)\delta] U'(Y - t(Y)) = (1 - pp) U'(Y - t(Y) - s(Y)),
\]

which we can rewrite as

\[
\frac{U'(Y - t(Y))}{U'(Y - t(Y) - s(Y))} = \frac{1 - pp}{(1 - p) + p(1 - \rho)\delta} \geq 1. \tag{10}
\]

The above condition is always consistent with the inequality constraint we mentioned at the beginning. Let us call \(\beta\) the constant on the right-hand side in (10), so that

\[
\frac{1}{\beta} U'(Y - t(Y)) = U'(Y - t(Y) - s(Y)),
\]

that is

\[
(U')^{-1} \left[ \frac{1}{\beta} U'(Y - t(Y)) \right] = Y - t(Y) - s(Y).
\]

The constraint is satisfied if \(Y - t(Y) - s(Y) \geq U^{-1}(\delta U(Y - t(Y)))\), therefore the compatibility requirement is

\[
(U')^{-1} \left[ \frac{1}{\beta} U'(Y - t(Y)) \right] \geq U^{-1}(\delta U(Y - t(Y))).
\]
Now, since $\delta$ and $1/\beta$ are less than 1, $U$ is increasing and $U'$ is decreasing we have

$$U^{-1}(\delta U(Y - t(Y))) \leq Y - t(Y) \leq (U')^{-1}\left[\frac{1}{\beta} U'(Y - t(Y))\right]$$

as wanted.
Appendix 3

The problem can be written as:

\[
\begin{align*}
\text{Max} & \int_0^\bar{Y} \left[ (1-p)U(Y - t(Y)) + ppU(Y - t(Y) - s(Y)) + p(1-\rho)\delta U(Y - t(Y)) \right] f(Y) dY \\
\text{s.t.} & \\
\rho &= \frac{B}{E - p \int_0^\bar{Y} s(Y)f(Y) dY} \\
\rho &\leq 1 \\
s(Y) &\leq Y - t(Y) - U^{-1}(\delta U(Y - t(Y))) = s^{\text{max}}(Y) \\
p \int_0^{s^{\text{max}}(Y)} f(Y) dY = Z
\end{align*}
\]

In this case, given that the tax revenue is fixed, we derive the welfare function only for \(\rho\) and \(s\).

As a first observation we can see that the derivative with respect to \(\rho\) of the objective function is always positive, provided the individual participation constraint is satisfied. In fact we have

\[
\frac{\partial L}{\partial \rho} = p \int_0^\bar{Y} \left[ U(Y - t(Y) - s(Y)) - \delta U(Y - t(Y)) \right] f(Y) dY
\]

This means that the problem is solved for a corner solution. In this respect we have two different cases:

- \(Z < E - B\) which implies \(\rho^* = \frac{B}{E - Z} < 1\); in this case the optimal choice is of course \(s(Y) = s^{\text{max}}(Y)\) for all \(Y\);

- \(Z > E - B\) which implies \(\rho^* = 1\). In this case the optimal \(s\) can be
found solving the following problem:

\[
\begin{align*}
\text{Max } & \int_0^Y U(Y - t(Y) - s(Y)) \ f(Y) dY \\
\text{s.t. } & \int_0^Y s(Y) f(Y) dY = E - B, \quad s(Y) \geq 0
\end{align*}
\]

The problem is equivalent to finding an optimal income tax schedule that does not allow for direct income transfers through the use of negative copayments\(^3\). The problem can be solved using the optimal control theory.

\(^3\)See Atkinson and Stiglitz (1980), pag. 396
References


