

SOME REFLECTIONS UPON THE CONCEPT OF EXCESS BURDEN

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Some Reflections upon the Concept of Excess Burden

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1. Introduction and Basic Definitions.

The purpose of this paper is to review some of the definitions of the excess burden of a tax, starting from the pioneering contribution of Barone (1912), in order to point out how the evolution of this concept sometimes departed from the original framework with the result of obscuring the nature of the concept itself. The proposed definitions reviewed in the paper have been chosen somewhat arbitrarily within a vast literature since the main aim is to compare the most recent definitions to the original ones.

After this review it is proposed a definition of excess burden which avoids some problems posed by other proposals.

Excess burden may be defined with reference to a wide class of problems; in the present paper, unless otherwise stated, it is considered the simplest conceivable i.e. the introduction of a single distorting tax or a lump sum tax starting from a no tax situation. This choice has the advantage to permit familiar graphical interpretation of the definitions.

2. Classical Definitions.

The first definition of the excess burden is due to Barone (1912) and his basic demonstration is shown in the following figure 1, where, for the moment, the reader is invited to ignore the dotted line with vertical intercept in G.

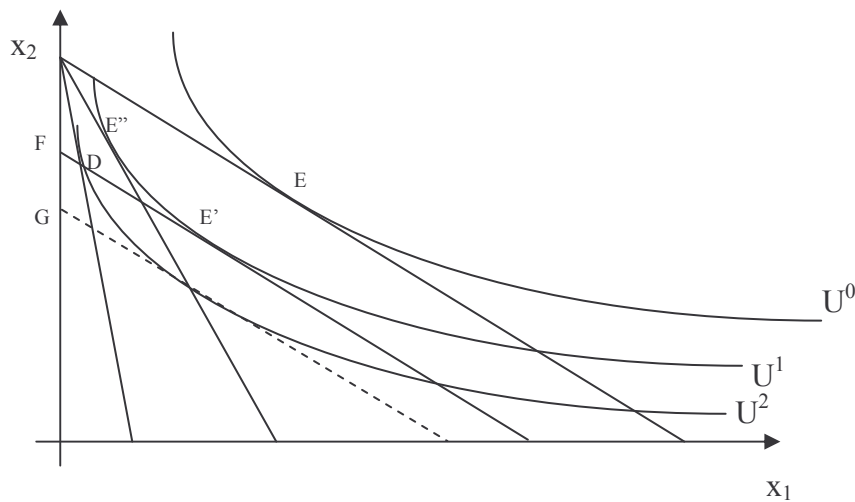


Figure 1

Initial equilibrium is point E where no tax is levied; E' indicates a new equilibrium induced by a lump sum tax while E'' and D are two equilibria resulting from the introduction of a distorting tax on good 1 only. In the case depicted, where the consumer has only a given exogenous income the lump sum tax may be interpreted as a direct tax and the distorting tax as an indirect tax.

What may be deduced from the above figure is a superiority of the lump sum tax over the distorting tax: if the two taxes are indifferent for the consumer (points E' and E'') the distorting tax yields less

revenue than the lump sum tax¹; alternatively, if the two taxes yield the same revenue the distorting tax puts the consumer on a lower indifference curve².

Ignoring Barone (1912), Joseph (1939) deals with the same problem even if the aim of the paper is a bit different, as results by the beginning of the paper:

“The purpose of this note is to analyse, by means of indifference curves, the *monetary value of the additional loss* imposed by the introduction of a commodity tax, as compared with an income tax of equivalent yield to the Exchequer”³.

The graphical analysis is identical to that of Barone with the difference that the excess burden is now represented, in money terms, by the vertical distance FG in figure 1 above; its interpretation is in terms of “the additional yield which the Exchequer could have obtained by substituting an income tax for the [indirect tax] without imposing any extra burden on the consumer”⁴.

It is interesting, for the following discussion, to note that the vertical distance FG in figure 1 may be also interpreted as the difference between the equivalent variation of the price change due to the distorting tax and its revenue; in this sense its interpretation is quite similar to the definition of excess burden due to Marshall and reported by Joseph (1939, p. 226) where the monetary loss is measured by the change in consumer surplus⁵ and excess burden is an approximate triangular area often called the “Harberger triangle”.

3. Modern Treatment.

Explicit reference to hicksian measures of the welfare change is made in a famous paper by Diamond and McFadden (1974); after observing that there are several different definitions of excess burden they define it as “the excess of the income we must give a consumer to restore him to his pretax indifference curve over the tax revenue collected from him”⁶. The first part of their definition implies, and their graphical exposition confirms, that the measure of the welfare loss is the hicksian compensating variation due to the change in prices induced by the tax. What is a bit puzzling is the definition of the tax revenue; in their words “While it is not clear that this is the most intuitive notion, for consistency we measure the tax revenue for this definition as the level collected at the consumer equilibrium after the consumer has been restored to his original indifference curve”⁷. Consequently they define the following compensated tax revenue function:

$$T(\mathbf{q}, \mathbf{p}, U^0) = \sum_{i=1}^N (q_i - p_i) h_i(\mathbf{q}, U^0) = \sum_{i=1}^N t_i h_i(\mathbf{q}, U^0) \quad (1)$$

where q_i and p_i are, respectively, consumer and producer price of good i , t_i is the tax rate on good i , h_i is the compensated demand function for good i and U^0 is the utility level achieved by the consumer before the introduction of any tax.

Using the above function (1) their definition of excess burden is the following:

$$e(\mathbf{q}, U^0) - e(\mathbf{p}, U^0) - T(\mathbf{q}, \mathbf{p}, U^0) \quad (2)$$

¹ The point of intersection of the budget lines tangent to U^1 guarantees the equal yield condition and it implies a larger demand for the taxed good with respect to E ”.

² The tangency point D is also the intersection point between the budget lines tangent to U^1 and U^2 indifference curves.

³ Joseph (1939, p. 226), italics not in the original text.

⁴ Joseph (1939, p. 227).

⁵ This is not the place to comment upon the use of consumer’s surplus; for an interesting treatment of the matter the reader is referred, among others, to Mohring (1971).

⁶ Diamond and McFadden (1974, p. 5).

⁷ Diamond and McFadden (1974, p. 5).

This measure is shown graphically, in the following figure 2 for the same tax change as in figure 1 above, as the vertical distance CB, since AB is the hicksian compensating variation while AC is the compensated revenue accruing to the government when the consumption of the taxed good is in E'', that is after compensating the consumer.

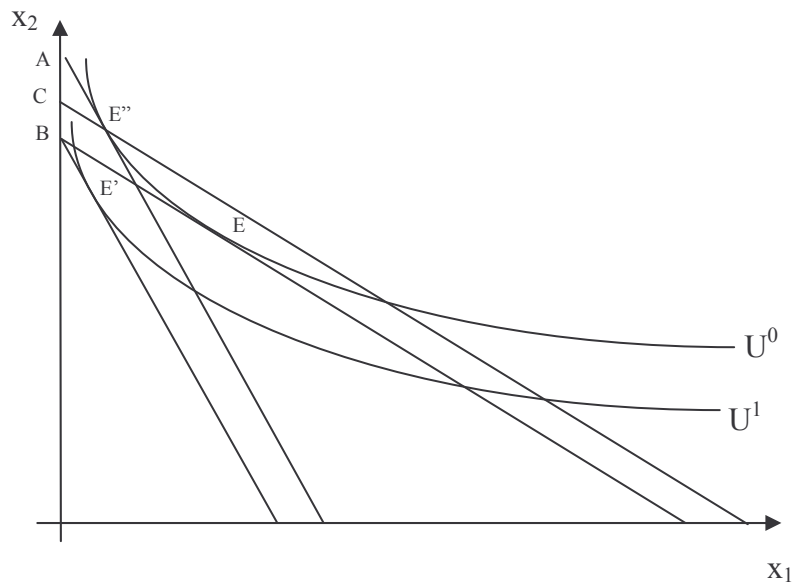


Figure 2

The puzzling aspect referred to above concerns the definition of the compensated tax revenue which is a hypothetical revenue which never accrues to the government.

Two points, which will be soon discussed, emerge from the Diamond and McFadden paper: the former is the definition (or measurement) of the excess burden as the difference between the welfare loss for the consumer and the (compensated) tax revenue; the latter is the introduction of the concept of compensated tax revenue which is poorly justified only in terms of a generic consistency⁸.

The same definition as in Diamond and McFadden (1974) is also found in a famous paper by Hausman (1981); even if the author defines the excess burden “to be the difference of the compensating variation minus the tax revenue collected”⁹ the graphical analysis seems to suggest that the author employs the concept of compensated revenue¹⁰.

Along the lines suggested by Diamond and McFadden we find also the most extensive treatment of the subject by Auerbach (1985) and its latest and updated version by Auerbach and Hines (2002).

The definition of excess burden given in Auerbach (1985, p. 67) is “that amount that is lost in excess of what government collects”, even if it is recognized that it is too vague and is consequently open to different interpretations.

Then two definitions are proposed employing the hicksian measures of welfare change; when considering the equivalent variation excess burden is defined with reference to the true tax revenue as follows:

$$e(\mathbf{q}, U^1) - e(\mathbf{p}, U^1) - (\mathbf{q} - \mathbf{p})\mathbf{x}(\mathbf{q}, R) \quad (3)$$

where the demand functions in the last term are marshallian demands and R is the exogenous income of the consumer. According to the above definition (1) this term might be defined as:

⁸ See the above quotation from Diamond and McFadden (1974).

⁹ Hausman (1981, p. 672-3).

¹⁰ It is used the term “seems” because the paper is a bit confusing on this point.

$$T(\mathbf{q}, \mathbf{p}, U^1) \tag{4}$$

Explicit reference to Diamond and McFadden (1974) is made for the other definition of excess burden, employing the compensating variation:

$$e(\mathbf{q}, U^0) - e(\mathbf{p}, U^0) - (\mathbf{q} - \mathbf{p})\mathbf{h}(\mathbf{q}, U^0) \tag{5}$$

which is identical to (2) given the definition in (1).

In the latest version Auerbach and Hines (2002) condense the two definitions into a single comprehensive definition where the utility level may be arbitrary:

$$e(\mathbf{q}, U^i) - e(\mathbf{p}, U^i) - (\mathbf{q} - \mathbf{p})\mathbf{h}(\mathbf{q}, U^i) \tag{6}$$

The use of the compensated tax revenue is justified in terms of the conceptual experiment¹¹ of holding constant the utility level as prices vary.

A feature of the definitions in (6) is their graphical interpretation in the case of a single price change like in figure 1 and figure 2 above. If it is assumed that the taxed good¹² is normal the relative position of hicksian and marshallian demands is shown in the following figure, where x and h denote, respectively, marshallian and hicksian demand and where the pre and post tax utility levels are, respectively, U^0 and U^1 :

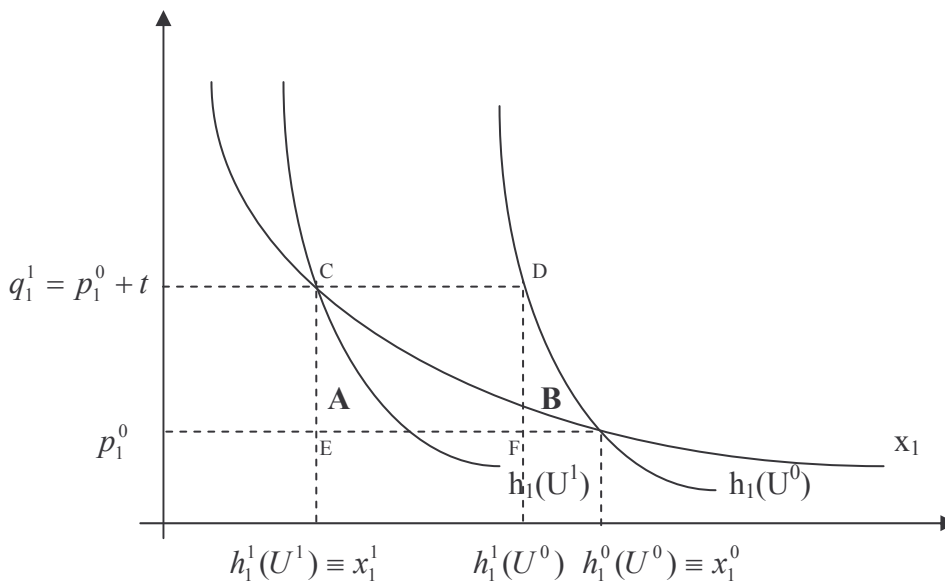


Figure 3

In this figure the compensating and equivalent variations are represented by the area to the left of the hicksian demand with utility level, respectively, U^0 and U^1 , between the two price levels; consequently the corresponding measures of excess burden, according to (6), are, respectively, area B and area A.

Given that the definition of excess burden presented in Auerbach (1985) and Auerbach and Hines (2002) seems to be widely accepted, let's propose a reflection before passing to examine the only critical view on this definition.

¹¹ Auerbach and Hines (2002, p. 1351).

¹² Good 1 in the above examples.

In my opinion it is meaningful that the definition of excess burden in the latter work is given with reference to the conceptual experiment which holds utility constant at the initial or final level¹³, while in the former the general definition is given in terms of comparison with the tax revenue. Explicit reference to the tax revenue, in fact, might led someone to propose a different definition of excess burden; the introduction of the tax brings the consumer from an initial equilibrium to a final equilibrium and two possible, and correct, measures of his welfare change are compensating and equivalent variation: within this framework excess burden would be the difference between any of this two measures and the tax revenue.

This, apparently indivisible, definition has two consequences one of which may be illustrated in the above figure 3: employing the equivalent variation excess burden is again represented by area A, while in the case of compensating variation an additional rectangular area (CDEF) enters the computation of excess burden. It is the second consequence to cast some doubts on the above definition of excess burden and it is illustrated in the following figure 4, which is identical to figure 3 with the single exception that the taxed good is an inferior good: this alters the relative positions of compensated demand functions with respect to marshallian demand.

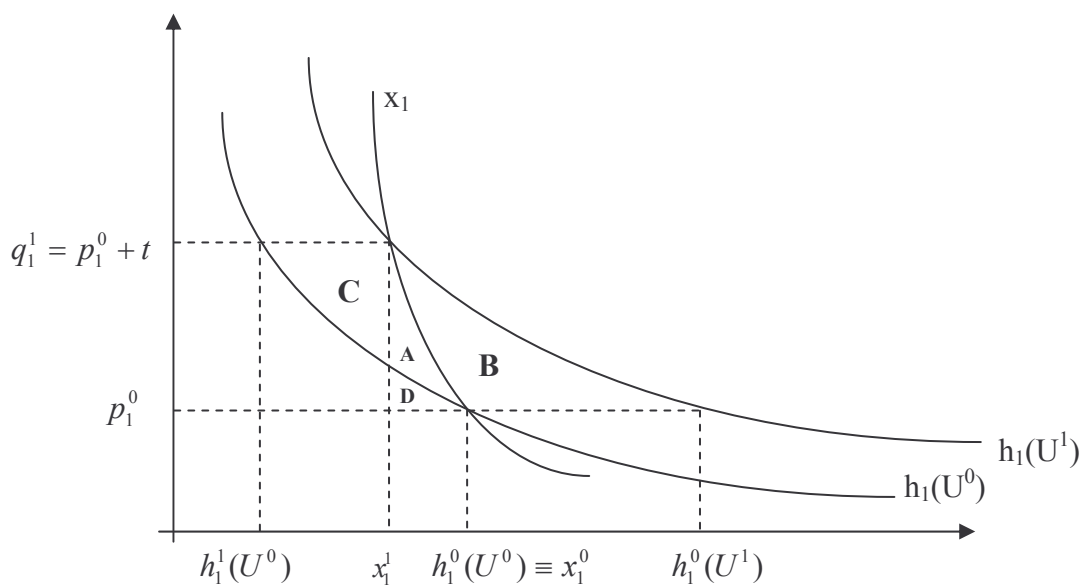


Figure 4

The equivalent variation measure of excess burden is again an approximate triangular area equal to the sum of areas A, B and D in the figure. Employing the compensating variation to measure the welfare loss and comparing it with the tax revenue gives a geometrical interpretation of the excess burden as the difference between areas D and C.

Taking, as we have done, the welfare loss with positive sign, in the case of figure 4 this measure of the excess burden would be negative, that is the introduction of the tax would be beneficial to the society as a whole or the tax would generate an excess benefit. This conclusion is clearly false since a positive excess burden simply reflects the fact that the post tax equilibrium is not pareto optimal as the initial no tax equilibrium.

4. Kay's Critique.

The only critical voice against the definition of excess burden proposed by Diamond and McFadden is a paper by Kay (1980)¹⁴. First of all the author points out the hypothetical nature of the

¹³ Or at some other arbitrary level.

¹⁴ Even if the paper is antecedent to Auerbach (1985) the critique applies as well to his definitions.

compensated tax revenue and observes that “As a result the measure of loss proposed is based on characteristics of the utility function evaluated at a point very different from that which the consumer is actually at, and the concept of compensated tax revenue differs from, and bears no close relationship to, the actual revenue which the system brings in”¹⁵.

A second critical observation is much deeper; a definition of excess burden should have the property of being minimized by an optimal tax structure. Kay’s fundamental result is that only the definition which employs the equivalent variation, when minimized subject to a revenue constraint, replicates the conditions for an optimal tax structure¹⁶ and must be thus considered the only admissible definition of excess burden:

$$e(\mathbf{q}, U^1) - e(\mathbf{p}, U^1) - (\mathbf{q} - \mathbf{p})\mathbf{x}(\mathbf{q}, R) \quad (7)$$

where, with Kay’s words, “ U^1 is the utility level achieved in the taxed equilibrium and \mathbf{x} is the vector of quantities actually purchased in that equilibrium”¹⁷.

The result of Kay’s paper explains why his initial definition of excess burden, in accordance with Diamond and McFadden, is given in terms of comparison between the welfare loss for the consumer and the tax revenue. This means that the above critical considerations would be still applicable but lose any force since the troublesome definition employing the compensating variation is ruled out by his fundamental argument.

Kay’s paper goes on considering some properties of the proposed measure of excess burden the first and most important of which is its non-negativeness; in expression (7) the first term may be written as $\mathbf{q}\mathbf{x}(\mathbf{q}, R)$ so that it may be simplified as:

$$\mathbf{p}\mathbf{x}(\mathbf{q}, R) - e(\mathbf{p}, U^1) \quad (8)$$

Given that the bundle \mathbf{x} is on the indifference curve labelled U^1 the above expression is non negative due to the definition of expenditure function.

This property introduces a further consideration which will be further discussed below; let’s consider the opposite change in tax: starting from the distorted equilibrium remove the tax and compute the excess burden. Given the above quotation and considered that initial utility level is now U^1 , Kay’s definition is non negative, as demonstrated above, and the welfare loss is now computed in terms of compensating variation.

Both features are, in my opinion, not satisfactory: the former because the removal of a distortionary tax should give rise to an excess benefit not an excess burden; the latter because the admissible measure of welfare change adopted in the above definition changes as the tax policy changes.

It is interesting noting that the opposite tax change is also considered by Auerbach and Hines (2002) but their treatment is a bit confusing as the following quotation witnesses:

“An alternative conceptual experiment is to begin with the tax already in place and then remove it, extracting from consumers in lump-sum fashion an amount that prevents them from changing their utility levels while the tax is removed. Because the initial tax is distortionary, it is necessary to extract more from consumers than the tax revenue, the difference representing the excess burden of the initial tax. [...] Again using the expenditure function to calculate the amount the government extracts in this case – the Hicksian equivalent variation, based on [the utility level U^1] - this exceeds the forgone revenue”¹⁸

Two the confusing features common to Kay’s definition: the former is a contradiction between the implied beneficial result of the reform and the positive sign of excess burden; the latter the measure

¹⁵ Kay (1980, p. 112-3).

¹⁶ Kay (1980, p.114-6).

¹⁷ Kay (1980, p. 115).

¹⁸ Auerbach and Hines (2002, p. 1352-53).

employed to compute the welfare change: if the reference utility level is U^1 , which is now the initial level with the distortionary tax in place, this should be a compensating and not an equivalent variation.

5. A New Proposal.

The motivation for the proposal which is developed in this section directly stems from the above critical considerations and aims at recovering the original spirit of Barone (1912). That paper assessed the consequences of two alternative taxes, introduced in a no tax situation; the superiority of the lump sum tax over the distortionary tax, and the implied excess burden of the latter, was given in terms both of greater revenue at a common utility level or of smaller welfare loss at a common tax revenue.

Even if the problem is not dealt with by Barone (1912) it may be argued that the same reasoning applies to the opposite tax change; given that the starting point is a distorted one the alternatives are, in this case, the removal of the distortionary tax and a lump sum transfer at distorted prices. The consequences are evaluated in the same ways as before and the proposal is built around the first of them, that is, a comparison of tax revenues at constant utility level.

Let's consider first the introduction of a tax in a no tax situation; let's suppose that a given distortionary tax brings the consumer from equilibrium E , at a utility level U^0 , to equilibrium E'' , at a utility level U^1 , as shown in the following figure which is essentially identical to figure 1 above.

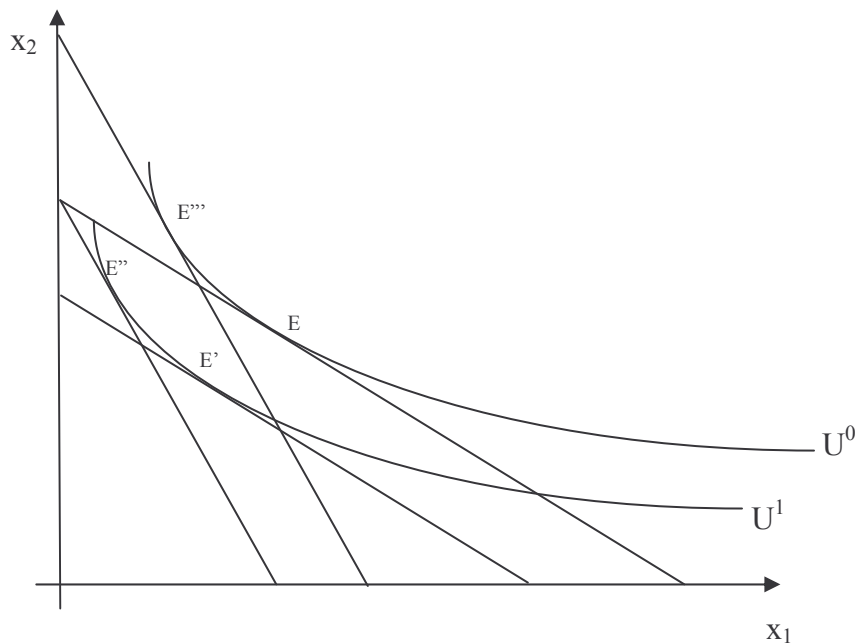


Figure 5

Alternatively a lump sum tax would bring the consumer at equilibrium E' at the same utility level U^1 . The proposed measure of excess burden compares the change in tax revenues in both cases and a positive excess burden emerges if the change in revenue from the lump sum tax is greater than the change in revenue due to the distortionary tax. Indicating the former with ΔT^{ND} and the latter with ΔT^D excess burden is given by:

$$\Delta T^{ND} - \Delta T^D = [e(\mathbf{q}, U^1) - e(\mathbf{p}, U^1) - 0] - [(\mathbf{q} - \mathbf{p})\mathbf{x}(\mathbf{q}, R) - 0] \quad (9)$$

which corresponds to the “traditional” measure of excess burden employing the equivalent variation to compute the welfare change for the consumer; in particular it corresponds to the definition suggested by Kay (1980) as the only correct one.

A fundamental difference emerges when the proposed definition of excess burden is applied to the opposite tax change. Now the initial situation is a distorted one¹⁹ and the alternatives are the removal of the distortionary tax, bringing the consumer back to equilibrium E, or a lump sum transfer, at distorted prices, bringing the consumer to equilibrium E’’. Excess burden is now given by:

$$\Delta T^{ND} - \Delta T^D = [-e(\mathbf{q}, U^0) + e(\mathbf{p}, U^0) + \mathbf{th}(\mathbf{q}, U^0) - \mathbf{tx}(\mathbf{q}, R)] - [0 - \mathbf{tx}(\mathbf{q}, R)] \quad (10)$$

where the first two terms, in the right hand side, are the lump transfer²⁰, the third is the tax revenue from the distortionary tax at final equilibrium and the fourth is total tax revenue in the initial equilibrium. After simplification (10) becomes:

$$\Delta T^{ND} - \Delta T^D = -e(\mathbf{q}, U^0) + e(\mathbf{p}, U^0) + \mathbf{th}(\mathbf{q}, U^0) \quad (11)$$

According to (11) excess burden is the opposite of the “traditional” definition employing, again, the equivalent variation to measure the welfare change, since now U^0 is the final utility level.

The advantage of the adoption of the proposed definition of excess burden is twofold: firstly it coincides with Kay’s definition when alternative taxes are introduced in a no tax situation and so shares the same theoretical justification; secondly when the initial situation is distorted it employs the same measure of welfare change and its sign is negative, indicating an excess benefit from the removal of the distortionary tax. Moreover, in this second case, it coincides²¹ with one of the measures proposed by Auerbach and Hines (2002) but their concept of compensated tax revenue enters now the computation of actual tax revenue accruing to the government and is no longer a hypothetical concept.

From a graphical point of view the definition corresponds, in any case, to an approximate triangular area as shown, in the case of a normal good, in the following figure where the subscripts are suppressed.

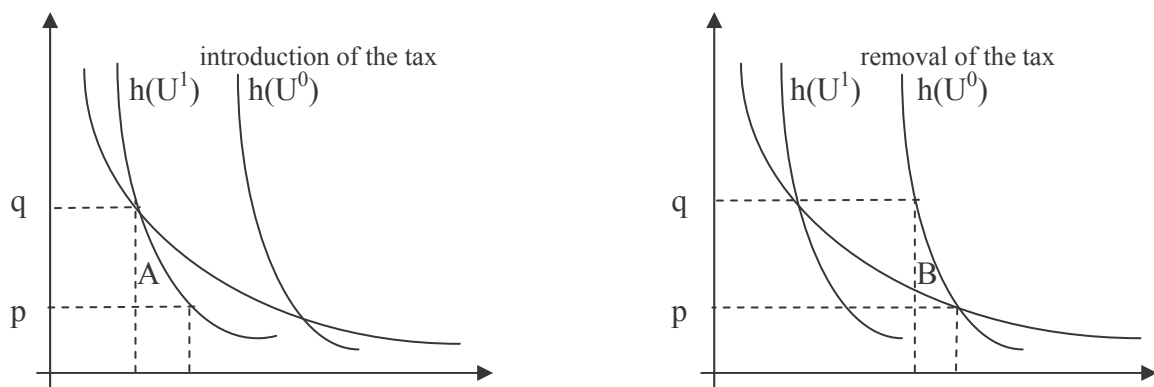


Figure 6

Area A is the excess burden of the introduction of the distortionary tax, while area B, with a negative sign, is the excess benefit from the removal of the distortionary tax with respect to a lump-sum transfer at distorted prices. The case of an inferior good is shown in the following figure.

¹⁹ Point E’’ in figure 5.

²⁰ And so a negative change in tax revenue.

²¹ Apart from the sign.

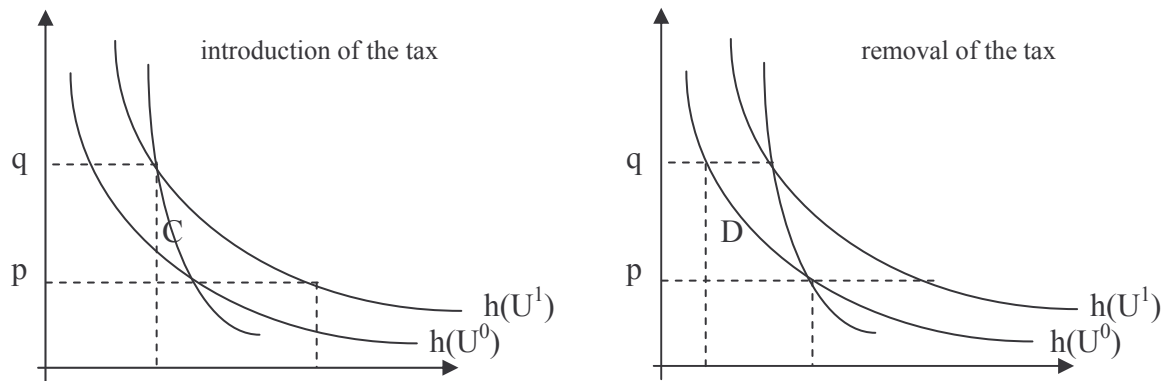


Figure 7

Now area C is the excess burden of the introduction of the distortional tax, while area D, with a negative sign, is the excess benefit from the removal of the distortional tax with respect to a lump-sum transfer with the distortional tax still in place.

It may be interesting to close this section with a brief description of a rather neglected work by Dodgson (1983), because it is strictly connected with an above argument. In the original paper of Barone (1912) the superiority of the lump-sum tax was demonstrated either by a larger revenue at constant utility level or by a smaller welfare loss at constant revenue; the proposal examined so far adopts the former definition. Dodgson (1983) defines the excess burden making reference to the latter, as shown in the following figure.

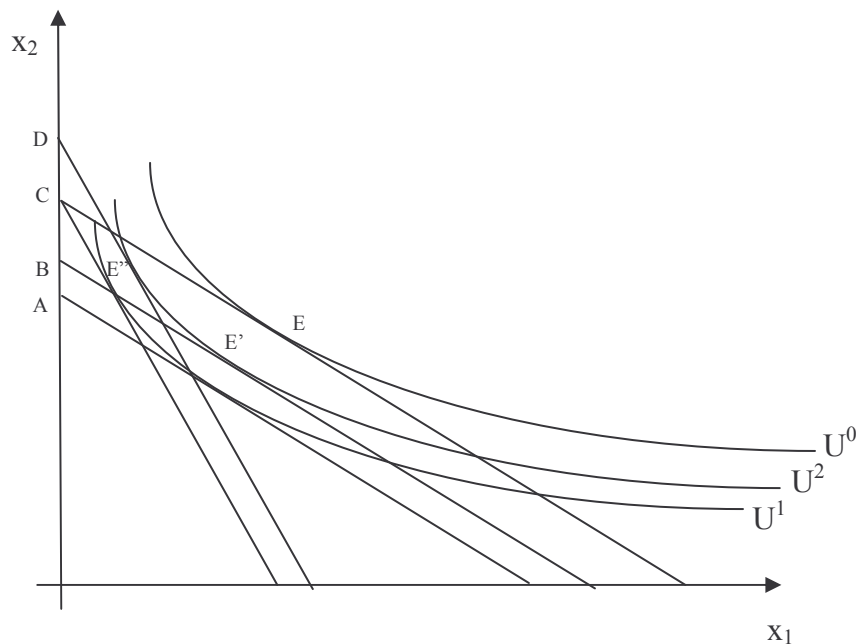


Figure 8

Points E' and E'' are the new equilibrium, respectively, with the lump sum tax and with a distortional tax of equal revenue and excess burden is defined by the difference between utility level U^2 with respect to utility level U^1 .

Two relevant features emerge from the Dodgson paper: the first is that this difference in utility levels may be computed either by the compensating or the equivalent variation, that is either at initial or at final prices. The second, and most important, is the interpretation of these two measures; on the one hand using the equivalent variation brings back to the usual definition of excess burden

as difference between the welfare loss and the tax revenue: in the above figure it may be seen that distance BA may be decomposed as the difference between CA²² and CB²³.

On the other hand, using the compensating variation the computation coincides with the corresponding definition by Diamond and McFadden²⁴ only when preferences are homothetic²⁵. When preferences are not homothetic the definition by Dodgson has the major shortcoming of not being interpretable as difference between areas to the left of an appropriate demand curve as is usual in the Harberger tradition²⁶.

6. Concluding Remarks.

This final section is devoted to sum up the problems which may be met with the definition of excess burden; the original paper by Barone (1912) offers two definitions of excess burden which are not controversial: a lump sum tax which is indifferent for the consumer guarantees a larger revenue than a distortionary tax or a lump sum tax of equal revenue causes a smaller welfare loss.

Perhaps the problems stem out of the Marshallian treatment of the problem using consumer's surplus to measure the welfare loss; it is natural, in this context, to define the excess burden as the difference between the welfare loss and the tax revenue.

Even if the early treatment by Joseph (1939) is not developed in such terms it is tempting to observe that the definition by Diamond and McFadden (1974) and its later extensions by Auerbach (1985) and Auerbach and Hines (2002) seem to pursue the aim of adopting the above definition in the context of correct welfare measures²⁷.

The shortcomings of such definitions are well pointed out by Kay (1980) whose conclusion is that the only correct definition of excess burden, still in terms of comparison between welfare loss and tax revenue, needs the equivalent variation as a measure of the welfare loss.

Even if the authors are not explicit on this issue, a common feature of these traditional measures of excess burden seems to be their positive sign even if the comparison is between the removal of the taxes and not their introduction; in this case such a measure should have a negative sign implying an excess benefit from the removal of the distortionary tax.

The definition of excess burden that is proposed in section 5 is in the spirit of Barone (1912) and is given in terms of comparison between the (change in) tax revenues at constant utility level; it is demonstrated that this definition is positive when the taxes are introduced and negative when are removed; furthermore in the former case it coincides with Kay's definition and in the latter with one of those proposed by Auerbach and Hines (2002) without the need to resort to the concept of compensated tax revenue. Moreover the definition is still interpretable as difference between areas under appropriate demand curves²⁸ and tax revenue, as in the Marshall-Harberger tradition.

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²² That is the welfare loss caused by the lump tax measured by the equivalent variation.

²³ The tax revenue from the distortionary tax.

²⁴ That is the difference between the welfare loss, measured by the compensating variation, and the compensated tax revenue of the distortionary tax.

²⁵ Dodgson (1983, p. 111).

²⁶ It is interpretable as difference between areas under two different compensated demand curves.

²⁷ Auerbach and Hines (2002, p.1351) make explicit reference to the "Harberger triangle".

²⁸ That is hicksian compensated demand functions.

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