HUMAN CAPITAL, PRODUCT MARKET POWER AND ECONOMIC GROWTH

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Abstract

We build a generalised growth model of horizontal product innovation with human capital accumulation in which the monopolistic mark-up set in the uncompetitive sector and the degree of returns to specialization are disentangled. We find that product market power has a positive growth effect when durables are employed in the production of the homogeneous consumers good. Thus, in this case the model allows to replicate one of the main results of the neo-Schumpeterian growth theory within a framework where innovation is both horizontal and deterministic and economic growth is driven by private incentives to invest in education. We also find that not only the type of technology employed in the final output production, but also the intensity of the inter-sectoral competition for the same resource (human capital) affect both the steady state level of growth and the relationship between market power and economic development.

Keywords: Human Capital, R&D, Product Market Power, Endogenous Growth

JEL Classification: J24, O31, O41

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Introduction

Human capital accumulation and Research and Development (R&D) activity are two primary determinants of economic growth. However, despite the fact that in the R&D-based growth literature an important research line has already investigated whether the presence of imperfect competition in the product market may be growth-enhancing or not,¹ this has not yet been done within an integrated economic growth model where agents (firms and individuals) may invest respectively in innovation and education activities and the growth engine is represented by the investment in human capital.

The aim of this paper is to combine in the simplest possible way the basic Lucas (1988) model of human capital accumulation with (a generalization of) the Grossman and Helpman (1991, Ch. 3, pp. 43/57) model of endogenous technical change without knowledge spillovers in order to fill this gap in the literature. The reason why we focus on the Grossman and Helpman model without knowledge spillovers is as follows. We are interested in studying the nexus between imperfect product market competition (measured by the elasticity of substitution among different varieties of capital goods) and economic growth within an economy where the lever to economic development is schooling investment and not the R&D externality.

Apart from introducing human capital accumulation à la Lucas, the structure of our model economy remains very similar to the basic Grossman and Helpman’s approach (1991, Ch. 3) mentioned above. In more detail, we postulate the existence of three vertically integrated sectors. A competitive final output sector produces a homogeneous consumers good. Depending on the value of a crucial parameter (that we interpret as the share of total income being devoted to the purchase of the available capital goods varieties) the final output sector technology may employ (with constant returns to scale) only human capital, only the existing varieties of intermediate goods or both (human capital and intermediates) as inputs.² The intermediate goods sector consists of monopolistically competitive firms, each producing a differentiated variety. We assume that the production of (whatever variety of) intermediate goods requires only human capital. Finally, the research activity produces designs (or blueprints) for new intermediate input varieties by employing only human capital, as well. When a new


² In the remainder of the paper we will use interchangeably such expressions as intermediate inputs, capital goods or durables. All these terms are supposed to have the same meaning.
blueprint is discovered in the competitive R&D sector, an intermediate-goods producer acquires the perpetual patent over it. This allows the intermediate firm to manufacture the new variety and practice monopoly pricing forever. Population is stationary and a representative household invests portions of its fixed-time endowment to acquire formal education. Hence, in our model human capital can be used in each sector to produce a homogeneous final output, capital goods, infinitely-lived patents and new human capital, respectively.

Our main conclusions are threefold. First of all, we find that there always exist (except when the technology for the production of the homogeneous consumers good is linear in human capital) a positive relationship between product market power and aggregate productivity growth. Secondly, we get that both the type of technology being used in the final output sector (Cobb Douglas versus CES) and the way the (growing) human capital is allocated to the different activities (what we term by inter-sectoral competition for skills) do affect the relationship between aggregate productivity growth and monopoly power. Lastly, we also show that the type of technology being used in the final output sector and the inter-sectoral competition for the (growing) human capital also influence the level of the equilibrium (long-run) growth rate.

This paper is especially related to two existing works. Bucci (2003b) also develops an endogenous growth model that integrates purposive R&D activity with human capital accumulation and where the engine of growth is represented by the investment in education activity. The present paper represents a generalisation of Bucci (2003b). The generalisation we propose here consists in writing the production technology in use in the downstream sector in such a way to disentangle the (equilibrium) monopolistic mark-up set in the intermediate sector and the degree of returns to specialization.3 Due to this generalization, in the present paper we have the possibility of studying in detail, and within the same framework, the relationship between imperfect competition and economic growth as emerging from two different classes of endogenous growth models: a) the Rebelo’s model (1991) with human (instead of physical) capital accumulation (or “AH model”), and b) the Grossman and Helpman’s model (1991) of endogenous technological change without knowledge spillovers and human capital investment (or “Lucas - Grossman and Helpman’s model”). In other words, the model we present in this paper enables us to analyse the potential implications (as for the monopoly power-growth relationship) of the seminal Rebelo’s (1991) and Grossman and Helpman’s

3 This point is made clear by Benassy (1998). According to him (Benassy, 1998, p.63) the degree of returns to specialisation “...measures the degree to which society benefits from specialising production between a larger number of intermediates n”.
(1991) models when a positive skills supply à la Lucas (1988) is explicitly introduced within them and to compare such implications with those stemming from Bucci (2003b).

The other paper which comes closer to ours is Bucci (2003a). This paper examines what happens to the market power-growth nexus within a model where there is no human capital accumulation (skills are in fixed supply) and the engine of growth is represented by the externality in the R&D activity. Unlike Bucci (2003a), in the present article we take an importantly different view, by considering an economy where the lever to economic growth is represented by a deliberate choice of investing in formal education by utility-maximizing agents.

The rest of the paper is organised as follows. Section 1 introduces the basic model. In Section 2 we study the general equilibrium of it and examine its steady-state properties. In Section 3 we compute the equilibrium output growth rate of the economy and solve for the inter-sectoral distribution of human capital. Section 4 presents the results concerning the steady-state predictions of the model concerning the relationship between the type of production function employed in the downstream sector, the sectoral distribution of human capital, product market power and economic growth in some special cases. Section 5 concludes.

1. The Basic Model.

In this economy three vertically integrated sectors produce respectively a homogeneous consumers good, intermediate inputs and ideas. In order to produce the undifferentiated consumers good, an aggregate production function combines, with constant returns to scale, human capital and intermediate inputs. These are available, at time $t$, in $n_t$ different varieties and are produced by employing human capital only. In the research sector, firms use only human capital to engage in innovation activity. Innovation consists in discovering new designs (or blueprints) for firms operating in the capital goods sector. The number of designs existing at a certain point in time coincides with the number of intermediate input varieties and represents the actual stock of non-rival knowledge capital available in the economy. Finally, unlike the traditional R&D-based growth models, we assume that the supply of human capital may grow over time. Following the pathbreaking papers by Uzawa (1965) and Lucas (1988), we postulate the existence of a representative household that devotes part of its own time-endowment to
educational activities. Thus, in the model human capital can be employed everywhere across sectors in order to produce consumer goods, intermediate inputs, ideas and new human capital.

**The Final Output Sector.**

In this sector atomistic producers engage in perfect competition. The technology to produce final goods \((Y)\) is given by:

\[
Y = \alpha \lambda \int_0^1 (x^\alpha)^\lambda \, dx, \quad A>0, \quad 0 \leq \lambda \leq 1, \quad 0 \leq \frac{\lambda}{1+\lambda} < \alpha < 1.
\]

As in Bucci (2003a), we have written the production technology in use in the downstream sector in such a way to disentangle the (equilibrium) monopolistic mark-up set in the intermediate sector and the degree of returns from specialization.\(^4\) Another reason why we employ the production function of equation (1) is that this technology allows to encompass as particular cases (and depending on the value of \(\lambda\) ) two recent models of endogenous growth\(^5\) (one of which is not R&D-based) that in their original version do not include human capital accumulation. Even with respect to these models we are interested to study in this paper their potential implications (as for the monopoly power-growth relationship) when a positive supply of skills is explicitly introduced in them. As already mentioned, unlike Bucci (2003a), we take here a different view by considering an economy where the lever to economic growth is human capital accumulation (and not the R&D externality).

According to equation (1), output at time \(t\) \((Y_t)\) is obtained by combining, through a constant returns to scale technology, human capital \((H_{Y_t})\) and \(n\) different varieties of intermediate inputs, each of which is employed in the quantity \(x_j\).

\(\alpha\), \(\lambda\) and \(A\) are technological parameters. The latter (total factor productivity) is strictly positive, whereas \(\lambda\) is (not strictly) between zero and one. In a momenti we will

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\(4\) Indeed, in a moment we will show that (under additional assumptions) the mark-up charged over the marginal cost by the monopolistic producers of intermediate inputs is \(1/\alpha\). At the same time, from equation (1), it is possible to see that in a symmetric equilibrium (in which the total production of intermediates, \(X\), is spread evenly between the \(n\) brands) the degree of returns to specialization (the exponent of \(n\)) is equal to \(\lambda(1/\alpha - 1)\). This one is clearly different from the monopoly power measure \((1/\alpha)\) and, more importantly, depends not only on \(\alpha\) but also on \(\lambda\). In this sense, the model we present here represents an extension of Bucci (2003b).

\(5\) Namely the Rebelo (1991) and Grossman and Helpman’s (1991, Ch. 3, pp. 43/57) models.
see that the restriction on $\alpha$ ensures that in a symmetric equilibrium the instantaneous profit accruing to a generic intermediate producer at a given point in time is inversely related to the number of varieties existing at that date.

Since the industry is competitive, in equilibrium each variety of intermediates receives its own marginal product (in terms of the *numeraire* good, the final output):

$$p_{j\mu} = A\lambda H_{ji}^{1-\lambda} \left[ \int_0^{n_j} (x_{j\mu}) \lambda dx_j \right]^{\lambda-1} (x_{j\mu})^{\alpha-1}. \quad (2)$$

In equation (2) $p_{j\mu}$ represents the inverse demand function faced, at time $t$, by the generic $j$-th intermediate producer. As it is common in the first generation *innovation-based* growth literature, the elasticity of substitution between two generic intermediates coincides with the price-demand elasticity faced by each capital goods producer and is equal to $1/(1-\alpha)$.\(^6\)

**The Intermediate Goods Sector.**

In the *intermediate sector*, capital good producers engage in *monopolistic competition*. Each firm produces one (and only one) horizontally differentiated intermediate good and must purchase a patented design before producing its own specialised durable. Following Bucci (2003b), we continue to assume that each local intermediate monopolist has access to the same technology, employing only human capital ($h_j$):

$$x_{j\mu} = B \cdot h_{j\mu}, \quad \forall j \in (0, n), \quad B>0. \quad (3)$$

This production function is characterised by constant returns to scale in the only input employed (human capital) and, according to it, one unit of skills is able to produce (at each time) the same constant quantity of whatever variety. $B$ measures the productivity of human capital employed in this sector. The generic $j$-th firm maximises (with respect to $x_{j\mu}$) its own instantaneous profit function under the (inverse) demand

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\(^6\) This result is obtained under the specific assumption that each firm producing intermediate inputs is so small that a marginal increase in the quantity of output it produces does not change the quantities produced by its market rivals and, then, total intermediate output.
constraint (equation (2)). Under the assumption that in the intermediate sector there exists no strategic interaction among firms, the resolution of this maximisation program gives the optimal price set by the generic $j$-th intermediate producer for one unit of its own output:

\[ p_{jt} = \frac{1}{\alpha} \frac{w_{jt}}{B}. \]

From equations (4) and (2), the wage rate accruing at time $t$ to one unit of human capital employed in the capital goods sector ($w_{jt}$) is equal to:

\[ w_{jt} = AB\alpha\lambda H_{jt}^{\frac{1}{\alpha}} \left[ \int_0^{1/n} (x_{jt})^{\alpha} \right]^{\frac{\lambda}{\alpha}} (x_{jt})^{\alpha-1}. \]

In a symmetric equilibrium (where $x_{jt} = x_j$, $\forall j \in (0, n_j)$), each local monopolist faces the same wage rate ($w_{jt} = w_i$, $\forall j \in (0, n_j)$) and equation (4) can be recast as:

\[ p_{jt} = \frac{1}{\alpha} \frac{w_{jt}}{B} = p_i, \quad \forall j \in (0, n_i). \]

The hypothesis of symmetry is dictated by the way through which each variety of intermediates enters the final output technology and by the fact that all the capital goods producers use the same production function (equation (3)). Hence, when all the capital goods firms are identical, they produce the same quantity ($x_j$), face the same wage rate accruing to intermediate human capital ($w_{jt}$) and fix the same price for one unit of their own output. The price is equal to a constant mark-up ($1/\alpha$) over the marginal cost ($w_{jt}/B$). In equilibrium the wage rate accruing to one unit of human capital employed in the intermediate sector ($w_{jt}$) will be the same (and equal to $w_i$) for all the sectors where this factor input is employed. This is due to the hypothesis that human capital is homogeneous in this model economy. Moreover, it is perfectly mobile across sectors.

Defining by $H_{jt} \equiv \int_0^n h_{jt}dj$ the total amount of human capital employed in the intermediate sector at time $t$ and under the assumption of symmetry among capital goods producers ($x_{jt} = x_j$, $\forall j \in (0, n_j)$), from equation (3) we obtain:
Finally, the instantaneous profit function of a generic $j$-th intermediate firm will be:

\[\pi_j = \left( p_i - \frac{1}{B}\cdot w_i \right) \cdot x_i = A\lambda(1-\alpha) \cdot H^\lambda_{n_t} \cdot (n_t) \cdot \left( B \cdot H_\mu \cdot \frac{n_t}{n_i} \right)^{\frac{\lambda-\alpha}{\alpha}} \pi_i, \quad \forall j \in (0, n_t).\]

Since we are dealing with a monopolistic competition market, $\pi$ will be decreasing in $n$ (the number of intermediate firms existing at time $t$) if and only if $\alpha > \lambda / (1 + \lambda)$. This explains the restriction on $\alpha$ we have introduced in equation (1).

Equation (6) says that, just as $x$ and $p$, so too the instantaneous profit is equal for each variety of intermediates in a symmetric equilibrium.

The Research Sector.

There are many competitive research firms undertaking R&D. These firms produce designs indexed by 0 through an upper bound $n \geq 0$ (thus, $n$ measures the total stock of society's knowledge). Designs are patented and partially excludable, but non-rival and indispensable for capital goods production. With access to the available stock of knowledge $n$, research firms use human capital to develop new blueprints. The production of new designs is governed by:

\[n_t = C \cdot H_n, \quad C > 0,\]

where $n_t$ denotes the number of capital goods varieties existing at time $t$, $H_n$ is the total amount of human capital employed in the sector and $C$ is the productivity of the research human capital input.

The production function of new ideas coincides with the one employed by Grossman and Helpman (1991) in their Chapter 3 model without knowledge spillovers (pp.43-57). In that model such a specification of the R&D process implies the cessation of growth in the log run. In our model, instead, this does not happen since in our economy the engine of growth is represented by human capital accumulation. In this last sense the model we present here shares the same conclusions of many others with purposive R&D activity and skill accumulation.\(^7\)

\(^7\) Notably Arnold (1998) and Blackburn et al. (2000).
As the research sector is competitive, imposing the zero profit condition amounts to setting:

\[ \frac{1}{C} w_n = V_n \tag{8} \]

\[ V_n = \int_{t}^{\infty} \exp \left[ - \int_{t}^{s} \pi_j \, ds \right] \, d\tau, \quad \tau > t. \tag{9} \]

In equations (8) and (9), \( w_n \) represents the wage rate accruing to one unit of human capital devoted to research; the term \( \exp \left[ - \int_{t}^{s} \pi_j \, ds \right] \) is a present value factor which converts a unit of profit at time \( \tau \) into an equivalent unit of profit at time \( t \); \( r \) is the real rate of return on the consumers’ asset holdings; \( \pi_j \) is the profit accruing to the \( j \)-th intermediate producer (once the \( j \)-th infinitely-lived patent has been acquired) and \( V_n \) is the market value of one unit of research output (the generic \( j \)-th idea allowing to produce the \( j \)-th variety of capital goods). Notice that \( V_n \) is equal to the discounted present value of the profit flow a local monopolist can potentially earn from \( t \) to infinity and coincides with the market value of the \( j \)-th intermediate firm (since there is a one to one relationship between number of patents and number of capital goods producers).

**Households**

Total output produced in this economy (Y) can be only consumed. Population is stationary and the available human capital is fully employed. For the sake of simplicity, we normalize population to one and postulate the existence of an infinitely-lived representative consumer with perfect foresight. This consumer owns, in the form of assets (\( a \)), all the firms operating in the economy and is endowed with one unit of time that he/she allocates (in the fraction \( u \)) to productive activities (research, capital goods and consumer goods manufacture), and (in the fraction \( 1-u \)) to non-productive activities (education). The representative consumer maximises, under constraints, the discounted value of his/her lifetime utility:8

\[ \max_{[c_t, w, \pi, A]} U_0 = \int_{0}^{\infty} e^{-\rho t} \log(c_t) \, dt, \quad \rho > 0 \tag{10} \]

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8 Following Grossman and Helpman (1991) we assume that the instantaneous utility function of the representative agent is logarithmic. Using more general isoelastic functions does not alter the results.
\[ s.t.: \]
\[ (11) \quad a_t = r_t a_t + w_t u_t h_t - c_t \]
\[ (12) \quad \dot{h}_t = \delta (1 - u_t) h_t, \quad \delta > 0, \quad a_0, h_0 \text{ given.} \]

The control variables of this problem are \( c_t \) and \( u_t \), and \( a_t \) and \( h_t \) are the two state variables. Equation (10) is the intertemporal utility function; equation (11) is the budget constraint and equation (12) represents the human capital supply function.\(^9\) The symbols used have the following meaning: \( \rho \) is the subjective discount rate; \( c \) denotes consumption of the homogeneous final good; \( w \) is the wage rate accruing to one unit of human capital\(^{10}\) and \( \delta \) is a parameter reflecting the productivity of the education technology.

With \( \mu_{1t} \) and \( \mu_{2t} \) denoting respectively the shadow price of the consumer’s asset holdings \((a)\) and human capital stock \((h)\), the first order conditions read as:

\[ (13) \quad \frac{e^{-\rho t}}{c_t} = \mu_{1t} \]
\[ (14) \quad \mu_{1t} = \mu_{2t} \frac{\delta}{w_t} \]
\[ (15) \quad \mu_{1t} r_t = -\mu_{1t} \]
\[ (16) \quad \mu_{1t} w_t u_t + \mu_{2t} \delta (1 - u_t) = -\mu_{2t}. \]

Equation (13) gives the discounted marginal utility of consumption, which satisfies the dynamic optimality condition in equation (15). Equation (14) is the static optimality condition for the allocation of time, equating the marginal benefit and the marginal cost of an additional unit of skills devoted to working. The marginal cost involves the cost associated with future reductions in human capital, as expressed by the other dynamic optimality condition in equation (16). Conditions (13) through (16) must satisfy the constraints (11) and (12), together with the transversality conditions:

\[ \lim_{t \to \infty} \mu_{1t} a_t = 0; \quad \lim_{t \to \infty} \mu_{2t} h_t = 0. \]

\(^9\) Notice that we assume no depreciation for human capital. This hypothesis is completely harmless in the present context and serves the scope of simplifying the analysis. Also notice that we consider the variant of the basic Lucas model (1988) in which spillovers from education are internalised. This is done because we are explicitly assuming that there exists only one agent and population is stationary.

\(^{10}\) The equilibrium wage rate accruing to human capital is unique since this factor input is perfectly mobile across sectors.

In this section we solve for the general equilibrium of the model and characterise its steady state under the symmetry hypothesis \( x_{jt} = B \cdot H_{jt} / n_j = x_j, \forall j \in (0, n_j) \). At this aim, after defining by \( u^* \) the optimal fraction of skills devoted by the representative consumer to production activities,\(^{11}\) the general equilibrium distribution of human capital between research, capital and consumer goods production can be obtained through solving simultaneously the following equations:

\[
(17) \quad H_y + H_j + H_n = u^* H, \quad \forall t \\
(18a) \quad w_j = w_n \\
(18b) \quad w_j = w_y
\]

Equation (17) is a resource constraint, saying that at any time \( t \) the sum of the human capital demands coming from each productive activity must be equal to the total stock of productive human capital available at the same time. Equations (18a) and (18b) together state that, due to human capital mobility across sectors, in equilibrium the wage earned by one unit of human capital is to be the same irrespective of the productive sector where that unit of skill is actually employed.

Moreover, since the total value of the representative agent’s assets \( (a) \) must equal the total value of firms, the next equation has also to be checked in a symmetric equilibrium:

\[
(19) \quad a = nV_n
\]

where \( V_n \) is given by equation (9) above and satisfies the asset pricing condition:

\[
(19a) \quad \dot{V}_n = rV_n - \pi_j
\]

with:

\[
(19b) \quad \pi_j = \frac{\lambda}{1-\lambda} (1-\alpha) \frac{H_y w_y}{n}, \quad \text{and} \quad (19c) \quad w_y = \frac{A(1-\lambda)}{H_y} n^{\frac{\lambda}{\alpha}} (\frac{BH_j}{n})^{\frac{\lambda}{\alpha}}.
\]

\(^{11}\) \( u^* \) is endogenous in the model and, as such, has to be determined.
In the model one new idea allows a new intermediate firm to produce one new variety of capital goods. In other words, there exists a one-to-one relationship between number of ideas, number of capital goods producers and number of intermediate input varieties. This explains why, in equation (19), the total value of the consumer’s assets \((a)\) is equal to the number of profit-making intermediate firms \((n)\) times the market value \((V_n)\) of each of them (equal to the market value of the corresponding idea). Equation (19a) simply suggests that the interest on the value of the \(j\)-th generic intermediate firm \((rV_n)\) should be equal, in equilibrium, to the sum of two terms:

- the instantaneous monopoly profit \((\pi_j)\) coming from the production of the \(j\)-th capital good;
- the capital gain or loss matured on \(V_n\) during the time interval \(dt\) \((\dot{V}_n)\).

In order to characterise the steady-state (or balanced growth path) equilibrium of the model presented so far, we start with a formal definition of it:

**Definition: Steady State (or Balanced Growth Path) Equilibrium**

A steady state (or balanced growth path) equilibrium is an equilibrium where the growth rate of all variables depending on time is constant, human \((H)\) and knowledge \((n)\) capital grow at the same rate and \(H, H_j, H_n\) all grow at the same constant rate as \(H\).

With this definition in mind we notice that, when \(g_H^H\) (the growth rate of \(H\)) is constant, then \(u\) is constant as well (see equation (12)). This means that, along the balanced growth path, the household will optimally decide to devote a constant fraction of its fixed time endowment to working \((u^*)\) and education \((1-u^*)\) activities. Solving explicitly the representative consumer’s problem, it is possible to show that the following results do hold in the steady state (mathematical derivation of such results can be obtained from the author upon request):

\[
(20) \quad r = \frac{\alpha \delta + \lambda (1-\alpha)(\delta - \rho)}{\alpha} ;
\]

\[
(21) \quad g_{H^H} = g_{H_j} = g_{H_n} = g_n = g_H = \delta - \rho ;
\]

\(^{12}\) Given our assumptions on the size of the representative household and the population growth rate, \(h \equiv H\) (which implies that we can use \(g_H^H\) instead of \(g_h\)).
According to result (20), the real interest rate \( r \) is constant. Equation (21) states that along the balanced growth path, the number of new ideas \( n \), the consumer’s total human capital stock \( H \) and the human capital stocks devoted respectively to the final output production \( H_j \), to the intermediate sector \( H_i \) and to research \( H_R \) all grow at the same constant rate, given by the difference between the schooling technology productivity parameter \( \delta \) and the subjective discount rate \( \rho \). Equation (22) gives the equilibrium growth rate of consumption and the consumer’s asset holdings. Equations (23) and (24), instead, give respectively the equilibrium values of the constant \( H_j/n \) and \( H_i/n \) ratios, whereas equation (25) represents the optimal and constant fraction of the representative agent’s time endowment that he/she will decide to allocate to working activities \( u^* \). For the growth rate of the variables in equations (21) and (22) to be positive and bounded, \( \delta \) should be strictly greater than \( \rho \) and bounded. The condition \( \delta > \rho \) also assures that \( 0 < u^* < 1 \).

### 3. Endogenous Growth and the Shares of Human Capital devoted to the different activities.

To compute the output growth rate of this economy in a symmetric, steady state equilibrium, first rewrite equation (1) as follows:

\[
Y_t = AH_{Ht}^{\lambda} \left( \frac{B \cdot H_j}{n_t} \right)^{\lambda} = \Psi H^{\lambda} n_{t}^{\lambda}, \quad \Psi \equiv A \left( \frac{B \cdot H_j}{n_t} \right)^{\lambda}.
\]

Then, taking logs of both sides of this expression, totally differentiating with respect to time and recalling that in the steady-state equilibrium \( g_{Ht} = g_n = g_H = \delta - \rho \) (see equation (21) above), we obtain:
Hence, economic growth depends only on $\alpha$ (the inverse of which can be easily interpreted as a measure of the monopoly power enjoyed by each intermediate local monopolist$^{13}$), $\lambda$ (which represents the share of total income being devoted in a symmetric equilibrium to the purchase of all the available capital goods varieties$^{14}$) and the accumulation rate of human capital ($g_{H}$). In this last respect the model supports the main conclusion of that branch of the endogenous growth literature pioneered by Uzawa (1965) and Lucas (1988)$^{15}$. As a consequence, and in line with this literature, our analysis does not display any scale effect, since $g_{r}$ depends neither on the absolute dimension of the economy (its total human capital stock), nor on the population growth rate (that, indeed, is equal to zero in our model)$^{16}$.

In equation (1a) the equilibrium growth rate of output depends on $\lambda(\beta - 1)$ that measures the returns to specialization. Such returns depend (positively) not only on $\beta$ (the monopoly power), but also on $\lambda$. The intuition behind this result is as follows: the higher the mark-up rate that can be charged over the marginal cost in the monopolistic sector and the share of national income spent on the intermediate inputs, the higher the return an intermediate producer may obtain from specializing in the production of the marginal variety of capital goods. Moreover, it is also worth pointing out that $\beta$ enters the equilibrium growth rate when (and only when) $\lambda$ is not equal to zero (i.e. when capital goods are an input in the production of the final good). This is clear when one considers that the only product market where imperfect competition prevails in the model is the intermediate one.

$^{13}$ The higher $\alpha$, the higher the elasticity of substitution between two generic intermediate inputs. This means that they become more and more alike when $\alpha$ grows and, as a consequence, the price elasticity of the derived demand curve faced by a local monopolist tends to be infinitely large when $\alpha$ tends to one. Thus, the inverse of $\alpha (1/\alpha)$ may be considered as a measure of how uncompetitive the capital goods sector is.

$^{14}$ $\lambda = \frac{\int (p_{j} \cdot x_{j})dj}{Y_{l}}$.

$^{15}$ Benhabib and Spiegel (1994), Islam (1995) and Pritchett (1996), among others, all suggest that, unlike Lucas (1988), international differences in per-capita growth rates depend on differences in the respective human capital stocks each country is endowed with. However, Jones (1995a,b) points out that the scale effect hypothesis should be rejected on empirical grounds.

$^{16}$ The no-scale-effect prediction is indeed shared by many other models (see Aghion and Howitt, 1998a, Chap. 12; Jones, 1999 and 2003 and Eicher and Turnovsky, 1999 for surveys).
Before being able to compute the shares of human capital devoted to the different economic activities, we first need to determine an expression for the equilibrium human to technological capital ratio \((R \equiv H / n)\). At this aim, we use equation (17), with \(u^* = \rho / \delta\), \(H_j / n = \alpha \delta / (1 - \alpha)C\) and \(H_Y / n = (1 - \lambda) \delta / \lambda C (1 - \alpha)\), and obtain:

\[
\frac{H_n}{n} = R \frac{\rho}{\delta} \left( \frac{\alpha \delta}{C(1 - \alpha)} \right) = \frac{(1 - \lambda) \delta}{(1 - \alpha) \lambda C} \Rightarrow
\]

\[
g_n = C \frac{H_n}{n} = RC \frac{\rho}{\delta} \left( \frac{\alpha \delta}{1 - \alpha} \right) \frac{(1 - \lambda) \delta}{\lambda (1 - \alpha)}
\]

Equating the last expression above to equation (21) yields:

\[
\frac{H_n}{n} = \frac{\delta(\delta - \lambda \rho (1 - \alpha))}{\lambda \rho (1 - \alpha) C}, \quad \forall t.
\]

Given \(R\), the shares of human capital devoted to each sector employing this factor input in the decentralised, symmetric balanced growth path equilibrium can be easily determined as follows:

\[
\begin{align*}
    s_j & \equiv \frac{H_j}{H} = \frac{H_j}{n} \cdot \frac{n}{H} = \frac{H_j}{nR} = \frac{\alpha \lambda \rho}{\delta - \lambda \rho (1 - \alpha)}; \\
    s_Y & \equiv \frac{H_Y}{H} = \frac{H_Y}{n} \cdot \frac{n}{H} = \frac{H_Y}{nR} = \frac{\rho (1 - \lambda)}{\delta - \lambda \rho (1 - \alpha)}; \\
    s_n & \equiv \frac{H_n}{H} = \frac{H_n}{n} \cdot \frac{n}{H} = \frac{H_n}{nR} = \frac{\lambda \rho (\delta - \rho)}{\delta (\delta - \lambda \rho (1 - \alpha))}, \quad \frac{H_n}{n} = \frac{\delta - \rho}{C}; \\
    s_H & \equiv \frac{H_H}{H} = 1 - u^* = \frac{\delta - \rho}{\delta}.
\end{align*}
\]

Thus, the shares of human capital devoted to each activity depend on the technological \((\lambda \text{ and } \delta)\) and preference \((\rho)\) parameters and, more importantly, on the degree of competition in the capital goods sector \((\alpha)\). As it would be outside the scope of this paper, in what follows we do not analyse the way these four variables (respectively \(\lambda\), \(\delta\), \(\rho\) and \(\alpha\)) may influence the distribution of human capital across the different sectors.\(^{17}\)

\[^{17}\text{See Bucci (2002) for some comparative statics results.}\]

All the results stated up to now have been obtained under the assumptions that $\delta$ is strictly greater than $\rho$.\textsuperscript{18} In the present section, while keeping this assumption, we study how the sectoral shares of human capital and the relationship between product market power and economic growth may change when $\lambda$ is assumed to be respectively equal to zero, one and $\alpha$ (i.e., when we allow the production function in the downstream sector to vary).

Case (a): $\lambda = 0$.

In this case the technologies adopted in each economic sector (in the symmetric, steady state equilibrium) are the following:

\[
Y_t = aH_t, \quad a = \frac{A\rho}{\delta}\quad \text{(for the final goods production)};
\]
\[
x_{it} = B \cdot h_{it}, \quad \forall j(0, n_t) \quad \text{(for the capital goods production)};
\]
\[
n_t = C \cdot H_{nt} \quad \text{(for research)};
\]
\[
h_t = (\delta - \rho) \cdot h_t \quad \text{(for human capital supply)},
\]

and the model we are dealing with is the Rebelo (1991)-Lucas (1988) one or “aH-model”. The main variables of the model take on the following values:

\[
s_j = 0; \quad s_Y = \frac{\rho}{\delta}; \quad s_n = 0; \quad s_H = \frac{\delta - \rho}{\delta}; \quad r = \delta ;
\]

(32) $g_{ht} = g_{ht} = g_{ht} = g_a = g_H = g_e = g_a = g_Y = \delta - \rho$.

As is well known, both in Rebelo (1991) and Lucas (1988), technical progress happens through devoting resources to physical (human) capital accumulation rather than a deliberate R&D activity aimed at expanding the set of available (horizontally differentiated) capital goods. In case (a) this is reflected in the fact that the intermediate inputs do not enter the final goods production technology and $s_j = s_n = 0$. Thus, all the human capital is distributed between the final output ($s_Y$) and education ($s_H$) sectors.

\textsuperscript{18} This hypothesis assures that the human capital accumulation rate is positive.
Since capital goods are not productive inputs, market power \((1/\alpha)\), which in the model outlined in the previous sections arises from the intermediate sector, does not play any role on the growth rate of output \((g_y)\). As in Lucas (1988), this last coincides with the growth rate of human capital and is equal to the difference between the productivity of the schooling technology \((\delta = r)\) and the subjective discount rate \((\rho)\).19 Finally, it is worth noticing that, in a long run equilibrium where each sector gets a constant fraction of the available stock of human capital, \(s_y\) affects only the level of output \((Y_i = As_yH_i)\), whereas its growth rate is solely driven by \(s_H\) \((g_y = \delta \cdot s_H)\).

**Case (b):** \(\lambda = 1\).

In this case the technologies employed in each economic sector (in the symmetric steady state equilibrium) are the following:

\[
Y_i = A \left[ \int_0^n \left( x_{ij} \right)^\alpha dj \right]^{1/\alpha}, \quad \text{(for the final goods production)};
\]
\[
x_{ij} = B \cdot h_{ji}, \quad \forall j(0,n_i) \quad \text{(for the capital goods production)};
\]
\[
n_t = C \cdot H_{nt} \quad \text{(for research)};
\]
\[
h_t = (\delta - \rho \cdot h_j) \quad \text{(for human capital supply)},
\]

and the model we are dealing with is the Lucas (1988)-Grossman and Helpman (1991, Chap. 3, pp. 43/57) one. The main variables of the model now assume the following values:

\[
s_j = \frac{\alpha \rho}{\delta - \rho (1 - \alpha)}; \quad s_y = 0; \quad s_n = \frac{\rho(1 - \alpha)(\delta - \rho)}{\delta (\delta - \rho (1 - \alpha))}; \quad s_H = \frac{\delta - \rho}{\delta};
\]
\[
r = \frac{\alpha \delta + (1 - \alpha)(\delta - \rho)}{\alpha}; \quad g_{u_t} = g_{u_j} = g_{u_n} = g_n = g_H = \delta - \rho;
\]
\[
(33) \quad g_h = g_n = g_y = \frac{\delta - \rho}{\alpha}
\]

In this case human capital enters only indirectly (through the capital goods) the final output technology, whereas it continues to be employed in all the remaining sectors \((s_y = 0 \text{ and } s_j, s_n \text{ and } s_H \text{ are all positive})\). As in the previous case, the accumulation

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rate of human capital is equal in equilibrium to $\delta - \rho$, but now the growth rate of output ($g_Y$) depends positively and unambiguously on the mark-up rate ($1/\alpha$). The reason is that in the present case $g_Y$ is a function not only of $s_{H}$, but also of $s_{n}$:

$$g_Y = \frac{(1-\alpha)\rho\delta s_n}{\rho(1-\alpha) - \delta s_n} + \delta s_H,$$

and it is easy to show that $\frac{\partial s_n}{\partial (1/\alpha)} > 0$ and $\frac{\partial g_Y}{\partial s_n} > 0$. In other words, in this particular case it is through allocating a higher share of human capital from the intermediate sector ($\partial s_j/\partial (1/\alpha) < 0$) towards the research sector that monopoly power positively affects growth.

**Case (c): $\lambda = \alpha$.**

The last special case we wish to deal with in this section is the case where $\lambda = \alpha$. Under this assumption the technologies adopted in each economic sector (in the symmetric long-run equilibrium) are the following:

$$Y_t = AH^{1-\alpha} \int_0^n \left( x_j \right)^\alpha d j,$$

(for the final goods production);

$$x_j = B \cdot h_j, \quad \forall j(0,n),$$

(for the capital goods production);

$$h_t = C \cdot H_{nt},$$

(for research);

$$h_t = (\delta - \rho) \cdot h_j$$

(for human capital supply).

The main variables of the model take on the following values:

$$s_j = \frac{\alpha^2 \rho}{\delta - \alpha \rho (1-\alpha)}; \quad s_y = \frac{\rho(1-\alpha)}{\delta - \alpha \rho (1-\alpha)}; \quad s_n = \frac{\alpha \rho (1-\alpha)(\delta - \rho)}{\delta [\delta - \alpha \rho (1-\alpha)]}; \quad s_H = \frac{\delta - \rho}{\delta};$$

$$r = \delta (2-\alpha) - \rho (1-\alpha); \quad g_{Ht} = g_{Hj} = g_{Hn} = g_n = g_H = \delta - \rho;$$

$$g_c = g_a = g_Y = (2-\alpha)(\delta - \rho).$$

---

20 This is the only case considered in Bucci (2003b), where the market power and the returns to specialization are not disentangled (they both depend exclusively on $\alpha$ in a symmetric, balanced growth path equilibrium).
In the present case human capital is employed in each economic sector. Thus, we can identify this case (unlike the two previous ones) as that in which the inter-sectoral competition for the same input (human capital) is tougher ($s_j, s_y, s_n$ and $s_H$ are all positive). As before, the accumulation rate of human capital is equal in equilibrium to $\delta - \rho$, but now (unlike case (b)) the relationship between $s_n$ and $1/\alpha$ is non-monotonic\(^{21}\) and the growth rate of output ($g_y$) is a positive and non-linear (concave) function of the mark-up rate ($1/\alpha$).\(^{22}\)

The main results of the model concerning the relationship between the type of production technology in use in the downstream sector, the inter-sectoral distribution of human capital, the degree of product market power and the aggregate growth rate can be summarised through the following propositions:

**Result 1**

In a generalised, integrated growth model of deterministic R&D activity and human capital accumulation where economic growth is sustained by a supply of skills à la Lucas (1988), as the one described by the equilibrium equations (20) through (31) and (1a), there always exists (except when $\lambda = 0$) a positive relationship between monopoly power ($1/\alpha$) and aggregate growth ($g_y$).

**Proof:**

See equations (1a), (32), (33) and (34).

The reason why there exists no relationship between market power and growth when $\lambda = 0$ is that in this case there is neither an intermediate sector, nor a research one (accordingly, the output growth rate is completely independent of the mark-up that in the model arises from the capital goods sector). What Result 1 seems to suggest is the following: as far as the steady state relationship between the degree of product market power and economic growth is concerned, we can replicate one of the most important results obtained in the basic Schumpeterian model of growth\(^ {23}\) by using a horizontal product differentiation approach where: 1) the engine of growth is human capital accumulation; 2) there exists no pecuniary externality from purposive R&D activity,

\(^{21}\) See Bucci (2003b), pp. 274-75 for an intuition.

\(^{22}\) See Bucci (2003b), pp. 277-78 for further details.

\(^{23}\) Namely, Aghion and Howitt (1992), where more product market power unambiguously spurs economic growth in the steady state.
and 3) human and technological capital grow at the same constant and positive rate in the long-run (balanced growth path equilibrium).

**Result 2**

In a generalised, integrated growth model of deterministic R&D activity and human capital accumulation where economic growth is sustained by a supply of skills à la Lucas (1988), as the one described by the equilibrium equations (20) through (31) and (1a), both the type of technology being used in the final output sector and the level of inter-sectoral competition for the (growing) human capital do affect the shape of the relationship between aggregate growth ($g_Y$) and monopoly power ($1/\alpha$).

Indeed, such a relationship is linear in case (b) - where human capital is not directly employed in the final output sector, whose technology is of the CES type - and concave in case (c) - where human capital is used everywhere and the final output technology is (an extension of) Cobb/Douglas. Similar results are obtained in a model where the growth engine is the R&D externality and there is no human capital accumulation (see Results 1 and 2 in Bucci (2003a)).

**Result 3**

In a generalised, integrated growth model of deterministic R&D activity and human capital accumulation where economic growth is sustained by a supply of skills à la Lucas (1988), as the one described by the equilibrium equations (20) through (31) and (1a), both the type of technology being used in the final output sector and the level of inter-sectoral competition for the (growing) human capital do affect the intensity of the equilibrium growth rate. This last is higher whenever the final output technology is CES and does not employ human capital.

**Proof:**

From equations (32), (33) and (34) one easily concludes that: $g_Y$ (case b) > $g_Y$ (case c) > $g_Y$ (case a).

This result parallels Results 3 and 4 of Bucci (2003a). Therefore, even when human capital is allowed to grow over time, the highest possible growth rate is obtained within a Grossman and Helpman-type economy. On the contrary, the lowest growth rate does prevail in a Rebello-Lucas-type-economy, where the final output technology is linear in
the human capital input and all the existing markets (final output and education) are perfectly competitive.

5. Concluding Remarks.

In this article we presented a generalization of Bucci (2003b). The generalisation we proposed here consists in writing the production technology in use in the downstream sector in such a way to disentangle the (equilibrium) monopolistic mark-up set in the intermediate sector and the degree of returns to specialization. Depending on the value of a specific parameter (the share of total income being devoted to the purchase of the available capital goods varieties), in the present paper we were able to study the relationship between product market power and economic growth as emerges from two different classes of endogenous growth models: a) the Rebelo’s model (1991) with human (instead of physical) capital accumulation, and b) the Grossman and Helpman’s model (1991, Chap.3) of endogenous technological change without knowledge spillovers and human capital investment. At the same time, the proposed generalization allowed us to encompass the model of Bucci (2003b) as a special case and to analyse the impact both the kind of technology in use in the downstream sector and the degree of inter-sectoral competition for the (growing) human capital have on the market power-growth nexus and the level of the equilibrium growth rate in the presence of human capital accumulation, the growth engine. In this last respect, we compared our results with those obtained in Bucci (2003a), where human capital is in fixed supply and economic growth is driven by the positive externality from R&D activity.

Our main findings were threefold. First of all, we found that the presence of more intense product market power within the sector producing capital goods turns out to have always positive growth effects (except when the share of national income spent on the purchase of capital goods is exactly equal to zero). This confirms one of the results found by Bucci (2003b), according to which it is possible to restore the Schumpeterian growth paradigm (positive relationship between market power and aggregate growth) provided that: 1) human capital accumulation (à la Lucas) is the engine of growth; 2) there exists no pecuniary externality from purposive R&D activity, and 3) human and technological capital grow at the same constant and positive rate in the long-run (balanced growth path equilibrium). Secondly, we obtained that, though positive, the relationship linking market power and economic growth may be linear or concave depending on the type of technology employed in the final output sector (CES versus
Cobb-Douglas) and the way human capital is distributed across sectors (whether this factor input is employed everywhere or not). Finally, we stated that these two elements (the type of technology and the intensity of the inter-sectoral competition for human capital) are also able to affect the level of the steady state growth rate. This is higher within a Grossman-Helpman-Lucas-type economy where the final output technology is CES and does not employ human capital directly.

Our findings depend on the hypothesis (common to all the first-generation innovation-based growth models) that there exists no strategic interaction among rivals on goods and factor markets. In the future it could be interesting to analyse how, within our simplified framework, the market power-growth relationship might change when one explicitly allows for the presence of some kind of interaction across firms.
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