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COOPERATION AND NEGOTIATIONS IN A THIN MARKET

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Cooperation and Negotiations in a Thin Market

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Abstract

This paper studies a thin market where one seller and two heterogeneous buyers may endogenously form coalitions in the attempt to enhance their bargaining position in a subsequent negotiation over an indivisible good. The game, of perfect information, is divided in two stages. In the coalition formation stage, coalitions are endogenously formed by non-cooperative bargaining among players. In the negotiation stage, the resulting coalitions, acting as single players, bargain over the price of the good by making offers according to a random order of proposers. The model differs from the one by Montero (1998), in that traders are impatient and there is neither exogenous probability of breaking down negotiations nor reselling. The main conclusions of the paper are as follows. First, if the reservation values of the buyers are not radically heterogenous and the traders are sufficiently patient, there are two possible SSPE in the coalition formation game, where all, and only, the two-persons coalitions have identical probability to be formed. It turns out that, in the limiting friction-less case, our SSPE payoffs converge to the Shapley Value of the corresponding cooperative game.

1 Introduction

This paper studies endogenous coalition formation in a decentralized thin market with one seller and two heterogeneous buyers. Coalitions are formed non-cooperatively in the attempt to gain stronger positions in subsequent trading negotiations.

The model has been closely inspired by the paper by Montero (1998).

In the model it is assumed that a valuable indivisible good may be traded among the players, while its allocation is determined by noncooperative bargaining. Before the bargaining process starts, however, all the players may form coalitions: if a coalition forms, it will act as a single player in the following trading negotiations. The coalition formation process is also modelled as a sequential bargaining procedure.

The game, of complete and perfect information, is divided in two stages: coalition formation and bargaining between coalitions. In the bargaining stage, a coalition is randomly selected to make a proposal to other coalitions. In the coalition formation stage, coalitions are endogenously formed by noncooperative bargaining among players, in the spirit of Hart and Kurz (1983), Chatterjee, Dutta, Ray and Sengupta (1993), Bloch (1996), Serrano and Vohra (1997) and Ray and Vohra (1999).

In particular, we follow Okada (1996) in adopting a bargaining procedure with a random selection of proposer at every round. In the first stage of our model, in fact, a player is selected randomly at every period to propose a coalition and a division of the surplus that the coalition will obtain in the subsequent bargaining stage. In the second stage, then, one of the coalitions that have been previously formed is, again, randomly selected at every period to propose the other coalitions still in the market to trade.

Montero (1998) studies a model where all the players are patient, and negotiations, both in the bargaining and in the coalition formation stage, may break down with an exogenous probability when a proposal is rejected. In contrast, we explore a different model where all the players are equally impatient and there is no risk of exogenous breakdown in the negotiations.

The paper focuses on the role of bargaining and coalitions on price formation in a thin market. In our model, a seller can sell an indivisible object to one of two potential buyers with different reservation prices. The allocation of the good and the payments to be made are determined by bargaining among players. The reservation prices are common knowledge.

A central question to explore may be whether the buyer with the lower reservation price should expect a positive payoff from exchange. Efficient trade, indeed, implies the weak buyer can never expect to get the good. However, intuitively one may argue that his presence in the market would clearly benefit the seller, who is likely to ask for an higher price as facing two buyers.

If coalitions may be formed before the bargaining process, the weak buyer may benefit from this influence. For instance, he may negotiate with the seller and get paid to be in the market. Alternatively, he may negotiate with the other buyer and get paid to be out of the market.

The question, then, is which coalition will form in equilibrium and how the expected payoffs for the three players look like.

The main conclusions of our paper are as follows. First, only if the reservation prices of the two potential buyers are sufficiently similar, the weaker buyer can in fact exploit his influence in the decentralized market by getting a strictly positive payoff.

Second, if the reservation values of the buyers are not radically heterogenous and the traders are sufficiently patient, there are two possible subgame perfect Nash equilibria in pure and stationary strategies in the coalition formation game.

In both equilibria all, and only, the two-persons coalitions have identical probability to be formed as the outcome of non-cooperative bargaining among the traders. In one equilibrium, in particular, the seller proposes to join a coalition with the weak buyer, the latter proposes to form a buyers' cartel, while the strong buyer proposes the seller to join a coalition.

In the other equilibrium, it is the seller to propose a coalition with the strong buyer, the latter proposes to join a buyers' cartel, while the weak buyer proposes to form a coalition with the seller.

Furthermore, if the buyers are still not too heterogeneous but impatience is higher, there is only one subgame perfect equilibrium in the coalition formation stage, in which not all the coalitions form. In particular, most of the times it occurs that both the seller and the strong buyer agree on joining a coalition, while, with minor probability, it is the weak buyer to propose to form a coalition with the seller.

When impatience is even lower, less dense coalition structures emerge in equilibrium, in which some of the traders always opts out to stay alone. When the buyers are also very heterogeneous in their reservation prices, no coalition at all is ever formed.

Therefore, we find that, even if the formation of a buyers' cartel happens at most one-third of the cases, as $\delta \to 1$, the traders on the long side of the market are able to appropriate approximately half of the potential surplus, in contrast with what happens in a friction-less Walrasian thin market.

Finally, it turns out that, as $\delta \to 1$, the stationary subgame perfect equilibrium payoffs of our non-cooperative bargaining and endogenous coalition formation game converges to the Shapley Value of the corresponding cooperative game.

The rest of the paper is organized with a description of the model in the next Section. The bargaining and the coalition formation stages will be solved in Section 2 and 3 respectively. In the last Section the implications of the present model and our results on a future experiment will be briefly discussed.

2 The Model

We study the simplest thin market, formed by a seller and two heterogeneous buyers. The set of agents is thus $N = \{S, B_1, B_2\}$. Agent S is a potential seller who owns one unit of an indivisible good, and derives zero utility from keeping it. Agents B_1 and B_2 are potential buyers whose reservation prices for the good are respectively 1 and λ, with $0 < \lambda < 1$. All players are risk neutral and all valuations are common knowledge. For notational simplicity, we always refer to the seller as a female, and to the two buyers as males.

Since a coalition is any non-empty subset of the set of players, in a game with n players we could have 2^n possible coalitions. In our case these are the empty one $\{\emptyset\}$, the singletons $\{S\}$, $\{B_1\}$, $\{B_2\}$, the two-players coalitions $\{S, B_1\}$, $\{S, B_2\}$, $\{B_1, B_2\}$, and, finally, the grand coalition $\{S, B_1, B_2\}$.

Because interactions between the different coalitions can be very complicate, in cooperative game theory is often used the simplifying assumption of transferable utility. That is, there is often assumed to be a commodity - called money - that players can freely transfer among themselves, such that any player's utility payoff increases one unit for every unit of money that he gets.

With transferable utility, the cooperative possibilities of a game can be described by a characteristic function v that assigns a number $v(K)$ to every coalition K: that is, a TU-game is defined by a couple (N, v) where $N = \{1, ..., n\}$ is the set of players and $v : 2^n \rightarrow R$ is the characteristic function.

Here $v(K)$ is called the worth of coalition $K \in 2^n$, and it represents the total amount of transferable utility that the members of S could earn without any help from the players outside of K. In any characteristic function, we let $v(\emptyset) = 0$, where $\emptyset$ denotes the empty set.

Hence it is easy to see that the characteristic function associated to the above situation in our thin market is the following: $v(\{\emptyset\}) = 0$, $v(\{i\}) = 0$ for any $i = S, B_1, B_2$, $v(\{S, B_2\}) = \lambda$, $v(\{S, B_1\}) = 1$, $v(\{B_1, B_2\}) = \emptyset$, $v(\{S, B_1, B_2\}) = 1$.

By means of such a characteristic function it is also possible to describe the Core of the corresponding cooperative TU-game, as the set of imputations x which are stable to any unilateral or group deviation, or, in other words, such that $\sum_{i \in K} x_i \geq v(K)$, for all $K \in 2^N \setminus \{\emptyset\}$. In fact, the Core of this game clearly consists of all the vectors x in R^3_+ such that $x(S) \geq \lambda$, $x(S) + x(B_1) = 1$ and $x(B_2) = 0$: graphically, the Core is the line segment in the $[S, B_1, B_2]$ 3-dimensional space with endpoints $(1, 0, 0)$ and $(\lambda, 1 - \lambda, 0)$.

It is important to notice that the Shapley Value of the corresponding cooperative game assigns to S a value $\frac{1}{2} + \frac{\lambda}{6}$, to the high valuation buyer $\frac{1}{2}$ ¡ $\frac{\lambda}{3}$, and to B_2 a value $\frac{\lambda}{6}$, and that therefore is not in the Core.

The main question we aim at investigating is whether buyer B_2 may expect a positive payo¤ in such a situation. The intuition about the driving forces behind this thin market is as follows. On the one hand, B_2 can never expect to buy the good form the seller, since for any price he is prepared to pay, buyer B_1 is willing to pay more.

On the other hand, buyer B_2 may a¤ect the resulting price of the trade. In fact, if B_2 were not in the market, the intuitive outcome would be the one emerging from a bilateral negotiation between the seller and the high-valuation buyer: B_1 is buying the good for a price close to $\frac{1}{2}$. But, if buyer B_2 is indeed in the market, intuitively the price cannot be lower than λ.

Hence, one may think that buyer B_2 will somehow exploit this power and, for example, he will agree on forming a buyers' cartel with B_1, getting some positive share of the gains derived form the cartel.

In fact, it may be proved (see also Galizzi (2003)) that, if buyers are allowed to meet before a random-matching bargaining with the seller, they always join a buyers' cartel as they both bene…t from sharing the coalitional surplus rather than entering individual negotiation with the seller.

One may object, however, that this should be a not very surprising result, given that it is assumed that the only buyers may always meet together, before negotiation with the seller, with no cost, delay, risk or any other restriction.

Thus, here we would like to explore whether the emergence of a buyers' cartel is indeed a robust …nding even in a more general and neutral environment.

To address this question and to investigate the role of the low-valuation buyer in the process of coalition formation, we model the trading in the described thin market as a two-stages game.

The allocation of the good and the payments are determined in a bargaining process with random proposers. Prior to this bargaining, in stage 1, all the players may form coalitions. In particular, if two players form a coalition, they will bargain with the third player as a single trader.

As in Montero (1998), it is crucially assumed that contracts specifying the division of the coalitional payo¤ are binding and can be enforced.

We now describe the game in more details.

2.1 First Stage: the Coalition Formation Game

The game starts with Nature selecting a proposer: each of the three players is selected with equal probability $\frac{1}{3}$. A proposal consists of a coalition to which the proposer belongs and a division of the coalitional payoff.

The coalitional payoff may be a monetary payment (for instance, the payoff for coalition $\{S\}$ would be the trading price), a consumer's surplus (for example, the payoff of coalition $\{B_2\}$ would be the difference between his reservation price λ and the price actually paid) or a sum of payments and consumer's surplus (for coalition $\{S, B_1\}$, for instance, the payoff would be the total value to be created, 1).

The coalitional payoff is determined at the end of the second stage of the game: players, however, can anticipate it by the usual backwards induction argument.

Because the game includes random selection at every stage, the coalitional payoff is not deterministic. Since all players are risk neutral, only expected payoff matter, and we assume that is the proposer of a coalition that bears all the risks from the chance moves.

Hence, we define a proposal as a pair (K, y) where K is a coalition to which the proposer belongs, and y is a $|K - 1|$-dimensional vector describing the deterministic payoffs to the remaining players in the coalition K.

The proposer is then understood to get the residual expected payoff. Note that if a player proposes a singleton coalition - that is, if proposes to stay alone - he does not need to specify any payoff division or accept his own proposal.

The key point to be underlined here is that the expected coalitional payoff depends crucially on which other coalitions form. To capture this idea, following Ray and Vohra (1999) we should better refer to the partition function of a coalition rather than to the characteristic function, described above, in which clearly the value of the coalition does not depend on which other coalition formed. In our thin market, the analysis is greatly simplified by the fact that, since there are only three players, the coalitions structure is automatically determined given a coalition K, unless K is a singleton.

Hence we denote with by $\varphi(K; \pi)$ the expected payoff for the coalition K when the coalition structure is $\pi = (K_1, ..., K_n)$. Given a coalition structure π we will denote by $\varphi(\pi)$ the payoff vector whose j-th element is the expected payoff of coalition K_j.

Once a proposal is made, the rest of the players in K accept or reject the proposal sequentially according to the order S, B_1, B_2. It is important to note that the order in which players accept or reject offers does not affect the results, since the first player to reject has no advantage over the other players given that the proposer in any stage is randomly selected.

If the proposal is accepted by all the players in the coalition, the coalition is formed and its members retire from this stage. This also implies that formed coalitions can not be enlarged in the following stages.

As soon as the proposal is rejected by one of the responder, all the players enter a new coalition formation stage.

Since it is reasonable to believe that traders are somehow impatient, all the players are assumed to discount the future payoff by the common intertemporal discount rate δ.

The coalition formation stage lasts until a coalition structure is formed, that is when all traders have accepted a proposal to join a coalition. Note that this may be represented by a collection of singletons as well, but also note that, since there are only three players in the game, the coalition formation stage ends automatically once a non-singleton coalition is formed.

Finally, note that there are two cases in which the whole game ends after this first stage. In fact, if the grand coalition forms, as a proposal to form a coalition includes a payoff sharing rule, the division of the value of the grand coalition has already been decided and nothing remains to be settled.

Analogously, if the coalition $\{S, B_1\}$ forms, it can achieve a payoff of 1 by itself, and then will never enter any further bargaining stage.

2.2 Second Stage: the Bargaining Game

The bargaining stage is played between the coalitions formed at the first stage. In this bargaining game between coalitions, each coalition acts as a single player. For instance, we may imagine that, like in Galizzi (2003), each coalition nominates a representative and sends him to the negotiation.

Thus, depending on the outcome of the first stage, bargaining takes place between a two-player coalition and a single player, or among all the three traders.

The bargaining process is as follows. First a coalition is randomly chosen by Nature to be the proposer: all coalitions are chosen with equal probability. This coalition makes a proposal about the allocation of the good and the transfers between coalitions.

The coalitions affected by the proposal, accept or reject sequentially, again according to the mentioned order in case of singletons.

If the proposal is accepted, the agreement is implemented, trade takes place and the game ends.

If a proposal is rejected, all the coalitions that have been formed enter a new bargaining stage. No further coalition formation stage is permitted. Again, all the players in the coalitions are assumed to discount the future payoff by the common intertemporal discount rate δ.

It is important to notice the difference between $\varphi(K;\pi)$, the expected payoff of a coalition in this bargaining process, and $v(K)$, the payoff the coalition gets by itself - that is when it is isolated form the other players. Indeed, the coalition will often be able to do better than the latter, and can improve upon $v(K)$ by reaching an agreement with other coalitions.

Consider for example the coalition $\{B_1, B_2\}$: since neither B_1 nor B_2 own the good, $v(\{B_1, B_2\}) = \emptyset$. However, as mentioned before, the coalition $\{B_1, B_2\}$ can act as a single buyer and reach an agreement with S about the price of the good: hence, as we will show below, $\varphi(\{B_1, B_2\}; [\{S\}, \{B_1, B_2\}]) = \frac{1}{2} > 0$.

Note that the coalition formation stage and the bargaining stage are formally very similar: in both stages players move sequentially and proposers are selected randomly.

However there are two main differences. First, the stages differ in the players, since the first stage is played among individuals, whereas the second one is played among coalitions. Secondly, the stages differ in the content of proposals, as in the first stage proposals include a coalition and a payoff division within the coalition, while in the second one a proposal consists of a payoff division among coalitions.

2.3 The Equilibrium Concept

The history of the game at a give moment consists of all proposers, proposals and responses so far. A strategy for a trader in the first stage, as well as for a coalition in the second stage, assigns proposals to all nodes at which the player is the proposer, and a response to all possible proposals at every node the player is a responder.

A strategy is stationary if it is independent of the past history, except all the payoff-relevant aspects, like the coalition structure that has been formed, and the current proposal.

A stationary subgame perfect equilibrium is a subgame perfect equilibrium in which each player employs a stationary strategy. To solve this game, we focus on stationary subgame perfect equilibria.

The game, then, can be solved by backward induction. First the equilibrium of the bargaining stage is found for each possible coalition structure. This determines the expected payoffs, described by the partition function $\varphi(K;\pi)$, that are used as an input to solve the coalition formation game.

3 Solving the Bargaining Game

The bargaining process in the second stage depends on the outcome of the first stage. Remember that if either the grand coalition $\{S, B_1, B_2\}$ or the coalition $\{S, B_1\}$ have been formed, nothing remains to be settled, as the total value 1 has already been divided by the members of the coalition. Hence there is no further bargaining in these cases, and the game ends.

Thus, there are only three bargaining processes to be considered, corresponding to the following coalition structures: the one where all the coalitions are singletons $[\{S\}, \{B_1\}, \{B_2\}]$, the one with a buyers' cartel $[\{S\}, \{B_1, B_2\}]$, and the one with a coalition between the seller and the low-valuation buyer $[\{S, B_2\}, \{B_1\}]$.

3.1 The Bargaining Stage with $\pi = [\{S\}, \{B_1\}, \{B_2\}]$

If the coalition structure resulting form the first stage is $\pi = [\{S\}, \{B_1\}, \{B_2\}]$, bargaining takes place among individual players.

Negotiations start by a chance move that selects the first proposer: each player is selected with equal probability $\frac{1}{3}$.

If the seller is selected, she can choose one of the two buyers and she can offer him the good for a proposed price. If one of the buyers is selected, he can propose the seller a price to buy the good. Then, in this model the matching between the seller and one of the buyer is completely endogenous and deterministic. Note also that the seller is restricted to address the offer only to one buyer, in contrast with a model of public offers negotiation in Osborne and Rubinstein (1990).

In contrast with Montero (1998) in our bargaining game among individuals, reselling the good from B_2 to B_1 is impossible. Then the outcome of the bargaining procedure may be inefficient, as it is by theory possible that B_2 rather than B_1 buys the good from the seller. Note, however, that our model embraces the reselling case considering the bargaining game with coalition structure $[\{S, B_2\}, \{B_1\}]$, so that all the possible emerging cases are explored.

It seems reasonable to expect that the presence of a second buyer in the thin market will not affect the price if the latter is not prepared to pay more than the amount the seller may obtain restricting her to bilateral negotiations with B_1.

On the other hand, with λ sufficiently high, intuitition should suggest that the seller will sell the good to B_1 at a price equal to λ.

Define $V(i)$ as the expected payoff by player $i = S, B_1, B_2$ from entering the bargaining stage. Then the following Propositions state that the above intuition seems to be partially supported.

Proposition 1 In the bargaining stage with coalition structure $\pi = [\{S\}, \{B_1\}, \{B_2\}]$, with no possibility of reselling, as $\lambda \geq \tilde{\lambda} = \frac{3\delta - 2\delta^2}{\delta^2 - 9\delta + 9}$, the following strategies constitute the unique subgame perfect equilibrium of the game:

- The seller S, when selected to be the proposer, always makes an offer to buyer B_1, asking for a price $p_S^* = 1 - \delta V(B_1)$, and, when she has to respond a proposal, she accepts any price $p_{B_i}^* \geq \delta V(S)$ from buyer B_i, $i = 1, 2$;

- Buyer B_1, when selected to be the proposer, always offers the seller a price $p_{B_1}^* = \delta V(S)$, and, when he has to respond, he accepts any price $p_S^* \leq 1 - \delta V(B_1)$;

- Buyer B_2, when selected to be the proposer, always offers the seller a price $p_{B_2}^* = \delta V(S)$, and, when he has to respond, always accepts any price $p_S^* \leq \lambda - \delta V(B_2)$.

Proof. In the Appendix. ■

Therefore, when the reservation prices of the buyers are not too heterogeneous, they both attempt to trade with the seller by offering his continuation payoff. On the other hand, the seller strictly prefers to negotiate exclusively with the strong buyer. Note that, in such a case, trade always occurs with no delays.

However, the outcome of the bargaining process may be allocatively inefficient as the low-valuation buyer gets the good with $\frac{1}{3}$ probability.

Proposition 2 In the bargaining stage with coalition structure $\pi = [\{S\}, \{B_1\}, \{B_2\}]$, with no possibility of reselling and $\lambda < \tilde{\lambda} = \frac{3\delta - 2\delta^2}{\delta^2 - 9\delta + 9}$, the following strategies constitute the unique subgame perfect equilibrium of the game:

- The seller S, when selected to be the proposer, always makes an offer to buyer B_1, asking for a price $p_S^* = 1 - \delta V(B_1)$, and, when she has to respond a proposal, she accepts any price $p_{B_i}^* \geq \delta V(S)$ from buyer B_i, $i = 1, 2$;

- Buyer B_1, when selected to be the proposer, always offers the seller a price $p_{B_1}^* = \delta V(S)$, and, when he has to respond, he accepts any price $p_S^* \leq 1 - \delta V(B_1)$;

- Buyer B_2, when selected to be the proposer, always offers the seller the lowest price satisfying $p_{B_2}^* < \delta V(S)$, and, when he has to respond, always accepts any price $p_S^* \leq \lambda - \delta V(B_2)$.

Proof. In the Appendix. ■

Hence, if $\lambda < \tilde{\lambda}$, the reservation price of the weak buyer is so low that, whenever he has the chance, he prefers to make unacceptable offers to the seller. The latter's continuation payoff, in fact, is unaffordable for him.

Therefore the entire bargaining process looks like a bilateral negotiation between the seller and B_1.

The outcome of the process is clearly allocatively efficient, since always the high-valuation buyer obtains the good. However, the outcome may be inefficient because of the delays in the trading due to the $\frac{1}{3}$ probability that B_2 is selected to make an offer.

Also note that $\tilde{\lambda} = \frac{3\delta - 2\delta^2}{\delta^2 - 9\delta + 9}$ is always strictly lower than 1, and just converges to 1 as $\delta \to 1$. That is, in a thin market with no frictions, there is no offer affordable by the weak buyer that can possibly be accepted by the seller: the latter will always wait a round more to have the chance to bargain with the strong buyer. Indeed, whenever $\delta \to 1$, trade with B_2 would only occur if the buyers were identical.

Notice also that, as intuition suggests, if $\lambda \geq \tilde{\lambda}$ the seller benefits from the presence of a second buyer, which, however, always expects zero payoffs. In fact, the following holds.

Corollary 3 In the bargaining stage with coalition structure $\pi = [\{S\}, \{B_1\}, \{B_2\}]$, and with no possibility of reselling, the expected payoffs $V(i)$ by the traders by entering the negotiation are the following:

- If $\lambda < \tilde{\lambda}$, $V(S) = V(B_1) = \frac{1}{3-\delta}$, while $V(B_2) = 0$. Clearly $\lim_{\delta \to 1} V(S) = \lim_{\delta \to 1} V(B_1) = \frac{1}{2}$.
- If $\lambda \geq \tilde{\lambda}$, $V(S) = \frac{3-2\delta}{\delta^2 - 9\delta + 9}$, $V(B_1) = \frac{3(1-\delta)}{\delta^2 - 9\delta + 9}$, and $V(B_2) = \frac{1}{3}\ \lambda - \frac{3\delta - 2\delta^2}{\delta^2 - 9\delta + 9}$. As $\delta \to 1$, $\lim_{\delta \to 1} V(S) = 1$, $\lim_{\delta \to 1} V(B_1) = 0$, and $\lim_{\delta \to 1} V(B_2)$ gets negative.

Proof. In the Appendix. ■

Note that, when $\lambda \geq \tilde{\lambda}$, as $\delta \to 1$, the seller is able to appropriate all the surplus from the trade. Also note that, when $\lambda \geq \tilde{\lambda}$, the low-valuation buyer is always able to get some positive surplus from this individual bargaining process as long as $\delta \neq 1$. Finally, as $\lambda \geq \tilde{\lambda}$, the seller greatly benefit from the presence of the low-valuation buyer in the thin market withy respect to a bilateral negotiation, while the high-valuation buyer is hurt to the same extent.

3.2 The Bargaining Stage with $\pi = [\{S\}, \{B_1, B_2\}]$

The second case arises when both buyers decide to enter a coalition to face the seller in the negotiations. When such a buyers' cartel is formed, everything is as if there is effectively only one buyer in the market: the two buyers nominate a representative of the coalition to bargain with the seller, the representative negotiate on trading price, the cartel's surplus from the trade is split according to the sharing rule agreed on at the first stage.

Negotiations start by a chance move that selects the first proposer: each coalition is selected with equal probability $\frac{1}{2}$.

If the $\{S\}$ coalition is selected, it can the buyers $\{B_1, B_2\}$ the good for a proposed price. If the buyers' cartel $\{B_1, B_2\}$ is selected, it can propose the seller a price to buy the good.

Notice that the two coalitions are symmetric in that the values of the characteristic function for both of them are zero. Hence, it seems reasonable to expect that each coalition will split equally the potential gain from the trade. Define $V(\{K\})$ the expected payoff by coalition K from entering the bargaining stage. The following Proposition describes the equilibrium.

Claim 4 In the bargaining stage with coalition structure $\pi = [\{S\}, \{B_1, B_2\}]$, the following strategies constitute the unique subgame perfect equilibrium of the game:

- Coalition $\{S\}$ always offers a price $p^*_{\{S\}} = 1 - \delta V(\{B_1, B_2\})$ when is selected to make an offer, and always accepts any price $p^*_{\{B_1,B_2\}} \geq \delta V(\{S\})$, when she has to respond.
- Buyers' cartel $\{B_1, B_2\}$ always offers a price $p^*_{\{B_1,B_2\}} = \delta V(\{S\})$ when is selected to make an offer, and always accepts any price $p^*_{\{S\}} \leq 1 - \delta V(\{B_1, B_2\})$, when it has to respond.

Proof. In the Appendix. ■

The following Proposition show that, as intuition suggests, in equilibrium the seller and the buyers' cartel split equally potential surplus from trade.

Claim 5 In the bargaining stage with coalition structure $\pi = [\{S\}, \{B_1, B_2\}]$, both the seller $\{S\}$ and the buyers' cartel $\{B_1, B_2\}$ expect a coalitional surplus equal to $\frac{1}{2}$.

Proof. In the Appendix. ■

Note the perfect equivalence with a Rubinstein bilateral bargaining with random selection of the proposer: the price of the good depends on which coalition is the proposer, as the seller always offers a price $1 - \frac{\delta}{2}$, while the buyers' cartel proposes a price of $\frac{\delta}{2}$.

3.3 The Bargaining Stage with $\pi = [\{S, B_2\}, \{B_1\}]$

The last case emerging in the bargaining process occurs when in the first stage the coalition structure $\pi = [\{S, B_2\}, \{B_1\}]$ emerges.

Negotiations start by a chance move that selects the first proposer: each coalition is selected with equal probability $\frac{1}{2}$.

If the $\{S, B_2\}$ coalition is selected, it can offer $\{B_1\}$ the good for a proposed price. If $\{B_1\}$ is selected, he can propose the other coalition a price to buy the good.

Consider the value of the characteristic function for the coalition between the seller and the low-valuation buyer, $v(\{S, B_2\}) = \lambda$: this may also be equivalently interpreted as the lowest value that the coalition $\{S, B_2\}$ would accept for selling the good.

In other words, the effect of formation of the $\{S, B_2\}$ coalition is the same as if the seller will raise her reservation price from zero up to λ: the coalition will accept only prices at least equal to λ to be willing to sell the good.

Then define $V(\{K\})$ the expected payoff by coalition K from entering the bargaining stage. Hence, the equilibrium strategies in this second stage are as described by the following Proposition.

Claim 6 In the bargaining stage with coalition structure $\pi = [\{S, B_2\}, \{B_1\}]$, the following strategies constitute the unique subgame perfect equilibrium of the game:

- Coalition $\{B_1\}$ always offers a price $p^*_{\{B_1\}} = \lambda + \delta V(\{S, B_2\})$ when is selected to make an offer, and always accepts any price $p^*_{\{S,B_2\}} \leq 1 - \delta V(\{B_1\})$, when he has to respond.

- Coalition $\{S, B_2\}$ always offers a price $p^*_{\{S,B_2\}} = 1 - \delta V(\{B_1\})$ when is selected to make an offer, and always accepts any price $p^*_{\{B_1\}} \geq \lambda + \delta V(\{S, B_2\})$, when it has to respond.

Proof. In the Appendix. ■

In fact, it can be argued that this case is equivalent to a situation where the low-valuation buyer has already bought the good from the seller and is then reselling it to the high-valuation buyer. Hence, it seems reasonable to expect the trading price being somewhere in the range $[\lambda, 1]$. The following Proposition show that this indeed the case.

Claim 7 In the bargaining stage with coalition structure $\pi = [\{S, B_2\}, \{B_1\}]$, the good is sold from the coalition to the strong buyer at a trading price equal to $p = \frac{1-\lambda}{2}$. Hence, the coalition $\{S, B_2\}$ expects a coalitional surplus equal to $\frac{1+\lambda}{2}$, while the strong buyer $\{B_1\}$ expects a surplus equal to $\frac{1-\lambda}{2}$.

Proof. In the Appendix. ■

Hence, the outcome of negotiation is as if the coalition resells the good to the high-valuation buyer at a price exactly in between the two reservation prices.

3.4 The Partition Function

Thus we have solved the bargaining game between coalitions for all the possible coalition structures. Keep also in mind that, for two coalition structures emerging at the first stage - the grand coalition and the $\pi = (\{S, B_1\}, \{B_2\})$ - no bargaining game is needed.

As described above, the partition function $\varphi(\pi)$ associated with this game assigns a payoff for each coalition in any coalition structure. The partition function, as resulting from the equilibrium of the bargaining game at the first stage, is given by

$$\varphi(\pi) = \begin{cases} \varphi(\{S, B_1, B_2\}) = 1 \\ \varphi(\{S, B_1\}, \{B_2\}) = [1, 0] \\ \varphi(\{S, B_2\}, \{B_1\}) = \left[\frac{1+\lambda}{2}, \frac{1-\lambda}{2}\right] \\ \varphi(\{S\}, \{B_1, B_2\}) = \left[\frac{1}{2}, \frac{1}{2}\right] \\ \varphi(\{S\}, \{B_1\}, \{B_2\}) = \begin{cases} \left[\frac{1}{3-\delta}, \frac{1}{3-\delta}, 0\right] & \text{if } \lambda < \tilde{\lambda}, \\ \left[\frac{3-2\delta}{\delta^2-9\delta+9}, \frac{3(1-\delta)}{\delta^2-9\delta+9}, \frac{1}{3}\lambda - \frac{3\delta-2\delta^2}{\delta^2-9\delta+9}\right] & \text{if } \lambda \geq \tilde{\lambda} \end{cases} \end{cases}.$$

Note that at the limit, as $\delta \rightarrow 1$, all but the last expression remain the same,while the latter converges to

$$\lim_{\delta \rightarrow 1} \varphi(\{S\}, \{B_1\}, \{B_2\}) \rightarrow \begin{cases} \left[\frac{1}{2}, \frac{1}{2}, 0\right] & \text{if } \lambda < \tilde{\lambda} \\ [1, 0, (< 0)] & \text{if } \lambda \geq \tilde{\lambda} \end{cases}.$$

We can now solve by backward induction the coalition formation game at the first stage, to find out which coalition structures will indeed form in equilibrium.

4 Solving the Coalition Formation Game

As in the bargaining stage, proposers are randomly selected: each players is chosen to make an offer with identical probability $\frac{1}{3}$. A proposal consists in a coalition to which the proposer belongs, and a fixed payment for the members of the coalition other than the proposer.

The proposer is understood to keep the remaining of the coalition's payoff. The coalitional payoff is not deterministic, but its expected value is given by the partition function described above, and it is anticipated by the players by the usual argument of backwards induction.

We treat the cases $\lambda \geq \tilde{\lambda}$ and $\lambda < \tilde{\lambda}$ separately. One may expect that non-singletons coalitions form only in the former case, since in the latter the reservation price of the weak buyer is too low either to make his presence in the thin market desirable for the seller, or to represent a serious threat for the strong buyer.

Then, we consider first the case $\lambda \geq \tilde{\lambda}$, referring to it as a case of mild asymmetry in the reservation prices.

Define $W(i|\overline{\pi})$, with $i = S, B_1, B_2$, the expected payoff by trader i by entering a stage of the coalition formation game, given the already existing coalition structure $\overline{\pi}$. Define the unconditional equivalent $W(i)$ as the expected payoff by trader i by entering the coalition formation game given that no coalition has already been formed.

4.1 Mild Heterogeneity: $\lambda \geq \tilde{\lambda}$

We now proved in the following lemmas that some coalition structures will never emerge in equilibrium. Hence these lemmas restrict the set of the candidate equilibria.

Claim 8 As $\delta > \overline{\delta}$, none of the players proposes to stay alone in a subgame perfect equilibrium.

Proof. In the Appendix. ■

In fact, choosing to stay singleton is always a strictly dominated strategy both for the seller and the strong buyer.

The intuition is that, by proposing to stay on his own, any of the three traders would give the remaining two the chance to join a coalition that will make himself worse off in the following bargaining stage. Hence, in a subgame perfect equilibrium, any player sufficiently patient always proposes a non-singleton coalition in the attempt to avoid the formation, in the next rounds of the first stage, of a coalition that he considers too dangerous for his bargaining position in the second stage of the game. The case of the seller and the buyers' cartel is the most intuitive example.

Hence the coalition structure $\pi = [\{S\}, \{B_1\}, \{B_2\}]$ can never be a subgame perfect equilibrium, due to the fact that each selected proposer may anticipate the coalitions that would emerge in the following rounds, and thus is better off by blocking them with an alternative proposal. Note the resemblance to the idea of the deviating blocking coalitions as lying beyond the concept of the Core.

Thus, only structures with one non-singleton coalition may enter the bargaining game in the second stage. To help to further restrict the set of the candidate equilibrium, the following lemma shows that neither a grand coalition will never form at the first stage.

Claim 9 As $\delta > \overline{\delta}$, none of the players proposes to form the grand coalition in a subgame perfect equilibrium.

Proof. In the Appendix. ■

In fact, when δ is high enough, does not make much sense either for the seller or for the high-valuation buyer to propose the formation of the grand coalition, since they can obtain the surplus 1 even without the consent of the low-valuation buyer. Indeed, proposing to join a grand coalition is always a strictly dominated strategy both for the seller and the strong buyer.

Then we have further restricted the set of the candidate equilibrium coalition structures. First note that, having ruled out all the proposals for singletons and the grand coalition, in a subgame perfect equilibrium must be the case that each trader proposes a two-players coalition.

Secondly, note that the last Proposition allows us to rule out the possibility that all three traders propose the same coalition in equilibrium. In fact we have already shown that all three players proposing the grand coalition cannot be a subgame perfect equilibrium; furthermore not all three players can propose the same two-players coalition, since a player has to belong to the coalition he proposes and since clearly no one has an interest to propose that the other two players form a coalition against him.

Hence, only two possible cases may arise in equilibrium: either in equilibrium each trader, when is selected to make an offer, proposes a different two-players coalition, or must be the case that there is exactly one two-players coalition that is proposed in equilibrium by all its members.

The following results show that, as the traders are sufficiently patient, the former is indeed the case.

Proposition 10 As $\lambda \geq \tilde{\lambda}$ and $\delta > \hat{\delta}$, there exist two alternative stationary subgame perfect equilibria of the coalition formation stage..

- In one equilibrium, the seller S proposes to form a $\{S, B_2\}$ coalition, the strong buyer B_1 proposes to form a $\{S, B_1\}$ coalition, and the weak buyer B_2 proposes to join a buyers' cartel $\{B_1, B_2\}$.

- In the second equilibrium, the seller S proposes to form a $\{S, B_1\}$ coalition, the strong buyer B_1 proposes to form a buyers' cartel $\{B_1, B_2\}$, and the weak buyer B_2 proposes to join a coalition $\{S, B_2\}$.

Proof. In the Appendix. ■

Hence, as traders are sufficiently patient, in the coalition formation game any two-persons coalition form in equilibrium with probability $\frac{1}{3}$. Therefore, with probability $\frac{1}{3}$, the entire game ends at the first stage, while with probability $\frac{2}{3}$ a negotiation stage is played between either the seller and the buyers' cartel, or the $\{S, B_2\}$ coalition and the strong buyer.

Note that in our equilibria, in contrast with Montero (1998), traders play pure strategies in the coalition formation stage. Also note that $\hat{\delta}$ is approximately 0.9, an intuitively appealing and realistical value as concerns experimental investigations.

Therefore, the expected payoffs by the traders in the above equilibria of the whole game can be described as follows.

Proposition 11 The expected payoffs of the traders in the described equilibria of our two-stages game are, respectively, such that:

Corollary 12

- in the first equilibrium, the seller expects a payoff equal to $W(S) = \frac{2\delta^2 - 5\delta + 6 + \lambda(3 - 2\delta)}{6(\delta^2 - 3\delta + 3)}$, the strong buyer expects a surplus equal to $W(B_1) = \frac{2\delta^2 - 8\delta + 9 + \lambda(\delta - 3)}{6(\delta^2 - 3\delta + 3)}$, and the weak buyer expects a surplus of $W(B_2) = \frac{2\delta^2 - 8\delta + 3 + \delta\lambda}{6(\delta^2 - 3\delta + 3)}$;
- In the second equilibrium, the seller expects a payoff equal to $W(S) = \frac{2\delta^2 - 8\delta + 9 + \delta\lambda}{6(\delta^2 - 3\delta + 3)}$, the strong buyer expects a surplus equal to $W(B_1) = \frac{2\delta^2 - 5\delta + 6 + \lambda(\delta - 3)}{6(\delta^2 - 3\delta + 3)}$, and the weak buyer expects a surplus of $W(B_2) = \frac{2\delta^2 - 5\delta + 3 + \lambda(3 - 2\delta)}{6(\delta^2 - 3\delta + 3)}$.

Proof. In the Appendix. ■

In the following, we underline an interesting property of both the equilibria we have found.

Corollary 13 In both the subgame perfect equilibria we have described above, the expected payoffs of the traders as $\delta \to 1$ converge to the Shapley Values of each player in the corresponding cooperative game: in fact,

$$\left\{ \begin{array}{l} \lim_{\delta \to 1} W(S) = \frac{1}{2} + \frac{\lambda}{6} \\ \lim_{\delta \to 1} W(B_1) = \frac{1}{2} - \frac{\lambda}{3} \\ \lim_{\delta \to 1} W(B_2) = \frac{\lambda}{6} \end{array} \right\}.$$

This provides a possible interpretation of our results in the spirit of non-cooperative implementation of the cooperative Shapley Value.

When traders are more impatient, a new equilibrium occurs in the coalition formation stage.

Claim 14 When $\lambda \geq \tilde{\lambda}$ and $\tilde{\delta} < \delta < \hat{\delta}$, there exists a unique stationary subgame perfect equilibrium where both the seller S and the strong buyer B_1 agree on joining a $\{S, B_1\}$ coalition, while weak buyer B_2 proposes to join a $\{S, B_2\}$ coalition.

Proof. In the Appendix. ■

Hence, if the buyers are still not too heterogeneous but get more impatient, there is only one equilibrium in the coalition formation stage, in which not all the coalitions form. In particular, most of the times it occurs that both the seller and the strong buyer agree on joining a coalition, while, with minor probability, it is the weak buyer to propose to form a coalition with the seller.

Corollary 15 In the described equilibrium for $\lambda \geq \tilde{\lambda}$ and $\tilde{\delta} < \delta < \hat{\delta}$, the seller expects a payoff equal to $W(S) = \frac{-5\delta+6+\delta\lambda}{2(\delta^2-9\delta+9)}$, the strong buyer expects a surplus equal to $W(B_1) = \frac{-8\delta+9+\lambda(2\delta-3)}{2(\delta^2-9\delta+9)}$, and the weak buyer expects a surplus of $W(B_2) = \frac{2\delta^2-5\delta+3+\lambda(3-3\delta)}{2(\delta^2-9\delta+9)}$.

Note that $\tilde{\delta}$ is approximately 0.8. Furthermore, note, incidentally, that as the expected payoffs of the traders as $\delta \to 1$ converge to $\left(\frac{1+\lambda}{2}, \frac{1-\lambda}{2}, 0\right)$, which not only lies clearly in the Core, but also represents the Tjis τ-value of the corresponding cooperative game.

Whenever, traders get more impatient, many other, less dense, coalition structures emerge in equilibrium, in whicheither the seller or the strong buyer, or both, may opt to stay singletons. However, we have not provided yet a full characterization of the equilibria when $\lambda \geq \tilde{\lambda}$ and $\delta < \tilde{\delta}$.

4.2 Strong heterogeneity: $\lambda < \tilde{\lambda}$

In the case $\lambda < \tilde{\lambda}$, the valuation of the weaker buyer is too low to represent an appealing trading alternative for the seller. Then any threat by the seller to ever sell the good to the low-valuation buyer is clearly not credible from the stronger buyer's perspective.

Hence, neither the seller not the high-valuation buyer will ever propose or accept to join a coalition with the weaker buyer, as his presence in the coalition would imply a positive, though possibly close-to-zero, share of the coalitional surplus, while it would add nothing to the latter.

Thus, in case of strong heterogeneity among the valuations of the buyers, the subgame perfect equilibrium of the described game will always involve singletons coalitions, so that the individual negotiation among the players will end up with the seller trading immediately with the high-valuation buyer. Hence the following Proposition may be proved.

Claim 16 *In a subgame perfect equilibrium with $\lambda < \tilde{\lambda}$, non-singleton coalitions are never formed and in the individual negotiation bargaining stage the seller trades immediately with the high-valuation buyer.*

Proof. In the Appendix. ■

5 Concluding Remarks

Some concluding remarks are in order.

First, only if the reservation prices of the two potential buyers are sufficiently similar, the weaker buyer can in fact exploit his influence in the decentralized market by getting a strictly positive payoff.

Second, if the reservation values of the buyers are not radically heterogenous and the traders are sufficiently patient, there are two possible subgame perfect Nash equilibria in pure and stationary strategies in the coalition formation game.

In both equilibria all, and only, the two-persons coalitions have identical probability to be formed as the outcome of non-cooperative bargaining among the traders. In one equilibrium, in particular, the seller proposes to join a coalition with the weak buyer, the latter proposes to form a buyers' cartel, while the strong buyer proposes the seller to join a coalition.

In the other equilibrium, it is the seller to propose a coalition with the strong buyer, the latter proposes to join a buyers' cartel, while the weak buyer proposes to form a coalition with the seller.

Furthermore, if the buyers are still not too heterogeneous but impatience is higher, there is only one subgame perfect equilibrium in the coalition formation stage, in which not all the coalitions form. In particular, most of the times it occurs that both the seller and the strong buyer agree on joining a coalition, while, with minor probability, it is the weak buyer to propose to form a coalition with the seller.

When impatience is even lower, less dense coalition structures emerge in equilibrium, in which some of the traders always opts out to stay alone. When the buyers are also very heterogeneous in their reservation prices, no coalition at all is ever formed.

Therefore, we find that, even if the formation of a buyers' cartel happens at most one-third of the cases, as $\delta \to 1$, the traders on the long side of the market are able to appropriate approximately half of the potential surplus, in contrast with what happens in a friction-less Walrasian thin market.

Finally, it turns out that, as $\delta \to 1$, the stationary subgame perfect equilibrium payoffs of our non-cooperative bargaining and endogenous coalition formation game converges to the Shapley Value of the corresponding cooperative game.

The latter consideration, altogether with the …ndings of some relations with the convergence to an element of the Core and, in particular, to the τ-value, suggests a further investigation of our model.

We should better devote some e¤ort to a deeper exploration of the possibility to also obtain any of the element of the Core through a more general speci…cation of our bargaining and coalition formation game, in the spirit of Perez-Castrillo (1994), Serrano (1995) and Serrano and Vohra (1997). For instance, one may guess that, by let varying the probability to be selected as proposer, it would be possible to span the equilibrium outcomes over all the points of the segment underlying within $(1, 0, 0)$ and $(\lambda, 1 - \lambda, 0)$, thus obtaining any of the Core allocations, and not only the speci…c $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$ described above.

Finally, an experimental investigation of the above proposed game urges to help shedding some light on the sensibility of the present approach to analyze the interrelations among bargaining and coalition formation in a thin market. As Bolton et al (2003) and Okada and Riedl (2004) show, there is a number of in‡uent factors a¤ecting coalitional bargaining, from the individual behaviour toward fairness and reciprocity to the communication structure in the negotiation process.

However, as far as we know, all the experimental analysis of coalitional bargaining has always focused on situations where negotiations among players are exclusively present in the coalition formation stage. In fact, in the above experiments it is usually assumed that three generical players, characterized by predetermined asymmetric bargaining positions, can negotiate over the division of some coalitional values, as de…ned by an exogenously given characteristic function. Our two-stages model, at the contrary, allows an experimental setup where, on the one hand, further insights into the process of strategic price formation in a thin market may be hopefully reached, and, on the other hand, both coalitional values and bargaining positions are endogenously determined by the players' behaviour. To this, in fact, will be next addressed our attention.

6 Appendix. Proofs

To be written

7 Appendix. Acknowledgements

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