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BARGAINING AND TAXATION IN VENTURE CAPITAL: A REAL OPTION APPROACH

MATTEO M. GALIZZI AND PAOLO M. PANTEGHINI

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Bargaining and Taxation in Venture Capital: a Real Option Approach

Matteo M. Galizzi and Paolo M. Panteghini
Department of Economics, University of Brescia

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1 Motivation

Even in European Union, an increasing number of companies, specially small medium enterprises (*SME*) and start-ups, has been involved in Venture Capital (*VC*) initiatives. Businessmen, economists and policy makers seem to agree on that Venture Capital may not only represent an extraordinary challenge for the traditional european economic model - largely based upon SME - but also speed up the diffusion of innovative entrepreneurship and start-ups. Experts and practitioners typically point out three salient aspects that can be radically innovated by Venture Capital.

The first clearly regards financial innovation. *VC* should be seen not simply as an injection of fresh funds into *SME* or start-ups to finance investments, dimensional growth or presence in the international markets. On the contrary, it should be regarded as a chance for firms to undertake a less naive and more professional approach to financial markets and innovations, dealing with private equity funds and professional investors and, thus, moving beyond the logic of traditional credit contracts with banks.

The second innovation lies in the opportunity for firms to open up their traditional governance to external shareholders and to deeply review their strategies and projects under the influence of a professional, internationally grown, management.

The third one, concerns the very nature of investments. The easier access to financial means and managerial skills suggests a better evaluation of costs and returns of risky and irreversible projects and a more precise estimation of the optimal timing of corresponding investments. Firms and start-ups typically face a trade-off when deciding on when to invest on new technologies or innovative projects: investing too early may mean a too high risk as the technical or technological profile of the investments is too much uncertain, while investing too late can clearly imply losing the leadership or most the business opportunities, as other competitors could also invest in the same innovation in the meanwhile.

It is the interaction of the above three main innovations that has convinced policy makers to investigate how to favour the diffusion of *VC* between firms and, specially, start-ups. Many European countries have been studying how to provide incentives for stimulating *VC* by means of corporate taxation. The issue of which corporate tax scheme can be more effective in favouring *VC* and how to design it, are still open questions. This paper aims at contributing to investigate along this line.

In particular, having in mind an entrepreneur (*ENT*) and a venture capital fund (*VCF*) starting up a company we attempt to model *VC* in such a way to capture three salient issues.

Firstly, start-up investments are risky and irreversible. In our model, *VCF* takes full responsibility for management and, in particular, for investment decisions. As investing time is the key concern, dynamic investment decisions are modelled following the *real option* approach.

Secondly, we model the relationship among entrepreneurs and *VC* funds as characterized by *symmetric information*. We therefore depart from Principal-Agent models which typically deal with asymmetric information among agents in *VC* and focus on the design of contracts to offer to the more informed agent. We do it as we believe that economic partners in real world start-ups behave in a very symmetric manner concerning information, or, at least, that it is difficult to claim *a priori* which party should act as a Principal. In fact, on the one hand, entrepreneurs have better information on technical and scientific profile of the project, but lacks of managerial experience. On the other hand, *VC* funds have better information on management's effort but are unable to fully assess the true technical potential of company's services. Therefore, we believe that, rather than offering or accepting a menu of *contracts* as Principal or Agent, both entrepreneurs and venture capital funds are more in a position to *bargain* on an equal foot. In particular, we model a strategic negotiation process among entrepreneurs and *VC* funds over profit shares as a non-cooperative *bargaining* game.

Finally, after having described the *VC* model with risky and irreversible investments and strategic bargaining, we characterize the (multiple) equilibria of the overall game, and compare them with the socially optimal solutions. We then discuss which corporate taxation scheme of start-ups is more effective in either selecting equilibria or in possibly reducing the gap between game's equilibria and efficient outcomes.

2 The Model

2.1 The Agents

In the model, there are three agents: *investors*, *venture capital* funds, and potential *entrepreneurs*.

We think at a potential *entrepreneur* (*ENT*) as a (female) scientist, researcher or academic professor employed by a research department or a university, where is paid a fixed salary w. In carrying on her investigation, she comes into an idea, a discovery or a patent which can be potentially incorporated into a product or a service and profitably sold in the market. Hence she is facing the choice whether to remain working full time for the research or academic institution or to start up a company as entrepreneur. However, given her different background, in the latter case she needs managerial and financial support. She, then, look for a venture capital partner with which she is ready to start up a firm: the entrepreneur then gives up her (full time) academic position and salary, contributes to the firms' assets with her entrepreneurial ideas and patents, and acts as scientific supervisor in exchange for a share $1-\alpha$ of all future profits by the start-up cpmpany.

On the other hand, a (male) *venture capital* fund (*VCF*) is willing to offer to the entrepreneur not only the expertise of his managerial and consulting team, but also any needed financial support, which he is able to collect from the financial markets. In particolar, *VCF* typically borrows money I from financial investors *FIN*, paying them an interest rate ρ_t at period t, while the capital I is only returned at the end of the investment. *VCF* then injects the collected money I, altogether with his managerial effort, into the start-up company, in exchange for a share α of all its future profits.

Finally, the role of financial *investors* (*INV*) is rather simple: they are just assumed to invest money I in *VCF* any time the rate of interest ρ_t paid by *VCF* is at least as high than the market's interest rate on a free-risk asset r.

2.2 The Assumptions

The agents' behaviour in the model is described by four features.

1. Firstly, all agents are assumed to be risk-neutral.

2. Secondly, the start-up project is affected by a double source of uncertainty, so that the company profits are *ex ante* unknown to both the entrepreneur and *VCF*. In fact, on the one hand, there is a first period of time ($t=0$) during which the project is risky in that it can either *succeed* (in the real option jargon, a *good state of the world*), with probability p, returning an annual profit R_t from $t=0$ on, or *fail* (*bad news*), with probability $1-p$, returning $L_t=0$ from $t=0$ on. When deciding on profit-sharing, both *ENT* and *VCF* only know the *ex ante* probability of success. As discussed below, after such a decision, *VCF* can also undertake managerial actions in order to increase the probability of success from p to q, with $q>p$.

   On the other hand, even when, in the next period ($t=1$) uncertainty is resolved and the project succeeds, the start-up annual profit, at any period from $t=1$ on, is a random variable R_t: R_t is independently and identically distributed in the space $\left[\underline{R},\overline{R}\right]$, according to a probability density function $f\left(R_t\right)$ having cumulated density function $F\left(R_t\right)$. Since, even in case of success, *ENT* and *VCF* do not know in advance the exact value of the profit R_t realized at any instant of time, they just formulate their strategies and decisions on the basis of R_t's *expected value* R, with $R=E\left[R_t\right]=$

$$\int_{\underline{R}}^{\overline{R}} R_t f(R_t)\, dR_t.$$

3. Thirdly, the expected profit from the start-up are divided between *ENT* and *VCF* after strategic negotiations. In particular *VCF* and *ENT* bargain over their share α and $1-\alpha$, respectively, of the expected profits taking into account both the future managerial actions by the former and the outside option for the latter represented by his fixed salary w. The conflict in negotiations clearly arises from the fact that the entrepreneur wants to be assigned a share large enough to compensate the loss from leaving his safe job in the research institution; while *VCF* not only needs to cover the financial costs of the debt, but also pretends larger shares for exerting higher managerial effort. Whatever the equilibrium outcome of the strategic bargaining is, it is assumed that the division of expected future profits between *ENT* and *VCF* is then signed up in a contract which is binding and enforceable, and can not be renegotiated.

4. Furthermore, once *ENT* and *VCF* have agreed on how to share the future profits, the start up firm is assumed to be run by the managerial team of *VCF* only. Therefore, it is imagined that the entrepreneur is just taking care of the technical and scientific aspects of the project, while the management decisions are exclusively in the *VCF*'s hands. In particular, as it will be discussed below, taking as given their share of the profits, the *VCF* managers may decide on the timing of the investments and on the extent of their effort.

5. Finally, note that when uncertainty on the success of the projects vanishes at $t=1$ and financial investors are in a competitive market, the return asked by *FIN* on I should equal in equilibrium the interest rate paid by a risk-free asset, that is $\rho_1 = r$, while the interest rate at $t=0$, in presence of uncertainty, would typically be higher, namely $\rho_0 > r$.

We now describe in more detail the timing of the model and the actions available to each agent.

2.3 The Timing

The model consists of two stages.

In the first stage, a potential entrepreneur and a *VCF* meet up to negotiate how to share the expected future profits of the start up firm: a share α goes to *VCF*, the remaining share $1-\alpha$ to the entrepreneur. The exact bargaining protocol is described below.

In the second stage, taking as given the division of the profits, *VCF* borrows money from the financial investors and decides which managerial action to undertake, concerning both the timing of the start up's investment and the managers' effort.

After *VCF*'s decision at the second stage, nothing is left to be decided: the start up invests as decided in the second stage, the uncertainty is solved, borrowed money are returned to the financial investors with the corresponding interest rate, and the eventual profits divided according to the shares agreed at the first stage.

2.3.1 The bargaining game at the first stage

In particular it is assumed they bargain over *VCF*'s share α of the profits, while the complementary share $1-\alpha$ is intended to go to the entrepreneur. The negotiations are modelled as a non-cooperative infinitely repeated game of bilateral bargaining with random order of proposers. We assume that agents are impatient and they share a common discount factor δ.

At any period of time, *VCF* and *ENT* has $\frac{1}{2}$ probability to be selected to formulate a proposal $0 \leq \alpha \leq 1$ about the division of the profit. The other part can then respond to the offer. If the offer is accepted, the future profits are divided according to the proposed α and the bargaining stage ends. If, instead, the offer is rejected, both *VCF* and the entrepreneur enter a new round of negotiations, in which a new random selection occurrs to determine who can make offers and who, instead, is called to respond. The game goes on in the same way until an offer would be accepted.

We will solve the bargaining game by looking at the Sub-game Perfect Nash Equilibria, focusing on pure and *stationary* strategies only (*PSSPNE*). That is, we only consider equilibria where players behave in such a way that, at any strategically equivalent node at which they are selected to make offers, they always propose the same offer, and, at any strategically equivalent node at which they are called for responding the same offer, they either always or never accept it.

Crucially, it is assumed that the equilibrium outcome from the bargaining process is immediately formalized in a contract on profit-sharing, and that such a contract is fully enforceable and not re-negotiable.

2.3.2 The VC manager's decision at the second stage

After having agreed with the entrepreneur on a share α of the start up profits, *VCF* takes full responsibility of the management and is called for investment decisions. The second stage of our overall *VC* game is therefore just a decision node, rather than a proper game. In particular, at the beginning of $t = 0$, *VCF* is called to decide on two irreversible binary options:

1. To exert or not extra managerial effort: exerting effort costs c to *VCF* managers, but it increases from p to q the investment's probability of success.

2. To invest at $t = 0$ or at $t = 1$: as discussed above, only at $t = 1$ uncertainty about the project's success is solved.

Concerning the latter binary option, *VCF* managers faces a typical trade-off in deciding the investment timing.

Infact, *if VCF invests at* $t = 0$:

- *VCF* needs to borrow money I from *FIN* before knowing whether the project is successful or not, and expects to repay back I at rate $\rho_0 > r$ in any case (whatever is the state of the world).
- Hovewer, *VCF* is the only firm in the market and can enjoy leader profits from $t = 0$ on.

On the other hand, *if VCF invests at* $t = 1$ instead:

- *VCF* only borrows money I from *FIN* after having known the good state of the world, and repays back it at rate $\rho_1 = r$ only with probability p (or q, if he decides to exert effort).
- However, if he waits until $t = 1$, competitors can also have entered the market, reducing expected profits, from $t = 1$ on, of a percentage $d \in [0, 1]$.[1]

Therefore, *VCF* has 4 actions he can take at the beginning of $t = 0$:

1. Invest Now and Exert Effort: *Now/Effort*
2. Invest Now and Exert No Effort: *Now/No Effort*
3. Invest Later and Exert Effort: *Wait/Effort*
4. Invest Later and Exert No Effort: *Wait/No Effort*

Invest now and Exert Effort: *Now/Effort* strategy If *VCF* managers decide to exert effort - whose cost is c - and to invest in the project at $t = 0$, the net present value (NPV) of the future profits from the start-up ($T\Pi_0^c$) is

$$T\Pi_0^c = q \sum_{t=0}^{\infty} \frac{R}{(1+r)^t} \tag{1}$$

which from standard properties of geometric series reduces to

$$T\Pi_0^c = qR\frac{1+r}{r}$$

[1] This assumption allows us to take into account, in a very simple way, competition among *VCF*s. In fact, parameter $d \in [0, 1]$ is as a proxy for competition: setting $d = 0$ implies that returns do not decrease when investment is postponed, in line with Dixit and Pindyck (1994). If instead $d = 1$, then *VCF* gross returns faced at $t = 1$, and therefore the option to delay, fall to zero.

Therefore, for a given profit share α, and taking into account the costs of effort and of debt, the NPV of the total payoffs by *VCF* ($V\Pi_0^c$) is

$$V\Pi_0^c = \alpha q R \frac{1+r}{r} - c - \rho_0 I$$

while the NPV expected by *ENT* ($E\Pi_0^c$) is simply

$$E\Pi_0^c = (1-\alpha)\, qR \frac{1+r}{r}.$$

Invest now but not Exert Effort: *Now/No Effort* strategy Analogously, if *VCF* managers decide to not exert effort but to still invest in the project at $t = 0$, the net present value (NPV) of the future profits from the start-up ($T\Pi_0$) is

$$T\Pi_0 = p \sum_{t=0}^{\infty} \frac{R}{(1+r)^t} = pR\frac{1+r}{r} \tag{2}$$

Therefore, for a given profit share α, taking into account the costs of debt, the NPV of the total payoffs by *VCF* ($V\Pi_0$) is

$$V\Pi_0 = \alpha p R \frac{1+r}{r} - \rho_0 I$$

while the NPV expected by *ENT* ($E\Pi_0$) is simply

$$E\Pi_0 = (1-\alpha)\, pR \frac{1+r}{r}.$$

Invest Later and Exert Effort: *Wait/Effort* strategy If, on the other hand, *VCF* managers decide to exert effort - whose cost is c - but to invest in the project at $t = 1$, the net present value (NPV) of the future profits from the start-up ($T\Pi_1^c$) is

$$T\Pi_1^c = q \sum_{t=1}^{\infty} \frac{(1-d)R}{(1+r)^t} \tag{3}$$

which from standard properties of geometric series reduces to

$$T\Pi_1^c = q\frac{(1-d)\,R}{r}$$

Therefore, for a given profit share α, and taking into account the costs of effort and the one of debt (only in case of success), the NPV of the total payoffs by *VCF* ($V\Pi_1^c$) is

$$V\Pi_1^c = \alpha q \frac{(1-d)\,R}{r} - c - q\rho_1 I$$

while the NPV expected by *ENT* ($E\Pi_1^c$) is simply

$$E\Pi_1^c = (1-\alpha)\, q \frac{(1-d)\,R}{r}.$$

Invest Later and Not Exert Effort: *Wait/No Effort* strategy If, finally, *VCF* managers decide to not exert effort and to invest in the project at $t = 1$, the net present value (NPV) of the future profits from the start-up ($T\Pi_1$) is

$$T\Pi_0 = p\sum_{t=1}^{\infty}\frac{(1-d)R}{(1+r)^t} = p\frac{(1-d)\,R}{r}. \tag{4}$$

Therefore, for a given profit share α, and taking the one of debt (only in case of success), the NPV of the total payoffs by *VCF* ($V\Pi_1$) is

$$V\Pi_1 = \alpha p\frac{(1-d)\,R}{r} - c - p\rho_1 I$$

while the NPV expected by *ENT* ($E\Pi_1$) is simply

$$E\Pi_1 = (1-\alpha)\,p\frac{(1-d)\,R}{r}.$$

3 Sketch of the Preliminary Results

The solution of the two-stages overall game uses *backward induction*:

1. We first solve *VCF* decision node at the second stage: we aim at finding optimal choices by *VCF* managers under different parameters' configurations, and at computing the corresponding start-up profits.

2. We then solve the strategic bargaining game between *ENT* and *VCF* over the shares of the specific start-up profits corresponding to any parameters' configuration found at the first stage. By solving the infinitely repeated bargaining game looking at the subgame perfect equilibria, we take advantage of our focus on stationary (and pure) strategies, which makes the problem much easier to deal with.

If the equilibrium shares at the first stage are fully compatible with *VCF* optimal decision at the first stage under a specific parameters configuration, we have therefore found a subgame perfect Nash equilibrium in pure and stationary strategies (*PSSPNE*) of our overall *VC* game.

3.1 Solving *VCF* investments decision at last stage

When solving *VCF* investment decision on timing and managerial effort at beginning of $t = 0$, we just need to compare, *for a given share* α, *VCF* expected payoffs from any of the four available options.

In fact, as seen above, at the beginning of $t = 0$, *VCF* face four options he can take in his decision node:

1. either to Invest Now and to Exert Effort: *Now/Effort*

2. or, to Invest Now and to not Exert Effort: *Now/No Effort*
3. or, again, to Invest Later and to Exert Effort: *Wait/Effort*
4. or, finally, to Invest Later and to not Exert Effort: *Wait/No Effort*

The *VCF* problem is simply to find

$$\max\{V\Pi_0^c, V\Pi_0, V\Pi_1^c, V\Pi_1\}$$

where, from above the expected payoffs for a given α are

$$\begin{aligned} V\Pi_0^c &= \alpha qR\frac{1+r}{r} - c - \rho_0 I \\ V\Pi_0 &= \alpha pR\frac{1+r}{r} - \rho_0 I \\ V\Pi_1^c &= \alpha q\frac{(1-d)\,R}{r} - c - q\rho_1 I \\ V\Pi_1 &= \alpha p\frac{(1-d)\,R}{r} - c - p\rho_1 I \end{aligned}$$

The easier way to proceed is to *compare* action payoffs *pairwise*. However, comparison inequalities will typically depend on α. For instance, it is immediate to see that

$$\begin{aligned} V\Pi_0^c &\geq V\Pi_0 \text{ iff} \\ \alpha qR\frac{1+r}{r} - c - \rho_0 I &\geq \alpha pR\frac{1+r}{r} - \rho_0 I \quad \text{i.e. iff} \\ \alpha &\geq \overline{\alpha_0^{c0}} = \frac{cr}{R(1+r)\,(q-p)}. \end{aligned}$$

Analogously,

$$\begin{aligned} V\Pi_0^c &\geq V\Pi_1^c \text{ iff} \\ \alpha qR\frac{1+r}{r} - c - \rho_0 I &\geq \alpha q\frac{(1-d)\,R}{r} - c - q\rho_1 I \quad \text{i.e. iff} \\ \alpha &\geq \overline{\alpha_{c1}^{c0}} = \frac{Ir\,(\rho_0 - qr)}{Rq(r+d)}. \end{aligned}$$

Again,

$$\begin{aligned} V\Pi_0^c &\geq V\Pi_1 \text{ iff} \\ \alpha qR\frac{1+r}{r} - c - \rho_0 I &\geq \alpha p\frac{(1-d)\,R}{r} - c - p\rho_1 I \quad \text{i.e. iff} \\ \alpha &\geq \overline{\alpha_1^{c0}} = \frac{cr + Ir\,(\rho_0 - pr)}{R\,[q(1+r) - p(1-d)]}, \end{aligned}$$

and so on and so forth.

Such threshold shares $\overline{\alpha^i_j}$, where i and j are any two managerial actions by *VCF* in the second stage, typically depend on strategies' primitive parameters c, d, $\rho_0 - r$, $q - p$.

Therefore all $\overline{\alpha^i_j}$ can be relatively ranked according to a specific configuration of parameters, as described by a set of restrictions on c, d, $\rho_0 - r$, q and p. Under a specific combination, an exhaustive relative ranking of $\overline{\alpha^i_j}$ implies a complete ranking of corresponding strategies. Therefore, for a specific configuration, it is possible to order in a unique way the different available strategies in terms of expected payoff conveyed to *VCF,* according to the relative distance of the profit share α from the corresponding thresholds. That is, for any set of restictions, one can identify the optimal decision by *VCF* according to his profit share α.

For the sake of brevity, here we can not provide a full characterization of all the possible ranking of $\overline{\alpha^i_j}$ under any parameters configuration. However, just an example can illustrate our typical findings.

When c high compared to $q - p$, it is possible to show that *Wait/Effort* is always a *dominated* strategy.

Hence by looking at VCF payoffs from the other actions, it is possible to consider three cases:

- For d low and interest rate spread $\rho_0 - r$ high compared to $q - p$:
  - Either *WaitNo Effort* if $\alpha \leq \alpha\prime$,
  - Or *Now/No Effort* if $\alpha\prime < \alpha \leq \alpha\jmath$
  - Or finally, *Now/Effort* if $\alpha > \alpha\jmath$.

  Where $\alpha\prime$, $\alpha\jmath$ are thresholds depending on primitive parameters d, p, q...
- Otherwise, for d low and $\rho_0 - r$ low compared to $q - p$
  - Either *Wait/No Effort* if $\alpha \leq \alpha\hat{}$,
  - Or *Now/Effort* if $\alpha > \alpha\hat{}$.
- Again, for d high and $\rho_0 - r$ low
  - Always better *Now/Effort*

  and so on and so forth.

3.2 Solving the bargaining game at the first stage

Moving backward, for each *VCF* optimal decision, we consider the corresponding start-up profit and solve the bargaining game among *VCF* and *ENT* over shares of such profit.

For instance, consider the start-up profit when *VCF* managers opt for investing at $t = 0$ and exerting effort - i.e. for the above called *Now/Effort* strategy.

Hence, whenever is selected to make offers, in a subgame perfect equilibrium of the bargaining game ENT should propose a share $\alpha(ENT)$ such that

$$\alpha(ENT)qR\frac{1+r}{r} - c - \rho_0 I = \delta W(VCF)$$

where $\delta W(VCF)$ is the discounted expected continuation value by VCF by entering a new round of negotiations. In fact, if ENT was offering in equilibrium a lower $\alpha(ENT)$, this would imply $\alpha(ENT)qR\frac{1+r}{r} - c - \rho_0 I < \delta W(VCF)$ and therefore such an offer would be rejected, returning ENT her own continuation value which - by using stationarity hypothesis - can be shown to be lower than $[1-\alpha(ENT)]\, qR\frac{1+r}{r}$. If, on the other hand, ENT was offering a higher $\alpha(ENT)$, this would imply that the offer would be immediately accepted by *VCF*, as $\alpha(ENT)qR\frac{1+r}{r} - c - \rho_0 I > \delta W(VCF)$, but that ENT would be giving *VCF* a strictly higher share than what is needed to convince him to accept, and hence that she would be adopting a strictly dominated strategy.

By the same line of arguments, it follows that whenever is selected to make offers, in a subgame perfect equilibrium of the bargaining game *VCF* should propose a share $\alpha(VCF)$ such that

$$[1-\alpha(VCF)]\, qR\frac{1+r}{r} = w + \delta W(ENT)$$

where $\delta W(ENT)$ is the discounted expected continuation value by ENT by entering a new round of negotiations.

It is then possible to explicitly work out the equilibrium expressions for the continuation payoffs by using the stationary hypothesis. In fact,

$$\begin{aligned} W(VCF) &= \frac{1}{2}\left[\alpha(VCF)qR\frac{1+r}{r} - c - \rho_0 I\right] + \frac{\delta W(VCF)}{2} \\ W(ENT) &= \frac{\delta W(ENT)}{2} + \frac{1}{2}\left[1-\alpha(ENT)\right] qR\frac{1+r}{r}. \end{aligned}$$

Therefore, by substituting

$$\begin{aligned} W(VCF) &= \frac{1}{2-\delta}\left[\alpha(VCF)qR\frac{1+r}{r} - c - \rho_0 I\right] \\ W(ENT) &= \frac{1}{2-\delta}\left[1-\alpha(ENT)\right] qR\frac{1+r}{r} \end{aligned}$$

into the above expressions for the proposed shares, we have two equations in two unknowns $\alpha(VCF)$ and $\alpha(ENT)$

$$\begin{aligned} \alpha(ENT)qR\frac{1+r}{r} - c - \rho_0 I &= \frac{\delta}{2-\delta}\left[\alpha(VCF)qR\frac{1+r}{r} - c - \rho_0 I\right] \\ \left[1-\alpha(VCF)\right] qR\frac{1+r}{r} &= w + \frac{\delta}{2-\delta}\left[1-\alpha(ENT)\right] qR\frac{1+r}{r} \end{aligned}$$

which can be solved for the equilibrium shares $\alpha^*(ENT)$ and $\alpha^*(VCF)$. Clearly, such *PSSPNE* solutions $\alpha^*(ENT)$ and $\alpha^*(VCF)$ differ according to who makes offer at the first stage, and depending both on δ, they look like rather cumbersome:

$$\alpha^*(ENT) = \frac{2qRr\delta^2 + 2wr\delta - 2qRr\delta - wr\delta^2 - 4cr + 6cr\delta - 4\rho_0 Ir - 2r\delta^2 c + 2\delta r\rho_0 I - 2qR\delta + 2qR\delta^2}{4qR\left(\delta + r\delta - 1 - r\right)}$$

$$\alpha^*(VCF) = \frac{2qRr\delta^2 + 4wr\delta - 6qRr\delta - wr\delta^2 + 2r\delta c + 2\delta r\rho_0 I - 6qR\delta + 2qR\delta^2 + 4qR - 2r\delta^2 c - 4wr}{4qR\left(1 + r\right)\left(\delta - 1\right)}$$

However, by noting that the discount rate can be equivalently written as $\delta = \frac{1}{1+r}$, the expressions greatly simplify. Furthermore, and more importantly, as standard in non-cooperative bargaining games, when $\delta \to 1$ - that is when $r \to 0$ - both $\alpha^*(ENT)$ and $\alpha^*(VCF)$ converge to the same $\alpha^*_{\delta\to1}$, corresponing to the limit case of frictionless negotiations and fully patient players and to the cooperative Nash bargaining solution.

For instance, under the same parameter configuration as for the example reported above, is it possible to compute the equilibrium limit shares for each management strategy start-up's profit:

$$\alpha^*_{\delta\to1}(Now/Effort) = \frac{1}{2} - \frac{w}{4qR} + \frac{I\rho_0}{2qR}$$

$$\alpha^*_{\delta\to1}(Wait/NoEffort) = \frac{1}{2} - \frac{w}{2pR(1-d)}$$

$$\alpha^*_{\delta\to1}(Now/NoEffort) = \frac{1}{2} - \frac{w}{4pR} + \frac{I\rho_0}{2pR}$$

As can be seen, in general the *VCF* share of profits is an inverse function of the *ENT* salary w and is increasing with the cost of debt.

The last step to solve the overall game implies to match the equilibrium solutions from the bargaining game in the first stage with the optimal decisions by the *VCF* management in the last stage. In particular, we need to compare equilibrium expressions for $\alpha^*_{\delta\to1}$ with the conditions sustaining each *VCF* optimal decision and to investigate in which range of parameters, if any, they are compatible.

For the sake of brevity, here we just provide an example.

The question, for instance, can be seen as follows: is $\alpha^*_{\delta\to1}(Now/Effort) = \frac{1}{2} - \frac{w}{4qR} + \frac{I\rho_0}{2qR}$ satisfying any of the conditions on the parameter configuration under which strategy *Now/Effort* is indeed an optimal choice by the *VCF* management? Namely,

either if $\alpha\prime < \alpha^*_{\delta\to1}(Now/Effort) \leq \alpha\rfloor$ for d low and interest rate spread $\rho_0 - r$ high compared to $q - p$

Or if $\alpha^*_{\delta\to1}(Now/Effort) > \alpha\hat{}$ for d low and interest rate spread $\rho_0 - r$ low compared to $q - p$

and so on and so forth.

After having repeated such an exercise over and over for all the candidate equilibrium profit shares, it is possible to find specific ranges of primitive parameters where *equilibrium shares* of the start-up profits *in the bargaining game exist* and are *also optimal solutions* of the corresponding *VCF management decision*, and therefore, are subgame perfect Nash *equilibria* shares (*PSSPNE*) *of the overall game*.

Here we can not provide a full characterization of all the subgame perfect equilibria. It should be enough to say that we have been able to find as many as 7 *PSSPN* equilibria, according to the values of primitive parameters c, d, $\rho_0 - r$, $q - p$. Rather it can be worthwhile to underline some of the most interesting qualitative results we have been able to find insofar.

1. In general, *ENT* needs to give *VCF larger shares* in order to provide incentives for the start-up entering immediately in the market.

2. In three cases, there exists a unique equilibrium in a specific range. However, in two specific configurations of parameters, typically when the parameters take values in their middle ranges, *multiple equilibria* emerge.

In particular *two equilibria co-exist*:

One in which *ENT* let *VCF* to get a larger share of profits, anticipating that *VCF* would exert effort: *Now/Effort* equilibria.

The other in which *ENT* is willing to keep a larger share of a smaller pie, so that in equilibrium *VCF* is not exerting effort in the management: either *Now/No Effort* or *Later/No Effort* equilibria.

We find these latter findings particularly interesting, as they seem to fit the European Union case where entrepreneurs, specially of SME, are still skeptical about the possible advantages of involving external managers and financial means in their companies. Therefore, whenever they actually opt for opening up their governance to Venture Capital funds or private equity investors, they still prefer to keep firms under their strong control, thus potentially trapping their companies in a "bad" equilibrium as investments are concerned.

3.3 About introducing corporate taxation

The emergence of multiple equilibria in the overall game opens up several alternative lines of investigation. However, our current attention is addressed to a public economics approach tending to introduce corporate taxation in the above frame.

In fact, a policy maker can consider as not desirable the co-existing equilibria where shares to *VCF* are small, as management is not in the condition to exert the optimal effort. Moreover, equilibria where start-ups wait to invest suggests start-up entrepreneurship is perceived as too risky, and this, too, can be not desirable from the government point of view, as it may delay investments that are actually needed: see, for instance, the oncoming policy discussion over the investments in the *Next Generation Network* in telecommunication markets.

To see the problem, it is possible to compute any, even rough, measure of Social Welfare, for instance simply as the sum of subjects' payoffs in our model. If one does it, it immediately turns out that several mismatches arise between socially optimal and equilibrium investments and levels of effort under some configurations of parameters.

This, in turn, could imply that some specifically designed scheme of corporate taxation can be used as an effective device either to select, from the multiple, only the good equilibria - typically the ones with high managerial effort and early investments - or even to fill the gap between equilibria and First Best outcome. To this is indeed addressed our current work.