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QUALITY AND ADVERTISING IN A DYNAMIC MONOPOLY

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Quality and Advertising in a Dynamic Monopoly*

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Abstract

I apply optimal control theory to analyze a dynamic setting where a vertically differentiated monopolist, through capital accumulation over time, may invest both in product quality and advertising campaigns. By comparing the monopolist's behavior to the social planner's, I show that, at the steady state: (i) the monopolist's bi-dimensional R&D portfolio is always distorted, along, at least, one dimension; (ii) the level of demand induced by the monopolist exactly coincides with the one a benevolent social planner would choose. The latter result is illustrated within a linear-quadratic technology.

Keywords: product quality, advertising, dynamic monopoly, vertical differentiation, capital accumulation.

JEL Classification: D3, L12, O31.

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1 Introduction

The existing literature on advertising dynamic models can essentially be partitioned into two main subsets (see Sethi, 1977). The first, à la Nerlove and Arrow (1962), considers advertising as an instrument to increase the stock of goodwill or reputation; the second, à la Vidale and Wolfe (1957), is characterized by a direct relationship between advertising expenditures and the rate of change in sales[1].

With regard to product quality provision, a dynamic set-up has prevalently been adopted in relation with advertising expenditures aimed at the formation of goodwill (see Kotowitz and Mathewson, 1979; Conrad, 1985; Ringbeck, 1985)[2]. A prominent exception is represented by Lambertini (2001), who investigates a differential duopoly game where firms supply goods of different quality, resulting from capital accumulation over time. The set-up is borrowed from static monopolistic models with vertical differentiation (see Spence, 1975; Mussa and Rosen, 1978). The basic insight common to these contributions is that a private monopolist designs the product to match the preference of the marginal consumer, while a benevolent social planner cares about the taste of the average one. Therefore, quality overprovision (underprovision) arises whenever the marginal willingness to pay of the latter results lower (higher) than the former's. In another paper, Lambertini (1997) shows that under largely acceptable hypothesis concerning the distribution of the population[3] and a well behaving cost function[4], quality distortions do not arise, while prices and quantities are upwards and downwards distorted, respectively. At a first glance, this result seems to be in contrast with the conventional wisdom on quality provision by a vertically differentiated monopolist (Spence, 1975). Actually, what Spence argues is that for given and *equal* output, compared to the social planning, the monopolist undersupplies product quality[5] (see Tirole, 1988). However, prices distortions are quite likely to happen in monopoly, not to use the term always. Consequently, one could be tempted to ask himself whether Spence's arguments are wrong. As Tirole (1988) well explains, when the monopoly structure is not questioned, reasoning for given quantities can be appropriate. But if the purpose of the analysis is to compare the monopoly to the social planning, such a reasoning is not acceptable, since it does not allow to take into account the global dead weight loss. Therefore, as usual, all depends on what one is interested in.

[1] For surveys, see Jϕrgensen (1982), Feichtinger, Hartl and Sethi (1994) and Dockner, Jϕrgensen, Van Long and Sorger (2000, ch.11)

[2] For surveys on dynamic advertising, see Sethi (1977); Jϕrgensen (1982); Feichtinger and Jϕrgensen (1983); Erickson (1991); Feichtinger, Hartl and Sethi (1994).

[3] An uniform density function is assumed.

[4] The total cost function is $C(x, q)$, with $C_x > 0$, $C_q > 0$, $C_{xx} \geq 0$, $C_{qq} \geq 0$, x and q being quantity and quality, respectively.

[5] A sufficient condition for this to happen is that the marginal willingness to pay for quality decreases with the quantity purchased, i.e., marginal and absolute willingness to pay for quality be positively linked.

I apply optimal control theory[6] to analyze a dynamic setting where a vertically differentiated monopolist, through capital accumulation over time, may invest both in product quality and advertising campaigns[7]. The first task of the analysis is to compare the monopolist's bi-dimensional R&D portfolio with the social optimum. To this aim, I start by applying a very general model where functional specifications are avoided. I solve both the monopolist and the social planner's problem for the implicit quality and advertising steady state levels, as well as for the related investments. Then, by comparing the implicit solutions, I conclude that the monopolist's bi-dimensional R&D portfolio is always distorted, along, at least, one dimension.

The reminder of this paper is structured as follows. The general model is laid out in section 2. In section 3, I focus on product quality investments, while in section 4, I focus on advertising campaigns. In section 5, aimed at obtaining explicit solutions, I employ a linear-quadratic technology, showing that, when both investments are jointly activated, a profit-maximizing monopolist provides the market with the first best quantity level. In section 6, I evaluate the resulting monopolist's equilibria in terms of welfare, suggesting which should be the optimal policies to cope with dynamic inefficiencies. Finally, in section 7, I provide concluding remarks.

2 The model

I consider a market for vertically differentiated products where a monopolist subject to no entry threat supplies a single variety q at price p in a number of units x and affects market preferences by means of advertising campaigns. Consumers are ordered along a support $S = \theta_1 - \theta_0$ on the basis of their quality appraisal, expressed by their marginal willingness to pay, θ. Without any loss of generality, let S be unitary, with $\theta_1 > 1$. The population is distributed according to the density $f(\theta)$. Suppose $f(\theta)$ be uniform. Since θ_i is the marginal willingness to pay characterizing consumer i, his gross surplus from the consumption of quality q is given by:

$$V(\theta_i, q) = \theta_i q \tag{1}$$

Net surplus simply amounts to:

$$S(\theta_i, q, p) = \theta_i q - p \tag{2}$$

Each consumer is confronted with the choice between buying or not buying one unit of a certain variety. These alternatives are equivalent if:

$$0 = \theta_i q - p \Rightarrow \theta_k = \frac{p}{q} \tag{3}$$

[6]See Chiang, 1992 or Seierstad and Sydsaeter, 1987. The former provides a good introduction to optimal control theory, while the latter, at a less introductory level, is reached of economic applications.

[7]The analysis of dynamic monopoly originates with Evans (1924) and Tintner (1937).

Therefore, total demand writes:

$$x = \theta_1 - \theta_k = \theta_1 - \frac{p}{q} \leq 1 \tag{4}$$

Production entails a variable cost, which is assumed to be convex in the quality level:

$$C^q = xq^2 \tag{5}$$

and the setup of advertising campaigns entails a fixed cost:

$$C^{\theta_1} = B + \frac{\gamma}{2}(\theta_1^{SS} - \theta_1)^2 \tag{6}$$

with $\theta_1^{SS} > \theta_1$ and $B, \gamma > 0$, where θ_1^{SS} denotes the *exogenous* advertising target, γ is a parameter weighing the square of the distance between such a target and the actual reservation price, and B is the amount of money to be spent in the case in which the target is achieved.

Instantaneous profits amount to:

$$\Pi = (p - q^2)(\theta_1 - \frac{p}{q}) - B - \frac{\gamma}{2}(\theta_1^{SS} - \theta_1)^2 \tag{7}$$

Instantaneous consumer surplus is:

$$CS = \int_{\theta_k}^{\theta_1} (\theta q - p) d\theta = \frac{1}{2} \frac{\theta_1^2 q^2 - 2p\theta_1 q + p^2}{q} \tag{8}$$

Instantaneous social welfare is obtained adding up consumer surplus and profits:

$$W = \Pi + CS \tag{9}$$

Now, let me introduce the time dimension into this setup. Suppose, first, that the market exists over $t \in [0, \infty)$. To keep things manageable, I assume that both the size and the distribution of the population remain constant over t. Accordingly, the demand function can be written:

$$x(t) = \theta_1(t) - \frac{p(t)}{q(t)} \leq 1 \tag{10}$$

In the remainder, I will consider the three following scenarios: (i) investments in product quality; (ii) investments in advertising campaigns; (iii) both jointly.

In (i), while $\theta_1(t)$ remains constant at $\widehat{\theta}_1$, in response of capital accumulation over time, $q(t)$ evolves according to the following kinematic:

$$\frac{\partial q(t)}{\partial t} = b\phi(k(t)) - \delta q(t) \tag{11}$$

In (ii), while $q(t)$ remains constant at $\widehat{q}$, in response of capital accumulation over time, $\theta_1(t)$ evolves according to the following kinematic:

$$\frac{\partial\theta_1(t)}{\partial t} = c\psi(l(t)) - \delta\theta_1(t) \tag{12}$$

where $k(t)$ and $l(t)$ denote the *specific* capital to be devoted to product quality improvements and advertising campaigns, respectively; $\phi(k)$ and $\psi(l)$ are differentiable and invertible functions defined in the set of real numbers, with $\frac{\partial\phi(k)}{\partial k} > 0$, $\frac{\partial\psi(l)}{\partial l} > 0$, $\frac{\partial\phi^2(k)}{\partial^2 k} \leq 0$, $\frac{\partial\psi^2(l)}{\partial^2 l} \leq 0$. $\delta \in [0,1]$ is the usual depreciation rate, supposed to be common to both dynamics for the sake of simplicity. Finally, b and c are positive real constants.

Let $r > 0$ be the price for $k(t)$ and $w > 0$ be the price for $l(t)$. Therefore, instantaneous investments costs in quality improvements and advertising campaigns are, respectively:

$$IC^q(t) = rh(k(t)) \tag{13}$$

$$IC^{\theta_1}(t) = wz(l(t)) \tag{14}$$

with $\frac{\partial h(k(t))}{\partial k(t)} > 0$, $\frac{\partial^2 h(k(t))}{\partial k(t)^2} \geq 0$, $\frac{\partial z(l(t))}{\partial l(t)} > 0$, $\frac{\partial^2 z(l(t))}{\partial l(t)^2} \geq 0$.

3 Product Quality

3.1 The Social Planner's equilibrium

The current value Hamiltonian for the social planner's problem (P) turns out to be:

$$H = e^{-\rho t}\left\{\begin{array}{c} \frac{1}{2}\frac{(\widehat{\theta}_1)^2 q^2 - 2p\widehat{\theta}_1 q + p^2}{q} + (p - q^2)(\widehat{\theta}_1 - \frac{p}{q}) + \\ -rh(k(t)) + \lambda_1(t)(b\phi(k(t)) - \delta q(t)) \end{array}\right\} \tag{15}$$

where $\lambda_1(t) = \mu_1(t)e^{\rho t}$, $\mu_1(t)$ being the co-state variable associated to $q(t)$. The feasible set is $F^P = \{\widehat{\theta}_1 \geq q,\ p \geq 0,\ q \geq 0,\ \frac{p}{q} \geq \theta_0\}$. To simplify notations, I neglect the index of time. By applying Pontryagin's maximum principle, necessary conditions for a path to be optimal are[8]:

$$H_p = 0 \rightarrow p = q^2 \tag{16}$$

$$H_k = 0 \rightarrow \frac{r\frac{\partial h(k)}{\partial k}}{\lambda_1 b} = \frac{\partial\phi(k)}{\partial k} \rightarrow \lambda_1 = \frac{r\frac{\partial h(k)}{\partial k}}{b\frac{\partial\phi(k)}{\partial k}} \tag{17}$$

[8]For a revision of the methodology have a look at Chiang's book (1992).

$$H_q = -\frac{1}{2}\frac{-(\widehat{\theta_1})^2q^2 - p^2 + 4q^3\widehat{\theta_1} - 2q^2p}{q^2} - \lambda_1\delta = \rho\lambda_1 - \dot{\lambda_1} \quad (18)$$

along with the transversality condition:

$$\lim_{t\to\infty} \mu_1 q = 0 \quad (19)$$

I apply Mangasarian's theorem to check whether these conditions are also sufficient: define $V^P = \frac{(\widehat{\theta_1})^2q^2 - p^2 - 2q^3\widehat{\theta_1} + 2q^2p}{2q} - rh(k)$; $v_1 = b\phi(k(t)) - \delta q(t)$. I compute: $\frac{\partial^2 V^P}{\partial^2 q} = -2\theta_1 - \frac{p^2}{q^3} < 0$ always; $\frac{\partial^2 V^P}{\partial^2 p} = -\frac{1}{q} < 0$ always. Then I compute: $\frac{\partial^2 v_1}{\partial^2 q} = 0$; $\frac{\partial^2 v_1}{\partial^2 k} = b\frac{\partial^2\phi(k)}{\partial^2 k} \leq 0$, as long as $b > 0$ and $\frac{\partial^2\phi(k)}{\partial^2 k} \leq 0$, which is always true by assumption. It remains to check that, in the optimal solution, the co-state variable be non negative: $\lambda_1 = \frac{r\frac{\partial h(k)}{\partial k}}{b\frac{\partial\phi(k)}{\partial k}} \geq 0$ by assumption. Therefore, the necessary conditions of the maximum principle are also sufficient.

By differentiating (17) w.r.t. time:

$$\dot{\lambda_1} = \frac{r\,\dot{k}\,[\frac{\partial^2 h}{\partial^2 k}\frac{\partial\phi}{\partial k} - \frac{\partial h}{\partial k}\frac{\partial^2\phi}{\partial^2 k}]}{b[\frac{\partial\phi}{\partial k}]^2} \quad (20)$$

By inserting (20), (16) and (17) into (18) and by solving for $\dot{k}$:

$$\dot{k} = \left(-\frac{(\widehat{\theta_1})^2}{2} + 2q\widehat{\theta_1} - \frac{3}{2}q^2 + \frac{r\frac{\partial h(k)}{\partial k}}{b\frac{\partial\phi(k)}{\partial k}}(\delta+\rho)\right)\frac{b(\frac{\partial\phi}{\partial k})^2}{r\left(\frac{\partial^2 h}{\partial^2 k}\frac{\partial\phi}{\partial k} - \frac{\partial h}{\partial k}\frac{\partial^2\phi}{\partial^2 k}\right)} \quad (21)$$

with $\frac{\partial^2 h(k)}{\partial^2 k}\frac{\partial\phi(k)}{\partial k} \neq \frac{\partial h(k)}{\partial k}\frac{\partial^2\phi(k)}{\partial^2 k}$, i.e., the ratio between second derivatives has to be different from the corresponding ratio between first derivatives. Since $\frac{\partial^2 h}{\partial^2 k}\frac{\partial\phi}{\partial k} > \frac{\partial h}{\partial k}\frac{\partial^2\phi}{\partial^2 k}$:

$$sign(\dot{k}) = sign(\frac{(4q\theta_1 - \theta_1^2 - 3q^2)}{2} + \frac{r(\delta+\rho)}{b}\quad(k))$$

Therefore, with $q < \frac{1}{3}\theta_1$ (see figure 1) if q increases k does likewise[9].

[9] If $q > \frac{2}{3}\theta_1$ the opposite holds. However, since the locus $\dot{q} = 0$ is represented by a function which is monotonically decreasing, $q > \frac{1}{3}\theta_1$ can not be an equilibrium.

The steady state conditions $\dot{k}=0$, $\dot{q}=0$ imply:

$$\dot{k}=0 \rightarrow \frac{\partial \phi(k)}{\partial k} = 2r\frac{\partial h(k)}{\partial k}\frac{\delta+\rho}{b\left(-4\widehat{\theta}_1 q + 3q^2 + (\widehat{\theta}_1)^2\right)} \tag{22}$$

$$\dot{q}=0 \rightarrow q = \frac{b\phi(k)}{\delta} \tag{23}$$

By solving (22) for q I find the following roots:

$$q_{1,2} = \frac{2}{3}\widehat{\theta}_1 \pm \frac{1}{3b}\sqrt{b^2(\widehat{\theta}_1)^2 + 6r(\delta+\rho)\Delta(k)} \tag{24}$$

where $\Delta(k) = \dfrac{\frac{\partial h(k)}{\partial k}}{\frac{\partial \phi(k)}{\partial k}}$, with $\Delta(k)' = \dfrac{h''\phi' - h'\phi''}{(\phi')^2} > 0$ since $h'' > 0, \phi' > 0, h' > 0, \phi'' \leq 0$ by assumption. Since $q'_{1,2} = \pm(-\frac{1}{3b}\frac{1}{2}(\theta^2 b^2 + 12br(\delta+\rho)\Delta)^{-1/2}12br(\delta+\rho)\Delta') \gtrless 0$, the root $q_1 = \frac{2}{3}\widehat{\theta}_1 + \frac{1}{3b}\sqrt{b^2(\widehat{\theta}_1) + 6r(\delta+\rho)\Delta(k)}$ is not admissible, being $q_1 > \widehat{\theta}_1$ for any $k > 0$[10]. What about concavity? It is easy to verify that $q''_2 > 0$ if $\Delta'' < 0$. A sufficient condition for $\Delta'' < 0$ to hold follows:

$$\frac{\phi'}{2\phi''}\frac{h'''\phi' - h'\phi'''}{h''\phi' - h'\phi''} > 1$$

Notice that when h''' and ϕ''' are nil the condition does not hold but this does not imply that $q'' < 0$. The necessary condition for $q'' > 0$ turns out to be:

$$\frac{\Delta''}{\Delta'} < \frac{6br(\delta+\rho)\Delta'}{\theta^2 b + 12r(\delta+\rho)\Delta}$$

Let me assume that the above disequality holds along the entire admissible range of k, implying that $q''_2 > 0$. Anyway, the qualitative analysis of the equilibrium is not affected by the concavity of the locus $\dot{q}=0$, even in the case in which the sign of the secon derivative changes within the admissible range of k.

I am interested in investigating the dynamics of the system in the positive quadrant of the space $\{k, q\}$, which is described in figure 1 (see the Appendix). The locus $\dot{k}=0$ corresponds to the decreasing curve which intersects the vertical axe at $q = \dfrac{\widehat{\theta}_1}{3}$. The economic interpretation of such a locus is easy. When q is high, the planner is willing to stop the accumulation process quickly, at a low level of k. Viceversa, when q is low, he has incentives to keep on accumulating until the optimal level of q is reached. Moving from the origin, the locus $\dot{q}=0$ draws a curve which with no

[10] q belongs to the feasible set as long as it is not greater than θ_1.

doubt intersects the locus $\dot{k}=0$ once. The planner's equilibrium is denoted with a P. Clearly, it is a saddle and it can be approached only along the north-west arm of the path. As usual, the initial conditions play a crucial role in determining the trajectory of both variables over time. By following the directions indicated by the arrows, if $q(0) < \frac{\widehat{\theta_1}}{3}$ the steady state can never be achieved. Similarly, if $q(0) >> \frac{\widehat{\theta_1}}{3}$, the descending trajectory crosses the locus $\dot{q}=0$ at the right of P and then becomes increasing.

With regard to the formal stability analysis, let me focus on the following interesting case: $\Delta(k) = \phi(k) = k$. This, among others, is the case of a linear quadratic technology. The resulting dynamic system described by (11) and (21) can be written in matrix form as follows:

$$\begin{bmatrix} \dot{q} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} -\delta & b \\ \frac{b}{r}(2\theta_1 - \frac{3}{2}q) & \delta+\rho \end{bmatrix} \begin{bmatrix} q \\ k \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{b}{2r}\theta_1^2 \end{bmatrix}$$

Since the determinant of the above 2×2 matrix is always negative in the relevant parameter range, the equilibrium is stable along a saddle path.

3.2 The Monopolist's equilibrium

The current value Hamiltonian for the monopolist's problem (M) turns out to be:

$$H = e^{-\rho t}\left\{(\theta_1 - \frac{p(t)}{q(t)})(p(t) - q(t)^2) - rh(k(t)) + \lambda_1(t)(b\phi(k(t)) - \delta q(t))\right\} \quad (25)$$

where $\lambda_1(t) = \mu_1(t)e^{\rho t}$, $\mu_1(t)$ being the co-state variable associated to $q(t)$. The feasible set is $F^M = \{q \leq \widehat{\theta}_1,\ q \geq \frac{p}{\widehat{\theta}_1},\ p \geq q^2, \frac{p}{q} \geq \theta_0\}$. To simplify notations, as before, I neglect the index of time. By applying Pontryagin's maximum principle, necessary conditions for a path to be optimal are:

$$H_p = 0 \rightarrow p = \frac{q(\widehat{\theta_1}+q)}{2} \rightarrow x^M = \frac{\widehat{\theta_1}-q}{2} \quad (26)$$

$$H_k = 0 \rightarrow \frac{r\frac{\partial h(k)}{\partial k}}{\lambda_1 b} = \frac{\partial \phi(k)}{\partial k} \rightarrow \lambda_1 = \frac{r\frac{\partial h(k)}{\partial k}}{b\frac{\partial \phi(k)}{\partial k}} \quad (27)$$

$$H_q = -2q\widehat{\theta_1} + \frac{p^2}{q^2} + p - \lambda_1\delta = \rho\lambda_1 - \dot{\lambda}_1 \quad (28)$$

along with the transversality conditions:

$$\lim_{t\to\infty} \mu_1 q = 0 \quad (29)$$

I apply Mangasarian's sufficiency theorem: define $V^M = (\theta_1 - \frac{p}{q})(p - q^2) - rh(k)$; $v_1 = b\phi(k(t)) - \delta q(t)$. I compute: $\frac{\partial^2 V^M}{\partial^2 q} = -2\frac{q^3\theta_1 + p^2}{q^3} < 0$; $\frac{\partial^2 V^M}{\partial^2 p} = -\frac{2}{q} < 0$; $\frac{\partial^2 V}{\partial^2 k} = -r\frac{\partial^2 h(k)}{\partial^2 k} \leq 0$ as long as $\frac{\partial^2 h(k)}{\partial^2 k} \geq 0$, which is always true by assumption. Therefore, the necessary conditions of the maximum principle are also sufficient.

By differentiating (27) w.r.t. time:

$$\dot{\lambda}_1 = \frac{r\,\dot{k}\,[\frac{\partial^2 h}{\partial^2 k}\frac{\partial \phi}{\partial k} - \frac{\partial h}{\partial k}\frac{\partial^2 \phi}{\partial^2 k}]}{b[\frac{\partial \phi}{\partial k}]^2} \tag{30}$$

By inserting (26), (27) and (30) into (28) and by solving for $\dot{k}$:

$$\dot{k} = \frac{b(\frac{\partial \phi}{\partial k})^2}{r(\frac{\partial^2 h}{\partial^2 k}\frac{\partial \phi}{\partial k} - \frac{\partial h}{\partial k}\frac{\partial^2 \phi}{\partial^2 k})}\left\{q\widehat{\theta_1} - \frac{1}{4}(\widehat{\theta_1})^2 - \frac{3}{4}q^2 + \frac{r\frac{\partial h}{\partial k}}{b\frac{\partial \phi}{\partial k}}(\delta + \rho)\right\} \tag{31}$$

with

$$sign(\dot{k}) = sign(\frac{(4q\theta_1 - \theta_1^2 - 3q^2)}{4} + \frac{r(\delta + \rho)}{b}\Delta(k))$$

As before, with $q < \frac{1}{3}\theta_1$ if q increases k does likewise.

The steady state conditions $\dot{k} = 0$, $\dot{q} = 0$ imply:

$$\dot{k} = 0 \rightarrow \frac{\partial \phi(k)}{\partial k} = 4r\frac{\partial h(k)}{\partial k}\frac{\delta + \rho}{b\left(-4\widehat{\theta}_1 q + 3q^2 + (\widehat{\theta_1})^2\right)} \tag{32}$$

$$\dot{q} = 0 \rightarrow q = \frac{b\phi(k)}{\delta} \tag{33}$$

By solving (32) for q I find the following admissible root:

$$q = \frac{2}{3}\widehat{\theta}_1 - \frac{1}{3b}\sqrt{b^2(\widehat{\theta_1})^2 + 12r(\delta + \rho)\Delta} \tag{34}$$

The necessary condition for $q'' > 0$ is:

$$\frac{\Delta''}{\Delta'} < \frac{6br(\delta + \rho)\Delta'}{\theta^2 b + 12r(\delta + \rho)\Delta}$$

Let me assume that this condition holds, hence $q'' > 0$.

The phase diagram of the dynamic system referred to the monopolist's problem is very close to the one already seen for the planner's, therefore omitted, as well as for the stability analysis. What is relevant to my purpose is to compare (34) with (24) in order to understand who between the monopolist and the planner invests more in product quality. (34) and (24) are two descending curves with common intercept, but different slopes. It is straightforward to assess that the curve which is referred to the monopolist's problem is always less-sloped than the one referred to the planner's. Since the locus $\dot{q}= 0$ is common to both, it follows that the monopolist ends up with undersupplying product quality. Figure 2 (see the Appendix) illustrates the previous discussion.

Proposition 1 *For any given $\widehat{\theta_1}$, the monopolist underinvests in product quality compared with the social optimum. If $q(0) \in (\widehat{\theta_1}/3, q_0)$ with q_0 sufficiently low, the resulting equilibria are stable along a saddle path.*

It is worth noting that, in order to have the monopolist producing the level of product quality which is socially desirable, the monopolist should face a richer market than the planner: the locus $\dot{k}= 0$ referred to the his problem should shift upwards so as to cross the locus $\dot{k}= 0$ referred to the planner's exactly at P.

4 Advertising Campaigns

4.1 The Social Planner's equilibrium

The current value Hamiltonian for the social planner's problem (P) turns out to be:

$$H = e^{-\rho t} \left\{ \begin{array}{c} \frac{1}{2}\frac{\theta_1^2\widehat{q}^2 - 2p\theta_1\widehat{q} + p^2}{\widehat{q}} + (p - \widehat{q}^2)(\theta_1 - \frac{p}{\widehat{q}}) - B+ \\ -\frac{\gamma}{2}(\theta_1^{SS} - \theta_1)^2 - wz(l(t)) + \lambda_2(t)(c\psi(l(t)) - \delta\theta_1(t)) \end{array} \right\} \tag{35}$$

where $\mu_2(t)$ being the co-state variable associated to $\theta_1(t)$. The feasible set is $F^P = \{\theta_1 \geq q,\, p \geq 0,\, q \geq 0,\, \frac{p}{q} \geq \theta_0\}$. To simplify notations, as before, I neglect the index of time. Necessary conditions for a path to be optimal are:

$$H_p = 0 \rightarrow p = \widehat{q}^2 \tag{36}$$

$$H_l = 0 \rightarrow \lambda_2 = \frac{w\frac{\partial z(l)}{\partial l}}{c\frac{\partial \psi(l)}{\partial l}} \tag{37}$$

$$H_{\theta_1} = \theta_1\widehat{q} - \widehat{q}^2 - \gamma(\theta_1 - \theta_1^{SS}) - \lambda_2\delta = \rho\lambda_2 - \dot{\lambda}_2 \tag{38}$$

along with the transversality condition:

$$\lim_{t\rightarrow\infty} \mu_2\theta_1 = 0 \tag{39}$$

I apply Mangasarian's theorem to check whether these conditions are also sufficient: define $V^P = \frac{(\widehat{\theta_1})^2 q^2 - p^2 - 2q^3\widehat{\theta_1} + 2q^2 p}{2q} - C^{\theta_1} - wz(l)$; $v_2 = c\psi(l(t)) - \delta\theta_1(t)$. I compute: $\frac{\partial^2 V^P}{\partial^2 \theta_1} = q - \gamma \leq 0$ as long as $\gamma \geq q$. Then I compute: $\frac{\partial^2 v_2}{\partial^2 \theta_1} = 0$; $\frac{\partial^2 v_2}{\partial^2 l} = c\frac{\partial^2 \psi(l)}{\partial^2 l} \leq 0$, as long as $c > 0$ and $\frac{\partial^2 \psi(l)}{\partial^2 l} \leq 0$, which is always true by assumption; $\frac{\partial^2 V}{\partial^2 p} = -\frac{2}{q} < 0$; $\frac{\partial^2 V}{\partial^2 l} = -w\frac{\partial^2 z(l)}{\partial^2 l} \leq 0$ as long as $\frac{\partial^2 z(l)}{\partial^2 l} \geq 0$, which is always true by assumption.It remains to check that, in the optimal solution, the co-state variable be non negative: $\lambda_2 = \frac{w\frac{\partial z(l)}{\partial l}}{c\frac{\partial \psi(l)}{\partial l}} \geq 0$ by assumption.Therefore, as long as $\gamma \geq q$, the necessary conditions of the maximum principle are also sufficient.

By differentiating (37) w.r.t. time:

$$\dot{\lambda}_2 = \frac{w \dot{l} [\frac{\partial^2 z}{\partial^2 l}\frac{\partial \psi}{\partial l} - \frac{\partial z}{\partial l}\frac{\partial^2 \psi}{\partial^2 l}]}{c[\frac{\partial \psi}{\partial l}]^2} \tag{40}$$

By inserting (37) and (40) into (38) and by solving for $\dot{l}$ I obtain:

$$\dot{l} = \left(-\theta_1\widehat{q} + \widehat{q}^2 + \gamma(\theta_1 - \theta_1^{SS}) + \frac{w}{c}(\delta+\rho)\beta(l)\right)\frac{c(\frac{\partial \psi}{\partial l})^2}{w(\frac{\partial^2 z}{\partial^2 l}\frac{\partial \psi}{\partial l} - \frac{\partial z}{\partial l}\frac{\partial^2 \psi}{\partial^2 l})} \tag{41}$$

with $\beta(l) = z'/\psi'$, $\beta(l)' = \frac{z''\psi' - z'\psi''}{(\psi')^2} > 0$. $\beta(l)'' > 0$ if $z'''\psi' > z'\psi'''$. Since $\frac{\partial^2 z}{\partial^2 l}\frac{\partial \psi}{\partial l} > \frac{\partial z}{\partial l}\frac{\partial^2 \psi}{\partial^2 l}$:

$$sign(\dot{l}) = sign(-\theta_1\widehat{q} + \widehat{q}^2 + \gamma(\theta_1 - \theta_1^{SS}) + \frac{w}{c}(\delta+\rho)\beta(l))$$

>From s.o.c., $\gamma > q$, so if θ_1 increases l does likewise.

The steady state condition $\dot{l} = 0$ implies:

$$\frac{\partial \psi}{\partial l} = w\frac{\partial z}{\partial l}\frac{\delta+\rho}{c(\widehat{q}(\theta_1 - \widehat{q}) - \gamma(\theta_1 - \theta_1^{SS}))} \tag{42}$$

which, under the assumption that at the steady state $\theta_1 = \theta_1^{SS}$, yields:

$$\theta_1^P = \widehat{q} + \frac{w(\delta+\rho)}{c\widehat{q}}\beta \Rightarrow x^P = w\frac{\delta+\rho}{c\widehat{q}}\beta \tag{43}$$

If $z'''\psi' > z'\psi'''$, then $\beta'' > 0$ and $\theta'' > 0$. For the time being, suppose this is the case.

The steady state condition $\dot{\theta}_1 = 0$ implies:

$$\dot{\theta}_1 = 0 \rightarrow \theta_1 = \frac{c\psi(l)}{\delta} \tag{44}$$

I am interested in investigating the dynamics of the system in the positive quadrant of the space $\{l, \theta_1\}$, which is described in figure 3 (see the Appendix). The locus $\dot{l} = 0$ corresponds to the increasing curve that intersects the vertical axe at $\theta_1 = \widehat{q}$. Moving from the origin, the locus $\dot{\theta}_1 = 0$ draws a curve that may or may not intersect the locus $\dot{l} = 0$. In the case depicted in figure 3, two equilibria arise: P_1 which is a saddle, and P_2 which is a stable equilibrium. Sufficient but not necessary conditions to have equilibrium unicity are: $\psi'' = 0$ and $\theta_1'' \leq 0$.

As a mere illustration of the stability analysis, I focus on the following simple case: $\beta(l) = \psi(l) = l$. This is, among others, the case of a linear quadratic technology, in which only one equilibrium arises. From (12) and (41), the dynamic system can be written in matrix form:

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{l} \end{bmatrix} = \begin{bmatrix} -\delta & c \\ \frac{c}{w}(-\widehat{q} + \gamma) & \delta + \rho \end{bmatrix} \begin{bmatrix} \theta_1 \\ l \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{c}{w}(q^2 - \gamma\theta_1^{SS}) \end{bmatrix}$$

Since the determinant of the above 2×2 matrix is always negative in the relevant parameter range, the equilibrium is stable along a saddle path.

4.2 The Monopolist's equilibrium

The current value Hamiltonian for the monopolist's problem (M) turns out to be:

$$H = e^{-\rho t} \left\{ \begin{array}{c} (\theta_1(t) - \frac{p(t)}{\widehat{q}})(p(t) - \widehat{q}^2) - B - \frac{\gamma}{2}(\theta_1^{SS} - \theta_1)^2 + \\ -wz(l(t)) + \lambda_2(t)(c\psi(l(t)) - \delta\theta_1(t)) \end{array} \right\} \tag{45}$$

where $\lambda_2(t) = \mu_2(t)e^{\rho t}$, $\mu_2(t)$ being the co-state variable associated to $\theta_1(t)$. The feasible set is $F^M = \{\theta_1 \geq q, \, q \geq \frac{p}{\theta_1}, \, p \geq q^2, \frac{p}{q} \geq \theta_0\}$. To simplify notations, I neglect the index of time. By applying Pontryagin's maximum principle, necessary conditions for a path to be optimal are:

$$H_p = 0 \rightarrow p = \frac{\widehat{q}(\theta_1 + \widehat{q})}{2} \rightarrow x^M = \frac{\theta_1 - \widehat{q}}{2} \tag{46}$$

$$H_l = 0 \rightarrow \frac{w\frac{\partial z(k)}{\partial l}}{\lambda_2 c} = \frac{\partial \psi(l)}{\partial l} \rightarrow \lambda_2 = \frac{w\frac{\partial z(l)}{\partial l}}{c\frac{\partial \psi(l)}{\partial l}} \tag{47}$$

$$H_{\theta_1} = p - \widehat{q}^2 - \gamma(\theta_1 - \theta_1^{SS}) - \lambda_2\delta = \rho\lambda_2 - \dot{\lambda}_2 \tag{48}$$

along with the transversality conditions:

$$\lim_{t\to\infty} \mu_2\theta_1 = 0 \tag{49}$$

I apply Mangasarian's sufficiency theorem: define $V^M = (\theta_1 - \frac{p}{q})(p-q^2) - C^{\theta_1} - wz(l)$; $v_2 = c\psi(l(t)) - \delta\theta_1(t)$. I compute: $\frac{\partial^2 V^M}{\partial^2\theta_1} = -\gamma \leq 0$ as long as $\gamma \geq 0$, which is always true by assumption; $\frac{\partial^2 V}{\partial^2 p} = -\frac{2}{q} < 0$; $\frac{\partial^2 V}{\partial^2 l} = -w\frac{\partial^2 z(l)}{\partial^2 l} \leq 0$ as long as $\frac{\partial^2 z(l)}{\partial^2 l} \geq 0$, which is always true by assumption. Therefore, the necessary conditions of the maximum principle are also sufficient.

By differentiating (47) w.r.t. time:

$$\dot{\lambda}_2 = \frac{w\,\dot{l}\,[\frac{\partial^2 z}{\partial^2 l}\frac{\partial\psi}{\partial l} - \frac{\partial z}{\partial l}\frac{\partial^2\psi}{\partial^2 l}]}{c[\frac{\partial\psi}{\partial l}]^2} \tag{50}$$

Similarly, by inserting (46), (47) and (50) into (48) and by solving for l I obtain:

$$\dot{l} = \frac{c[\frac{\partial\psi}{\partial l}]^2}{w(\frac{\partial^2 z}{\partial^2 l}\frac{\partial\psi}{\partial l} - \frac{\partial z}{\partial l}\frac{\partial^2\psi}{\partial^2 l})}\left\{-\frac{1}{2}\theta_1\widehat{q} + \frac{1}{2}\widehat{q}^2 + \gamma(\theta_1 - \theta_1^{SS}) + \frac{w\frac{\partial z}{\partial l}}{c\frac{\partial\psi}{\partial l}}(\delta+\rho)\right\} \tag{51}$$

with $\frac{\partial^2 z(l)}{\partial^2 l}\frac{\partial\psi(k)}{\partial k} \neq \frac{\partial h(l)}{\partial l}\frac{\partial^2\psi(l)}{\partial^2 l}$, i.e., the ratio between second derivatives has to be different from the corresponding ratio between first derivatives.

$$sign(\dot{l}) = sign(-\frac{1}{2}\theta_1\widehat{q} + \frac{1}{2}\widehat{q}^2 + \gamma(\theta_1 - \theta_1^{SS}) + \frac{w}{c}(\delta+\rho)\beta(l))$$

As before, given s.o.c., if θ_1 increases l does likewise.

The steady state condition $\dot{l} = 0$ implies:

$$\frac{\partial\psi}{\partial l} = 2w\frac{\partial z}{\partial l}\frac{\delta+\rho}{c(\widehat{q}(\theta_1 - \widehat{q}) - 2\gamma(\theta_1 - \theta_1^{SS}))} \tag{52}$$

By solving (53) for θ_1^M:

$$\theta_1^M = \widehat{q} + \frac{2w(\delta+\rho)}{c\widehat{q}}\beta \;\Rightarrow x^M = w\frac{\delta+\rho}{c\widehat{q}}\beta \tag{53}$$

It is worth noting that $x^M = x^P$ as long as $l^M = l^P$, that is, as long as the monopolist's advertising investment coincide with the planner's.

Assume $z'''\psi' > z'\psi'''$, implying that $\theta'' > 0$.

As before, the steady state condition $\dot{\theta}_1 = 0$ implies:

$$\dot{\theta}_1 = 0 \rightarrow \theta_1 = \frac{c\psi(l)}{\delta} \tag{54}$$

By looking at figure 4 (see the Appendix) it is possible to make a direct comparison between the monopolist's problem and the planner's in the case in which $\theta'' > 0$. The above discussion can be summarized by the following:

Proposition 2 *In all odd (even) equilibria the monopolist overinvests (underinvests) in advertising compared with the social optimum. As a consequence, in all odd (even) equilibria the monopolist oversupplies (undersupplies) the good. If the monopolist's advertising investments and the planner's coincide, then the monopolist ends up with providing the market with the first best quantity level.*

5 Product Quality and Advertising Campaigns

Let $\phi(k) = k$, $\psi(l) = l$ and $h(k) = \frac{k^2}{2}$, $z(l) = \frac{l^2}{2}$ throughout this section; this implies: $\Delta(k) = k$, $\beta(l) = l$. More generally, let $\phi(k) = \Delta(k)$ and $\psi(l) = \beta(l)$.

5.1 The social planner's equilibrium

The Hamiltonian function the social planner faces is:

$$H = e^{-\rho t} \left\{ \begin{array}{c} \frac{\theta_1^2 q^2 - 2p\theta_1 q + p^2}{2q} + (p - q^2)(\theta_1 - \frac{p}{q}) - B - \gamma(\theta_1^{SS} - \theta_1)^2 + \\ -r\frac{k(t)^2}{2} - w\frac{z(t)^2}{2} + \lambda_1(t)(bk(t) - \delta q(t)) + \lambda_2(t)(cl(t) - \delta\theta_1(t)) \end{array} \right\} \tag{55}$$

where $\lambda_1(t) = \mu_1(t)e^{\rho t}$, $\mu_1(t)$ being the co-state variable associated to $q(t)$; $\lambda_2(t) = \mu_2(t)e^{\rho t}$, $\mu_2(t)$ being the co-state variable associated to $\theta_1(t)$. By using (21) and (41) the following dynamics obtain:

$$\frac{\partial k}{\partial t} = k(\rho + \delta) + \frac{b}{r}\frac{4q\theta_1 - \theta_1^2 - 3q^2}{2} \tag{56}$$

$$\frac{\partial l}{\partial t} = l(\rho + \delta) + \frac{c}{w}(q(q - \theta_1) + \gamma(\theta_1 - \theta_1^{SS})) \tag{57}$$

The dynamic system (57) and (58) along with the equations of motion (11) and (12) yields the following steady states[11]:

$$\theta_1^{PSS} = \frac{9.9037}{2b^2} r\delta(\delta + a) \tag{58}$$

$$q^{PSS} = \frac{2.7808}{2b^2} r\delta(\delta + a) \tag{59}$$

$$x^{PSS} = \frac{3.5614}{b^2} r\delta(\delta + a) \tag{60}$$

5.2 The monopolist's equilibrium

The Hamiltonian function the monopolist faces is:

$$H = e^{-\rho t} \left\{ \begin{array}{c} (p - q^2)(\theta_1 - \frac{p}{q}) - B - \gamma(\theta_1^{SS} - \theta_1)^2 + \\ -r\frac{k(t)^2}{2} - w\frac{z(t)^2}{2} + \lambda_1(t)(bk(t) - \delta q(t)) + \lambda_2(t)(cl(t) - \delta\theta_1(t)) \end{array} \right\} \tag{61}$$

where $\lambda_1(t) = \mu_1(t)e^{\rho t}$, $\mu_1(t)$ being the co-state variable associated to $q(t)$; $\lambda_2(t) = \mu_2(t)e^{\rho t}$, $\mu_2(t)$ being the co-state variable associated to $\theta_1(t)$. To ease notations, I drop the indication of time. By using (27) and (51), the following dynamic system obtains:

$$\frac{\partial k}{\partial t} = k(\rho + \delta) + \frac{b}{r}\frac{4q\theta_1 - \theta_1^2 - 3q^2}{4} \tag{62}$$

$$\frac{\partial l}{\partial t} = l(\rho + \delta) + \frac{c}{w}\frac{q(q - \theta_1) + 2(\gamma(\theta_1 - \theta_1^{SS})}{2} \tag{63}$$

The above system along with the equations of motion (11) and (12) yields:

$$\theta_1^{MSS} = \frac{9.9037}{b^2} r\delta(\delta + a) \tag{64}$$

$$q^{MSS} = \frac{2.7808}{b^2} r\delta(\delta + a) \tag{65}$$

$$x^{MSS} = \frac{3.5614}{b^2} r\delta(\delta + a) \tag{66}$$

with $b > 1.8872\sqrt{r\delta(\delta + a)}$ for $x < 1$.

Proposition 3 *Within a linear quadratic technology, the monopolist invests both in product quality and advertising campaigns twice as much compared to the social planner. As a consequence, he provides the market with the first best quantity level.*

[11] To ease calculations I assume that $b = c$ and $r = w$.

6 Policy implications

The task of this section is to make explicit the properties of the linear quadratic technology in terms of welfare and discuss them from a policy perspective. The focus is on efficiency. Before proceeding, let me remind you that, at the steady state, welfare is simply given by the sum between consumers surplus and profits steady state values, as follows:

$$W^{MSS} = CS^{MSS} + \pi^{MSS} \tag{67}$$

First, I compute the level of welfare at the steady state under monopoly regime. By applying (14) and (10), respectively, and by taking into account that, at the steady state, capital accumulation stops:

$$W^{MSS} = r^2\delta^2 (\delta + a)^2 \frac{80.44\,7r\delta(\delta + a) - 9.903\,6b^2}{b^6} - B \tag{68}$$

where

$$\begin{aligned} CS^{MSS} &= \frac{1}{4}(\frac{(\theta_1^{MSS})^2}{2}q^{MSS} - \theta_1^{MSS}(q^{MSS})^2 + \frac{(q^{MSS})^3}{2}) \\ &= \frac{17.636}{b^6}r^3\delta^3 (\delta + a)^3 \end{aligned} \tag{69}$$

$$\pi^{MSS} = 0.002475\,9r^2\delta^2 (\delta + a)^2 \frac{25369r\delta(\delta + a) - 4000b^2}{b^6} - B \tag{70}$$

with B small enough to respect the usual non negative constraints and $b < 2.850\,1\sqrt{r\delta (\delta + a)}$ for W^{MSS} to be positive.

The corresponding computation under social planning is omitted, since it is not possible to compare the welfare arising from markets in which the intercept term of the demand functions differs, as it is in our case. Rather, I focus on W^{MSS}, aimed at investigating which are its responses to different policy regulations. Assume that the government may choose between two kinds of policies: (i) to affect the instantaneous investment costs, through r; (ii) to affect the accumulation process, through b. Notice that, in order to obtain manageable solutions, I have assumed that $c = b$ and $w = r$. As a consequence, b enters the accumulation of both quality and reservation price, while r denotes the price of both specific inputs, k and l. Consider, first, policy (i). Its effect is given by:

$$\frac{\partial W^{MSS}}{\partial r} = 232\,533\,(\delta + \rho)^3\, r^2 \frac{\delta^3}{b^6} > 0 \tag{71}$$

Therefore, a marginal increase in r yields welfare improvements, since it hinders the acquisition of capital.

As to policy (ii), its effect is given by:

$$\frac{\partial W^{MSS}}{\partial b} = -465\,066\,(\delta + \rho)^3\, r^3 \frac{\delta^3}{b^7} < 0 \tag{72}$$

Therefore, a marginal decrease in b yields welfare improvements, since it slows down the accumulation of capital.

In order to understand which of the two policies should be preferred, I consider the following equation:

$$\left|\frac{\partial W^{MSS}}{\partial b}\right| = \frac{2r}{b}\left|\frac{\partial W^{MSS}}{\partial r}\right| \tag{73}$$

It is immediate to draw from it:

Proposition 4 *As long as input prices are sufficiently high,* $r > \frac{b}{2}$*, it is optimal to reduce the accumulation of capital through the equations of motion, both w.r.t. consumers'surplus and profits. This amounts to saying that regulation (ii) should be preferred to regulation (i).*

7 Concluding remarks

The comparison between static and dynamic inefficiencies which are due to a monopolistic structure dates back to Schumpeter (1942). In accordance with him, I have shown that, when both product quality and advertising investments are activated, a monopolist is more innovator than a social planner. More surprisingly, I have shown that his traditional tendency to undersupply quantities is not robust to a dynamic setting. The basic implication of this result is that, in weighing the pros and cons of a monopolistic structure, one should be aware of making explicit the time horizon at which he refers itself. Indeed, quantity distortions are present in the short run, as we know from static analyses, but they vanish in the long run, within a linear-quadratic technology, at least. Further developments are needed to understand the extent to which a vertical differentiated monopolist may provide the market with the first best quantity level, a result which, per se, deserves some attentions. As to product quality provision, the existing literature dealing with static models has not reached homogenous results, while dynamic analyses have been almost completely absconding. The result I have obtained is that, for any given reservation price, the monopolist always underinvests compared to the social planner. In my paper, I have also studied the appropriate policy implications w.r.t. the investments in R&D jointly considered: for a relatively low level of input prices, I have shown that it is optimal for the government increasing them up to a critical threshold, above which it becomes optimal to slow down the accumulation of capital.

8 Graphical Appendix

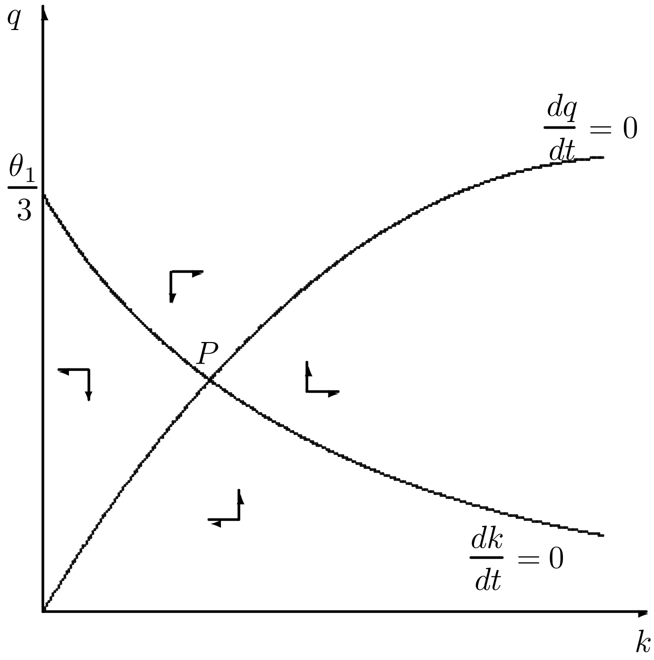

Figure 1 : Planner's Phase Diagram

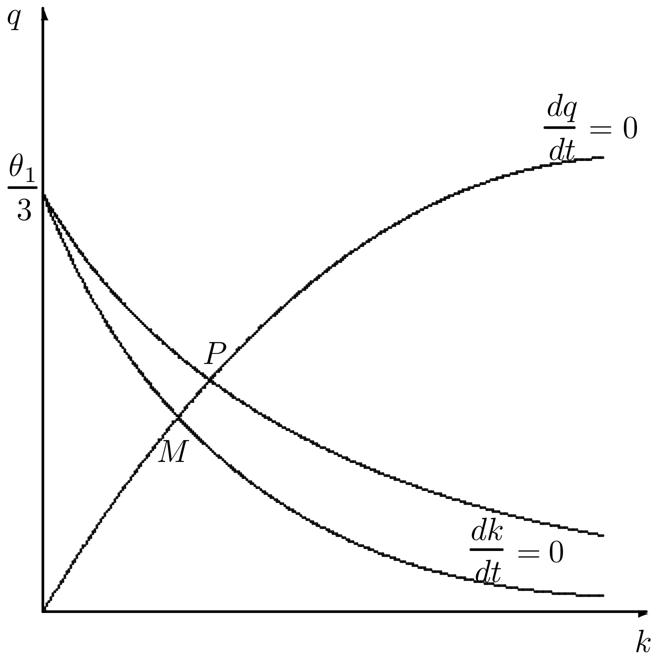

Figure 2 : Monopolist vs Planner

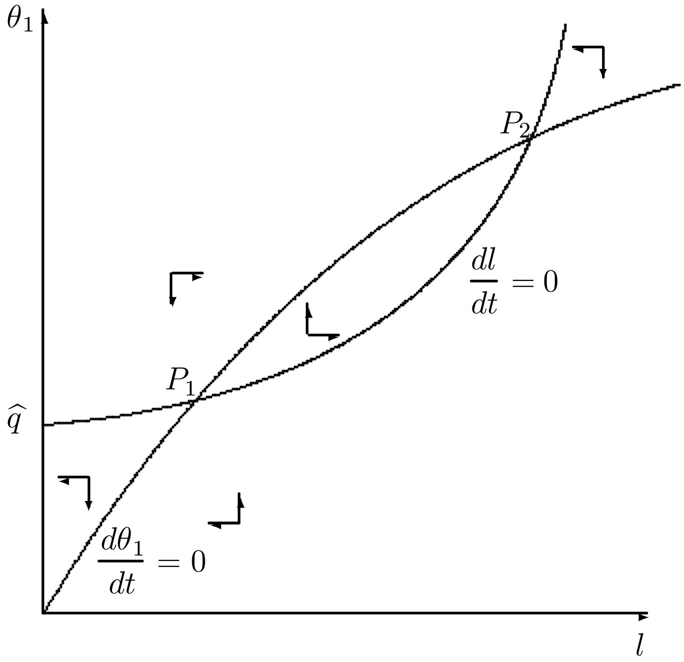

Figure 3 : Planner's Phase Diagram

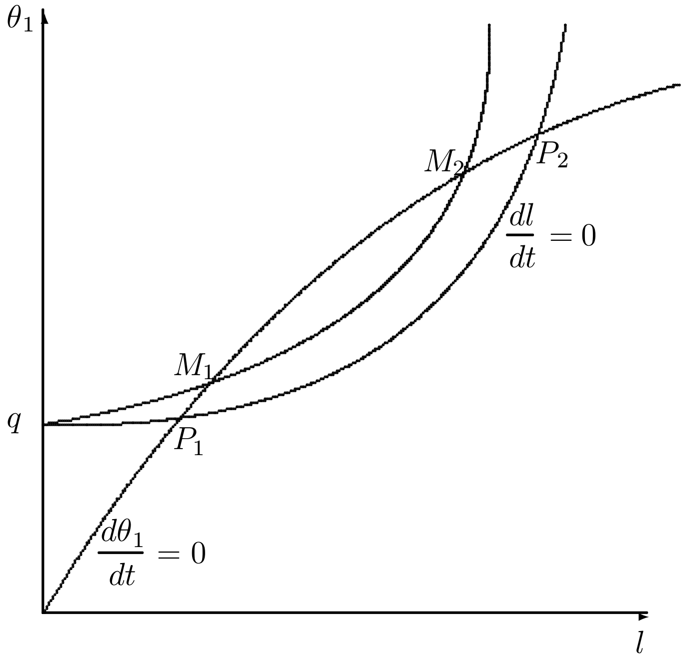

Figure 4 : Monopolist vs Planner

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