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Search and Taxation in a Model of Underground Economic Activities

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Abstract

We develop a simple and flexible general equilibrium model of an economy with underground production and trade. Because of the furtive nature of underground activities, information about trading opportunities in the irregular sector is less than perfect — hence, agents devote some time to locate trading partners in the black economy and then bargain over the terms of trade. The model allows a unified treatment of a number of issues: optimal taxation and enforcement, dynamic responses to tax increases and political incentives for taxation and enforcement. We also argue that the main predictions of the model are in line with stylised facts and empirical regularities.

Keywords: Taxation, Tax Enforcement, Underground Economy, Search, Optimal Policy, Dynamic Policy, Political Incentives, Redistribution.

JEL Code: H26, H21, D72.

1 Introduction

It is widely recognized that one major effect of taxation is to stimulate activities in the so-called "underground" (or "shadow") economy. This existence of underground activity is a significant source for concern. The possibility of evading taxes makes taxation less efficient; by undermining the tax base this possibility may jeopardize long-run scope for financing adequate public services. Furthermore, it implies that official measures of e.g. income and unemployment, which form the basis for policy, will necessarily be biased.

The purpose of this paper is to suggest a simple framework for studying the relationship between tax policy and underground economic activity. The analysis is motivated by a set of empirical regularities. First, the size of the underground economy varies greatly across countries. Schneider and Enste (2000) report that the size of the shadow economy as percent of GDP in the OECD countries in 1996 ranged from 7.5 percent in Switzerland to 28.5 percent in Greece.

Second, participation in underground economic activities at the individual level, as well as aggregate underground activities, appears to be positively related to tax levels. At the individual level, Clotfelter (1983), using data from the 1969 Taxpayer Compliance Measurement Program, considered the impact of (total) marginal tax rates on evasion and found the impact to be positive and significant. The subsequent literature — surveyed in Andreoni et al. (1998) and Slemrod and Yitzhaki (2002) — has generally, but not exclusively, corroborated Clotfelter's finding. At the aggregate level, Schneider (2000) provides cross-country estimates of total tax burden alongside estimates of the size of the shadow economy for the OECD countries. Though there is a positive correlation, it is not overwhelmingly strong. However, as noted by Schneider, tax implementation is also likely to play an important role.[1]

Third, the underground economies in the western countries appear to have been growing over time (see Schneider, 2000). Part of this growth can be explained by increasing taxes. However, note that e.g. effective tax rates on labour, while generally increasing in the OECD region during the 1980s, did not increase in all countries during the 1990s.[2] One possibility is that the dynamic effects of tax increases exceed the short-run effects.

Fourth, worker participation in the underground economy appears to be related to outcomes in the regular labour market. In particular, in an important paper, Lemieux et al. (1994) found that labour supply to the underground economy comes disproportionately from workers with weak labour market attachment, e.g. unemployed, young, benefit recipients, and, more generally, from individuals with low incomes from the regular labour market.

A defining characteristic of underground economic activities is that they are illegal.[3] The

[1] Indeed, Friedman et al. (2000) provide estimates for 69 countries, including transition economies and developing countries, and find negative (but not significant) effect of taxes on underground activity. Rather they find that tax implementation, regulation and corruption generate black activities — see also Johnson et al. (1998). Johnson et al. (1997), who study the determinants of unofficial economic activity in East-European countries, also stress tax implementation, the legal system, corruption and regulation facing entrepreneurs.

[2] See e.g. Liebfritz et al. (1997) and Martinez-Mongay (2000).

[3] We exclude household production from our definition; also, legal activities aimed at reducing the tax liability are excluded.

traditional approach, starting with the seminal contribution of Allingham and Sandmo (1972) characterizes tax evasion as a risky activity.[4] This approach, by focusing largely on the individual decision to evade taxes as a portfolio choice, does not particularly highlight that underground economic activity involves trade. We argue that, while underground economic activity and regular activity are fundamentally identical and can coexist, one effect of the illegal status of the former is to differentiate the amount of information that is available in the two market segments: while information about trading opportunities in regular markets can be assumed to be perfect, since underground activity is, by definition, furtive, information about trading opportunities in this segment will be less than perfect.

Traditionally search theory has been used to capture the notion of imperfect information in markets. This has a natural interpretation in the current context: while trading in the regular market can be regarded as frictionless, an agent wishing to trade in the underground sector must devote some time and effort to locating "trustworthy" trading partners with whom he/she can trade illegally at no risk. Hence we model underground trade as the outcome of a standard bilateral search process where buyers and sellers devote some time to locating each other (and implicitly verifying each others' "trustworthiness").[5] This process — while individually rational when facing taxes — is a purely socially wasteful activity since it subtracts time from physical production.

Our model, while being a complete general equilibrium model, is both flexible and tractable, lending itself to the study of a variety of issues. In order to highlight this, we first use the model to show that one can derive an optimal tax formula, as well as a rule for the optimal level of costly tax enforcement, in a static environment; we also discuss the properties of optimal policy, with the aid of a numerical example. We then include dynamics by allowing underground networks to have a "stock" feature, and study the response to a permanent tax increase, showing that the effects in terms of increased underground activity tend to propagate over time; this is consistent with the observed delay between tax rises and black sector expansion in the OECD countries referred to above.[6] Finally, we use the model to consider the political incentives for taxation

[4] See Cowell (1990) for a general discussion — both theoretical and empirical — of tax evasion.

[5] See Boadway et al. (2000) for an interesting analysis on how agents wishing to trade in the underground economy face the problem of verifying each others' trustworthiness.

[6] An alternative explanation could be based on evolving endogenous social norms. For example, Gordon (1989) considers how preferences for honesty affect how evasion responds to taxes; Myles and Naylor (1995) have shown how social norms can give rise to multiple equilibria.

and for tax enforcement. A standard prediction in the literature on voting over redistributive taxes based on the labour-leisure framework is that increased inequality should increase taxes.[7] Our political analysis, while confirming the above prediction, also suggests that higher taxes are in a political equilibrium combined with stricter tax enforcement. This fact may partly explain why the observed link between taxes and the size of the underground economy in cross-country comparisons is perhaps weaker than one would expect.

The paper is organized as follows. Section 2 sets up the static model. Section 3 considers optimal taxation and tax enforcement. Section 4 then shows how one can introduce dynamics. Section 5 turns to a political economy analysis. Finally, Section 6 concludes.

2 The Basic Model

Consider an economy with two modes of supply: "regular" and "black". There is no difference in technologies, only black sales are hidden from the tax authorities. We will not be concerned with heterogenous goods. However, in order to ensure that there is a rationale for trade, we will implicitly model diversity of goods by assuming that each consumer buys at most one unit of output from each producer.

The production technology is linear and uses effective labour as only input. Workers differ in productivities only: a type-w agent produces w units of output per hour. w has a distribution F on a support $\mathcal{W} = [w^-, w^+]$ with a strictly positive lower bound w^-; and the associated density f is strictly positive on the entire support. The mean wage is denoted $\overline{w}$ and the median wage is denoted w^m. The price of a unit of output sold in the regular market is normalized to unity, whereas the black market price is π.

2.1 Agent's Behaviour

Each agent in the economy can be thought of as an entrepreneur. He decides how much to produce, and how to sell his output (regular or black). He is also a consumer with a very simple objective: to maximize his total consumption, denoted y. There is a linear income tax in place, with marginal tax rate $\tau \geq 0$ and lump-sum subsidy $S \geq 0$.[8]

Each agent has H hours to allocate. Let l denote the hours of labour devoted to regular production, and let η denote the amount of output the agent sells "black". Disposable income

[7] See Romer (1975), Meltzer and Richard (1981), and Persson and Tabellini (2000).

[8] More generally, S can be taken as a measure of per-capita public expenditure.

is therefore $(1-\tau) wl + \pi\eta + S$. In order to trade in the black market the agent must spend some time locating suppliers to buy from, and consumers to sell to.[9] Thus let α denote the time that the agent spends "searching" for black suppliers; it is natural to assume that there are diminishing returns to these efforts: by spending α hours searching, the agent locates $\phi(\alpha)$ black suppliers, where $\phi(\cdot)$ is increasing and concave (and is described in more detail below). Similarly, in order to sell in the black market, the agent must find buyers; this requires some search effort ξ. By searching ξ hours, the agent locates $\eta(\xi)$ buyers, where $\eta(\cdot)$ is also increasing and concave. To satisfy this demand he must spend η/w hours producing for the black market. Thus we can write the consumer's time constraint as

$$l + \alpha + \xi + \frac{\eta}{w} = H. \tag{1}$$

Since the agent buys ϕ units in the black market at the lower price π, his total consumption y exceeds his disposable income by the amount $\phi(1-\pi)$. Hence, substituting for disposable income and for regular labour supply, the agent's time-allocation problem, and the achieved consumption $y(w)$, can be written

$$y(w) = \max_{\alpha,\xi} \ (1-\tau) w \left(H - \alpha - \xi - \frac{\eta(\xi)}{w} \right) + \pi\eta(\xi) + \phi(\alpha)(1-\pi) + S, \tag{2}$$

where l, α and ξ must all be chosen non-negatively.

2.2 Bargaining over the Black Market Price

The black market price π is endogenous. Consider a buyer and a seller who have located each other and trust that they can trade (at no risk) without paying the tax. These agents find themselves in a bilateral monopoly situation: if they trade one unit of output they can save the associated tax. Suppose that an initial "search stage" is followed by a "bargaining stage". Then, if bargaining breaks down their fallback position is to trade legally (or equivalently, reject each other and trade with others in the regular market).

If the price π is agreed, the seller earns π. On the other hand, if they disagree he can earn the net income $1-\tau$ from selling the output on the regular market. Turning to the buyer, at the agreed price π he saves $1-\pi$ which he uses to buy additional consumption (at the price of unity) in the regular market. Hence the gain to the buyer is $1-\pi$. Assuming (generalized) Nash bargaining, the bargained π solves

$$\max_{\pi} \ (1-\pi)^b (\pi - (1-\tau))^{1-b},$$

[9] We assume that there is no "double coincidence of wants" although this is not crucial.

where b and $1-b$ denote the bargaining power of the buyer and the seller respectively, with $b \in (0,1)$. The solution is simply

$$\pi = 1 - b\tau. \tag{3}$$

Note that π decreases with τ. In other words, the relative price of black market supply to regular supply decreases in the tax rate. Eq. (3) shows that the black market price splits the private benefit from tax evasion according to a simple sharing rule.[10]

2.3 Matching

We postulate the existence of a constant returns to scale aggregate matching technology. However, in order to capture the idea that it becomes increasingly difficult for a single individual to locate trustworthy trading partners we assume that individual hours of search translate into *effective search* at a diminishing rate. For simplicity we adopt an isoelastic specification: an agent's *effective search* for buyers and sellers is thus α^{ε} and ξ^{ε} respectively, where $\varepsilon \in (0,1)$ is the elasticity of effective search with respect to the corresponding time-input. The aggregate effective search by buyers is then $A \equiv \int [\alpha(w)]^{\varepsilon} \mathrm{d}F$, while aggregate effective search by sellers is $\Xi \equiv \int [\xi(w)]^{\varepsilon} \mathrm{d}F$. It is useful to define $\theta \equiv \Xi / A$.

The total number of meetings between buyers and sellers is given by $km(A, \Xi)$, where $m(\cdot,\cdot)$ has constant returns to scale. Further defining $q(\theta) = m(1/\theta, 1)$ it follows that the number of black market sellers located by an individual agent is $\phi(\alpha) = \alpha^{\varepsilon} k \theta q(\theta)$. Similarly, the number of black market buyers located by an agent is $\eta(\xi) = \xi^{\varepsilon} k q(\theta)$. Note that both ϕ and η are increasing and concave as postulated above, and naturally $\phi(0) = \eta(0) = 0$.

We will treat the matching parameter k as a policy variable for the government that can be reduced at a cost. Let $\chi(k)$ be the spending on tax enforcement required to achieve k. We assume that k has a finite upper bound $k^{+} > 0$ associated with zero spending on enforcement, and lower bound $k^{-} \geq 0$ associated with infinite spending on enforcement. Since reductions in k are costly, $\chi(\cdot)$ is a strictly decreasing function on the domain $\mathcal{K} = [k^{-}, k^{+}]$.[11]

[10] This possibility of determining the cash-sale price as the solution to a bargain struck between the buyer and the seller was noted in Gordon (1990).

[11] Our interpretation of enforcement is akin to the one prevailing in the literature on tax avoidance — see e.g. Slemrod (2001). Tax avoidance is a costly but riskless activity which enables the individual to hide part of the tax base from the fiscal authorities: correspondingly, tax enforcement is to be seen as an attempt on the government's part to reduce the profitability of this activity.

2.4 Equilibrium

In *laissez-faire* (or for that matter in any first-best allocation), there would be no black markets. However, introducing distortionary taxes to achieve e.g. some redistribution will naturally give rise to black market activities. The government has three policy parameters at its disposal; the matching-parameter k, the tax rate τ, and the lump-sum transfer S. Hence a *policy* is a triple $z = (k, \tau, S)$. Each agent, when solving (2) takes the policy z, as well as π and θ, as given. Replacing π using (3) yields the following solution:

$$\alpha(w) = \left[\frac{b\varepsilon\tau k\theta q(\theta)}{(1-\tau)w}\right]^{\frac{1}{1-\varepsilon}}, \tag{4}$$

and

$$\xi(w) = \left[\frac{(1-b)\varepsilon\tau k q(\theta)}{(1-\tau)w}\right]^{\frac{1}{1-\varepsilon}}. \tag{5}$$

Note that α and ξ are strictly positive whenever $\tau > 0$.[12] In order to characterize the equilibrium we consider first the role of θ. Recalling that b measures the bargaining power of buyers, when b is large an agent will tend to devote more effort searching for sellers rather than searching for buyers. To restore the equilibrium θ adjusts. Consequently θ, which measures the ratio of search by sellers relative to search by buyers, naturally decreases in b. Direct calculations immediately yields that

$$\theta = \left(\frac{1-b}{b}\right)^{\varepsilon} \tag{6}$$

when all agent optimize. Hence, importantly, θ will not depend on policy.

Lemma 1 *The equilibrium θ is independent of τ and k and is given by (6).*

Also, using (6) to replace θ in (4) and (5) immediately yields that each agent makes equally many black market purchases as black market sales:

Lemma 2 *$\phi(w) = \eta(w)$ for all w.*

[12] We will only consider cases where all agents supply positive amounts of regular labour; doing otherwise would complicate the analysis without adding valuable insights. Alternatively, we might have used a modified isoelastic search function (shifted downwards so that it has an intercept on the horizontal axis) with which corner solutions never occur.

Both results hinge on the iso-elastic specification and are somewhat restrictive in that they rule out some potentially interesting general equilibrium effects; on the other hand, they have the benefit of allowing us two major simplifications. First, θ can be treated as a constant, and therefore, policy will not affect behaviour indirectly through θ; second, we can talk about an agent's level of black market trade as a single-dimensional variable.

Consider how black market trade varies with policy and with ability.[13] Turning first to the effect of policy, straightforward differentiation of (4) and (5) immediately yields the following expressions for the elasticities of α and ξ with respect to τ and k,

$$e_{\alpha,\tau} = e_{\xi,\tau} = \frac{1}{(1-\varepsilon)(1-\tau)}; \quad e_{\alpha,k} = e_{\xi,k} = \frac{1}{(1-\varepsilon)}. \tag{7}$$

Thus, we see that black market activities respond monotonically to the policy variables: α and ξ increase in τ and k. The intuition is quite general and very simple: as τ and k go up, the returns to black market activities go up, and therefore the agents become more active in the black market. Thus, in comparison to e.g. the portfolio approach following Allingham and Sandmo (1972), the current model unambiguously predicts that taxes increase tax evasion. Note also that taxes reduce aggregate physical output; by inducing black-market search (which is privately rational but socially wasteful) taxation reduces the effort spent producing consumption goods. Hence taxes reduce not only *officially measured* output, but also *actual* output.

As for the effect of ability, it is also immediate to see that

$$e_{\alpha,w} = e_{\xi,w} = -\frac{1}{(1-\varepsilon)}; \tag{8}$$

that is, search time decreases monotonically as ability increases. An immediate corollary of this is that the amounts traded in the black market, $\eta(w)$ and $\phi(w)$, are strictly decreasing in the agent's type w, while regular labour supply is strictly increasing in w. The properties of the model are thus consistent with one of the empirical regularities noted by Lemieux et al. (1994): agents with low regular income tend to participate more intensely in the underground economy.

3 Optimal Policy in the Static Model

Suppose that the government's objective is given by a Paretian, concave function of the agents' consumption levels. More specifically, assume that the government chooses a policy $z = (k, \tau, S)$

[13] It can be shown that the comparative statics signs generalise to a non-isoelastic specification.

to maximise a social welfare function

$$W = \int g[y(w)]\mathrm{d}F, \tag{9}$$

where

$$y(w) = (1-\tau)\,wl + (1-b\tau)\,\eta(\xi) + b\tau\phi(\alpha) + S, \tag{10}$$

and where it is implicit that α, ξ and l are optimised by the agents. Concavity of g implies a preference for equity and hence redistribution: in *laissez-faire*, higher-w agents are better off.

The choice of policy is subject to a budget balance condition:

$$\int \tau wl(w)\,\mathrm{d}F - S - \chi(k) \geq 0. \tag{11}$$

A policy is said to be *feasible* if it satisfies the revenue constraint (11); the set of feasible policies is denoted $\mathcal{Z}$. An optimal policy then maximizes (9) by choice of $z \in \mathcal{Z}$.

3.1 Optimal Taxation and Tax Enforcement

Let $\omega = (1-\tau)\,w$ denote the net wage, and let ν be the Lagrange multiplier associated with the revenue constraint (11); differentiating the Lagrangian of the optimal tax problem with respect to τ and S, and manipulating the resulting first order conditions in a standard way, we get an optimal linear income tax formula,

$$\frac{\tau}{1-\tau} = \frac{\operatorname{cov}(g'/\nu, wl)}{-\int wle_{l,\omega}\mathrm{d}F} = \frac{\operatorname{cov}[g'/\nu,\ wl)]}{\int w(\alpha+\xi)e_{\alpha+\xi,\omega}\mathrm{d}F + \int \eta e_{\eta,\omega}\mathrm{d}F}, \tag{12}$$

where $e_{l,\omega}$, $e_{\alpha+\xi,\omega}$ and $e_{\eta,\omega}$ are the elasticities of, respectively, regular labour supply, total search and black market production with respect to the net wage, and the second equality sign follows from the fact that $e_{l,\omega} = -[(\alpha+\xi)e_{\alpha+\xi,\omega} + (\eta/w)e_{\eta,\omega}]/l$.

This is a variant of the rule derived by Dixit and Sandmo (1977) for the case of linear income taxation without black markets and by Slemrod (1994) for the case of tax avoidance. As expected, we have a covariance term, capturing equity considerations, at the numerator, and a weighted average of the elasticities of labor supply or equivalently the elasticities of total search and black production, capturing efficiency considerations, at the denominator.

We can show that both the numerator and the denominator are negative, so that, at the optimum, $0 < \tau < 1$. The covariance term is negative because $g'(y)$ and wl vary in opposite directions as w increases: concavity of g implies that $g'(y)$ is decreasing in w (because y is increasing), whereas wl is increasing in w (because l is increasing). Note that, unlike in the

Dixit-Sandmo model, but similarly to the Slemrod model, only a fraction of total income appears in the covariance term, due to the presence of black market. The elasticity term is negative also by the signs of the above comparative statics. In our setting, the welfare loss is made of two components. The first is an efficiency loss, here expressed as the elasticity of total search time, which plays the same role as leisure in the original Dixit-Sandmo framework. The second is given by the elasticity of black market production and represents the revenue reduction due to erosion of the income tax base; it corresponds to a component of Slemrod's optimal tax rule with a similar interpretation.

Consider then enforcement. Differentiating the Lagrangian of the optimal tax problem with respect to k, and manipulating, yields

$$\int \tau w \left(\frac{\partial \alpha}{\partial k} + \frac{\partial \xi}{\partial k} + \frac{1}{w} \frac{\partial \eta}{\partial k} \right) \mathrm{d}F = \frac{\tau}{k} \mathrm{cov} \left(\frac{g'}{\nu}, \eta \right) - \chi'. \tag{13}$$

The l.h.s. measures the efficiency gains of tax enforcement; stricter enforcement, i.e. a reduction of k, implies a reduction of search time and black market production, and allows the government to raise more revenue. On the r.h.s, we have the marginal cost of tax enforcement; a reduction of k implies that the government has to bear more direct costs (χ goes up) and at the same time induces a negative equity effect. This is captured by the first term on the r.h.s., which has the same sign as the covariance between the net social marginal valuation of income and black market meetings ($\eta(w) = \phi(w)$ for all w by Lemma 2). We know that low-wage types trade more in the black sector than high-wage types; hence the covariance is positive, as both g' and η are decreasing in w. It follows that the equity term is positive also, i.e. an increase of tax enforcement is bad for equity, for the simple reason that the poor are more engaged in tax dodging. Thus stricter enforcement is socially optimal if and only if the efficiency gains compensate for the equity losses and the direct cost of tax enforcement.

3.2 A Numerical Example

In order to illustrate the properties of optimal policy, we resort to a numerical simulation. The main insight is that social optimality requires that increases in the tax rate are matched by stricter enforcement policies; as a result, we find that increased progressivity does not necessarily lead to a larger underground sector. This is in line with the experience of e.g. the Scandinavian countries, that despite having some of the largest redistributive programs among European countries, score in the middle group for the size of the shadow economy — see Schneider and Este (2000).

		B	I	II	III	IV	V	VI
1	μ	0.00					-0.16	0.065
2	σ	0.70					0.90	0.60
3	ϱ	1.00			2.00	0.75		
4	mean w	2.28			3.28	2.03		
5	median w	2.00			3.00	1.75	1.85	2.07
6	modal w	1.61			2.61	1.36	1.37	1.74
7	$w <$ mean %	63.6					67.3	61.8
8	$w <$ mode %	24.1					18.4	27.4
9	υ	-2	-1	-3				
10	k	1.501	2.080	1.289	1.911	1.399	1.141	1.902
11	τ	0.386	0.278	0.442	0.346	0.396	0.477	0.309
12	winners %	50.2	52.8	48.9	54.1	48.7	54.8	48.4
13	black GDP %	7.96	9.29	7.43	4.87	9.48	7.21	8.80
14	$\chi(k)$ %	2.25	0.99	3.00	0.88	2.91	3.70	1.29
15	excess burden %	2.44	1.75	2.86	1.27	3.01	3.18	1.93
16	black time %	9.73	11.43	9.04	5.38	12.21	9.57	10.31
17	search time %	3.05	2.20	3.58	1.42	4.00	4.37	2.31

Table 1: Optimal Taxation and Enforcement

The model has been parametrized as follows. The wage distribution, $f(w)$, is a lognormal with parameters (μ, σ, ϱ), where ϱ is a rightward shift (so that the lowest wage is strictly positive).[14] The effective search function is $5\alpha^{\varepsilon}$ and $5\xi^{\varepsilon}$ for buyers' and sellers' search, respectively.[15] We take the matching technology to be Cobb-Douglas, namely $m(A, \Xi) \equiv \sqrt{A\Xi}$; hence $q(\theta) \equiv \sqrt{1/\theta}$. The cost of enforcement is $\chi(k) = 1.25(k^{+} - k)/(k - k^{-})$. Finally, the social welfare function is isoelastic, $W = \int (y^{\upsilon}/\upsilon)\mathrm{d}F$, $\upsilon \,\Box\, 1$ ($\upsilon = 1$ gives the utilitarian case, whereas $\upsilon \to -\infty$ approaches the Rawlsian case). We set $H = 25$, $\varepsilon = 0.5$, $b = 0.5$, $k^{+} = 3$ and $k^{-} = 0$ throughout.

Results are reported in Table 1. Rows 1–9 describe the model, while the remaining rows give the results. In particular, rows 10–11 describe the optimal policy; row 12 shows the percentage of population that, compared to *laissez faire*, gains from redistribution. Next, we give black output and enforcement costs as percentages of GDP, i.e. $\int wl\mathrm{d}F + \int \eta \mathrm{d}F$, regular plus black output. In row 15, we have the excess burden of tax evasion, measured by the opportunity cost (in terms of forgone production) of search, i.e. $\int w(\alpha + \xi)\mathrm{d}F$, and expressed as a percentage of aggregate potential output, $\int wH\mathrm{d}F$. Finally, the last two rows show the time allocation: hours devoted to black production and search are given as percentages of total available time H.

Column B contains the benchmark case. The remaining columns of Table 1 make some comparative statics. Columns I and II show the impact of changing the social welfare function. As expected, the optimal marginal tax rate falls as the social welfare function becomes less egalitarian (υ increases); notice that tax progressivity and enforcement are complementary tools in the sense that as τ increases, k becomes smaller. The excess burden of tax evasion, as well as search time, increase with tax progressivity. Black GDP and black time, however, are decreasing in $-\upsilon$, which highlights the fact that an egalitarian policy does not necessarily leads to a larger underground sector since it is accompanied with a strict tax enforcement.

The impact of changes in the income distribution is examined in columns III–VI. In particular, in columns III and IV we compare "rich" and "poor" economies, for a given degree of

[14] Following Slemrod et al. (1994), we employ a discrete approximation of the wage distribution in an economy with 1000 agents. The lowest wage, w^{-}, corresponds to the cumulative frequency of 0.0005, and the highest wage, w^{+}, to the cumulative frequency of 0.9995 (the intermediate wages are computed using a step of 0.001 along the cumulative distribution).

[15] For the numerical simulation it is convenient to have a constant (equal to 5 in this case) multiplying the effective search functions. Since our analytical results do not depend on this constant, and in order to save on notation, we dropped the constant in the theoretical model.

inequality, by shifting the wage distribution (i.e. changing ϱ). A "rich" economy (column III, $\varrho = 2$) chooses less redistribution and also less enforcement whereas a "poor" one (column IV, $\varrho = 0.75$) opts for more progressivity and enforcement (again a pattern of complementarity). In this case, lower progressivity in the rich economy is accompanied by a smaller size of the underground economy. We finally compare economies with the same per capita GDP but different inequality in the income distribution: in column V wage inequality is higher than in the benchmark case, in column VI is lower.[16] Not surprisingly, higher inequality is associated to more tax progressivity; the complementarity pattern with tax enforcement is also confirmed, as is the fact that higher progressivity may be associated with a smaller size of the underground economy (even though the relation is rather weak).[17]

4 Introducing Dynamics

One benefit to the current model is that it can readily be generalized to a dynamic setting. This allows us to consider e.g. dynamic responses to policy, and also optimal dynamic policy. Formally, let time be discrete and let the horizon be infinite, $t = 0, 1, \ldots$. In each period an agent has H hours to allocate to search and production as above. Black market contacts have a "stock" feature. New meetings add to the agent's underground "network"; on the other hand existing contacts dissolve at an exogenous rate. Hence, black market "search" can be thought of as a private investment activity. On the other hand, there are no savings.

We begin by describing the equilibrium for a given policy path $\{z_t\}_{t=0}^{\infty} = \{k_t, \tau_t, S_t\}_{t=0}^{\infty}$. Each agent formally chooses a sequence of time allocations; however, it turns out to be more convenient to work with "meetings" as control variables. Thus let

$$a \equiv k\theta q(\theta)\alpha^{\varepsilon}; \quad x \equiv kq(\theta)\,\xi^{\varepsilon}. \tag{14}$$

Then $\alpha = c_a a^{1/\varepsilon}$ and $\xi = c_x x^{1/\varepsilon}$ where $c_a \equiv (k\theta q(\theta))^{-1/\varepsilon}$ and $c_x \equiv (kq(\theta))^{-1/\varepsilon}$. In general c_a and c_x will be time-dependent since k and, possibly also θ, may be time-dependent. The number

[16] The mean of the lognormal distribution is equal to $\mu_w = \varrho + \exp(\mu + \frac{1}{2}\sigma^2)$; the standard deviation is equal to $\sigma_w = e^{\mu}\sqrt{\exp(\sigma^2)[\exp(\sigma^2) - 1]}$. To obtain a "mean preserving spread" of the benchmark distribution, we then change σ and accordingly adjust μ so that μ_w is kept constant.

[17] It should be noted that changes in the income distribution induce a "composition" effect that tends to increase the size of the hidden economy. The "underground economic activity" (either measured as time, or production, or both) as a function of wage-type is convex (poor individuals devote a larger proportion of their time to black market activities); hence a mean-preserving spread of wages will directly increase the size of the underground sector by changing the composition of the population.

of trades an agent makes, ϕ_t and η_t, are now stock variables with an exogenous "retention rate" $r \in (0,1)$. Hence their motions are

$$\phi_{t+1} = a_t + r\phi_t; \quad \eta_{t+1} = x_t + r\eta_t. \tag{15}$$

Each agent maximizes his discounted flow of consumption, with the discount factor $\rho \in (0,1)$:

$$\max_{\{a_t,x_t\}} \sum_{t=0}^{\infty} \rho^t \left[(1-\tau_t)\, w \left(H - c_{a,t} a_t^{1/\varepsilon} - c_{x,t} x_t^{1/\varepsilon} - \frac{\eta_t}{w} \right) + \pi_t \eta_t + (1-\pi_t)\, \phi_t + S_t \right], \tag{16}$$

given (15) and some initial values ϕ_0 and η_0. In equilibrium each agent acts optimally conditional on the path of policy, and on the equilibrium prices and matching rates.

Definition 1 *Given a policy path $\{z_t\}_{t=0}^{\infty}$, an* equilibrium *is a collection of paths $\{\pi_t\}_{t=0}^{\infty}$, $\{\theta_t\}_{t=0}^{\infty}$ and $\{a_t, x_t\}_{t=0}^{\infty}$ (one for each agent) such that for each agent $\{a_t, x_t\}_{t=0}^{\infty}$ solves (16), and, for each period t, π_t is the outcome of buyer-seller bargaining and $\theta_t = \Xi_t / A_t$.*

We start by making a couple of simple observations. Note first that, due to the strictly decreasing returns to individual search, problem (16) has a unique solution. Second, note that, in each period, the outside option in buyer/seller bargaining is to trade regularly; hence the private surplus to be shared is simply the current tax and consequently $\pi_t = 1 - b\tau_t$ for all t.

Introducing co-state variables λ_t and δ_t for the state variables ϕ_t and η_t and formulating the discrete time Hamiltonian $\mathcal{H}$ we obtain the following first order conditions:

$$\frac{\partial \mathcal{H}}{\partial a} = -\rho^t (1-\tau_t) \frac{w c_{a,t}}{\varepsilon} a_t^{\frac{1-\varepsilon}{\varepsilon}} + \lambda_t = 0, \tag{17}$$

$$\frac{\partial \mathcal{H}}{\partial x} = -\rho^t (1-\tau_t) \frac{w c_{x,t}}{\varepsilon} x_t^{\frac{1-\varepsilon}{\varepsilon}} + \delta_t = 0. \tag{18}$$

The motion for the co-state variables are,

$$\lambda_{t-1} = \frac{\partial \mathcal{H}}{\partial \phi} = \rho^t b\tau_t + \lambda_t r, \tag{19}$$

$$\delta_{t-1} = \frac{\partial \mathcal{H}}{\partial \eta} = \rho^t (1-b)\, \tau_t + \delta_t r, \tag{20}$$

where we used that $\pi_t = 1 - b\tau_t$. In the static analysis we found that each worker traded equally many units as buyer and seller (Lemma 2). It is thus natural to assume that each agent has been optimising prior to time 0 so that, at that date, $\phi_0(w) = \eta_0(w)$ for all agents. We can then show that each agent preserves this equality along the equilibrium path. In particular, the following holds in the general case.

Lemma 3 *In equilibrium, (i)* $\theta_t = [(1-b)/b]^{\varepsilon}$ *for all* t*, and for all* t *and* w*, (ii)* $\lambda_t(w)/\delta_t(w) = b/(1-b)$ *and (iii)* $\phi_t(w) = \eta_t(w)$*.*

Proof. See the Appendix. ■

4.1 The Response to a Permanent Change in Tax Policy

The dynamic model can be used to consider a number of policy experiments (e.g. temporary tax amnesties, promises of stricter enforcement from some future date). We will restrict ourselves to deriving the response to an unanticipated permanent tax increase.

Thus suppose that the tax has been τ_0 for some time prior to period t_0 and that the economy is in steady state. At time t_0 the tax is permanently increased to $\tau > \tau_0$, with the revenue used e.g. to increase the lump-sum transfer S. Consider then how the agents respond immediately and over time. There are no terminal conditions for ϕ and η; hence we impose the transversality conditions $\lambda_t \to 0$ and $\delta_t \to 0$ as $t \to \infty$. Then solving (19) and (20) for the co-state variables, using that $\pi = 1 - b\tau$, we then obtain:

$$\lambda_t = \frac{\rho^{t+1}}{(1-\rho r)} b\tau; \quad \delta_t = \frac{\rho^{t+1}}{(1-\rho r)}(1-b)\tau. \tag{21}$$

Consider now the meeting rates a and x; using (21) to replace λ_t and δ_t in (17) and (18) immediately shows that a and x do not depend on time — in other words, the meeting rates immediately adjust to their new long-run levels, both of which are trivially increasing in τ:

$$a_t(w) = a^*(w) = \left(\frac{\rho}{(1-\rho r)} \frac{b\varepsilon}{w} \frac{\tau}{(1-\tau)}\right)^{\frac{\varepsilon}{1-\varepsilon}} (k\theta q(\theta))^{\frac{1}{1-\varepsilon}}, \tag{22}$$

and

$$x_t(w) = x^*(w) = \left(\frac{\rho}{(1-\rho r)} \frac{(1-b)\varepsilon}{w} \frac{\tau}{(1-\tau)}\right)^{\frac{\varepsilon}{1-\varepsilon}} (kq(\theta))^{\frac{1}{1-\varepsilon}}. \tag{23}$$

Using the constancy of the meetings rates, we can solve for the agent's networks using (15):

$$\phi_t(w) = \frac{a^*(w)}{(1-r)} + \phi_0(w) r^t; \quad \eta_t(w) = \frac{x^*(w)}{(1-r)} + \eta_0(w) r^t.$$

Over time, the amounts of trades done by an agent of type w converges to the steady state values:

$$\phi^*(w) = \frac{a^*(w)}{(1-r)}; \quad \eta^*(w) = \frac{x^*(w)}{(1-r)}.$$

Since $a^*(w)$ and $x^*(w)$ increase in τ (for all w), and since the economy was initially in steady state, all agents will increase their underground trade over time. We can thus summarize the

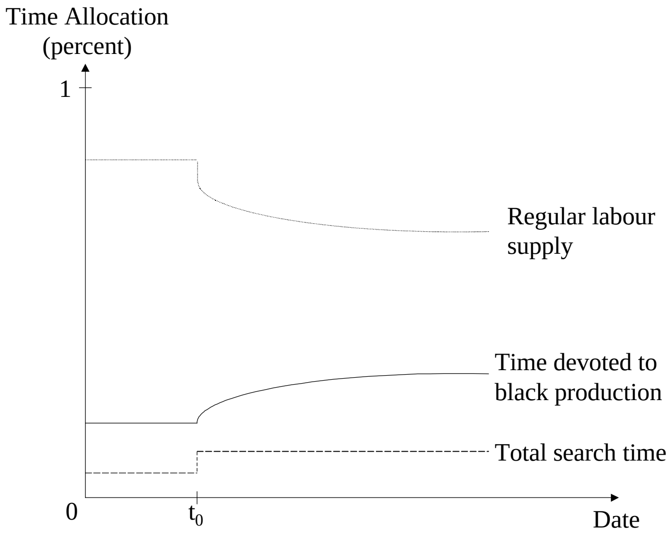

Figure 1: The effects of a permanent tax increase

response to a permanent (unanticipated) tax increase as follows: immediately when the tax is increased, the agents increase the time they spend establishing contacts for black market trade. This effect immediately reduces each agent's regular labour supply. However, black market production and trade grow only over time as the underground networks are expanded; as the underground production expands, less labour is used for regular production as more and more trade is moved into the underground sector; hence regular trade (and thus tax revenue) decreases not only instantaneously, but also gradually over time.

Proposition 1 *The dynamic response, by each agent, to an unanticipated permanent tax increase involves (i) an immediate upward adjustment of search efforts to their new long-run levels, and (ii) a gradual increase of underground production and trade, and hence (iii) an immediate, plus a gradual, decrease in regular labour supply.*

Figure 1 illustrates how a typical agent adjust his allocation over time in response to an unanticipated permanent tax increase. Thus the model suggests that the dynamic responses to increased taxation can exceed the direct response and continue even after the tax rate has stabilized. Moreover, simply repeating the above steps shows that the response to a permanent unanticipated increase in k — that is, a permanent relaxation of the tax enforcement — would generate a completely isomorphic impulse response.

The model could also be used to consider optimal dynamic policy; the optimal steady state policy would have a very similar structure to the optimal policy in the static framework but would also highlight the role of the underground network retention rate r; naturally the lower is r, the smaller is the distortionary effect of taxation. Thus, the model illustrates how the persistence of underground networks affects the government's ability to redistribute income. Conversely, it points to a particular role for government auditing policy, namely to make it more difficult to maintain underground contacts: if r can be reduced in a cost-effective manner by e.g. a policy of random auditing, redistributive policy will be facilitated.

5 Political Incentives for Taxation and Tax Enforcement

A standard result in the political economy of redistributive taxation is that income inequality generates political pressure for redistribution. However, it is usually assumed that whatever tax is put in place is also perfectly enforced. In this section, we return to the static model from Section 2 and consider the joint political incentives for taxation and tax enforcement.

The main insights from the analysis can be summarized as follows. Conditional on a fixed level of enforcement, low-wage individuals have an incentive to support high taxation since it entails redistribution in their direction. Similarly, conditional on a positive tax rate the low-wage agents have an incentive to keep enforcement low since they are disproportionately active in the underground economy. However, if both policy dimensions are included in the political process, complementarities come into play which generally lead the majority preference relation to be intransitive; thus voting cycles generally exist, making majority voting (and sincere voting in particular) an unattractive assumption.

Hence we consider a more structured model of the political process: the citizen-candidate model due to Osborne and Slivinski (1996) and Besley and Coate (1997). In that model different types of equilibria may occur; however, since median voter equilibria have a central role in the empirical literature on redistribution we devote particular attention to verifying that an equilibrium where a median type agent is elected and implements her ideal policy is likely to exist.

5.1 Direct Democracy

As an initial step, let us restate the agents' indirect utility/consumption $y(w)$ defined in (10). Noting that $\phi(w) = \eta(w)$ holds for each agent in equilibrium, and formally incorporating policy as an argument, we can write agent w's achieved consumption as

$$y(z;w) = w(H - \alpha - \xi) - \tau wl + S, \tag{24}$$

where it is understood that α, ξ and l are optimized and hence depend on z as well as w. The terms in (24) have natural interpretations: the first is the agent's physical production; the second and third measure how he is affected by the redistributive tax system.

From (24) it immediately follows that

$$\frac{\partial y(z;w)}{\partial k} = \tau \frac{\eta(w)}{k}; \quad \frac{\partial y(z;w)}{\partial \tau} = -wl(w). \tag{25}$$

Note that monotonicity of the time allocations implies that a "single-crossing" condition holds in each of the two dimension. More precisely, with S being determined through the budget balance condition (11), the following two conditions hold:

Condition 1 *(Single-crossing in τ) Suppose voting is over τ only with k arbitrarily fixed. Then for any two taxes, τ' and τ'' such that $\tau' > \tau''$ and any two voters, w' and w'' such that $w'' > w'$, if w' prefers τ'' to τ', then w'' also prefers τ'' to τ'.*

Condition 2 *(Single-crossing in k) Suppose voting is over k only with $\tau > 0$ arbitrarily fixed. Then for any k' and k'' such that $k' > k''$ and any two voters, w' and w'' such that $w'' > w'$, if w' prefers k'' to k', then w'' also prefers k'' to k'.*

Applying the separation argument due to Gans and Smart (1996) then immediately verifies the existence of a Condorcet winner policy in each dimension.

Proposition 2 *Existence of a Condorcet winner policy with unidimensional policy decisions:*

1. *For a given $k \in [k^-, k^+]$ a Condorcet winner tax τ^* exists and coincides with τ^* being the median type's most preferred tax.*

2. *For a given $\tau > 0$ a Condorcet winner k^* exists and coincides with k^* being the median type's most preferred level.*

Proof. See the Appendix. ■

While the Condorcet winner τ has the expected structure, the structure of the political support for the Condorcet winner k^* is more peculiar. In particular, the existence of a Condorcet winner k^* hinges on the *low-wage* agents supporting more *lenient* enforcement and *high-wage* agents supporting *stricter* enforcement.

When both policy dimensions, k and τ, can be adjusted simultaneously, complementarities forcefully come into play. Moreover, the agents will in general trade off taxes and enforcement at different rates, implying that the majority preference relation will generally fail to be transitive, and consequently voting cycles will generally exist.[18] To see how such cycles can come about, let the lump-sum transfer S be determined by the government budget constraint. An example of a cycles is then when, starting from the median's ideal policy (k^m, τ^m), a reform $dk > 0$, $d\tau < 0$ to (k', τ') defeats (k^m, τ^m).[19] The high-wage agents benefit from the lower tax rate, while the low-wage agents benefit from relaxed tax enforcement. However, conditional on τ', political support can be obtained for restoring k to k^m: given that this move is supported by the median it will also be supported by all types $w > w^m$ (see Condition 2). Finally, from the policy (k^m, τ'), political support can be obtained for restoring τ to τ^m: since the median supports this move at least all wage types $w < w^m$ also support it (see Condition 1), completing the cycle.

The problem of voting cycles has been frequently observed in the literature on voting over redistributive taxation, not least in the literature on voting over non-linear taxes (see e.g. Marhuenda and Ortuno-Ortin (1995) and Hindriks (2001)). Hence we can add the result that voting cycles will generally be a problem even with linear taxation if there is scope for varying the degree to which the tax is enforced.

5.2 Representative Democracy

The non-existence of a Condorcet winner when all feasible policies are considered is striking, but not surprising. Indeed, a common response to similar non-existence problems is that the space of policies that are considered is in some sense too large. As a response, some authors have suggested imposing more structure on the political process. A notable example of this approach is the citizen-candidate model. Since this model assumes that a policy-maker will, once in office, select a policy guided by self-interest, it implies that a particular feasible policy z is relevant to the political process if and only if it is ideal for some agent in the population.

The citizen-candidate model is attractive because it has very good equilibrium existence

[18] Grandmont's (1978) "intermediate preference condition" will generally not hold.

[19] It can be shown that a small reform $(dk, d\tau)$ that is orthogonal to the strictly positive vector $\left(-\tau^m \varepsilon \left(\xi\left(w^m\right)\right)^{\varepsilon-1} \xi'\left(w^m\right) q\left(\theta\right), l\left(w^m\right) + wl'\left(w^m\right)\right)$ will generally beat the median's ideal policy.

properties while making few assumptions about specific institutions. The timing of the model is as follows. (i) Any agent, of any type, can enter as candidate at a (small) cost ζ. (ii) An election is held which selects a policy maker from the set of candidates; the candidate who receives the most votes is wins. (If there is a tie, a coin is flipped.). (iii) Once in office, the elected policy maker selects a policy $z \in \mathcal{Z}$. If nobody runs, a default policy $z_0 \in \mathcal{Z}$ is implemented.

As shown by Besley and Coate, a political equilibrium always exists, but may involve mixed strategies. However, since the focus in the empirical literature on redistributive policy has been primarily on median voter equilibria, we will devote particular attention to investigating whether, in the current context, there will exist an equilibrium where the median type's ideal policy gets implemented.[20]

Let $\widehat{w} \in \mathcal{W}$ denote the identity of the policy maker (to be determined in the political process). Each type $w \in \mathcal{W}$ has some *ideal policy*, denoted $z(w)$; formally $z(w)$ maximizes $y(z; w)$ over z in the set of feasible policies $\mathcal{Z}$. Since a policy will be relevant to the process if and only if it is ideal for some worker it is useful to define $\mathcal{Z}^* \equiv \{z | z = z(w) \text{ for some } w \in \mathcal{W}\}$.

The first thing to note is that all agents with above average wages will oppose redistribution and hence will not favour any spending on enforcement. Agents with below average wages, on the other hand, will support some redistribution and possibly some positive spending on enforcement.

Lemma 4 *Preferences for redistribution:*

1. $\tau(w) = 0$, $k(w) = k^{+}$ and $S(w) = 0$ for all $w \geq \overline{w}$,

2. $\tau(w) > 0$, $k(w) \square k^{+}$and $S(w) > 0$ for all $w < \overline{w}$.

Proof. See the Appendix. ■

Lemma 4 reflects the natural complementarity between the tax rate and the level of tax enforcement. If there exists a citizen-candidate equilibrium where a median type gets elected it will generally be an equilibrium where a median type agent runs uncontested. For small enough entry cost ζ, there exists a citizen-candidate equilibrium where a type-w agent runs uncontested if and only if $z(w)$ is a Condorcet winner in the set $\mathcal{Z}^*$ (see Corollary 1 in Besley and Coate, 1997).

[20] See e.g. Milanovic (2000), Lindert (1996) and Benabou (1996) for a discussion of the empirical evidence.

	w^-	w^{25}	w^m	w^{75}	w^+
w	1.044	1.464	1.851	2.561	17.47
ideal k	0.644	0.805	1.540	3.000	3.000
ideal τ	0.701	0.609	0.358	0.000	0.000
$y(w^-)$	40.21	39.63	35.32	26.10	26.10
$y\left(w^{25}\right)$	42.76	43.21	41.66	36.59	36.59
$y\left(w^m\right)$	45.35	46.73	47.67	46.28	46.28
$y\left(w^{75}\right)$	50.34	53.38	58.86	64.04	64.04
$y\left(w^+\right)$	161.0	198.5	297.7	436.7	436.7

Table 2: Agents' ideal policies

Consider the voters' induced preferences over the identity of the policy maker. Let $y\left(\widehat{w};w\right)$ denote the indirect utility achieved by a type-w agent when the policy is $z\left(\widehat{w}\right)$ — that is, the policy preferred by type $\widehat{w}$. Formally, $y\left(\widehat{w};w\right)$ is obtained from $y\left(z;w\right)$ defined in (24) by letting the policy be $z\left(\widehat{w}\right)$. Since $\widehat{w}\in\mathcal{W}$ and $w\in\mathcal{W}$ where $\mathcal{W}$ is unidimensional and ordered, there exists a Citizen-Candidate equilibrium where a median-type agent runs uncontested whenever $y\left(\widehat{w};w\right)$ satisfies the following natural *single-crossing* condition: for any two candidates $\widehat{w}'$ and $\widehat{w}''$ such that $\widehat{w}'>\widehat{w}''$, and any two voters w'' and w' such that $w''>w'$, if w' prefers $\widehat{w}'$ to $\widehat{w}''$ then w'' also prefers $\widehat{w}'$ to $\widehat{w}''$.[21]

Lemma 5 *If $y\left(\widehat{w};w\right)$ is single-crossing, and if the entry cost ζ is sufficiently small, then there exists a Citizen-Candidate equilibrium where a median type agent runs uncontested.*

Proof. See the Appendix. ■

5.3 A Numerical Example

We present now some numerical simulations in order to illustrate some features of the political equilibria and compare them with the optimal policy outcomes. Table 2 shows in its columns the ideal policies of the poorest agent, the richest, the median, and of the agents located at the

[21] Single-crossing is plausible but may not always be easy to check. It is however possible – using the connection between single-crossing and supermodularity – to provide a sufficient condition which can be more easily verified in specific examples. Write $y\left(z;w\right)=\psi\left(k,\tau;w\right)+S$, where $\psi\left(k,\tau;w\right)\equiv w\left(H-\alpha-\xi\right)-\tau wl$ does not depend on S, and define $\psi(\widehat{w};w)=\psi(k(\widehat{w}),\tau(\widehat{w});w)$. One can then show that $y\left(\widehat{w};w\right)$ is single-crossing if $\psi\left(\widehat{w};w\right)$ satisfies $\partial^2\psi/\partial\widehat{w}\partial w\geq 0$ for all $(\widehat{w},w)$.

	B	III	IV	V	VI
k	3.000	3.000	3.000	1.540	3.000
τ	0.151	0.198	0.136	0.358	0.129
winners %	58.0	58.4	57.9	57.1	56.7
black GDP %	8.86	5.58	10.19	7.92	7.07

Table 3: Median voter's ideal policy

25th and 75th percentile of the income distribution. The simulated model is number V of Table 1. In accordance with Lemma 4, the agents with above-the-mean income (w^{75} and w^{+}) do not support any kind of redistribution; as for the other, the ideal levels of tax progressivity and tax enforcement are, as expected, decreasing in the wage rate. For each agent, Table 2 also shows net income as a function of the various ideal policies. It can be checked numerically that policy preferences satisfy single-crossing, hence Lemma 5 applies and we can be sure that there exists a political equilibrium featuring the median's ideal policy.

In Table 3 we report the median type's preferred policy for the models simulated for the optimal tax analysis. The row labelled "winners%" gives the majority supporting the median type's ideal policy over *laissez faire*. First, notice how the results confirm the complementarity pattern. In all cases but one the tax rate is pretty low — less then 20% — and tax enforcement is nil; in the high-inequality economy of model V we have a rather large tax rate and a positive enforcement level. Also, notice that for all computed models tax progressivity and tax enforcement are much lower in the political equilibrium than in the benchmark optimal tax policy ($\upsilon = -2$), while much closer to the case of $\upsilon = -1$.[22] This reflects the fact that the majority supporting the implemented policy is made by agents with less then average wage. These agents want some enforcement to boost redistribution, but not too much (as they are relatively more active in the illegal sector). It is also noteworthy that there does not exist a clear relation between the income tax rate prevailing in the political equilibrium and the size of the underground economy, as measured in the last row of Table 3; that is, similar policy patterns are compatible with different extensions of the black sector.

[22] As usual, the median's ideal tax is larger than the optimal utilitarian tax, which in the current model is trivially equal to zero.

6 Concluding Remarks

We have developed a general equilibrium model of an economy with underground production and trade, and used the model to study a variety of issues, including optimal policy, dynamic tax responses and political incentives for redistribution and tax enforcement. We have highlighted a crucial characteristic of underground economic activity, namely that it involves trade in an imperfect information setting — the imperfection being attributable to its illegal, and therefore necessarily furtive, nature. Underground trading is therefore modelled as a bilateral search process followed by a bargaining stage. The model reveals a good flexibility, allowing us to treat the above-mentioned issues in a unified manner and permitting useful comparisons between the various approaches (e.g. between the optimal policy results and the political equilibria). We report three sets of results.

First, we provide a characterisation of optimal income taxation and tax enforcement. The degree of tax progressivity is shown to depend crucially on the reactivity of time devoted to black production and trade to the tax rate, while tax enforcement is shown to be potentially in conflict with equity concerns, due to the fact that the poor are relatively more engaged in black activities than the rich. By means of a numerical example we emphasize that the two policy instruments are strongly complementary, as increases in the tax rate always call for stricter enforcement.

Second, we investigate the response to a permanent (unanticipated) tax increase using a dynamic version of the basic model. We find that when the tax is increased, the agents increase the time they spend establishing contacts for black market trade; we have thus an immediate reduction in labour supply. Moreover, black market production and trade grow over time; hence, less labour is used for regular production as more and more trade is moved into the underground sector. That is, we find that the effect of a tax increase tend to propagate over time — increasing taxes today implies a steady growth of the black economy in the future.

Third, we study the political economy of redistributive taxation in the presence of tax enforcement. We find that in conventional direct democracy models when there is simultaneous voting over the tax level and the level of tax enforcement voting cycles are a major problem; the majority preference relation cannot be expected to be transitive in the general case. We then resort to representative democracy models of the citizen-candidate variety, identifying a condition which ensures the existence of a citizen-candidate equilibrium where the median type's preferred policy is implemented. We show by means of a numerical example that complementarity between taxation and enforcement is again key, and that the simultaneous choice of a tax level and enforcement spending may imply that there no clear relation between the income tax level and size of the underground economy.

Appendix

Proof of Lemma 3. Part (i). The co-state variable λ_0 measures the discounted value of meeting one extra seller at time 0, while δ_0 measures the corresponding value of meeting one extra buyer. Focusing first on λ_0, note that the contact yields a payoff of $b\tau_t$ in period t if the contact is still intact at that date. Hence $\lambda_0 = b\sum_{t=0}^{\infty} \tau_t \rho^t \vartheta_t$ where ϑ_t is the probability that the contact is still intact at time t. Similarly, if the contact with the buyer is still intact at time t it yields an instantaneous gain of $(1-b)\,\tau_t$. Hence $\delta_0 = (1-b)\sum_{t=0}^{\infty} \tau_t \rho^t \vartheta_t$ where we used that both types of contacts deteriorate at the same rate. (Note also that both sums converge due to discounting and since the contacts dissolve with positive probability.) Thus $\lambda_0/\delta_0 = b/\,(1-b)$. Assume then that $\lambda_{t-1}/\delta_{t-1} = b/\,(1-b)$. Combining (19) and (20) then yields

$$\frac{b}{1-b} = \frac{\rho^t b\tau_t + \lambda_t \delta}{\rho^t (1-b)\,\tau_t + \delta_t \delta}. \tag{26}$$

Rearranging and crossing terms then immediately gives that $\lambda_t/\delta_t = b/\,(1-b)$. Hence the ratio holds also for t, and thus, by induction on t, it holds for every period.

Part (ii). Dividing (17) by (18), and simplifying using part (i) gives that

$$a_t^{\frac{1-\varepsilon}{\varepsilon}} = \frac{b}{1-b}\theta_t^{\frac{1}{\varepsilon}} x_t^{\frac{1-\varepsilon}{\varepsilon}}, \tag{27}$$

for all agent and all t. Then using (14), together with the definitions of A and Ξ gives that

$$\theta_t = \frac{\Xi_t}{A_t} = \left(\frac{b}{1-b}\theta_t\right)^{\frac{-\varepsilon}{1-\varepsilon}}. \tag{28}$$

Solving gives that $\theta_t = [(1-b/b)]^{\varepsilon}$ for all t.

Part (iii) Using part (ii) to replace θ_t in (27) immediately gives that $a_t = x_t$ for all t; the result then follows from the motions in (15) and the assumption that $\phi_0 = \eta_0$. ■

Proof of Proposition 2. As noted by Gans and Smart (1996) each single-crossing condition, 1 and 2, is equivalent to a more familiar Spence-Mirrlees condition. In particular, with voting over τ, and with S being determined through the budget constraint, $S = S(\tau)$, Condition 1 is equivalent to the marginal rate of substitution,

$$MRS_{\tau S}(w) = -\frac{\partial y/\partial \tau}{\partial y/\partial S} = wl(w), \tag{29}$$

being increasing in w. But this is true since $l(w)$ is increasing in w. Similarly, with voting over k, and with S being determined through the budget constraint, $S = S(k)$, Condition 2 is equivalent to the marginal rate of substitution,

$$MRS_{kS}(w) = -\frac{\partial y/\partial k}{\partial y/\partial S} = -\tau\frac{\eta(w)}{k}, \tag{30}$$

being increasing in w; but this is true since $\eta(w)$ is decreasing in w.

Using condition 1 and Condition 2 it is then easy to verify that the median type is pivotal. Consider e.g. voting over τ given a fixed k and let τ^* be the tax preferred by the median. Then every voter $w > w^m$ prefers τ^* to any other tax $\tau < \tau^*$ while every voter $w < w^m$ prefers τ^* to any other tax $\tau > \tau^*$. Hence τ^* defeats every other tax in a pairwise vote and is thus the unique Condorcet winner. The same argument holds for voting over k given a fixed $\tau > 0$. (See Gans and Smart (1996) for details.) ■

Proof of Lemma 4. We find each agent's ideal policy by maximising his indirect utility w.r.t. to τ and k (second order conditions are taken to be satisfied throughout). Solving the revenue constraint (11), we obtain an expression $S(\tau, k) = \tau \int wl\mathrm{d}F - \chi(k)$, where $S(0, k^+) = 0$; inserting $S(\cdot)$ in the indirect utility function (24) and maximising over τ and k under the constraints that $\tau \geq 0$, $k \geq k^-$ and $k \square k^+$ yields the first order conditions

$$-wl + \frac{\partial S}{\partial \tau} \square 0, \quad \tau \geq 0, \, \tau\left(-wl + \frac{\partial S}{\partial \tau}\right) = 0; \tag{31}$$

$$\frac{\tau}{k}\eta + \frac{\partial S}{\partial k} - \varkappa \square 0, \quad k \geq k^-, \, \left(k - k^-\right)\left(\frac{\tau}{k}\eta + \frac{\partial S}{\partial k} - \varkappa\right) = 0; \tag{32}$$

$$k^+ - k \geq 0, \quad \varkappa \geq 0, \, \varkappa\left(k^+ - k\right) = 0, \tag{33}$$

where $\varkappa$ is a Lagrange multiplier and

$$\frac{\partial S}{\partial \tau} = \tau \int w \frac{\partial l}{\partial \tau}\mathrm{d}F + \int wl\mathrm{d}F; \quad \frac{\partial S}{\partial k} = \tau \int w \frac{\partial l}{\partial k}\mathrm{d}F - \chi'. \tag{34}$$

At $\tau = 0$ all agents will be fully employed in the regular sector, and, trivially, there would no revenue, $S = 0$ and $k = k^+$; hence, at a corner solution, $(\tau, k) = (0, k^+)$, we have:

$$wl\left(0, k^+; w\right) = wH; \quad \frac{\tau}{k}\eta\left(0, k^+; w\right) = 0; \tag{35}$$

$$\frac{\partial S}{\partial \tau}\left(0, k^+; w\right) = \int wH\mathrm{d}F; \quad \frac{\partial S}{\partial k}\left(0, k^+; w\right) = -\chi'\left(k^+\right). \tag{36}$$

Then, we see that for $(\tau, k) = (0, k^+)$ to be a solution, it must be that:

$$-w + \int w\mathrm{d}F \equiv \overline{w} \square 0; \tag{37}$$

$$-\chi'\left(k^+\right) = \varkappa > 0. \tag{38}$$

This simply says that all agents whose wage is above or equal to the average wage prefer zero taxation and hence no spending on enforcement. The condition is instead violated by all agents with below average wage, who prefer positive taxation (and therefore positive public spending), and possibly some spending on enforcement. ■

Proof of Lemma 5. Applying the separation argument used in the proof of Lemma 2 immediately shows that the median type is a Condorcet winner among the set of all types, i.e. a median-type candidate would beat any other candidate in a plurality vote. Thus if a median type agents enter the election, no other voter has an incentive to do so. Moreover, given that the default policy z_0 is sufficiently unattractive, and the entry cost ζ is small, a median type agent will want to enter if no one else does so. ■

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