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THE ARCHITECTURE OF GOVERNMENTS: FISCAL FEDERALISM AND ENDOGENOUS LOBBIES' FORMATION

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The Architecture of Governments: Fiscal Federalism and Endogenous Lobbies' Formation

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Preliminary version

Abstract

Does decentralization induce more lobbying behavior? And if this is true, does it follow that we should refrain to decentralize in order to avoid wasteful lobbying behavior? We address this question in a model with two regions, a policy which can be assigned to local or central government and two (possible) regional lobbies. We find that under decentralization lobbies will lobby both regions if policy makers have strong preferences for lobbies contributions. Lobbies make more profits under decentralization than under centralization, even though public good supply is the same under the two regimes. On the contrary, lobbying will be undertaken only locally if policy makers care more about social welfare than about contributions. Both lobbies' profits and public good supply are smaller under decentralization. Contributions to policy makers are always smaller under decentralization.

Keywords: Fiscal federalism, Lobbying, Pressure groups.

JEL classification: D70, H23, H77

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1 Introduction

Does decentralization induce more lobbying behavior? Should governments refrain to decentralize in order to avoid wasteful lobbying behavior? In a world in which the number of lobbies is given exogenously, decentralization should make lobbying easier as it is less costly to influence a small local government than a large central one. However, when the number of lobbies is endogenous and organizing a lobby is costly, it is not clear whether decentralization increases lobbying behavior.[1] The organizational costs for lobbying may be worth to be sustained only above a certain threshold; i.e. it may be worth to organize a lobby to influence industrial policy nationwide, but not locally. Given that organizing lobbies is costly, it may be possible that decentralization will result in a lower number of lobbies and higher social welfare in equilibrium.

We examine these issues in a model in which there are two regions and a firm — eventually undertaking lobbying activities— in each region. The public sector provides a local public good in each region that, by stimulating the demand of a consumption good, bears a positive impact on firms' profits. Hence, the latter have an incentive to lobby for higher public good provision.

We compare lobbying activity under two institutional frameworks: a *centralized system*, in which the decision on both public goods is taken by a single policy maker, and a *decentralized system*, in which the decision on public good supply is taken by two independent regional polic makers. Each firm produces only in its own region, but both sell, competing *à la* Cournot, in both regions. Therefore, under decentralization each firm has to decide not only whether to lobby but also *where* to lobby: the "home" region, the other region ("abroad"), or both.

We find that under decentralization, on the one hand, firms will lobby both regions only when policy makers are sufficiently "greedy", in the sense that they assign a high weight to lobbies contributions over social welfare. On the other hand, firms will lobby only the home region when policy makers are sufficiently benevolent, meaning that they care much about social welfare. Comparing centralization and decentralization, we find that when firms lobby both regions under decentralization, profits are higher under decentralization than under centralization, even though public good supply turns out to be the same under the two regimes. When firms lobby only the home region, both firms' profits and public goods supplies are smaller under decentralization. In all cases, contributions to policy makers are smaller under decentralization.

Our results highlight the "pure" role of lobbying activity for the comparison between centralized and decentralized policy making. In fact, we depart from the existing literature by abstracting from standard elements entering the centralization versus decentralization debate, such as spillover effects across regions from public good provision or differences in tastes.

(Survey of existing and related literature on lobbying and fiscal federalism to be added)

The outline of the paper is as follows. In section 2 we set up the model. In section 3 we examine the policy makers' choices in the benchmark situation of no lobbying activity. In section 4 we address the core issues of the paper, by examining and comparing

[1] There is a growing literature body of literature on endogenous lobby formation, see e.g. Besley and Coate (2001), Felli and Merlo (2001), and Redoano (2002).

lobbying behavior under centralization and decentralization. Two appendices develop the analytical details of the model and provide proofs to the propositions.

2 The model

The economy is composed of two identical regions indexed by $r \in \{a, b\}$. There are four goods in the economy: two private consumption goods, x and z, a production factor, y, and a public investment good, g. The latter is purely local, meaning that there is a distinct provision in each region with no spillover effects across regions.[2] In each region live a continuum of identical consumers with a mass of unity, not moving across regions. Consumers are endowed with a fixed quantity $\bar{y} > 0$ of the production factor and derive utility from the consumption of goods x and z. Public good g positively affects the demand of good x; its consumption, however, creates a negative consumption externality. In each region there is a firm producing good x. Firms are indexed by $\rho \in \{\alpha, \beta\}$, where α and β are the firms located in regions a and b, respectively. Firms α and β have an incentive to lobby the policy maker for a higher provision of the public investment good, given that good g increases firms' profits by stimulating the demand of x. We compare lobbying behavior under two institutional frameworks. One is a *centralized system*, in which a single policy maker chooses the supply of public goods in both regions. The other is a *decentralized system*, in which each region is characterized by an independent policy maker that chooses the level of the public good.

In discussing the model, it is convenient to focus on four conceptually distinct stages. At the first stage, firms α and β decide whether or not to organize a lobby. At the second stage, upon entry, firms lobby the policy maker(s). At the third stage, policy choices are made. Finally, at the fourth stage market equilibrium is determined. We proceed by describing and solving the model backwards, starting with the analysis of market equilibrium.

2.1 The market of good z

We take good z to be the *numeraire* and its market to be perfectly competitive. Technology is assumed to be linear and units are normalized so that the production of one unit of z requires one unit of input y. These assumptions imply that in equilibrium profits in the production of good z are zero and that its supply is perfectly elastic. Moreover, the market price of factor y is equal to one.

2.2 The consumption sector

Each consumer in region r has the following utility function:

$$u_r(x_r, z_r, g_r) = x_r - \frac{x_r^2}{2g_r} + z_r - e_r(x_r). \tag{1}$$

[2] Spillover effects in public good provision are a common ingredient in the literature on fiscal federalism (see e.g. Besley and Coate, 2002). In order to highlight the role of lobbying under centralized and decentralized provision of public goods, we abstract from spillover effects.

The first three terms represent the "private" utility of consumption, while the last one represents the negative externality that derives from the aggregate consumption of good x. We assume that $e_r' \geq 0$, $e_r'' > 0$.[3]

We assume that firms α and β distribute all their net profits to consumers, and that firms α and β are entirely owned by consumers living in regions a and b, respectively.[4] Hence, consumers' income is composed of two terms: the market value of the fixed endowment of good y, and the distributed firms' profits. Let π_ρ denote distributed net profits of firm ρ and let p_r be the price of good x. Taking g_r, π_ρ, and the externality level as given, each consumer in region r solves:

$$\begin{aligned} \max_{x_r, z_r} \quad & x_r - \frac{x_r^2}{2g_r} + z_r, \\ \text{s.t.} \quad & p_r x_r + z_r \leq \bar{y} + \pi_\rho, \end{aligned}$$

from which we immediately obtain the inverse demand function for good x_r as:

$$p_r(x_r, g_r) = 1 - \frac{x_r}{g_r}. \tag{2}$$

From (2) it is apparent that for any given quantity $x_r > 0$, an increase in g_r increases the marginal willingness to pay for good x_r.[5]

2.3 The market of good x

In each region the market for good x is a duopoly, with one of the firms located within the region and the other one located outside it. Firms maximize profits and compete *à la* Cournot.[6] Good y is the only input into production and technolgy is assumed to be linear, so that marginal costs are constant. There is however a source of asymmetry between firms. When a firm supplies to its own regional market, the production function is $x = y/c$ (the marginal cost is $c > 0$). When a firm supplies "abroad", the production function becomes $x = y/(\delta c)$, $\delta \geq 1$ (the marginal cost is δc), so that the home firm has a cost advantage over its competitor.[7]

Let $x_{\rho r}$ be the quantity sold by firm ρ in region r, so that we can write aggregate sales in regions a and b as $x_a = x_{\alpha a} + x_{\beta a}$ and $x_b = x_{\alpha b} + x_{\beta b}$. We denote by $\Pi_{\rho r}$ the

[3] The externality created by the consumption of good x in region r is purely local. As discussed above, we rule out externality spillovers.

[4] Note that, given quasi-linearity of the utility function, by which all income effects fall on the demand of good z, the equilibrium of the economy is independent of the distribution of profits across consumers and across regions.

[5] An example of such a kind of public investment good is represented by roads: roads increase the benefits and hence the use of cars, but also increase the level of pollution. However, in some circumstances, e.g. when existing roads are highly congested, an increase in the supply of roads may *reduce* pollution.

[6] Firms' managers maximize profits even though under imperfect competition this means that consumers' welfare (which includes producers' profits) is not maximized. There are several institutional features of real economies that justify this assumption, like the separation between ownership and control in large firms, or the need to give the right incentives to managers.

[7] For instance, the parameter δ (strictly speaking, $\delta - 1$) can be interpreted as representing the extra transportation costs needed to transfer one unit of good x across regions. We take the industrial structure as given. In particular, we do not allow for a firm located in one region to open a new plant in the other region so as to avoid paying the extra cost.

gross profits earned by firm ρ in region r. Profits are subject to a proportional tax according to the source principle, where $t_r \in [0,1)$ is the tax rate applied in region r. Using the demand functions (2) and the notation introduced above, firm α solves:

$$\begin{aligned}
&\max_{x_{\alpha a}, x_{\alpha b}} \quad (1-t_a)\Pi_{\alpha a} + (1-t_b)\Pi_{\alpha b} = \\
&= (1-t_a)\left(1 - \frac{x_{\alpha a} + x_{\beta a}}{g_a} - c\right) x_{\alpha a} + (1-t_b)\left(1 - \frac{x_{\alpha b} + x_{\beta b}}{g_b} - \delta c\right) x_{\alpha b}. \qquad (3)
\end{aligned}$$

Notice that there is no link between the two regional markets through the demand side. From the two first order conditions of problem (3), and the other two obtained from the symmetric problem for firm β, we obtain the equilibrium quantities

$$\begin{aligned}
&x^*_{\alpha a} = h g_a, \quad x^*_{\beta b} = h g_b, \quad x^*_{\beta a} = f g_a, \quad x^*_{\alpha b} = f g_b, \\
&x^*_a = (h+f) g_a, \quad x^*_b = (h+f) g_b, \qquad (4)
\end{aligned}$$

and the equilibrium prices

$$p^*_a = p^*_b = p^*, \quad p^* = 1 - (h+f),$$

where

$$h = \frac{1 + \delta c - 2c}{3}, \qquad f = \frac{1 + c - 2\delta c}{3}, \qquad h + f = \frac{2 - (1+\delta)c}{3}. \qquad (5)$$

In order to ensure that the quantities supplied by each firm in both regions are non-negative, we introduce the following restrictions on parameters c and δ:

Assumption 1 $0 < c < 1$ *and* $1 \leq \delta \leq \delta_{\max} = \dfrac{1+c}{2c}$.

Parameter δ allows for a wide range of market structures. When $\delta = 1$, then $h = f = (1-c)/3$, so that there is a *symmetric duopoly* in each region, since the "home" firm has no cost advantage over its "foreign" rival. At the other extreme, when $\delta = \delta_{\max}$, then $h = (1-c)/2$ and $f = 0$. The cost advantage of the "home" firm is so high that the "foreign" firm does not enter the market, meaning there is a *monopoly* in each region. A continuum of intermediate cases is obtained for $\delta \in (1, \delta_{\max})$.

Equilibrium gross profits are linearly increasing in public good provision:

$$\Pi^*_\alpha = \Pi^*_{\alpha a} + \Pi^*_{\alpha b} = h^2 g_a + f^2 g_b, \quad \Pi^*_\beta = \Pi^*_{\beta a} + \Pi^*_{\beta b} = f^2 g_a + h^2 g_b. \qquad (6)$$

From these expressions it is immediate to see that the firms' managers have an incentive to lobby the policy maker(s) for an expansion in the provision of the public goods.[8]

[8] Consumers may have an incentive to lobby as well. However, it may not always be in their interest to lobby for an increase in public good supply. For any given g, on the one hand, the market is too small because of imperfect competition; on the other hand, it is too large because of the negative externality. While "double" lobbying is an interesting issue (e.g. large corporations vs. environmental organizations), here we assume that free riding impedes the consumers to get organized as a lobby.

2.4 The public sector

The public sector provides the public goods g_a and g_b, financing their production with the taxation of profits at source.[9]. Technology is linear and uses factor y as the only input; $\phi \geq 0$ denotes the marginal cost. Policy makers care about social welfare and contributions from the lobbies.

The budget of the public sector must be separately defined for the cases of centralized and decentralized decision making. Under a centralized system, a single decision maker chooses g_a and g_b and sets a uniform tax rate across regions, so that $t_a = t_b = t$. The budget constraint she faces is:

$$\phi(g_a + g_b) = t(\Pi_\alpha^* + \Pi_\beta^*). \tag{7}$$

Substituting for equilibrium profits from (6) and solving for t, one can observe that the tax rate is independent of (g_a, g_b) and is equal to:

$$t = \frac{\phi}{h^2 + f^2}. \tag{8}$$

The following assumption is sufficient to ensure that, for any given c, $t < 1$ for all $\delta \in [1, \delta_{\max}]$.[10]

Assumption 2 $\phi < \phi_{\max} = (1 - c)^2/5$.

Under a decentralized system, each regional policy maker independently and simultaneously chooses public good provision in her own region. In each region, public expenditure is financed through a local tax on profits at source.[11] Formally, the regional budget constraints are:

$$\phi g_a = t_a(\Pi_{\alpha a}^* + \Pi_{\beta a}^*), \quad \phi g_b = t_b(\Pi_{\beta b}^* + \Pi_{\alpha b}^*). \tag{9}$$

Substituting for equilibrium profits from (6) and solving for t_r, we obtain again the tax rate (8). Hence, the tax rate is identical under centralization and decentralization, and therefore in the latter case we can drop the subscript r from regional tax rates.

2.5 Market clearing in markets for goods z and y

Given our simplified framework (in particular, the assumption of quasi-linear preferences), it is immediate to check that the markets for good z and factor y clear. The supply of good z is perfectly elastic (see section 2.1) and thus its equilibrium quantity is determined by aggregate demand, z^d, from consumers. As for factor y, aggregate

[9] The tax on profits at source can be interpreted as an earmarked tax based on the benefit principle. Our qualitative results carry over to the case in which the profit tax is based on the residence principle, or to the case in which public goods are financed through a comprehensive income tax that includes consumers' factor income $\bar{y}$ in the tax base.

[10] One can show that for any given $c \in (0, 1)$, the function $\mathcal{F}(c, \delta) = h^2 + f^2$ has a minimum at $\delta^\circ = (1 + 4c)/(5c) < \delta_{\max}$ and its minimum value function is $\mathcal{F}(c, \delta^\circ) = (1 - c)^2/5$.

[11] As an alternative, local public goods under decentralization could be financed through a common pool. The impact of lobbying on local public goods provision when the latter are financed through a common pool in a centralized system is examined in Persson (1998).

supply from consumers is inelastic, $y^s = 2\bar{y}$. The demand for y comes from three sources: the public sector (y^d_{PS}), the firms producing good z (y^d_Z), and the firms α and β ($y^d_{\alpha+\beta}$). By Walras' law, given that the public sector's budget constraint balances, it follows that $y^d_Z + y^d_{PS} + y^d_{\alpha+\beta} = y^s$.[12]

3 Optimal policy without lobbying

Before introducing lobbying by firms, we examine policy choices in the benchmark case of no lobbying. Substituting the equilibrium values for x^*_r and z^*_r into the utility function (1), social welfare in region r becomes:

$$W_r = x^*_r - \frac{{x^*_r}^2}{2g_r} - p^* x^*_r + \bar{y} + \pi^*_\rho - e_r(x^*_r).$$

In order to be able to derive closed form solutions, we let

Assumption 3 *$e_r(x_r) = x_r^2/2$.*

By substituting x^*_r from (4), $\pi^*_\rho = (1-t)\Pi^*_\rho$, with Π^*_ρ defined in (6) and t defined in (8), we obtain:

$$W_r(g_a, g_b) = \frac{(h+f)^2(1-g_r)g_r + 2(h^2 g_r + f^2 g_{-r})}{2} - \frac{\phi(h^2 g_r + f^2 g_{-r})}{h^2+f^2} + \bar{y}. \quad (10)$$

Aggregate social welfare, $W = W_a + W_b$, is:

$$W(g_a, g_b) = \frac{(h+f)^2[(1-g_a)g_a + (1-g_b)g_b] + 2(h^2+f^2-\phi)(g_a+g_b)}{2} + 2\bar{y}. \quad (11)$$

Under centralization, by maximizing (11) we get both for g_a and g_b:[13]

$$\hat{g}^C = \frac{(h+f)^2 + 2(1-t)(h^2+f^2)}{2(h+f)^2}. \quad (12)$$

Under decentralization, each regional policy maker simultaneously maximizes (10) with respect to g_r, obtaining:

$$\hat{g}^D = \frac{(h+f)^2 + 2(1-t)h^2}{2(h+f)^2}. \quad (13)$$

Using (12) and (13) one can state the following

Proposition 1 *If $\delta \in [1, \delta_{\max})$, then $\hat{g}^C > \hat{g}^D$. If $\delta = \delta_{\max}$, then $\hat{g}^C = \hat{g}^D$.*

[12] Formally, $y^d_{PS} = \phi(g_a+g_b)$, $y^d_Z = z^d = 2\bar{y} + \pi^*_\alpha + \pi^*_\beta - p^*(x^*_a + x^*_b)$, $y^d_{\alpha+\beta} = c(x^*_{\alpha a} + \delta x^*_{\alpha b} + x^*_{\beta b} + \delta x^*_{\beta a})$.

[13] Whenever solutions are symmetric we drop subscripts r and ρ to save on notation.

Proof. It follows from $f^2 > 0$ if $\delta \in [1, \delta_{\max})$ and $f^2 = 0$ if $\delta = \delta_{\max}$, and by comparison of (12) and (13). ■

Without lobbying public good supply is never lower under centralization than under decentralization. When a regional policy maker increases public good supply, she does not internalize as social welfare the profits gains made by the non-resident firm. Hence, when both firms sell in both regions, public good supply is lower under decentralization. On the contrary, a centralized policy maker internalizes the entire firms' profit gains, and hence she has a greater incentive to expand public good supply. The incentives of centralized and decentralized policy making are the same when the resident firm is a monopoly within its own region, and hence $\hat{g}^C = \hat{g}^D$.

By using (6), (12) and (13), the equilibrium profits of each firm under centralization and decentralization are respectively:

$$\hat{\pi}^C = (1-t)(h^2 + f^2)\hat{g}^C, \tag{14}$$

$$\hat{\pi}^D = (1-t)(h^2 + f^2)\hat{g}^D. \tag{15}$$

Given (14) and (15) it is straightforward to show that the following holds:

Corollary 1 *Without lobbying, profits are larger under centralization.*

4 Lobbying

In this section we examine lobbying behavior by firms α and β assuming that the decision to form a lobby has taken place. We apply the common agency approach developed by Bernheim and Whinston (1986), and popularized in the lobbying literature by Dixit et al. (1997). We examine first the case of a centralized system.

4.1 Centralization

A lobby maximizes after-tax profits net of contributions to the policy maker, who in turn maximizes a weighted average of social welfare and lobbists' contributions. First, each firm ρ independently and simultaneously offers the policy maker a contribution schedule $S_\rho(g_a, g_b, \pi_\rho)$, defining the monetary contribution as a function of public good provision. Second, upon acceptance of the lobbies contributions, the policy maker chooses public goods supply.

Following Dixit et al. (1997), we focus on *truthful equilibria*, in which each lobby offers the policy maker a *compensating contribution schedule*, shaped along the firm iso-profit curve.[14] Firm ρ net profits are defined as $\pi_\rho = (1-t)\Pi^*_\rho - s_\rho$, where s_ρ is the value of the contribution, which is not deductible from the profit tax. The compensating contribution schedule is defined as:

$$S_\rho(g_a, g_b, \pi_\rho) = (1-t)(h^2 g_r + f^2 g_{-r}) - \pi_\rho. \tag{16}$$

[14] Since the common agency game usually has a multiplicity of subgame perfect Nash equilibria, the concept of truthful equilibrium has been introduced in the literature as an equilibrium refinement. Importantly, truthful equilibria are shown to be Pareto efficient (see Dixit et al., 1998, proposition 4).

Contributions must be non-negative.[15] We check *ex post* in the computed equilibria whether contributions are non-negative, and we rule out all equilibria with negative contributions.

The policy maker's objective function is:

$$V^C(g_a, g_b, \pi_\alpha, \pi_\beta) = \mu W + (1-\mu)(S_\alpha + S_\beta). \tag{17}$$

The parameter μ, $0 < \mu \leq 1$, captures the degree of benevolence of the policy maker. We assume that the politician never cares about contributions only, i.e. $\mu \neq 0$. Note that in (17) the expression for social welfare, W, defined in (11), includes profits net of taxes but gross of lobbists' contributions. This is often the case in the literature (see e.g. Persson, 1998). The reason is that contributions are a pure transfer from lobbists to politicians; hence, their net effect on social welfare is nil.

By solving the lobbying game, one can show that under centralization public good supply is:[16]

$$\tilde{g}^C = \hat{g}^C + \frac{m(1-t)(h^2+f^2)}{(h+f)^2}, \tag{18}$$

where:

$$m = \frac{1-\mu}{\mu}. \tag{19}$$

Unsuprisingly, public good supply is greater under lobbying than without it, provided that the policy maker is not fully benevolent ($m = 0$).

The equilibrium net profits and contributions are:

$$\tilde{\pi}^C = \hat{\pi}^C + \psi m(h^4 + f^4 + 4h^2 f^2), \tag{20}$$

$$\tilde{s}^C = \psi m(h^4 + f^4). \tag{21}$$

where:

$$\psi = \frac{(1-t)^2}{2(h+f)^2}. \tag{22}$$

Eq. (20) shows that profits under lobbying are equal to profits without it, $\hat{\pi}^C$, plus a profit gain from lobbying.[17] Notice that equilibrium contributions are always non-negative. As expected, if the policy maker does not care about lobbists' contributions, $\tilde{\pi}^C = \hat{\pi}^C$ and $\tilde{s}^C = 0$, since $m = 0$.

4.2 Decentralization

Under decentralization, each firm has three possible strategies: lobby both regions (B), lobby only the "home" region (H) and, finally, no lobby (N).[18] For each strategy pairs

[15] In the real world, transfers from the politician to the pressure groups typically do not pass through the kind of monetary contributions considered here.

[16] We refer the reader to Appendix A for all the analytical details. Notice that a "hat" denotes the solutions obtained without lobbying, whereas a "tilde" denotes the corresponding solutions under lobbying.

[17] One can show that the equilibrium in which both firms lobby for both public goods gives rise to higher net profits than those in which firms lobby for one public good only.

[18] In fact there is a fourth action: lobby only "abroad". We do not consider it from the outset since it turns out to be always dominated by lobbying only "at home".

we solve the lobbying game, deriving the optimal public goods supply and equilibrium profits.[19]

Consider first the case in which both firms lobby both regions (denoted as BB). Let $S_{\rho r}(g_r, \pi_{\rho r})$ be the compensating contribution schedule that firm ρ offers the policy maker of region r. Policy makers maximize:

$$V_a^{DBB} = \mu(W_a - S_{\alpha b} + S_{\beta a}) + (1-\mu)(S_{\alpha a} + S_{\beta a}), \tag{23}$$
$$V_b^{DBB} = \mu(W_b - S_{\beta a} + S_{\alpha b}) + (1-\mu)(S_{\alpha b} + S_{\beta b}), \tag{24}$$

where $S_{\alpha a} = (1-t)h^2 g_a - \pi_{\alpha a}$, $S_{\beta a} = (1-t)f^2 g_a - \pi_{\beta a}$, $S_{\alpha b} = (1-t)f^2 g_b - \pi_{\alpha b}$ and $S_{\beta b} = (1-t)h^2 g_b - \pi_{\beta b}$. Notice that between-regions contributions are no longer pure transfers without any effect on social welfare. The contribution of firm α to the policy maker of region b counts as a welfare loss in region a but as a welfare gain in region b, and viceversa. This implies that lobbies' contributions do not have the same weight into the politicians' preferences. One unit of contribution a firm makes abroad counts as $-\mu$ in the home region but as 1 in the receipient region. One unit of contribution a firm makes at home counts as $1-\mu$ in the home region and nothing abroad.

The optimal public good supply is (see Appendix A):

$$\tilde{g}^{DBB} = \hat{g}^D + \frac{(1-t)f^2}{(h+f)^2} + \frac{m(1-t)(h^2+f^2)}{(h+f)^2}. \tag{25}$$

By (18) and (25), it is $\tilde{g}^C = \tilde{g}^{DBB}$, meaning that public good supply is the same both under centralization and decentralization. Moreover, $\tilde{g}^C - \hat{g}^C < \tilde{g}^{DBB} - \hat{g}^D$, which means that the increase in the supply of the public good due to lobbying is bigger under decentralization. This follows from the fact that in a decentralized system lobbies are able to influence public policy even when the social planner is fully benevolent ($\mu = 1$). In fact, even if the policy maker does not place any value on contributions *per se*, contributions offered by the "foreign" firm enter social welfare and hence influence her choices, as represented by the second term in (25).

The equilibrium profits and contributions ("abroad" and "at home", respectively) are:

$$\tilde{\pi}_{\rho(-r)}^{DBB} = (1-t)f^2\hat{g}^D + \psi\left[(1+m)f^4 + 2mh^2f^2\right], \tag{26}$$

$$\tilde{s}_{\rho(-r)}^{DBB} = \psi(1+m)f^4, \tag{27}$$

$$\tilde{\pi}_{\rho r}^{DBB} = (1-t)h^2\hat{g}^D + \psi\left[mh^4 + 2(1+m)h^2f^2 + (1+m^{-1})f^4\right], \tag{28}$$

$$\tilde{s}_{\rho r}^{DBB} = \psi\left[mh^4 - (1+m^{-1})f^4\right]. \tag{29}$$

[19] There being two principals (firms α and β) lobbying two agents (policy makers a and b), our framework is a hybrid between the common agency model and the one-principal many-agents model (on the latter, see Mookherjee, 1984, and Ma, 1988).

Total net profits of each firm ρ are:

$$\tilde{\pi}^{DBB} = \hat{\pi}^{D} + \psi\left[m(h^4 + f^4 + 4h^2f^2) + 2f^2(h^2 + f^2) + m^{-1}f^4\right]. \tag{30}$$

We now turn to the case in which both firms lobby their home region only (HH). The policy makers' objective functions are:

$$V_a^{DHH} = \mu W_a + (1-\mu)S_{\alpha a}, \tag{31}$$
$$V_b^{DHH} = \mu W_b + (1-\mu)S_{\beta b}, \tag{32}$$

and, as it is shown in Appendix A, public good supply is:

$$\tilde{g}^{DHH} = \hat{g}^{D} + \frac{m(1-t)h^2}{(h+f)^2}, \tag{33}$$

and net profits (home plus abroad) and contributions (home) are:

$$\tilde{\pi}^{DHH} = \hat{\pi}^{D} + \psi m(h^4 + 2h^2f^2), \tag{34}$$

$$\tilde{s}^{DHH} = \psi m h^4. \tag{35}$$

The equilibria corresponding to all other strategy pairs —one firm lobbying both regions and the other the home region only (BH), one firm lobbying both regions and the other not lobbying (BN), and finally one firm lobbying the home region and the other not lobbying (HN)— are shown in Appendix A.

Notice from (29) that in the lobbying game BB one needs to check the conditions under which contributions in the home region are non-negative. As it is shown in Appendix A, Table 2, the same is true for the lobbying games BH and BN. The following proposition illustrates the existence conditions of the lobbying games equilibria.

Proposition 2 *An equilibrium with non-negative contributions of the lobbying games HH and HN exists for all $\mu \in (0,1]$. An equilibrium with non-negative contributions of the lobbying game BB exists if and only if $\mu \in (0, \tilde{\mu}]$, where*

$$\tilde{\mu} = 1 - \frac{\left(\sqrt{4h^4 + f^4} - f^2\right)f^2}{2h^4}. \tag{36}$$

If an equilibrium with non-negative contributions of the lobbying games BH and BN exists, then $\mu \in (0, \tilde{\mu}]$.

Proof. See Appendix B. ■

According to the above proposition, on the one hand, when policy makers assign a sufficiently high weight to contributions over social welfare (i.e. $\mu \leq \tilde{\mu}$), it is profitable for both firms to lobby both "at home" and "abroad". Firms are willing to offer both policy makers positive contributions for an expansion of public good provision. On the other hand, when policy makers do not care much about contributions (i.e. $\mu > \tilde{\mu}$), each firm does not find it profitable to lobby both regions. The equilibria in which firms lobby only at home, on the contrary, exist no matter the politician's degree of benevolence.[20]

The intuition of why firms may not lobby both regions rests on the following argument. Consider region b and suppose that social welfare is $\check{W}_b$ when only firm α is lobbying both regions. If firm β as well starts lobbying in both regions, then social welfare in region b falls to $\tilde{W}_b$, since more lobbying increases the upward distortion in public good supply. In this scenario, the minimum contribution that firm β must be willing to offer in order to successfully lobby region b is:

$$\tilde{s}_{\beta b} = -\frac{\mu}{1-\mu}\underbrace{\left(\tilde{W}_b - \check{W}_b - \tilde{s}_{\beta a}\right)}_{\Delta W_b < 0} - \frac{1}{1-\mu}\underbrace{\left(\tilde{\Pi}_{\alpha b} - \check{\Pi}_{\alpha b}\right)}_{\Delta \Pi_{\alpha b} > 0}, \tag{37}$$

where $\tilde{\Pi}_{\alpha b}$ and $\check{\Pi}_{\alpha b}$ are the after-tax profits earned by firm α in region b when both firms lobby and when only firm α lobbies, respectively. Eq. (37) shows that the optimal contribution is made up of two terms. The first one is the amount that firm β must pay the politician to reward her for the welfare loss caused by its decision to lobby (notice that the welfare loss includes the contribution firm β pays to lobby region a). However, as the second term in (37) shows, since firm α is already lobbying region b, firm β can take a free ride on the increase in contributions that firm α pays to region b. Whenever the latter is not sufficient to make up for the welfare loss, $\tilde{s}_{\beta b} > 0$ and therefore firm β has an incentive to lobby both regions. Otherwise, since both firms try to free ride on each other contributions, an equilibrium with both firms lobbying both regions does not exist.

From (37) one can see why an equilibrium BB exists only when μ is relatively small ($\mu \leq \tilde{\mu}$). The smaller μ (i.e. the greedier the policy maker) the larger the distortion in public good supply and, hence, the welfare loss and the profit gains for firm α. However, since the latter are linear while the welfare losses are quadratic in g, $\tilde{s}_{\beta b}$ turns out to be positive. On the contrary, an equilibrium does not exist ($\tilde{s}_{\beta b} < 0$) when the politician is sufficiently benevolent ($\mu > \tilde{\mu}$) since the distortions induced by lobbying are small. The same kind of argument provides an intuition for why for μ sufficiently high the equilibria BH and BN do not exist. Notice finally that when firms lobby only in their home region, the second term in (37) disappears, implying that $\tilde{s}_{\beta b} > 0$ for all μ, and therefore that an equilibrium HH or HN always exists.

The following proposition describes firms' decisions on where to lobby in a decentralized world.

Proposition 3 *For $\mu \in (0, \tilde{\mu}]$ the strategy pair BB is the unique Nash equilibrium in truthful strategies of the "where-to-lobby" game under decentralization. Otherwise, the unique Nash equilibrium is HH.*

Proof. See Appendix B. ■

From proposition 2 we know that BB and HH equilibria co-exist only for $\mu \leq \tilde{\mu}$, i.e. when the politician is sufficiently greedy. In this case, however, firms make more profits by lobbying both regions. On the contrary, for $\mu > \tilde{\mu}$ only the equilibria HH

[20]Notice that the threshold $\tilde{\mu}$ is an increasing function of δ, with $\tilde{\mu} = (3-\sqrt{5})/2 \approx .38$ at $\delta = 1$ and $\tilde{\mu} = 1$ at $\delta = \delta_{\max}$.

and HN exist; however, the former strategy pair dominates the latter, as each firm has an incentive to lobby its home region.

4.3 Centralization versus decentralization

By using the above results it is possible to compare lobbying activity under centralization and decentralization through the following:

Proposition 4 *In the presence of lobbying, for $\mu \in (0, \tilde{\mu}]$ firms' net profits are higher under decentralization than under centralization, whereas public good supply is the same. For $\mu \in (\tilde{\mu}, 1]$ both firms' net profits and public good supply are smaller under decentralization. For all $\mu \in (0, 1]$ contributions to policy makers are smaller under decentralization.*

Proof. See Appendix B. ■

The proposition shows that when regional policy makers are sufficiently greedy to be captured by both lobbies, the latter are better off under decentralization, given that the same distortion in public good supply can be obtained via smaller contributions. On the contrary, firms are better off under centralization whenever the politician is benevolent enough. In fact, under decentralization firms can successfully lobby only the home region, hence inducing a small distortion in public policy.

Until now we implicitly assumed away the existence of lobbying costs and therefore all kinds of expenses a lobbist sustains over contribution payments. We now remove this assumption, by introducing an exogenously fixed lobbying cost, C, independent of the institutional framework (centralization, decentralization and, in the latter case, whether a firm lobbies both regions or just one). The introduction of lobbying costs allows us to discuss in the following proposition the entry decision by firms in the lobbying arena.

Proposition 5 *For $\mu \in (\tilde{\mu}, 1]$ lobbying occurs under centralization but not under decentralization if $C \in [C_1, C_2]$. For $\mu \in (0, \tilde{\mu}]$ lobbying occurs under decentralization but not under centralization if $C \in [C_2, C_3]$, where*

$$C_1 = m\psi(h^4 + 2h^2 f^2), \quad C_2 = C_1 + m\psi f^2(f^2 + 2h^2), \quad C_3 = C_2 + \frac{1}{m}\psi f^4.$$

In all other cases either there is no lobbying ($C > C_3$) or there is lobbying ($C < C_1$) both under centralization and under decentralization.

Proof. See Appendix B. ■

Unsuprisingly, the cost thresholds are decreasing in μ, meaning that as the social planner cares less about the lobbists contributions, lobbying is less likely to occur. In particular, if $\mu = 1$, it is $C_1 = C_2 = C_3 = 0$ and no lobbying occurs.

A Appendix: Derivation of the lobbying equilibria

The common-agency lobbying-games are solved by the same logic as in proposition 3 in Dixit et al. (1997).

A.1 Centralization

From the first order conditions for maximizing (17),

$$\mu\frac{\partial W}{\partial g_r}+(1-\mu)(h^2+f^2)(1-t)=0,$$

we obtain $\tilde{g}^C$ in (18) for both g_a and g_b. To compute the equilibrium profits of firm β we need first to solve the problem in which firm α is lobbying and firm β is not lobbying. Hence, the policy maker maximizes $V^C_{-\beta}=\mu W+(1-\mu)S_\alpha$. From the corresponding first order conditions:

$$\mu\frac{\partial W}{\partial g_a}+(1-\mu)(1-t)h^2=0,\qquad \mu\frac{\partial W}{\partial g_b}+(1-\mu)(1-t)f^2=0,$$

we obtain the optimal public good supplies:

$$\tilde{g}^C_{a(-\beta)}=\hat{g}^C+\frac{1-\mu}{\mu}\frac{(1-t)h^2}{(h+f)^2},\qquad \tilde{g}^C_{b(-\beta)}=\hat{g}^C+\frac{1-\mu}{\mu}\frac{(1-t)f^2}{(h+f)^2}.$$

Writing the equation $V^C\left(\tilde{g}^C_a,\tilde{g}^C_b,\pi_\alpha,\pi_\beta\right)=V^C_{-\beta}\left(\tilde{g}^C_{a(-\beta)},\tilde{g}^C_{b(-\beta)},\pi_\alpha\right)$ and solving for π_β, we obtain the equilibrium profits $\tilde{\pi}^C_\beta$ shown in (20). By symmetry, $\tilde{\pi}^C_\alpha=\tilde{\pi}^C_\beta$. Finally, by substituting (18) and (20) into (16), we compute the equilibrium contributions.

A.2 Decentralization

We solve the lobby game for each possible strategy pair occurring under decentralization. Throughout this section, V^{Dij}_r denotes the preferences of policy maker r when firms α and β are choosing action i and j, respectively, $i,j\in\{B,H,N\}$. The results of the analysis are summarized in Table 1 (equilibrium profits) and Table 2 (equilibrium contributions).

Both firms lobbying both regions (BB)

When both firms lobby both regions, the policy makers' objective functions are (23) and (24) in the text. By maximizing (23) with respect to g_a and (24) with respect to g_b, we obtain the symmetric solution $\tilde{g}^{DBB}$ in (25). To compute the equilibrium profits, assume that firm α lobbies both regions (B) and firm β does not lobby (N). Policy makers maximize:

$$V^{DBN}_a=\mu(W_a-S_{\alpha b})+(1-\mu)S_{\alpha a},\tag{38}$$
$$V^{DBN}_b=\mu(W_b+S_{\alpha b})+(1-\mu)S_{\alpha b}.\tag{39}$$

Optimal public goods supplies are:

$$\tilde{g}^{DBN}_a=\hat{g}^D+\frac{m(1-t)h^2}{(h+f)^2},\tag{40}$$
$$\tilde{g}^{DBN}_b=\hat{g}^D+\frac{(1+m)(1-t)f^2}{(h+f)^2}.\tag{41}$$

		Profit at home $(1-t)h^2\hat{g}^D+$	Profit abroad $(1-t)f^2\hat{g}^D+$
firm ρ	B	$\psi\left[mh^4+2(1+m)h^2f^2+(1+m^{-1})f^4\right]$	$\psi\left[(1+m)f^4+2mh^2f^2\right]$
firm $-\rho$	B	$\psi\left[mh^4+2(1+m)h^2f^2+(1+m^{-1})f^4\right]$	$\psi\left[(1+m)f^4+2mh^2f^2\right]$
firm ρ	B	$\psi\left[mh^4+2(1+m)^{-1}h^2f^2+(1+m^{-1})f^4\right]$	$\psi\left[(1+m)f^4+2mh^2f^2+2m(1+m)^{-1}h^2f^2\right]$
firm $-\rho$	H	$\psi\left[mh^4+2(1+m)h^2f^2\right]$	$2\psi mh^2f^2$
firm ρ	B	$\psi\left[mh^4+2(1+m)^{-1}h^2f^2+(1+m^{-1})f^4\right]$	$\psi\left[(1+m)f^4+2m(1+m)^{-1}h^2f^2\right]$
firm $-\rho$	N	$2\psi(1+m)h^2f^2$	$2\psi mh^2f^2$
firm ρ	H	ψmh^4	$2\psi mh^2f^2$
firm $-\rho$	H	ψmh^4	$2\psi mh^2f^2$
firm ρ	H	ψmh^4	0
firm $-\rho$	N	0	$2\psi mh^2f^2$

Table 1: Firms' net profits under decentralization

Solving

$$V_a^{DBB}\left(\tilde{g}_a^{DBB},\tilde{g}_b^{DBB},\pi_{\alpha a},\pi_{\beta a},\pi_{\alpha b}\right)=V_a^{DBN}\left(\tilde{g}_a^{DBN},\tilde{g}_b^{DBN},\pi_{\alpha a},\pi_{\alpha b}\right),$$
$$V_b^{DBB}\left(\tilde{g}_a^{DBB},\tilde{g}_b^{DBB},\pi_{\alpha b},\pi_{\beta b},\pi_{\beta a}\right)=V_b^{DBN}\left(\tilde{g}_a^{DBN},\tilde{g}_b^{DBN},\pi_{\alpha b}\right),$$

for $\pi_{\beta a}$ and $\pi_{\beta b}$, we get the equilibrium profits shown in (26) and (28). Equilibrium contributions are obtained from substitution of optimal public good supplies and profits into the contribution functions, i.e. $\tilde{s}_{\beta a}^{DBB}=(1-t)f^2\tilde{g}^{DBB}-\tilde{\pi}_{\beta a}^{DBB}$ and $\tilde{s}_{\beta b}^{DBB}=(1-t)h^2\tilde{g}^{DBB}-\tilde{\pi}_{\beta b}^{DBB}$.

One firm lobbying both regions and one lobbying the home region only (BH)

Suppose that firm α chooses B and firm β chooses H. Policy makers maximize:

$$V_a^{DBH}=\mu(W_a-S_{\alpha b})+(1-\mu)S_{\alpha a}, \tag{42}$$
$$V_b^{DBH}=\mu(W_b+S_{\alpha b})+(1-\mu)(S_{\alpha b}+S_{\beta b}), \tag{43}$$

from which:

$$\tilde{g}_a^{DBH}=\hat{g}^D+\frac{m(1-t)h^2}{(h+f)^2}, \tag{44}$$
$$\tilde{g}_b^{DBH}=\hat{g}^D+\frac{(1-t)f^2}{(h+f)^2}+\frac{m(1-t)(h^2+f^2)}{(h+f)^2}. \tag{45}$$

		Contributions at home	Contributions abroad
firm ρ	B	$\tilde{s}_1^{DBB} = \psi\left[mh^4 - (1+m^{-1})f^4\right]$	$\tilde{s}_2^{DBB} = \psi(1+m)f^4$
firm $-\rho$	B	$\tilde{s}_1^{DBB} = \psi\left[mh^4 - (1+m^{-1})f^4\right]$	$\tilde{s}_2^{DBB} = \psi(1+m)f^4$
firm ρ	B	$\tilde{s}_1^{DBH} = \psi\left[mh^4 - (1+m^{-1})f^4 - 2(1+m)^{-1}h^2f^2\right]$	$\tilde{s}_2^{DBH} = \psi\left[(1+m)f^4 - 2m(1+m)^{-1}h^2f^2\right]$
firm $-\rho$	H	$\tilde{s}_1^{DHB} = \psi mh^4$	—
firm ρ	B	$\tilde{s}_1^{DBN} = \psi\left[mh^4 - (1+m^{-1})f^4 - 2(1+m)^{-1}h^2f^2\right]$	$\tilde{s}_2^{DBN} = \psi\left[(1+m)f^4 - 2m(1+m)^{-1}h^2f^2\right]$
firm $-\rho$	N	—	—
firm ρ	H	$\tilde{s}_1^{DHH} = \psi mh^4$	—
firm $-\rho$	H	$\tilde{s}_1^{DHH} = \psi mh^4$	—
firm ρ	H	$\tilde{s}_1^{DHN} = \psi mh^4$	—
firm $-\rho$	N	—	—
$\tilde{s}_k^{Dij}$ denotes the contribution made at home ($k=1$) or abroad ($k=2$) by the firm playing strategy i when the other firm is playing strategy j, where $i,j \in \{B,H,N\}$.			

Table 2: Firms' contributions under decentralization

Assume now that firm α does not lobby. Policy makers maximize:

$$V_a^{DNH} = \mu W_a, \tag{46}$$
$$V_b^{DNH} = \mu W_b + (1-\mu)S_{\beta b} \tag{47}$$

and optimal public goods supplies are:

$$\tilde{g}_a^{DNH} = \hat{g}^D, \tag{48}$$
$$\tilde{g}_b^{DNH} = \hat{g}^D + \frac{m(1-t)h^2}{(h+f)^2}. \tag{49}$$

Solving

$$V_a^{DBH}\left(\tilde{g}_a^{DBH}, \tilde{g}_b^{DBH}, \pi_{\alpha a}, \pi_{\alpha b}\right) = V_a^{DNH}\left(\tilde{g}_a^{DNH}, \tilde{g}_b^{DNH}\right),$$
$$V_b^{DBH}\left(\tilde{g}_a^{DBH}, \tilde{g}_b^{DBH}, \pi_{\beta b}, \pi_{\alpha b}\right) = V_b^{DNH}\left(\tilde{g}_a^{DNH}, \tilde{g}_b^{DNH}, \pi_{\beta b}\right),$$

for $\pi_{\alpha a}$ and $\pi_{\alpha b}$ we get the equilibrium profits that a firm lobbying both regions makes at home and abroad when the other firm is lobbying only at home (see Table 1). To compute the equilibrium profits of firm β, assume now that α chooses B while β chooses N. Policy makers maximize (38) and (39) and the solutions are (40) and (41). Solving the equation

$$V_b^{DBH}\left(\tilde{g}_a^{DBH}, \tilde{g}_b^{DBH}, \pi_{\beta b}, \pi_{\alpha b}\right) = V_b^{DBN}\left(\tilde{g}_a^{DBN}, \tilde{g}_b^{DBN}, \pi_{\alpha b}\right),$$

for $\pi_{\beta b}$ we get the equilibrium profits that a firm makes at home when lobbying only at home while the other firm is lobbying both regions (see Table 1). Finally, equilibrium contributions for the game BH (see Table 2) are computed by substitutions of net profits and public good supplies into the compensating contribution schedules.

One firm lobbying both regions and the other no lobbying (BN)

Suppose that firm α chooses B and β chooses N. Policy makers maximize (38) and (39) and the solutions are (40) and (41). Assume now that firm α is not lobbying. Policy makers maximize $V_a^{DNN} = \mu W_a$ and $V_b^{DNN} = \mu W_b$. The solution is the no-lobbying optimal public good supply $\hat{g}^D$ for both g_a and g_b. Solving

$$\begin{aligned} &V_a^{DBN}\left(\tilde{g}_a^{DBN}, \tilde{g}_b^{DBN}, \pi_{\alpha a}, \pi_{\alpha b}\right) = V_a^{DNN}\left(\hat{g}_a^D, \hat{g}_b^D\right), \\ &V_b^{DBN}\left(\tilde{g}_a^{DBN}, \tilde{g}_b^{DBN}, \pi_{\alpha b}\right) = V_a^{DNN}\left(\hat{g}_a^D, \hat{g}_b^D\right), \end{aligned}$$

for $\pi_{\alpha a}$ and $\pi_{\alpha b}$we get the equilibrium profits that a firm lobbying both regions makes at home and abroad when the other firm is not lobbying (see Table 1). Profits at home and abroad of the no-lobbying firm β are computed by substituting the optimal public good supplies into the corresponding profit functions. Finally, equilibrium contributions for the game BN (see Table 2) are computed by simple substitutions of net profits and public good supplies into the compensating contribution schedules of firm α.

Both firms lobbying only the home region (HH)

When both firms lobby only the home region, the policy makers' objective functions are (31) and (32) in the text. By maximizing (31) with respect to g_a and (32) with respect to g_b, we get the optimal public good supplies $\tilde{g}^{DHH}$ in (33). To compute the equilibrium profits, assume that α lobbies at home (H), while β does not lobby (N). The problem solved by policy maker a is unchanged. She maximizes (31) with respect to g_a, obtaining $\tilde{g}^{DHH}$. Policy maker b maximizes $V_b^{DHN} = \mu W_b$ with respect to g_b, and the solution is the no-lobby public good supply $\hat{g}^D$. Solving the equation $V_b^{DHH}\left(\tilde{g}_a^{DHH}, \tilde{g}_b^{DHH}, \pi_{\alpha a}\right) = \mu W_b\left(\tilde{g}_a^{DHH}, \hat{g}_b^D\right)$ for the home profits $\pi_{\alpha a}$, and then adding the "abroad" profits, $(1-t)f^2\tilde{g}^{DHH}$, we get total profits $\tilde{\pi}^{DHH}$ in (34). Equilibrium contributions are $\tilde{s}^{DHH} = (1-t)h^2\tilde{g}^{DHH} - \tilde{\pi}^{DHH} = m\psi h^4$.

One firm lobbying the home region and the other no lobbying (HN)

Suppose finally that firm β chooses H and firm α chooses N. Policy makers maximize (46) and (47) and the solutions are (48) and (49). Assuming that firm β is not lobbying, policy makers maximize $V_a^{DNN} = \mu W_a$ and $V_b^{DNN} = \mu W_b$. The solution is the no-lobbying optimal public good supply $\hat{g}^D$ for both g_a and g_b. Solving

$$V_b^{DHN}\left(\tilde{g}_a^{DNH}, \tilde{g}_b^{DNH}, \pi_{\beta b}\right) = V_b^{DNN}\left(\hat{g}_a^D, \hat{g}_b^D\right),$$

for $\pi_{\beta b}$ we get the equilibrium profits that a firm makes at home when lobbying only at home while the other firm is not lobbying (see Table 1). Firm β's profits abroad and profits at home and abroad of the no-lobbying firm α are computed by substituting the optimal public good supplies into the corresponding profit functions. Equilibrium contributions for the game HN (see Table 2) are computed by substituting net profits and public good supplies into firm β's compensating contribution schedule.

B Appendix: Proofs of propositions

B.1 Proof of proposition 2

From Table 2 one can see that $\tilde{s}_1^{DHH}$ and $\tilde{s}_1^{DHN}$ are always non-negative, proving the first part of the proposition on the lobbying games HH and HN. As for the other cases, while $\tilde{s}_1^{DHB} \geq 0$ and $\tilde{s}_2^{DBB} \geq 0$, non-negativity conditions must be found for $\tilde{s}_1^{DBB}$, $\tilde{s}_1^{DBH} = \tilde{s}_1^{DBN}$ and $\tilde{s}_2^{DBH} = \tilde{s}_2^{DBN}$. This is done through the following lemma.

Lemma 1 *$\tilde{s}_1^{DBB} \geq 0$ iff $\mu \in (0, \tilde{\mu}]$ with $\tilde{\mu}$ defined in (36).*

Proof. Substituting for $m = (1-\mu)/\mu$, $\tilde{s}_1^{DBB}$ can be written as:

$$\tilde{s}_1^{DBB}(\mu) = \psi \left(\frac{1-\mu}{\mu} h^4 - \frac{1}{1-\mu} f^4 \right). \tag{50}$$

From (50) it is $\lim_{\mu \to 0} \tilde{s}_1^{DBB} = +\infty$, $\lim_{\mu \to 1} \tilde{s}_1^{DBB} = -\infty$. Moreover, $\tilde{\mu}$ is the unique root of eq. (50) for $\mu \in (0,1)$. Hence, by continuity and monotonicity of $\tilde{s}_1^{DBB}(\mu)$, $\tilde{s}_1^{DBB} \geq 0$ iff $\mu \in (0, \tilde{\mu}]$. ■

Lemma 2 *$\tilde{s}_1^{DBH} \geq 0$ and $\tilde{s}_1^{DBN} \geq 0$ iff $\mu \in (0, \tilde{\mu}_1]$, where $\tilde{\mu}_1 \leq \tilde{\mu}$.*

Proof. Substituting for $m = (1-\mu)/\mu$, $\tilde{s}_1^{DBH}$ and $\tilde{s}_1^{DBN}$ can be written as:

$$\tilde{s}_1^{DBi}(\mu) = \psi \left(\frac{1-\mu}{\mu} h^4 - \frac{1}{1-\mu} f^4 - 2\mu h^2 f^2 \right), \quad i \in \{H, N\}. \tag{51}$$

From (51) it is $\lim_{\mu \to 0} \tilde{s}_1^{DBi} = +\infty$, $\lim_{\mu \to 1} \tilde{s}_1^{DBi} = -\infty$. Moreover, from (50) and (51) it is $\tilde{s}_1^{DBi}(\mu) - \tilde{s}_1^{DBB}(\mu) = -2\mu h^2 f^2 \leq 0$ for all $\mu \in (0,1]$. Hence, by monotonicity, $\tilde{s}_1^{DBi}(\mu)$ has a unique root $\tilde{\mu}_1 \leq \tilde{\mu}$ in the interval $\mu \in (0,1)$. ■

Lemma 3 *For $h^2 \geq 2f^2$, $\tilde{s}_2^{DBH} \geq 0$ and $\tilde{s}_2^{DBN} \geq 0$ iff $\mu \in (0, \tilde{\mu}_2]$ and $\mu \in [\tilde{\mu}_3, 1]$, $\tilde{\mu}_2 \leq \tilde{\mu}_3$, where:*

$$\tilde{\mu}_2 = \frac{1}{2} - \frac{\sqrt{h^2 - 2f^2}}{2h}, \qquad \tilde{\mu}_3 = \frac{1}{2} + \frac{\sqrt{h^2 - 2f^2}}{2h}. \tag{52}$$

If $h^2 < 2f^2$ then $\tilde{s}_2^{DBH} > 0$ and $\tilde{s}_2^{DBN} > 0$ for all $\mu \in (0,1]$.

Proof. Substituting for $m = (1-\mu)/\mu$, $\tilde{s}_2^{DBH}$ and $\tilde{s}_2^{DBN}$ can be written as:

$$\tilde{s}_2^{DBi}(\mu) = \psi f^2 \left(\frac{f^2}{\mu} - 2(1-\mu) h^2 \right), \quad i \in \{H, N\}. \tag{53}$$

From (53) it is $\lim_{\mu \to 0} \tilde{s}_2^{DBi} = +\infty$, $\tilde{s}_2^{DBi}(1) = \psi f^4 \geq 0$. Moreover, $\tilde{s}_2^{DBi}(\mu)$ has two roots in the interval $\mu \in (0,1]$, shown in (52). From these it follows that if $h^2 \geq 2f^2$ then $0 \leq \tilde{\mu}_2 \leq \tilde{\mu}_3 \leq 1$, with $\tilde{s}_2^{DBi}(\mu) \geq 0$ for $\mu \in (0, \tilde{\mu}_2]$ and $\mu \in [\tilde{\mu}_3, 1]$. If $h^2 < 2f^2$ then $\tilde{s}_2^{DBi}(\mu) \geq 0$ for all $\mu \in (0,1]$. ■

From lemma 1–3, since $\tilde{\mu}_1 \leq \tilde{\mu}$, the condition $\mu \in (0, \tilde{\mu}]$ is necessary for an equilibrium with non-negative contributions of the lobbying games BH and BN to exist. This completes the proof. ■

B.2 Proof of proposition 3

Table 1 allows us to compute the aggregate (home plus abroad) net profits earned by firms for each possible strategy pairs. The resulting symmetric normal form of the where-to-lobby game is represented in Table 3.Each cell shows the payoff of the row player, firm α, at the top, and that of the column player, firm β, at the bottom. $\Delta\pi^{ij}$ denotes the profit gains from lobbying of the firm playing strategy i when the opponent is playing j, with $i, j \in \{B, H, N\}$. From Table 3, for $f \neq 0$ (i.e. $\delta < \delta_{\max}$), it is $\Delta\pi^{BB} > \Delta\pi^{HB} > \Delta\pi^{NB}$, $\Delta\pi^{BH} > \Delta\pi^{HH} > \Delta\pi^{NH}$ and $\Delta\pi^{BN} > \Delta\pi^{HN} > \Delta\pi^{NH}$. Hence, when $\mu \leq \tilde{\mu}$ and the equilibria BB, BH and BN of the lobbying game exist, then B is a dominant strategy for each player. Thus BB is the unique Nash equilibrium of the "where-to-lobby" game. When the equilibria BB, BH and BN do not exist ($\mu > \tilde{\mu}$) the game is restricted to the 4 lower-right cells, in which H is a dominant strategy for each player, and HH is the unique Nash equilibrium of the "where-to-lobby" game. ■

B.3 Proof of proposition 4

Net profits under centralization, $\tilde{\pi}^C$, are given by (20). Consider first the case $\mu \in (0, \tilde{\mu}]$. Net profits of the Nash equilibrium BB under decentralization, $\tilde{\pi}^{DBB}$, are shown in (30). It is then immediate to see that $\tilde{\pi}^{DBB} - \tilde{\pi}^C = \psi m^{-1} f^4 \geq 0$. By comparing (18) and (25) it follows that public good supply is the same under centralization and decentralization. As for contributions, using (21), (27) and (29), one gets $\tilde{s}^C - \tilde{s}^{DBB} = \psi m^{-1} f^4 \geq 0$. Next consider the case $\mu \in (\tilde{\mu}, 1]$. Net profits of the Nash equilibrium HH under decentralization, $\tilde{\pi}^{DHH}$, are shown in (34). From (20) and (34) it is $\tilde{\pi}^C - \tilde{\pi}^{DHH} = \psi f^2[2(h^2 + f^2) + m(f^2 + 2h^2)] \geq 0$. By the comparison of (18) and (33) it follows that public good supply is smaller under decentralization. Finally, from (21) and (35) it is $\tilde{s}^C - \tilde{s}^{DHH} = \psi m f^4 \geq 0$. ■

B.4 Proof of proposition 5

Assuming without loss of generality that when indifferent between lobbying and no lobbying a firm does lobby, under centralization firms lobby only if $\tilde{\pi}^C - \hat{\pi}^C \geq C$. Using (20) it follows that firms lobby under centralization only if $C \leq C_2 = m\psi(h^4 + f^4 + 4h^2 f^2)$. Under decentralization, if $\mu \in (0, \tilde{\mu}]$, firms lobby only if $\tilde{\pi}^{DBB} - \hat{\pi}^D \geq C$. Using (30) one obtains the lobbying condition $C \leq C_3 = m\psi(h^4 + f^4 + 4h^2 f^2) + \psi m^{-1} f^4$. Finally, if $\mu \in (\tilde{\mu}, 1]$ firms lobby only if $\tilde{\pi}^{DHH} - \hat{\pi}^D \geq C$, from which, using (34), it must be $C \leq C_1 = m\psi(h^4 + 2h^2 f^2)$ for lobbying to occur. It is then $C_1 \leq C_2 \leq C_3$ and hence the proposition holds. ■

Table 3: The where-to-lobby game under decentralization

		firm β		
		B	H	N
α	B	$\Delta\pi^{BB} = \psi\left[mh^4 + 2(1+2m)h^2f^2 + (1+m)^2m^{-1}f^4\right]$, $\psi\left[mh^4 + 2(1+2m)h^2f^2 + (1+m)^2m^{-1}f^4\right] = \Delta\pi^{BB}$	$\Delta\pi^{BH} = \psi\left[mh^4 + 2(1+m)h^2f^2 + (1+m)^2m^{-1}f^4\right]$, $\psi\left[mh^4 + 2(1+2m)h^2f^2\right] = \Delta\pi^{HB}$	$\Delta\pi^{BN} = \psi\left[mh^4 + 2h^2f^2 + (1+m)^2m^{-1}f^4\right]$, $2\psi(1+m)h^2f^2 = \Delta\pi^{NB}$
	H	$\Delta\pi^{HB} = \psi\left[mh^4 + 2(1+2m)h^2f^2\right]$, $\psi\left[mh^4 + 2(1+m)h^2f^2 + (1+m)^2m^{-1}f^4\right] = \Delta\pi^{BH}$	$\Delta\pi^{HH} = \psi\left[mh^4 + 2mh^2f^2\right]$, $\psi\left[mh^4 + 2mh^2f^2\right] = \Delta\pi^{HH}$	$\Delta\pi^{HN} = \psi mh^4$, $2\psi mh^2f^2 = \Delta\pi^{NH}$
	N	$\Delta\pi^{NB} = 2\psi(1+m)h^2f^2$, $\psi\left[mh^4 + 2h^2f^2 + (1+m)^2m^{-1}f^4\right] = \Delta\pi^{BN}$	$\Delta\pi^{NH} = 2\psi mh^2f^2$, $\psi mh^4 = \Delta\pi^{HN}$	0, 0

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