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DISTRIBUTIVE POLITICS AND FEDERATIONS

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Distributive politics and federations*

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Abstract

We integrate the distributive politics literature with the political economy literature of country unions or federations. We consider a simple model of legislative bargaining by specifing the behavior of a central legislature composed of an odd number of representatives elected by regions whose preferences over local public goods differ. Representatives have to make a decision by majority voting on how to allocate the amount of local public goods financed by a linear income tax or, furthermore, by a region specific tax. One of the representative is randomly chosen as agenda setter and makes a policy proposal of local public goods to be provided. We also investigate whether the credible threath of secession by any region modifies the agenda setter proposal and hence the outcome of the legislative bargaining game. A key finding is that the way in which the bargaining process inside the legislature is modelled and the way of financing local public goods provision affect the probability of federation or secession. Thus, we make a specific case of multidimensional political conflict among regions and therefore the politcy outcomes are "structure induced" rather than "preference induced".

1 Introduction

The main objective of the paper is to integrate the distributive politics literature with the political economy literature of country unions or federations. We use the term of federation as synonimus of union of heterogeneous constituencies (or regions) which associate themselves to centralize some policy choices or to decide together some common policies. Obviously, talking of constituencies we may think of regions in a country, or countries in a "supranational level".

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Recently, we have observed two related phenomena. First, in many countries (for instance Italy, Spain, UK, Cananda) the demand for more autonomy - if not secession - has been put forward by some regions or locally based political groups. Second we have seen the creation or the strengthening of country unions to enforce some common policies, the European Union being the most striking example but of course we may think of trade agreements or military agreements.

Many works in the literature have provided important insights on the economic factors or determinants that can be very helpful in explaining these phenomena[1]. However, these models still do not consider any explicit and more realistic case of a decision making process as for constituencies' decisions to join the union. Rather, in these models, the decision to separate from the union or to join the union is simplified along a single dimension and the regional median voters or the union median voter solve the conflict and determine the policy outcome (namely separation or unification). Influential scholars in economic theory, such as Inman and Rubinfeld (1997a) have stressed the importance of the politics of decision making for countries' stability. But, to the best of our knowledge, this question has not yet been formally analyzed. Therefore, in the paper, we choose to proceed in this direction by specifying the behavior of a central legislature (composed of representatives elected by constituencies) borrowing from the political science literature on distributive politics.[2]

A distributive policy is a political decision that concentrates benefits in a specific geographic district or region and finance expenditure through generalized taxation[3]. In other words, a distributive policy is one which benefits the citizens of one district or jurisdiction, but whose costs are borne by citizens of all districts. Classical examples are local public goods which are financed by central taxation. The main insight of this literature has been to predict the possibility of minimum winning coalition among members of the national legislature in order to provide local public goods whose benefits are concentrated within geographic districts to which members of the minimum winning coalition belong[4]. Since these local public goods are financed through general taxation, members of the legislature tend to demand more of these local public goods than the social optimum. However, to the best of our knowledge, the literature still does not contemplate that the inefficiency in local public goods choice generated by such decision making process may affect the stability and eventually induce the breaking up of the national legislature. We address this issue. We postulate

[1] Recenlty Alesina, Angeloni, Etro (2001a; 2001b).

[2] Notice that how to approach decision making in the legislature is still a key issue in the literature, since there is no standard model. However the distributive politics approach has been considered an appropriate description of the decision-making process on local public goods provision by the central legislature. See Persson and Tabellini (2000).

[3] For this definition of distributive politics, see Weingast, Shepsle and Johnsen (1981); moreover they write: "While it is clear that all policies have a geographic incidence of benefits and costs, what distinguishes a distributive policy is that benefits are geographically targeted... geography is the hallmark of distributive politics: programs and projects are geographically targeted, geographically fashioned, and may be independly varied. Importantly, geography is also the basis for political organization and representation" (p. 644).

[4] See, for instance, Ferejohn, Fiorina and McKelvey (1987).

that secession may take place because of interjurisdictional conflict over local public goods provision. To investigate the issue we build on Persson's contribution (1998), who proposes a clear case of a national legislature composed of representatives elected by country's regions and whose preference parameters over local public goods provision differ[5]. Representatives have to make a decision, by majority voting, on how to allocate the amount of local public goods financed out of a common pool of tax revenues. It is well known that local public goods financed nationally have redistributive effects and therefore it may turn out that there is no voting equilibrium inside the legislature. Borrowing from Baron and Ferejohn's seminal political model (1989), Persson avoids cycles in majority voting by imposing an "agenda structure" on the legislature via a "budget process": one of the representatives is randomly chosen as the agenda setter and makes a policy proposal on the amount of local public goods to be provided, financed out of a uniform national tax. To be implemented, such a policy proposal has to win the majority support inside the legislature, otherwise the default outcome of zero public good provision prevails. If the latter occurs, the "budget process" starts again and a new agenda setter is randomly chosen. Under this setting, Persson shows that the agenda setter always formulates his policy proposal in order to obtain a majority. In other words, in the bargaining model described in Persson's model, the agenda setter plays a non-cooperative game with other regions selecting the policy proposal that maximises her utility under the incentive compatibilty constraint holding for a majority of regions. A key result is that the agenda setter always obtains "rent" from other members in the legislature and majority voting within a legislature leads to an asymmetric distribution of benefits and costs from local public goods provision among districts. Furthermore, the minimum winning coalition is always composed of representatives with the highest preference for the local public goods since they are the cheapest to buy off for the agenda setter.

Therefore, a first concern motivating our analysis is to consider how likely is that a union of districts or regions governed by the decision making rule analyzed in Persson's model will survive or even be formed[6]. To explore this issue we assume that if a region does not join the union, it can constitute an indipendent political unit and it provides efficiently the local public good, but it bears a finite, positive, cost from being independent. As for this cost, let us consider that the indipendent region should provide a new defence system or a new legal system. Under this setting, we show under which conditions, regions which anticipate the outcome of the bargaining game inside the national legislature are willing to join the union or rather they prefer to stand alone. To explore this eventuality (that we call ex-ante secession) we compare for each representative the expected utility from the bargaining game inside the national legislature with the utility of being an independent political unit.

Furthermore, whereas Persson confines itself to discuss the outcome of a legislative making process when local public good provision is financed through

[5] Note that throught the paper we use constituencies, districts and regions in an interchangeably way.

[6] This question is not formally analysed by Persson (1998).

linear income taxation, we consider also the possibility of lump sum district specific taxation and we show that, in contrast to case of linear income tax, the expected utility for each district is independent of who is going to be the agenda setter and is the same across districts.

Secondly, we extend Persson's analysis by investigating whether the credible threat of secession by any district in the unicameral legislature modifies the agenda setter proposal and hence the outcome of the legislative bargaining game. So we modify the bargaining game allowing for an opt-out option by assuming that breaking-up of the union is a feasible option via a peaceful secession[7]. This eventuality (that we call ex post secession) allows us to examine the outcome of the bargaining game when the agenda setter solves her maximisation problem under incentive compatibility as well as non-secession constraints. This issue is also investigated taking into account the case of linear income taxation as well as the case of district specific taxation. Notice that for the non-secession constraints we assume that, in order to stay in and to accept the outcome of the bargaining game, each district should get a level of utiliy at least equal to the utility of being independent. We show that non secession constraints place a limit on the amount of rent extraction by the agenda setter, neverthless we also show that the agenda setter still prefers to respect the secession constraints rather than the breaking up of the union.

To sum up, it is worth noting that, in our work, by integrating the distributive politics literature into the issue of country formation and stability, we make a specific case of multidimensional political conflict among regions and therefore the policy outcomes (federation or secession) are endogenously determined by the specific decision making rules adopted by a national legislature: the equilibrium outcomes are "structure induced" rather than "preference induced".

The paper is organized as follows. Section 2 introduces the analytical framework. As background, Section 3 provides a brief review of the legislative bargaining game. Section 4 introduces the issue of secession into the legislative bargaining game. Section 5 presents the case of ex-ante secession by regions. Sections 6 considers the case of legislative bargaining when the opt-out option is allowed. Section 7 concludes and names some further extensions.

2 The analytical framework

We consider a country with a number of distinct districts or regions $i = 1, ...n$ where n is any odd number equal to or larger than 3. Each region i is populated by a number of identical citizens with unit mass. The preference of the citizens in district i over a private consumption good c and a public good g are represented by the utility function:

$$U_i = c_i + \alpha_i H(g_i) \tag{1}$$

[7] Let us think that the Constitution contains a clause allowing for peaceful secession by regions.

The function $H(g_i)$ is increasing and strictly concave and satisfying $H(0) = 0, H'(g) > 0, H''(g) < 0, g \geq 0$, and $\lim_{g \to 0} H'(g) = \infty, \lim_{g \to \infty} H'(g) = 0$.

The α_i term is the public good preference parameter for district i. Citizens in districts with higher $\alpha_i\ 's$ have both absolute and marginal higher valuations for the public good. We assume that citizens have the same exogenous income levels[8] in any district, i.e. $y_i = y$.

In the country, decisions over the provision of local public goods are delegated to a unicameral legislature consisting of one elected representative from each district[9]. Since we assume that income levels are the same across districts, then districts in this country can be divided into n different "types" according to their public good preference parameter α_i. This means that the composition of the unicameral legislature is characterized by the vector $\boldsymbol{\alpha} \equiv (\alpha_i)_{i=1}^{n}$.

The unicameral legislature is governed by majority rule[10] and has to decide the amount of public good to be offered in each district i. Therefore in this country, local public good decisions are delegated to a central unicameral legislature. However, for local public goods financing, in what follows we distinguish explicitly two cases: namely, the linear income tax and the district specific tax.

2.1 The national lump-sum tax or a linear income tax

Firstly, it is possible to assume that local public goods are financed centrally "by an economy wide-pool of tax revenue, with equal contribution from each group" (Persoon, 1998) which we will denote by t. So, the local public good g_i is financed through a common lump-sum tax or national lump-sum tax t levied on each citizen in district i. Stated differently, since we are assuming that income levels are the same across districts (i.e. $y_i = y$) imposing a linear income tax or a national lump-sum tax implies the same "tax price" for each district. Thus, the budget constraint for citizens in district i is written as

$$c_i = y - t \tag{2}$$

and substituting the constraint (2) into (1) we obtain

$$U_i = y + \alpha_i H(g_i) - t \tag{3}$$

[8] Without loss of generality it is possible also to set $y = 1$.

[9] Since citizens in each district are identical we are assuming that each region coincides with a voting district and is represented by exactly one legislator. For a model on strategic voting by districts whose citizens are heterogeneous see Coate (1997) and Besley and Coate (2000).

[10] Another legislative rule traditionally considered in the literature is the universalistic coalition, which is composed by all members of the legislature (Weingast, Shepsle and Johnsen, 1981; Niou and Ordeshook, 1985). Under this rule, each member of the legislature defers to the choice of all other legislators. "If any legislator or group of legislators fail to defer, the norm requires that all legislatore penalize the defectors by denying their first choices. This norm of deference — You scratch my back, I'll scratch yours — results in legislative proposals that are approved unanimously" (Inman and Rubinfeld, 1997a, p. 90).

Also, the government budget constraint is

$$nt \equiv \sum_i g_i \equiv G \tag{4}$$

where G denotes aggregate expenditure (i.e. the set of local public goods that have to be financed). So, combining the government budget constraint with (1) and (3), the payoff to any citizen in district i or representative i, can be written as follows:

$$U_i = y + \alpha_i H(g_i) - \frac{1}{n}\sum_i g_i \tag{5}$$

Note that this proposed "centralized choice procedure", through which local public goods are provided, is a standard case of distributive politics: the unicameral legislature uses a common pool of tax revenues to finance some local public goods whose benefits are completely concentrated to well defined districts.

2.2 The district specific tax or lump sum district specific taxes

As already mentioned, we do not restrict our attention to local public goods financed only through linear income taxation, but we consider (as a second case) that local public goods are financed through district specific taxes. In fact, the assumption of uniform tax rates across regions - which leads to uniform tax burdens when income is the same across regions – is often justified on the ground of administrative complexity. Still regions may differ with respect to other dimensions, or characteristics, than income. Just to exemplify, suppose regions, with the same income, differ in the effort exterted to curb tax elusion and evasion. If the national legislator is aware of such differences, by adopting a uniform tax rate he is implicitly imposing heterogeneous tax burdens. In our view, this offers some support for the assumption of region specific lump sum taxes. So, we consider such cases too and we denote as τ_i the region or district specific tax. In this second way of financing local public goods provision, the government budget constraint can be written as

$$g_1 + g_2 + ...g_n = \tau_1 + \tau_2 + ...\tau_n \tag{6}$$

And the payoff to any citizen or representative i can be written as

$$U_i = y + \alpha_i H(g_i) - \tau_i$$

2.3 Decentralized choices and local public goods provision

Before discussing in detail the political process and the outcomes of these two centralized choice procedures or legislative bargaining, conducted in a unicameral legislature, it is worth mentioning other "collective choice procedures" relating to local public goods provision, which can be thought of as benchmarks for evaluating the outcomes of the proposed legislative bargaining games.

- The social optimum or "optimal decentralization".

Suppose the policy vector $\mathbf{g} \equiv (g_i, ..g_n)$ of local jurisdiction-specific public goods is financed by local jurisdiction specific lump-sum taxes $\boldsymbol{\tau} \equiv (\tau_{i,..}\tau_n)$ so that

$$g_1 + g_2 + ...g_n = \tau_1 + \tau_2 + ...\tau_n \tag{7}$$

The utilitarian optimum is obtained by maximizing the welfare function $\sum_i u_i$ subject to the resource constraint $\sum_i (g_i + c_i) = ny$ and the government budget constraint (7). The solution to this problem implies efficient provision in each district: the average marginal benefit in each district equals the marginal social cost of unity. In fact, from first order conditions with respect to g_i and t_i:

$$\alpha_i H'(g_i^*) = 1 \tag{8}$$

where g_i^* denotes the efficient level of local public good provided in each district while the aggregate spending associated with this allocation is $G^* = \sum_i g_i^*$.

The optimal policy could emerge under an "appropriate federal instrument assignment" which decentralizes both taxation and spending over local public goods. But, "in the real world, however, it is often impossible to design the tax system so that the tax payers who finance a local public good are also those who benefit from it." (Persson and Tabellini, 1999, p. 58).

- Decentralized spending or "soft budget constraint"

Another possibility to consider is that each district decides individually the supply of the public good about which it cares, while the tax rate is residually determined. This is a case of soft budget constraint or decentralized spending with linear income taxation. Notice that decentralized spending entails a non-cooperative Nash equilibrium. In fact, each region maximises its own utility subject to the resource constraint and taking equilibrium expenditure by all other regions as given.

So, the equilibrium spending for each district satisfies

$$\alpha_i H'(g_i) = \frac{1}{n} \tag{9}$$

As the R.H.S of (9) is smaller than one, all districts overspend compared to the social optimum expressed in (8). Each region fully internalizes the benefit of the local public good, but, as taxes are shared, it internalizes only the fraction $1/n$ of the social marginal cost of higher taxes. Thus, concentration of benefits and dispersion of costs lead to excessive spending compared to the social optimum.

3 The legislative bargaining model

As already stated, in the case of distributive politics, a national legislature uses a common pool of tax revenues to finance some local public goods whose benefits are completely concentrated to well defined districts. The predictions of the above distributive politics model are challenging: the redistributive nature of these policies implies that with majority rule there is no voting equilibrium and Condorcet cycles unavoidably result, unless the voting agenda is restricted in some way. It is well known that when the space of alternative is multidimensional (as the distributive politics implies) there is no Condorcet winner or there is no voting equilibrium[11]. However, in a seminal work, Baron and Ferejohn (1989) show that any distribution of benefits among members of the legislature can be supported as a majority voting equilibrium if the sequential nature of the policymaking process is explicitly considered and only if "punishment strategies" are allowed.

The model they propose differs to some extent from traditional bargaining models[12] and they propose instead a "legislative choice model".

It is useful to summarize their framework:

(1) The legislature consists of n members, each of whom represents a legislative district. The task assigned to the members is to choose a nonnegative distribution of one unit of benefit among the districts according to majority rule.

(2) A representative is randomly selected to be the agenda setter. Thus each representative has probability $\frac{1}{n}$ of being selected as the agenda setter.

(3) The agenda setter proposes the amount of benefit to be offered to each district. Formally, a proposal by representative i is a distribution $x^i = (x^i_1, ..., x^i_n)$ such that $\sum\limits_{j=1}^{n} x^i_j \leqslant 1$.

All the representatives simultaneously vote for or against a proposal. If a proposal receives the support of a strict majority of the representatives (minimum winning coalition), it is implemented. Otherwise the default outcome of zero public spending prevails and the legislature moves to the next session where again one representative is randomly selected to be the agenda setter (i.e. closed rule)[13].

[11] See appendix A for an example.

[12] Denoted in Baron and Ferejohn's terminology as "bilateral exchange models". Baron and Ferejohn explain: "Legislative choice differs significantly from bilateral exchange in several respects. First, bilateral exchange requires unanimous consent for an outcome, and this requirement gives each party veto power that is reflected in the equilibrium outcomes. In a majority rule legislature, no member possesses veto power. Second, in bilateral bargaining, if agents are identical and make alternating offers, equilibrium distributions approach equality as impatience diminishes, as Rubinstein (1982) showed. In the legislature model considered here, majority rule equilibria do not generally tend to equal distributions, and equilibria in which some members receive nothing may occur even as impatience goes to zero" (p.1182).

[13] Note that in the paper, Baron and Ferejohn analyse furtermore equilibria in the legislature under "open rule", that is, allowing amendment to the proposal on the floor by another member of the legislature. If the amendment obtains a majority, the amendment becomes the

Baron and Ferejohn show that at the proposal stage, each representative selects a minimum winning coalition of representatives to support the proposal she makes. The proposal provides benefits only to the districts associated with representatives belonging to the minimum winning coalition. The benefit levels chosen by the agenda setter is such that each representative in the minimum winning coalition (with the exception of the proposer) is provided with just enough utility to induce her to support the proposal. If a member recognized by the proposal to be in the minimum winning coalition fails to vote for the proposal, that member runs the risk that in the next session a proposal could be passed allocating no benefits to her district; therefore: "this provides an incentive for the member to vote for the proposal on the floor if it provides an allocation to the member's district at least as great as can be expected from future legislative sessions" (p.1185). Thus, time preference of members plays a crucial role in legislative behavior.[14]

To see that, suppose (as in Baron and Ferejohn) that members in the legislature have a common discount factor $\delta < 1$ while the preferences of member j are represented by the utility function $u^j(x^k, t) = \delta^t x_j^k$ where t is the session in which the legislature adopts the distribution x^k (p. 1186).

Assume, without loss of generality, that there are three representatives and a two-session legislature. The agenda setter always obtains a majority vote in the first session by proposing to receive $1 - \delta/3$ and to offer $\delta/3$ to one of the other two members. As the game ends after the second session, the member recognized to be agenda setter at the beginning of session 2 proposes to take all the benefits and obtain a majority. In fact, in session 2 members will vote for any of the proposed allocation is at least as great as the member's continuation value[15] (i.e. zero). As each member has probability 1/3 to be recognized as agenda setter in the second and last session, in session one any member of legislature accepts the agenda setter's proposal if it grants her at least $\delta(\frac{1}{3})$. On the other hand competition among members to enter into the minimum winning coalition entails that each member is willing to accept a proposal which grants her no more than $\delta(\frac{1}{3})$. This establishes the equilibrium outcome $(1-\frac{\delta}{3}; \frac{\delta}{3})$. Note that if all the three members have the same discount factor the model does not predict the identity of the minimum winning coalition members besides the agenda setter. Whereas, if the discount factor differs across members, the agenda setter will propose to share the benefit with the member with the lowest δ.

To sum up, the main result of the model is that, under majority voting, a minimum winning coalition is formed and therefore a proposal implemented. The possibility of Condorcet cycles is thus avoided. Finally, it should be noted that minimum winning coalitions will include those representatives whose votes

new proposal on the floor. To the aim of our work we restrict our attention to a closed rule.

[14] The time preference parameter may represent also the probability that each member has to return in office at the next election.

[15] Formally, in the paper the continuation value is defined as $\delta v_i(t, g)$ which is the value if the legislature moves to subgame g after t session. Obviously, since the game ends after the second session the continuation value for each player for all subgames g after the second session is equal to zero.

are cheapest to buy off: in the context above, the minimum winning coalition would be composed by the members in the legislature with the highest degree of impatience.

Baron and Ferejohn's seminal political model has been applied in a number of economic contexts. Persson's contribution (1998) is a "one-shot version" of Baron and Ferejohn' s model. In Persson's model, the legislative bargaining approach developed in Baron and Ferejohn is applied to a national legislature composed by representatives of different districts making a decision on how to allocate local public goods among districts.

The analytical framework in Persson model is the same as expressed in Section 2 and more specifically it refers to the linear income tax case. So in Persson model the policy vector $\mathbf{g} \equiv (g_{i,\dots}g_n)$ of local public goods and the national lump sum tax t has to be approved by a majority of the districts according to following "budget process" or sequence of events:

a) One of the representatives i is randomly chosen to be the agenda setter, denoted a.

b) The agenda setter a makes a policy proposal on the amount of public goods to be offered (in other words the agenda setter chooses the policy vector $\mathbf{g}, t$) and has to gain the support of at least $n/2$ other representatives in order to implement the proposal. The supply of public good is then financed through a linear income tax. If the proposal is not accepted the status quo of zero public goods prevails. The status quo outcome is denoted as $(g_i) = t = 0$.

The utility each representative i derives from the local public good consumed in her districts, is:

$$U_i(g) = y + \alpha_i H(g_i) - (\sum_i g_i)/n \tag{10}$$

The agenda setter anticipates that only proposals which give a representative no less than the status quo will win her approval. Thus the "incentive compatibility constraint" for each representative is expressed as:

$$U_i(g) - U_i(g_s) \equiv \alpha_i H(g_i) - (\sum_i g_i)/n \geq 0 \tag{11}$$

The agenda setter maximises (10) subject to (11) holding for a majority coalition M including at least $n/2$ other legislators, and subject to the non negativity constraints that $g_i \geq 0$ for all i. The solution to this problem , in Persson analysis, is expressed by the following conditions:

$$\alpha_a H'_g(g_a) = \frac{1}{n - \sum_{i \in M} \frac{1}{\alpha_i H'_g(g_i)}} \tag{12}$$

$$
\begin{aligned}
\alpha_i H(g_i) &= (g_a + \sum_{i \in M} g_i)/n = t, \quad i \in M \qquad (13) \\
|M| &= n/2 \\
g^i &= 0, \qquad i \notin M.
\end{aligned}
$$

4 Legislative bargaining and secession

To sum up, in the framework adopted so far, representatives of different districts in the central legislature form coalitions and decide policies on local public goods supply (financed through national taxation) subject only to the constraint that such policies have to receive the majority's support in the unicameral legislature.

Two strong implications from this decision making model, as clearly stressed by Persson's analysis (1998), are:

1. the agenda setter obtains rent from other members in the legislature;
2. representatives with the highest preference for the local public goods are always in the minimum winning coalitions since they are the cheapest to buy off for the agenda setter.

The above consideration raises the question whether the very nature of the legislature decision making process on local public goods constitutes an incentive for districts or regions to leave the country. As each district stands only one chance over n of being the agenda setter, the question is: how likely is it that a union of districts or regions governed by the rule analysed in Persson's model will survive or even be formed?

So, in the following, we address the issue by investigating first (section 5) under which conditions a district that anticipates the outcome of the legislative bargaining as described in Persson (1998) will join other districts in forming the legislature. Next (section 6) we ask whether the threat of secession by any district in the unicameral legislature modifies the agenda setter's policy proposal and hence the outcome of the legislative bargaining game, so affecting in turn the choice of whether or not joining the legislature.

To address these questions we adopt the following framework or sequence of events:[16]

- In period 0, absent ex-ante side payment, each district i decides whether or not to join other districts in the legislature. If a district opts for not to join, we refer to this as *ex ante secession.*
- In period 1, legislative bargaining takes place.

[16] For simplicity we ignore a discounting factor between periods.

- In period 2, each district i decides whether to accept the outcome of the legislative bargaining or rather abandon the legislature. If the latter occurs, we refer to it as *ex post secession.*

In addition, to complete the description of the "economic environment" we assume that:

a) Each district decides to secede if the utility of opting out from the legislature is greater than the utility of "staying in". We assume that if a district secedes, it forms an independent political unit and the local public good provision is efficient. Nevertheless it bears a finite, positive cost from being independent or standing alone. We denote such cost by k. Notice that models on secession assume that there are some "efficiency losses" associated with secession. These efficiency losses may be due, for instance, to a potential reduction in international trade, or to the cost of providing a new defence system or a new legal system. Stated in more general terms k can be thought of a "federal public good" or, in other words, a benefit for a region arising from being member of the union. Thus, the net benefit for region i from standing alone is denoted as

$$R(\alpha_i) \equiv y + \max_{g} \{\alpha_i H(g_i) - g_i\} - k \tag{14}$$

We do not restrict $R(\alpha_i)$ to be non negative. Obviously, for each district, the utility of "staying in" depends on the outcome of the legislative bargaining game.

c) Also note that whereas Persson's model confines itself to discuss the outcome of a legislative making process when local public goods provision is financed through linear income taxation, we consider, as already stated, also the possibility of lump sum district specific taxation.

Therefore, to explore the issues already mentioned, in the rest of the paper we consider in turn:

i) how the decision making process influences ex ante districts' decision to participate to the legislative game when ex post peaceful secession is not possible (section 5).

ii) next we assume that ex post secession is a viable option and we investigate the outcomes of legislative bargaining in this context (section 6).

5 Join or not to join: legislative bargaining and ex ante secession

In this section we consider that no region can threat to secede ex post, i.e. after the deliberation in the unicameral legislature on the local public good provision and financing. At this stage we are simply saying that ex post secession can be infinitely costly due for instance to a potential conflict resolution.[17] It follows that, if this is the case, then each district has the option to secede only ex-ante.

However, for a district to decide to not secede ex-ante, it must be the case that the expected utility from the legislative bargaining game is greater than the utility of standing alone. Recall that we have already defined the utility of standing alone as $R(\alpha_i) \equiv y + \max_g \{\alpha_i H(g_i) - g_i\} - k$.

We will explore the argument of ex ante secession evaluating the expected utility from the legislative bargaining game for each region's representative when local public goods are financed through district specific lump sum taxes, namely τ_i (section 5.1) as well as through linear income taxation, t (section 5.2).

To explore such cases we need to introduce the notion of a *stable coalition of regions*. We define that a coalition or union of regions (i.e. the country) denoted by S is stable if nobody wants to leave or join it[18]. It is worth recalling that for regions in S, the local public goods are centrally provided through legislative bargaining, whereas each region outside the country can choose its own level of the public goods, but, as already said, it bears some finite, positive fixed costs k.Then, formally let $EU_i(S)$ be the expected utility (to be calculated explicitly below) that region i gets in a coalition with other regions in S.

It is possible to say that S is a *stable coalition of regions* if

$$\begin{aligned} EU_i(S) &\geq R(\alpha_i), \quad i \in S \\ R(\alpha_i) &\geq EU(S \cup \{i\}), \quad i \notin S \end{aligned}$$

5.1 Ex ante secession with lump sum district specific taxes

To look at the ex ante secession with lump sum district specific taxes we need firstly to analyse the outcomes of the legislative bargaining game in such context.

For the sake of simplicity and without loss of generality, suppose $n = 3$. Thus, the legislature consists of three representatives, each characterized by a different α_i, namely the preference parameter for the local public good.

Let us also call:

- a the representative randomly chosen to be the agenda setter
- cp the coalition partner of a in the minimum winning coalition
- ncp the representative not included in the minimum winning coalition

We assume that districts excluded from the minimum winning coalition contribute to the provision of public goods to other districts at most by an amount

[17] Notice that this assumption shapes the outcome of the legislative bargaining game as the agenda setter's behaviour is constrained only by incentive compatibility, i.e. any proposal which gives to $n/2$ regions a benefit larger or equal to zero (the status quo) is approved independently of the net benefit the same regions could reap by standing alone.

[18] This notion of "stability" we employ is quite standard in the literature on this issue; see Alesina and Spolaore (1997).

equal to z. In other words, we impose an "upper bound" on the amount of resources the agenda setter can extract from the non-coalition partner.

In this setting, the policy proposal by the agenda setter a solves

$$\max_{g_a, g_{cp}, g_{ncp}, \tau_a, \tau_{cp}, \tau_{ncp},} y + \alpha_a H(g_a) - \tau_a \tag{15}$$

subject to the feasibility constraint

$$g_a + g_{cp} + g_{ncp} - \tau_a - \tau_{cp} - \tau_{ncp} = 0 \tag{16}$$

and the incentive compatibility constraints respectively for the coalition partner (i.e *cp*) and the non-coalition partner (i.e. *ncp*)

$$\begin{aligned} y + \alpha_{cp} H(g_{cp}) - \tau_{cp} &\geqslant y && (17) \\ y + \alpha_{ncp} H(g_{ncp}) - \tau_{ncp} &\geqslant y - z && (18) \end{aligned}$$

where τ_i, as already said, denotes the district specific lump sum tax. The F.O.C. for the above problem are

$$\begin{aligned} -1 + \lambda &= 0 && (19) \\ \lambda - \mu_{cp} &= 0 \\ \lambda - \mu_{ncp} &= 0 \\ \alpha_a H'(g_a) - \lambda &= 0 \\ \mu_{cp} \alpha_{cp} H'(g_{cp}) - \lambda &= 0 \\ \mu_{ncp} \alpha_{ncp} H'(g_{ncp}) - \lambda &= 0 \end{aligned}$$

where μ_{cp} is the Lagrange multiplier associated with the incentive compatibility constraint for the coalition partner and μ_{ncp} is the Lagrange multiplier associated with the incentive compatibility constraint for the non coalition partner. From above we obtain

$$\alpha_a H'(g_a) = \alpha_{cp} H'(g_{cp}) = \alpha_{ncp} H'(g_{ncp}) = 1 \tag{20}$$

Denote $\tilde{g}_i$ the solution to (20), where $i = a, \quad cp, \quad ncp$. Note that (20) implies an efficient supply of local public goods – as a comparison with (8) makes immediately clear.

Note that from inspections of F.O.C conditions, given that $\mu_{cp} = \mu_{ncp} > 0$, constraints (17) and (18) are binding. In fact, if $\mu_{cp} = \mu_{ncp} = \lambda = 0$ then $H'(g_a) = 0$, implying therefore infinite taxes (as $g_a = \infty$) and then (17) and (18) would be violated. As (17) and (18) bind, we have

$$\begin{aligned} \tau_{cp} &= \alpha_{cp} H(\tilde{g}_{cp}) && (21) \\ \tau_{ncp} &= \alpha_{ncp} H(\tilde{g}_{ncp}) + z && (22) \end{aligned}$$

Substituting (21) and (22) into (16) we have

$$\tau_a = \tilde{g}_a + \tilde{g}_{cp} + \tilde{g}_{ncp} - \alpha_{cp} H(\tilde{g}_{cp}) - \alpha_{ncp} H(\tilde{g}_{ncp}) - z \tag{23}$$

Thus, the equilibrium payoff for the agenda setter a is[19]

$$\begin{aligned} U_a &= [\alpha_a H(\tilde{g}_a) - \tilde{g}_a] + [\alpha_{cp} H(\tilde{g}_{cp}) - \tilde{g}_{cp}] \\ &\quad + [\alpha_{ncp} H(\tilde{g}_{ncp}) - \tilde{g}_{ncp}] + z \end{aligned} \tag{24}$$

from (24) we see that the agenda setter is indifferent about the identity of the coalition partner. This result then is in contrast with Persson's model. In other words, in the case in which the agenda setter makes a policy proposal on the amount of public goods provision, financed through linear income taxation, the coalition partner's identity is always defined. In contrast, under district specific taxes, the agenda setter is indifferent about the coalition partner: the agenda setter expropriates completely both the coalition and the non-coalition partner. Moreover local public goods provision is efficient.

So, as the agenda setter under this setting is indifferent about the identity of the coalition partner, to break the tie, we postulate that any agenda setter tosses a coin to select the coalition partner. It follows that representative i's expected utility from being a member of the unicameral legislature and therefore from participating to the bargaining legislative game is:

$$EU_i = \frac{1}{3}[U_a] + \frac{1}{3}[0] + \frac{1}{3}[-z] \tag{25}$$

where the second term on R.H.S. of (25) applies when i is a coalition partner and the third term when he is not a coalition partner. It is worth noting that the expected utility for each district i from the legislative bargaining game is independent of who is going to be the agenda setter and is the same across districts.

At this point let us consider a district with a high preference parameter for the public good, denoted by α_h, a district with a medium preference, α_m, and with a low preference, α_l (so $i = h, m, l$) and $S = (h, m, l)$ then by substituting (24) into (25), we obtain

$$EU_i(S) = \frac{1}{3}\{[\alpha_h H(\tilde{g}_h) - \tilde{g}_h] + [\alpha_m H(\tilde{g}_m) - \tilde{g}_m] + [\alpha_l H(\tilde{g}_l) - \tilde{g}_l]\} \tag{26}$$

Before considering each district's choice whether to secede or not, we shall recall the payoff obtained by each district in the event of being alone:

$$R(\alpha_i) \equiv \max_g \{\alpha_i H(g_i) - g_i\} - k \quad i = h, m, l \tag{27}$$

We are now in a position to check whether given the outcome of the bargaining game, expressed by (25), a district or region will join or will not join the country. To address this issue observe first that region i joins the country only if

[19] Notice also that the equilibrium payoff for the coalition partner *cp* is $U_{cp} = y$, and for the non-coalition partner *ncp* is $U_{ncp} = y - z$. However, without loss of generality, in the analysis, henceforth we omit the constant y.

$$EU_i(S) \geq R(\alpha_i), \qquad i \in S \tag{28}$$

We assume that if indifferent, region i joins the union and let us call k_i^* the value of k such that $EU_i(S) = R(\alpha_i)$ for any i. In other words, let us call k_i^* the threshold value of union benefit such that region i joins the country. So from (28)

$$k_i^* = \max_g \{\alpha_i H(g_i) - g_i\} - EU_i(S) \tag{29}$$

and we know that $\alpha_h > \alpha_m > \alpha_l$, then $R(\alpha_h) > R(\alpha_m) > R(\alpha_l)$ whereas $EU_h(S) = EU_m(S) = EU_l(S)$. Hence from (29) we can see that $k_h^* > k_m^* > k_l^*$. This implies, as will be clear below, that if region h joins the union, it follows that also regions m and l will do so. So, region h is pivotal in the determining the union formation and it joins the country only if the efficiency gain from being a union's member exceeds the costs that the decision making process imposes on it. So, more generally k_i^* is increasing in the preference parameter for the public good (i.e. α_i) and notice that if we consider $i = h, m, l$ it is possible to show that $k_h^* > 0$, $k_m^* \lesseqgtr 0$ depending on the preference parameters values, whereas $k_l^* < 0$. We wish to point out the argument that a decrease in the dispersion of the α_i 's, increases the possibility of union's formation[20]. In other words, if regions are more heterogeneous in terms of their preference parameter for the public good a higher level of k (or union benefit) is necessary to establish the union ex ante. In fact, let us write explicitly the threshold value of union benefit for h (i.e. k_h^*) such that h will join the country. We know from (28) that region h will join the country whenever

$$\frac{1}{3}\{[\alpha_h H(\tilde{g}_h) - \tilde{g}_h] + [\alpha_m H(\tilde{g}_m) - \tilde{g}_m] + [\alpha_l H(\tilde{g}_l) - \tilde{g}_l]\} \geq \{[\alpha_h H(\tilde{g}_h) - \tilde{g}_h] - k\} \tag{30}$$

and from (30) it is possible to write

$$\frac{1}{3}\{[\alpha_h H(\tilde{g}_h) - \tilde{g}_h] - [\alpha_m H(\tilde{g}_m) - \tilde{g}_m]\} +$$

$$+\frac{1}{3}\{[\alpha_h H(\tilde{g}_h) - \tilde{g}_h] - [\alpha_l H(\tilde{g}_l) - \tilde{g}_l]\} = k_h^* \leq k \tag{31}$$

To sum up (31) describe also the threshold level of "federal public good" such that the unicameral legislature is composed by all three representatives and illustrates that a decrease in the dispersion of the $\alpha_i's$ increases the possibility of union's formation.

5.2 Ex ante secession and the linear income taxation case

In what follows we turn to the case already discussed in which, as in Persson's model (1998), local public goods are financed "by an economy-wide pool of

[20] Or, stated differently, a lower level of k (i.e. the union's benefit) is required.

equal contributions from each groups" (p. 36). We have seen thet in such case the agenda setter always lists among the coalition partners the regions with the highest preference parameter for the public good, the reason being that they are the least demanding in terms of amount of public good required to cast their vote in favor of the agenda setter's proposal. The model predicts that only coalition members receive a positive amount of public goods, whereas members and non-members alike contribute to the finance of the public goods provision. Before considering in more detail the expected utilities for representatives in the linear income taxation case it is useful to clarify the argument by means of the following example.

Let us assume again that $i = h, m, l$, and $S = (h, m, l)$ and that district h has been randomly selected as the agenda setter. Then, let us present the maximization problem whereby h selects the coalition partner.

Suppose h chooses l as coalition partner; he has to choose g_h, g_m, g_l to maximize his own utility[21]:

$$U_h = \alpha_h H(g_h) - (g_h + g_m + g_l)/3 \tag{32}$$

subject to the incentive compatibility constraint for representative l

$$U_l = \alpha_l H(g_h) - (g_h + +g_m + g_l)/3 \geqslant 0 \tag{33}$$

and g_h, g_m, $g_l \geqslant 0$. Let us call P_l this maximization problem faced by h.

Or, alternatively, he may decide to chose m as coalition partner. In this case, h has to maximize

$$U_h = \alpha_h H(g_h) - (g_h + g_m + g_l)/3 \tag{34}$$

subject to the incentive compatibility constraint for representative m

$$U_m = \alpha_m H(g_m) - (g_h + g_m + g_l)/3 \geqslant 0 \tag{35}$$

and g_h, g_m, $g_l \geqslant 0$. Let us call the latter maximization problem P_m. It is possible to show that h will always choose m as coalition partner. In fact, let $(\tilde{g}_h, \tilde{g}_l)$ be the solution to P_l. We first show that $(\tilde{g}_h, \tilde{g}_l - \varepsilon)$ is a feasible solution to P_m for ε small enough. Note that $(\tilde{g}_h, \tilde{g}_l)$ must satisfy the incentive compatibility constraint in P_l with equality

$$0 = \alpha_l H(\tilde{g}_l) - (\tilde{g}_h + \tilde{g}_l)/3 \tag{36}$$

Also as $\alpha_m > \alpha_l$ and for ε small enough

$$\alpha_l H(\tilde{g}_l) - (\tilde{g}_h + \tilde{g}_l)/3 < \alpha_m H(\tilde{g}_l - \varepsilon) - (\tilde{g}_h + (\tilde{g}_l - \varepsilon))/3 \tag{37}$$

So, from (36) and (37) we have

$$0 < \alpha_m H(\tilde{g}_l - \varepsilon) - (\tilde{g}_h + (\tilde{g}_l - \varepsilon))/3$$

[21] As in the previous section, we omit the constant of y.

i.e. $\tilde{g}_l - \varepsilon$, $\tilde{g}_h$ is feasible in P_m. In other words, to solve P_m, representative h has to offer to m a smaller amount of public good (which is $\tilde{g}_l - \varepsilon$) compared to the amount that he should offer to l (which is $\tilde{g}_l$) in order to satisfy P_l.

It follows also that the tax paid by each representative is less in P_m, rather than in P_l as $(\tilde{g}_h + \tilde{g}_l)/3 > (\tilde{g}_h + (\tilde{g}_l - \varepsilon))/3$.

So, representative h will choose m as coalition partner: for h it is easier to satisfy the incentive compatibility constraint for m rather than for l.

By the same argument, it is possible to show that if m is randomly chosen as the agenda setter she will choose h as coalition partner, and if l is randomly chosen as the agenda setter she will choose also h as coalition partner.

We have verified that the agenda setter will choose as partner for the minimum winning coalition the representative with the highest values of α_i since she is the cheapest to buy off. Thus in a three district framework, the representative with the lowest preference parameter, with probability 1/3 obtains a positive amount of local public good and with probability 2/3 she pays to finance the public goods provided to districts m and h. Besides, as we will see shortly, district h gets the higher level of expected utility compared to other districts from being member of the legislature and therefore from participating in the bargaining game[22].This raises a natural question, would region l join a country when public good provision is financed through linear income taxation? In more general terms, would the union $S = (h, m, l)$ be stable under the assumption of linear income taxation?

To address this issue, we have to compute the expected utility from the bargaining game for each region and compare it with the utility from standing alone. Thus, let us denote by g_i^a the amount of public good the agenda setter a's proposal specifies for region i.The expected utilities for each district $i = h, m, l$ can be written as follows:

$$EU_h(S) = \frac{1}{3}\left[\alpha_h H(g_h^h) - (g_h^h + g_m^h)/3\right] + \frac{1}{3}\left[0\right] + \frac{1}{3}\left[0\right] \tag{38}$$

$$EU_m(S) = \left[\alpha_m H(g_m^m) - (g_m^m + g_h^m)/3\right] + \frac{1}{3}\left[0\right] + \frac{1}{3}\left[-(g_l^l + g_h^l)/3\right] \tag{39}$$

$$EU_l(S) = \frac{1}{3}\left[\alpha_l H(g_l^l) - (g_l^l + g_h^l)/3\right] + \frac{1}{3}\left[-(g_h^h + g_m^h)/3\right] + \frac{1}{3}\left[-(g_m^m + g_h^m)/3\right] \tag{40}$$

As in the previous section, each region i will join the union if and only if

$$EU_i(S) \geq R(\alpha_i), \quad i = h, m, l \tag{41}$$

Thus we can once again compute the threshold value of k (which in the case we are considering, namely the linear income taxation case, we denote as $\bar{k}_i^*$) which

[22] Whereas, as we have seen in the previous section in the case of lump sum specific taxes, the expected utility from the legislative bargaining game is the same for every district.

follows from expression below

$$EU_i(S) = R(\alpha_i), \quad i = h, m, l \tag{42}$$

Moreover, we are unable to establish whether $k_i^* \gtreqless \bar{k}_i^*$(i.e. under which taxation regime coalition formation is more likely).To see why this is so, observe that a non coalition partner is at least as well of under uniform taxation than under region specific taxes. As taxes are set equal across regions, the agenda setter may fail to expropriate completely the non-coalition partner (for instance it can be that $-\frac{1}{3}\sum_i g_i \geq -z$).

For the very same reason, the agenda setter is not better off under uniform taxes than under region specific ones. A third element plays a key role: the taxation regime affects a region's chances of being a coalition partner. In fact, region h is always a member of the minimum winning coalition under uniform taxation, whereas in the other case we have seen that, whenever is not the agenda setter, she has only 1/2 chances of being a coalition partner. In contrast, region l sees his chances of being a coalition partner decreased under the uniform tax regime.

Next, in order to look explicitly at the expected utility for each representative we now make the assumption that region i utility function is linear, namely

$$H(g_i) = \begin{cases} g_i & \text{if } 0 \leq g_i \leq 1 \\ 1 & \text{if } g_i > 1 \end{cases} \tag{43}$$

As for the preference parameters we have already assumed that $\alpha_h > \alpha_m > \alpha_l$ and we now make two further assumptions:

A1: $\alpha_h > \alpha_m > \alpha_l > 1$

A2: $\alpha_l > \dfrac{\alpha_h}{2}$

If h is the agenda setter we have already shown that in this case representative h will choose representative m as coalition partner. In particular, the agenda setter h has to choose g_h^h, g_m^h, g_l^h to maximize his own utility

$$U_h = \alpha_h H(g_h^h) - (g_h^h + g_m^h + g_l^h)/3$$

subject to the incentive compatibility constraint for district m, which is

$$\alpha_m H(g_m^h) - (g_h^h + g_m^h + g_l^h)/3 \geq 0$$

and we look for the following solutions $0 \leq g_l^h, g_m^h, g_h^h \leq 1$ to the above problem; thus F.O.C are

$$\alpha_h - 1/3 - 1/3\mu_m \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \quad \text{as} \quad g_h^h \left\{ \begin{array}{c} = 1 \\ \in [0,1] \\ = 0 \end{array} \right. \tag{44}$$

$$-1/3 + \mu_m(\alpha_m - 1/3) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \quad \text{as} \quad g_m^h \left\{ \begin{array}{c} = 1 \\ \in [0,1] \\ = 0 \end{array} \right. \tag{45}$$

where μ_m is the Lagrange multiplier associated with the incentive compatibility constraint for district m. We look for a solution where $g_h^h = 1$ and $g_m^h \in [0,1]$. In fact, from (45) it is easy to see that

$$\mu_m = \frac{1}{3\alpha_m - 1} \tag{46}$$

which is greater than zero. So by substituting (46) into (44);

$$\alpha_h - 1/3 - 1/3\mu_m = \alpha_h - \frac{\alpha_m}{3\alpha_m - 1} \tag{47}$$

Now under assumptions A1 and A2 we know that $\alpha_h > \frac{\alpha_m}{3\alpha_m - 1}$; so from (47) $\alpha_h - 1/3 - 1/3\mu_m > 0$; and from (44), we see that $g_h^h = 1$. To find g_m^h, we know from the incentive compatibility constraint that

$$\alpha_m g_m^h - \frac{1}{3}(g_h^h + g_m^h + g_l^h) = 0 \tag{48}$$

and as $g_h^h = 1$ and $g_l^h = 0$ thus the incentive compatibility constraint implies a solution for g_m^h. In fact, $g_m^h = \frac{1}{3\alpha_m - 1}$. It is easy to see that $g_m^h \in [0,1]$.

By using the same argument we now are able to show the solutions to the bargaining game when m is the agenda setter and when l is the agenda setter[23].

So when m is the agenda setter she will chose h as coalition partner and solution to the bargaining game are: $g_m^m = 1$ and $g_l^m = 0$ it follows $g_h^m = \frac{1}{3\alpha_h - 1}$; observe that $g_h^m \in [0,1]$.

Finally, to consider solutions when l is the agenda setter, recall that representative l also will choose h as coalition partner. Thus solutions are: $g_l^l = 1$ and $g_m^l = 0$ it follows $g_h^l = \frac{1}{3a_h - 1}$; also $g_h^l \in [0,1]$.

At this stage by substituting the above solutions of the bargaining game into (38), (39) and (40) the expected utility for each district can be expressed as follows:

$$\begin{aligned} EU_h(S) &= \frac{\alpha_h + \alpha_m - 3\alpha_h\alpha_m}{3 - 9\alpha_m} \\ EU_m(S) &= \frac{2\alpha_h + \alpha_m - 3\alpha_h\alpha_m}{3 - 9\alpha_h} \\ EU_l(S) &= \frac{\alpha_l + \alpha_m - 3\alpha_l\alpha_m + \alpha_h(2 - 9\alpha_m + 3\alpha_l(3\alpha_m - 1))}{3(3\alpha_h - 1)(3\alpha_m - 1)} \end{aligned} \tag{49}$$

Note that in contrast to the case of lump sum district specific tax, now the expected utility is different across districts.

[23] We omit the maximization problem when m is the agenda setter and when l is the agenda setter as obviously the problem is solved using the same argument when h is the agenda setter.

Also, similarly to the case with the lump sum district specific taxes we have now to check under what circumstances, given regions $i = h, m, l$, the union $S = (h, m, l)$ will be established ex-ante. This event occurs if

$$EU_i(S) \geq R(\alpha_i), \quad i \in S \tag{50}$$

Recall that we have expressed $R(\alpha_i)$ as the net benefit for region i from standing alone, i.e.

$$R(\alpha_i) \equiv \max_g \{a_i H(g_i) - g_i\} - k, \quad i = h, m, l \tag{51}$$

which in the case of the assumed linear utility function such net benefit can be written as

$$R(\alpha_i) \equiv \max\{\alpha_i - 1, 0\} - k, \quad i = h, m, l \tag{52}$$

and assumption A1 must hold. Therefore from (50) we can identify for each region i the threshold value of union benefit k (namely $\bar{k}_i^*$) such that each region will join the union, that is

$$\bar{k}_i^* = \max\{\alpha_i - 1, 0\} - EU_i(S) \qquad i = h, m, l \tag{53}$$

So, recall the expected utility for each player expressed in (49). From (53) it follows that h will join ex ante the union whenever the union's benefit is such that

$$k \geq \bar{k}_h^* = \frac{3 - 2\alpha_h - 8\alpha_m + 6\alpha_h\alpha_m}{9\alpha_m - 3} \tag{54}$$

Region m will join whenever

$$k \geq \bar{k}_m^* = \frac{3 - 7\alpha_h - 2a_m + 6\alpha_h\alpha_m}{9\alpha_h - 3} \tag{55}$$

For region l

$$k \geq \bar{k}_l^* = \frac{\alpha_l(2 - 6\alpha_m) - 3 + 8\alpha_m + \alpha_h(7 - 18\alpha_m + 6\alpha_l(3\alpha_m - 1)}{3(3\alpha_h - 1)(3\alpha_m - 1)} \tag{56}$$

At this stage, to establish a ranking of $\bar{k}_i^*$ and then to infer which region is pivotal in union formation - when linear income taxation case is considered - we make use of numerical simulations under assumptions A1 and A2.

The results of the numerical simulations are illustrated in the Tables which follow. Note that for the numerical simulation we have assumed for each region the following preference parameters for the local public good:

- $\bar{\alpha} + \varepsilon$ for district h;
- $\bar{\alpha}$ for district m;
- $\bar{\alpha} - \varepsilon$ for district l.

More specifically $\bar{\alpha}$ is the mean value for the three preference parameters[24] and ε is a dispersion parameter which measures the extent of heterogeneity among regions in terms of the preference parameter for the public good. So, from (54), (55) and (56) and considering the different values of $\bar{\alpha}$ as well as of ε the numerical simulations indicate the values of $\bar{k}_i^*$ listed in Tables 1, 2 and 3

Table 1

$\alpha = 2$	$\varepsilon = 0.001$	$\varepsilon = 0.125$	$\varepsilon = 0.25$	$\varepsilon = 0.5$
$\bar{k}_h^*$	0.467	0.55	0.633	0.8
$\bar{k}_m^*$	0.599	0.596	0.594	0.589
$\bar{k}_l^*$	0.732	0.646	0.560	0.389

Table 2

$\alpha = 3$	$\varepsilon = 0.001$	$\varepsilon = 0.125$	$\varepsilon = 0.25$	$\varepsilon = 0.5$
$\bar{k}_h^*$	1.125	1.208	1.291	1.458
$\bar{k}_m^*$	1.249	1.248	1.247	1.245
$\bar{k}_l^*$	1.374	1.290	1.205	1.037

[24] As it is clear in the Tables which follow we have considered three possible values of $\bar{\alpha} = 2, 3, 4$.

Table 3

$\alpha = 4$	$\varepsilon = 0.001$	$\varepsilon = 0.125$	$\varepsilon = 0.25$	$\varepsilon = 0.5$
$\bar{k}_h^*$	1.7885	1.871	1.954	2.121
$\bar{k}_m^*$	1.909	1.908	1.907	1.906
$\bar{k}_l^*$	2.029	1.946	1.862	1.694

From the results shown in the Tables we observe that as long as heterogeneity increases a higher level of "union benefit" is necessary to establish the union.

On the other hand, if regions are more homogeneous district l rather than h is pivotal in establishing the union. This is in strong contrast with the case of lump sum district specific taxes where always region h is pivotal in establishing the union[25].

To shed further light on what has already been asserted we can consider again the lump sum district specific taxes under the assumption of the linear utility function (43). Recall that the expected utility for each player i in the lump sum case is written as

$$EU_i(S) = \frac{1}{3}\left\{[\alpha_h H(\tilde{g}_h) - \tilde{g}_h] + [\alpha_m H(\tilde{g}_m) - \tilde{g}_m] + [\alpha_l H(\tilde{g}_l) - \tilde{g}_l]\right\} \qquad i = h, m, l$$

It is easily understood that under the assumption of the linear utility function and under assumption A1, the expression above can be written as

$$EU_i(S) = \frac{1}{3}[\alpha_h + \alpha_m + \alpha_l - 3] \tag{57}$$

Let us call $\bar{\alpha} = (\alpha_h + \alpha_m + \alpha_l)/3$; where obviously $\bar{\alpha}$ is the mean value of the preference parameters for the public good. So (57) becomes

$$EU_i(S) = \bar{\alpha} - 1 \tag{58}$$

We know also the with the linear utility function, the benefit from being alone is

$$R(\alpha_i) \equiv \max\{\alpha_i - 1, 0\} - k, \quad i = h, m, l; \tag{59}$$

We can identify at this stage the threshold value of k for each region such that it will join the union and that in the case of lump sum taxes we denoted as k_i^*. So from (58) and (59):

$$\begin{aligned} k_i^* &= \max\{\alpha_i - 1, 0\} - (\bar{\alpha} - 1); \qquad i = h, m, l \\ &= \alpha_i - \bar{\alpha} \end{aligned} \tag{60}$$

[25] Recall previous section.

Furthermore, recall that for the above numerical simulations we assumed the preference parameters listed as follows:

$$\begin{aligned} \alpha_h &= \bar{\alpha} + \varepsilon \\ \alpha_m &= \bar{\alpha} \\ \alpha_l &= \bar{\alpha} - \varepsilon \end{aligned} \tag{61}$$

From (60) and (61) it turns out that

$$\begin{aligned} k_h^* &= \varepsilon \\ k_m^* &= 0 \\ k_l^* &= -\varepsilon \end{aligned} \tag{62}$$

So from (62) we observe again that with lump sum taxes: i) region h is always pivotal in forming the union, ii) as long as regions are more heterogeneous a larger level of union benefit is necessary to establish the union.

Finally, under the assumption of the same utility function and the same values for the preference parameters, it is possible to observe that with lump sum taxes the union is a more likely event ($k_h^* < \bar{k}_h^*$).

6 To opt out or not to opt out: legislative bargaining and ex-post secession

In what follows we now extend Persson' s model considering also the possibility that the agenda setter takes into account the secession constraint for each district in the legislature. To be more clear on this point, recall that in the bargaining game described in Persson's model the agenda setter plays a non-cooperative game with the other regions selecting the policy proposal that maximises her utility under the incentive compatibility constraint holding for a majority of regions and ignoring ex-post secession constraints. In other words, in Persson'model, ex-post secession is not allowed since it would imply for instance infinite resource costs devoted to a "conflict resolution". We assume, at the other extreme, that ex post secession is a feasible option via a peaceful secession and then we examine the outcome of the bargaining game when the agenda setter solves her maximisation problem under incentive compatibility as well as non-secession constraints. So we postulate that representatives not included in the minimum winning coalition can credibly threat to leave the country.

This has important implications for the outcome of the bargaining game. If the agenda setter wishes to prevent the breaking up of the country, she has to satisfy the incentive compatibility constraint for representatives belonging to the minimum winning coalition, but also a participation constraint or non secession constraint for each district. This observation begs two questions: would any agenda setter be willing to prevent ex post secession? As the threat of ex post secession modifies the outcome of the bargaining game and thus expected utility from taking part into the unicameral legislature, are regions more or less inclined to join a union? We address these issues in the following sections. In this respect, we consider again first the lump sum district specific taxes case. Then, we turn to analyse the linear income taxation case.

6.1 Ex post secession with lump sum taxes

We already denoted as τ_i the lump-sum tax on region i and let us consider $i = h, m, l$. For the sake of clarity and without loss of generality, let us suppose that region h is the agenda setter[26]. The secession constraints for m and l can generally be written as

$$\alpha_m H(g_m) - \tau_m \geq \bar{u}_m \quad (63)$$
$$\alpha_l H(g_l) - \tau_l \geq \bar{u}_l \quad (64)$$

where $\bar{u}_m$ and $\bar{u}_l$ are some payoffs.

In particular if $i \in \{m, l\}$, $\bar{u}_i = \max\{0, R(\alpha_i)\}$ if i is a coalition partner of h, and $\bar{u}_i = R(\alpha_i)$ otherwise.[27] Thus, the agenda setter h solves the following problem

$$\max_{g_h, g_m, g_l, \tau_h, \tau_m, \tau_l} \alpha_h H(g_h) - \tau_h \quad (65)$$

subject to (63) and (64) and to the feasibility constraint

$$\tau_h + \tau_m + \tau_l - g_h - g_m - g_l = 0 \quad (66)$$

The F.O.C. for the above problem are

$$\begin{aligned} -1 + \lambda &= 0 \quad (67) \\ \lambda - \mu_m &= 0 \\ \lambda - \mu_l &= 0 \\ \alpha_h H'(g_h) - \lambda &= 0 \\ \mu_m \alpha_m H'(g_m) - \lambda &= 0 \\ \mu_l \alpha_l H'(g_l) - \lambda &= 0 \end{aligned}$$

where μ_m and μ_l are the Lagrange multipliers associated respectively with m and l. These first order conditions imply that:

$$\alpha_h H'(g_h) = \alpha_m H'(g_m) = \alpha_l H'(g_l) = 1 \quad (68)$$

[26] Again, note that, without loss of generality, in the following we omit the constant y.

[27] The intuitions for such payoffs are as follows: we have already said we do not restrict $R(\alpha_i)$ to be non negative. Suppose that $R(\alpha_m) > R(\alpha_m) > 0$; in this case h is indifferent about his coalition partner as long as both m and l, in order to stay in, want to receive $\bar{u}_i = [\alpha_i H(\tilde{g}_i) - \tilde{g}_i] - k \quad i = m, l$. However if $0 > R(\alpha_m) > R(\alpha_m)$, h strictly prefers m as coalition partner, in fact m is willing to take part in the union for $\bar{u}_i = 0$ and therefore h can extract the surplus of m which is greater than l (recall $a_m > \alpha_l$).

Note that (68) implies an efficient supply of local public good. Denote $\tilde{g}_i$ the efficient supply level. As (63) and (64) bind we have

$$\tau_m = \alpha_m H(\tilde{g}_m) - \bar{u}_m \quad (69)$$
$$\tau_l = \alpha_l H(\tilde{g}_l) - \bar{u}_l \quad (70)$$

and combining equations (69) and (70) with the feasibility constraint we see that

$$\tau_h = \tilde{g}_h + \tilde{g}_m + \tilde{g}_l - \alpha_m H(\tilde{g}_m) + \bar{u}_m - \alpha_l H(\tilde{g}_l) + \bar{u} \quad (71)$$

Then, we can write the equilibrium payoff for the agenda setter as

$$\tilde{U}_h = [\alpha_h H(\tilde{g}_h) - \tilde{g}_h] + [\alpha_m H(\tilde{g}_m) - \tilde{g}_m] + [\alpha_l H(\tilde{g}_l) - \tilde{g}_l] - [\bar{u}_m + \bar{u}_l] \quad (72)$$

From (72), we see that the agenda setter selects her coalition partner (cp) to minimise the sum $\bar{u}_m + \bar{u}_l$. Observe that

$$\bar{u}_m + \bar{u}_l = \max\{0, R(\alpha_m)\} + R(\alpha_l) \text{ if } m \text{ is a cp} \quad (73)$$
$$\bar{u}_m + \bar{u}_l = \max\{0, R(\alpha_l)\} + R(\alpha_m) \text{ if } l \text{ is a cp} \quad (74)$$

Thus we can list three cases:

1. If $R(\alpha_m) > R(\alpha_l) > 0$ the agenda setter h is indifferent about his coalition partner;

2. If $R(\alpha_m) > 0 > R(\alpha_l)$ the agenda setter strictly prefers m as coalition partner;

3. If $0 > R(\alpha_m) > R(\alpha_l)$ again the agenda setter strictly prefers m as his coalition partner.

Therefore:

Proposition 1 *If h is the agenda-setter she is either indifferent about her coalition partner or strictly prefers m. If m is the agenda-setter, she is either indifferent about her coalition partner or strictly prefers h as her partner. If l is the agenda-setter, she is either indifferent about her coalition partner or strictly prefers h as her coalition partner.*

In other words, when the "opt-out" option is available under region specific taxes, the agenda setter is no longer indifferent about her coalition partner.

We are now able to address the question whether any agenda setter will act so as to prevent the breaking of the union. Again, let us suppose that h is the agenda-setter, and $j \in \{m, l\}$ secedes.

It is possible to solve the problem exactly as before, except that there is only one other region in the country. Also with majority vote interpreted in the strict sense, this other region $i \neq j$ must prefer $h's$ proposal to the status quo. So, the payoff to h in this case is

$$\hat{U}_h = [a_h H(\tilde{g}_h) - \tilde{g}_h] + [\alpha_i H(\tilde{g}_i) - \tilde{g}_i] - \max\{R(\alpha_i), 0\} \tag{75}$$

and

$$\tilde{U}_h - \hat{U}_h = [\alpha_j H(\tilde{g}_j) - \tilde{g}_j] + \max\{R(\alpha_i), 0\} - [\bar{u}_m + \bar{u}_l] \tag{76}$$

Suppose l secedes, (i.e. $i = l$) then

$$\hat{U}_h = [a_h H(\tilde{g}_h) - \tilde{g}_h] + [\alpha_m H(\tilde{g}_m) - \tilde{g}_m] - \max\{R(\alpha_m), 0\} \tag{77}$$

Therefore the loss for h if l secedes can be written as

$$\tilde{U}_h - \hat{U}_h = [\alpha_l H(\tilde{g}_l) - \tilde{g}_l] + \max\{R(\alpha_m), 0\} - [\bar{u}_m + \bar{u}_l] \tag{78}$$

since $[\bar{u}_m + \bar{u}_l] = \max\{0, R(\alpha_m)\} + R(\alpha_l)$,

$$\tilde{U}_h - \hat{U}_h = k \tag{79}$$

By the same argument, if m secedes, the loss for the agenda setter is k. Therefore:

Proposition 2 *The agenda-setter (no matter her identity) never wishes any region to secede, and will therefore always set her agenda to respect the secession constraints.*

To sum up, we have established that when the threat of ex post secession by any region is credible, secession never takes place as the agenda setter will respect the secession constraints.

6.2 Ex post secession and the linear income taxation case

As far as the linear income taxation case is concerned, let us recall the analytical framework we are considering. We denoted as t the uniform tax to be paid by each district, so that $t = \frac{1}{3}\sum_i g_i$ and we have assumed the following linear utility function

$$H(g_i) = \begin{cases} g_i & \text{if } 0 \leq g_i \leq 1 \\ 1 & \text{if } g_i > 1 \end{cases}$$

Again let us consider the union $S = (h, m, l)$. As in the previous section we have firstly to consider the secession constraints for regions that we can write as follows:

$$\alpha_i H(g_i) - t \geq \bar{u}_i \qquad i = h, m, l \tag{80}$$

and $\bar{u}_i$ are the payoff or utilities associated with the event of staying out of the union.

So as before, we can write $\bar{u}_i = \max\{0, R(\alpha_i)\}$ if $i, j = h, m, l$ is a coalition partner of j (and $i \neq j$) and $\bar{u}_i = R(\alpha_i)$ otherwise. Note also (recall Section

5.2) that in the case of the assumed linear utility function the benefit for each region in the event of being alone is written as

$$R(\alpha_i) \equiv \max\{\alpha_i - 1, 0\} - k$$

and also assumptions A1 and A2 must hold.

Let us consider now that h is the agenda setter, but m and l can credibly threaten to leave the legislature. Under this setting the agenda setter h has to choose g_l^h, g_m^h, g_h^h to maximize his own utility[28]

$$U_h = \alpha_h H(g_h^h) - (g_l^h + g_m^h + g_h^h)/3 \tag{81}$$

subject to the non secession constraints (80). We look for the following solutions: $0 \leq g_l^h, g_m^h, g_h^h \leq 1$ to the above problem; F.O.C are

$$\alpha_h - \frac{1}{3} - \frac{1}{3}\mu_m - \frac{1}{3}\mu_l \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \quad \text{as } g_h^h \left\{ \begin{array}{c} = 1 \\ \in [0,1] \\ 0 \end{array} \right. \tag{82}$$

$$-\frac{1}{3} + \alpha_m \mu_m - \frac{1}{3}\mu_m - \frac{1}{3}\mu_l \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \quad \text{as } g_m^h \left\{ \begin{array}{c} = 1 \\ \in [0,1] \\ 0 \end{array} \right. \tag{83}$$

$$-\frac{1}{3} + \alpha_l \mu_l - \frac{1}{3}\mu_m - \frac{1}{3}\mu_l \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \quad \text{as } g_l^h \left\{ \begin{array}{c} = 1 \\ \in [0,1] \\ 0 \end{array} \right. \tag{84}$$

and μ_m and μ_l are the Lagrange multipliers associated with m and l. At this stage we make a further assumption

A3: $3\alpha_i a_j - \alpha_i - \alpha_j > 0 \qquad i, j = h, m, l, \qquad i \neq j$

We look for the following solutions: $g_h^h = 1$, $g_m^h \in [0,1]$, $g_l^h \in [0,1]$. From (83) and (84) it follows that the Lagrange multipliers are

$$\begin{aligned} \mu_m &= \frac{\alpha_l}{3\alpha_l\alpha_m - \alpha_l - \alpha_m} \\ \mu_l &= \frac{\alpha m}{3\alpha_l\alpha_m - \alpha_l - \alpha_m} \end{aligned}$$

which under assumption A3 are both greater than zero. By substituting them into (82):

$$\alpha_h - \frac{1}{3} - \frac{1}{3}\mu_m - \frac{1}{3}\mu_l = a_h - \frac{\alpha_l}{3(3\alpha_l\alpha_m - \alpha_l - \alpha_m)} - \frac{\alpha_m}{3(3\alpha_l\alpha_m - \alpha_l - \alpha_m)} \tag{85}$$

Now from A1-A3

$$a_h > \frac{\alpha_l}{3(3\alpha_l\alpha_m - \alpha_l - \alpha_m)} + \frac{\alpha_m}{3(3\alpha_l\alpha_m - \alpha_l - \alpha_m)}$$

[28] Also notice that we have already denoted as g_i^a the amount of public good the agenda setter a's proposal specifies for region i.

so from (85), $\alpha_h - \frac{1}{3} - \frac{1}{3}\mu_m - \frac{1}{3}\mu_l > 0$ and from (82) we see that $g_h^h = 1$. To get the solutions for g_m^h and g_l^h let us consider that $R(\alpha_m) > R(\alpha_l) > 0$ and we know that the non secession constraints must hold, so

$$\alpha_m g_m^h - \frac{1}{3}(g_h^h + g_m^h + g_l^h) = \bar{u}_m \tag{86}$$

$$\alpha_l g_l^h - \frac{1}{3}(g_h^h + g_m^h + g_l^h) = \bar{u}_l \tag{87}$$

As $g_h^h = 1$ it is possible to solve equations (86) and (87) and so we get the amount of local public good obtained by regions m and l when h is the agenda setter and h has to satisfy the non secession constraints, namely:

$$\begin{aligned} g_m^h &= \frac{\alpha_l + \alpha_m - 3\alpha_l\alpha_m + 3\alpha_l k}{\alpha_l + \alpha_m - 3\alpha_l\alpha_m} \\ g_l^h &= \frac{\alpha_l + \alpha_m - 3\alpha_l\alpha_m + 3\alpha_m k}{\alpha_l + \alpha_m - 3\alpha_l\alpha_m} \end{aligned} \tag{88}$$

Note that solution $g_m^h \in [0,1]$ requires the following restriction as for the union benefit k, namely

$$\frac{3\alpha_l\alpha_m - \alpha_l - \alpha_m}{3\alpha_l} \leq k \leq 0 \tag{89}$$

and considering the solution $g_l^h \in [0,1]$ the restriction on k is

$$\frac{3\alpha_l\alpha_m - \alpha_l - \alpha_m}{3\alpha_m} \leq k \leq 0 \tag{90}$$

Looking at the above solutions to the bargaining game with the opt out options (namely g_h^h, g_m^h, g_l^h), one might conjecture that if there is no benefit from the union (i.e. $k = 0$); every player should get one unit of local public good in order to stay in the union. The agenda setter h in this case does not extract any rent from other players: being alone or being in the union implies the same benefit for each player. However, as long as there is a positive benefit from the union, the agenda setter still can extract a rent from the other players. From the same level of k it follows $g_m^h \in [0,1]$ which is greater than $g_l^h \in [0,1]$.

Using the same argument it is possible to show solutions referring to the amount of local public goods chosen by the legislature when m is the agenda setter and satisfies the non secession constraints for both regions (i.e. h and l) and finally the solutions when l is the agenda setter.

So, when m is the agenda setter $g_m^m = 1$ and

$$\begin{aligned} g_h^m &= \frac{\alpha_h + \alpha_l - 3\alpha_h\alpha_l + 3\alpha_l k}{\alpha_h + \alpha_l - 3\alpha_h\alpha_l} \\ g_l^m &= \frac{\alpha_h + \alpha_l - 3\alpha_h\alpha_l + 3\alpha_h k}{\alpha_h + \alpha_l - 3\alpha_h\alpha_l} \end{aligned}$$

and the solution $g_h^m \in [0,1]$ requires the following restriction on k

$$\frac{3\alpha_h\alpha_l - \alpha_l - \alpha_h}{3\alpha_l} \leq k \leq 0$$

whereas for the solution $g_l^m \in [0,1]$; the restriction is

$$\frac{3\alpha_h\alpha_l - \alpha_l - \alpha_h}{3\alpha_h} \leq k \leq 0$$

Finally, when representative l is the agenda setter $g_l^l = 1$

$$\begin{aligned} g_m^l &= \frac{\alpha_h + \alpha_m - 3\alpha_h\alpha_m + 3\alpha_h k}{\alpha_h + \alpha_m - 3\alpha_h\alpha_m} \\ g_h^l &= \frac{\alpha_h + \alpha_m - 3\alpha_h\alpha_m + 3\alpha_m k}{\alpha_h + \alpha_m - 3\alpha_h\alpha_m} \end{aligned}$$

For the solution $g_m^l \in [0,1]$, the restriction on k is

$$\frac{3\alpha_h\alpha_m - \alpha_h - \alpha_m}{3\alpha_h} \leq k \leq 0$$

For the solution $g_h^l \in [0,1]$, the restriction on k is

$$\frac{3\alpha_h\alpha_m - \alpha_h - \alpha_m}{3\alpha_m} \leq k \leq 0$$

At this stage, having established the outcome of the bargaining game in the linear income taxation case when the opt-out options are considered we may ask whether the agenda setter will act to prevent the breaking up of the union. To address the issue, let us start by considering that h is the agenda setter and region l secedes. In this case the agenda setter has to choose $0 \leq g_m^h,\ g_h^h \leq 1$ to maximise his own utility (91) subject to the secession constraint by only one other region (92) namely m, and the problem can be written as follows

$$U_h = \alpha_h H(g_h^h) - (g_m^h + g_h^h)/2 \tag{91}$$

$$\alpha_m H(g_m) - (g_m^h + g_h^h)/2 \geq \bar{u}_m \tag{92}$$

We look for the solutions $g_h^h = 1$ and $g_m^h \in [0,1]$; F.O.C are

$$\alpha_h - \frac{1}{2} - \frac{1}{2}\mu_m \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \text{ as } g_h^h \left\{ \begin{array}{c} =1 \\ \in [0,1] \\ =0 \end{array} \right. \tag{93}$$

$$-\frac{1}{2} + \alpha_m\mu_m - \frac{1}{2}\mu_m \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \text{as } g_m^h \left\{ \begin{array}{c} =1 \\ \in [0,1] \\ =0 \end{array} \right. \tag{94}$$

and μ_m is the Lagrange multiplier for m. To look for the solutions $g_h^h = 1$ and $g_m^h \in [0,1]$ from (94);

$$\mu_m = \frac{1}{2\alpha_m - 1} \tag{95}$$

which is greater than zero under assumption A1. Substituting (95) into (93) we can write

$$\alpha_h - \frac{1}{2} - \frac{1}{2}\mu_m = \alpha_h - \frac{\alpha_m}{2\alpha_m - 1}$$

Under assumption A.1 we know that $\alpha_h > \frac{\alpha_m}{2\alpha_m - 1}$ so $\alpha_h - \frac{1}{2} - \frac{1}{2}\mu_m > 0$ and from (93) we see that $g_h^h = 1$. To find g_m^h, we know than the secession constraint must hold. Then by substituting $g_h^h = 1$ into (92) we see that

$$g_m^h = \frac{1 - 2\alpha_m + 2k}{1 - 2\alpha_m}$$

and the solution $g_m^h \in [0, 1]$ requires as a restriction on k

$$\alpha_m - \frac{1}{2} \leq k \leq 0$$

To investigate whether the agenda setter h will act to prevent secession, we have to compare the utility in the case in which the secession constraints for region m and l are respected (which we denote as $\hat{U}_h$) with the utility in the case in which l secedes (which we denote as $\tilde{U}_h$).

To compare the utilities in the two cases, let us consider the taxes to be paid in both cases. Firstly we know that the tax $t = (g_h^h + g_h^h + g_h^h)$. Consider the solutions to the bargaining game with the opt out option when h is the agenda setter (recall (88)). In this case it turns out that the tax to be paid by all districts is

$$t = \frac{\alpha_l + \alpha_m - 3\alpha_l\alpha_m + \alpha_l k + \alpha_m k}{\alpha_l + \alpha_m - 3\alpha_l\alpha_m} \tag{96}$$

Considering the solutions to the bargaining game when l secedes,[29] the tax (which we denote $\tilde{t}$) to be paid by district m and h is

$$\tilde{t} = \frac{1 - 2\alpha_m + k}{1 - 2\alpha_m} \tag{97}$$

Therefore we can write the utilities as follows

$$\hat{U}_h = \alpha_h H(g_h^h) - \frac{\alpha_l + \alpha_m - 3\alpha_l\alpha_m + \alpha_l k + \alpha_m k}{\alpha_l + \alpha_m - 3\alpha_l\alpha_m} \tag{98}$$

and

$$\tilde{U}_h = \alpha_h H(g_h^h) - \frac{1 - 2\alpha_m + k}{1 - 2\alpha_m}$$

It is possible to show[30] that $\hat{U}_h > \tilde{U}_h$.

We consider finally the case in which for instance h secedes and l is the agenda setter. In this case l has to choose $0 \leq g_m^l$, $g_l^l \leq 1$ to maximise his own

[29] Namely $g_h^h = 1$ and $g_m^h = \frac{1-2\alpha_m+2k}{1-2\alpha_m}$.

[30] See the Appendix B.

utility (99) subject to the non secession constraint for region m (100) As before the problem is written as follows:

$$U_l = \alpha_l H(g_l^l) - (g_m^l + g_l^l)/2 \tag{99}$$

$$\alpha_m H(g_m) - ((g_m^l + g_l^l)/2 \geq \bar{u}_m \tag{100}$$

and we look for the solutions $g_l^l = 1$ and $g_m^l \in [0,1]$. F.O.C are:

$$\alpha_l - \frac{1}{2} - \frac{1}{2}\mu_m \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \text{ as } g_l^l \left\{ \begin{array}{c} = 1 \\ \in [0,1] \\ = 0 \end{array} \right. \tag{101}$$

$$-\frac{1}{2} + \alpha_m \mu_m - \frac{1}{2}\mu_m \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \text{as } g_m^l \left\{ \begin{array}{c} = 1 \\ \in [0,1] \\ = 0 \end{array} \right. \tag{102}$$

where μ_m is the Lagrange multiplier. To look for the solutions $g_l^l = 1$ and $g_m^l \in [0,1]$ from (102);

$$\mu_m = \frac{1}{2\alpha_m - 1} \tag{103}$$

which is greater than zero under assumption A1. Substituting (103) into (101) we can write

$$\alpha_l - \frac{1}{2} - \frac{1}{2}\mu_m = \alpha_l - \frac{\alpha_m}{2\alpha_m - 1}$$

and under assumptions A1-A2 we know that $\alpha_l > \frac{\alpha_m}{2\alpha_m - 1}$; and so $\alpha_l - \frac{1}{2} - \frac{1}{2}\mu_m > 0$ thus from (101) we see that $g_l^l = 1$. To find g_m^l we know that the secession constraint must hold and then by substituting $g_l^l = 1$ into (100) we see that $g_m^l = \frac{1-2\alpha_m+k}{1-2\alpha_m}$.

At this stage we have again to compare the utility of l when l does not violate the secession constraint ($\hat{U}_l$) with the utility in the case in which l secedes ($\tilde{U}_l$) Then, as before, by considering the taxes in both cases it is possible to see that

$$\hat{U}_l = \alpha_l H(g_l^l) - \frac{\alpha_h + \alpha_m - 3\alpha_h\alpha_m + \alpha_h k + \alpha_m k}{\alpha_h + \alpha_m - 3\alpha_h\alpha_m}$$

and

$$\tilde{U}_l = \alpha_l H(g_l^l) - \frac{1 - 2\alpha_m + k}{1 - 2\alpha_m}$$

Thus, as before it follows that $\hat{U}_l > \tilde{U}_l$.

To conclude, as in the case of lump sum district taxes, the agenda setter respects the secession constraints.

7 Concluding remarks and extensions

In this work we have attempted to provide through a very simple model of legislative bargaining some insights into country formation or secession. To this aim, in contrast to other models on break-up of countries where the political conflict whether to secede or not is always unidimensional and where two regions are assumed, we have considered a specific case of multidimensional political conflict among an odd number of regions.

In our setting, the multidimensional political conflict arises from the redistributive nature of local public goods whose provision is decided inside a national legislature composed by representatives of regions. To overcome Condorcet cycles, which unavoidably result in this case, we have imposed an agenda structure in the legislative process following Baron and Ferejohn (1989) and Persson model (1998) which is our point of departure. So, to address secession and breaking up of the union, we have also made very simply assumptions. In fact, in our work, regions differ among themselves only with respect the preference parameter for the local public good, whereas regions are equal in income and size. In addition, we assume that population inside each region is homogeneous. To sum up, heterogeneity among regions is discussed only looking at the preferences for the local public good. Moreover, we have simply assumed that each region is represented by one representative in the national legislature and such representative has the same preference as any citizen of his region. So, we have not considered procedures by which representatives are selected from regions. Also we have ignored problems of externalities across district[31].

Despite these simple assumptions, some conclusions can be drawn. A key finding is that the bargaining process inside the legislature and the way of financing local public good provision does affect the probability of regions' union ex ante or break-up. In fact, we have seen that, whereas the benefit of centralization is the internalization for each district of the cost of being independent, however the decision making process on local public goods provision is such that expose members of the union to the risk of expropriation.

Firstly, we have considered that in the case in which peaceful secession is not possible, secession or federation can only take place ex-ante, in this setting when district specific taxes are assumed, the expected utility from the bargaining game is the same for each region and therefore only the region with the highest preference parameter for local public good is pivotal in determining the union formation. Moreover, if regions are more heterogeneous in terms of preference parameter for the public good a higher level of "union benefit" is necessary to establish the union ex ante. This result, can also be compared, to some extent, with the standard results in the literature on decentralization. In fact also in this case the degree of heterogeneity of preferences is in the direction of emphasizing the trade off between centralization and decentralization. On the other hand, when linear income taxation is considered we have shown that such taxation regime affects a region's chance of being a coalition partner and so the

[31] For models allowing such possibilities see Lockwood (1998) and Besley and Coate (1988).

expected utility from the bargaining game varies among regions. Notably, if regions are more homogeneous in terms of preference for local public good, it must be the case that the region with the lowest preference parameter is pivotal in determining the union formation.

Secondly, we have also considered that we may be in a world where peaceful secessions are possible and we have investigated such an eventuality too by considering a bargaining game with an opt out option. We have shown that peaceful secession or an *opt out option* by any region may place a limit on the amount of surplus expropriated from other representatives by the agenda setter. In other words, credible threat of secession reduces the amount of expropriation in the case in which local public goods are financed with distric specific taxes as well as with linear income tax.[32]. Finally, our results also suggest that in both ways of financing local public goods the agenda setter still prefers to respect the secession option to the outcome of independence. The reason is that there is still, to some extent, some amount of expropriation. Further insights may derive from this result, worthy of future research, such as the case of ex ante secession by regions when the opt out option is allowed for in the bargaining process. It might be possible that whenever opt out options are allowed, ex ante secession never takes place and the regions union is always established.

To conclude, our results seem worthy of further analysis and a more general treatment of the issue addressed here would be useful. In fact the issue can also be thought somehow in the light of the current debate in Europe on enlargement to new countries as well as on the future "European Constitution". Among other questions to be discussed such as the issue of the division of responsibilities of power among levels of government one issue to be discussed may be whether a secession clause has to be included or not in the future European constitution.

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8 Appendix A

We have already defined the payoff to representative i from district i, in the linear income taxation case, as:

$$U_i = y + \alpha_i H(g_i) - \frac{1}{n}\sum_i g_i$$

where now $i = h, m, l$.

These three representatives in the unicameral legislature have to make a decision on the amount of public goods to be offered. All the public goods are financed through linear income taxation and so $t = \frac{1}{n}\sum_i g_i; \quad i = h, m, l$.

It is possible to consider eight policy proposals that could be chosen by the three representatives, as follows: [33]:

$w = (g_h, g_m, g_l)$
$z = (g_h, g_m, 0)$
$x = (0, g_m, g_l)$
$n = (g_h, 0, g_l)$
$y = (0, 0, g_l)$
$h = (0, g_m, 0)$
$j = (g_h, 0, 0)$
$o = (0, 0, 0)$

Also assume $g_h > g_m > g_l > 0$. Considering the configuration of the payoff for each representative over the alternatives, it is easy to show that alternative x is strictly preferred to alternatives n and z by representatives m and l respectively.[34] If follows that xPn and xPz. Also, yPh as representatives h and l vote for y while yPj, as l and m vote for y.

So, at this stage, we can restrict our attention to the alternatives x, w, y, o. Again, considering the representatives' preferences over these alternatives, it follows that:

1. xPw as m and l vote for x
2. yPx as h and l vote for y
3. oPy as h and m vote for o.

Thus, it is possible to infer that, under majority rule, preferences over the alternatives w, x, y, o yield the social order $oPyPxPw$. However, note that all three representatives unanimously prefer w to o; thus we conclude that the final outcome is the cyclic social order $oPyPxPwPo$.

[33] Recall that the taxes to be paid are $\sum_i g_i/3$ and $g_i = g_h, g_m, g_l$.

[34] Formally for instance the preference relation P by representatives $i = m, l$ over the alternatives $(x,\, n)$ can be defined as $xPn \Leftrightarrow \#\left\{i \mid U_i(x) > U_i(n)\right\} > \#\left\{i \mid U_i(n) > U_i(x)\right\}$.

9 Appendix B

We now turn our attention to show that in the case of ex post secession and the linear income taxation case the agenda setter h respects the secession contraint. Therefore we have to show that:

$$\hat{U}_h > \tilde{U}_h$$

we know that:

$$\hat{U}_h = \alpha_h H(g_h^h) - \frac{\alpha_l + \alpha_m - 3\alpha_l\alpha_m + \alpha_l k + \alpha_m k}{\alpha_l + \alpha_m - 3\alpha_l\alpha_m}$$

and

$$\tilde{U}_h = \alpha_h H(g_h^h) - \frac{1 - 2\alpha_m + k}{1 - 2\alpha_m}$$

it is possible to write:

$$\alpha_h H(g_h^h) - \frac{\alpha_l + \alpha_m - 3\alpha_l\alpha_m + \alpha_l k + \alpha_m k}{\alpha_l + \alpha_m - 3\alpha_l\alpha_m} \geq \alpha_h H(g_h^h) - \frac{1 - 2\alpha_m + k}{1 - 2\alpha_m}$$

and the inequality above can be written also

$$-\left(\frac{\alpha_l + \alpha_m - 3\alpha_l\alpha_m + \alpha_l k + \alpha_m k}{\alpha_l + \alpha_m - 3\alpha_l\alpha_m}\right) \geq -\left(\frac{1 - 2\alpha_m + k}{1 - 2\alpha_m}\right) \qquad \text{(b1)}$$

and from (b1) we can also write

$$-\left[(1 - 2\alpha_m)\left(\alpha_l + \alpha_m - 3\alpha_l\alpha_m\right)\right] - k(\alpha_l + \alpha_m)(1 - 2\alpha_m) =$$

$$= -(1 - 2\alpha_m)(\alpha_l + \alpha_m - 3\alpha_l\alpha_m) - k(\alpha_l + \alpha_m - 3\alpha_l\alpha_m) \qquad \text{(b2)}$$

and from (b2) it follows

$$-k(\alpha_l - 2\alpha_l\alpha_m + \alpha_m - 2(\alpha_m)^2) + k(\alpha_l + \alpha_m - 3\alpha_l\alpha_m) = 0 \qquad \text{(b3)}$$

we rewrite (b3) as follows

$$k(-\alpha_l + 2\alpha_l\alpha_m - \alpha_m + 2(\alpha_m)^2 + \alpha_l + \alpha_m - 3\alpha_l\alpha_m) = 0$$

which becomes

$$k[2(\alpha_m)^2 - \alpha_l\alpha_m] = 0 \qquad \text{(b4)}$$

it is easy to see that, under assumption A1-A2, and $k > 0$; expression (b4) is positive, this implies that $\hat{U}_h > \tilde{U}_h$.