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**PLATFORM COMPETITION
IN TWO-SIDED MARKETS**

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Platform Competition in Two-Sided Markets

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Abstract

Many if not most markets with network externalities are two-sided. To succeed, platforms in industries such as software, portals and media, payment systems and the Internet, must “get both sides of the market on board”. Accordingly, platforms devote much attention to their business model, that is to how they court each side while making money overall. The paper builds a model of platform competition with two-sided markets. It unveils the determinants of price allocation and end-user surplus for different governance structures (profit-maximizing platforms and not-for-profit joint undertakings), and compares the outcomes with those under an integrated monopolist and a Ramsey planner.

1 Introduction

Buyers of videogame consoles want games to play on; game developers pick platforms that are or will be popular among gamers. Cardholders value credit or debit card only to the extent that these are accepted by the merchants they patronize; affiliated merchants benefit from a widespread diffusion of cards among consumers. More generally, many if not most markets with network externalities are characterized by the presence of two distinct sides whose ultimate benefit stems from interacting through a common platform.

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Platform owners or sponsors in these industries must address the celebrated “chicken-and-egg problem” and be careful to “get both sides on board”. Despite much theoretical progress made in the last two decades on the economics of network externalities and widespread strategy discussions of the chicken-and-egg problem, two-sided markets have received scant attention. The purpose of this paper is to start filling this gap.

The recognition that many markets are multi-sided leads to new and interesting, positive and normative questions. Under multi-sidedness, platforms must choose a price structure and not only a price level for their service. For example, videogame platforms such as Sony, Sega and Nintendo make money on game developers through per-unit royalties on games and fixed fees for development kits and treat the gamers side as a loss leader. Interestingly, operating system platforms for the PC and handheld devices have adopted the opposite *business model* and aim at making money on consumers. The choice of a business model seems to be key to the success of a platform and receives much corporate attention. Table 1 provides a few illustrations¹ of the two-sided markets and shows that platforms often treat one side as a profit center and the other as a loss leader, or, at best, as financially neutral. A number of these illustrations are discussed in “mini-case studies” in section 2. And Table 2 lists a few important segments of the new economy that will be searching for a proper business model in the next few years. Such conventional wisdom

¹There are of course other illustrations, for example *scientific journals*, that must match readers and authors. Interestingly, the *Bell Journal of Economics* for a number of years after it was launched was sent for free to anyone who requested it. There is currently much discussion of how the business model for scientific journals will evolve with electronic publishing. The list of *social gatherings* examples of cross-subsidization could be extended to include dating or marital agencies which may charge only one side of the market.

A couple of explanations regarding markets that will not be discussed in section 2: *Social gatherings*: celebrities often do not pay or are paid to come to social happenings as they attract other participants (who may then be charged an hefty fee); similarly, in some conferences, star speakers are paid while others pay. *Real estate*: In many countries buyers are not charged for visiting real estate properties and thus marginal visits are heavily subsidized. To be certain, the sale price reflects the real estate agency fee, but this does not imply that the arrangement is neutral (see section 3). *Shopping malls*: shoppers are subsidized. They don't pay for parking; in France they can also buy gasoline at a substantial discount. *Discount coupon books*: These are given away to consumers. Intermediaries charge merchants for the service. *Browsers*: The picture given in Table 1 is a bit simplistic. In particular, Netscape initially made about one third of its revenue on the client side before giving the software away. But Netscape always viewed the software running on top of the operating system on the web servers as a major source of profit.

about business models found in the trade press and summarized in Table 1 is of course subject to criticism. To reason in terms of profit centers, costs are often “intuitively,” but arbitrarily allocated to either side of the market. Yet, the conventional wisdom points at some more fundamental logic related to prices and surpluses on both sides of the market. A major objective of our paper is to unveil this logic and the determinants of the choice of a business model.

ILLUSTRATIONS OF EXISTING BUSINESS MODELS

Product	loss leader/break-even segment/ subsidized segment	profit-making segment/ subsidizing segment
SOFTWARE		
Videogames	consumers (consoles)	software developers
Streaming media	consumers	servers
Browsers	users	web servers
Operating systems (Windows; Palm, Pocket PC)	application developers (development tools, support, functionality,...)	clients
Text processing	reader/viewer	writer
PORTALS AND MEDIA		
Portals	“eyeballs”	advertisers
Newspapers	readers	advertisers
(Charge-free) TV networks	viewers	advertisers
PAYMENT SYSTEMS		
Credit and differed debit cards (Visa, MasterCard, Amex,...)	cardholders	merchants
On-line debit cards	merchants	cardholders
Settlement systems (TARGET)	receiver	sender

(continued...)

(continued...)

OTHERS...		
Social gatherings	celebrities in social happenings	other participants
Shopping malls	consumers (free parking, cheap gas,...)	shops
Discount coupon books (Want Advertizer)	consumers	merchants
(Legacy) Internet	websites	dial-up consumers
Real estate	buyers	sellers

Table 1: existing business models

LOOKING FORWARD: KEEP POSTED ON:

Platform	Two sides	Instruments of cost allocation or cross-subsidization
B2B	buyers / sellers	design of auctions, information flows,...
Internet backbone services	consumers / websites	termination (settlement) charges
Pools and standards	relevant sides	level of royalties, inclusiveness of pools,...
Software as a service (.Net vs Java,...)	consumers / application developers	development tools and other efforts to create an applications development environment, backward compatibility, pricing,...

Table 2: prospective applications

From both a positive and a normative viewpoints, two-sided markets differ from the textbook treatment of multiproduct oligopoly or monopoly. The interaction between the two sides gives rise to strong complementarities, but the corresponding externalities are not internalized by end users, unlike in the multiproduct literature (the same consumer buys the razor and the razor blade). In this sense, our theory is a cross between network economics, which emphasizes such externalities, and the literature on (monopoly or competitive) multiproduct pricing, which stresses cross-elasticities. For example, socially optimal “Ramsey” prices are not driven solely by superelasticity formulae but also reflect each side’s contribution to the other side’s surplus.

Some new questions raised by two-sided markets are more specific to the existence of competition between platforms. In a number of markets, a fraction of end users on one or the two sides connect to several platforms. Using the Internet terminology, we will say that they *multihome*. For example, many merchants accept both American Express and Visa; furthermore, some consumers have an Amex and a Visa cards in their pockets. Many consumers have the Internet Explorer and the Netscape browsers installed on their PC, and a number of websites are configured optimally for both browsers. Readers may subscribe to multiple newspapers, B2B exchange members may buy or sell their wares on several exchanges, and real estate sellers and buyers may use the services of multiple real estate agencies. Competitive prices on one market then depend on the extent of multihoming on the other side of the market. For example, when Visa reduces the (transaction-proportional) charge paid by the merchants,² merchants become more tempted to turn down the more costly Amex card as long as a large fraction of Amex customers also owns a Visa card. More generally, multihoming on one side intensifies price competition on the other side as platforms use low prices in an attempt to “steer” end users on the latter side toward an exclusive relationship.³

²The mechanism through which this reduction operates is indirect and is described in section 2.

³The occurrence of steering is easiest to visualize in those illustrations in which platforms charge per-enduser-transaction fees: The seller of a house or a *B-2-B* supplier may only list the house or the wares on the cheapest platform. An ISP or a website may buy most or all of its Internet interconnection capacity

The paper studies how the price allocation between the two sides of the market is affected by a) platform governance (for-profit vs not-for-profit), b) end users' cost of multihoming, c) platform differentiation, d) platforms' ability to use volume-based pricing, e) the presence of same-side externalities, and f) platform compatibility. It also investigates how privately optimal pricing structures compare with socially optimal ones.

The paper is organized as follows. Section 2 presents some mini case studies. Section 3 makes a distinction between one- and two-sided markets. Section 4 describes the model, in which interactions between the two sides of the market can be mediated through either one of two platforms. Section 5 analyzes the monopoly benchmarks in which a single entity (profit maximizer or Ramsey planner) operates the two platforms in a coordinated fashion. Section 6 derives equilibrium behavior when the two platforms compete and are either for-profit or run as not-for-profit associations. Section 7 derives some comparative statics. Section 8 compares the price structures in the case of linear demands. Section 9 extends the model and results in a number of relevant dimensions, and section 10 concludes by summarizing the main insights and discussing alleys for future research.

As we discussed, our work puts network economics and multiproduct pricing together. From the early work of Rohlfs (1974) to the recent theoretical advances and applications to antitrust through the pioneering work of Katz-Shapiro (1985, 1986) and Farrell-Saloner (1985, 1986), a large body of literature has developed on network industries. To make progress, however, this literature has ignored multisidedness and the price allocation question. In contrast, the competitive multiproduct pricing literature (e.g., Baumol et al 1982, Wilson 1993) has carefully described the interdependency of pricing decisions but it has from the cheapest backbone (in that industry, multihoming is currently motivated either by a small difference in on-net and off-net quality of connectivity or by a probability of a network's breakdown).

In industries in which platforms do not charge per-enduser-transaction fees, steering is more subtle as it operates through effort substitution. For example, a software platform offering better software development kits, support, and application programming interfaces not only encourages the development of applications optimized to this platform, but is also likely to induce application developers to devote less attention to rival platforms. A portal or TV network's cut in advertising rates induces advertisers to advertise more on their medium and to substitute away from other media. A shopping mall's cut in rental prices or improved layout may induce a shop to increase its size or appeal and lead the latter to neglect or abandon its outlets in rival shopping malls. And so forth.

not considered the affiliation externalities that lie at the core of the network economics literature.

Our paper is most closely related to the recent theoretical literature on chicken-and-egg problems. This literature however assumes either that there is a monopoly platform (Baye-Morgan 2000, Rochet-Tirole 2000, Schmalensee 2000) or that platforms are fully interconnected (Laffont et al 2001) and so end-users enjoy the same level of connectivity regardless of the platform they select. Caillaud and Jullien (2001) study competition among intermediaries. In their model, platforms act as matchmakers and can use sophisticated pricing (registration fees, and possibly transaction fees provided the intermediaries observe transactions). Indeed, one of their contributions is to show that dominant firms are better off charging transactions rather than registrations when deterring entry. They also show that competition is more intense when platforms cannot deter multihoming. Their contribution is complementary to ours. For example, it assumes homogeneous populations on either side, and thus abstracts from the elasticity-related issues studied in our paper. Last, in a model related to that of Caillaud-Jullien, Jullien (2001) shows that an entrant represents a much stronger competitive threat on an incumbent platform when third-degree price discrimination is feasible. The ability to “divide and conquer” forces profit down, so much so that the incumbent may prefer platform compatibility.

2 Mini case studies

2.1 Videogames

The videogame market is a typical two-sided one. A platform cannot sell the console without games to play on and cannot attract game developers without the prospect of an installed base of consumers. In its thirty years of existence, the video game industry has had four leading platforms, Atari, Nintendo and Sega, and finally Sony. The business model that has emerged uses consoles as the loss leader and draws platform profit from applications development. To be certain, history has repeatedly shown that technically

impressive platforms (e.g., Mattel in 1981, Panasonic in 1993, and Sega in 1985 and after 1995) fail when few quality games are written for them. But attracting game developers is only a necessary condition. In fact, the business model currently employed by Nintendo, Sega and Sony is to charge software developers a fixed fee together with a per-unit royalty on the games they produce.⁴ Microsoft will release in the fall of 2001 the Xbox in competition with Sony's dominant PlayStation 2. Interestingly, Microsoft intends to manufacture the Xbox console and use it as a loss leader. While courting the developers,⁵ by using the familiar X86 chip and Windows platform and by not charging for the Xbox Prototype kit, Microsoft has stated that it intends to draw revenue from royalties.

Although the industry's business model involves drawing revenue from developers, platforms can only go so far in taxing the latter. A key factor in Sony's PlayStation's victory over the Sega Saturn and Nintendo 64 was that Sony offered a development platform and software application that was much cheaper (about \$10,000 per seat) and much easier to use (as it was PC based) than its two rivals'.⁶

2.2 Streaming-media technology

Streaming-media platforms incorporate encoding, compression, scripting and delivery technologies to allow the delivery of streaming content, facilitate content creation and permit interactivity; for example, it is central to conferencing and Webcast. The current competition is among the RealNetworks, Microsoft and Apple platforms.

The streaming-media industry is still in its infancy and it is probably too early to point at "the" business model. The current business mostly, but not exclusively, subsidizes the client side. RealNetworks and Apple offer two clients, a basic, free client and a better,

⁴Initially, Nintendo placed a chip in its console. The console would not work unless an authenticating chip was present in the game cartridge. Encryption techniques allow platform manufacturers to meter game sales.

⁵In September 2000, 157 developers were working on Xbox games. The Xbox will be launched with 26 games. Interestingly, Electronic Arts (the maker of FIFA, SimCity and James Bond) was able to impose special conditions on Microsoft.

⁶See Cringely (2001) for more detail. Sony sold its console below cost and made the money back on game royalties.

non-free one. RealNetworks, the current leader charges significant amounts on the server side for RealServer Plus and its upgrades. Apple in contrast is currently free on the server side, but has the disadvantage on running only on Macs. Microsoft's Windows Media is free (bundled with the operating systems).

2.3 Credit and debit cards

Historically, the business model for credit and differed debit cards has been to attract cardholders and induce them to use their cards. Visa and MasterCard are not-for-profit associations owned by over 6,000 bank (and nonbank) members. The associations centrally set interchange fees to be paid by acquirers (the merchants' banks) to issuers (the cardholders' banks). These interchange fees are proportional to transaction volume. A higher interchange fee is, via the competition among issuers, partly or fully passed through to consumers in the form of lower card fees and higher card benefits, which encourages card ownership and usage; and, via the competition among acquirers, partly or fully passed through to merchants, who pay a higher merchant discount (the percentage of the sale price that the merchant must pay the acquirer), which discourages merchant acceptance. The associations' choice of interchange fees have typically favored cardholders over merchants.⁷

American Express, a for-profit closed system, works on the same business model, with an even higher degree of cross-subsidization. Traditionally, it has charged a much higher merchant discount.⁸ Interestingly for the theory developed below, the gap between Amex's and the associations' merchant discounts has narrowed in the 1990s as more and more Amex customers got also a Visa card or MasterCard. Such "multihoming" by a fraction of cardholders made it less costly for merchants to turn down Amex cards.

The on-line debit card market in the US has adopted an entirely different business

⁷Looking forward, it is likely that merchant card acceptance will become more elastic with the (ongoing) advent of on-line debit and the (future) introduction of Webplatforms.

⁸And thus implicitly a much higher "interchange fee". For Amex, the interchange fee is only implicit, since the company is vertically integrated and performs the three roles of issuer, system and acquirer.

model. Rather than courting consumers, it has wooed merchants through a low interchange fee. One key difference with credit and differed debit cards is that consumers indeed do not need to be courted (they already have in their pocket an ATM card, that they can use as an on-line debit card), while merchants, to perform on-line debit, must install costly pinpads (which most of them have not yet done).⁹

2.4 Operating systems

Both sides in the Microsoft browser case agreed that a key competitive advantage of Windows over competing operating systems is its large installed base of applications. Windows' business model is basically the opposite of that of videogame platforms. Windows makes money on users and as a first approximation does not make or loses money on applications developers.¹⁰ It fixes the Applications Programming Interfaces 3 or 4 years in advance (a costly strategy) and invests heavily in developer support. This strategy proved profitable in its contest with Apple and IBM's OS/2. Apple has no integrated developers system tools allowing developers to test their programs; the latter had to buy an IBM workstation and a compiler. IBM viewed developer kits as a profit center.¹¹ While other factors undoubtedly played a role in the competition among the three platforms, observers usually agree that Microsoft's choice of business model helped Windows establish dominance.

⁹The on-line offerings were first made by regional ATM networks. A number of these networks have now been consolidated and converted into a for-profit platform (Concord ESF).

¹⁰We are unaware of "hard data" on this and just report the industry's conventional wisdom. Nor do we have any hard data for handheld computer operating systems. Handheld computers operating systems, dominated by Palm's platform (75% market share in the US) and Microsoft's Pocket PC software, have adopted a business model that is similar to Windows for PC operating systems. Palm and Microsoft apparently charge about 10% of the hardware's wholesale price (\$5 to \$15) to hardware manufacturers. Both provide standard user interfaces and central support and development tools for developers of third-party software. For more detail, see <http://www5.zdnet.com/zdnn/stories/news/0,4586,2714210,00.html?chkpt-zdhpnews01>.

¹¹Software developer kits were sold at about \$600.

2.5 Internet

In the Internet, the instrument of cross-subsidization is the termination or settlement charge (or lack thereof) between backbones (the platforms). The termination charge for off-net traffic between two backbones is, as its name indicates, the price paid by the originating backbone to the terminating backbone for terminating the traffic handed over. A high termination charge is indirectly borne by end users, like websites, whose volume of outgoing traffic far exceeds the volume of incoming traffic, and benefits end users, such as dial-up customers, who mostly receive traffic (downloads).

The Internet is still mostly run by pre-deregulation legacy arrangements, according to which the backbones charge nothing to each other for terminating traffic. This business model is currently being reexamined and it is quite possible that, as is the case for regular telephony, positive termination charges will be levied in the future. The legacy arrangements may well have made sense in an epoch in which the posting of content on the Web had to be encouraged. A key question now is whether a change in industry conditions motivates a move toward paying settlements.

2.6 Text processing

A key issue confronting purchasers of text processing software is whether they will be able to “communicate” with people who don’t make the same choice. Commercial software vendors have in this respect converged on the following business model: They offer a downgraded version of the paying software as “freeware”. This free version allows “nonusers” to open, view and print, but not edit documents prepared with the paying software, and copy information from those documents to other applications. Examples of such free viewers are Word Viewer, PDF Viewer, and Scientific Viewer.¹²

¹²For Scientific Word, a mathematics software program adding a user interface and various other functions on to L^AT_EX.

2.7 Portals and media

The business model of (non-pay) TV and to a large extent newspapers has been to use viewers and readers as a loss leader, who attract advertizers. This business model has been adopted with a vengeance by Internet portals, which have supplied cheap or free Internet access as well as free content (share quotes, news, e-mail, etc....) to consumers. The profit center has been advertising revenue, including both fixed charges for banner placement and proportional referral fees.¹³

Interestingly the portal industry is considering whether to stick to this business model or move in particular to for-fee content. For example, Yahoo! is now starting to charge fees for services such as real-time share-quote services or auction services. A number of content sites have appeared that charge substantial fees for on-line content.¹⁴

3 One- and two-sided markets

Our premise is that many (probably most) markets with network externalities are two- (or multiple-) sided markets. The examples listed above make this premise look self-evident. Yet, it is not, as we now argue.

3.1 One-sided markets

We will say that a market with network externalities is a one-sided market if platforms cannot effectively cross-subsidize between different categories of end users that are parties to a transaction. That is, the volume of transactions on and the profit of a platform depend only on the total price charged to the parties to the transaction and not on its decomposition. There are two reasons why platforms may be unable to perform such cross-subsidization; the first is obvious and the second requires some more thinking.

a) *Both sides of the market coordinate their purchases .*

¹³See Elfenbein-Lerner (2001) for a thorough analysis of contracts in recent Internet Portal Alliances.

¹⁴See, e.g., the Economist (April 14, 2001, p65) for more details.

A debit card platform negotiating with a government for the handling of inter-agency financial transactions, an Internet operator offering an Intranet solution to a company, or a streaming-media platform offering streaming audio and video to a firm primarily for internal use all deal with a single party. A subsidization of the client side by the server side for example does not affect the total price of the software service and, *ceteris paribus*, does not affect the demand for the platform.¹⁵

b) *Pass-through and neutrality.*

Even when end users on the two sides of the market act independently, monetary transfers between them may undo the redistributive impact and prevent any cross-subsidization. The value-added tax is an epitome of the possibility of neutrality. First-year economic students are taught that it really does not matter whether the seller or the buyer pays the tax. In the end, prices adjust so that any tax paid by the seller is passed through to the consumer. If such neutrality holds, then the markets discussed above should be treated as one-sided markets, that is markets in which only the total per transaction price charged by the platform matters and not its decomposition between end users.

Remark: “Neutrality” refers to the pass-through property and a priori bears no connotation with respect to the well-being of end users and platforms and to social welfare. While neutrality reduces the number of instruments at the disposal of a given platform, it is not clear whether it helps or hurts the platforms in their rivalry. Similarly, neutrality a priori may be good or bad for end users and social welfare.

¹⁵Mobile and fixed telephone services, for which most users are both callers and receivers, cannot be treated as one-sided markets. A high termination charge raises the marginal cost of calls and lowers the marginal cost of call receptions. In other words, the termination charge is an instrument of cross-subsidization similar to the interchange fee in credit card markets. Telephone users are on both sides of the market for *different* communications only. For a given communication, end users are on a single side and (unless they are engaged in a repeated relationship) their consumption behaviors depend on their own price (calling price for the caller, receiving price for the receiver). As a consequence, the choice of termination charge is not neutral. See Jeon et al (2001) for more detail.

3.2 Foundations of two-sidedness

Let us say right away that thinking of the markets listed in Sections 1 and 2 as one-sided markets just runs counter to evidence. First, the platform owners in all these industries devote much attention to their price-allocation business model: Is it more important to woo one side or the other? The quest for “getting both sides on board” makes no sense in a world in which only the total price for the end user interaction, and not its decomposition, matters. And the trade press would not contain so many descriptions of “chicken- and-egg” problems. Second, the end users themselves are also very sensitive to the allocation of cost between them, indicating that some actual redistribution is taking place. Merchants vocally object to increases in interchange fees, and website operators will do so if settlement charges are introduced in the Internet. End users would not react in this way if charges were passed through.

Let us investigate the (industry-specific) reasons why neutrality does not hold:

- *Transaction costs.* “Transaction costs” refer to a broad range of frictions that make it costly for one side of the market to pass through a redistribution of charges to the other side. Often, these transaction costs are associated with small stakes for individual transactions (which can become substantial when applied to a large number of transactions). The cost of thinking about including the pass-through, writing it into a contract, advertizing it to the other side and enforcing the covenant may then be prohibitive. For example, contractual relationships between a supplier, a buyer and their banks may not specify on which payment system the settlement of the transaction will occur.¹⁶

A second type of transaction cost has to do with the absence of a low-cost billing system. Suppose that an academic downloads a PDF file of another academic’s paper. The micropayments that would be required for pass-through would probably

¹⁶As mentioned earlier, TARGET (the European large value payment system) charges the sender (while its US counterpart, Fedwire, splits the charge equally between sender and receiver). TARGET in general does not involve the same costs for the sending and receiving banks as the competing payment system.

require a costly third-party billing system to be developed cooperatively by Internet backbones and service providers.¹⁷

A third transaction cost is the impossibility of monitoring and recording the actual transaction or interaction. In the portal and media example, neutrality would imply that when the platform (portal, TV network, newspaper) raises the price of advertising, this price increase translates into a smaller amount of money given by the advertiser to the consumer for “listening to the ad”. But “listening” is not easily measurable (except for the monitoring of clicks in the Internet, and even then it is impossible to measure whether the consumer pays genuine attention). In practice therefore, the viewer/reader receives no compensation from the advertiser and neutrality does not obtain.

- *Volume-insensitive costs.* Neutrality also fails when at least one side of the market incurs costs that a) are influenced by the platform and b) are not proportional to the number of transactions on the platform. For example, while software developers incur some costs, such as the per-game royalties paid by game developers, that are proportional to sales, many costs are insensitive and affected by the platform: The fixed development cost is influenced by platform through software design, and so is the fixed charge for the development kit. On the user side, getting familiar with the platform’s user interface may also involve some fixed costs.¹⁸ End user transaction-insensitive prices and non-price attributes of a platform affect the number of end users or applications, but not directly the terms of the transactions between the end users.

The portal-and-media and real estate example offers another illustration of this phenomenon. The advertising cost of locating a potential buyer ought to be treated

¹⁷See Laffont et al (2001) for a demonstration that termination charges are neutral in the Internet in the absence of the frictions considered in this section.

¹⁸Similarly, end users seem to be averse to being “nickelled and dimed” by Internet portals (perhaps because they have a hard time thinking through the total amounts at stake) and flat fees are still quite popular in that industry.

by the seller as a sunk cost when choosing the price to offer to the potentially interested buyer.

- *Platform-determined constraints on pass-through.* The platform may also take steps that limit the extent of pass-through. A case in point is the no-discrimination-rule adopted by credit card associations (Visa, MasterCard) and for-profits (Amex).¹⁹ Merchants do not pass the merchant discount through only to cardholders. So there is only a partial passthrough between the two sides.

4 The model

4.1 Assumptions

The two-sided markets described heretofore differ in some respects, and we therefore should not aim at capturing all specificities of all industries. Our strategy will be to include a number of key ingredients common to our illustrations in a basic model, and then to investigate some generalizations in order to extend the relevance of our analysis to various two-sided markets. In particular, the basic model is a good representation of examples, such as the credit card market, in which the platform implements (and not only enables) transactions and charges proportional fees for that service; the reader may want to keep this in mind, although it will be clear that the insights have much broader generality.²⁰

Economic value is created by “interactions” or “transactions” between pairs of end users, buyers (superscript B) and sellers (superscript S). Such transactions are mediated by a platform. Two such platforms compete to attract both sides of the market. The platforms’ marginal cost of a transaction is denoted by $c \geq 0$.

¹⁹In the US, the associations’ no-discrimination-rule takes a weaker form. Namely, merchants are not allowed to impose surcharges on card payments; but they can offer discounts for cash purchases! That very few do is an interesting fact, that is probably related to the transaction costs category. In Rochet-Tirole (2000), we abstract from such transaction costs and show that the level of the interchange fee is irrelevant if the no-discrimination rule is lifted.

²⁰Note that a “transaction” between a cardholder and a merchant means that the payments is by card rather than by cash.

Buyers and sellers are heterogenous: Their benefits from transacting vary across the two populations and are private information. These benefits are denoted b_i^B for the buyer (when the transaction takes place on platform i) and b^S for the seller, and are drawn from continuous distributions.²¹

For the moment, we neglect fixed costs borne by buyers or sellers (either fixed fees charged by the platforms or fixed usage costs). The proportional fees charged by platform i are p_i^B for buyers and p_i^S for sellers. A buyer with gross surplus b_i^B from transacting on platform i is willing to use that platform provided that $b_i^B \geq p_i^B$. However, the buyer prefers to transact on platform j if $b_j^B - p_j^B > b_i^B - p_i^B$. Similarly, a seller with type b^S is willing to trade on platform i provided that $b^S \geq p_i^S$, and prefers to trade on platform j if $p_j^S < p_i^S$.

Consider a (buyer, seller) pair. Without loss of generality we can assume that each such pair corresponds to one potential transaction. A transaction however occurs only if the two sides have at least one platform in common; that is, there exists at least one platform on which both are willing to trade. If both “multihome” (are affiliated with both platforms), the choice of platform is a priori indeterminate. In accordance with our illustrations, we introduce a slight asymmetry between the two sides of the market by assuming that, whenever a seller is affiliated with the two platforms, the buyer chooses the one on which the transaction takes place.²²

4.2 Buyer’s behavior

Let us derive the demands faced by a seller on platform i when singlehoming on that platform and multihoming. The buyers’ behavior described in Section 4.1 generates “quasi-

²¹For simplicity, we assume that the seller’s gross surplus does not depend on the platform where the transaction takes place.

²²This assumption is satisfied by most of our illustrations: a cardholder selects the card when the merchant accepts multiple cards, the reader or viewer selects the newspaper, portal or TV network, the videogame user selects the platform if the game is written for several consoles, etc.

demand functions”:

$$D_i^B = D^B(p_i^B) = \Pr(b_i^B - p_i^B > 0), \quad (1)$$

and

$$d_i^B(p_1^B, p_2^B) = \Pr [b_i^B - p_i^B > \max(0, b_j^B - p_j^B)]. \quad (2)$$

D_i^B is the proportion of buyers who are willing to use platform i when the seller is affiliated only with platform i . Similarly, d_i^B is the proportion of buyers who are willing to trade on platform i when the seller multihomes. By construction, these functions satisfy the following properties:

$$d_i^B \leq D_i^B \leq d_1^B + d_2^B. \quad (3)$$

We assume that the distribution of (b_1^B, b_2^B) is symmetric, which implies that demand functions are also symmetric: $D_1^B(p^B) = D_2^B(p^B) \equiv D^B(p^B)$ and $d_1^B(p_1^B, p_2^B) \equiv d_2^B(p_2^B, p_1^B)$. When prices are equal $p_1^B = p_2^B = p^B$, we will use the simplified notation:

$$d^B(p^B) \equiv d_i^B(p^B, p^B).$$

Finally, for a symmetric price configuration, we will denote by σ the following index:

$$\sigma = 2 - \frac{D^B}{d^B}. \quad (4)$$

Given property (3), σ belongs to the interval $[0, 1]$. It measures the proportion of “unique consumers” who are only willing to trade on their preferred platform: we will call it the “singlehoming index”. This singlehoming index is equal to 0 when buyer demand faced by the seller is independent of whether the seller multihomes ($D^B = 2d^B$). In contrast, the singlehoming index is equal to 1 when demand doubles when the seller multihomes ($D^B = d^B$).

4.3 Sellers' behavior

Suppose that platform 1 is cheaper for sellers: $p_1^S < p_2^S$. A seller of type b^S has three possibilities: no trade, affiliation with platform 1 only, affiliation with both platforms. The first possibility is optimal whenever $b^S \leq p_1^S$. The choice between the other two possibilities involves a trade-off between a lower volume (when affiliated with platform 1 only) and an obligation to trade on the most expensive platform (when affiliated with both platforms). The corresponding expected net surpluses of a seller of type b^S are respectively $(b^S - p_1^S)D_1^B(p_1^B)$ and $(b^S - p_1^S)d_1^B(p_1^B, p_2^B) + (b^S - p_2^S)d_2^B(p_1^B, p_2^B)$. The seller chooses to multihome when b^S is large enough, more precisely when

$$b^S > \hat{b}_{12} \equiv \frac{p_2^S d_2^B - p_1^S (D_1^B - d_1^B)}{d_2^B - (D_1^B - d_1^B)}. \quad (5)$$

We can now summarize sellers' optimal decisions:

- sellers with low types ($b^S \leq p_1^S$) do not trade,
- sellers with high types ($b^S \geq \hat{b}_{12}$) trade on both platforms,
- sellers with intermediate types ($p_1^S < b^S < \hat{b}_{12}$) only trade on the less expensive platform (here, platform 1).

By undercutting the rival platform, each platform thus induces some sellers (those with intermediate types) to stop multihoming, a strategy known as “steering”. The formulae for $p_1^S > p_2^S$ are obtained by permutation of indices²³. When p_1^S and p_2^S converge to the same price p^S , \hat{b}_{12} and \hat{b}_{21} both converge also to p^S , which establishes continuity of the formulae giving \hat{b}_{12} and \hat{b}_{21} .

4.4 Transaction volumes

We are now in a position to determine the volume of transactions on each platform as a function of prices p_i^B and p_i^S . We restrict ourselves to the case $p_1^S \leq p_2^S$ (the case $p_2^S < p_1^S$

²³Notice however that the expression of \hat{b}_{12} is not symmetric: for arbitrary buyers' prices, $\hat{b}_{12} \neq \hat{b}_{21}$

is obtained by symmetry). Let us denote by D^S the sellers' "quasi-demand function":

$$D^S(p^S) = \Pr(b^S > p^S).$$

From the affiliation decisions derived above, a proportion $D^S(\hat{b}_{12})$ of sellers multihome, while a proportion $D^S(p_1^S) - D^S(\hat{b}_{12})$ are affiliated only with platform 1. Assuming that the probability of a meeting between a buyer and a seller is independent of their types, the total expected volumes of transactions on the platforms are:

$$Q_1 = d_1^B(p_1^B, p_2^B)D^S(\hat{b}_{12}) + D_1^B(p_1^B) \left\{ D^S(p_1^S) - D^S(\hat{b}_{12}) \right\}, \quad (6)$$

for platform 1, and:

$$Q_2 = d_2^B(p_1^B, p_2^B)D^S(\hat{b}_{12}), \quad (7)$$

for platform 2, where \hat{b}_{12} is given by formula (5). As already noticed, these formulae are continuous across the "diagonal" $p_1^S = p_2^S$:

$$\lim_{\substack{p_1^S \rightarrow p^S \\ p_2^S \rightarrow p^S}} Q_i = d_i^B(p_1^B, p_2^B)D^S(p^S).$$

5 The platform monopoly benchmark

Before studying the outcome of competition between the two platforms, let us consider the benchmark case where both platforms are controlled by the same entity. We consider in turn the case of a private monopoly, and that of a public monopoly maximizing social welfare subject to budget balance.

5.1 Private monopoly

We focus on symmetric prices. That is, a private monopoly chooses equal prices on both platforms, and selects these prices so as to maximize total profit:

$$\pi = 2(p^B + p^S - c)d^B(p^B)D^S(p^S),$$

using our simplified notation:

$$d^B(p^B) = d_1^B(p^B, p^B) = d_2^B(p^B, p^B).$$

Assuming that d^B and D^S are log concave, it is easy to see that π is also log concave (jointly in (p^B, p^S)). Its maximum is characterized by the first-order conditions:

$$\begin{aligned} \frac{\partial(\log \pi)}{\partial p^B} &= \frac{1}{p^B + p^S - c} + \frac{(d^B)'}{d^B} = 0, \\ \frac{\partial(\log \pi)}{\partial p^S} &= \frac{1}{p^B + p^S - c} + \frac{(D^S)'}{D^S} = 0. \end{aligned}$$

In particular:

$$(d^B)'D^S = d^B(D^S)'.$$

This condition characterizes the values of p^B and p^S that maximize volume for a given total price p : The volume impact of a small (absolute) variation of prices has to be the same on both sides. If we introduce the elasticities of quasi-demands:

$$\eta^B = -\frac{p^B(d^B)'}{d^B} \quad \text{and} \quad \eta^S = -\frac{p^S(D^S)'}{D^S},$$

the private monopoly prices can be characterized by a two-sided formula that is reminiscent of Lerner's formula:

$$p^B + p^S - c = \frac{p^B}{\eta^B} = \frac{p^S}{\eta^S}. \quad (8)$$

In fact, the total price $p = p^B + p^S$ chosen by the private monopoly is given by the classical Lerner formula:

$$\frac{p - c}{p} = \frac{1}{\eta}, \quad \text{or} \quad p = \frac{\eta}{\eta - 1}c, \quad (9)$$

where $\eta = \eta^B + \eta^S$, the total volume elasticity, is assumed to exceed 1. What is new in formula (8) is the way in which this total price is allocated between the two sides of the market:

$$p^B = \frac{\eta^B}{\eta}p = \frac{\eta^B}{\eta - 1}c, \quad (10)$$

and

$$p^S = \frac{\eta^S}{\eta} p = \frac{\eta^S}{\eta - 1} c. \quad (11)$$

From (8), the more elastic side gets a *higher* share of the total price. Therefore when η^B increases, while η^S is maintained constant, the global impact on p^B is a priori ambiguous: the total price p decreases, but the share allocated to buyers increases. Indeed, from (10),

$$p^B = \frac{\eta^B c}{\eta^B + \eta^S - 1} = \frac{c}{1 + \frac{\eta^S - 1}{\eta^B}},$$

and so p^B decreases with η^B if and only if $\eta^S < 1$. When elasticities exceed 1, the monopoly price for buyers (resp. for sellers) increases with the elasticity of buyers' (resp. sellers') quasi-demand. To obtain some intuition for this result, note that an increase in the buyers' demand elasticity, say, has two opposite effects. The “price structure effect”, embodied in (8), says that the ratio of the buyer price over the seller price increases. The “price level effect”, embodied in (9), implies that the overall price decreases. When the sellers' demand elasticity is large, though, the price level is already close to total marginal cost and the price level effect is small, and so the price structure effect dominates.

Proposition 1 : (i) A monopoly platform's total prices, $p = p^B + p^S$, is given by the standard Lerner formula for elasticity equal to the sum of the two elasticities, $\eta = \eta^B + \eta^S$:

$$\frac{p - c}{p} = \frac{1}{\eta}. \quad (9)$$

(ii) The monopoly price charged to one side of the market increases with the elasticity of demand on that side if and only if the elasticity of demand on the other side exceeds 1. The price structure is given by the ratio of elasticities (and not inverse elasticities):

$$\frac{p^B}{\eta^B} = \frac{p^S}{\eta^S}. \quad (12)$$

5.2 Ramsey pricing

We consider now the case of a Ramsey monopolist maximizing welfare subject to budget balance, and derive the Ramsey formulae in our context²⁴. We can again focus on symmetric prices ($p_1^B = p_2^B = p^B, p_1^S = p_2^S = p^S$). The net surpluses on each side for an average transaction are given by standard formulae:

$$V^S(p^S) = \int_{p^S}^{+\infty} D^S(t)dt$$

for sellers, and

$$\begin{aligned} V^B(p_1^B, p_2^B) &= \int_{p_1^B}^{+\infty} d_1^B(t_1, p_2^B)dt_1 + \int_{p_2^B}^{+\infty} D_2^B(t_2)dt_2 \\ &= \int_{p_2^B}^{+\infty} d_2^B(p_1^B, t_2)dt_2 + \int_{p_1^B}^{+\infty} D_1^B(t_1)dt_1 \end{aligned}$$

for buyers.

Under budget balance, social welfare is equal to the sum of both sides' net surpluses:

$$W = V^S(p^S)[d_1^B(p^B, p^B) + d_2^B(p^B, p^B)] + V^B(p^B, p^B)D^S(p^S)$$

or, using our simplified notation

$$W = 2V^S(p^S)d^B(p^B) + V^B(p^B, p^B)D^S(p^S),$$

which has to be maximized subject to the constraint:

$$p^B + p^S = c.$$

The first-order, “cost allocation” condition is:

$$\frac{\partial W}{\partial p^B} = \frac{\partial W}{\partial p^S}.$$

This gives:

$$2V^S(d^B)' - 2d^B D^S = -2D^S d^B + V^B(D^S)'$$

After simplification, we obtain a characterization of Ramsey prices:

²⁴A similar formula is derived in Laffont et al. (2001) in a model in which network externalities are reaped through platform interconnection rather than through end-user multihoming.

Proposition 2 : *Ramsey prices embody the average surpluses created on the other side of the market and are characterized by two conditions:*

$$p^B + p^S = c \quad (\text{budget balance}) \quad (13)$$

and

$$\frac{p^B}{\eta^B} \left[\frac{V^B}{2d^B} \right] = \frac{p^S}{\eta^S} \left[\frac{V^S}{D^S} \right] \quad (\text{cost allocation}). \quad (14)$$

Condition (14) characterizes the price structure that maximizes social surplus for a given total price p . Returning to the formula yielding the private monopolist's price structure,

$$\frac{p^B}{\eta^B} = \frac{p^S}{\eta^S}, \quad (8')$$

the additional terms in formula (14) (the bracketed terms) reflect the average surpluses per transaction for buyers and sellers.

6 Competing platforms

From now on we assume that the two platforms are controlled by competing entities, either profit-maximizing firms or not-for-profit associations. Important examples of such associations can be found in the payment card industry (Visa and MasterCard). In such associations, prices for buyers and sellers are determined by competition (both intra and inter platforms) on downstream markets (issuing banks on the buyers' side, and acquirers on the sellers' side). The treatment of for-profit platforms and cooperatives are very similar. Under some assumptions, their price *structure* formulae are identical, as both attempt to maximize volume for a given total price. The difference between the two is then the price level.

6.1 Competition between proprietary platforms

Proprietary platforms choose prices so as to maximize profit. Consider for example platform 1's profit:

$$\pi_1 = (p_1^B + p_1^S - c)Q_1. \quad (15)$$

As in the case of a monopolist, this maximization can be decomposed into the choice of a price level, $p_1 = p_1^B + p_1^S$, and that of a price structure given a price level. The first-order conditions are:

$$Q_1 + (p_1^B + p_1^S - c) \frac{\partial Q_1}{\partial p_1^B} = Q_1 + (p_1^B + p_1^S - c) \frac{\partial Q_1}{\partial p_1^S} = 0,$$

or

$$\frac{\partial Q_1}{\partial p_1^S} = \frac{\partial Q_1}{\partial p_1^B} = -\frac{Q_1}{p_1^B + p_1^S - c}. \quad (16)$$

Recall the expressions of volumes on both systems, when, say, $p_1^S \leq p_2^S$:

$$Q_1 = d_1^B(p_1^B, p_2^B)D^S(\hat{b}_{12}) + D_1^B(p_1^B)\{D^S(p_1^S) - D^S(\hat{b}_{12})\}, \quad (6)$$

$$Q_2 = d_2^B(p_1^B, p_2^B)D^S(\hat{b}_{12}), \quad (7)$$

where

$$\hat{b}_{12} = \frac{p_2^S d_2^B - p_1^S (D_1^B - d_1^B)}{d_2^B - (D_1^B - d_1^B)}. \quad (5)$$

We focus on symmetric equilibria ($p_i^S \equiv p^S, p_i^B \equiv p^B$), for which volumes have simpler expressions:

$$Q_i = d_i^B(p^B, p^B)D^S(p^S).$$

While

$$\frac{\partial Q_1}{\partial p_1^B} = \frac{\partial d_1^B}{\partial p_1^B}(p^B, p^B)D^S(p^S), \quad (17)$$

the first derivative in formula (17) is not necessarily well defined since volumes have a different expression according to whether $p_1^S \leq p_2^S$ or $p_1^S > p_2^S$:

$$Q_1 = d^B(p^B)D^S(\hat{b}_{12}) + D^B(p^B)\{D^S(p_1^S) - D^S(\hat{b}_{12})\}$$

when $p_1^S < p_2^S$, and

$$Q_1 = d^B(p^B)D^S(\hat{b}_{21})$$

when $p_1^S > p_2^S$. Interestingly, Q_1 turns out to be differentiable. The left- and right-derivatives of Q_1 with respect to p_1^S at $p_1^S = p_2^S = p^S$ (implying $\hat{b}_{12} = \hat{b}_{21} = p^S$) are:

$$\left(\frac{\partial Q_1}{\partial p_1^S}\right)_L = (D^S)' \frac{\partial \hat{b}_{12}}{\partial p_1^S} [d^B - D^B] + (D^S)' D^B,$$

and

$$\left(\frac{\partial Q_1}{\partial p_1^S}\right)_R = (D^S)' \frac{\partial \hat{b}_{21}}{\partial p_1^S} d^B.$$

Moreover

$$\frac{\partial \hat{b}_{12}}{\partial p_1^S} = -\frac{D^B - d^B}{2d^B - D^B}, \quad \text{and} \quad \frac{\partial \hat{b}_{21}}{\partial p_1^S} = \frac{d^B}{2d^B - D^B}.$$

And so

$$\begin{aligned} \left(\frac{\partial Q_1}{\partial p_1^S}\right)_L &= (D^S)' \left[\frac{(D^B - d^B)^2}{2d^B - D^B} + D^B \right] = (D^S)' \frac{(d^B)^2}{2d^B - D^B} \\ \left(\frac{\partial Q_1}{\partial p_1^S}\right)_R &= (D^S)' \frac{(d^B)^2}{2d^B - D^B}. \end{aligned}$$

Thus Q_1 is differentiable with respect to p_1^S and:

$$\frac{\partial Q_1}{\partial p_1^S} = (D^S)' \frac{(d^B)^2}{2d^B - D^B}. \quad (18)$$

Using (21), (17) and (18) we obtain a simple form for the first-order condition for a symmetric equilibrium:

$$\frac{\partial d_i^B}{\partial p_i^B} D^S = (D^S)' \frac{(d^B)^2}{2d^B - D^B},$$

or:

$$\left(\frac{2d^B - D^B}{d^B}\right) \left(-\frac{\partial d_i^B / \partial p_i^B}{d^B}\right) = -\frac{(D^S)'}{D^S}.$$

The first term on the left-hand side of this latter formula is the singlehoming index σ defined earlier, which measures the proportion of “unique customers”. The second term is the ratio of the own-brand elasticity of demand for buyers

$$\eta_o^B = -\frac{p^B \partial d_i^B / \partial p_i^B}{d^B}$$

over the buyers’ price p^B . Finally, the last term is the ratio of the elasticity of sellers’ demand over sellers’ price. Thus we can state:

Proposition 3 : *A symmetric equilibrium of the competition between proprietary platforms is characterized by:*

$$p^B + p^S - c = \frac{p^B}{\eta_o^B} = \frac{p^S}{\eta^S} \sigma.$$

The formulae are thus the same as in the monopoly platform case, except that a) on the buyer side, the demand elasticity η^B is replaced by the (higher) own-brand elasticity η_o^B , and b) on the seller side, the demand elasticity η^S is replaced by the equivalent of an own-brand elasticity η^S/σ . When all buyers singlehome ($\sigma = 1$), the own-brand elasticity and the demand elasticity coincide. But as multihoming becomes more widespread (σ decreases), the possibility of steering increases the own-brand elasticity η^S/σ .

6.2 Competition between associations

When platforms are run by not-for-profit cooperatives owned by members (operators on the buyer and seller sides), prices paid by the end users are set by the members and not by the platforms. Platforms however have an important say in the price structure, especially if competition among members is intense on both sides of the market. In our model, an association’s only strategic decision is the choice of access charges between members. Neglecting platform costs, the zero-profit condition implies that these access “charges” exactly offset each other as one side receives the charge paid by the other side. For example in the payment card industry the access charge is called the interchange fee and is paid

by acquirers (the sellers' banks) to issuers (the buyers' banks).²⁵ This section studies the access charge chosen by competing associations and compares the corresponding prices for final users (buyers and sellers) with those resulting from competition between profit-maximizing systems.

The members compete on two downstream markets, the buyer and the seller downstream markets. Given access charge a_i on platform i , the net marginal costs for a member of platform i of serving a buyer and a seller, respectively, are $c^B - a_i$ and $c^S + a_i$, where c^B and c^S represent the gross marginal costs incurred by the members on each side of the market. We make the simplifying assumption that intraplatform competition results in constant equilibrium margins charged by members on downstream markets: m^B on the buyers' side and m^S on the sellers' side. Equilibrium prices are thus given by:

$$p_i^B = c^B - a_i + m^B, \quad p_i^S = c^S + a_i + m^S.$$

This assumption is for example satisfied if (a) members belong to a single association and are differentiated in a direction orthogonal to that of platform differentiation;²⁶ and (b) members on a given platform are little differentiated. Intense intraplatform competition then results in Hotelling competition between members taking as given (as a first-order approximation) the number of end users on the platform (which is basically determined by the platforms' access charges given that the members' markups are small).²⁷

Under this simplifying assumption, the profits of all members of an association are proportional to the volume of transactions on the association's platform. The interests of

²⁵The determination of access charges within associations has so far only been studied in the context of the payment card industry and under the assumption of a monopoly platform (Rochet-Tirole (2000), Schmalensee (2000)).

²⁶Mathematically, in a generalized Hotelling framework, the "transportation cost" for an end-user when selecting a (platform, member) pair is the sum of the transportation cost to the platform and that to the member.

²⁷If members have dual membership instead (eg. they are both affiliated with Visa and MasterCard, or they provide support or write applications for two cooperatively designed operating systems or videogame platforms), then requirement (b) is unnecessary in that margins are constant even if member differentiation is not small relative to platform differentiation: See Hausman et al. (2001). But one must then inquire into the associations' governance structure. Our treatment carries over as long as governance leads each association to maximize its volume.

all members are thus completely aligned. Regardless of its exact structure the association selects the access charge so as to maximize its volume. Furthermore the total price on each system is constant:

$$p_i^B + p_i^S = c + m, \quad (19)$$

where $m = m^B + m^S$ is the total margin on downstream markets and $c = c^B + c^S$.

Last, in order to be able to compare the association with the cases of a monopolist and of competing proprietary platforms, we must assume that the quasi-demand functions are the same. That is, the members are only selling the varieties of each platform that the proprietary platforms were selling. Because we kept quasi-demand functions quite general, there is no difficulty in assuming this is indeed the case.

The outcome of the competition between the two associations is characterized by two price vectors (p_i^B, p_i^S) , $i = 1, 2$, such that: for all i , (p_i^B, p_i^S) maximizes the volume Q_i on system i subject to (19), taking as given the price vector (p_j^B, p_j^S) on the other system.

The first-order conditions for a symmetric equilibrium are given by

$$p^B + p^S = c + m, \quad (20)$$

(condition on total price) and the equivalent of condition (12):

$$\frac{\partial Q_i}{\partial p_i^B} = \frac{\partial Q_i}{\partial p_i^S}, \quad (21)$$

(same impact on volume of a marginal price increase on each side of the market).

The analysis of the price structure is therefore identical to that for proprietary platforms. The price *level* is lower for associations with healthy competition among their members but may exceed the proprietary platforms price level if double marginalization is strong.

Proposition 4 : *A symmetric equilibrium of the competition between associations, is characterized by*

$$p^B + p^S = c + m$$

and

$$\frac{p^B}{\sigma\eta_o^B} = \frac{p^S}{\eta^S}. \quad (22)$$

Comparing now Proposition 2 and 4, we see that even when downstream markets are perfectly competitive (the margin m converges to zero) and so the price level is socially optimal, competition between not-for-profit associations need not generate an efficient outcome. Indeed, the condition for an efficient price structure (given in Proposition 2) is:

$$\frac{p^B}{\eta^B} \left[\frac{V^B}{2d^B} \right] = \frac{p^S}{\eta^S} \left[\frac{V^S}{d^S} \right], \quad (14)$$

while the condition characterizing competition between associations is different:

$$\frac{p^B}{\sigma\eta_o^B} = \frac{p^S}{\eta^S}. \quad (22)$$

This is natural, as (a) the associations do not internalize the end-users' surpluses, and (b) the associations aim at steering sellers (which is reflected by the presence of σ) and stealing buyers (as indicated by the presence of η_o^B) away from the rival association, while market share considerations play no role in a Ramsey program. It is therefore perhaps remarkable that the two conditions coincide in the special case of linear demands, which we now explore in detail.

7 Comparative statics

Let θ be a parameter that affects the volume of transactions on the platforms. In this section, we consider the impact of a small variation in θ on user prices p^B and p^S , depending on industry structure (monopoly or duopoly) and on the platforms' governance structure (for-profits or associations). We concentrate on two examples:

Marquee buyers: In the first example θ represents a (small) uniform shift in sellers' surpluses, due to the presence of marquee buyers on the other side of the market. As a result, the sellers' demand function becomes:

$$D^S(p^S, \theta) = D^S(p^S - \theta).$$

Installed bases/captive buyers: In the second example, θ represents the (small) mass of buyers who are loyal to their platform, independently of prices. Such buyers say are tied by long-term contracts. As a result, the buyers' demand functions become:

$$d_i^B(p_1^B, p_2^B, \theta) = d_i^B(p_1^B, p_2^B) + \theta, \quad D^B(p^B, \theta) = D^B(p^B) + \theta.$$

We now consider the impact of small variations of θ on the prices p^B and p^S . We start with the case of a monopoly.

7.1 Monopoly

As before we restrict ourselves to the symmetric case, in which the prices chosen by the monopoly are the same on the two platforms:

$$p_1^B = p_2^B = p^B; \quad p_1^S = p_2^S = p^S.$$

Total volume is then given by

$$Q = 2d^B(p^B, \theta)D^S(p^S, \theta).$$

We assume that d^B and D^S are (strictly) log concave with respect to prices, so that the first-order conditions are sufficient for the maximization of volume under a constant margin (case of an association) and for the maximization of profit (case of a for-profit monopoly).

7.1.1 Monopoly association

The buyer price p^B induced by an association is characterized by

$$\varphi(p^B, \theta) = \lambda^B(p^B, \theta) - \lambda^S(c + m - p^B, \theta) = 0,$$

where $\lambda^B(p^B, \theta) = -(d^B(p^B, \theta))' / (d^B(p^B, \theta))$ and $\lambda^S(p^S, \theta) = -(D^S(p^S, \theta))' / (D^S(p^S, \theta))$ denote the “sensitivities” of demands and $c + m$ is the (fixed) total price. We can apply the implicit function theorem to φ , given that $\frac{\partial \varphi}{\partial p^B} > 0$. This is because the strict log

concavity of demands implies that sensitivities are increasing. Thus p^B is differentiable in θ and $\frac{dp^B}{d\theta}$ has the same sign as $(-\frac{\partial\varphi}{\partial\theta})$.

We just have to compute $\frac{\partial\varphi}{\partial\theta}$ in our two examples:

Marquee buyers:

$$\begin{aligned}\varphi(p^B, \theta) &= \lambda^B(p^B) - \lambda^S(c + m - \theta - p^B) \\ \frac{\partial\varphi}{\partial\theta} &= (\lambda^S)' > 0 \quad (\text{since } D^S \text{ is log concave}).\end{aligned}$$

Thus $\frac{dp^B}{d\theta}$ is negative.

Captive buyers:

$$\begin{aligned}\varphi(p^B, \theta) &= \frac{-(d^B)'}{d^B + \theta} - \lambda^S(c + m - p^B) \\ \frac{\partial\varphi}{\partial\theta} &= \frac{(d^B)'}{(d^B + \theta)^2} < 0.\end{aligned}$$

And so $\frac{dp^B}{d\theta}$ is positive.

7.1.2 For-profit monopoly

The maximum of the (log) profit is characterized by two conditions:

$$\begin{cases} \lambda^B(p^B, \theta) - \frac{1}{p^B + p^S - c} = 0 \\ \lambda^S(p^S, \theta) - \frac{1}{p^B + p^S - c} = 0. \end{cases}$$

Denoting by $\varphi(p, \theta)$ the (vector) function on the left-hand side, we can apply the implicit function theorem (this time in R^2) given that the Jacobian $\frac{D\varphi}{Dp}$ is nonsingular (by the concavity of $\log \pi$). Thus we obtain:

$$\frac{dp}{d\theta} = - \left(\frac{D\varphi}{Dp} \right)^{-1} \frac{\partial\varphi}{\partial\theta},$$

where

$$\left(\frac{D\varphi}{Dp} \right)^{-1} = \frac{1}{\det \frac{D\varphi}{Dp}} \begin{bmatrix} \frac{\partial\lambda^S}{\partial p^S} + \frac{1}{(p-c)^2} & -\frac{1}{(p-c)^2} \\ -\frac{1}{(p-c)^2} & \frac{\partial\lambda^B}{\partial p^B} + \frac{1}{(p-c)^2} \end{bmatrix}$$

and

$$\frac{\partial\varphi}{\partial\theta} = \begin{pmatrix} \frac{\partial\lambda^B}{\partial\theta} \\ \frac{\partial\lambda^S}{\partial\theta} \end{pmatrix}.$$

Marquee buyers: $\frac{\partial \lambda^B}{\partial \theta} = 0$, $\frac{\partial \lambda^S}{\partial \theta} < 0$

$$\begin{pmatrix} \frac{dp^B}{d\theta} \\ \frac{dp^S}{d\theta} \end{pmatrix} = \frac{-\frac{\partial \lambda^S}{\partial \theta}}{\det \frac{D\varphi}{Dp}} \begin{bmatrix} -\frac{1}{(p-c)^2} \\ \frac{\partial \lambda^B}{\partial p^B} + \frac{1}{(p-c)^2} \end{bmatrix}.$$

Thus $\frac{dp^B}{d\theta} < 0$ and $\frac{dp^S}{d\theta} > 0$.

Captive buyers: $\frac{\partial \lambda^B}{\partial \theta} < 0$, $\frac{\partial \lambda^S}{\partial \theta} = 0$, and so:

$$\begin{pmatrix} \frac{dp^B}{d\theta} \\ \frac{dp^S}{d\theta} \end{pmatrix} = \frac{-\frac{\partial \lambda^B}{\partial \theta}}{\det \frac{D\varphi}{Dp}} \begin{bmatrix} \frac{\partial \lambda^S}{\partial p^S} + \frac{1}{(p-c)^2} \\ -\frac{1}{(p-c)^2} \end{bmatrix}$$

Thus $\frac{dp^B}{d\theta} > 0$ and $\frac{dp^S}{d\theta} < 0$.

We can therefore summarize our results in a single proposition.

Proposition 5 : *In the case of a monopoly platform (for-profit or association) and with log concave demand functions, the seller price increases when there are marquee buyers and decreases when there are captive buyers. The buyer price moves in the opposite direction.*

7.2 Duopoly

We restrict ourselves to the case of associations, for which the equilibrium buyer price is characterized by:

$$\lambda_0^B(p^B, \theta)\sigma(p^B, \theta) - \lambda^S(c + m - p^B, \theta) = 0,$$

where

$$\lambda_0^B(p^B, \theta) = \frac{-\frac{\partial d_1^B}{\partial p_1^B}}{d_1^B}(p^B, p^B, \theta)$$

is the “own-price sensitivity” of buyer demand and

$$\sigma(p^B, \theta) = 2 - \frac{D^B(p^B, \theta)}{d^B(p^B, \theta)}.$$

In order to determine the monotonicity properties of p^B with respect to θ , we apply the implicit function theorem to the left-hand side of the above equation:

$$\psi(p^B, \theta) = \lambda_0^B(p^B, \theta)\sigma(p^B, \theta) - \lambda^S(c + m - p^B, \theta).$$

However we need additional assumptions to ensure that $\frac{\partial \psi}{\partial p^B} > 0$, so that $p^B(\theta)$ is indeed (locally) unique and differentiable, for two reasons:

- Possible nonexistence of equilibrium, due to the fact that the volume on system i is not necessarily quasiconcave with respect to (p_i^B, p_i^S) . Section 8 will observe that the candidate for equilibrium (i.e. the solution of $\psi = 0$) may sometimes be destabilized by “double deviations” of the form $(p^B + \varepsilon, p^S - \varepsilon)$.
- The possible presence of strategic complementarities that may generate a multiplicity of equilibria.

We will assume away these difficulties and postulate that $\frac{\partial \psi}{\partial p^B} > 0$ (regularity condition).²⁸ In this case, $p^B(\theta)$ is (locally) unique, differentiable, and $\frac{dp^B}{d\theta}$ has the same sign as $-\frac{\partial \psi}{\partial \theta}$. We then just have to determine the sign of $\frac{\partial \psi}{\partial \theta}$:

Marquee buyers:

$$\begin{aligned}\psi(p^B, \theta) &= \lambda_0^B(p^B)\sigma(p^B) - \lambda^S(c + m - \theta - p^B) \\ \frac{\partial \psi}{\partial \theta} &= (\lambda^S)' > 0.\end{aligned}$$

Captive buyers:

$$\begin{aligned}\psi(p^B, \theta) &= \left(\frac{-\frac{\partial d_1^B}{\partial p_1^B}}{d_1^B + \theta} \right) \left(2 - \frac{D^B + \theta}{d^B + \theta} \right) - \lambda^S(c + m - p^B) \\ \frac{\partial \psi}{\partial \theta} &= -\frac{\lambda_0^B \sigma}{d^B + \theta} - \frac{d^B - D^B}{(d^B + \theta)^2} \lambda_0^B = \frac{-\lambda_0^B}{d^B + \theta} \left[\sigma + \frac{d^B - D^B}{d^B + \theta} \right] < 0.\end{aligned}$$

An increase in the number of captive buyers has two opposite effects. First, and as in the monopoly case, the captive customers reduce the elasticity of buyer demand, calling for a higher buyer price. Second, captive customers make steering more attractive, which pushes toward a higher seller price. The first effect dominates the second.

Proposition 6 : *In the case of competing associations and under log concave demands and the regularity condition $\frac{\partial \psi}{\partial p^B} > 0$, the seller price increases when there are marquee*

²⁸This regularity condition is satisfied when $\frac{\partial \lambda_0^B}{\partial p^B}$ and $\frac{\partial \sigma}{\partial p^B} > 0$.

buyers and decreases when there are captive buyers. The buyer price moves in the opposite direction.

8 Linear demands

We illustrate the results obtained so far in a variant of the Hotelling model, where a buyer's preferences for platforms are represented by his location x on a line. Platform 1 and 2 are symmetrically located at a distance $\Delta/2$ of the origin of the line ($x = -\frac{\Delta}{2}$ for platform 1 and $x = \Delta/2$ for platform 2). The number Δ parametrizes the degree of substitutability between platforms. Buyers have also access to outside options, represented conventionally by two other symmetric platforms (denoted 1' and 2'), located further away from the origin ($x = -\frac{\Delta}{2} - \delta$ and $x = \frac{\Delta}{2} + \delta$) and charging the same, exogenous, price p_0 . The number δ will serve us as a measure of the weight of "unique consumers". When using a platform located at distance d , buyers incur a quadratic transportation cost $\frac{1}{2}d^2$, (the transportation cost parameter is normalized to 1 without loss of generality).

8.1 Comparing price structures

Assuming a uniform distribution of buyers on the line, and letting $T = p_0 + \frac{\delta(\Delta+\delta)}{2}$, the quasi-demands are given by:²⁹

$$d_1^B(p_1^B, p_2^B) = \frac{p_2^B - p_1^B}{\Delta} + \frac{T - p_1^B}{\delta}, \quad (23)$$

and

$$D_1^B(p_1^B) = (T - p_1^B) \left(\frac{1}{\delta} + \frac{1}{\Delta + \delta} \right). \quad (24)$$

²⁹The expressions of quasi-demands are easily deduced from the locations of marginal buyers:

- x_1 is indifferent between 1 and 1': $p_1^B + \frac{1}{2} (x_1 + \frac{\Delta}{2})^2 = p_0 + \frac{1}{2} (x_1 + \frac{\Delta}{2} + \delta)^2$, which gives:
 $x_1 = \frac{p_1^B - p_0}{\delta} - \frac{\Delta + \delta}{2}$;
- x_2 is indifferent between 1 and 2: $p_1^B + \frac{1}{2} (x_2 + \frac{\Delta}{2})^2 = p_2^B + \frac{1}{2} (x_2 - \frac{\Delta}{2})^2$, which gives: $x_2 = \frac{p_2^B - p_1^B}{\Delta}$;
- x_3 is indifferent between 1 and 2': $p_1^B + \frac{1}{2} (x_3 + \frac{\Delta}{2})^2 = p_0 + \frac{1}{2} (x_3 - \frac{\Delta}{2} - \delta)^2$, which gives:
 $x_3 = \frac{p_0 - p_1^B}{\Delta + \delta} + \delta/2$.

The expressions of d_2^B and D_2^B are obtained by symmetry. Due to the linearity of these expressions, several simplifications appear. For example, the singlehoming index is price independent:

$$\sigma = 2 - \frac{D_1^B(p^B)}{d_1^B(p^B, p^B)} = \frac{\Delta}{\Delta + \delta}. \quad (25)$$

Similarly the expression of the marginal seller (who is indifferent between multihoming and singlehoming with the cheapest platform), does not depend on buyers' prices. For example, when $p_1^S \leq p_2^S$, formula (5) gives:

$$\hat{b}_{12} = p_2^S \frac{d_2^B}{d_1^B + d_2^B - D_1^B} + p_1^S \frac{d_1^B - D_1^B}{d_1^B + d_2^B - D_1^B}.$$

Using (23) and (24) we get (for $p_1^B = p_2^B = p^B$):

$$\hat{b}_{12} = p_2^S + \frac{\delta}{\Delta}(p_2^S - p_1^S). \quad (26)$$

and so \hat{b}_{12} does not depend on p^B . Furthermore, steering is particularly powerful (in that undercutting induces many sellers to stop multihoming) when most consumers multihome, that is when σ is low.

Another simplification that appears when buyers' quasi-demand is linear is that the ratio of total elasticity to own-brand elasticity is equal to the single homing index σ :

$$\frac{\eta^B}{\eta_o^B} = \frac{\frac{\partial d_1^B}{\partial p_1^B} + \frac{\partial d_1^B}{\partial p_2^B}}{\frac{\partial d_1^B}{\partial p_1^B}} = \frac{\frac{1}{\delta}}{\frac{1}{\Delta} + \frac{1}{\delta}} = \frac{\Delta}{\Delta + \delta} = \sigma.$$

This property implies that the price structure under platform competition (between for-profits or between associations) is the same as under a monopoly platform:

$$\frac{p^B}{\eta^B} = \frac{p^S}{\eta^S}.$$

Consider for example a decrease in Δ . As the platforms become more substitutable, buyer multihoming increases (σ falls); this induces platforms to steer, resulting in low prices on the seller side. However, competition also becomes more intense on the buyer

side, resulting in lower buyer prices (p^B falls) and thereby in a higher opportunity cost ($c - p^B$) of servicing sellers. For linear demand on the buyer side, these two effects offset.

Last, let us compare the common price structure with that of the Ramsey optimum. A useful property of linear demands is that the revenue (price times quantity) is equal to twice the product of the net surplus and the elasticity of demand. This property implies that if seller's quasi-demand is linear as well, (14) is equivalent to (12), and so the common price structure is Ramsey optimal.

We gather the findings of this section in:

Proposition 7 : *Suppose that the buyers' quasi-demand is described by an Hotelling model, with uniform distribution and outside options with distance Δ between the two platforms and distance δ between each platform and its nearest outside option, and that the market is not covered (not all potential buyers buy).*

(i) • *The buyer singlehoming index is equal to:*

$$\sigma = \Delta / (\Delta + \delta),$$

and decreases when the platforms become more substitutable.

• *The platforms' ability to steer (discourage through undercutting sellers from multi-homing) decreases with the buyer singlehoming index.*

• *On the buyer side, total elasticity is equal to own-brand elasticity times the single-homing index:*

$$\eta^B = \eta_o^B \sigma.$$

(ii) *The price structure is the same under a monopoly platform, competing proprietary platforms and competing associations. It satisfies*

$$\frac{p^B}{\eta^B} = \frac{p^S}{\eta^S}.$$

(iii) *If furthermore seller demand is linear, then the price structure in the three environments is Ramsey optimal.*

8.2 Second-order conditions

The study of second-order conditions under platform competition is complex in general. We content ourselves with the case of linear demand on both sides. So let us we assume that the sellers' quasi-demand is also linear:

$$D^S(p^S) = A - p^S. \quad (27)$$

The volume on platform 1 when $p_1^S \leq p_2^S$ is:

$$\begin{aligned} Q_1 = & \left(\frac{p_2^B - p_1^B}{\Delta} + \frac{T - p_1^B}{\delta} \right) \left[A - p_2^S - \frac{\delta}{\Delta} (p_2^S - p_1^S) \right] \\ & + \left(\frac{1}{\delta} + \frac{1}{\Delta + \delta} \right) (T - p_1^B) (p_2^B - p_1^S) \left(1 + \frac{\delta}{\Delta} \right). \end{aligned} \quad (28)$$

When $p_1^S > p_2^S$, the expression is simpler:

$$Q_1 = \left[A - p_1^S - \frac{\delta}{\Delta} (p_1^S - p_2^S) \right] \left[\frac{p_2^B - p_1^B}{\Delta} + \frac{T - p_1^B}{\delta} \right]. \quad (29)$$

In Proposition 2 we have shown that a symmetric equilibrium between competing associations must satisfy condition (22):

$$\frac{p^B}{\sigma \eta_o^B} = \frac{p^S}{\eta^S}. \quad (22)$$

Using formulae (23), (25) and (27) and after simplifications, this condition becomes:

$$p^B - p^S = T - A. \quad (30)$$

Recall that this condition is necessarily satisfied in a symmetric equilibrium between competing platforms, independently of their governance structure. However the value of the total price is different:

$$p^B + p^S = c + m$$

for associations, and

$$p^B + p^S - c = \frac{\Delta}{\Delta + \delta} (T - p^B) = \frac{\Delta}{\Delta + \delta} (A - p^S),$$

for proprietary platforms. The resulting equilibrium prices are:

$$p_A^B = \frac{1}{2}(c + m + T - A), \quad p_A^S = \frac{1}{2}(c + m - T + A), \quad (31)$$

for associations, and

$$p_P^B = \frac{c - A + T \left(1 + \frac{\Delta}{\Delta + \delta}\right)}{2 + \frac{\Delta}{\Delta + \delta}}, \quad p_P^S = \frac{c - T + A \left(1 + \frac{\Delta}{\Delta + \delta}\right)}{2 + \frac{\Delta}{\Delta + \delta}} \quad (32)$$

for proprietary systems.

These formulae only give necessary conditions for a symmetric equilibrium. However, sufficient conditions are easy to check, given that volume Q_i is bilinear with respect to prices (p_i^B, p_i^S) in each region $p_1^S < p_2^S$ and $p_1^S > p_2^S$.

Proposition 8 : *The price vectors given in formulae (31) and (32) satisfy the second-order conditions for an equilibrium if and only if $\frac{\delta}{\Delta}$ is smaller than $\frac{1+\sqrt{5}}{2}$.*

Proof: See Appendix.

When $\frac{\delta}{\Delta} > \frac{1+\sqrt{5}}{2}$, there is no symmetric equilibrium in pure strategies. Starting from the price vectors given in formula (31), a proprietary platform can obtain a (second order) increase in volume (and thus profit) by a “double deviation”: a small increase of p_1^B combined with a small decrease of p_1^S (while maintaining the total price constant). The same is true for associations, at the price vectors given in formula (32).

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APPENDIX: Proof of Proposition 6

$$\begin{aligned}
\pi_i &= (p_i^B + p_i^S - c)Q_i; \\
\frac{\partial \pi_i}{\partial p_i^B} &= Q_i + (p_i^B + p_i^S - c)\frac{\partial Q_i}{\partial p_i^B}; \\
\frac{\partial \pi_i}{\partial p_i^S} &= Q_i + (p_i^B + p_i^S - c)\frac{\partial Q_i}{\partial p_i^S}; \\
\frac{\partial^2 \pi_i}{(\partial p_i^B)^2} &= 2\frac{\partial Q_i}{\partial p_i^B}; \quad \frac{\partial^2 \pi_i}{(\partial p_i^S)^2} = 2\frac{\partial Q_i}{\partial p_i^S}; \\
\frac{\partial^2 \pi_i}{\partial p_i^B \partial p_i^S} &= \frac{\partial Q_i}{\partial p_i^B} + \frac{\partial Q_i}{\partial p_i^S} + (p_i^B + p_i^S - c)\frac{\partial^2 Q_i}{\partial p_i^B \partial p_i^S}.
\end{aligned}$$

At a symmetric equilibrium of the game between competing proprietary platforms, we have

$$\frac{\partial Q_i}{\partial p_i^B} = \frac{\partial Q_i}{\partial p_i^S} = -(p - c)\frac{(\Delta + \delta)^2}{\Delta^2 \delta} \equiv \alpha < 0.$$

Therefore the second-order condition is satisfied whenever the Hessian determinant of π_i is nonnegative:

$$\begin{aligned}
H &= \frac{\partial^2 \pi_i}{(\partial p_i^B)^2} \cdot \frac{\partial^2 \pi_i}{(\partial p_i^S)^2} - \left(\frac{\partial^2 \pi_i}{\partial p_i^B \partial p_i^S} \right)^2 \\
H &= 4\alpha^2 - (2\alpha + \beta)^2 = -\beta(4\alpha + \beta),
\end{aligned}$$

where

$$\beta \equiv (p_i^B + p_i^S - c)\frac{\partial^2 Q_i}{\partial p_i^B \partial p_i^S}$$

has a different expression in the two regions:

$$\begin{aligned}
\beta_1 &= (p - c)\frac{(\Delta^2 + \delta\Delta - \delta^2)}{\Delta^2 \delta} \text{ when } p_1^S < p_2^S, \text{ and} \\
\beta_2 &= (p - c)\frac{(\Delta + \delta)^2}{\Delta^2 \delta} \text{ when } p_1^S > p_2^S.
\end{aligned}$$

The second-order condition is always satisfied in the second region, since $\beta_2 = -\alpha > 0$ so that $H = 3\alpha^2 > 0$. In the first region, it is easy to see that $\beta_1 + 4\alpha$ is always negative. Thus the second-order condition is satisfied if and only if $\beta_1 \geq 0$, which is equivalent to

$$\delta^2 - \delta\Delta - \Delta^2 \leq 0$$

or

$$\frac{\delta}{\Delta} \leq \frac{1 + \sqrt{5}}{2}.$$

When this condition is not satisfied, there is no symmetric equilibrium in pure strategies. The only equilibrium candidate (p^B, p^S) can be destabilized by a “double-deviation”, where one of the platforms (say platform 1) increases p_1^B by ε and simultaneously decreases p_1^S by the same amount. The first order increase in profit is zero (as guaranteed by the first-order conditions) but the second-order increase is positive:

$$\frac{\partial \pi_1}{\partial \varepsilon} = \left[\frac{\partial^2 \pi_1}{(\partial p_1^B)^2} + \frac{\partial^2 \pi_1}{(\partial p_1^S)^2} - 2 \frac{\partial^2 \pi_1}{\partial p_1^B \partial p_1^S} \right] \varepsilon^2 = -2\beta_1 \varepsilon^2 > 0.$$