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**QUANTITY CONTROLS IN IMPERFECTLY  
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# Quantity controls in imperfectly competitive economies

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## Abstract

In this note we study the optimal design of quantity controls in a two-consumption-good economy where one good is produced by a competitive industry and the other by a monopolist. We show that, when price controls are not used, it is always possible to design the quantity controls (either in the competitive or in the non-competitive sector) in such a way that the inefficiently high price of the non-competitive good is reduced; in some cases, we are able to identify precisely the circumstances in which consumption *floors* or *ceilings* are best suited to this end. We also argue that when the use of price controls introduces additional distortions in the economy, the design of quantity controls may be altered, as it may be preferable to use them to reduce the welfare cost of price controls rather than directly against market imperfections.

*JEL Classification:* D42, H21, H42

*Keywords:* quantity controls, imperfect competition, rationing, in-kind transfers, optimal indirect taxes

## 1 Introduction

Two facts concerning imperfectly competitive markets on which most of us would agree are that they are endemic in important areas of real-world economies, and that their presence calls for some form of public intervention for remedying the ensuing inefficiencies. These two facts alone

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would justify a large public finance literature on optimal fiscal and public expenditure policy in imperfectly competitive economies; instead, works on this subject are not particularly common.

There are of course exceptions. For our purposes here, a notable exception is provided by Myles (1987, 1989), who studies optimal indirect taxation in models with imperfect competition.<sup>1</sup> The main insight conveyed by Myles' work is that optimal taxes can indeed be designed so as to reduce the distortions induced by the absence of perfect competition; in particular, he studies the conditions under which it is desirable to subsidize the price of the goods produced by the imperfectly competitive industries. Given that non-competitive goods are sold at an inefficiently high price and the social planner is restricted to using commodity taxes, the best it can hope to do is to reduce that price by subsidizing it. Since however this requires collecting revenue by taxing the goods traded on the competitive markets, it is not always the case that the benefits of such policy exceed its costs; Myles is however able to identify a set of sufficient conditions for that.

In this paper, we study optimal policies in economies with non-competitive sectors assuming that the planner has at its disposal not only *price* controls (taxes and subsidies), but also *quantity* controls (rations and transfers). Real-world governments tend quite often to intervene not only altering the price at which some goods are exchanged, but also ruling that the consumption of certain goods cannot go below a given floor (e.g. compulsory education) or above a given ceiling (e.g. restrictions to the use of fuels). While there is by now a well-established theory of quantity controls in perfectly competitive economies,<sup>2</sup> the question of their usefulness in the case of imperfect competition is still largely unexplored. In particular, intuition suggests that, when markets are non-competitive, quantity controls may have a role in correcting market imperfections, quite independently of whether price controls are used or not; in the competitive market case, instead, quantity controls can only be used alongside tax instruments.

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<sup>1</sup>Other contributions on indirect taxation in imperfectly competitive markets include Guesnerie and Laffont (1978), Stern (1987), Gabszewicz and Grazzini (1999) and Reinhorn (2000). Others have studied the taxation of intermediate goods – see e.g. Colangelo and Galmarini (2001) and the references therein.

<sup>2</sup>The seminal contribution is Guesnerie and Roberts (1984); recent papers include Munro (1991), Boadway and Marchand (1995) and Blomquist and Christiansen (1995) – see however Balestrino (2000) and Boadway and Keen (2000) for reviews and more complete lists of references.

## 2 The model

Consider an economy consisting of a large number of identical individuals. There are two consumption goods,  $x$  and  $y$ , both produced using labour only. Each consumer is endowed with a unit of labour; the latter is traded on a competitive market and is taken to be the *numeraire*.<sup>3</sup> Good  $x$  is produced by a competitive industry, while good  $y$  is produced by a monopolist.<sup>4</sup>

To avoid the problem concerning the distribution of profits to consumers and their effect upon firm's demand, we assume that the government imposes a 100% tax on profits. Besides that, the government has at its disposal four policy instruments: it can impose per unit taxes/subsidies<sup>5</sup> and quantity constraints on both goods. The latter may take the form of consumption *floors* or *ceilings*. In the first case, which we refer to as an *in-kind transfer scheme*, the minimum consumption level can be topped up freely, but the good cannot be resold. In the second case, which we refer to as a *rationing scheme*, the maximum consumption level cannot be supplemented. These assumptions are needed to ensure the effectiveness of quantity controls, i.e. to make them distinguishable from cash transfers. Of course, their plausibility may be questioned for at least some types of goods – after all, secondary markets may be hard to deter in practice, unless the commodity in question is immaterial (a service) or difficult to transport and/or store. Since the imperfectly competitive good may or may not be amenable to quantity controls, it then becomes relevant to investigate also quantity controls for perfectly competitive goods which are linked to the non-competitive one via a complementarity/substitutability relationship. Indeed, since the analysis of this latter case is less straightforward and requires more detailed explanations, we present it first and then rely on it to illustrate more briefly the other case.

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<sup>3</sup>Normalisation in imperfectly competitive economies is far from innocuous, as it has real effects on the equilibrium – see Gabsewicz and Vial (1972). However, it can be shown that “the equilibrium is invariant to any normalisation rule that is defined as a function of competitive goods prices but is independent of the prices of goods traded on imperfectly competitive markets” (Myles 1995, p. 351); our selection of the *numeraire* follows Myles (1987, 1989).

<sup>4</sup>The extension to an oligopolistic market using a conjectural variations approach is relatively straightforward, although it would require some technical assumptions to make the analysis viable.

<sup>5</sup>As it is well-known, specific and ad valorem taxes are not equivalent under imperfect competition – see e.g. Delipalla and Keen (1992). A discussion of this issue is however beyond the scope of the present paper.

## 2.1 Consumers

All individuals' preferences are represented by the same quasi-concave and increasing utility function  $U(x, y, h)$ , with  $h$  denoting leisure. The consumer prices for the two goods are denoted  $q_z$ ,  $z = x, y$  – we will discuss in a moment how they are determined. As a point of reference, we consider the free (i.e. without quantity controls) maximization problem. In that case, the consumer chooses  $x$ ,  $y$  and  $h$  so as to

$$\{\max U(x, y, h) \mid q_x x + q_y y + h = 1\}. \quad (1)$$

Let  $\{\hat{x}, \hat{y}, \hat{h}\}$  denote the solution to problem (1); for the reasons given above, we focus first on the case in which the government operates its quantity controls only in the market for good  $x$ , with  $\bar{x}$  denoting the government-controlled quantity.<sup>6</sup> For our discussion of policy design later on, it is useful to assume that, initially, the government fixes the constraint at  $\bar{x} = \hat{x}$ . This way, the behavior of the consumer is unaffected – the quantity control is effectively non-operative. The introduction of a quantity control will take the form of infinitesimal variations around  $\hat{x}$ ; a marginal increase will represent a move towards an in-kind transfer scheme, whereas a marginal reduction will represent a move towards a rationing scheme. Let  $z(\bar{x}, q_x, q_y)$ ,  $z = x, y$ , denote the demand for good  $z$  arising from the constrained problem; since at  $\bar{x} = \hat{x}$  this problem will generate the same allocation as the free problem, we have that<sup>7</sup>

$$x(\hat{x}, q_x, q_y) = 0; \quad \frac{\partial x(\cdot)}{\partial \hat{x}} = -1. \quad (2)$$

Furthermore, we can use the theory of behavior under rationing (Neary and Roberts, 1980) to argue that

$$\text{sign} \left( \frac{\partial y(\cdot)}{\partial \hat{x}} \right) = -\text{sign}(S_{xy}), \quad (3)$$

where  $S_{xy}$  denotes the cross-price Slutsky term for the free maximization problem.<sup>8</sup> Intuitively, when  $x$  and  $y$  are Hicksian complements (substitutes), an increase in the government-imposed quantity will increase (reduce) the demand for the other good.

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<sup>6</sup>The case in which  $y$  is subject to quantity control is perfectly symmetric – we will sketch it in Section 4.

<sup>7</sup>To avoid clutter, we will slightly abuse the notation throughout the paper and write  $\partial \gamma(\cdot) / \partial \hat{z}$  to denote the derivative of a generic function  $\gamma$  w.r.t. the quantity control evaluated at  $\bar{z} = \hat{z}$ .

<sup>8</sup>Let  $\pi$  denote the *virtual* price, i.e. the one at which the consumer would have bought exactly the government-controlled quantity if she were free to choose. Using a result in Neary and Roberts (1980), we have that

$$\frac{\partial y}{\partial \hat{x}} = \underbrace{\frac{\partial \tilde{y}^c}{\partial \pi} \frac{\partial \pi}{\partial \hat{x}}}_{\text{substitution effect}} + (\pi - q_x^*) \underbrace{\left( \frac{\partial y}{\partial B} - \frac{\partial \tilde{y}^c}{\partial \pi} \frac{\partial \pi}{\partial \hat{x}} \frac{\partial \hat{x}}{\partial B} \right)}_{\text{income effect}},$$

Substituting the solution to the constrained problem back into the maximand gives indirect utility,  $V(\widehat{x}, q_x, q_y)$ , with derivatives

$$\frac{\partial V}{\partial \widehat{x}} = \frac{\partial U}{\partial \widehat{x}} - \alpha q_x = 0; \quad \frac{\partial V}{\partial q_x} = -\alpha(\widehat{x} + x(\cdot)); \quad \frac{\partial V}{\partial q_y} = -\alpha y(\cdot), \quad (4)$$

where  $\alpha$  is the marginal utility of income.

## 2.2 Producers

Good  $x$  is produced within a constant-returns-to-scale competitive sector; it takes  $p$  units of labour to produce one unit of  $x$ . Let  $t_x$  denote the tax rate on good  $x$ . Thus, its consumer price is immediately determined as

$$q_x^* = p + t_x. \quad (5)$$

The consumer price of good  $y$  is instead obtained by the outcome of the profit-maximization problem of the monopolist. From the solution of the consumer problem, we can invert the demand for  $y$  to obtain  $q_y = f(\widehat{x}, t_x, y)$ . The technology is again constant-returns-to scale, with  $c$  denoting the average and marginal cost. Let  $t_y$  denote the tax rate on good  $y$ . The monopolist's profit is therefore  $\pi = (f(\cdot) - t_y - c)y$ . Maximizing over  $y$  yields

$$\frac{\partial \pi}{\partial y} = f(\cdot) - t_y - c + y \frac{\partial f}{\partial y} = 0, \quad (6)$$

with second order condition

$$\frac{\partial^2 \pi}{\partial y^2} = 2 \frac{\partial f}{\partial y} + y \frac{\partial^2 f}{\partial y^2} < 0. \quad (7)$$

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where the “tilde” denotes the “unconstrained” demand, that is the one that would be generated if the virtual price would prevail (as opposed to the “constrained” demand, that is the one arising from the actual presence of a quantity control), the superscript  $c$  denotes the compensated demand, and  $B$  is lump-sum income (zero in our model). Neary and Roberts (1980) show that all the properties of the constrained demand function may be expressed in terms of the properties of the unconstrained demand function, evaluated at the virtual price. The above expression is just an application of this general rule. The substitution effect, if  $x$  and  $y$  are net substitutes (complements), tends to reduce (increase) the demand for  $y$ . The income effect instead is zero in our case: for a marginal variation of the control around the equilibrium quantity  $\widehat{x}$ , the virtual price equals the actual price ( $\pi = q_x^*$ ), and thus the second term on the r.h.s. is zero. Now, note that when  $\pi = q_x^*$ , the constrained demand equals the unconstrained demand by definition of the two; we then have

$$\frac{\partial y}{\partial \widehat{x}} = \left( \frac{\partial \widetilde{y}^c}{\partial \pi} \frac{\partial \pi}{\partial \widehat{x}} \right) \Big|_{\pi=q_x^*} = \left( S_{yx} \frac{\partial \pi}{\partial \widehat{x}} \right) \Big|_{\pi=q_x^*},$$

from which eq. (3) immediately follows since the virtual price is clearly decreasing in the quantity control.

Let the solution to (6) be  $y^*(\hat{x}, t_y, t_x)$ ; using the inverse demand function determines an equilibrium price on the market for  $y$ ,

$$q_y^* = f(\hat{x}, t_x, y^*(\hat{x}, t_y, t_x)). \quad (8)$$

### 2.3 Incidence

It will be useful to know the effects of the policy instruments on the consumer price of goods  $x$  and  $y$ . For  $q_x^*$ , it is immediate to see that

$$\frac{\partial q_x^*}{\partial t_x} = 1; \quad \frac{\partial q_x^*}{\partial t_y} = \frac{\partial q_x^*}{\partial \hat{x}} = 0. \quad (9)$$

Accordingly, a tax on good  $x$  is shifted 100% forward and the consumer price of good  $x$  is affected neither by the tax on good  $y$  nor by the quantity control because of the assumption that the producer price is fixed in the competitive industry.

As for  $q_y^*$ , we begin by totally differentiating (6) to find the impact on the equilibrium quantity  $y^*$ :

$$\left( \frac{\partial f}{\partial \hat{x}} + y \frac{\partial^2 f}{\partial y \partial \hat{x}} \right) d\hat{x} + \left( \frac{\partial f}{\partial t_x} + y \frac{\partial^2 f}{\partial y \partial t_x} \right) dt_x - dt_y + \left( 2 \frac{\partial f}{\partial y} + y \frac{\partial^2 f}{\partial y^2} \right) dy = 0. \quad (10)$$

From (10), the effect of each tax instrument (taken in isolation) on  $y^*$  obtains as

$$\frac{\partial y^*}{\partial t_y} = \frac{1}{2 \frac{\partial f}{\partial y} + y \frac{\partial^2 f}{\partial y^2}}; \quad \frac{\partial y^*}{\partial t_x} = - \frac{\frac{\partial f}{\partial t_x} + y \frac{\partial^2 f}{\partial y \partial t_x}}{2 \frac{\partial f}{\partial y} + y \frac{\partial^2 f}{\partial y^2}}; \quad \frac{\partial y^*}{\partial \hat{x}} = - \frac{\frac{\partial f}{\partial \hat{x}} + y \frac{\partial^2 f}{\partial y \partial \hat{x}}}{2 \frac{\partial f}{\partial y} + y \frac{\partial^2 f}{\partial y^2}}. \quad (11)$$

Now, from (8), we can identify the expressions yielding the incidence effects of the policy tools:

$$\frac{\partial q_y^*}{\partial t_y} = \frac{\partial f}{\partial y} \frac{\partial y^*}{\partial t_y}; \quad \frac{\partial q_y^*}{\partial t_x} = \frac{\partial f}{\partial t_x} + \frac{\partial f}{\partial y} \frac{\partial y^*}{\partial t_x}; \quad \frac{\partial q_y^*}{\partial \hat{x}} = \frac{\partial f}{\partial \hat{x}} + \frac{\partial f}{\partial y} \frac{\partial y^*}{\partial \hat{x}}, \quad (12)$$

where the derivatives of  $y^*$  w.r.t. the policy tools are given by (11).

The first expression in (12), termed the “degree of forward shifting” of  $t_y$  (*cf.* Myles, 1995), is positive by (7). The situation in which  $\partial q_y^*/\partial t_y$  exceeds (falls short of) unity is usually called “overshifting” (“undershifting”), meaning that any increase in the tax determines a more (less) than proportional increase in the consumer price. It is easy to see that overshifting occurs if the inverse demand function exhibits a sufficiently high degree of convexity –  $\partial^2 f/\partial y^2$  must be positive and large enough, otherwise  $\partial q_y^*/\partial t_y$  cannot exceed unity. For concave and moderately convex inverse demand functions, we will have undershifting.<sup>9</sup>

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<sup>9</sup>See Delipalla and O’Donnell (2001) for a recent empirical analysis on tax incidence, in the case of the European cigarette industry. For a group of countries with broadly similar cigarette industries, they show that there is evidence of undershifting of both ad valorem and specific taxes.

The second and third expressions are instead generally ambiguous in sign.<sup>10</sup> In the policy analysis, we will consider several alternative combinations of the signs of the incidence effects of  $\hat{x}$  and  $t_x$ , and, in some cases, we will use restrictions on the form of the utility function to gain more insights on the matter.

### 3 Quantity controls in the competitive industry

We now study the design of quantity controls under the two alternative assumptions that such controls are used in isolation or jointly with price controls, i.e. indirect taxes. Since all individuals are identical, social welfare is simply represented by the indirect utility function of a representative consumer,  $V(\hat{x}, q_x^*, q_y^*)$ . Tax revenue evaluated at  $\hat{x}$  is given by

$$t_x (\hat{x} + x(\hat{x}, q_x^*, q_y^*)) + t_y y(\hat{x}, q_x^*, q_y^*) = 0, \quad (13)$$

where we assume a zero revenue requirement, so that non-zero tax rates will emerge at the optimum only if they correct some pre-existing distortion (also we will always have one negative and one positive tax).

The policy problem is then to maximise indirect utility subject to (13). Setting up the Lagrangian of the government problem and then differentiating it w.r.t. to the quantity control gives us the following expression (evaluated at  $\hat{x}$ ):

$$\frac{\partial V}{\partial \hat{x}} + \frac{\partial V}{\partial q_y^*} \frac{\partial q_y^*}{\partial \hat{x}} + \mu \left( t_x \left( 1 + \frac{\partial x(\cdot)}{\partial \hat{x}} + \frac{\partial x}{\partial q_y^*} \frac{\partial q_y^*}{\partial \hat{x}} \right) + t_y \left( \frac{\partial y}{\partial \hat{x}} + \frac{\partial y(\cdot)}{\partial q_y^*} \frac{\partial q_y^*}{\partial \hat{x}} \right) \right). \quad (14)$$

Using (4) and (2) this simplifies to

$$\frac{\partial V}{\partial q_y^*} \frac{\partial q_y^*}{\partial \hat{x}} + \mu \left( t_x \frac{\partial x}{\partial q_y^*} \frac{\partial q_y^*}{\partial \hat{x}} + t_y \left( \frac{\partial y(\cdot)}{\partial \hat{x}} + \frac{\partial y}{\partial q_y^*} \frac{\partial q_y^*}{\partial \hat{x}} \right) \right). \quad (15)$$

In fact, (15) gives us the total impact of quantity controls on social welfare as the sum of a welfare effect (the first term) and a revenue effect (the second term). If the sum of these effects is positive (negative), the government will find it desirable to increase (decrease) marginally the quantity constraint, i.e. to operate an in-kind transfer (rationing) scheme for  $x$ .<sup>11</sup>

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<sup>10</sup>It would be tempting to argue i) that they should have opposite sign (an increase in the quantity control should work in the same way as a price reduction) and, more specifically, ii) that complementarity between  $x$  and  $y$  should work in favour of making  $\partial q_y^*/\partial t_x$  positive and  $\partial q_y^*/\partial \hat{x}$  negative (with substitutability working in the opposite direction). It turns out, however that the changes in  $t_x$  and  $\hat{x}$  generate a rather complex chain of effects on the equilibrium price of  $y$ , and that neither statement is true in general.

<sup>11</sup>This is a local result. In general, we do not know whether the maximisation problem is globally concave in  $\hat{x}$ . However, this limitation is common to virtually the whole literature on second-best policy.



### 3.1 Welfare-improving quantity controls

Consider first the case in which taxes are not used. At  $t_x = t_y = 0$ , (15) reduces to the welfare term. Since  $\partial V/\partial q_y^* < 0$ , the term has the opposite sign of  $\partial q_y^*/\partial \hat{x}$ . Then, we can easily deduce the following: first, since  $\partial q_y^*/\partial \hat{x}$  is in general non-zero, quantity controls are almost always welfare-improving;<sup>12</sup> second, whether in-kind transfers or rationing schemes should be used depends exclusively on the sign of  $\partial q_y^*/\partial \hat{x}$ .

The intuitive reasoning behind the first point is immediate: since a quantity control can in general reduce the inefficiently high price at which  $y$  is sold, it is welfare-improving. This makes clear that market imperfections may be corrected using quantity controls in the competitive sector. To properly design them, however, one has to identify the determinants of the sign of  $\partial q_y^*/\partial \hat{x}$ . Unfortunately, no general results are available, since we saw in Section 2 that  $\partial q_y^*/\partial \hat{x}$  is ambiguous in sign. To proceed, we impose (only within this sub-section) some structure on the model, assuming that the utility function is separable between  $x$  and  $y$ . In that case, it can be shown that the incidence effect of the quantity control reduces to:

$$\frac{\partial q_y^*}{\partial \hat{x}} = \frac{\partial f}{\partial \hat{x}} \left( 1 - \frac{\partial q_y^*}{\partial t_y} \right), \quad (16)$$

where  $\partial q_y^*/\partial t_y$  is of course evaluated at  $t_y = 0$ . Note now that, since  $\partial f/\partial \hat{x} = (\partial f/\partial y)(\partial y/\partial \hat{x})$  and  $\partial f/\partial y < 0$ , (3) implies that

$$\text{sign}(S_{xy}) = \text{sign} \left( \frac{df}{d\hat{x}} \right). \quad (17)$$

That is,  $\partial f/\partial \hat{x}$  is positive when  $x$  and  $y$  are *net* substitutes and negative if they are *net* complements. Then, it is immediate to see, using (17), that:

- when  $\partial q_y^*/\partial t_y < 1$  (undershifting), then if  $x$  and  $y$  are net complements (substitutes), the equilibrium price of good  $y$  is reduced (raised) by a marginal increase in the government-controlled quantity;
- when  $\partial q_y^*/\partial t_y > 1$  (overshifting), then if  $x$  and  $y$  are net complements (substitutes), the equilibrium price of good  $y$  is raised (reduced) by a marginal increase in the government-controlled quantity.

We expect the most common case to be the first, since overshifting requires a high degree of convexity which is not especially likely to hold. Hence, we deduce:

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<sup>12</sup>Note that the presence of imperfect competition is crucial for this; when all markets are perfectly competitive, marginal quantity controls have no welfare effect, as  $\partial q_y^*/\partial \hat{x} = 0$  (*cf.* Guesnerie and Roberts, 1984).

**Proposition 1** *Assume that there are no indirect taxes. Then, if the utility function is separable in  $x$  and  $y$  and there is undershifting of  $t_y$ , an in-kind transfer (rationing) scheme for good  $x$  is welfare-improving when  $x$  and  $y$  are Hicksian complements (substitutes).*

The intuition for Proposition 1 is straightforward. In the market for the non-competitive good, the equilibrium is reached at a point where consumption is “too low” and the price is “too high” (relative to a competitive equilibrium). Then, something can be gained by increasing the consumption of the non-competitive good, and reducing its price. An indirect way of doing this is forcing the individuals to consume more of goods which are complementary to the non-competitive good, or less of goods which are substitutable for it.

While this intuition is direct enough to suggest that the result should have a quite general validity, it has to be emphasized that it may fail for two reasons; first, for more general utility functions the sign of  $\partial q_y^*/\partial \hat{x}$  cannot be easily determined; second, even under separability, a sufficiently convex inverse demand function for  $y$  would induce overshifting, thereby turning the result in Proposition 1 on its head.

### 3.2 The optimal tax problem

We return now to a general utility function for the analysis of the case in which both price and quantity controls are used. As a first step, we characterise the optimal choice of the tax instruments  $t_x$  and  $t_y$ , taking quantity controls as given.<sup>13</sup> The first order conditions of the planner’s maximisation are:

$$\frac{\partial V}{\partial q_x^*} + \frac{\partial V}{\partial q_y^*} \frac{\partial q_y^*}{\partial t_x} + \mu \left( (\hat{x} + x(\cdot)) + t_x \left( \frac{\partial x}{\partial q_x^*} + \frac{\partial x}{\partial q_y^*} \frac{\partial q_y^*}{\partial t_x} \right) + t_y \left( \frac{\partial y}{\partial q_x^*} + \frac{\partial y}{\partial q_y^*} \frac{\partial q_y^*}{\partial t_x} \right) \right) = 0; \quad (18)$$

$$\frac{\partial V}{\partial q_y^*} \frac{\partial q_y^*}{\partial t_y} + \mu \left( y + t_x \frac{\partial x}{\partial q_y^*} \frac{\partial q_y^*}{\partial t_y} + t_y \frac{\partial y}{\partial q_y^*} \frac{\partial q_y^*}{\partial t_y} \right) = 0, \quad (19)$$

where we used (9) and the derivatives of  $q_y^*$  are given by (12).

The necessary conditions (18) and (19) together with the government budget constraint (13) can be solved for  $t_y$  to give:

$$t_y = \left[ y \left( \frac{\partial V}{\partial q_x^*} + \frac{\partial V}{\partial q_y^*} \frac{\partial q_y^*}{\partial t_x} \right) - (\hat{x} + x(\cdot)) \frac{\partial V}{\partial q_y^*} \frac{\partial q_y^*}{\partial t_y} \right] a^{-1}, \quad (20)$$

where

$$a = \frac{\partial q_y^*}{\partial t_y} \left[ \frac{\partial V}{\partial q_y^*} \left( \frac{\partial y(\cdot)}{\partial q_x^*} - \frac{\partial x(\cdot)}{\partial q_x^*} \frac{y(\cdot)}{\hat{x} + x(\cdot)} \right) - \frac{\partial V}{\partial q_x^*} \left( \frac{\partial y(\cdot)}{\partial q_y^*} - \frac{\partial x(\cdot)}{\partial q_y^*} \frac{y(\cdot)}{\hat{x} + x(\cdot)} \right) \right].$$

<sup>13</sup>The derivation of the optimal taxes follows Myles (1989).

In general, the sign of  $t_y$  is ambiguous. Myles (1989) is however able to find sufficient conditions for which  $t_y < 0$ . Assume that goods  $x$  and  $y$  are gross substitutes ( $\partial x/\partial q_y^* > 0$  and  $\partial y/\partial q_x^* > 0$ ); then  $\partial q_y^*/\partial t_y > 0$  implies that  $a < 0$ . Furthermore, by using Roy's identity in (4), (20) can be rewritten as

$$t_y = \left[ (\hat{x} + x(\cdot)) \left( \frac{\partial q_y^*}{\partial t_y} - 1 \right) - \frac{\partial q_y^*}{\partial t_x} y(\cdot) \right] a^{-1} \alpha y(\cdot). \quad (21)$$

Accordingly, the optimal tax on good  $y$  is negative if the term in square brackets is positive. A sufficient condition for this to be true is that (i)  $\partial q_y^*/\partial t_y > 1$  and (ii)  $\partial q_y^*/\partial t_x < 0$ . That is, the non-competitive industry should be subsidized and the competitive industry should be taxed when the tax on good  $y$  is overshifted, and the tax on good  $x$  has a negative effect on the consumer price of good  $y$ . Intuitively, when the monopolistic firm overshifts taxes on consumers does the same also for a subsidy. Thus, a subsidy on good  $y$  will lead to a reduction in its consumer price. This effect is reinforced when a tax on good  $x$  leads to a further reduction in the consumer price of good  $y$ . In this case, indirect taxation will counteract the distortion associated with the presence of a monopolist, i.e. an inefficiently high price for good  $y$ .

As mentioned above, Myles' result holds when the inverse demand curve for  $y$  is sufficiently convex to generate overshifting. We note here that if the inverse demand is approximately linear or indeed concave, it becomes more likely that the optimal tax policy involves a tax on  $y$ . Indeed, if we have  $\partial q_y^*/\partial t_y < 1$  and  $\partial q_y^*/\partial t_x > 0$ , then the term in square bracket in (21) is negative, and therefore  $y$  is optimally taxed at the optimum. This result has also a clear interpretation: if there is undershifting, a tax or subsidy on  $y$  will not alter  $q_y^*$  much; if, at the same time, the price of  $y$  goes down when a subsidy on  $x$  is introduced, the most effective way of boosting the demand for  $y$  is indeed to raise revenue by taxing it and use the proceeds to fund the subsidisation of  $x$ .

For future use, we state the above results formally:

**Lemma 2** (Myles, 1989) *If  $x$  and  $y$  are gross substitutes, and i)  $\partial q_y^*/\partial t_y > 1$  and ii)  $\partial q_y^*/\partial t_x < 0$ , then at the optimum  $t_y < 0$  and  $t_x > 0$ .*

**Lemma 3** *If  $x$  and  $y$  are gross substitutes, and i)  $\partial q_y^*/\partial t_y < 1$  and ii)  $\partial q_y^*/\partial t_x > 0$ , then at the optimum  $t_y > 0$  and  $t_x < 0$ .*

### 3.3 The simultaneous use of quantity and price controls

If commodity taxes are set at their optimal level, both terms in (15) are present. We will focus our discussion on the cases in which either Lemma 2 or Lemma 3 applies, under the assumptions

that i)  $\partial q_y^*/\partial \hat{x} > 0$ , and ii)  $x$  and  $y$  are net (as well as gross) substitutes<sup>14</sup>. We can then show that, under Lemma 2, the revenue effect is positive, and the welfare effect is negative; instead, under Lemma 3, both the revenue effect and the welfare effect are negative.

To see the first point, note that net substitutability implies  $\partial y/\partial \hat{x} < 0$  by (3). Hence, the signs of the various elements of the welfare and revenue term are as follows:

$$\underbrace{\frac{\partial V}{\partial q_y^*}}_{-} \underbrace{\frac{\partial q_y^*}{\partial \hat{x}}}_{+} + \mu \left[ \underbrace{t_x}_{+} \underbrace{\frac{\partial x(\cdot)}{\partial q_y^*}}_{+} \underbrace{\frac{\partial q_y^*}{\partial \hat{x}}}_{+} + \underbrace{t_y}_{-} \left( \underbrace{\frac{\partial y}{\partial \hat{x}}}_{-} + \underbrace{\frac{\partial y(\cdot)}{\partial q_y^*}}_{-} \underbrace{\frac{\partial q_y^*}{\partial \hat{x}}}_{+} \right) \right], \quad (22)$$

which proves the point. The welfare term calls for a rationing scheme, while the revenue term calls for an in-kind transfer; depending on the relative strength of two effects, either scheme can be welfare-improving. We have already provided an interpretation for the welfare term which is equally valid here: any scheme which counteracts the distortion imposed by the presence of a monopolist (an excessively high price for  $y$ ) is desirable, and absent taxes, rationing would do the job in this case. However, the indirect tax system, which is also correcting the market imperfections, introduces an additional distortion by depressing the demand for  $x$  in order to encourage that of  $y$ . Then, if the welfare loss associated with the tax policy is large enough (i.e. if the revenue effect exceeds the welfare effect in absolute value), it may become desirable to actually reverse the direction of the quantity control in order to push the consumption of  $x$  up.

When Lemma 3 applies, it is easy to identify the signs of the components of the welfare and revenue effect as follows:

$$\underbrace{\frac{\partial V}{\partial q_y^*}}_{-} \underbrace{\frac{\partial q_y^*}{\partial \hat{x}}}_{+} + \mu \left[ \underbrace{t_x}_{-} \underbrace{\frac{\partial x(\cdot)}{\partial q_y^*}}_{+} \underbrace{\frac{\partial q_y^*}{\partial \hat{x}}}_{+} + \underbrace{t_y}_{+} \left( \underbrace{\frac{\partial y}{\partial \hat{x}}}_{-} + \underbrace{\frac{\partial y(\cdot)}{\partial q_y^*}}_{-} \underbrace{\frac{\partial q_y^*}{\partial \hat{x}}}_{+} \right) \right]. \quad (23)$$

Clearly, (23) is negative, that is a rationing scheme is desirable. The intuition is again immediate: by forcing the consumption of  $x$  down, the government is able to reduce the price of  $y$  as well as to counteract the upward distortion in the consumption of  $x$  induced by the tax system (recall that  $t_x < 0$  under Lemma 3). We may state this result formally as

**Proposition 4** *When i)  $\partial q_y^*/\partial \hat{x} > 0$  and ii)  $x$  and  $y$  are net substitutes, and Lemma 3 applies, a rationing (in-kind transfer) scheme for good  $x$  is always (never) welfare-improving.*

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<sup>14</sup>Other assumptions would yield different results, but the method of analysis we employ in the text would be equally valid.

## 4 Quantity controls in the non-competitive industry

Now we consider the case in which the government operates its quantity controls only in the market for good  $y$ . We expect that, at least in the absence of indirect taxation, the government will always want to operate an in-kind transfer scheme for good  $y$ , in order to force individuals to consume more of this good, and accordingly to reduce its price. On the contrary, we expect that rationing will never be optimal since its effects would operate in the opposite direction, i.e. decreasing consumption and pushing price up. This section will verify formally the validity of this intuitive argument, and consider the extension to a situation in which taxes are used.

We start by briefly re-working the model, and then proceed to sketch the welfare analysis; details are omitted, since they can be deduced from the analysis in the previous sections. Suppose that, initially, the government fixes the constraint at  $\bar{y} = \hat{y}$ . Then (2) would continue to hold with  $\hat{y}$  replacing  $\hat{x}$ . Also, we have that  $\text{sign}(\partial x(\cdot)/\partial \hat{y}) = -\text{sign}(S_{yx})$ . From the solution of the consumer problem, we can invert the demand for  $y$  to obtain  $q_y = f(y + \hat{y}, t_x)$ . Accordingly, the monopolist's profit is  $\pi = (f(\cdot) - t_y - c)y$ ; letting the solution to the monopolist's problem be  $y^*(\hat{y}, t_x, t_y)$ , we can then use the inverse demand function to determine an equilibrium price on the market for good  $y$ ,  $q_y^* = f(y(\hat{y}, t_x, t_y) + \hat{y}, t_x)$ . The incidence analysis will then show, using the second order condition of the monopolist's maximization problem, that

- the effects of the policy parameters on the equilibrium price of good  $x$  are as in (9), with  $\hat{x}$  replaced by  $\hat{y}$ ;
- $\partial q_y^*/\partial t_y$  is positive, while  $\partial q_y^*/\partial t_x$  is ambiguous in sign;
- $\partial q_y^*/\partial \hat{y} < 0$ , as expected, when the inverse demand function is concave in  $\hat{y} + y(\cdot)$ , or when it is moderately convex; for future use, we take it that  $\partial q_y^*/\partial \hat{y} < 0$  is always verified (this is equivalent to ruling out overshifting – see the discussion in Section 2).

Social welfare is represented by the indirect utility function of a representative consumer,  $V(\hat{y}, q_x^*, q_y^*)$ , and tax revenue evaluated at  $\hat{y}$  is given by  $t_x x(\hat{y}, q_x^*, q_y^*) + t_y (\hat{y} + y(\hat{y}, q_x^*, q_y^*)) = 0$ . By differentiating the Lagrangian of the government problem w.r.t.  $\bar{y}$  and evaluating it at  $\hat{y}$ , we obtain, after some simplifications:

$$\frac{\partial V}{\partial q_y^*} \frac{\partial q_y^*}{\partial \hat{y}} + \mu \left( t_x \left( \frac{\partial x(\cdot)}{\partial \hat{y}} + \frac{\partial x}{\partial q_y^*} \frac{\partial q_y^*}{\partial \hat{y}} \right) + t_y \frac{\partial y(\cdot)}{\partial q_y^*} \frac{\partial q_y^*}{\partial \hat{y}} \right). \quad (24)$$

This expression tells us whether a quantity control on  $y$  is desirable, and which form it should take.

Absent indirect taxes, (24) reduces to its first term. Since  $\partial V/\partial q_y^* < 0$  and  $\partial q_y^*/\partial \hat{y} < 0$  (see our discussion of incidence above), the following proposition results:

**Proposition 5** *Assume that there are no indirect taxes. Then, an in-kind transfer (rationing) scheme for good  $y$  is always (never) welfare-improving.*

What if there are indirect taxes set at their optimal level? Note first that the tax analysis is then basically the same as in Section 3, with  $\hat{y} + y(\cdot)$  replacing  $y(\cdot)$ , and  $x(\cdot)$  replacing  $\hat{x} + x(\cdot)$ ; in particular, Lemmas 2 and 3 are still valid. We however ruled out overshifting in order to ensure that  $\partial q_y^*/\partial \hat{y} < 0$ . That leaves Lemma 3: if it applies and  $x$  and  $y$  are net substitutes (so that  $\partial x/\partial \hat{y} < 0$ ), the revenue effect can be easily checked to be positive:

$$\underbrace{\frac{\partial V}{\partial q_y^*}}_{-} \underbrace{\frac{\partial q_y^*}{\partial \hat{y}}}_{-} + \mu \left[ \underbrace{t_x}_{-} \left( \underbrace{\frac{\partial x}{\partial \hat{y}}}_{-} + \underbrace{\frac{\partial x(\cdot)}{\partial q_y^*}}_{+} \underbrace{\frac{\partial q_y^*}{\partial \hat{y}}}_{-} \right) + \underbrace{t_y}_{+} \underbrace{\frac{\partial y(\cdot)}{\partial q_y^*}}_{-} \underbrace{\frac{\partial q_y^*}{\partial \hat{y}}}_{-} \right] \quad (25)$$

Under these circumstances, the result in Proposition 5 carries over to a situation in which taxes are set optimally:

**Proposition 6** *If Lemma 3 applies and  $x$  and  $y$  are net substitutes, then an in-kind transfer (rationing) scheme for good  $y$  is always (never) welfare-improving also when taxes are set optimally.*

## 5 Concluding remarks

We employed a two-consumption-good model in which one good is produced by a competitive industry and the other by a monopolist to study the optimal design of quantity controls in imperfectly competitive economies. We have argued that, when price controls are not used, it is almost always desirable to use quantity controls in the competitive sector, since this helps reduce the price of the non-competitive good; and that an in-kind transfer (rationing) scheme in the non-competitive sector is always (never) welfare-improving. The rationale for these results is that the consumption of the non-competitive good is inefficiently low; therefore, quantity controls should be designed so as to boost the demand for it, thereby correcting the market imperfection.

An important difference relative to the case in which all markets are competitive is that quantity controls may be used also in the absence of taxes. If these are used to counteract

the market imperfections, however, additional distortions are introduced, and this may alter the design of quantity controls in a significant way. For example, when the demand for the non-competitive good is encouraged through a price subsidy, then it must be the case that the competitive good is taxed, and therefore discouraged. This calls for an in-kind transfer scheme to support the demand for the competitive good, and if this effect is large enough, it may determine the desirability of such a scheme even when rationing would have been used in the absence of price controls. Even when quantity controls are not directly aimed at the market imperfections, they may serve a useful role in that they can alleviate the welfare cost of price controls.

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