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**PRIVATISATIONS AS PRICE REFORMS:  
AN ANALYSIS OF  
CONSUMERS' WELFARE CHANGE IN THE UK**

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# **Privatisations as price reforms: an analysis of consumers' welfare change in the UK**

*by Rinaldo Brau\* and Massimo Florio\*\**

(preliminary draft: not to be cited without permission).

## **1. Introduction**

The British privatisation policy carried out under the 18 years (since 1979 to 1997) of Conservative governments in the United Kingdom has represented the largest and more famous episode of a more general new attitude towards the role of the state in the economy which has interested most of the developed economies. In addition, British privatisations have anticipated by several years the analogous initiatives in the other countries, so that they constitute a consolidated benchmark for guess the main economic consequences for those countries where the process still is in its infancy.

In its own, such an important policy justifies an analysis based on strictly economic grounds because of the wide monetary transfers it has implied. Given the high number and the variety of the economic agents involved (consumers, shareholders, workers, taxpayers), the evaluation of the overall welfare impacts represents a very extensive task. As matter of fact, this paper is part of a wider project (see Florio, 2001) aimed at attaining this goal. In particular, our restricted objective here is that of assessing the welfare effects of British privatisations on the demand side.

For this purpose, the approach we follow here is a one which sees the privatisation of a national company within the broader framework of the “policy reform” theory, in line with a setting up originally suggested by Drèze and Stern (1990). Mainly known in its more restrict form of “tax reform”, it is well known that this approach allows for a normative analysis of policy regimes changes in a second best framework in the presence of small deviations from the status quo

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(Dixit, 1975). Whether or not the cases under examination can be considered a “small reform” with reference to the ex ante and ex post market values can be assessed from the price and expenditure values reported on the top of tables 1 and 2. In any case, a (tax) reform approach maintains its validity also in case of large reforms because it only entails the knowledge of the characteristics of the economic system in the neighbourhood of the starting point (which should be known by definition).

By adopting this viewpoint, it is well-known that the change in consumers’ welfare “with” and “without” privatisations can be studied by means of the tools of applied welfare economics. The crucial variables are the prices, quantities and some characteristics of the demand functions (once opportune definitions or assumptions about them have been done).

Indeed, when looking through the seminal contributions by Guesnerie (1977) and Ahmad and Stern (1984), the tax-policy reform theory turns out to be a “price reform” theory, that is an approach where the value judgement are contingent to the “direction” (and the “length” in the case of non marginal reforms) of the price vectors considered in the analysis. As a consequence, also the evaluation of privatisation effects on consumers welfare must be first of all based on the scrutiny of the market price behaviour.

However, the individuation of the price variations constitutes a necessary but not a sufficient condition for a reliable welfare analysis. First, industry market prices have to be compared with the consumer price index, in order to check for increases or decreases in “real terms”. Second, prices variations have to be compared with productivity changes determined by technological shocks or changes in demand conditions (so that in these cases it would be wrong to attribute to privatisation effects which would have happened even in public firms.). Third, when privatisation is accompanied by a change in market structure, for example from a statutory monopoly to a system of oligopolistic competition, changes in prices could be wholly or partially attributable to increased competitive pressure and not to the change in ownership; similar considerations can be made for the particular system of public control of prices or quantities supplied, which for example may change from being a quite lax or discretionary cost-plus regime to being a stricter one (e.g. of a price cap “RPI-X” with a high “X” element). Fourth, by themselves, price and quantity variations do not capture welfare effects due to endogenous changes in the quality of the products.

All in all, the credibility of traditional welfare analyses is conditioned to the close scrutiny of the determinants of price variations. However, once these problems are solved, the advantage of a price reform approach consists of being able to provide the required welfare assessment with a limited set of information on the characteristics of demand functions.

An unsatisfactory element regarding welfare analyses is that they often appear more an academic exercise, carried out with complex econometric methods and heavy investments, rather than an effective tool in the hands of the regulators or governments for promptly assessing the effects of privatisation programs which have been already carried out or simply designed. Indeed, this is in contrast with the “reform theory philosophy” and de facto limits the real relevance of applied welfare economics. It is an aim of this paper to foster the possibility of an easy implementation of this kind of analysis by showing that a series of “easy-to-implement” welfare measures are available once one becomes ready to accept a few “reasonable” assumptions. With the term “easy-to-implement” we mean a method which only requires the collection of information (at an aggregate level) easily accessible from the most common statistical sources, or at worse limited elaboration starting from them.

## **2. Welfare effects of price changes**

For the aim of this paper, we restrict the following analysis to the welfare effects referable to the price changes which can be associated to the transfer of some production from the public to the private sector. As a consequence, henceforth the term “price change” will be referred to a hypothetical “net privatisation effect”.

By defining these (consumption) price changes with the symbol  $dq$ , the welfare variation  $dW$  can be recovered starting from a generic Bergson-Samuelson Social Welfare Function, which arguments are the following individual (indirect) utility functions

$$v^h \equiv v^h(\mathbf{q}(\mathbf{p}), y^h, \mathbf{z}^h), \quad (1)$$

where the vectors  $\mathbf{q}$  and  $\mathbf{p}$  respectively refer to consumption and production prices,  $y^h$  is the exogenous personal disposable income and  $\mathbf{z}^h$  is a vector of social-demographic characteristics.

### *Marginal price variations*

A variation of consumption prices implies a welfare variation equal to:

$$dW = \frac{\partial W}{dq_i} dq_i = \left( \sum_h \sum_j \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial q_j} \frac{dq_j}{dq_i} \right) dq_i,$$

which, by exploiting Roy's identity, becomes

$$dW = \frac{\partial W}{dq_i} dq_i = \left( \sum_h \sum_j \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial y^h} \frac{dq_j}{dq_i} x_j^h \right) dq_i. \quad (2)$$

By introducing the concept of marginal social value of household  $h$  income and defining it as

$\beta^h \equiv \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial y^h}$ , equation (2) becomes:

$$dW = \frac{\partial W}{dq_i} dq_i = \left( \sum_h \beta^h \sum_j x_j^h \frac{dq_j}{dq_i} \right) dq_i. \quad (3)$$

It is customary to consider irrelevant the cross demand effects. In this case the welfare variation reduces to

$$dW = \frac{\partial W}{dq_i} dq_i = \sum_h \beta^h x_i^h dq_i. \quad (4)$$

Equation (4) represents a first workable formula for the case of privatisations with marginal effects on prices. From an empirical viewpoint, it is important to remark that, as long as marginal variations are considered, the assessment of the welfare effects of a price change does not need any behavioural parameter such as price or income elasticities. Remark that in case no different weights are given to households, equation (4) reduces to a Laspeyres price index

Previous two equations also make clear that, by adopting a Bergson-Samuelson Social Welfare Function, the value of welfare changes reduces to a double weighting of the price variation, where these weights consist of a market datum (the individual demand) and a value judgement (the welfare weight  $\beta^h$ ).

A different way of expressing formulae (3) and (4) which is not constrained to the use of household level data, namely those referring to the individual demand of goods, relates the

welfare variation to the so-called distributional characteristic (henceforth DC). By defining the latter as

$$d_i \equiv \frac{1}{X_i} \sum_h \beta^h x_i^h \quad (5)$$

and substituting it into (4) we get:

$$dW \equiv -d_i X_i dq_i, \quad (6)$$

which in principle can be immediately calculated with aggregate data only provided that the information on the DC of good  $i$  is available from some other sources.

As it is well known (e.g. Stern, 1987), the DC can be expressed in covariance form between consumption shares and welfare weights, so that equation (6) can also be written as:

$$dW = -X_i \left( \bar{\beta} + \text{cov} \left( \frac{x_i^h}{\bar{x}_i}, \beta^h \right) \right) dq_i, \quad (7)$$

where  $\bar{\beta} = \sum_h \frac{\beta^h}{H}$  is the average of welfare weights over the households, which value depends on the scale adopted for the  $\beta^h$  s.

Sometimes, a DC normalised for  $\bar{\beta}$  (e.g. Newbery, 1995) is used, that is:

$$\tilde{d}_i \equiv \frac{1}{X_i} \sum_h \frac{\beta^h}{\bar{\beta}} x_i^h. \quad (5a)$$

In this case the equivalent of expressions (6) and (7) are:

$$dW = -\bar{\beta} \tilde{d}_i X_i dq_i, \quad (6a)$$

and

$$dW = -X_i \left( 1 + \text{cov} \left( \frac{x_i^h}{\bar{x}_i}, \frac{\beta^h}{\bar{\beta}} \right) \right) dq_i. \quad (7a)$$

The interpretation of (7) and (7a) is quite straightforward: the larger the share of a good consumed by the poorest households (those with a high  $\beta^h$  associated to them) the larger is the welfare loss.

The implementation of these kind of formulae clearly requires the specification of the social weights  $\beta^h$ . As it is well-known, the most used parameterisation is derived from the following utilitarian additive social welfare function of iso-elastic utility functions, originally proposed by Atkinson (1970):

$$\begin{aligned}
 W &= \sum_{h=1}^H k \frac{(E^h)^{1-e}}{1-e}, \quad \text{for } e \neq 1 \\
 &= \sum_{h=1}^H k \ln E^h, \quad \text{for } e = 1
 \end{aligned} \tag{8}$$

where  $E^h$  is the personal expenditure by individual  $h$ . The parameter  $k$  is usually chosen in order to take into account the number of equivalent adults within each household or in order to assign a weight equal to 1 to the individual with the lowest expenditure or the average expenditure (which yields respectively weights of the form  $\beta^h = (E^h/E^1)^{-e}$  and  $\beta^h = (E^h/\bar{E})^{-e}$ ). The choice of  $e$  determines the degree of “inequality aversion”. It is customary to undertake some sensitivity analysis by considering values ranging from 0 (the Benthamian case) to 5 (very high inequality aversion).

A reference value is often represented by  $e=1$ , which involves the value judgment that a marginal transfer to someone at half the expenditure level of another has a social value of twice that of the reference person. By setting  $k=1/H$ , in this case we obtain  $\beta^h = 1/E^h$  and  $\bar{\beta} = 1/\text{HM}(E^h)$ , where  $\text{HM}(E^h)$  is the harmonic mean of individual consumption. Hence, the expression (6a) reduces to:

$$dW = -X_i \left( 2 - \frac{\bar{E}_i}{\text{HM}(E_i^h)} \right) dq_i, \tag{9}$$

which can be additionally simplified by setting  $\bar{\beta} = 1$ , that is  $\beta^h = \frac{1/E^h}{\bar{E}_i}$ .

As a general comment, it must be pointed out that these “socially weighted measures” do not account for one of the most elusive effects to capture in the transition from public to private firm, that is the change in the regime of price discrimination.<sup>1</sup>

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<sup>1</sup> In the case of multi-product firms, sometimes both public and private firms are capable of charging different prices to different consumers for the same product. In turn, the different types of consumers could have different functions of demand, so that variation in welfare due to the change in regime would need to be ascertained for the different types of consumers.

## Large price variations

For the case of large price variations, we make an option for the use of second order approximations,<sup>2</sup> which can refer both to money metric measures (Harberger, 1964) and more general welfare measures allowing for distributional considerations (Banks, Blundell and Lewbel, 1996).

Unlike the case of the evaluation of small reforms, the assessment of large price variations usually requires information about individual demand elasticities. For example, the second order welfare approximation proposed by Banks *et al.* (1996) takes the form (with redundant notation in order to highlight the elasticities required for its implementation):

$$\frac{\Delta W}{\Delta q_i} \approx -\sum_h \beta^h x_i^h \left[ 1 + \frac{1}{2} \frac{\Delta q_i}{q_i} \left( \frac{\partial x_i^h}{\partial q_i} \frac{q_i}{x_i^h} + \frac{\partial \beta^h}{\partial q_i} \frac{q_i}{\beta^h} \right) \right], \quad (10)$$

whilst the equivalent money metric approximation by Harberger (1964) is:

$$\frac{\Delta X_i}{\Delta q_i} \approx -\sum_h x_i^{hc} \left[ 1 + \frac{1}{2} \frac{\Delta q_i}{q_i} \left( \frac{\partial x_i^{hc}}{\partial q_i} \frac{q_i}{x_i^{hc}} \right) \right]. \quad (11)$$

Reliable estimation of individual elasticities requires the availability of microdata of adequate quality and the imposition of some identifying assumptions.<sup>3</sup> For the case in which these two conditions cannot be ensured by the analyst, a few additional restrictions or approximations are to be adopted in order to allow for an evaluation based on aggregated data only.

The simplest solution for avoiding the use of household level information is to give up assigning different welfare weights in the equation (7) above. As a matter of fact, if we impose  $\beta^h = 1$  for all  $h$ , we get:

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<sup>2</sup> The typical advantage of welfare approximate measures consists of their reduced informational requirements, as compared to the computation of "exact" measures based on the estimation of household expenditure functions. In Starrett words: "...the method gives up on collecting hypothetical information concerning demand conditions in unobserved parts of the economic environment and instead extrapolates to those areas using curvatures at the status quo" (Starrett, 1988, p. 246). In addition, exact welfare measures suffer major problems in aggregation (Auerbach, 1985).

<sup>3</sup> With  $H$  individuals, the estimation of  $H$  parameters would of course be impossible. In order to overcome this problem, somewhat ad hoc (identifying) hypotheses must be done about the constancy among the individuals of some values. For example, in the well known case of the AIDS model by Deaton and Muellbauer (1980), the identifying assumptions are that:  $\frac{\partial w_i^h}{\partial \ln q_i} = \gamma_i$  and  $\frac{\partial w_i^h}{\partial \ln(x^h/P)} = \beta_i$  for  $h=1, \dots, H$ .

$$\frac{\Delta W}{\Delta q_i} \approx - \left[ \sum_h x_i^h + \frac{\Delta q_i}{2} \sum_h \frac{\partial x_i^h}{\partial q_i} \right],$$

from which:

$$\frac{\Delta W}{\Delta q_i} \approx -X_i \left[ 1 + \frac{\Delta q_i}{2q_i} \eta_{X_i, q_i} \right]. \quad (12)$$

Hence, a second order approximation which keeps apart distributional concerns only requires information about the aggregate demand and demand elasticity.

The results are slightly more complex in the general case  $\beta^h \neq 1$ , for some  $h$ . Let us first rewrite equation (10) as

$$\frac{\Delta W}{\Delta q_i} \approx -X_i \left[ \sum_h \frac{\beta^h x_i^h}{X_i} + \frac{\Delta q_i}{2q_i} \eta_{X_i, q_i} \frac{1}{H} \sum_h \left( \beta^h \frac{\partial x_i^h}{\partial q_i} / \frac{\partial \bar{x}_i}{\partial q_i} \right) \right], \quad (13)$$

where an additional approximation has been done by assuming the price invariance of welfare weights.<sup>4</sup> By using the definition of distributional characteristic in (5) and exploiting again the definition of covariance we have:

$$\frac{1}{H} \sum_h \left( \beta^h \frac{\partial x_i^h}{\partial q_i} / \frac{\partial \bar{x}_i}{\partial q_i} \right) = \bar{\beta} + b, \quad (14)$$

where  $b = \text{cov} \left( \left( \frac{\partial x_i^h}{\partial q_i} / \frac{\partial \bar{x}_i}{\partial q_i} \right), \beta^h \right)$ .

Hence, by substituting (14) into (13) we finally get:

$$\frac{\Delta W}{\Delta q_i} = -X_i \left[ d_i + (\bar{\beta} + b) \frac{\Delta q_i}{2q_i} \eta_{X_i, q_i} \right]. \quad (15)$$

In the case the use of  $\tilde{d}_i$  is preferred, it easy to verify that the equivalent to equation (15) is:

$$\frac{\Delta W}{\Delta q_i} = -X_i \bar{\beta} \left[ \tilde{d}_i + (1 + \tilde{b}) \frac{\Delta q_i}{2q_i} \eta_{X_i, q_i} \right], \quad (15a)$$

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<sup>4</sup> This actually is an usual assumption in the applied welfare analyses, although theoretically weak (e.g., Roberts, 1980; Banks *et al.*, 1996). For an empirical study in which welfare weights price elasticity is taken into account see Ray (1999).

$$\text{where } \tilde{b} = \text{cov} \left( \left( \frac{\partial x_i^h}{\partial q_i} / \sqrt{\overline{\partial x_i^h}} \right), \frac{\beta^h}{\beta} \right).$$

Hence, the estimation of a second order welfare approximation based on aggregated data needs, again, some information about the DC associated to the good which provision is being privatised. In addition, an estimate of the aggregate demand elasticity to price is required. The importance of this behavioural parameter is enhanced or tempered by the presence of a second distributive parameter ( $b$ ), which takes smaller values when the demand by the poorest is more rigid.

Unfortunately, differently from all the other parameters, the covariances  $b$  and  $\tilde{b}$  can be computed only after having estimated the individual demand derivatives, which requires the use of microeconomic estimation techniques. In alternative, functional form assumptions are needed.

For example, in the case of linear Engel curves, expression (15) clearly simplifies since we have  $b=0$ . The parameter  $b$  is also equal to 0 when the individual demand functions are linear, an hypothesis which can be seen as a good approximation of the real shape of the demand function in a neighbourhood of the starting point.<sup>5</sup>

*Second order approximations in the case of Working-Leser Engel curves.*

Besides this limited case, it is of interest to infer what the sign of  $b$  could be. A much more accepted Engel curve structure is the Working-Leser one, which is implied by the well known “TRANSLOG” and “AIDS” demand systems.

In the case of price demand response estimation by means of these demand systems, the identifying assumption for the individual price response is:

$$\frac{\partial w_i^h}{\partial \ln q_i} = \gamma, \quad \text{for all } h. \quad (16)$$

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<sup>5</sup> Hence, this hypothesis is typically reliable only in case of small price reforms.

Depending on  $\gamma$  being smaller or greater than zero, the demand for good  $i$  will be “rigid” ( $|\eta_{x_i^h, q_i}| < 1$ ) or “elastic” ( $|\eta_{x_i^h, q_i}| > 1$ ), given that the price elasticity in this models is approximated to:

$$\eta_{x_i^h, q_i} = \frac{1}{w_i^h} \frac{\partial w_i^h}{\partial \ln q_i} - 1.$$

Rearranging, we can write:

$$\frac{\partial w_i^h}{\partial \ln q_i} = \gamma = w_i^h (1 + \eta_{x_i^h, q_i}),$$

from which, after a some algebra,

$$\frac{\partial x_i^h}{\partial q_i} = (\gamma - w_i^h) \frac{E^h}{q_i^2},$$

that is:

$$\frac{\partial x_i^h}{\partial q_i} = \gamma \frac{E^h}{q_i^2} - \frac{x_i^h}{q_i}. \quad (17)$$

For  $\gamma < 0$ , i.e. if the demand is elastic, both terms on the right hand side are negative. This implies that (provided that  $i$  is not an inferior good), the individual demand responses become higher in absolute terms as the individual expenditure increases. As a consequence, the distribution of the individual demand response to price is negatively correlated with the distribution of the social weights. Hence, for the Working-Leser family of Engel curves, the term  $b$  in the equation (15) is negative (positive) if the demand for the good to which the price reform is referred is elastic (inelastic).

We can therefore conclude that equation (15), in the limiting case in which the parameter  $b$  is set to 0, represents an upper (lower) bound of a second order welfare approximation if the demand of the good under scrutiny is elastic (inelastic).

It also interesting to see which are the implications of setting  $e=1$  for the second order approximated welfare measures when combined with a Working-Leser functional form assumption. As it was seen in the previous section, in this case the social weights can be set equal to  $\beta^h = 1/E^h$ . As for the covariance term  $\tilde{b}$  in equation (15a), which can be rewritten as:

$$\tilde{b} = \frac{1}{H} \left( \frac{\partial \bar{x}_i^h}{\partial q_i} \right)^{-1} \sum_{h=1}^H \frac{\partial x_i^h}{\partial q_i} \frac{\beta^h}{\bar{\beta}} - 1,$$

the application of the relationships between the parameters we saw above yields:

$$\tilde{b} = \frac{1}{H} \left( \frac{\partial \bar{x}_i^h}{\partial q_i} \right)^{-1} \sum_{h=1}^H \frac{1}{q_i^2} (\gamma - w_i^h) \frac{\beta^h}{\bar{\beta}} - 1.$$

Without lack of generality we can set  $\bar{\beta} = 1$ . By substituting the  $\beta^h = 1/E^h$  into the previous equation we therefore get, after a few algebraic passages:

$$\tilde{b} = \frac{1}{q_i^2} \left( \frac{\partial \bar{x}_i^h}{\partial q_i} \right)^{-1} (\gamma - \bar{w}_i^h) - 1.$$

It is quite straightforward to show then that equation (15a) reduces to:

$$\frac{\Delta W}{\Delta q_i} = -\tilde{d}_i X_i \left[ 1 + \frac{\Delta q_i}{2q_i} \eta_{X_i, q_i} \right]. \quad (15b)$$

Overall, the value of a second order approximation is always smaller than a first order one because some efficiency effects are captured by the aggregated elasticity parameter, which usually is negative. The proximity to the first order approximation is partially restored when the demand for the good is elastic.

### *Money metric approximations*

Finally, let us see how we can limit the informational requirements of the second order money metric approximation. In this respect, we can apply Slutsky equation to equation (13) in order to get:

$$\frac{\Delta X_i}{\Delta q_i} \approx - \left[ \sum_h x_i^{hc} + \frac{\Delta q_i}{2} \sum_h \frac{\partial x_i^h}{\partial q_i} + \frac{\partial x_i^h}{\partial m^h} \right].$$

With a few simple manipulations we have:

$$\frac{\Delta X_i}{\Delta q_i} = -X_i \left( 1 + \frac{1}{2} \frac{\Delta q_i}{q_i} \eta_{X_i, p_i} \right) + \frac{1}{2} \Delta q_i \sum_h \frac{\partial x_i^h}{\partial m^h},$$

which becomes:

$$\frac{\Delta X_i}{\Delta q_i} = -X_i \left( 1 + \frac{1}{2} \frac{\Delta q_i}{q_i} \eta_{X_i, p_i} \right) + H \frac{\Delta q_i}{2} \frac{\overline{\partial x_i^h}}{\overline{\partial m^h}} \quad (18)$$

Previous formula shows that the money metric approximated measure does not suffer from aggregation problems. Hence, provided that an estimate of the per capita aggregated income (or expenditure) elasticity ( $\eta_{\bar{x}_i, \bar{m}}$ ) is available, the total welfare variation can be computed as:

$$\frac{\Delta X_i}{\Delta q_i} = -X_i \left( 1 + \frac{1}{2} \frac{\Delta q_i}{q_i} (\eta_{X_i, q_i} + \eta_{\bar{x}_i, \bar{m}}) \right). \quad (19)$$

Hence, the computation of a money metric second order welfare approximation only needs aggregated behavioural parameters and does not need any “distributional” or “individual” information.

To summarise, in this section we showed how and under what conditions the estimation of the effects on consumers’ welfare of the price changes referable to the privatisation of some previously publicly provided goods can be based on aggregated data. In the next section, we briefly report on the British privatisation policies which welfare effects we would like to estimate.

### **3. Price trends in the British privatised industries**

The privatised industries which we are examining in this paper are 7: Telecommunications, Railways, Bus services, Electricity, Gas, Water and Coal. All of them were privatised by the British Conservative Governments in a period ranging from the early Eighties to mid Nineties (the reference years for each privatisation are reported on the top of table 1).

This large spread among the changes of ownership dates has of course entailed a certain degree of heterogeneity in the forms of public disinvestments which were chosen and in the following regulatory initiatives adopted. However, on the whole the common feeling has been that of a unique policy (“privatisation”) carried out in successive steps. As it appears, we are also taking this perspective as well, but the limits of this preliminary choice should be borne in mind.

A case which is particularly affected by this lack of synchronicity is represented by the analysis of the prices trends in the various industries after privatisation. In fact, while for some sectors such as telecommunications and gas industry we dispose of about 15 yearly observations since their privatisations, for the rail and coal industries the “after privatisation” period reduces to 4 years only.<sup>6</sup> As a consequence, while for some cases a clear-cut judgement could be reliably expressed (e.g., “telephony prices have decreased”, or “water is more expensive than it was before privatisation”), for some other sectors a definite trend has not emerged yet.

While leaving to Florio (2001, ch. 7) for a more detailed description of the evolution of the prices and of the institutional aspects, such as the price cap systems adopted by the sectors’ regulators, here we briefly focus on a point particularly important when analysing privatisation policies as “price reforms”, i.e., the fact that trends in nominal prices “before” and “after” privatisations in the UK does not show a clear structural break, at least until the end of the “Conservative era”.

More in detail, in the case of electricity -- which privatisation started in 1990,<sup>7</sup> -- prices had been falling for over a decade under public ownership and they increased in preparation for privatisation and in the years that followed, especially prices for the residential users. Subsequently they started falling again in a manner not too different from the long term trend up to 1995, date which seems to mark the starting point of a more clear decreasing trend (of course still in its infancy).

In the case of gas there was a net drop in prices after privatisation in 1986, but prices had been falling with respect to the Retail Price Index also in the Seventies when British Gas was a nationalized industry. Conversely, a relative increase was again registered by considering the 5 years preceding the privatisation of British Gas. As in the case of electricity, an apparent stronger decreasing trend has started since the mid Nineties.

The privatisation of British Telecom dates back to 1984. Although the construction of a price index for this industry represents a very difficult task (due to the variety of contracts, the changes

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<sup>6</sup> Being 1999 the last year for which adequate information has been made available.

<sup>7</sup> In that year the twelve Regional Electricity Companies in England and Wales were privatised, followed by the two Scottish companies in 1991.

in the pricing methods and the difficulty in obtaining disaggregated data for each category of user), the existing aggregate estimates suggest that after privatisation there has been a reduction in the unit cost for business users and, for a number of years, an increase in the unit cost for domestic and public phone users. More recently (and a couple of years before the aforementioned cases), a generalised reduction in tariffs has been taking place, following a change in the regulatory constraint and increased competition.<sup>8</sup>

No clear trend emerge also for those services which have registered relative price increases since privatisation. In the case of water, the tariffs rose considerably after privatisation, but the lack of adequate previous information does not allow us to control for the behaviour of the prices under the public ownership. Also in the case of buses and rail the price of the service increased after privatisation, but the same had happened before.

#### ***4. Empirical analysis: consumers' net gains from price changes referable to the British privatisation policy.***

In this section we present a few preliminary estimates of the effects of the British privatisation policy on consumers' welfare. These estimates were carried out by applying most of the measures introduced in section 2. As a matter of fact, although the analyst's preference should be given to the second order welfare change approximations, at least in the presence of adequate information, those measures mainly suited to marginal changes still remain useful for comparison purposes and evaluating the likely magnitude of the errors associated to first order approximations.

The first datum from which to start is the direction of the market price changes. The time series by ONS (various editions) display a dichotomous behaviour, with a relative reduction for telephony, electricity and gas and an increase for water and transport. The size of these variations and the overall families' expenditure in the related sectors is, however, not homogenous. Our broad calculations suggest a diminution of 16% of the prices of the privatised sector as compared

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<sup>8</sup> For more details see Florio (2001).

to the path of the Retail Price Index (RPI) since 1987 to 1999.<sup>9</sup> To set 1987 as the starting year for evaluating the price effects of the British privatisation policy as a whole is, of course, an ad hoc hypothesis. The same applies for those assumptions which define the percentage of the price variation actually attributable to the change of ownership. This clearly is always a difficult task, which becomes particularly critical when looking to privatisation as a price reform.

In the preliminary estimates we are presenting in the next subsection, we substantially escape this problem -- which is typically handled by outlining a counterfactual scenario<sup>10</sup> -- by simply considering the percentage deviation of the prices of the privatised industries with respect to the RPI.<sup>11</sup>

## **4.2. Industry level welfare analysis**

In this subsection we discuss the results of the analysis carried out at the single industry level and presented in tables 1 and 2. The first part of these tables also contains the information used for the implementation of the various measures reported therein.

In particular, the privatisation year<sup>12</sup> has been used for determining the interval to which to refer the “price reform”. Having taken into consideration 1999 as the final year of our analysis, for each commodity the price reform has been calculated as the difference between the percentage variation of the related prices index and the overall RPI in the interval privatisation year-1999.<sup>13</sup>

As for the sector expenditures, they are expressed in “median year ” (between privatisation and 1999) prices. Given that this year is not the same among the various industries, caution should be paid before aggregating the single-good welfare measures we report.

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<sup>9</sup> We obtained this value by building an aggregate Laspeyres index of the privatised sector centered on 1987 by using the industry-level ONS time series.

<sup>10</sup> The interested reader is referred to Florio (2001) for an in-dept analysis of this issue in the British case.

<sup>11</sup> The preliminary nature of our analysis is accentuated by the simplifications regarding the timing and the persistency of the price changes. In fact, by considering the overall price change variation which has occurred between the privatisation year and 1999, we are assuming that the change is entirely taking place in one period only. Besides, when transforming the welfare change in a perpetuity value, we are implicitly assuming that no significant discrepancies between the RPI and the privatised sectors prices are to be expected.

<sup>12</sup> For those cases in which the privatisation has been carried out in subsequent steps, an “average” year has been considered.

<sup>13</sup> Of course the caveat expressed in note 11 applies here.

The second order measures need the imputation of aggregate demand price and income elasticities (the latter only in the case of the “money metric” approximation). For the aims of this paper, we have limited our choice regarding price elasticities to the values reported in Florio (2001). As for the elasticity of the demand to income (or total expenditure), the lack of adequate information has led us to make an ad hoc assumption of linear homogeneity in income, which implies an income elasticity equal to 1. Hence, the money metric approximations reported in this draft are presently not reliable and simply complete the list of measures which the researcher can easily compute by means of aggregate level data only.

A few caveats are also needed with respect to the distributional characteristics which we have used. As we said in the introduction, this paper aims to promote the adoption of “easier” approaches in applied welfare analyses in which, in principle, only the use of aggregated data should be required. Under this perspective, also well-defined distributional value judgements (such as DC) on the consumption of single commodities can be considered as an aggregate information. Consistently with these approaches, in this draft we have given up using microdata for estimating the distributional impact of reforms. If, on the one hand, this choice highlights the direct applicability of our approach whenever one (let us say the “regulator”) wish to easily estimate privatisations effects on consumers welfare, on the other hand has exposed our results to some additional drawbacks concerning the relative mismatch between the definition of the goods and services under scrutiny and the synthetic information about DC which was available from previous studies. In fact, our source (Newbery, 1994) reports specific DC for gas and “electricity” only, which has forced us to make use of the DC of the commodity aggregations to which the privatised industries belong to.<sup>14</sup>

Following the more consolidated approaches, the social weights have been derived by using an Atkinson’s Social Welfare Function like that in (8) and scaled so that to set their mean  $\bar{\beta}$  equal to 1, which makes the implementation of formulae like (15) easier. For sake of brevity, the DC

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<sup>14</sup> In particular, the DC used for weighting the expenditures in telephony has been that of the broader group “postage and telephone”; the DC referring to “other fuels” was used in the case of coal expenditures; the DC of the aggregate “travel” was used for both “rail” and “bus” expenditures; and the DC referring to “housing” expenditures was adopted when analysing the price variations related to water supply. In the latter two cases, we are probably overestimating the regressivity of our “privatised basket”, given that the group “travel” also collects expenses for “taxi” and “air” which typically are luxury goods, while the aggregate housing badly represents the nature of necessity referable to water consumption. Overall, these drawbacks outline the need for a future check of the reliability of our results by means of a direct use of the Family Expenditure Survey microdata, at least for a reference year. Besides, it certainly appears desirable the inclusion of a table of the DCs in the yearly reports on families’ expenditure presently published by the central statistical offices.

used in the tables are only those corresponding to a “coefficient of inequality aversion” equal to one.

As said when we derived equation (15-15a), the only very micro level datum which we would need when implementing a second order welfare approximation are the covariance “ $b$ ” and  $\tilde{b}$ . In order to overcome this problem, a Working-Leser functional form assumption has been done with respect the latent Engel curves of the observed demands so that, having set to 1 the coefficient of inequality aversion, equation (15b) was used when computing the second order welfare approximation.

Table 1 and 2 summarise the results we obtained by carrying out an analysis based on the welfare measures introduced in section 2. In order to assess the robustness of the outcomes, the two tables are built by taking the expenditures in the various industries respectively at the privatisation and the median year.<sup>15</sup> We also have aimed to keep a direct “monetary” interpretation of the welfare measures, by scaling price indexes in order to set the price of the reference year equal to 1.<sup>16</sup> This allows us to consider the recorded expenditures as if they were “quantities”. Finally, in order to facilitate the comparison of the two tables, the expenditures at the privatisation year have been computed at the median year prices. Overall, the values reported in these tables are millions of pounds so that, in principle, they could allow for an immediate comparison of the other net gains referable to British privatisations.

Due to the dissimilarity in the direction of the price changes, there have been losses and gains. The larger gain is clearly referable to telephony, surely determined by the relative importance of this service within the privatised basket, but also by the substantial price decrease recorded since its privatisation. The second privatised industry which has substantially contributed to the increase of consumers’ welfare has been the gas one. As it can be seen by comparing the two tables, the relative importance of the benefits deriving from this sector has been decreasing in the recent years, as a result of its reduced weight in consumers’ expenditures. The opposite has happened with respect to the “water” sector, which presently represents the larger source of losses for British consumers among the privatised utilities.

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<sup>15</sup> Of course, when a more general cost-benefit study is being carried out (see Florio, 2001), the choice of a specific reference year could be affected by the need of ensuring the consistency of such overall analysis.

<sup>16</sup> To make an example which refers to table 2: if the price indexes for the privatisation and the final year (1999) of the gas sector were respectively  $p_1=1.02$  and  $p_2=0.71$ , they were inserted as 1 and  $0.71/1.02$ .

Besides being an important part of the consumers' privatised good basket, this sectors exhibit price variations of more than 30% in absolute terms. It is in these cases that it becomes particularly important to focus on second order measures, given the high monetary values which can be involved. As a matter of fact, the difference between first and second order approximations can reach values close to 15% (e.g., see in table 2 the row reporting the "first order error" between the first and second order "socially weighted" measures, which indicate underestimates of the welfare gains of about 11% for the "phone" industry and an overestimate of about 14% for the water sector).<sup>17</sup>

As for the distributional corrections, in general their size is substantially determined by the value of the DC also in the case of the second order approximation.<sup>18</sup> Moreover, in these tables exercise even the slight differences existing in the general case disappear because of the Working-Leser form hypothesis regarding the "latent" Engel curves of the goods under examination. It should also remembered that this hypothesis allows us to apply the simplified formula (15b).

## **4.2 Some conjectural calculations.**

After having calculated the effects on consumer welfare referable to each good or utility, we repeat here a "mental experiment" carried out by Florio (2001, ch. 7) aimed at determining a rough indicator of the aggregate effects to be used in a simplified cost-benefit analysis.

Let us start from the fact that the households' aggregate expenditure for the privatised sector has been fluctuating around 8% of overall private expenditure in the period corresponding to the British privatisation policy. In particular, it was equal to 7.8% in an intermediate year like 1992. Being private consumption in that year equal to about £ 377 billions, we are left with a figure of nearly £ 29.5 bn referable to the privatised industries. As for the prices of the privatised sectors, we already pointed out that they decreased of 16% between 1987 and 1999 with respect to the retail price index. By simply multiplying this variation for the purchases in the intermediate

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<sup>17</sup> It can be easily seen that this percentage value is nearly independent of the size of the price variation.

<sup>18</sup> This is immediate when looking how the DC enters equation (15). For first order approximation this relationship is trivial.

year,<sup>19</sup> we would obtain a welfare increase of £ 4.7 bn. If finally the critical hypothesis is made that half of this relative price change can be attributed to privatisations,<sup>20</sup> we end up with an estimate of 2.35 bn of yearly benefits, which can be converted in a perpetuity value of £ 47 bn by adopting a real social discount rate of 5%.

Table 3 allows us to compare this result with those emerging from the adoption of the battery of approximated measures we have proposed in section 2. To keep things simple, again distributional considerations have been limited to the case of an unitary coefficient of inequality aversion. In this case the arithmetic mean of the DC associated to the commodities of the privatised industries is equal to 0.805, which is the value we have used for our calculations. As for the aggregate price elasticity, a value of  $-0.62$  (see Florio, 2001) was adopted.

While repeating this simple calculation, a few caveats should be pointed out. The presentation of the results in the form of a perpetuity is obviously a simplification. We are not pretending to say that the real price changes can be always considered as a discrete event. However, at this stage we are unable to say more on future trends and prefer to focus on actual price changes. Our procedure is equivalent to say that in the counterfactual the nationalized industries would have been able to offer the same prices as their private counterparts, but with a delay of some years.

As in the previous subsection, we have also computed those measures which do not allow for distributional considerations. At an aggregate level, these approximations of the Marshallian surplus show a limited differences between first and second order measures (about 2.5%).<sup>21</sup> Although the non marginal nature of the price variation should contribute to produce substantial monetary values, the perpetuity value is somewhat modest (as compared to the dimension of the policy under examination), if one considers that £ 47 bn traduce in a per-capita perpetuity value of about 770 pounds.

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<sup>19</sup> Notice that this “rough” measure, which is a “modified” version of the indicator used by Waddams-Price and Hancock (1998) is just a monotone transformation of a first order approximation to welfare change, where the “scalar” used for the transformation is just the aggregate price of the privatised sector and the demand taken into consideration is that of an intermediate year. An alternative interpretation is that this is just the first order approximation for the case in which that intermediate year is chosen as the base for the price index.

<sup>20</sup> In Florio (2001), this is just a benchmark hypothesis, halfway between assuming that privatisations were responsible for all price changes or the counterfactual of continued public ownership would have generated exactly the same price changes as those observed.

<sup>21</sup> Remember that, to this difference, one should add the likely difference existing between approximated and “exact” measures. As a touchstone, in the well-known Banks *et al.* (1996) exercise the difference between second order approximations and exact measures (in the form of a Quadratic Almost Ideal Demand System expenditure function), amounted only to 0.3%.

When considering the measures containing the distributional parameters, the first order approximation still proves to be a good indicator of the overall welfare variation, given that its relative discrepancy with respect to the second order approximation remains more and less the same. As for the entity of the distributional correction, this is of course equal, in relative terms, to 1 minus the distributional characteristic in case of first and second order approximated measures.<sup>22</sup>

## **5. Conclusions**

This paper has investigated the possibilities of applying a (price) reform approach to the analysis of the effects of the British privatisation policy on consumers' welfare. To this aim, a series of first order and second order approximated measures has been discussed and introduced, fostering the use of distributional characteristics as an aggregate indicator of the distributional implications of policy reforms.

The implementation of the proposed measures has revealed the presence of relevant effects at the single industry level, although with a heterogeneous behaviour. In fact, our preliminary analysis shows that industry level effects nearly compensate each other at an aggregate level. This lead us to conclude that the overall contribution to British consumers' welfare of the privatisation policy has been modest.

Of course what is lacking at this stage of our investigation is the construction of more plausible counterfactual scenarios regarding price evolution under the preservation of a national ownership regime. Once these scenarios are somewhat determined, the researcher (or the regulator) can easily go through welfare computations. For this purpose, a series of "ready-to-implement" formulae are available also for large "price reforms", and they do not need the estimation of a micro-level demand system, once one is ready to give up determining an "exact measure" of the welfare change. Besides, at an aggregate level, even a first order approximation (which does not take into account of the curvatures of aggregate demands) seem to provide a quite accurate

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<sup>22</sup> Of course, this exact correspondence depends on the Working-Leser functional form assumption.

indication of the overall welfare effects. Conversely, the use of the more complex measures seems to be needed when an industry level analysis is being carried out.

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Table 1: Welfare variations (in millions of pounds) for each privatised industry (computed at privatisation year expenditures).

	Privatised utilities						
	Phone	Rail	Bus	Electricity	Gas	Water	Coal
Privatisation year	1984	1995	1989	1990	1986	1990	1995
Median year	1991	1997	1994	1994	1992	1994	1997
E1	3655	2333	2256	6472	4299	2226	624
P*	0.88	1.19	1.14	1.01	0.85	1.52	0.82
P1	0.98	1.18	1.04	0.97	1.02	1.14	0.85
P2	0.6	1.22	1.19	0.8	0.71	1.7	0.8
$\eta_{X_i, q_i}$	0.6	0.8	0.9	0.5	0.7	0.5	0.2
$\eta_{\bar{x}_i, \bar{m}}$	1	1	1	1	1	1	1
$\tilde{d}_i$	0.811	0.703	0.703	0.954	0.889	0.716	0.86
<b>Welfare measures</b>							
<b>First order approximations</b>							
"Laspeyres index": $M = X_1(p_1 - p_2)$	1417.284	-79.086	-325.351	1134.297	1306.565	-1093.629	36.678
"Socially weighted": $dW \equiv -d_i X_i dq_i$	1149.418	-55.598	-228.722	1082.119	1161.536	-783.038	31.543
Distributive correction	-18.90%	-29.70%	-29.70%	-4.60%	-11.10%	-28.40%	-14.00%
<b>Second order approximations</b>							
<b>"Unweighted"</b>							
$\Delta W \approx -X_i \left[ 1 + \frac{\Delta q_i}{2q_i} \eta_{X_i, q_i} \right] \Delta q_i$	1582.152	-78.014	-304.235	1183.996	1445.547	-959.324	36.894
<b>"Socially weighted" (Working-Leser form)</b>							
$\Delta W = -\tilde{d}_i X_i \left[ 1 + \frac{\Delta q_i}{2q_i} \eta_{X_i, q_i} \right] \Delta q_i$	1283.125	-54.844	-213.877	1129.532	1285.092	-686.876	31.729
First order "error"	11.63%	-1.36%	-6.49%	4.38%	10.64%	-12.28%	0.59%
Distributive correction	-18.90%	-29.70%	-29.70%	-4.60%	-11.10%	-28.40%	-14.00%
<b>Second order money metric approx.</b>							
$\Delta X_i = -X_i \left( 1 + \frac{1}{2} \frac{\Delta p_i}{p_i} (\eta_{X_i, p_i} + \eta_{\bar{x}_i, \bar{m}}) \right) \Delta p_i$	1307.373	-79.355	-327.698	1084.598	1247.001	-1227.934	35.815
First order "error"	-7.76%	0.34%	0.72%	-4.38%	-4.56%	12.28%	-2.35%

Table 2: Welfare variations (in millions of pounds) for each privatised industry (computed at “median years” expenditures).

	Privatised utilities							
	Phone	Rail	Bus	Electricity	Gas	Water	Coal	
Privatisation year	1984	1995	1989	1990	1986	1990	1995	
Median year	1991	1997	1994	1994	1992	1994	1997	
E1 (at median year prices)	4154	1961	1979	6408	5058	1465	760	
P*	0.88	1.19	1.14	1.01	0.85	1.52	0.82	
P1	0.98	1.18	1.04	0.97	1.02	1.14	0.85	
P2	0.6	1.22	1.19	0.8	0.71	1.7	0.8	
$\eta_{X_i, q_i}$	0.6	0.8	0.9	0.5	0.7	0.5	0.2	
$\eta_{\bar{x}_j, \bar{m}}$	1	1	1	1	1	1	1	
$\tilde{d}_i$	0.811	0.703	0.703	0.954	0.889	0.716	0.86	
<b>Welfare measures</b>								
<b>First order approximations</b>								
"Laspeyres index": $M = X1(p1 - p2)$	1610.551	-66.459	-285.396	1123.066	1537.135	-719.493	44.730	
"Socially weighted": $dW = -d_i X_i dq_i$	1306.156	-46.721	-200.633	1071.405	1366.513	-515.157	38.467	
Distributive correction	-18.90%	-29.70%	-29.70%	-4.60%	-11.10%	-28.40%	-14.00%	
<b>Second order approximations</b>								
"Unweighted"								
$\Delta W \approx -X_i \left[ 1 + \frac{\Delta q_i}{2q_i} \eta_{X_i, q_i} \right] \Delta q_i$	1797.900	-65.558	-266.873	1172.273	1700.644	-631.134	44.993	
"Socially weighted" (Working-Leser form)								
$\Delta W = -\tilde{d}_i X_i \left[ 1 + \frac{\Delta q_i}{2q_i} \eta_{X_i, q_i} \right] \Delta q_i$	1458.097	-46.087	-187.611	1118.348	1511.873	-451.892	38.694	
First order "error"	11.63%	-1.36%	-6.49%	4.38%	10.64%	-12.28%	0.59%	
Distributive correction	-18.90%	-29.70%	-29.70%	-4.60%	-11.10%	-28.40%	-14.00%	
<b>Second order money metric approx.</b>								
$\Delta X_i = -X_i \left( 1 + \frac{1}{2} \frac{\Delta p_i}{p_i} (\eta_{X_i, p_i} + \eta_{\bar{x}_j, \bar{m}}) \right) \Delta p_i$	1485.651	-66.685	-287.454	1073.860	1467.060	-807.851	43.677	
First order "error"	-7.76%	0.34%	0.72%	-4.38%	-4.56%	12.28%	-2.35%	

Table 3. Aggregate welfare effects (in billions of pounds)

		welfare change	Perpetuity value
Privatisation year	1987		
Median year	1992		
E*	29.5		
Price variation	8%		
$\eta_{X_i, q_i}$	0.62		
$\eta_{\bar{x}_i, \bar{m}}$	1		
$\tilde{d}_i$	0.805		
<b>Welfare measures</b>			
<b>First order approximations</b>			
"Modified WP-H": $M = E^* (p1 - p2)/p^*$		2.35	46.97
$dW \equiv -d_i X_i dq_i$		1.89	37.81
Distributive correction	-19.50%		
<b>Second order approximations</b>			
<b>"Unweighted approximation"</b>			
$\Delta W \approx -X_i \left[ 1 + \frac{\Delta q_i}{2q_i} \eta_{X_i, q_i} \right] \Delta q_i$		2.41	48.13
First order "error"	2.48%		
<b>"Socially weighted" (Working-Leser form)</b>			
$\Delta W = -\tilde{d}_i X_i \left[ 1 + \frac{\Delta q_i}{2q_i} \eta_{X_i, q_i} \right] \Delta q_i$		1.94	38.75
First order "error"	2.48%		
Distributive correction	-19.50%		
<b>Second order money metric approximation</b>			
$\Delta X_i = -X_i \left( 1 + \frac{1}{2} \frac{\Delta p_i}{p_i} (\eta_{X_i, p_i} + \eta_{\bar{x}_i, \bar{m}}) \right) \Delta p_i$		2.31	46.25
First order "error"	-1.52%		