

## THE TAX MIX CHOICE IN ITALIAN MUNICIPALITIES

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## Abstract

This paper investigates the economic, socio-demographic and political determinants of the local tax mix choice based on a panel data of 4931 Italian municipality observed along a period of 4 years (1999-2002). Estimation is performed by a system of reduced form equations where the revenue share of each tax instrument (personal income surcharge, property tax and administration fees) represents the dependent variable while political economic and socio demographic characteristics of each municipality are the exogenous variable. The results indicate that the local tax mix choice is affected by the grants from higher level of government, the size of the tax bases, some specific demographic characteristics as the share of elderly and young population and finally local government's ideology.

## 1 Introduction

A common feature of local government finance structures is represented by the possibility for decentralized authorities to raise own revenues from a number of different sources (Sutherland et al., 2005). The typical "tax mix" includes revenues from taxing various types of property, income and consumption bases, as well as charging households and businesses for the use of local public services.

The most obvious and important implication of such widespread institutional arrangement is that the one-tax government model that is often employed in theoretical as well as empirical local public finance research runs the risk of overlooking the crucial fiscal dilemmas faced by local governments.

This paper aims at investigating empirically the determinants of the local tax mix choice based on unique and comprehensive panel data on the Italian municipal governments. The Italian system of municipal government is characterized by a large number of authorities (over 8,000), a high degree of fiscal decentralization, and a notable variety of local revenue sources, including property taxes, personal income surcharges, and user fees. This implies that the municipal tax mix choice has non-negligible allocative, distributive and political consequences.

Over the past two decades, a complex and still to complete process of fiscal decentralization occurred in Italy, starting from the introduction of a local property tax in the early 1990s, followed a few years later by the introduction of a municipal (and regional) surcharge on the national personal income tax, and culminated in the 2001 constitutional reform that further enlarged (in principle) the fiscal autonomy of all subcentral tiers of government. However, the fiscal decentralization process came to a halt during the 2000s, after the introduction of severe state limitations on local government income and property tax discretion.

This paper analyzes the tax mix determination process in the Italian municipalities during the years (1999-2002) when local fiscal discretion was at its highest. Understanding what drives the local tax mix choice is of key importance to evaluate the efficiency and distributive consequences of fiscal decentralization reforms that significantly enlarge the set of tax instruments of local governments (as the ones that occurred in Italy in the 1990s), as well as to the implementation of effective equalization mechanisms.

The empirical analysis of the local tax structure relies on the well-known Hettich and Winer (1984) political economy framework, and largely follows the econometric approach employed by Kenny and Winer (2006) in their cross-country investigation of the state tax mix. In particular, we focus on two crucial aspects of the local tax mix determination process. The first concerns the investigation of the economic, socio-demographic and political determinants of the observed tax mix: we estimate a system of reduced-form equations where the revenue shares from each of the available tax instruments (personal income surcharge, property tax and user fees) are jointly regressed on a common set of relevant characteristics of each municipality. The second aspect concerns the test of the so-called “scale effect” hypothesis, according to which an exogenous increase in the total size of the municipal budget requirement should generate a more intense use of all available tax instruments.

The results of the empirical analysis - based on a panel data set of 4931 Italian municipalities observed along a period of 4 years (1999-2002) - show that the main variables affecting the municipal tax mix choice are grants from higher levels of government, tax base sizes, demographic characteristics (shares of elderly and young population) and local government ideology. On the other hand, the scale effect hypothesis only receives weak support.

The paper is organized as follows. Section 2 provides a review of the most recent empirical literature on the local tax setting process, section 3 outlines the theoretical framework, and section 4 describes the Italian system of local government finance. Section 5 illustrates the data and the empirical approach, and presents the estimation results. Section 6 concludes.

## 2 Literature

The fiscal federalism literature has long and deeply explored the issue of whether and what tax revenue sources should be assigned to local governments. However, much less attention has been devoted to the empirical investigation of the factors that explain the local tax mix choice in the presence of multiple sources of own revenues.

Early studies on US local governments focused on the choice between income and property taxes for funding local spending. Nechyba (1997) and Oates and Schwab (2004) analyzed the optimal choice between the property tax and income tax instruments in USA cities. The property tax has long been the primary source of tax revenue for local government in the USA but at the same time this tax has long been a contentious issue. The tax, which finance the public schools, has been the source of continuing dissatisfaction because the tax base is distributed unequally among local jurisdictions, giving rise to unjustifiable fiscal disparities. In order to verify a possible gain in equity introducing an income tax the results of Nechyba 's model indicate that property taxation may be a dominant local tax strategy when community planners take migration and general equilibrium price changes into account but this result is strictly rely on the mobility of local resident as the costs to mobility rise the significance of the conclusion of his model falls. The empirical results of Oates and Schwab 's model are different in fact an introduction of local income taxation would imply a sizable reduction in tax base disparities. However significant differences in tax base across school district remain. Even if the empirical results stress an improvement in equity introducing local income tax, from a theoretical point of view the superiority of one tax base over the other is not clear.

More recent empirical research has investigated the determinants of local governments' decisions to adopt "new" taxes. Ashworth et al. (2006) in particular have explored the setting of new environmental taxes in Flemish municipalities over the period 1991-1999. The empirical results, in the Flemish experience, have shown that political factors are highly significant in the introduction of the tax. The presence of an election discourages innovation but post-election years represent the most favorite time to adopt a green tax. In their work there is evidence than coalition governments are more likely to set the new tax that fragmented government. An other crucial variable which determines the introduction of a new tax is the neighbors: the greater the adoption of the tax amongst neighbors, the greater the probability that a given municipality will introduce the tax. The general result is that the adoption of green taxes seems to depend more heavily on the political institutional context than the environmental situation on the jurisdiction as the green tax is not more likely to be adopted in highly polluted municipalities.

Fiva and Rattso (2007) use data on local governments in Norway, where the local authority can choose whether or not to introduce a tax on residential property. They show that the chosen tax structure is affected by yardstick competition, in the sense that the probability of having a residential property

tax in a jurisdiction is affected by the decisions made by nearby authorities. Their analysis also shows that grants have no effect on the propensity to have property taxation, while political factors are important.

However, the setting of the overall local tax mix choice has received limited empirical attention both at the national level and at the decentralized tier. The few explorations of the tax mix at the national level concern the analyses of taxation structure of 100 democratic and non democratic regimes (Kenny and Winer, 2006) and the tax system of a large sample of OECD countries for the period 1965-1995 (Volkerink and De Hann, 1999).

Based on the Hettich and Winer (1999) model, Kenny and Winer (2006) obtain the following main results: a) utilization of each tax source increases as the government expands; b) tax systems rely more heavily on relative larger tax bases; c) lower costs of administrations lead to increased reliance on the corresponding revenue source; d) democracies rely more on personal income taxation.

On the other hand, the aim of the Volkerink and De Hann (1999) research is to develop and test the influence of political and institutional variables on the tax ratio and on the shape of the tax mix of OECD countries. The main conclusion of their results is that political and institutional factors are not important explanatory factors for the shape of tax structure for the last year of considered period.

The empirical works on the overall tax mix choice at local government level have concerned a sample of large US cities (Inman, 1986) and Flemish municipalities (Geys and Revelli, 2009).

Inman (1986) estimated a model of the institutional, political and economic determinant of local tax policy. The Inman model assumes that the mayor in choosing the tax mix instruments receives pressures from three sources: a city council interested in providing core government services with the lowest tax rate possible, city agencies interested in providing agency services with the lowest agency fee possible and competitive tax payer coalitions interested in shifting the aggregate burden of local services from themselves to other taxpayers. The mayor tax mix choice is obtained by maximizing the sum of the three previous agents with respect to economic and legal constraints. Through this model Inman analyzes how redistributive politics affect the local tax policy and the obtained results cast serious doubt on the validity of the representative or average taxpayer approach to behavioral modeling of fiscal policy for large income diverse governments.

Geys and Revelli (2009) analyze the determinants of tax mix choice in Flemish municipalities based on the Hettich and Winer model of taxation, where the tax mix choice is a part of a broader political equilibrium in which political parties are assumed to maximize their expected vote. The empirical analysis has investigated the economic and political determinants of the local governments in Flemish region over the period 1995-2002. The results of empirical analysis suggest that the tax mix choice strongly reflects the needs and socio-economic characteristics of the municipalities and is only weakly affected by political biases.

As far as the Italian case is concerned, the local tax determination process has been studied in a number of recent papers. Bordignon et al. (2003) use data on the municipalities in the region Lombardia. The results of estimation of a spatial property tax setting equation show that positive spatial auto-correlation emerges only with regard to mayors having electoral concerns, while mayors facing a binding term limit as well as the ones that are confident of re-election do not appear to be affected by their neighbors' policies. The hypothesis of yardstick competition in setting the property tax is tested also by Padovano (2008), who considers a comprehensive dataset of Italian cities over the period 1993-2001.

Other empirical works concerning the local choice of property taxation in the Italian case are provided by Rizzi (2000) and Fedeli and Giannoni (2004). Rizzi (2000) analyzes the determinants of the choice of the local property tax rate in the municipalities of an Italian province (Udine) in the first year of introduction of the property tax (1993). The econometric analysis is based on a theoretical model in which a municipality chooses the property tax rate in the presence of transfers from other institutions and local income taxes. The most interesting result of the econometric analysis is the link between the level of tax rate and the variables which represent the structure of the budget, while socio-demographic and political variables that should capture municipalities' preferences turn out not to be significant.

Different results are obtained by Fedeli and Giannoni (2004), who analyze the choice of the property tax rate based on the entire set of Italian municipalities in the period 1998-2001. Their results show that the mayor's choice of setting the property tax rate is both significantly determined by variables which represent the structure of the municipality budget (state transfers and other sources of own tax revenue) and socio-demographic variables.

Finally, the choice between property tax and income tax has been previously analyzed in the Italian case by Bordignon and Piazza (2009), who investigated the effects of the Italian municipal finance reform of the late 1990s on tax setting behavior and local politicians' turnover. The main assumption of the theoretical model is that the surcharge on personal income tax, introduced in 1999 at the municipality level, is "less transparent" than the property tax (the main source of local finance).<sup>1</sup> Their theoretical analysis suggests that the less transparent tax instrument should give incompetent mayors a less costly way to "pool" with competent ones, allowing them to be more easily re-elected. Empirical results based on a sample of Piedmont municipalities tend to confirm the main theoretical hypotheses.

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<sup>1</sup> As clarified by authors the surcharge on personal income tax is less transparent than the property tax in sense of allowing for a less precise attribution of responsibility to the different levels of governments. In fact while the choice concerning the property tax could clearly attributed to the municipal government this is not the case for the surcharge on personal income tax. The administration of this latter tax is in fact shared between central state and municipality.

### 3 The economic theory of the tax mix choice

The theoretical background employed to analyze the tax mix choice of Italian municipalities builds on the theory of revenue structures developed by Hettich and Winer (1984, 1988, 1999). The equilibrium tax mix, in the Hettich and Winer model, is a part of a broader political equilibrium in which competition between parties for support from heterogeneous voters forces the government to choose a tax structure that minimizes the loss in terms of electoral support associated with the different tax sources. An important assumption of the model is that the government's sole objective is to be reelected and it pursues this aim by choosing policies that maximize total expected support across a heterogeneous electorate.

There are two determinants of the probability of support: the provision of public good with a positive effect, and the raising of taxes needed to finance the public output with a negative effect. A crucial assumption is that individual taxpayers do not see connection between the level of service provided and their own tax burden. This implies the lack of direct link between expenditure and tax structure, although the expenditure decision is endogenous in the model and affects tax structure indirectly through the government budget constraint.

Voters are affected by taxation in two basic ways: directly as taxes reduce their disposable income, and indirectly as there is a welfare loss due to the economic adjustments made in response to taxation. The sum of these effects can be defined as the loss in full income and taxation is positively related to this loss. Moreover, voters are assumed to be heterogeneous in several respects: they have different evaluations of public output and also different taxable activities and behaviors in order to escape taxation.

Given these assumptions, it is possible graphically illustrate the equilibrium tax structure resulting from the government's optimization problem which objective it is to maximize the probability to be re-elected by making appropriate choices concerning the taxation of different activities and the level of public output.

Figure 1 shows a stylized balanced-budget case with two taxes.

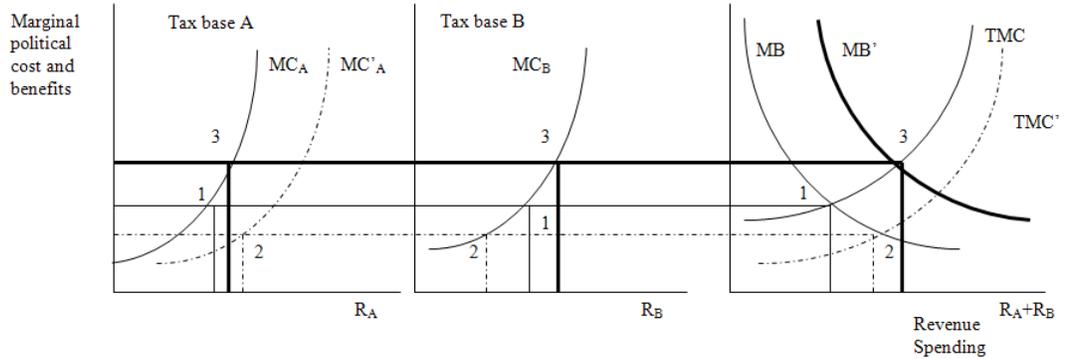
In the first and second panel of figure 1 are illustrated two marginal cost curves ( $MC_A$ ,  $MC_B$ ) which reflect the government's expected marginal vote loss from levying taxes on the two tax bases respectively. The two marginal political cost differ across the two bases reflecting the different voter evaluation of the economic effects of taxation levied on each activity. The total marginal cost curve (TMC), in the third panel on the right hand side of figure 1, is obtained by horizontally summing the two marginal cost curves.

The marginal cost curves are upward sloping because taxation becomes increasingly costly when the amount of revenue raised increases.<sup>2</sup> The total marginal benefit curve (MB curve in the third panel) is obtained by vertical sum of the two individual marginal benefit curves (which are not drawn in the graph for

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<sup>2</sup>See Hettich and Winer (1984), p. 73

Fig.1



ease of exposition) and is downward sloping to indicate that increasing amounts of government spending become progressively less desirable to the electorate.

The initial tax structure of equilibrium is at point 1 where the total marginal benefits equals the total marginal cost. This equilibrium results from a classical optimization problem in which the government, for any level of expenditures will adjust the tax rates until the marginal cost are equalized over the different tax instruments.

In the general case of  $N$  available tax instruments and a vector  $x$  of exogenous variables which determine the characteristics of marginal cost curves, the governments solve the optimization problem equalizing the marginal costs across all  $N$  bases. In this optimization problem, the result is given by a vector of optimal tax revenues as function of all exogenous variables in the model.

The model provides empirical predictions that can be tested, and figure 1 reports two examples of how exogenous shocks could affect the equilibrium tax mix.

An exogenous shock which affects the marginal cost curve is given, for example, by an increase of tax base A as depicted in the first panel of the figure 1. In this case the marginal cost associated to this tax base shift down and in turn this imply the same shift in the total marginal cost curve. The main reasons which explain the shift of marginal cost curve is that now it is possible collect the same revenue from base A with a lower rate. The new tax structure of equilibrium is at point 2 and, as result, reliance on base A increases while the revenues collected from base B declines. In the opposite case we should obtain a decrease in reliance on a base whose relative size has fallen (Hettich and Winer define these effects as "base effect"). A shift of marginal cost curves could also be determined by an increase (decrease) in the costs of administering a tax base. In this case the effects ("administration cost effects") on the tax structure of equilibrium are similar to "base effect".

The second shock considered concerns an exogenous increase in the level of

the benefits associated with the provision of public good, this is the case, for example, of an exogenous increase of community needs. The new equilibrium is at point 3 and in this case the reliance of different tax source depends on the shape of the individual political cost curve but the general result is that all bases are used more heavily as total revenues grow ("scale effect"). The "scale effect" is also evident in the presence of grants from higher level of governments. In this case (for ease of exposition this example is not drawn in the figure 1) the marginal benefit of public output funded by own taxes declines and the new resulting equilibrium will be opposite to equilibrium described at point 3. In fact in this case we should observe a reduction in the level of tax revenues and a reduced reliance on both available tax instruments.

The above stylized model indicates the main factors playing a role in determining the nature of the tax mix in a competitive political equilibrium, so it can be used to assess how exogenous changes in the socio-economic, demographic and political aspects of a community affect the optimal choice of tax structure. In section 5, the empirical predictions provided by the model are tested on panel data. In particular, in section 5.1 the empirical work is devoted to analyze whether economic, socio-demographic and political factors affect the tax mix choice in Italian municipalities, while we empirically evaluate the "scale effect" in section 5.2.

## 4 The Italian system of municipal finance

The Italian system of local government consists of four tiers: central, regional, provincial and municipal. There are 20 regional governments, 109 provincial governments and more than 8000 municipalities. Each local government is basically autonomous in pursuing its policies and, in particular, there are very few financial links between the different levels of sub-national governments. On the contrary, each local government has a direct financial link with the central government, which at least partly finances current and capital expenditures with grants.

Municipalities and provinces are administrative bodies. Municipalities, by far the most important of the two, are responsible for managing services such as local police, public hygiene, social welfare, solid waste collection, street cleaning, urban planning, urban public transportation, street maintenance, zoning and regulation of trade, supply of gas and electricity, parks and sports facilities, and the provision and maintenance of buildings for primary and secondary education.

Municipal finance in Italy has been historically characterized by a large share of the financial needs covered by grants from the central governments. Up to the early nineties transfers resulted from yearly negotiations with central government; the aim of fund allocation was to compensate for individual differences between past expenditures and own revenues. The recursive link between State transfers and past expenditures created several inefficiencies and a deterioration of the overall fiscal framework weakening local administrators' budget

constraints and generating overspending.

The monetary unification process, over the nineties, forced Italy to engage in budget consolidation. Decentralization was considered an important instrument to achieve fiscal discipline and Low-tiers financing system was substantially changed: the reforms introduced in this period renewed the intergovernmental transfers system and allowed municipalities to levy own taxes.

The municipality revenue structure was reformed in 1992, with the assignment of a property tax along with the rationalization of transfers from State, which became largely unconditional. The new criteria for grant allocation reflected structural parameters (i.e. demographic, socioeconomic and fiscal indicators) rather than past expenditures. Further changes occurred in 1997, when some minor revisions of the allocation criteria were put in place, in 1998 with the introduction of a surcharge on the personal income tax and in 2001 when the Parliament approved a constitutional reform which modified the powers of sub national governments and their financial relationships with the central government

Starting from 1999, the main sources of own revenues for Italian municipality have been the Property tax rate and a surcharge on the national personal income tax.

The local property tax rate, named ICI (Imposta Comunale sugli Immobili), was introduced together with a reform of the municipal electoral system, in order to increase the administrative power and the accountability of city governments.

ICI applies to both domestic and business properties. While the property tax base is defined by national procedures and regulations, as it is determined essentially on cadastral income and it is therefore uniformly determined across local jurisdictions, municipalities are free to choose different tax rates on domestic and business dwellings.

The range of the tax rates varies in the interval from a minimum of 0.4% up to a maximum of 0.7%. However, domestic property taxation may be accompanied by lump sum deduction, so that the statutory domestic tax rate does not coincide with the effective domestic tax rate.<sup>3</sup>

The municipal personal income surcharge was introduced in 1999, as additional tax instrument to increase municipality tax autonomy. Its tax base consists in the total taxable income for the national income tax declared by municipality residents. The range of tax rate goes from a minimum of 0% to a maximum of 0.5% but municipalities were made to levy the maximum tax rate of 0.2% per year. However over the period 2003-2006 the national government imposed a freeze on the personal income surcharge and this event justifies the choice of the period 1999-2002 in my analysis.

The residual third component of municipalities own source revenue is represented by fees and user charge.

Summary statistics are useful to highlight some aspects of Italian local finance and the following tables consist in a data processing of Italian municipal-

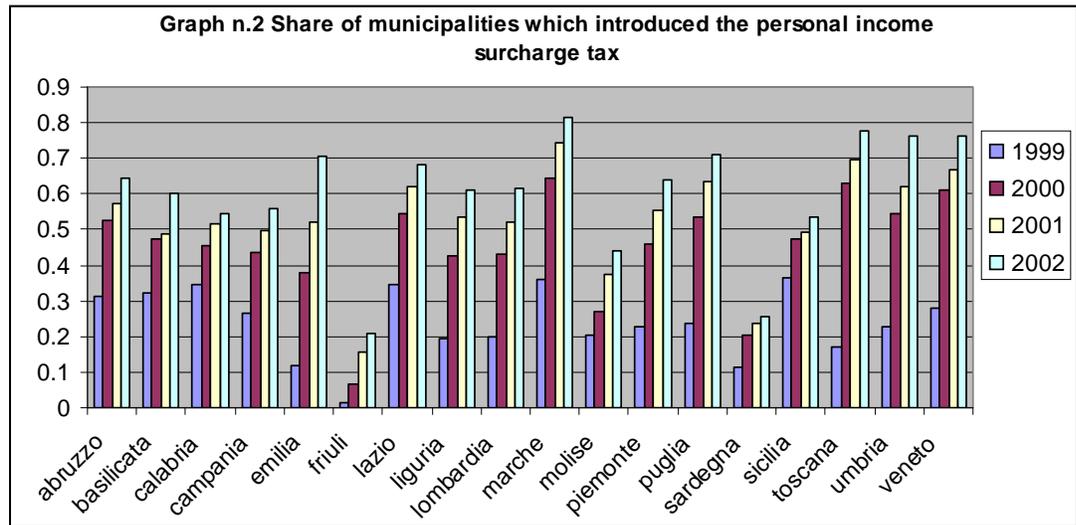
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<sup>3</sup> Actually in 2007 important reforms renewed this tax. In particular this tax was abolished on the property considered main dwellings.

ities balance sheet provided by Italian Ministry of Internal Affairs.

Figure 2 presents the regional mean of municipalities which levied a positive personal income surcharge tax over the period 1999-2002.

Except for two special region which actually did not introduce this the surcharge gradually became an instrument extensively employed by Italian municipalities, in fact in 2002 in almost all the regions the share of municipalities that introduced this tax was over the 50%.



Tables 1 and 2 present respectively the structure of municipality own revenues (table 1) and the share of own tax revenues and current transfers on total revenue of municipality (table n.2).

The data reported in table 1 show that over the period considered property tax was the most important source of own revenues for Italian municipalities in all regions but, the reliance of personal income surcharge tax on total tax revenue constantly increased except for one special region (Valle d’Aosta).

The data of table 2 indicate that, for Italian municipality, share of total own tax revenue on total revenue increased from a national average of 12% in 1999 to a national average of 25% in 2002, but it is also evident that in most regions the share of total current grants on total revenue is greater than share of total own tax revenue on total revenue confirming the crucial importance of current grants in funding the expenditure of municipalities.

The Italian local electoral system was reformed in 1993 with the introduction of the direct election of the mayors. The municipality governmental structure consists of three main bodies: the council (consiglio), the giunta and the mayor. The council is the representative body; it exercises law- and policy-making powers; its members are elected by universal suffrage for a five-year term. The giunta is the executive body; it is made up of a number of assessors which may vary

Tab. N.1 Share of property tax (ICI) and personal income surcharge tax on total own tax revenues (regional mean)

Reg.	1999		2000		2001		2002	
	ICI/ Tot. rev	Surch/ Tot. rev	ICI/ Tot. rev	Surch/ Tot. rev	ICI/ Tot. rev	Surch/ Tot. rev	ICI/ Tot. rev	Surch/ Tot. rev
ABR	0.511	0.017	0.511	0.038	0.546	0.054	0.498	0.073
BAS	0.402	0.018	0.411	0.035	0.455	0.051	0.379	0.058
CAL	0.412	0.020	0.415	0.033	0.439	0.046	0.395	0.053
CAM	0.460	0.012	0.468	0.029	0.515	0.042	0.456	0.052
EMI	0.594	0.006	0.611	0.023	0.647	0.041	0.551	0.051
FVG	0.530	0.001	0.544	0.005	0.554	0.013	0.524	0.019
LAZ	0.476	0.016	0.505	0.036	0.548	0.053	0.616	0.066
LIG	0.586	0.010	0.582	0.028	0.601	0.048	0.538	0.054
LOM	0.548	0.012	0.550	0.033	0.605	0.050	0.508	0.058
MAR	0.477	0.023	0.501	0.057	0.557	0.087	0.444	0.096
MOL	0.518	0.014	0.523	0.021	0.562	0.033	0.528	0.040
PIE	0.633	0.014	0.577	0.037	0.591	0.059	0.506	0.066
PUG	0.483	0.013	0.472	0.037	0.504	0.057	0.449	0.066
SAR	0.426	0.007	0.415	0.015	0.467	0.022	0.456	0.024
SIC	0.455	0.020	0.438	0.037	0.462	0.053	0.488	0.057
TOS	0.556	0.008	0.552	0.036	0.568	0.059	0.489	0.065
UMB	0.500	0.012	0.480	0.039	0.515	0.059	0.476	0.072
VEN	0.564	0.022	0.570	0.051	0.635	0.075	0.694	0.083
TOT	0.529	0.013	0.526	0.033	0.565	0.050	0.517	0.057

Source: Italian Ministry of Internal Affairs

(for the local bodies) depending on demographic density. The mayor is the chief executive and holds office for 4 year but in 2000 this interval was further extended to five years. Mayors can run only for two consecutive terms and to be reelected again he have to stay out for one or more legislatures.

Tab. N.2 Share of total own tax revenues and total current grants on total revenues (regional mean)

Reg.	1999		2000		2001		2002	
	Tot tax rev/tot rev	Tot cur grants /tot rev	Tot tax rev/tot rev	Tot cur grants/ tot rev	Tot tax rev/to t rev	Tot cur grants /tot rev	Tot tax rev/to t rev	Tot cur grants /tot rev
ABR	0.102	0.163	0.106	0.151	0.097	0.153	0.210	0.265
BAS	0.055	0.198	0.058	0.175	0.058	0.189	0.112	0.311
CAL	0.067	0.198	0.074	0.195	0.070	0.187	0.154	0.367
CAM	0.084	0.160	0.092	0.144	0.083	0.143	0.158	0.231
EMI	0.165	0.099	0.165	0.085	0.148	0.094	0.330	0.126
FVG	0.098	0.156	0.093	0.158	0.356	0.453	0.199	0.304
LAZ	0.106	0.146	0.115	0.136	0.107	0.140	0.225	0.260
LIG	0.158	0.124	0.158	0.109	0.128	0.087	0.296	0.171
LOM	0.148	0.115	0.156	0.108	0.137	0.111	0.348	0.178
MAR	0.080	0.111	0.092	0.116	0.083	0.122	0.209	0.208
MOL	0.062	0.148	0.080	0.166	0.083	0.172	0.151	0.318
PIE	0.146	0.132	0.148	0.123	0.127	0.112	0.304	0.183
PUG	0.116	0.166	0.142	0.176	0.117	0.146	0.256	0.258
SAR	0.046	0.194	0.063	0.245	0.062	0.240	0.097	0.408
SIC	0.070	0.224	0.080	0.226	0.074	0.222	0.150	0.430
TOS	0.157	0.113	0.158	0.095	0.142	0.105	0.325	0.160
UMB	0.090	0.102	0.110	0.113	0.099	0.119	0.192	0.189
VEN	0.166	0.122	0.175	0.109	0.152	0.118	0.347	0.168
TOT	0.119	0.141	0.126	0.137	0.118	0.145	0.253	0.233

Source: Italian Ministry of Internal Affairs

## 5 Data and empirical approach

### 5.1 Data description

The dataset employed in this empirical work contains financial, socio-economic and political information for all Italian municipalities (8,100) over the period 1999 -2002.<sup>4</sup> In particular, it contains the main balance sheet items of Italian municipalities - total current grants from the state, property tax revenues, and personal income surcharge revenues. The socio-economic characteristics of each municipality include population, population density, shares of elderly and young population, the unemployment rate (measured at the provincial level) and the income tax base. Political data concern the presence of election in a particular year, the municipality government's ideology (left, center or right), and the share of female participation in the municipal executive body. Finally, we have information on whether a municipality participates to intermunicipal cooperation organizations for the provision of public services ("Unione di Comuni"), and whether it suffers budget limitations due to the domestic stability pact subordination.

After dropping units with missing or incongruous data, we end up with 19724 observations (a balanced panel dataset with 4931 observations per year).<sup>5</sup> Table 3 provides a synthetic description of the data and their respective sources.

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<sup>4</sup>The dataset has been managed and collected by Professor Fabio Padovano with the collaboration of Ilaria Petrarca.

<sup>5</sup>The municipalities located in two special regions, Valle d'Aosta and Trentino Alto Adige, cannot to be included in the analysis as those regions did not introduce the municipal surcharge on personal income.

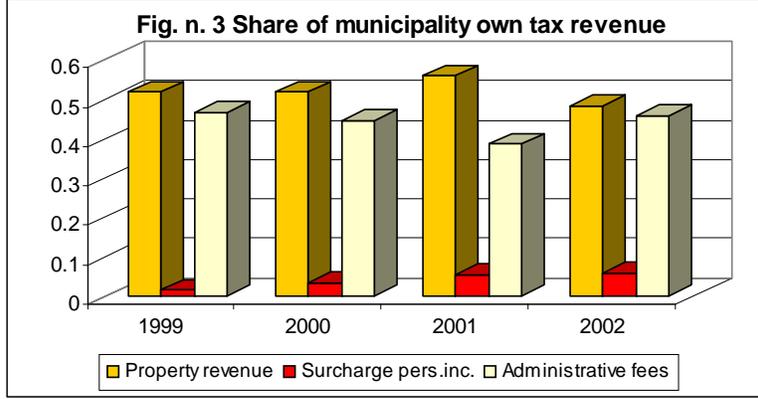
Tab. n.3

Data	Source
Income tax revenue	Italian Ministry of Internal Affairs
Property tax revenue	
Administrative fees and user charge	
Intergovernmental current grants	
Property tax base	
Share of female partic. in executive body .	
DSP (Domestic stability pact - dummy variable)	
Participation an upper tier of government ("Unione dei comuni" -dummy variable)	
Political variables (election year, left center or right coalition - dummies variables)	
Share of elderly and young population	ISTAT
Provincial unemployment rate	
Municipality population	
Jurisdiction area	
Personal income tax base	Ministry of Economy and Finance

Summary statistics for the panel dataset of 19,724 observations are provided in table 4 and figure 2. Table 4 shows that the property tax is the main source of own municipal revenue over the period 1999-2002, and that reliance on the personal income surcharge has significantly increased over time, with fee revenues remaining fairly stable.

Table 4: Tax revenue shares				
(averages across all municipalities)				
Revenue shares	1999	2000	2001	2002
Property tax	0.518	0.518	0.560	0.483
share	(0.114)	(0.133)	(0.13)	(0.131)
Personal income	0.015	0.035	0.054	0.059
surcharge share	(0.028)	(0.046)	(0.060)	(0.058)
Charges and fees	0.467	0.447	0.386	0.458
share	(0.112)	(0.131)	(0.127)	(0.127)

Note: N=4931, standard deviation in brackets



## 5.2 Empirical approach

Our empirical strategy consists in the estimation of a reduced-form system of tax revenue share equations, where a common vector of time-varying independent variables observed in municipality  $m$  in year  $t$  ( $\mathbf{x}_{mt}$ ) is intended to capture the municipality characteristics having an effect on the local tax mix choice.

In particular, given the fiscal instruments available to Italian municipalities - personal income tax surcharge ( $i$ ), property tax ( $p$ ) and user charges ( $u$ ) - the system consists of the following three revenue share equations:

$$rs_{mti} = \alpha_i + \mathbf{x}'_{mt}\beta_i + c_{mi} + q_{ti} + \nu_{mti} \quad (1)$$

$$rs_{mtp} = \alpha_p + \mathbf{x}'_{mt}\beta_p + c_{mp} + q_{tp} + \nu_{mtp} \quad (2)$$

$$rs_{mtu} = \alpha_u + \mathbf{x}'_{mt}\beta_u + c_{mu} + q_{tu} + \nu_{mtu} \quad (3)$$

where the share of revenues from each tax instrument ( $rs_{mtk}$ ,  $k = i, p, u$ ) is obtained as the ratio between total revenues from each tax source ( $r_{mtk}$ ) and total revenues from all municipality tax sources ( $r_{mt} = \sum_k r_{mtk}$ ).  $\alpha_k$  ( $k = i, p, u$ ) are the constant terms, while  $c_{mk}$  and  $q_{tk}$  represent municipal-specific and time-specific effects, and are treated as fixed.

The vector  $\mathbf{x}_{mt}$  includes a first set of variables measuring the availability of external resources (per capita total current grant) as well as the ability to raise own revenues, namely personal income and property tax bases.

The second set of variables included in  $\mathbf{x}_{mt}$  consists of socio-demographic characteristics that might affect the administrative and political costs of raising taxes. These variable are population, density of population, shares of elderly and young population, and the unemployment rate.

As for the political traits of a municipality, three variables are included to indicate the government's ideology (left, center and right ideology), a dummy variable indicating the presence of an election year, and a variable accounting

for the share of women in the executive body of the municipality. The political variables allow for the potential influence of parties' political ideology on the local tax mix choice, and in particular the inclusion of the last political variable intends to check if the gender is an indicator of policy preferences as shown in a number of recent studies<sup>6</sup> (e.g. , Funk and Gathmann, 2008; Edlun and Pande, 2002; Lott and Kenny 1999).

Finally, in order to allow for external constraints on a municipality's policy, we include a dummy variable "domestic stability pact" which indicates if a municipality is subject to that fiscal rule<sup>7</sup>, and a dummy variable equaling 1 if a municipality joins an intermunicipal tier of government<sup>8</sup>.

The system of three equations is characterized by the same set of regressors in each equation and the adding up constraint. In fact, the revenue shares from each tax source must sum to 1, which requires that the sum of the constant term coefficients across the equations equal one, that the column sum of the coefficients on each variable  $j$  in the vector  $\mathbf{x}_{mt}$  equal zero, and that the residuals sum to zero:<sup>9</sup>

$$\sum_k \alpha_k = 1 \tag{4}$$

$$\sum_k \beta_{kj} = 0 \tag{5}$$

$$\sum_k v_{mtk} = 0 \tag{6}$$

The system of revenue share equations constitutes a seemingly unrelated regression model. To make the model operational, we must impose the restrictions (4) and (5) and solve the problem of singularity of the disturbance variance-covariance matrix of the share equation implied by restriction (6). The

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<sup>6</sup>Other studies have also showed that female representation often affects the size of public sector and the composition of public spending (e.g. Pande, 2003; Svaleryd, 2009)

<sup>7</sup>As a consequence of the Stability and Growth Pact, many European countries have introduced fiscal rules to limit the expenditures of local administrations.

In Italy the law 448/1998 prescribes the implementation of a Domestic Stability Pact (DSP) which limits the budget deficit of local governments from the year 1999 onwards.

The scope of the law spans over all levels of the Italian territorial administrative structure: regions, provinces and municipalities. However, from the year 2001 municipalities with a population smaller than 5000 inhabitants were excluded from the DSP. In the year 2002 and from the year 2005 onwards, there has been an extension of the scope of the DSP, imposing also a limit to the growth rate of expenditures.

<sup>8</sup>Italian law (law n. 142/90 and following amendment) permits some forms of partnership among local councils; one of them is called Unioni di Comuni. Municipalities belonging to this form of upper tier of government can allocate essential function and the provision of public services to Unione. Empirical works concern the experience of unione dei comuni in a particular Italian region (Marche, e.g Erimi Santolini, 2006; Ermini Salvucci, 2005)

<sup>9</sup>For a review of literature concerning the systems of demand equations:Greene W. H. *Econometric Analysis*- ch. 14, Prentice Hall (fifth edition).

Wooldridge J.M. *Econometric Analysis of Cross Section and Panel Data*- ch. 7, MIT Press.

traditional approach consists in dropping one of the three equations and estimating the remaining ones.<sup>10</sup> The parameters not estimated directly are computed using (4), (5) and (6). As shown in the Appendix, OLS estimation equation by equation provides identical parameter estimates.

The employment of panel data and the correlation assumption between municipality fixed effects and regressors requires the within transformation of the three equation in order to estimate the model. With  $\bar{sr}_{mk} = T^{-1} \sum_{t=1}^T sr_{mtk}$ , the within-transformed dependent variable is:  $\tilde{sr}_{mtk} = sr_{mtk} - \bar{sr}_{mk}$ . Table 4 reports the estimation results.

Table 4 TAX REVENUE SHARE SYSTEM

Within variables	Income share		Property share		Fees share	
Grants	-0.0074***	(-4.59)	0.018***	(4.3)	-0.011**	(-2.56)
Income tax base	0.0029***	(5.9)	-0.0051***	(-3.81)	0.0021	(1.60)
Property tax base	-0.0038**	(-2.13)	0.004	(0.83)	-0.0001	(-0.04)
Population (,000)	-0.00007	(-0.1)	-0.001	(-0.51)	0.001	(0.54)
Density	-0.086	(-0.23)	0.14	(0.14)	-0.06	(-0.06)
Unemployment	0.029**	(2.58)	-0.029	(-0.98)	0.0006	(0.02)
Elderly pop.	-0.0267	(-0.88)	-0.105	(-1.28)	0.132	(1.58)
Female repres.	-0.0063**	(-2.18)	0.002	(0.26)	0.0043	(0.54)
Upper tier of gov.	0.0063***	(4.68)	-0.0025	(-0.69)	-0.0037	(-1.03)
Dom.stab. pact	0.0068***	(8.89)	-0.0111***	(-5.3)	0.0043**	(2.03)
CenterLeft	0.0051**	(4.18)	-0.002	(-0.68)	0.0029	(-0.85)
Right	-0.004***	(-3.7)	0.014***	(4.99)	-0.01***	(-3.57)
Young	0.059	(1.57)	-0.507***	(-4.93)	0.45***	(4.28)
Elect. year	0.0001	(0.25)	0.003	(1.66)	-0.0031*	(-1.72)
Time effect	yes		yes		yes	
Municip. effect	yes		yes		yes	
Observations	19724 (4931)		19724 (4931)		19724 (4931)	

Note: t statistic in brackets; coefficients significant at level \*\*\*1% ; \*\*5% , \*10%

Consider first the effect of grants. The results of estimation of the three-equation system show that the revenue shares from income surcharges and fees rise as current grants decrease, while the property tax revenue share decreases accordingly. Since a decrease in grants implies that, in order to provide the same level of public services, local governments need to levy higher taxes, the estimation results indicate that Italian local governments tend to increase income tax and fees rather than property taxes. In terms of the theoretical model in section 3, this suggests that the property tax marginal cost curve is steeper than the income and fee ones, making revenues from property taxation fairly inelastic with respect to changes in budget requirements.

<sup>10</sup> As shown in the Appendix, equivalent estimates of parameters can be obtained by different approaches.

In fact, given that until 1999 the property tax was the only own revenue source for Italian municipalities, property tax rates were frequently set close to or at the tax rate limits set by central government, implying that most municipalities could hardly manoeuvre it and were facing an indeed very steep marginal political cost curve. On the other hand, the newly introduced tax instrument (the personal income surcharge) allowed local governments substantial more flexibility in reacting to exogenous changes in budget requirements. In addition, it has been argued that higher reliance on the income surcharge might be due to it being less transparent than the property tax, in sense of allowing for a less precise attribution of responsibility to the different level of government.<sup>11</sup>

As for the availability of tax bases, the theoretical model suggests that a tax base increase for tax instrument  $k$  flattens its marginal cost function, thereby raising the reliance on instrument  $k$  and decreasing the revenues collected from other tax instruments. The estimation results indicate that the share of the income tax (property tax) rises (decreases) when per capita income tax base increases, and it decreases (increases) when the property tax base increases. While in the income surcharge share equation the coefficients of property tax and income tax base are both significant, in the property share equation only the coefficient of income tax base is significant.

Political variables turn out to have a significant direct effect on the share of income tax revenues. In particular, the presence of left-wing parties has a positive effect on personal income surcharge revenue share, while, unlike what emerges in some recent literature, the proportion of females in the executive body has a negative effect.<sup>12</sup>

Moreover the personal income surcharge revenue share is positively affected by the presence of the domestic stability pact and by the fact that the municipality belongs to an upper tier of government. The former effect can be justified by the municipality attempt to solve the pressing budget constraint imposed by domestic stability pact levying taxes with less political cost (surcharge tax and administrative fees), while a positive effect of participation in intermunicipal cooperation on the surcharge share can be justified in terms of fiscal competition. The level of fiscal competition in municipalities belonging to an upper tier of government decreases, and this can justify the choice of higher taxes.

The proportion of young population negatively affects the revenue share of property tax but positively the share of administrative fees. The negative effect of young population on the property revenue share but the positive impact on the other two revenue sources can be justified by the fact that young population implies more needs which in turn require more funding.

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<sup>11</sup>In fact, while the property tax is entirely controlled by municipality, citizens might have some difficulty in discerning in the total personal income tax they have to pay, the part which is due to municipal decision (Bordignon and Piazza, 2009).

<sup>12</sup>Some recent empirical research in fact, has shown that female representation significantly increases reliance on income taxes. This evidence is justified following the argument that women tend to be more egalitarian and socially aware (Funk and Gathmann, 2008; Edlund and Pande, 2002).

### 5.3 The scale effect

The theoretical model, as illustrated in figure 1, predicts that an exogenous increase in the total size of the budget requirement should generate a more intense use of all tax instruments ("scale effect").

The "scale effect" is also evident in the presence of grants from higher level of governments. When grants increase the marginal benefit of public output funded by own taxes declines and we should observe a reduction in the total level of tax revenues and a reduced reliance of all available tax instruments.

To test this hypothesis I firstly estimate the total scale effect by equation (7) and then the scale effect of all tax instruments by a system of three equations (8).

In the first equation, the dependent variable represents per capita total tax revenue of municipality  $m$  and the explanatory variables are the same as employed above.

The scale effect is captured by the coefficient on grants which is expected to be negative indicating that an increase of current transfers should generate a reduction of total tax revenue.

In the system of three equations (8), the dependent variables are per capita tax revenues in municipality  $m$  and year  $t$  from each tax source. According to the theoretical predictions, all grant coefficients are expected to be negative.

$$pcr_{mt} = \gamma + \mathbf{x}'_{it}\boldsymbol{\lambda} + f_m + h_t + \varepsilon_{mt} \quad (7)$$

$$\begin{aligned} pcr_{mti} &= \gamma_i + \mathbf{x}'_{mt}\boldsymbol{\lambda}_i + f_{mi} + h_{ti} + \varepsilon_{mti} \\ pcr_{mtp} &= \gamma_p + \mathbf{x}'_{mt}\boldsymbol{\lambda}_p + f_{mp} + h_{tp} + \varepsilon_{mtp} \\ pcr_{mtu} &= \gamma_u + \mathbf{x}'_{mt}\boldsymbol{\lambda}_u + f_{mu} + h_{tu} + \varepsilon_{mtu} \end{aligned} \quad (8)$$

The fixed effects linear regression results, reported in table 5, show that grants have a negative, but not significant impact on total tax revenues. In fact, the coefficient is virtually zero and is indicative of a "flypaper effect." It is worth to notice that the estimated coefficient of -0.0013 represents the sum of the estimated grant coefficients in the three-equation system.

Table 5 SCALE EFFECT

Within variables	total scale effect (1)	three equations system (2)		
	per capita total tax revenue	per capita income surc.	per capita property rev.	per capita fees
Grants	-0.0013	-0.0027***	0.00685***	-0.0054**
Income tax base	0.0092***	0.0021***	0.00435***	0.0027***
Property tax base	0.011***	-0.0007	0.0114***	0.0003
Population (,000)	-0.0014	-0.00004	0.004	0.0009
Density	0.124	0.069	0.05	0.1421
Unemployment	0.013***	0.006**	0.002	0.005
Elderly pop.	0.129***	-0.0049	-0.003	0.137***
Female repres.	0.0077	-0.001	0.003	0.0059
Upper tier of gov.	-0.0044	0.00121***	-0.0014	-0.0042**
Dom.stab. pact	-0.002	0.0033***	-0.006***	0.0007
Left-center	0.0015	0.0015***	0.0008	0.0009
Right	-0.112***	-0.002***	-0.0015	-0.007***
Young	0.39***	0.105***	0.063	0.215***
Elect. year	-0.0013	-0.0001	0.00012	-0.0013
Time effect	yes	yes	yes	yes
Municip. effect	yes	yes	yes	yes
Observations	19724 (4931)	19724 (4931)	19724 (4931)	19724 (4931)

Note: coefficients significant at level \*\*\*1% ; \*\*5% , \*10%

In fact, the estimated grant coefficients in the system are all significant but do not confirm the theoretical scale effect hypothesis. Only two coefficients exhibit the expected sign (per capita personal income surcharge revenue and per capita administrative fees), while the grant coefficient in the property equation presents a positive sign. Taken literally, the estimated results suggest that an increase in currents grants from higher level of government determines a decrease of personal income surcharge revenue and administrative fees but an opposite effect on property tax revenues.

## 6 Conclusions

The empirical work performed in this paper has explored the economic, socio-demographic and political determinants of the local tax mix choice in the Italian municipalities. In the last two decade Italy has been involved in a complex and still incomplete process of decentralization and in this contest, in 1999, an important local finance reform introducing a new tax, a surcharge on their resident's personal income tax, allowed municipalities to use a larger set of local tax instruments.

The empirical analysis, based on panel data on 4931 Italian municipalities over the period 1999-2002, has been performed by estimating a system of reduced form tax share equations, where the revenue shares of each tax instrument (personal income surcharge, property tax and administration fees) represent the dependent variable, while political economic and socio-demographic characteristics of each municipality are the independent variables.

The results of the empirical analysis show that several variables affect the tax mix choice of Italian municipality.

An important role is exerted by economic variables, specifically grants from higher level of government, and the size of the tax bases play a determinant role in affecting the tax choice in local government. Likewise, other socio-demographic characteristics of municipalities as the share of young population seem to have a determinant effect on the tax mix choice in affecting the property tax revenue and administrative fees.

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## APPENDIX

OLS equation by equation gives identical results as SURE. This evidence is justified by following demonstration:

I suppose to consider a system of  $n$  share equations whit the same set of regressors  $\mathbf{X}_{(n*m)}$ . Applying OLS equation by equation the estimation of parameters is given by

$$\mathbf{b}_1 = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{s}_1$$

$$\mathbf{b}_2 = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{s}_2$$

.

.

$$\mathbf{b}_n = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{s}_n$$

where  $\mathbf{b}_n$  is the parameter vector of the  $n$  equation and  $\mathbf{s}_n$  is the vector of dependent variable.

By construction the vector  $\mathbf{s} = \mathbf{s}_1 + \mathbf{s}_2 + \dots + \mathbf{s}_n = \mathbf{i}$

Summing  $\mathbf{b}_1 + \mathbf{b}_2 + \dots + \mathbf{b}_n$  we obtain a vector  $\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{i}$  which can be rewritten separating the regressors matrix from the constant term vector

$$(A) \begin{bmatrix} \mathbf{b}_0 \\ \mathbf{b}_{m-1} \end{bmatrix}_{(m*1)} = \left( \begin{bmatrix} \mathbf{i} & \mathbf{X}_{m-1} \end{bmatrix}'_{(n*m)} \begin{bmatrix} \mathbf{i} & \mathbf{X}_{m-1} \end{bmatrix}_{(n*m)} \right)^{-1} \begin{bmatrix} \mathbf{i} & \mathbf{X}_{m-1} \end{bmatrix}'_{(n*m)} \mathbf{i}_{(n*1)}$$

where  $\mathbf{b}_0$  represent the constant term parameter,  $\mathbf{b}_{m-1}$  the vector of other parameters and  $\mathbf{X}$  the regressor matrix without the constant term. The expression (A) can be rewritten as

$$(B) \begin{bmatrix} \mathbf{b}_0 \\ \mathbf{b}_{m-1} \end{bmatrix} = \begin{bmatrix} \mathbf{i}'\mathbf{i} & \mathbf{i}'\mathbf{X}_{m-1} \\ \mathbf{X}'_{m-1}\mathbf{i} & \mathbf{X}'_{m-1}\mathbf{X}_{m-1} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{i}'\mathbf{i} \\ \mathbf{X}'_{m-1}\mathbf{i} \end{bmatrix}$$

$$(C) \begin{bmatrix} \mathbf{b}_0 \\ \mathbf{b}_{m-1} \end{bmatrix} = \frac{1}{\mathbf{i}'\mathbf{X}'_{m-1}\mathbf{X}_{m-1}\mathbf{i} - \mathbf{i}'\mathbf{X}_{m-1}\mathbf{X}'_{m-1}\mathbf{i}} \begin{bmatrix} \mathbf{X}'_{m-1}\mathbf{X}_{m-1} & -\mathbf{i}'\mathbf{X}_{m-1} \\ -\mathbf{X}'_{m-1}\mathbf{i} & \mathbf{i}'\mathbf{i} \end{bmatrix} \begin{bmatrix} \mathbf{i}'\mathbf{i} \\ \mathbf{X}'_{m-1}\mathbf{i} \end{bmatrix}$$

We have to demonstrate that  $\mathbf{b}_0=1$  and  $\mathbf{b}_1=0$ , in fact

$$\mathbf{b}_0 = \frac{\mathbf{X}'_{m-1}\mathbf{X}_{m-1}\mathbf{i}'\mathbf{i} - \mathbf{i}'\mathbf{X}_{m-1}\mathbf{X}'_{m-1}\mathbf{i}}{\mathbf{i}'\mathbf{X}'_{m-1}\mathbf{X}_{m-1}\mathbf{i} - \mathbf{i}'\mathbf{X}_{m-1}\mathbf{X}'_{m-1}\mathbf{i}} = 1$$

$$\mathbf{b}_{m-1} = \frac{-\mathbf{X}'_{m-1}\mathbf{i}'\mathbf{i} + \mathbf{i}'\mathbf{X}'_{m-1}\mathbf{i}}{\mathbf{i}'\mathbf{X}'_{m-1}\mathbf{X}_{m-1}\mathbf{i} - \mathbf{i}'\mathbf{X}_{m-1}\mathbf{X}'_{m-1}\mathbf{i}} = \mathbf{0}_{((m-1)*1)}$$