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PUBLIC GOOD AND INCREASING COST

AMEDEO FOSSATI – MARCELLO MONTEFIORI

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Public good and increasing cost

Amedeo Fossati – Marcello Montefiori

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Abstract

We suppose a federation with two tiers of government. The central authority wishes to maximize the social welfare while the local authorities aim at individualistic utility maximization. There exist two types of local governments characterised by different utility, cost and income. The centre lacks precise information concerning what type of cost (high or low) each region is. This study examines how asymmetric information affects the design of the transfer scheme. We show that when the recipient region highlights an income greater than the contributor, then, disregarding the production cost for the public good, a first best outcome is attainable and the incentive compatibility constraints are not binding and therefore the complete information scenario provides the same results as the incomplete one. On the other hand if the contributing region is wealthier with respect to the receiver, then just a second best outcome is reachable in order not to violate the incentive compatibility constraint.

1. *Introduction*

Fiscal federalism is a widespread structure that regards both federation of States such as US or EU and single Countries characterized by local jurisdictions which represent regional or even municipal governments.

Local governments are allowed to collect local taxes and responsible for the provision of local public goods. The central government usually aims at implementing a redistribution policy to ensure optimal level of public goods to maximize the social welfare. In our study we assume that the center has the only means of money transfer (which is the most common system practically adopted) in order to pursue his goal.

The identification of the optimal transfer design suitable to implement the social welfare is a hard task for the policy maker when local jurisdictions differ in utility, public good provisional cost and disposable income and asymmetry of information between the center and the local authorities is assumed.

Because of the afore mentioned reasons, the literature shows a steady interest on modern fiscal federalism and redistribution.

The standard literature results assess that when a public good is privately provided, then the level of its provision turns to be at a lower level with respect to the optimal socially desirable one, even if Williams (1966) claims that "the complex interactions that occur even in highly simplified situations make it impossible to predict a priori whether undersupply or oversupply will generally result".

However it is possible to affect the private decision on how much to provide via income redistribution among individuals as long as they differ in their marginal propensity to contribute to the public good. Warr (1983) shows that the overall level of public good individually supplied might be independent from income redistribution. This important result, known as "the Warr theorem" holds even when individual utility functions are not the same but it is required that individuals behave as atomistic utility maximizers, the redistribution of income has to take place among current contributors of the public good and under the condition of interior solution to the individual utility maximization problem. Furthermore the Warr model assumes that individuals face an identical (and linear) price for the public good. When one of the afore mentioned conditions is relaxed, then the Warr theorem ceases to hold and an income redistribution comes back to affect the overall public good production level and consequently the social welfare.

The discussion on the Warr neutrality theorem has been revived by the paper of Kemp (1984) which extends the theorem to the case of more than one public good and Bergstrom et al. (1986) which "analyze the extent to which government provision of a public good "crowds out" private contributions".

Recent and growing literature on fiscal federalism relates with the implications of information asymmetry when local jurisdictions face different cost for the provision of public good (Cornes and Silva, 2002; Huber and Runkel, 2006).

This paper, consistently with this latter, concerns the transfer design suitable to implement the social welfare and the efficient public provision of public good taking into account both asymmetric information and cost structure.

Our model shows a close relation with the work of Huber and Runkel (2006), but with important differences: first of all, they assume a separable utility function, whereas our analysis is more general and the separability condition is not required. Secondly, they implicitly assume the same utility function for any jurisdictional type, while in the present paper the utility is allowed to vary from region to region according to the jurisdiction's type (high or low cost). As well as their setting we adopt a fairly general cost function $e_i = E^i(x_i, \vartheta_i)$ for the public good *x,* where θ is the cost parameter; whereas their model counts for two local public goods and a private one, we account for two types of good only: a private and a public one. The reason, however, is that, in shaping the federal transfer scheme, the crucial aspects do not concern the fact that the interested good is a private or a public one. Then, in as much as the models are similar, we are able to trust on H&R outcome as far as individual agents may be involved, while our public goods might present externalities on the other jurisdictions, i.e. might be non local public goods. Thus, the model participates the framework of the private provision of public goods as well as the relationships among central and local governments in a federal setting, because we suppose a federation of states with two tiers of Government.

We assume that regional utility directly represents the preferences of citizens, in as mush as the local governments aim at individualistic utility maximization; central government uses the redistribution of resources among the members of the federation to maximize the social welfare which is given, as usual, by the sum of regional utility.

As far as the informational structure is concerned, the centre knows that there are different types of regions characterized by different cost, income and utility. However, it lacks information concerning the type to which each region belongs. Thus, the central government's key informational problem concerns the regional costs and quantities with regard both to the public and the private good. Indeed we assume that the centre can observe the expenditure levels but neither the costs nor the outputs associated with those expenditure levels.

Finally, the policy instrument facing the central government consists mainly of lump sum transfers conditional to a certain amount of expenditure for the public good, but other types of grants are used as well.

In comparison with the current literature, the present paper contributes to the topic in two ways. Firstly, in our fairly general setting (no particular assumptions on utility and cost functions are required) we are able, under perfect information, to highlight the conditions which call for a transfer from the high to the low cost region or vice versa. Secondly, in an asymmetric information setting, we show how central governments might be limited by incentive compatibility constraints, both when the public goods are local and nation wide as well.

2. *The model regarding local public goods*

In this section we assume that *x* is a local public good. Consider an economic federation consisting of two tiers of government: a central government (the State) and a given number of regional governments. We assume that in each region two goods are provided: the private good *y* and the public good *x*. The production cost for the private good *y* is identical among jurisdictions and set, for simplicity, equal to 1. On the other hand the cost for the public good *x* differs according to the jurisdiction's type. We distinguish between two jurisdictional types: the low cost region's type and the high one, denoting the former by the *l* index and the latter by the *h* index. In the economic federation the number of low cost type identical regions is $L>1$, $l=1....L$ while the number of high cost identical regions is $H>1$, $h=1...H$.

The type $i \in \{l,h\}$ region faces an expenditure cost $E^i(\theta_i, x_i)$ on x which depends and increases both on the quantity of the public good x_i provided, and on the θ_i cost parameter, assuming $\theta_h > \theta_l$. The latter characteristic is rendered explicit by the following derivatives: $E_x^i; E_\theta^i > 0$; $E_{xx}^i; E_{x\theta}^i \geq 0$ (the subscript indicates the variable with respect to which the E cost function has been derived, either at first or second order). The maximization problem that faces the region type $i \in \{l,h\}$ is given by $Max\ U^i = f^i(y_i, x_i)$, subject to the budget constraint $R^i + \tau^i = y_i + E^i$, where R^i is the region's i income and τ^i is a lump-sum transfer (either positive or negative) set by the central government. We adopt standard assumptions for the U(·) function: it is increasing in y and x and strictly quasiconcave, as well as that all goods are normal. In order to maximize the utility function subject to the budget constraint, the region chooses the amount of y and x to be provided, so that the correspondent FOCs are[1]

$U_x^i = U_y^i E_x^i$ or equivalently $-SMS_{x,y}^i = E_x^i$

$$R^i + \tau^i = y^i + E^i(\vartheta^i, x^i)$$

Using the implicit function theorem, the optimal values[2] for x and y can be defined:

$$J^i(x^i, y^i) = E_x^i \Rightarrow \begin{cases} y^{i*} = Y^i(\vartheta^i, R^i + \tau^i) \\ x^{i*} = X^i(\vartheta^i, R^i + \tau^i) \end{cases} \qquad (1)$$

Shifting our attention from local to federal government, we assume that central government is concerned about efficiency, and it is interested in the provision of good x, both for reasons regarding competition in the common federation market and homogeneity of rights to be ensured to all the citizenship by the central government. Thus, we can get its efficiency conditions by solving the standard maximization problem

$$\underset{x_i,\, y_i}{Max}\ W = \sum_{i=1}^{L+H} U^i(x^i, y^i)$$

s.t

$$\sum_{i=1}^{L+H} \left[y^i + E^i(\vartheta^i, x^i)\right] = \sum_{i=1}^{L+H} R^i$$

For $-SMS_{x,y}^i = U_x^i / U_y^i$, as usual, the first order conditions are:

$$-SMS_{x,y}^i = E_x^i \quad ; \ i = 1,2, \dots, H+L \qquad (2)$$

$$U_y^i = U_y^j \quad ; \ i = 1,2, \dots, H \ ; \ j = 1,2, \dots, L \qquad (3)$$

Conditions (3) imply that marginal utility of good y be equal among high and low cost regions. Together, eq. (2) and (3) imply

$$U_x^i / E_x^i = U_x^j / E_x^j \quad ; \ i = 1,2, \dots, H \ ; \ j = 1,2, \dots, L \qquad (4)$$

At this stage we assume that the only means available to the central government to get its policy goal consists on a lump sum transfer, equal in amount and sign among all the same type region but different in sign according to the type of region. The central government maximization goal implies to solve the problem:

$$\underset{\tau^i}{Max} \sum_{i=1}^{L+H} U^i(x^{i*}, y^{i*})$$

s.t

$$\sum_{i=1}^{L+H} \tau^i = 0$$

where x^{i*}, y^{i*} are the equilibrium values from eq. (1).

From the lagrangian function $\underset{\tau^i}{L} = \sum_{i=1}^{L+H} U^i(x^{i*}, y^{i*}) - \lambda \sum_{i=1}^{L+H} \tau^i$, we get the condition

[1] The subscript indicates the derivative with respect to that variable, i.e., for instance $U_x \equiv \partial U(.)/\partial x$

[2] Which represent as well the demand function along the optimal path

$$\frac{\partial U^h}{\partial \tau^h} = -\frac{\partial U^l}{\partial \tau^l} \qquad (5)$$

because from the constraint $\sum_{i=1}^{L+H} \tau^i = 0$ we know that high and low cost region must have their τ opposite in sign. The economic hint underlying this condition is straightforward: the central government transfers money from one type region to the other as long as the marginal utility of the receiving jurisdiction is higher, in absolute value, with respect to the giver's. The optimal point is reached when (5) is satisfied.

Using (1), we know that $\frac{\partial U^i}{\partial \tau^i} = \frac{\partial U^i}{\partial x^{i*}}\frac{\partial x^{i*}}{\partial \tau^i} + \frac{\partial U^i}{\partial y^{i*}}\frac{\partial y^{i*}}{\partial \tau^i}$, and, using (2), we can write $\frac{\partial U^i}{\partial \tau^i} = U_y^i\left(E_x^i \frac{\partial x^{i*}}{\partial \tau^i} + \frac{\partial y^{i*}}{\partial \tau^i}\right)$. By differentiating the regions budget constraints $R^i + \tau^i = y^{i*} + E^i(\vartheta^i, x^{i*})$ we get $d\tau^i = \frac{\partial y^i}{\partial \tau^i} d\tau^i + E_x^i \frac{\partial x^i}{\partial \tau^i} d\tau^i$, or $1 = \frac{\partial y^i}{\partial \tau^i} + E_x^i \frac{\partial x^i}{\partial \tau^i}$. It follows that eq. (5) implies eq. (3), i.e. that in the Pareto equilibrium, which is characterized by $U_y^{h*} = U_y^{l*}$, the optimum transfer outlays $\tau^h = -\tau^l$ induces equal marginal utilities for good y for all the regions. This condition is quite important, in so far its economic meaning is that central government for its maximisation policy has to control income marginal utilities of the jurisdictions, which are attainable by the correct use of the transfers τ by the central government.

3. *The identification of the receiving & giving regions with local public goods*

For central government it is not straightforward to determine which type of region (high or low cost) is going to receive the transfer or to pay the tax. The point is that the decision depends both on the regions' cost structures and on their utility functions: the problems may arise both from the informational structure and from cost and utility functions. For the moment the problem of asymmetry in information is ignored since we assume full information, but still the transfer characteristics have to be investigated.

From now onwards, to make things simpler, we will limit the analysis at only to two jurisdictions: a *high* cost region *h* and a *low* cost region *l*. Then, as stated in the previous section, the Pareto equilibrium can be induced by the value (positive or negative) of τ that lead to

$$U_y^{h*}[x^{h*}(\vartheta^h, R^h + \tau^h), y^{h*}(\vartheta^h, R^h + \tau^h)] = U_y^{l*}[x^{l*}(\vartheta^l, R^l - \tau^l), y^{l*}(\vartheta^l, R^l - \tau^l)]$$

If both regions are in their Nash equilibriums with $\tau=0$, in order to attain the maximum welfare, the central government has to implement a positive transfer towards the region with a higher marginal utility for good *y* and, conversely, negative transfer from the low marginal utility region. The point is that, since the utility functions are different, it might be that $U_y^{h*} >,=,< U_y^{l*}$, and, accordingly, $\tau >,=,< 0$.

In order to solve that problem, let's assume provisionally that at the initial equilibria both the individual utility and the social welfare are maximized, in other words both Nash and Pareto conditions' are jointly satisfied. This hypothesis is admittable, in the simplest scenario, when the two regions have identical income, utility and cost functions[3]: in particular, $\theta^h = \theta^l$. If the afore-mentioned condition is met, then the optimal transfer, in order to satisfy eq. 3, is $\tau = 0$.

Afterwards, we assume that an exogenous shock alters the ϑ^h parameter, such that its value exceeds the low cost region's one: $\vartheta^h > \vartheta^l$. Thus, the exogenous shock which occurs to ϑ^h may either increase or decrease the marginal utility of the high cost region with respect to good *y*, i.e.,

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2 To note that this is not the only possible case and that the identity of income, cost and utility functions of the two type regions is not a necessary condition.

$$\frac{\partial U_y^{h*}}{\partial \vartheta^h}[x^{h*}(\vartheta^h, R^h + \tau^h), y^{h*}(\vartheta^h, R^h + \tau^h)] >,< 0$$, which in turn implies (by eq. 2)

$$\frac{\partial \left(\frac{U_x^{h*}[\cdot]}{E_x^{h*}[\cdot]} \right)}{\partial \vartheta^h} >,< 0 \quad \Rightarrow \quad \frac{\partial U_x^{h*}}{\partial \vartheta^h} E_x^{h*} - \frac{\partial E_x^{h*}}{\partial \vartheta^h} U_x^{h*} >,< 0 \quad \Rightarrow \quad \varepsilon >< 1 \qquad (6)$$

where $$\varepsilon = \frac{\frac{\partial U_x^{h*}}{\partial \vartheta^h}}{U_x^{h*}} \Big/ \frac{\frac{\partial E_x^{h*}}{\partial \vartheta^h}}{E_x^{h*}}. \qquad (7)$$

Thus, $\varepsilon > 1$ if $d\vartheta^h$ let the *per cent* variation of marginal utility be higher than the corresponding variation of *per cent* marginal cost of public good x[4]. In that case it is $U_y^{h*} > U_y^{l*}$ and the social welfare is maximized by a transfer τ>0, i.e. the low cost region must finance the high cost one.

Vice versa, if it is $\varepsilon < 1$, then it is the high cost region that has to finance the low cost one; finally if it is ε=1, then τ has to be set equal to *0*. This result has its identical in H&R statement that the sign of τ depends on the elasticities $\eta_W = \frac{\partial (U_x)}{\partial \theta}\frac{\theta}{U_x} >,=,< \frac{\partial (E_x)}{\partial \theta}\frac{\theta}{E_x} = \eta_E$. In fact it is sufficient to multiply both the numerator and the denominator of eq. 7 (which provides the definition of ε), to realize that condition $\varepsilon >,=,< 1$ becomes condition $\eta_W >,=,< \eta_E$. However, H&R obtain their result only under the assumptions that utilities are identical and separable, and as a consequence in their framework the quantity of good y is constant. On the contrary, for us the utility functions are not required to be separable, and the result may be generalized to the case that utilities are different between the jurisdictions. Actually, let's consider two regions characterized by different income, utility and cost function, and in particular by $\vartheta^h \neq \vartheta^l$. The initial equilibrium point for $\tau = 0$ is not a Pareto optimum; the result is that the two regions have diverging marginal utility in the equilibrium point with respect to income (the good *y*). Let's assume, without loss of generality, $U_y^{i*} > U_y^{j*}$, $i, j \in \{l, h\}, i \neq j$.

Thus, in order to attain the Pareto optimum, the central government intervention must consist on a positive transfer $\tau > 0$ to region *i* and on a negative's one to region *j*, while it might be $\vartheta^i > \vartheta^j$ or $\vartheta^i < \vartheta^j$. The positive transfer to region type *i* (and a negative transfer to region type *j*), is a mere consequence of the fact that $U_y^{i*} > U_y^{j*}$, which is equivalent to say

$$\Delta U_y^* = U_y^{i*} - U_y^{j*} > 0 \qquad (8)$$

In consequence of the fact that the two regions behave according to the Nash maximization rule, i.e., they move along the individual optimal path, the individual equilibrium point reflects eq. 2, or the (first order) condition $U_y^{m*} = \frac{U_x^{m*}}{E_x^{m*}}$ $m \in \{l, h\}$, eq. 8 can then be rewritten as follows:

$$\Delta U_y^* = \frac{U_x^{i*}}{E_x^{i*}} - \frac{U_x^{j*}}{E_x^{j*}} = \frac{U_x^{i*} E_x^{j*} - U_x^{j*} E_x^{i*}}{E_x^{i*} E_x^{j*}} \qquad (9)$$

Hence the initial hypothesis $\Delta U_y^* > 0$ turns out to be met when the numerator of equation 9 is greater than zero ($U_x^{i*} E_x^{j*} - U_x^{j*} E_x^{i*} > 0$) given that the denominator is always positive.

Assuming:

$\Delta E_y^* = E_x^{i*} - E_x^{j*} >,< 0$ and $\Delta U_x^* = U_x^{i*} - U_x^{j*} >,< 0$, it follows that:

[4] or equivalently $\eta^H_{U_{x\vartheta}} - \eta^H_{E_{x\vartheta}} >,< 0$, where $\frac{\partial U_x^{h*}}{\partial \vartheta^h}\frac{\vartheta^h}{U_x^{h*}} = \eta^h_{U_{x\vartheta}}$; $\frac{\partial E_x^{h*}}{\partial \vartheta^h}\frac{\vartheta^h}{E_x^{h*}} = \eta^h_{E_{x\vartheta}}$.

$\Delta U_y^* > 0$ if $\left(\Delta U_x^* + U_x^{j*}\right)E_x^{j*} - \left(\Delta E_x^* + E_x^{j*}\right)U_x^{j*} > 0 \quad \Rightarrow \quad \Delta U_x^* E_x^{j*} - \Delta E_x^* U_x^{j*} > 0 \Rightarrow \Delta U_x^* E_x^{j*} > \Delta E_x^* U_x^{j*}$ or $\frac{\Delta U_x^*}{U_x^{j*}} > \frac{\Delta E_x^*}{E_x^{j*}}$.

To sum up, the transfer has to move from *j* towards *i* when $\frac{\Delta U_x^*}{U_x^{j*}} > \frac{\Delta E_x^*}{E_x^{j*}}$, i.e. when the *per cent* difference between utilities is higher than the *per cent* difference between costs.

Defining $\Delta\theta = \theta^i - \theta^j > 0$, and dividing for $\Delta\theta$ both members of $\frac{\Delta U_x^*}{U_x^{j*}} > \frac{\Delta E_x^*}{E_x^{j*}}$ we can write $\frac{\frac{\Delta U_x^*}{\Delta\theta}}{U_x^{j*}} > \frac{\frac{\Delta E_x^*}{\Delta\theta}}{E_x^{j*}}$. Then, defining $\varepsilon^\circ = \frac{\frac{\Delta U_x^*}{\Delta\theta}}{U_x^{j*}} \Big/ \frac{\frac{\Delta E_x^*}{\Delta\theta}}{E_x^{j*}}$, the result we have obtained boils down to the statement that the transfer has to move from *j* towards *i* if $\varepsilon^\circ > 1$; on the opposite way when $\varepsilon^\circ < 1$ and finally it has to be set equal to zero when $\varepsilon^\circ = 1$ (this latter implies that a Pareto optimum has already been attained). It should be noted that the conditions just stated are independent in their effectiveness (interpretation) from the initial values assumed by the two cost parameters ϑ^i and ϑ^j.

So far the τ sign has been identified but the amount of it has still to be investigated. To derive this information we can use eq. 3 assuming that the central government hasn't set any transfer yet and that each region acts according to its individual interest (in other words it moves along the Nash optimal path). Thus assuming $\vartheta^h \neq \vartheta^l$, $\tau^h = \tau^l = \tau$ and different income, utility and cost function between the two type regions, as consequence it cannot be otherwise that the initial equilibrium point were not a Pareto optimum (eq. 3 is not satisfied).

Setting:

$$U_y^{h*}[x^{h*}(\vartheta^h, R^h + \tau), y^{h*}(\vartheta^h, R^h + \tau)] = \Omega^h(\vartheta^h, R^h + \tau)$$

$$U_y^{l*}[x^{l*}(\vartheta^l, R^l - \tau), y^{l*}(\vartheta^l, R^l - \tau)] = \Omega^l(\vartheta^l, R^l - \tau)$$

It is required, to reach the Pareto optimum:

$$\Omega^h(\vartheta^h, R^h + \tau) - \Omega^l(\vartheta^l, R^l - \tau) = 0$$

Thus using the implicit function theorem and solving for τ, it is possible to obtain the optimal value of transfer, i.e., that τ which enables the meet eq. 3

$$\tau^* = \Omega^*(\vartheta^h, \vartheta^l, R^h, R^l)$$

4. *Asymmetry of information with local goods*

In this section the incomplete information case is considered. The central government knows both the utility and the cost functions of jurisdictions, but information about the quantity provided for the public and the private good is not available. The center observes the expenditure on the private good (*y*) and the expenditure on the public good (*E*) that local jurisdictions face, but quantities are unverifiable. Furthermore the center is aware of the fact that there are low cost regions and high cost ones but it is prevented from associating the right type to each one.

For simplicity, let's assume there are two jurisdictions only: the first one (that we denote by *h*) is characterized by high cost in producing the public good, and the second *(l)* which is characterized by low production cost for good *x*.

In the case of complete information, center sets the transfer outlays as modelled in the previous section, in order to induce local governments to behave so to attain the best outcome in terms of social welfare.

Within the present scenario information is incomplete; the central government offers a contract to the local governments that consists in granting a positive transfer τ conditional on "ad hoc" expenditure on good *x* or, as option, a negative transfer $-\tau$ free from auditing. This contract scheme differs from the one suggested by Huber and Runkel (2006) where the two contracts L^* and H^* with associated well defined values for τ and e are presented. Jurisdictions can opt for the option they prefer. The central government, in order to attain the welfare maximization goal, must implement a contract such that local governments have no incentive to cheating. In other words, the proposed contract must grant to both jurisdictions benefits at least as great as in their next best alternative, otherwise they would behave strategically showing to be the other type.

Hence central government observes both E^i and y^i, for $i=h,l$ and has complete information about the income, the utility and the cost structure of the two jurisdictions: it lacks simply the information whether each jurisdiction is high or low cost as far as good x is concerned. Thus, the proposed contract should be $[+\tau^*; E^*]$, whereas the alternative is $[-\tau^*]$ and no auditing; the contract, however, may take the form of $[+\tau^*; y^*]$ as well, because the central government knows $R^i = E^i + y^i$, for $i=h,l$.

Both local governments may get a positive transfer τ^*, conditional on spending y^* for the private good y (or equivalently, spending E^* for the public good x), or a negative transfer (i.e., a tax) $-\tau^*$. The transfer τ^* is selected, in a first best scenario, according to the optimal expenditure y^* of the jurisdiction that, in the Nash previous equilibrium, must receive a transfer outlay according to the fact that it is $\varepsilon > 1$ or $\varepsilon < 1$, as formerly discussed[5]. However, central government cannot offer contracts with the Paretian y^{h*} or y^{l*} because of cheating: the jurisdiction that has to pay the transfer, could pretend to be the other.

For central government, the problem is to find a value for τ that maximises the sum $U^l(x^l, y^l) + U^h(x^h, y^h)$, when both the local government may opt for receiving the transfer τ^* conditional on spending y^*, or paying the tax ($-\tau^*$).

The point is that central government observes E^i and y^i, but not x^i; thus, instead of paying the tax, one of the jurisdictions might pretend to be the other one in order to get the positive transfer, but it could not spend the E^j correspondent to his true parameter cost θ^j; it has to exhibit the expenditure that corresponds to the cost θ^i of the other jurisdiction: if not, central government detects its cheating. This is tantamount to say that, owing to the fact that central government knows utility and cost functions (it does not know whom such functions are labelled to) the incentive compatibility constraints may be written, for each jurisdiction, as the utility of optimal $U^j(x^j, y^j)$ higher (or equal) than its utility corresponding to the $U^j(x_j^i, y^i)$ of the other jurisdiction, where x_j^i is the quantity of public good that the cheating region j has to provide in order not to be detected by the central government. It is derived as follows:

$E^i(x^i, \vartheta^i) = e^i$ is the expenditure on public good that the receiving region set in order to maximize its utility. The cheating region j will set its expenditure on public good $E^j(x_i^j, \vartheta^j) = e^j$ so that its cheating cannot be established. This goal is attained when $E^j(x_i^j, \vartheta^j) = e^j = e^i$. Using the implicit function theorem and naming x_i^j the public good provided by region j when pretending to be the other type: $x_i^j = \psi_x^j(e^i, \vartheta^j)$.

Let's summarize the central government maximization problem.
The welfare objective function can be defined as:

$$\underset{\tau}{Max} W[x^i, y^i, x^j, y^j] = U^i[x^i, y^i] + U^j[x^j, y^j]$$

subject to the following constraints:

[5] To note that in the present scenario the central government is able to know not only the Paretian $E_h^\circ; y_h^\circ; E_l^\circ; y_l^\circ$, but any optimal choice of the local government under the imposed constraints.

- budget constraint

$$y^i + E^i(\vartheta^i, x^i) = R^i + \tau \qquad \text{(associated lm: } \lambda^i\text{)} \qquad (10$$

$$y^j + E^j(\vartheta^j, x^j) = R^j - \tau \qquad \text{(associated lm: } \lambda^j\text{)} \qquad (11)$$

- incentive compatible constraints

$$U^i[x^i, y^i] \geq U^i\left\{\psi_x^i[E^j(x^j, \vartheta^j), \vartheta^i], y^j\right\}\Big|_{R^i > R^j} \qquad \text{(associated lm: } \mu^i\text{)} \qquad (12)$$

$$U^j[x^j, y^j] \geq U^i\left\{\psi_x^j[E^i(x^i, \vartheta^i), \vartheta^j], y^i\right\}\Big|_{R^j > R^i} \qquad \text{(associated lm: } \mu^j\text{)} \qquad (13)$$

- non negativity constraints

$$x^i, y^i, x^j, y^j \geq 0 \qquad (14)$$

The incentive compatible constraints are required to avoid that the region had the incentive to cheat, that is to mimic the other type region. The following constraints simply state that the utility deriving to the *i* type region when it sincerely reveals its type, has to be not lower than the utility deriving to that region from cheating, i.e. when it falsely declares to be of the other type. The goal is to avoid any incentive to lye. The incentive compatibility constraint for the receiving region is not binding in this scenario and it can be ignored (constraint of eq.12).

The first order conditions (foc) required for efficiency are reported in appendix 1

Let's assume that the center knows (see previous sections) that the sign of the transfer τ has to be set greater than zero: $\tau > 0$. This latter implies that the *j* region is taxed while the *i* region is subsidized. This assumption allows us to set $\mu^i = 0$ given that the eq.12 incentive compatibility constraint is not binding. Thus provided that region *i* receives the subsidy while region *j* is taxed, the Lagrange multiplier μ^i assumes the zero value because the *i* region has no advantages to misrepresenting its type declaring to be the other type.

Let's consider two jurisdictions: the low cost *(l)* and the high cost one *(h)*. In the trivial case when it is $U_y^h = U_y^l$ the first best and the second best allocation coincides and the transfer τ is set equal to zero. This latter coincides with the case where $\eta_E = \eta_W$ in the Huber and Runkel (2006) paper.

Hence at the initial equilibrium it is either $U_y^h > U_y^l$ or $U_y^h < U_y^l$, i.e., it is respectively $\varepsilon^\circ > 1$ or $\varepsilon^\circ < 1$.

In the first case, when it is $U_y^h > U_y^l$, in order to maximise social welfare it is necessary that U^h rises, i.e. central government has to tax *(l)* and must give to *(h)* the correspondent transfer outlay. The Central government's goal is to induce the condition $U_y^h = U_y^l$ to be implemented by the transfer *τ*. If the condition is met then the social welfare has been maximized. Unfortunately the central government has to consider the risk of strategic behaviour that the taxed region may carry out in order to take advantage from its dishonest disclosure of type. To this extent the incentive compatible constraint alternatively of eq.12 or eq.13 is required.

Jurisdiction *(l)* is indifferent to pay the tax τ and choose its optimal expenditure *(E^l, y^l)* or to receive the transfer τ conditional to the expenditure *(E^h, y^h)*, which is the optimal expenditure for the jurisdiction *(h)* taking the transfer τ. Actually, as we have noted above, this is tantamount to say that for jurisdiction *(l)* the positive transfer τ is conditional to the quantities *(x_h^l, y^h)*.

In the opposite case, when it is $U_y^h < U_y^l$, in order to maximise the social welfare it is necessary that U^l rises, i.e. central government has to tax *(h)* and must give to *(l)* the correspondent transfer outlay.

The incentive compatible constraints have to grant that for *(h)* it is indifferent to pay the tax τ and freely choose the expenditure (E^h, y^h) or to receive the transfer τ conditional to the expenditure (E^l, y^l), in other words for jurisdiction *(h)* the positive transfer τ is conditional to the quantities (x_l^h, y^l).
To be noted that in the case that the production frontier of the contributing region lies below the receiving region production constraint, then a first best policy may be implemented by the central government given that the high cost region would be unable to implement a cheating strategy given its (binding) budget constraint. The constraint that really binds is the budget ($y^j + E_x^j = R^i + \tau$)rather than the incentive compatibility one (eq.1 and eq.2). In the afore mentioned scenario, it happens that the income of the contributing region $(R^i+\tau)$ is not sufficient to match (E^j, y^j). This assertion can be generalized as follows:
if region *j* is the cheating region, i.e., the jurisdiction that misrepresents its type in order to receive the subsidy, then its budget constraint has to meet the condition:

$$R^j + \tau \geq y^i + E^i \qquad (15)$$

where *i* is the receiving region.
The receiving region meets its budget constraint, in order to maximize its utility, by equality, i.e.,

$$y^i + E^i = R^i + \tau \qquad (16)$$

It clearly emerges from eq.15 and eq.16 that the binding budget constraint which allows the donor region to declare to be the other type and meet the individual budget constraint can be simply synthesized by the condition:

$$R^j \geq R^i \qquad (17)$$

<u>The study of the first order conditions</u>
Analysing a.2 and a.4 we find that, with reference to the *j* region (which is the contributing one), the condition $SMS_{x,y}^j = E_x^j$ has to be met. This condition coincides with that identified for efficiency in the scenario of complete information. This result is also obtained in Huber and Runkel (2006) paper.
The transfer τ has to be set (see a.5) by the central government in order to equalize the shadow price of income of the two regions, or in other words to get: $\lambda^i = \lambda^j$, but this latter in turn implies that the difference in marginal terms between the two type of jurisdiction with respect to the private good cannot be settled up.
Indeed, from a.3 and a.4 clearly emerges that: $U_y^i > U_y^j$, i.e., the incentive compatible constraint does not permit to join the condition of equality between the marginal utility (with respect to the private good) for the two jurisdiction types. The final equilibrium outcome will turn out to be a second best outcome. In fact further improvement might be obtained by a different resource allocation, but this goal is avoided by the incentive compatibility constraint. The efficient equilibrium outcome where $U_y^i = U_y^j$ is not attainable in presence of asymmetry of information because this condition creates favourable condition for *j* to lie about its type and thus to adopt a strategic behaviour.

It is interesting to compare the outcome of the first best scenario obtained in the case of complete information with this latter characterized by a lack of information.
Moving from first order conditions, other interesting information might be derived. Looking at a.6 and a.8 we note that the marginal utility for the receiving jurisdiction i with respect both to the public and the private good, is greater if compared with the complete information scenario, and as a consequence, the quantity for the two goods will be lower. Noting by * (star) the outcome emerging from the complete information case and by ° (circle) the outcome in the incomplete scenario, we can summarize as follows:
$U_{x^i}^{i\circ} > U_{x^i}^{i*}$; $U_{y^i}^{i\circ} > U_{y^i}^{i*}$ and $x^{i\circ} < x^{i*}$; $y^{i\circ} < y^{i*}$. The Samuelson rule might not be satisfied in the recipient. This result sensibly differs from that of Huber and Runkel (2006), in fact we allow for no

distortion in the recipient region if $U^j_{\psi^j}\psi^j_{e^i} = U^j_{y^i}$ (see a.6 and a.8). On the other hand if this equality is not verified then a distortion emerges. In particular we may note underconsumption of good x (with respect to the Samuelson rule) if $\frac{U^j_{\psi^j}}{U^j_{y^i}} = \frac{1}{\psi^j_{e^i}}$ or overconsumption in the opposite case.

In general it is possible to state, with reference to the receiving region and with respect to the first best outcome, that it is detectable a underprovision for the goods x and y. That result comes as a direct consequence of the incentive compatible constraint which binds the central government to a suboptimal amount for the τ : $\tau^{\circ} < \tau^{*}$.

On the other hand, a.7 and a.9 imply the condition that $U^{j\circ}_{x^j} < U^{j*}_{x^j}$; $U^{j\circ}_{y^j} < U^{j*}_{y^j}$ and hence $x^{j\circ} > x^{j*}$; $y^{j\circ} > y^{j*}$. The contributor consumption of good x and y in the asymmetric information context exceeds the first best one.

5. *A pure public good*
6. *A pure public good and asymmetry of information*
7. *Concluding remarks*

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Appendix 1

Central Government maximization problem with local public good and asymmetric information; first order conditions (focs):

$$\frac{\partial L}{\partial x^i} = U^i_{x^i} - \lambda^i E^i_{x^i} + \mu^i U^i_{x^i} - \mu^j U^j_{\psi^j} \psi^j_{e^i} E^i_{x^i} \le 0, \qquad x^i \ge 0, \; x^i(\frac{\partial L}{\partial x^i}) = 0 \qquad \text{(a.1)}$$

$$\frac{\partial L}{\partial x^j} = U^j_{x^j} - \lambda^j E^j_{x^j} - \mu^i U^i_{\psi^i} \psi^i_{e^j} E^j_{x^j} + \mu^j U^j_{x^j} \le 0, \qquad x^j \ge 0, \qquad x^j(\frac{\partial L}{\partial x^j}) = 0 \qquad \text{(a.2)}$$

$$\frac{\partial L}{\partial y^i} = U^i_{y^i} - \lambda^i + \mu^i U^i_{y^i} - \mu^j U^j_{y^i} \le 0, \qquad y^i \ge 0, \; y^i(\frac{\partial L}{\partial y^i}) = 0 \qquad \text{(a.3)}$$

$$\frac{\partial L}{\partial y^j} = U^j_{y^j} - \lambda^j - \mu^i U^i_{y^j} + \mu^j U^j_{y^j} \le 0, \qquad y^j \ge 0, \qquad y^j(\frac{\partial L}{\partial y^j}) = 0 \qquad \text{(a.4)}$$

$$\frac{\partial L}{\partial \tau} = \lambda^i - \lambda^j = 0 \qquad \text{(a.5)}$$

$$\frac{\partial L}{\partial \lambda^i} = y^i + E^i(\vartheta^i, x^i) - R^i - \tau = 0 \qquad \text{(a.6)}$$

$$\frac{\partial L}{\partial \lambda^j} = y^j + E^j(\vartheta^j, x^j) - R^j + \tau = 0 \qquad \text{(a.7)}$$

$$\frac{\partial L}{\partial \mu^i} = U^i[x^i, y^i] - U^i\{\psi^i_x[E^j(x^j, \vartheta^j), \vartheta^i], y^j\} \ge 0, \quad \mu^i \ge 0, \quad \mu^i(\frac{\partial L}{\partial \mu^h}) = 0 \qquad \text{(a.8)}$$

$$\frac{\partial L}{\partial \mu^j} = U^j[x^j, y^j] - U^j\{\psi^j_x[E^i(x^i, \vartheta^i), \vartheta^j], y^i\} \ge 0, \quad \mu^j \ge 0, \quad \mu^j(\frac{\partial L}{\partial \mu^j}) = 0 \qquad \text{(a.9)}$$

Appendix 2

Central Government maximization problem with federal public good and asymmetric information; first order conditions (focs):

$$\frac{\partial L}{\partial X} = U_X^i + U_X^j + \sigma + \mu U_{x^j}^j = 0 \tag{b.1}$$

$$\frac{\partial L}{\partial x^i} = -\sigma - \lambda^i E_{x^i}^i - \mu[U_{x^i}^j + U_{\psi^j}^j \psi_{e^i}^j E_{x^i}^i] \leq 0, \quad x^i \geq 0, \; x^i(\frac{\partial L}{\partial x^i}) = 0 \tag{b.2}$$

$$\frac{\partial L}{\partial x^j} = -\sigma - \lambda^j E_{x^j}^j \leq 0, \qquad x^j \geq 0, \qquad x^j(\frac{\partial L}{\partial x^j}) = 0 \tag{b.3}$$

$$\frac{\partial L}{\partial y^i} = U_{y^i}^i - \lambda^i - \mu U_{y^i}^j \leq 0, \qquad y^i \geq 0, \; y^i(\frac{\partial L}{\partial y^i}) = 0 \tag{b.4}$$

$$\frac{\partial L}{\partial y^j} = U_{y^j}^j - \lambda^j + \mu^j U_{y^j}^j \leq 0, \qquad y^j \geq 0, \qquad y^j(\frac{\partial L}{\partial y^j}) = 0 \tag{b.5}$$

$$\frac{\partial L}{\partial \tau} = \lambda^i - \lambda^j = 0 \tag{b.6}$$

$$\frac{\partial L}{\partial \lambda^i} = y^i + E^i(\vartheta^i, x^i) - R^i - \tau = 0 \tag{b.7}$$

$$\frac{\partial L}{\partial \lambda^j} = y^j + E^j(\vartheta^j, x^j) - R^j + \tau = 0 \tag{b.8}$$

$$\frac{\partial L}{\partial \mu^j} = U^j[X, y^j] - U^j\{(x^i + \psi_x^j[E^i(x^i, \vartheta^i), \vartheta^j]), y^i\} \geq 0, \quad \mu^j \geq 0, \quad \mu^j(\frac{\partial L}{\partial \mu^j}) = 0 \tag{b.9}$$